COLLUSION AND THE CHOICE OF AUCTION

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September 1984

UCLA Working Paper #340

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Abstract

Auctions are used in many different markets, including the leasing of a substantial fraction of the natural resources in the United States. The procedures used in the auctions conducted by the Federal government have been a continuing source of controversy. One of the worries has been the possibility of collusion among bidders.

In this paper, several theorems are proven regarding cartels and auction methods. It is shown, under a wide variety of assumptions, that cartels are stable if the seller uses English (oral) or sealed-bid second-price auction methods, but not in sealed high-bid auctions.

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1. Introduction

Auctions are used in many different markets, including the leasing of a substantial fraction of the natural resources in the United States. The procedures used in the auctions conducted by the Federal government have been a continuing source of controversy. One of the worries has been the possibility of collusion among bidders.[1]

Despite an extensive, and thriving, literature on auction theory,[2] the effect of different auction methods on the probability and stability of cartels among bidders has not been formally analyzed. Mead [1967] stated that oral auctions were more vulnerable to collusion than was sealed bidding, but he did not discuss clearly why this would be so or which of the features of oral auctions encourages collusion. In other contexts, Stigler [1962] and Alchian [1977] argued that sealed bid auctions themselves promote collusion.

In this paper, several theorems are proven regarding cartels and auction methods. It is shown, under a wide variety of assumptions, that cartels are stable if the seller uses English (oral) or sealed-bid second-price auction methods, but not in sealed high-bid auctions.

The paper is structured as follows. In Section 2, four commonly discussed auction methods are defined and the previous literature is briefly reviewed. The cartel is modeled and the theorems are proven in Section 3. The behavior of a cartel is examined using different assumptions regarding preferences, the possibility of profit-sharing, and the informational environment, but the results regarding cartel stability are shown to be robust. Section 4 summarizes the results and draws conclusions.

2. Definitions and Review of the Literature

A growing body of literature on auction theory has developed from a seminal paper by William Vickery [1961]. Auction markets are characterized by a fundamental asymmetry: a single seller of an indivisible object faces a number of potential buyers. The literature which I will discuss focuses on the type of auction — the rules by which bids are presented and the level of payments are calculated from the bids.

Four types of auctions are commonly considered: English (or oral), Dutch, sealed-bid first-price (or high bid), and sealed-bid second price. In English auctions, the price rises incrementally until only one bidder remains, who wins the object for that price. For ease of analysis, a pure form of the auction is considered: everyone interested in purchasing the object would raise his hand, dropping it only when the auctioneer had increased the price beyond that which they were willing to pay. The auction would end when only one hand was up. In Dutch auctions, the price starts at a very high level, then falls until someone bids. That person wins the prize. It is usually assumed that bidders can observe each other in English and Dutch auctions. In sealed-bid first-price auctions, potential buyers submit closed offers, with the award going to the high bidder. The same is true in second-price auctions, except that the high bidder pays only what the second-highest bidder offered.

Two well-established results[3] are that the Dutch and sealed-bid first-price auctions are strategically equivalent and that, when bidder's preferences are independent of each other, English and second-price auctions have identical unique dominant-strategy equilibria.[4] The need for the caveat on the latter result arises because much more information regarding the preferences of other bidders is revealed prior to a final decision during an

English auction than in a sealed-bid second-price auction. In the open auction, the winning bidder sees the reservation prices of all lower bidders and, if preferences are not independent, will modify his bid accordingly.

In all four of these auctions, only the winner makes a payment to the seller. However, many potential types of auctions require more than one bidder to make a payment to the seller. For example, the seller could demand an entrance fee.

Riley and Samuelson [1981] and Myerson [1981] prove a revenue-equivalence theorem which states that, under the assumptions of risk-neutrality, independence of valuations, and symmetry of beliefs, any two auction rules within a broad class generate the same expected revenue for the seller if, in equilibrium, the incentives to participate in the auction remain the same. Thus, under these assumptions, all four of the common auctions yield the same expected revenue for the seller if they have the same announced reservation price.

In many common situations, such as natural resource auctions, the assumption that the bidder's valuations of the object are independent of each other is unacceptable. An alternative set of assumptions is that the value of the object is the same to all bidders, but that each bidder has an independent and identically distributed <u>estimate</u> of this value. This common value or mineral rights model implies that bidders should be aware of the "winner's curse";[5] winning the auction means that your original value estimate was probably too optimistic.

Milgrom and Weber [1982] prove that in the common value model, when bidders are symmetric and risk neutral but the value estimates are "affiliated" (roughly speaking, positively correlated), oral auctions yield the seller higher expected revenue than sealed second price auctions, which,

in turn, do better than sealed high bid. The intuition is that the English auction conveys more information (all lower value estimates) than does the second price auction, which in turn conveys more information than the sealed high bid or Dutch auctions. The positive correlation means that more information raises the expected selling price by increasing competition.

Given this last result, it might seem surprising that sealed bid auctions are used so frequently in situations where there might be a common value: offshore oil leases, construction contracts, etc. Recall, however, that Milgrom and Weber assumed symmetry, risk neutrality, and noncooperative behavior among bidders. The implications of relaxing this last assumption, for both the independent values and common value models, are explored in the next section.[6]

3. Auction Type and Cartel Stability

Before discussing the interaction between the type of auction and cartel formation, it is important to examine the strategy of a cartel of bidders in an auction and the obstacles to stable cartel formation in different bidding environments.[7] A cartel is assumed to try to maximize joint expected profits. In an auction of a single indivisible item, joint profits will be maximized if the cartel pays the minimum price that will purchase the item whenever its value to the cartel exceeds the purchase price. The cartel is assumed to select from among its members a "designated winner" (who should be the member with the highest valuation if they differ) and to recommend that he follow a particular bidding strategy while requesting other cartel members to be inactive in the bidding. For all of the four common auction rules, as well as many other rules, a strategy of this type will maximize joint expected profits for at least some strategy of the designated winner, even if the cartel does

not include all potential bidders.

There are several obstacles to forming such a cartel. There may be negotiation costs involved in reaching an agreement. Much more significantly for what follows, the cartel has an enforcement problem. The cartel members, particularly the designated losers, may find it in their private interest to deviate from the reommended strategy, that is, to cheat. In order for the prospective cartel to be stable, the recommended cartel strategies should be incentive-compatible (i.e., bidders should find it in their interest to follow them), at least in the weak sense that some other strategy for an individual bidder not be strictly preferred by that bidder given what the others are doing.

In the remainder of this section, the stability of cartels will be analyzed for both the independent values and the common value models under different assumptions regarding the information available to the bidders and the possibility of making payments after the auction is completed. It will be assumed that the cartel exists for one auction only: allowing for repeated auctions would increase the stability of a cartel as cheaters would risk future profits. The cartel is assumed to have no enforcement powers, though I shall consider the case where the cartel can make a credible commitment to give its members an agreed-upon share of the realized profits from the auction.

It is necessary to specify the information the bidders possess regarding the value of the object and the valuations of the other bidders. Two different cases will be considered. In the first set of results, every bidder's information is known to every other bidder. In this situation, no bidder would refuse to join a cartel which had no enforcement power and did not require any payment from them (whether they would adopt the strategy suggested

by the cartel is another matter). Later results are derived for the case when each bidder knows his own valuation or value estimate and the distribution of the valuations of the other bidders, but not their realization. The cartel is assumed to force all members to credibly reveal their private information to the other members upon entering the cartel. The cartel, in order to be stable under these conditions, must give bidders an incentive to join and reveal their information prior to the auction, as well as to follow the recommended strategy during the auction. Despite the differences among these conditions, one basic conclusion obtained regarding the relationship between collusion and auction type seems robust.

English and second-price auctions are much more susceptible to stable cartel formation than sealed-bid first-price auctions. In second-price auctions, the best strategy for the cartel to follow is to have the designated winner submit a bid equal to the cartel's valuation of the object. All other members of the cartel are expected either not to enter the bidding or to bid the seller's reservation price. If none of the members of the cartel cheat, and if no competitive fringe exists, the cartel will win the prize at the reservation price of the seller. If the cartel is outbid, it will not regret its bid since it expects that the winner will not make money. There is no incentive for a cartel member to break ranks with a bid. If the firm outbids the cartel, it will not make money on average (unless the firm has private, optimistic information). If the firm underbids the cartel, it will cost the cartel money without gaining anything for itself. If the firm expects to receive a fraction of the cartel's ex-post profits or if making bids is costly, it will strictly prefer to play along with the cartel. Otherwise, the designated winner will strictly prefer to follow the cartel strategy, while all other cartel members will, in a non-repeated game, be indifferent between

following the cartel and cheating on the cartel. Similarly, firms will not be caused to regret cartel participation by the existence of non-participants, for the same reason the cartel does not regret its bid; either the cartel will lose with no profits for anyone, or the cartel will win and earn maximum available profits. For the English auction, the cartel will instruct the designated winner to keep bidding until all bidders drop out or the cartel's valuation is reached. The discussion of the second-price auction carries through, with the additional point that the cartel can identify deviators and perhaps exact retribution.

For the sealed-bid first-price auction, there is great incentive to cheat. If the cartel did not have to consider the competitive fringe, it would maximize profits by having the designated winners bid the seller's reservation price, assuming that it was less than the cartel's valuation. By secretly bidding a fraction higher than this, a cartel member could capture the entire difference between the cartel's valuation and the seller's reservation price. At any price that the cartel chooses below its valuation, a cheater would make positive expected profits. If, however, the cartel bid its valuation it would have zero expected profits, by definition. Even if the cartel managed to overcome this difficulty (unlikely in a one-shot deal), it would still have to worry about the competitive fringe. The cartel, if it bids below the valuation, has a positive probability that it will regret its bid because the winning call was between the cartel's bid and valuation. If the cartel is risk-averse, this potential loss will cause it to bid closer to its valuation in the presence of the competitive fringe, thus increasing landowner revenue.

These results are formalized in the following theorems. The cartel's valuation (or the highest valuation in the cartel in the case of independent

values) is V, which is assumed for simplicity to be greater than the seller's reservation price R, and the designated winner is assumed to be bidder 1 (who in the independent values case has the highest valuation). All n potential bidders are assumed to be risk-neutral and to maximize expected profits.

THEOREM 1: In the common value (or mineral rights) model, if all cartel members have the same valuation V:

(i) bidder 1 bidding V and all other cartel members bidding R or not entering a bid is a stable cartel equilibrium[8] (in the sense that bidder 1 is following a dominant strategy and all other bidders are following Nash strategies in the second price auction, and

(ii) bidder 1 having a dropout price V and all other cartel members having dropout prices of R or not entering the auction is a stable cartel equilibrium in exactly the same sense as (i) for the English auction, but

(iii) no Nash equilibrium exists where bidder 1 bids less than V and all other cartel members bid less than or equal to 1's bid in the sealed bid first price auction. In fact, no Nash equilibrium exists in which any cartel members make positive profits.

THEOREM 2: In the independent values model, when bidder 1 is known to have the highest valuation among the cartel members:

(i) bidder 1 bidding V and all other cartel members bidding R or not bidding is a stable cartel equilibrium in the sense above for the second-price auction, and

(ii) bidder 1 having a dropout price V and all other cartel members having a dropout price of R or not entering the auction in a stable cartel equilibrium in the above sense for the English auction, but

(iii) no Nash equilibrium exists where bidder 1 bids below the second highest valuation in the sealed-bid first-price auction.

PROOF: In the second-price and English auctions, bidder 1's dominant strategy is to bid his own valuation in either the independent values or the common value models. The conditions are sufficient to insure that no other bidder will make positive profits, given that bidder 1 is bidding V. Since bidding R (which was assumed less than V) or not entering the auction will assure any bidder non-negative profits, all of the other bidders might as well follow these strategies. Not entering the auction is certainly not the unique profit maximizing strategy; many other bids would guarantee the designated losers zero profits, including bidding V. However, once bidder 1 plays his dominant strategy, any other bidder might as well not enter the auction, no matter what the other bidders do.

In the sealed-bid first-price auction, on the other hand, there is an incentive to cheat on the cartel. If bidder 1 and bidders 3 through n bid less than V, with bidder 1 the high bidder, in the common value model, bidder 2 obviously strictly prefers to bid between bidder 1 and V, since that is the only way to earn positive expected profits. No Nash equilibrium exists in the common value model where every bidder bids below V, since any loser would want to change his bid given what the others were doing. In the independent values model, the bidder with the second highest valuation could make positive expected profits by outbidding bidder 1 if he bid below that value, in which case bidder 1 would want to alter his bid. No cartel solution

is a Nash equilibrium in a sealed-bid first-price auction for the independent values model, either. Q.E.D.

An important condition of the above theorems was that there was no private information. Under that condition, there was no incentive to cheat on a cartel in the English or second-price auctions even if an agent's share was independent of cartel profits and if the auction was not part of a repeated game. There is, however, no reason to follow the cartel either. One way to assure compliance in the second-price and English auctions is to pay bidders a fraction of the ex-post profits of the cartel; even this would not lead to a stable cartel in the sealed high bid auction. A positive cost for entering a bid would yield similar results.

If bidders have private information, they will frequently have positive expected profits even if they do not cooperate. For a stable cartel to be formed, every member must prefer both participation and compliance. In the independent values model with a second-price or English auction, there is no problem as long as the person with the highest valuation is the designated winner and valuations are credibly revealed upon joining the cartel. In the common value model, if bidders are symmetric and if the value estimates will be credibly revealed to the other cartel members if the cartel is joined, a cartel solution guaranteeing every member an equal share of either the ex-ante or the ex-post cartel profit will be a Nash equilibrium in the second-price or English auctions. The sealed high bid auction still does not have a Nash equilibrium in this case. The condition that information must be credibly revealed is necessary or bidders might lie and then outbid the cartel.

These results are formalized in the following theorems. A <u>cartel solution</u> for a given auction is the set of bidding strategies for that auction

which are described in Theorems 1 and 2. As before, bidder 1 is one of n risk-neutral potential bidders and is the designated winner. In the independent values model, bidder 1 has the highest valuation. Cartel profits are the difference between the joint expected V and what bidder 1 pays in the mineral rights model; they are the difference between the second-highest valuation and what bidder 1 pays in the independent values model.

THEOREM 3: In either the common value or the independent values models, where all valuations are public, and where a positive fraction of the cartel profits (ex-post) are paid to all cartel members,

(i) a cartel solution is a stable cartel equilibrium in the second-price or English auction, in the sense that bidder 1 is following a dominant strategy and the losers are following Nash strategies. Moreover, the losers strictly prefer, given bidder 1's strategy, to bid no more than R (i.e., a cartel solution).

(ii) no cartel solution is a Nash equilibrium in the sealed-bid first-price auction.

PROOF: For the second-price and English auctions, any bid by the designated losers which is greater than R will lower the cartel profits. Since under the conditions of the theorem, not participating in the cartel or outbidding the designated winner will result in at most zero expected profits, and since the losers will have positive expected profits if the cartel wins, the designated losers will strictly prefer the cartel solution, once bidder 1 follows his dominant strategy.

In the sealed-bid first-price auction, any single loser in the common-value model could capture all of the profits by barely outbidding bidder 1.

Since not every loser can be given all of the cartel profits, at least one loser will prefer this to the cartel solution. In the independent values model, bidder 2 (with the second highest valuation) will prefer to outbid bidder 1 unless he receives all the cartel profits (recall that cartel profits in this case are the difference between the second highest valuation and the bid). If bidder 2 gets all the cartel profits, bidder 3 will prefer to outbid bidder 1 if he bids below 3's valuation (and bidder 1 would like to cheat on himself!) The condition that all cartel members receive a positive share of the profits means that no cartel solution is a Nash equilibrium in either case. Q.E.D.

THEOREM 4: In either the common value or the independent values model, where all valuations are public and bidding is costly:

(i) the cartel solution where all the designated losers do not enter the auction is a stable cartel equilibrium for the English and second-price auctions. Moreover, the losers strictly prefer not to enter the auction given bidder 1's strategy.

(ii) no cartel solution where cartel profits are greater than the cost of entering a bid is a Nash equilibrium in the sealed bid first-price auction.

PROOF: The proof is straightforward. Bidding by the designated losers will result in negative profits, once bidder 1 follows his dominant strategy in the second-price and English auctions; not participating guarantees zero profits. In the sealed high bid auction, positive profits for at least one loser are available if by outbidding bidder 1 the cost of participating can be covered.

Q.E.D.

THEOREM 5: In the common value model, when all bidders are symmetric and have private information in the form of value estimates which are drawn independently from a common distribution and which are unbiased estimates of the value, and if these estimates are revealed to all cartel members if the cartel is joined, then

(i) a cartel solution which pays an equal share of either the ex-ante expected cartel profits or the ex-post cartel profits to all cartel members will be a stable cartel equilibrium in the second-price or English auction. All bidders will strictly prefer to join the cartel and, if ex-post profits are shared, will strictly prefer not to bid more than $\bar{R}$ if they are a designated loser, given the designated winner's strategy.

(ii) no cartel solution is a Nash equilibrium in the sealed-bid first-price auction.

PROOF: In the common-value model when bidders are symmetric, each bidder, if they behave non-cooperatively, will expect to win the auction 1/nth of the time. If they have a private value estimate which they adjust for winner's curse, they expect positive profits when they win the auction. If a subset of the potential bidders join to form a cartel in the second-price or English auction, total cartel profits will be greater than the number of members times the expected profit of each member (which is the same, ex ante, because of symmetry). The cartel's strategy would be to have the designated winner bid the expected value conditional on the information the cartel has, on winning the auction, and on paying the highest conditional valuation among the competitive fringe if that is below the cartel's estimate. In symbols, the cartel solution is

$$b_1 = E[V|v_1,v_2,\ldots,v_m;\ b_1 > \sup(b_{m+1},\ldots,b_n)],\ b_2,\ldots,b_m < R \qquad (1)$$

where m is the number of cartel members, v_i is the value estimate and b_i is the bid (or the dropout price in the English auction) of the ith bidder, and V is the value of the item being auctioned. The expected profit of the cartel is more than the sum of the expected profits of the members for two reasons. First, there is no competition from the other cartel members, so the expected cost for the designated winner (though not the expected cost for the competitive fringe) will be lower. Secondly, the cartel will have a better estimate of the value than when the members behaved non-cooperatively. Not only will this directly raise expected profits, but the fringe will need to compensate by bidding conservatively.

Since per capita cartel profits are larger than non-cooperative profits however many bidders join the cartel, it is a dominant strategy in these auctions for all bidders to join a cartel which credibly promises an equal division of the expected cartel profits, either before or after the auction. Once the cartel has been joined and the information revealed to the other members, the expected profit for a designated loser to follow anything other than the cartel solution strategy is non-positive, as in Theorem 1. By Theorem 3, if ex-post expected cartel profits are shared, the designated losers will strictly prefer the cartel solution strategy and, since profits are shared equally, the designated losers would not prefer to be the winner.

In the sealed-bid auctions, by Theorem 3, no cartel solution with positive profits is a Nash equilibrium even if ex-post profits are shared. Since the bidders have positive expected profits outside the cartel, no cartel solution will be a Nash equilibrium for this auction type. Q.E.D.

All five of these theorems go to show that cartels are more stable under second-price or English auctions than under sealed-bid first-price auctions since cheaters on a cartel make positive expected profits in the latter but not in the former. This does not mean that collusion will always occur in a second-price or English auction or that there cannot be a cartel in a sealed high bid auction. Collusion entails bargaining costs and may run the risk of prosecution. The cartel solution is only one of many Nash equilibria for the English auction when ex-post payments cannot be made. On the other hand, in a repeated-game context, cartels can (and do) form in the sealed high bid case. Potential cheaters may be deterred by the loss of long-run profits. The results of this section just show that the probability of collusion is higher under second-price or English auctions. Since successful cartels reduce seller revenue, the theorems suggest reasons for a seller's choice of the sealed-bid first-price type of auction.

Even if a cartel would exist under either auction type, the seller still might have higher expected revenue using a sealed-bid first-price auction. If the members of the cartel were risk-averse, and they were concerned about the possibility of an outside bidder, they would increase their bid in the sealed high bid auction, while their strategy in the second-price or English auction would not be altered. This is shown for a special case in the following theorem.

THEOREM 6: If a stable cartel exists, if the cartel is risk-averse, and if the cartel's valuation of the object is independent of the competitive fringe, then the seller's revenue will be higher in a sealed high bid auction than in a second-price or English auction.

PROOF: This theorem is just a corollary to the result, proven by Riley and Maskin [1982] among others, that expected seller revenue rises in the independent values model when bidders are risk averse. Note, however, that the cartel's valuation is more likely to be independent of the competitive fringe. All that is necessary for the theorem is that one bidder, in this case the cartel, be risk-averse. The risk is that the cartel will lose the object when the winning bid was below its value. Q.E.D.

4. Conclusions

It has been shown that a cartel is stable — in the sense that the cartel solution is a Nash equilibrium — for the English and sealed-bid second-price auction when information is public whether in the independent values or common value model. The cartel is even stronger in these auctions if it can credibly promise members a fraction of the ex-post profits of the cartel. The cartel is still stable even if members have private information in the common value model, so long as it can credibly promise members a positive fraction of cartel profits. In none of these situations is a cartel stable in a sealed high bid (or, for that matter, a Dutch) auction.

What was critical for cartel stability was not the detection of cheating (in Dutch auctions, cheaters could be seen while they could not in sealed second-price auctions) but rather the incentive to cheat. It would be interesting to extend the results to other auction types.

The type of auction is only one of the important parameters of an auction; other variables include the use of a minimum price and the form of payment between buyer and seller. Robinson [1984] explores, among other questions, how the possibility of collusion affects the seller's optimal choice along these other dimensions.

The results of this paper offer one justification[9] for the prevalence of sealed high bid auctions. They also suggest that when oral or sealed second price auctions are used, the seller should be especially alert to the possibility of collusion among bidders. As emphasized in the previous section, however, the results do not imply that collusion will always occur in oral auctions nor that cartels will never form in sealed high bid auctions.

Footnotes

*Discussions with Robert Hansen, Benjamin Klein, David Levine, Paul Milgrom, Michael Porter, Steven Salant, John Shoven and David Starrett are gratefully acknowledged. Michael Boskin, Timothy Bresnahan and John Riley made many useful suggestions. The usual caveat applies. Financial support was received from the Center for Economic Policy Research (Stanford), the Foundation for Research in Economics and Education (UCLA), and the Centre of Policy Studies (Monash).

[1]See Hearings Before the House Committee on Agriculture [1977] and Johnson [1979].

[2]See Engelbrecht-Wiggans [1980] and Milgrom and Weber [1982].

[3]See, for example, Riley and Samuelson [1981] and Milgrom and Weber [1982].

[4]The caveat was emphasized by Milgrom and Weber [1982].

[5]See Wilson [1977].

[6]For a discussion of the consequences of relaxing the assumption of risk-neutrality, see Maskin and Riley [1982] for the independent values model and Milgrom and Weber [1982] for the common value model. Hansen [1984] and Milgrom and Weber [1981] discuss auctions when there is asymmetry of information, while Riley [1983] considers asymmetry of beliefs regarding other bidder's values.

[7]Stigler [1964] takes a similar view that the principal problem facing the cartel is policing the agreement. He also looks at the conditions affecting the feasibility of a cartel. Since he is considering an oligopoly of sellers, each producing large quantities over a period of time, the main variable affecting cartel stability is the time until detection. In the one-

time auction considered in this section, detection would be too late, so the agreement must be self-enforcing in a more immediate sense. It is worth noting that Stigler states that "sealed bids, publicly opened with full identification of each bidder's price and is the ideal instrument for the detection of price-cutting" (p. 48). The results in this paper suggest that an oral or second-price auction would be even more conducive to cartel formation.

[8]This is a somewhat stronger equilibrium than a simple Cournot-Nash. Bidder 1 is following a dominant strategy; each other bidder is following a Nash strategy regardless of what any bidder save bidder 1 is doing.

[9]This is not, of course, the only justification (see the references in footnote 6). In addition, the possibility of collusion may not be sufficient to outweigh the considerations of Milgrom and Weber [1982].

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