LIMITED COUNTERCYCLICAL POLICY IN A CORRIDOR THEORY OF CONSUMPTION

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Working Paper #344
October 1984

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ABSTRACT

The dominant strains of macroeconomic research lead to the conclusion that the design of optimal countercyclical policies is beyond our reach at the current state of economic knowledge. In addition, there are claims in the literature that, even along the optimal time path, the settings of policy variables should be fairly constant. In this paper, we suggest what a more fruitful approach to countercyclical policy analysis involves the analysis and comparison of simple policy rules. We suggest constraints on the class of admissible rules that make the analysis of the rules tractable.

We analyze some simple tax rules in the context of a two period model. We find that when policymakers are very uncertain about the true state of the economy, the optimal policy is akin to the Friedman k-percent rule, that is, policy variables do not respond to the perceived current state of the economy. Alternatively, when policymakers have accurate information about the state of the economy, "fine tuning" is optimal, that is, policy variables respond vigorously to current conditions. When uncertainty about the state of the economy is significant but not overwhelming, we find that the optimal policy involves discrete switching between policy regimes. Within each regime, the optimal policy requires that policy instruments remain fairly constant.

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I. Introduction

The postwar period has seen dramatic change in the views of macroeconomists on the role of countercyclical policy. Apparent policy successes in the 1950's and the early 1960's led some macroeconomists to claim that the major issues of countercyclical policy analysis had been resolved. However, policy failures in the succeeding years coinciding with the emergence of the so-called "rational expectations" school provided a major challenge to Keynesian countercyclical policy recommendations. This, in recent years, has led macroeconomists to question whether or not countercyclical government policies have *any* effect on output and employment, and, if so, whether the potential effects of alternative policies can be accurately assessed in advance.

In this paper, we present an alternative to the current view of macroeconomic policy. Our analysis deviates from the standard treatment of policy analysis in two crucial ways. First, we deviate from the standard treatment by considering what we call *limited countercyclical policies*. These are policies that are determined by simple rules and are limited in the econometric sense that policy instruments are allowed to take on only a limited set of values. In addition, the contingencies which constitute the domain of the policy rules are also limited. Advocacy of limited countercyclical policies is based on the uncertainty surrounding policymaking. First, limiting the range of values a policy instrument may take reduces the complexity and uncertainty underlying the expectations formation by agents with regard to policy variables. Second, policy rules should be designed to produce acceptable outcomes even when our economic models are misspecified; that is, policy rules should be robust with respect to uncertainty about the structure of the economy. It is our conjecture that limited policy rules can be designed to achieve this goal.

Our second major deviation from the standard treatment of policy analysis is allowing for occasional disruptions of crucial markets. This is based on a form of the corridor theory of Leijonhufvud (1981). In this theory, the macroeconomy is well-described by equilibrium models as long as aggregate variables remain within a "corridor" of stable solutions. Occasionally, however, events push the economy outside of the corridor and into a region where the usual relationships between variables break down. The credit rationing that arises in severe recessions is the type of disruption of crucial markets that we have in mind. This corridor theory of the economy fits in well with limited countercyclical policies. One can easily imagine a limited policy calling for one policy rule in "normal" periods, when the economy is within the corridor, and another policy rule in "constrained" periods when the economy is outside the corridor.

Our policy focus in this context is the choice of the time path of income or consumption tax rates. Kydland and Prescott (1980) have argued that income and consumption tax rates should not respond to variations in economic conditions. Their argument is based on the standard optimal tax theory from public finance which suggests that variation of tax rates across close substitutes will lead to large deadweight losses. Given their equilibrium model in which there is considerable intertemporal substitutibility present (particularly in terms of labor supply), tax rates must be kept stable over time in order to minimize the deadweight loss associated with raising a given amount of tax revenue.

Our corridor hypothesis provides an alternative point of view. When the economy is inside the corridor (when for instance credit markets are operating normally), our analysis suggests that tax rates should be non-contingent as recommended by Kydland and Prescott. However, our analysis suggests that tax rates should be discretely different for those periods when the economy drifts outside the corridor. In other words, we recommend a scheme for switching between two different policy rules based upon whether the economy is within or outside the corridor.

That optimal tax policy might call for varying the tax rate between normal (within the corridor) and constrained (outside the corridor) years is not surprising. However, what makes this analysis of interest is the assumption that the government must make these policy decisions under uncertainty with respect to whether the economy is within or outside the corridor. The potential errors made by the government associated with this uncertainty leads to interesting tradeoffs between how frequently the government should vary its policy rule and the magnitude of the difference between the tax rates. The characterization of these tradeoffs within a limited countercyclical policy perspective is the main focus of the analysis that follows.

The following two sections contain a highly simplified illustration of limited countercyclical policy analysis along the lines described above. First, we review some empirical evidence on the permanent income hypothesis. We use these empirical findings as the basis of a simple two period model of consumption that allows for a small probability of credit constraints. In this model, limited countercyclical policies are found to improve welfare. Next, we compare the behavior of the model under three different tax policies. We find that the outcomes are highly sensitive to the degree of uncertainty that faces the government. We close by summarizing the main results of our analysis.

II. A Corridor Theory of the Permanent Income Hypothesis

Before proceeding to our formal theoretical analysis, we briefly examine the empirical evidence on consumption to determine whether the data provides support for the corridor hypothesis in this context. This empirical analysis is intended to be suggestive rather than conclusive as our use of aggregate data permits only crude tests of the corridor hypothesis. A more persuasive test of this hypothesis can probably only be obtained by examining individual household data. Such a test is beyond the scope of the present analysis. For the present, our modest objective is to examine the empirical evidence to determine whether a corridor theory of consumption is defensible.

We begin our empirical analysis by considering the rational expectations version of the permanent income theory of consumption as exposited and tested by Hall (1978), Flavin (1981), and others.[1] Following Hall, consider an economy consisting of a single infinitely-lived representative individual. Each period, this individual is endowed with an income Y_t and chooses a level of consumption C_t. We denote the individual's rate of time preference by δ and the interest rate by r (both assumed constant over time), and we specify the individual's lifetime welfare by

$$W = \sum_{i=t}^{\infty} (1+\delta)^{t-i} U(C_i) \tag{2.1}$$

where $U(\ )$ is the within-period utility function and the C_t's are the welfare maximizing consumption levels. Consider decreasing C_t by a small amount x and raising C_{t+1} by just enough to satisfy the budget constraint. The first-order condition for maximizing welfare with respect to x implies that

$$U'(C_{t+1}) = \frac{1+\delta}{1+r} U'(C_t) \tag{2.2}$$

If consumption changes slowly over time (and if δ is approximately equal to r), this can be approximated by

$$C_{t+1} = C_t + \varepsilon_t \tag{2.3}$$

where the error term reflects both the small approximation error and any innovation to permanent income that shifts the consumption path. Thus, Hall reasons, if the rational expectations version of the permanent income hypothesis is correct, no variable observed in period t or earlier should help to forecast C_{t+1} after the influence of C_t has been taken into account. In other words, consumption should follow a random walk.

If, alternatively, the Keynesian theory of the consumption function is correct, then lagged values of income should help to predict current consumption (since they help to predict current income -- the proximate cause of current consumption). Hall regresses consumption on its own lagged value and lagged values of income. He finds that the income variables are jointly insignificant, as predicted by the permanent income hypothesis.

Following Hall's paper, Flavin (1981) and others have retested this random walk implication of the permanent income hypothesis. Most of these researchers have found that current consumption does respond to predictions of current income even after the influence of lagged consumption has been taken into account. Several hypotheses have been advanced to explain this so-called excess sensitivity of consumption to current income.

A leading candidate has to do with the potential misclassification of durable and nondurable goods in the data. Since current expenditures on durables bear little relationship to the current flow of consumption services from the stock of durables, it is important to exclude expenditures on durables from the measure of consumption. Darby (1972) has noted that many of the goods classified as non-durables in the National Income and Product Accounts may, from the point of view of consumers, be more like durable goods.

The corridor theory of economic fluctuations suggests another hypothesis to help explain the apparent excess sensitivity of consumption to current income. Imagine that the permanent income hypothesis is correct, but that there are occasional "credit crunches" — occasional periods where it is impossible to finance additional current consumption.[2,3] The consumer's maximization problem becomes[4]

$$\max_{C_t, C_{t+1}, \ldots} \mathcal{L}_W = W + \lambda \sum_{i=t}^{\infty} (1+r)^{t-i} (Y_i - C_i) + \sum_{j=t}^{\infty} \gamma_j (A_j + Y_j - C_j) \tag{2.4}$$

where Y_t is the endowment in period t, A_t is the beginning of period stock of assets, λ is the Lagrange multiplier for the lifetime budget constraint and the γ_j are the multipliers for the within-period budget constraints, that is, the γ_j are identically zero except in those periods where borrowing is prohibited. Again consider reducing C_t by an infinitesimal amount x and increasing C_{t+1} by just enough to satisfy the budget constraint. If the credit market is open in period t, then equation (2.2) holds as before. If however, period t is a credit constrained period, then the first-order condition for maximizing welfare with respect to x implies that

$$U'(C_{t+1}) = \frac{1+\delta}{1+r} U'(C_t) - \frac{1+\delta}{1+r} \gamma_t \tag{2.5}$$

Since γ_t is a non-linear function of current income and assets, a Hall-style regression may well show a statistically significant influence of lagged values of income and/or measures of wealth (such as stock prices).[5] By the same token, the γ_t term is non-zero only during credit-constrained periods, and the functional relationship between γ_t, Y_t and A_t is not clear *a priori*. Thus, it is not surprising that the "excess sensitivity" of consumption to income may fail to appear for some sample periods and some definitions of the variables.[6]

Table 1 contains estimates of some regressions designed to assess this credit-constrained version of the permanent income hypothesis. Regression 1 is a reestimate of Hall's null hypothesis: expenditures on non-durable consumption goods is regressed on a constant and its own value lagged once. In regression 2, disposable income is added to the right hand side of the equation. This equation is estimated by the instrumental variables technique to correct for the endogeneity of current disposable income. Four lagged values of disposable income are used as the instruments.

Regression 3 contains an approximation to the appropriate specification under the credit-constrained version of the permanent income hypothesis. γ_t is a presumably non-linear function of current income and beginning of period asset holdings. We approximate γ_t by a quadratic function $\gamma_t \simeq \mathrm{CONSTR}_t + (\mathrm{CONSTR}_t)Y_t + ((\mathrm{CONSTR}_t)Y_t)^2$ where CONSTR_t is a dummy variable that flags constrained periods. Again we estimate using instrumental variables. In this case we add the squares and cross products of lagged income to the instrument list. These instruments are added because we include the square of disposable income as a regressor.

Of course, we have no way of knowing which periods in the sample, if any, were credit-constrained. We chose to set CONSTR_t, the dummy indicator for constrained periods, to one for all quarters between one NBER reference cycle peak and the succeeding trough. We believe that this specification overstates the frequency of credit constraints. As a result, under <u>our</u> null hypothesis, our estimates of the impact of credit constraints should be biased towards zero. Regression 4 is a combination of regressions 2 and 3; both disposable income and the quadratic approximation to γ_t are entered on the right hand side.

Table 1

Regression

	1	2	3	4	5
Constant	-0.006 (0.006)	-0.004 (0.006)	0.004 (0.008)	0.004 (0.009)	-0.002 (0.006)
C_{t-1}	1.007 (0.002)	0.970 (0.027)	1.004 (0.003)	0.998 (0.043)	1.007 (0.002)
Y_t		0.029 (0.020)		0.005 (0.033)	
$CONSTR_t$			-0.370 (0.180)	-0.335 (0.302)	-0.017 (0.003)
$(CONSTR_t \times Y_t)$			0.215 (0.115)	0.193 (0.192)	
$(CONSTR_t \times Y_t)^2$			-0.031 (0.018)	-0.028 (0.029)	
Standard Error of the Regression	0.0143	0.0138	0.0135	0.0134	0.0128

Note: Dependent Variable is C_t for all regressions. Period of estimation is 1949:1 through 1981:4 . Standard errors in parentheses.

Data Source: Citibase.

Definition of variables:

C_t = Per Capita Expenditures on Non-Durables and Services, 1972 dollars.

Y_t = Per Capita Disposable Income, 1972 dollars.

$CONSTR_t$ = 1 , for all quarters following an NBER reference cycle peak and through the succeeding trough

0 , otherwise

The F-tests reported in Table 2 show that, in these data, there is little, if any, sensitivity of consumption to current income. The marginal significance of the fitted value of income in regression 2 is 0.1597. If, the credit-constrained permanent income hypothesis (regression 3) is taken as the null hypothesis, the current income has no predictive value for consumption. Its marginal significance level increases to 0.885. The quadratic approximation to γ_t does significantly improve predictions of future consumption. (If regression 1 is the null hypothesis and regression 3 is the alternative, the marginal significance level of γ_t is 0.0001. The marginal significance level of γ_t when regression 2 is tested against regression 4 is 0.0001.)[7]

These results should be interpreted with caution. For one thing, the marginal significance level of γ_t is overstated because of the inclusion of a separate constant term for constrained periods. By using the NBER reference cycle peaks and troughs as the delimiters for our constrained periods, we guarantee that $CONSTR_t$ will be significant. After all, $CONSTR_t$ equals 1 precisely during those quarters when output and employment fell below the NBER's business cycle dating committee's retrospective view of normal growth. Thus, the estimated residual in regression 1 is negative during 22 of 26 of the quarters we designate as constrained.

In regression 5, we report the results of regressing consumption on a constant, its own value lagged one period, and $CONSTR_t$. Comparing regression 5 to regression 3 allows us to assess the <u>additional</u> predictive power contributed by the income variables in the approximation of γ_t. The marginal significance level of these two variables is 0.0643.

These results suggest that the corridor view of the permanent income hypothesis is defensible. While the approach used in this section is simple

Table 2

F-tests

Null Regression	Alternative Regression	F-Statistic	Prob (x > F)
1	2	2.00	0.1597
1	3	12.45	0.0001
2	4	8.80	0.0001
3	4	0.02	0.885
5	3	2.80	0.0643

and straightforward, we do not feel that much more light would be thrown on the empirical standing of this hypothesis by utilizing a more sophisticated model of aggregate consumption. After all, credit constraints occur on the household (and firm) level. Some borrowing takes place even during constrained periods. Likewise, some agents are constrained even during normal times. As suggested above, a more persuasive test of this hypothesis can probably only be obtained by examining household data.[8]

III. Countercyclical Policy in a Potentially Constrained World

Let us accept for the moment the credit-constrained version of the permanent income hypothesis. Can we devise simple contingent policy rules that will produce higher welfare than would occur under a non-contingent policy rule? Of course, it is trivial to show that in a world of perfect information a fully optimal policy rule will be contingent on current economic variables in this model. After all, if consumers cannot currently borrow, the government can improve their welfare by "borrowing" for them, for example, by temporarily reducing tax rates. But this result neglects the inherent uncertainty faced by policymakers. The government may be uncertain with respect to whether significant credit rationing is occurring and may be uncertain about the current level of income. In the remainder of this section, we concentrate on the potential uncertainty with respect to whether credit markets are open or not.

Given uncertainty with respect to the state of credit markets, two types of errors and associated welfare losses are possible. First, the government may mistakenly perceive that the credit market is closed even though it is open and lower tax rates. We call this a type I error. The welfare loss associated with a type I error is due to the distortion of intertemporal after tax relative prices. Alternatively, the government may mistakenly perceive that the credit market is open and fail to intervene appropriately. We call this a type II error. The welfare loss associated with a type II error is due to the government's failure to intervene when it would have been optimal to do so.

The potential welfare losses associated with type I and type II errors makes the government's problem one of simultaneously choosing how much to lower tax rates during perceived credit market failures and how often to impose such a countercyclical policy. In the analysis that follows we find that the optimal policy involves a tradeoff between the magnitude of the credit crunch tax reduction and the frequency of imposing such a policy. We also find that the optimal policy under uncertainty about the state of the credit market is still quite complicated and is subject to criticism because it is not robust to other potential sources of uncertainty. Hence, we consider simplified versions of the optimal policy that may introduce some inefficiency in the current context but are sufficiently simple that they are robust to other potential sources of uncertainty. Overall, then, our approach is to use the characterization of the optimal policy to identify general principles to use in designing tax policy and then to suggest simple tax policies that follow the general principles of the optimal policy if not the complicating details.

Formally, we consider a two period model with a single representative consumer and a government. The consumer enters period 1 with real assets A_o and receives endowments X_1 and X_2 in periods 1 and 2 respectively. The consumer chooses consumptions C_1 and C_2 and faces tax rates on consumption of θ_1 and θ_2. If the credit market is operating in period 1, the consumer solves the following problem:

$$\max_{C_1, C_2, \lambda} \mathcal{L}_N = U(C_1) + \frac{1}{1+\delta} U(C_2) + \lambda[A_o + X_1 + \frac{1}{1+r} X_2 - (1+\theta_1) C_1 - \frac{1+\theta_2}{1+r} C_2]$$

Where, as before, δ is the rate of time preference and r is the real interest rate. If the credit market is closed in the first period, the consumer solves

$$\max_{C_1, C_2, \lambda^C, \gamma} \mathcal{L}_C = \mathcal{L}_N + \gamma[A_o + X_1 - (1+\theta_1) C_1].$$

The government is required to raise a fixed amount of revenue, R, over the two periods. However, the government can always borrow (at the same rate of interest, r, that faces the consumer), so there is no requirement to match the timing of tax receipts and government expenditures.[9] The government's goal, as in the standard optimal taxation literature, is to maximize the welfare of the representative consumer subject to satisfying its own revenue constraint. We assume that the government knows the values of A_o, X_1, and X_2, but it does not know whether the consumer's credit market is open or closed in period 1.

It is important to be clear about the sequence of events in this model. Before period 1 starts, both the consumer and the government know the consumer's initial wealth, the pattern of endowments, and the government's revenue requirement. However, the government must choose θ_1, the period 1 tax rate, before it knows whether or not the consumer will be able to borrow. After θ_1, is announced, the state of the consumer's credit market is revealed. The government has no further choices to make; θ_2, the second period tax rate, is determined by the revenue constraint. Thus the consumer knows both tax rates before choosing C_1 and C_2.

This admittedly artificial model and rigid sequence of decisions and revelations helps us to capture several important features of the real world (and to defer consideration of some real world complications). First, the ponderous nature of the tax machinery makes it necessary for the government to select a tax rate before all information about a period is available. Indeed, the sooner the government can commit itself to a time path of tax rates, the less uncertainty is faced by private agents who must themselves make important intertemporal choices. On the other hand, policy mistakes made today must be paid for eventually. In our simple model, "eventually" is just the second period. By specifying that the consumer's income is received as an endowment, we are able to avoid the complications that arise from the effect of taxes on the time path of labor supply.

While the government does not know the state of the credit market when it chooses θ_1, we do not assume that it is totally ignorant of credit market conditions. Let z be an unobservable variable that characterizes the state of the credit market. If $z < z^*$, then the credit market is closed; if $z > z^*$, the credit market is open. We assume that $z \sim N(0,1)$ independent of $\varepsilon \sim N(0,\sigma_\varepsilon^2)$. The government cannot observe z but we assume that it can observe y where

$$y = z + \varepsilon. \tag{3.1}$$

The government chooses some threshold value of y, call it y^*. If $y > y^*$, the government imposes θ_{1N} (where the "N" is mnemonic for a "normal" year). If $y < y^*$, the government imposes θ_{1C} (where the "C" denotes a constrained year.)

There are four possible states of the world in this model. The government may choose to impose either θ_{1C} or θ_{1N}. Given the government's choice the consumer's credit market may be open or closed. The probability of each state is completely determined by the threshhold values y^* and z^*. The value of z^* is given by nature, but the government can choose y^*, θ_{1C} and θ_{1N} to maximize expected welfare.

Let us denote the probability of a particular state of the world by P_{ij}, $(i,j = C,N)$. The i subscript denotes the tax regime chosen by the government, and the j subscript denotes the state of the credit market. The consumer's welfare in state (i,j) can be written as

$$W_{ij} \equiv U(C_{1j}) + \frac{1}{1+\delta} U(C_{2j}) + \eta_{ij} \left[\theta_{1i} C_{1j} + \frac{1}{1+r} \theta_{2ij} C_{2j} - R\right] \tag{3.2}$$

In (3.2), C_{1j} and C_{2j} are the consumer's consumption functions when the true state of the world is j. η_{ij} is a Lagrange multiplier for the revenue constraint when θ_{1i} is chosen and the true state of the economy is j (otherwise, $\eta_{ij} = 0$). θ_{2ij} is the second period tax rate when θ_{1i} is chosen and the true state is j.

Thus, the government's choice of y^*, θ_{1i} and θ_{2ij} is governed by:

$$\max_{\substack{y^*, \theta_{1i}, \theta_{2ij}, \eta_{ij} \\ i,j = C,N}} \mathcal{L}_G = P_{CC} W_{CC} + P_{CN} W_{CN} + P_{NC} W_{NC} + P_{NN} W_{NN}$$

The first-order conditions are:

$$\frac{\partial \mathcal{L}_G}{\partial y^*} = \frac{\partial P_{CC}}{\partial y^*} W_{CC} + \frac{\partial P_{CN}}{\partial y^*} W_{CN} + \frac{\partial P_{NC}}{\partial y^*} W_{NC} + \frac{\partial P_{NN}}{\partial y^*} W_{NN} = 0 \tag{3.3}$$

$$\frac{\partial \mathcal{L}_G}{\partial \theta_{1i}} = P_{iC} \frac{\partial W_{iC}}{\partial \theta_{1i}} + P_{iN} \frac{\partial W_{iN}}{\partial \theta_{1i}} = 0 \tag{3.4}$$

$$\frac{\partial \mathcal{L}_G}{\partial \theta_{2ij}} = P_{ij} \frac{\partial W_{ij}}{\partial \theta_{2ij}} = 0 \tag{3.5}$$

$$\frac{\partial \mathcal{L}_G}{\partial \eta_{ij}} = \theta_{1i} C_{1j} + \frac{1}{1+r} \theta_{2ij} C_{2j} - R = 0 \tag{3.6}$$

First, consider condition (3.3). In Appendix A, we demonstrate that

$$\frac{\partial P_{CC}}{\partial y^*} = -\frac{\partial P_{NC}}{\partial y^*}, \quad \frac{\partial P_{CN}}{\partial y^*} = -\frac{\partial P_{NN}}{\partial y^*}$$

which allows us to rewrite (3.3) as:

$$\frac{W_{CC} - W_{NC}}{W_{NN} - W_{CN}} = \frac{\partial P_{CN}/\partial y^*}{\partial P_{CC}/\partial y^*} \tag{3.7}$$

The left hand side of (3.7) is the ratio of the welfare cost of the two types of errors the government can make.

Consider the right hand side of (3.7). From Appendix A, we also know that

$$\frac{\partial P_{CN}/\partial y^*}{\partial P_{CC}/\partial y^*} = \frac{1 - \Phi[g(y^*)]}{\Phi[g(y^*)]} \tag{3.8}$$

where

$$g(y^*) \equiv \frac{\sqrt{1+\sigma_\varepsilon^2}}{\sigma_\varepsilon}\left(z^* - \frac{y^*}{(1+\sigma_\varepsilon^2)}\right). \tag{3.9}$$

and $\Phi(.)$ is the standard normal distribution function. Note that the right hand side of (3.8) is a strictly increasing function of y^*. Thus as the welfare cost of type II error rises relative to the cost of a type I error, the optimal y^* threshold increases. This relationship leads to the following proposition.

<u>Proposition 1</u>: If the welfare cost of a type I and a type II error are equal, then the optimal value of y^* is such that

$$z^* \lesseqgtr 0 \Rightarrow P(y < y^*) \lesseqgtr P(z < z^*)$$

In other words, if the objective probability that the credit market is constrained is less than $\frac{1}{2}$, then the government will choose to impose θ_{1C} less frequently than credit constraints actually occur.

Proof: See Appendix A.

In our view, z^* must certainly be negative. After all, the corridor theory is a theory of occasional, intermittent market failures. In the data used for our consumption regressions, contraction took place in only 19.7 percent of the quarters in the sample, and even this number must surely overstate the proportion of quarters in which credit markets were seriously constrained. As a result, if the welfare cost of both types of error is the same, the government will choose to impose θ_{1C} less frequently than credit constraints actually occur.

Proposition 1 describes how frequently the government will choose θ_{1C} (relative to the frequency of credit constraints) if the welfare costs of Type I and Type II errors happen to be equal. We would like to know, however, how frequently the government will in fact impose θ_{1C}. To determine this it is helpful to rearrange equation (3.9) as:

$$\frac{y^*}{\sqrt{1 + \sigma_\varepsilon^2}} = z^*\sqrt{1 + \sigma_\varepsilon^2} - \sigma_\varepsilon \Phi^{-1}(1-w) \tag{3.10}$$

where

$$w \equiv \frac{W_{CC} - W_{NC}}{(W_{CC}-W_{NC}) + (W_{NN}-W_{CN})} \tag{3.11}$$

Note that the objective probability of a credit constraint is $\Phi(z^*)$ while the probability that the government chooses to impose θ_{1C} is $\Phi(y^*/\sqrt{1+\sigma_\varepsilon^2})$. Equation (3.11) indicates the optimal y^* conditional on w.

The optimal policy is the solution for y^* and the θ's defined by the highly nonlinear first order conditions (3.3)-(3.6). However, it is relatively easy to calculate w for any arbitrary rule for the θ's. Hence the optimal y^* conditional on any rule for the θ's is also easily calculated. With this in mind, let us consider a simple, but illuminating special case. Imagine that the government chooses its countercyclical policy as though it knew the state of the credit market with certainty. For purposes of exposition we consider an explicit functional form of the utility function in order to facilitate the comparison of tax rates across credit market regimes. Letting $U(C) = \log(C)$, it is easily demonstrated that the optimal tax rates under certainty about the state of the credit market are given by:[10]

$$\theta_{1N} = \theta_{2N} \equiv \theta_N = \frac{R}{Y-R} \tag{3.12}$$

$$\theta_{1C} = \theta_N - \frac{[Y - \frac{2+\delta}{1+\delta}(A_o+X_1)]}{Y-R}, \tag{3.13}$$

$$\theta_{2C} = \theta_N + (1+\delta)\frac{[Y - \frac{2+\delta}{1+\delta}(A_o+X_1)]}{Y-R}. \tag{3.14}$$

where

$$Y \equiv A_o + X_1 + \frac{1}{1+r}X_2 \tag{3.15}$$

Observe that tax rates are equalized across periods in normal years. This is in accord with the intuition given by Kydland and Prescott (1980) that tax rates must be kept stable over time in order to avoid the deadweight loss associated with distorting intertemporal relative prices. In contrast, observe that optimal tax policy under certainty calls for lowering the tax rate below the normal level in credit constrained years. Indeed, as demonstrated in Appendix B, θ_{1C} and θ_{2C} are such that $W_{CC} = W_{NN}$ -- the government's countercyclical policy completely undoes the effects of the credit constraints.

Now consider what the optimal choice of y^* would be in the general problem conditional on the θ's being chosen in this manner.

<u>Proposition 2</u>: If the government chooses θ_{1C} and θ_{1N} as though it knew the state of the credit market with certainty, then the optimal value of y^*

is such that

$$\frac{y^*}{\sigma_y} < z^*$$

that is, θ_{1C} will be chosen less often than the credit market is actually constrained.

Proof: See Appendix B.

Proposition 2 hints at the tradeoff the government faces between the magnitude and frequency of countercyclical policy. Recall that the variation of tax rates under the certainty solution is such that if the credit market were closed its effect would be completely negated. Since such a policy is relatively powerful, it is not surprising that the government would impose this policy infrequently if there were a chance that a mistake about the state of the credit market was being made.

The optimal tax rates in the certainty case establish a bound for the optimal problem under uncertainty. That is, the θ_{1C} given in equation (3.13) provides a lower bound for the optimal θ_{1C} under uncertainty. This can be seen by reexamining the optimality conditions for the general problem. The optimal θ_{1C} from (3.3-3.6) will be that given in (3.13) only if $P_{CN} = P_{NC} = 0$, i.e., the government never makes a mistake about the state of the credit market. Thus, it will not choose to lower the tax rate as much as that suggested by (3.13) in a period it perceives (perhaps incorrectly) that the credit market is closed. Following similar reasoning, an upper bound for θ_{1C} is easily determined. Suppose that $P_{CC} = 0$ instead of $P_{CN} = 0$, (but $P_{NC} = 0$ still holds). In this limiting case in which the government is always mistaken when it perceives the credit market is closed it is easily demonstrated that optimal tax rates satisfy $\theta_{1C} = \theta_{2C} = \theta_N$.

The intuition underlying the upper and lower bounds for θ_{1C} is straightforward. On the one hand, it is not optimal to keep tax rates stable during perceived credit crunches as long as there is a positive probability that this perception is correct. On the other hand, it is not optimal to lower the tax rate in such periods as much as would be optimal under certainty as long as there is a positive probability that the perception is incorrect. While this general principle is relatively simple, the actual conditions defining the optimal choice of tax rates under uncertainty are quite complicated. To obtain a better perspective on the optimal solution, in the next section we provide numerical solutions to the optimal problem under alternative parameterizations. These numerical solutions provide insights into the robustness of the optimal policy to sources of uncertainty other than the state of the credit market. Simultaneously, we provide numerical characterizations of simple alternatives to the optimal policy (such as the policy suggested in Proposition 2).

IV. Numerical Analysis of Alternative Tax Policies

In the last section, we considered a particular tax policy in detail. This policy sets taxes so as to completely undo the effects of a credit constraint. Let us call this policy the "certainty policy" because it sets tax rates as though the government knew the state of credit markets with certainty.

The certainty policy requires that θ_1 be set to a constant, "permanent" value during normal periods. During constrained periods on the other hand, θ_1 is an increasing function of $(A_0+X_1)/Y$, the period 1 share of total income. In other words, the tax rate is "fine tuned" during constrained periods. Given this tax policy, it turns out that the optimal switching rule involves imposing θ_{1C}, the constrained period tax rate, less often than constrained periods actually occur.

It is of interest to compare the certainty policy to two other possible policy rules. The first is the "constrained optimal" policy. This policy leaves θ_{1N} set to its permanent value just as in the certainty policy. Subject to this simplifying constraint, θ_{1C} and y^* are chosen to maximize expected welfare. In the second policy, the "globally optimal" policy θ_{1N}, θ_{1C}, and y^* are chosen simultaneously to maximize expected welfare.[11]

Unlike in the certainty policy, we cannot find closed form solutions (or even bounds) for the decision variables under either the constrained optimal or the globally optimal policies. As a consequence, we must calculate the optimal values of these variables numerically in order to compare the policies. Of course, numerical optimization requires us to specify values for all the other parameters in the model. Some of these parameters can be specified by referring to previous studies. The values chosen for some other parameters have little effect on the results. However, there are two parameters that require some discussion -- the period 1 share of total income, $(A_0+X_1)/Y$, and the error variance of the credit market indicator, σ_ε^2.

In the introduction to this paper, we proposed examining countercyclical policies that are limited in two ways. First, the rule that determines switching between regimes (e.g., normal and constrained) is a simple function of a small number of observable variables. Second, within each regime, policy variables take on a limited number of values. The certainty policy does not completely satisfy either of these criteria because of the complicated dependence of both θ_{1C} and y^* on the period 1 share. In the real economy, the government must set tax rates prior to knowing both the state of the credit market and current period income, hence the certainty policy is particularly difficult to implement. Indeed, if the period 1 share and the state of the credit market are correlated, as seems likely, then the switching rule analyzed in the previous section is not necessarily optimal.

Clearly, then, we need to compare these three policies -- certainty, constrained optimal, and globally optimal -- at many different values of the period 1 share. Of particular interest is the dependence of the globally optimal policy variables to the period 1 share. If the policy variables are highly sensitive to the period 1 share, then a limited countercyclical policy will not approximate the optimal policy very well. On the other hand, if, within each regime, the policy variables do not respond very much to the period 1 share, then a limited countercyclical policy can provide a close approximation to the optimal policy.

The period 1 share is relevant only within a certain range. If the period 1 share rises above $(1+\delta)/(2+\delta)$, then potential credit constraints are not binding and all the policies collapse to the normal period solution. If the period 1 share falls below $[(1+\delta)/(2+\delta)]^2$, then the certainty policy may not be feasible. If θ_{1C} is mistakenly imposed when credit markets are open, it will not be possible to meet the revenue requirement.[12] To compare the three policies, we solve each model for 200 different values of the period 1 share in the relevant range.

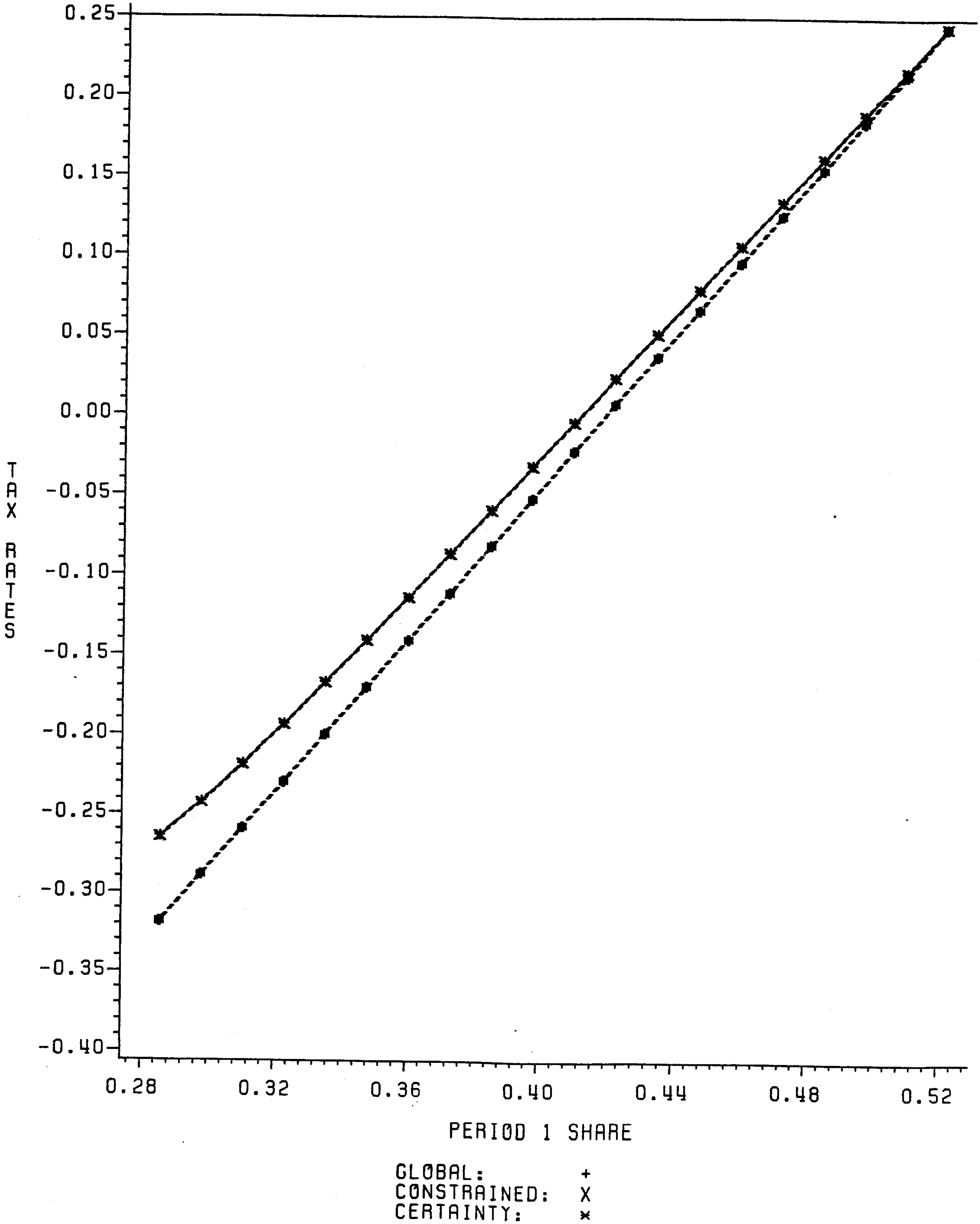
In the model set out above, all the uncertainty faced by the government is captured by σ_ε^2, the error variance of the credit market indicator. (Remember that z, the true state indicator, has been normalized.) To assess the sensitivity of all three policies to uncertainty, we allow σ_ε^2 to take on the values 0.01, 1.0, and 4.0. Since we solve the model for each of the three tax policies at three different values of σ_ε^2 and 200 different values for period 1 share, we end up solving 1800 different versions of our two period model.

The other parameters in the model remain fixed. We set both the interest rate, r, and the rate of internal discount, δ, equal to 0.10. Total income, Y, appears only as a scale factor, so it is normalized to 1.0. The government's revenue requirement, R, is set to 0.20, twenty percent of total income. Finally, z^* is set so that the objective probability of a credit constraint is ten percent.

Figures 1 through 3 display the results of the numerical optimizations. Each figure consists of a pair of plots. In the first plot of the pair, the values of θ_{1C} for all three of the policies is plotted against the period 1 share. A horizontal line is drawn at 0.25, the value of θ_{1N} under the certainty and constrained optimal policies, as a reference. In the second plot of the pair, the probabilities of choosing θ_{1C} in each policy are plotted against the period 1 share. Again, a horizontal line is drawn as a reference; this time it is drawn at 0.10, the objective probability of a credit constraint. The values plotted in Figures 1, 2, and 3 are calculated with σ_ε^2 set equal to 0.01, 1.0, and 4.0 respectively.

Figure 1 compares the policies in the case when the government has very accurate information about the state of the credit market ($\sigma_\varepsilon^2 = 0.01$). As a result, both the constrained and globally optimal policies closely resemble the certainty policy.[13] As we found in the previous section, θ_{1C} declines sharply as the period 1 share falls. At the same time the probability of imposing θ_{1C} is always less than the objective probability that credit markets are closed and this probability also falls as the period 1 share falls. (This last feature partially reflects the assumed independence between z and the period 1 share.) The decline in the probability of

FIGURE 1A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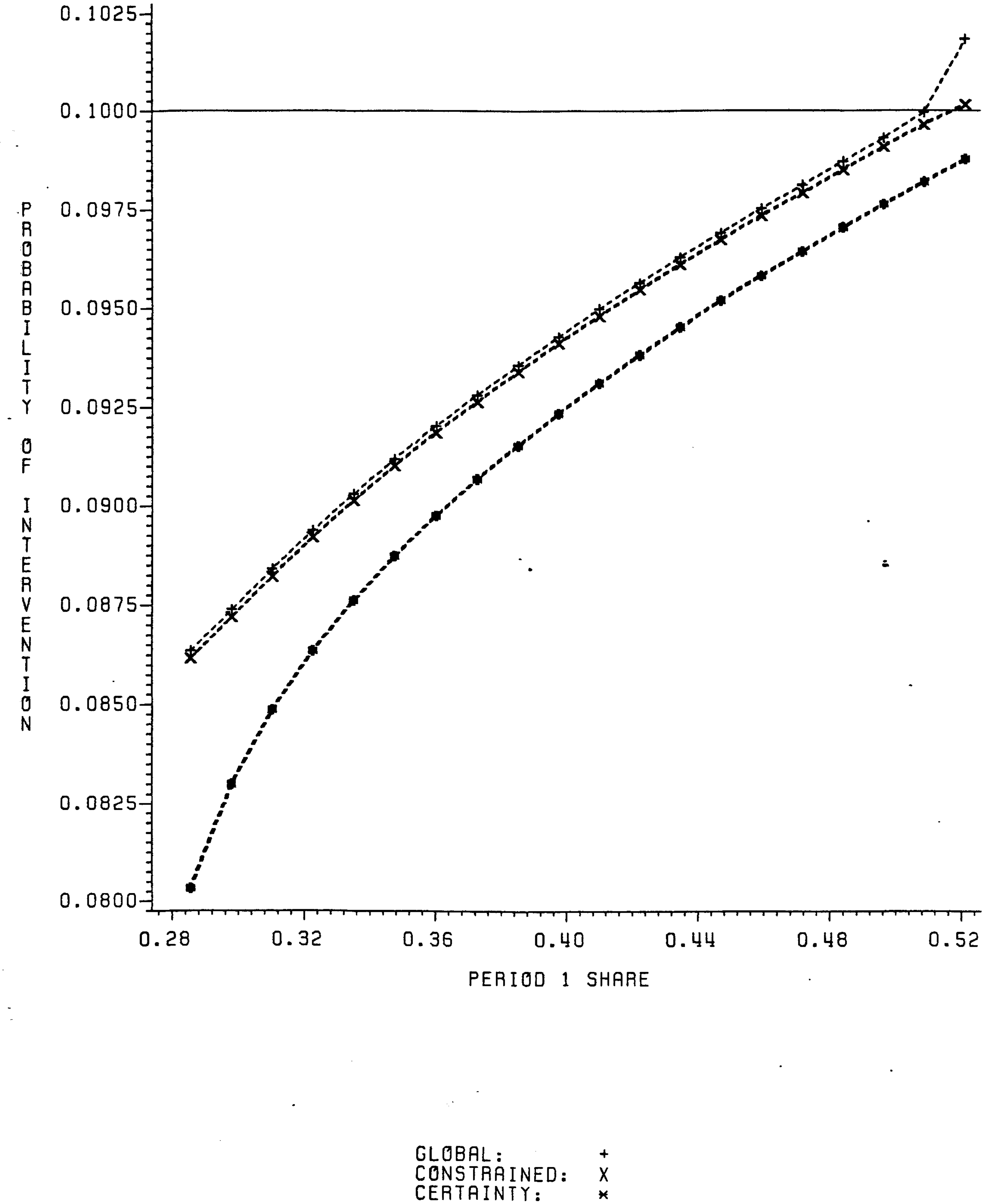
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
TAX RATES
0.28
0.32
0.36
0.40
0.44
0.48
0.52
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: ⋇

FIGURE 1B



PROBABILITY OF INTERVENTION
0.1025
0.1000
0.0975
0.0950
0.0925
0.0900
0.0875
0.0850
0.0825
0.0800
0.28
0.32
0.36
0.40
0.44
0.48
0.52
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: *

imposing is not as precipitous for the constrained optimal and globally optimal policies as it is for the certainty policy. For the former two policies, this probability never goes below 8.4 percent.

Figure 2 depicts the three policies when half the variability of the credit market indicator, y, is due to the "noise" that obscures the true state of the market. In this case, the constrained and globally optimal policies call for constrained regime tax rates that are far less generous than the values specified by the certainty policy (although they are still substantially lower than θ_{1N}). In other words, the globally optimal policy will only partially undo the effects of credit constraints even when the government believes there is a constraint. In addition, θ_{1C} is much less sensitive to the value of the period 1 share under the constrained optimal and globally optimal policies than under the certainty policy. The value of θ_{1C} under the former policies never goes below 0.06 and never goes above 0.25 in Figure 2(a).

Figure 2(b) reveals the surprising result that the constrained and globally optimal policies call for imposing θ_{1C} more frequently than credit constraints actually occur. Again, for these two policies, this probability is not highly sensitive to the period 1 share. (The probability of imposing θ_{1C} under these two policies goes from 14.3 percent to 17.1 percent.) Thus, at this intermediate level of uncertainty, the constrained and globally optimal policies are strongly activist even though frequent interventions guarantee that θ_{1C} will be "mistakenly" imposed a fair amount of the time. In the case depicted in Figure 2, the constrained and globally optimal policies can be well approximated by a limited countercyclical policy that ignores completely the value of the period 1 share.[14]

FIGURE 2A

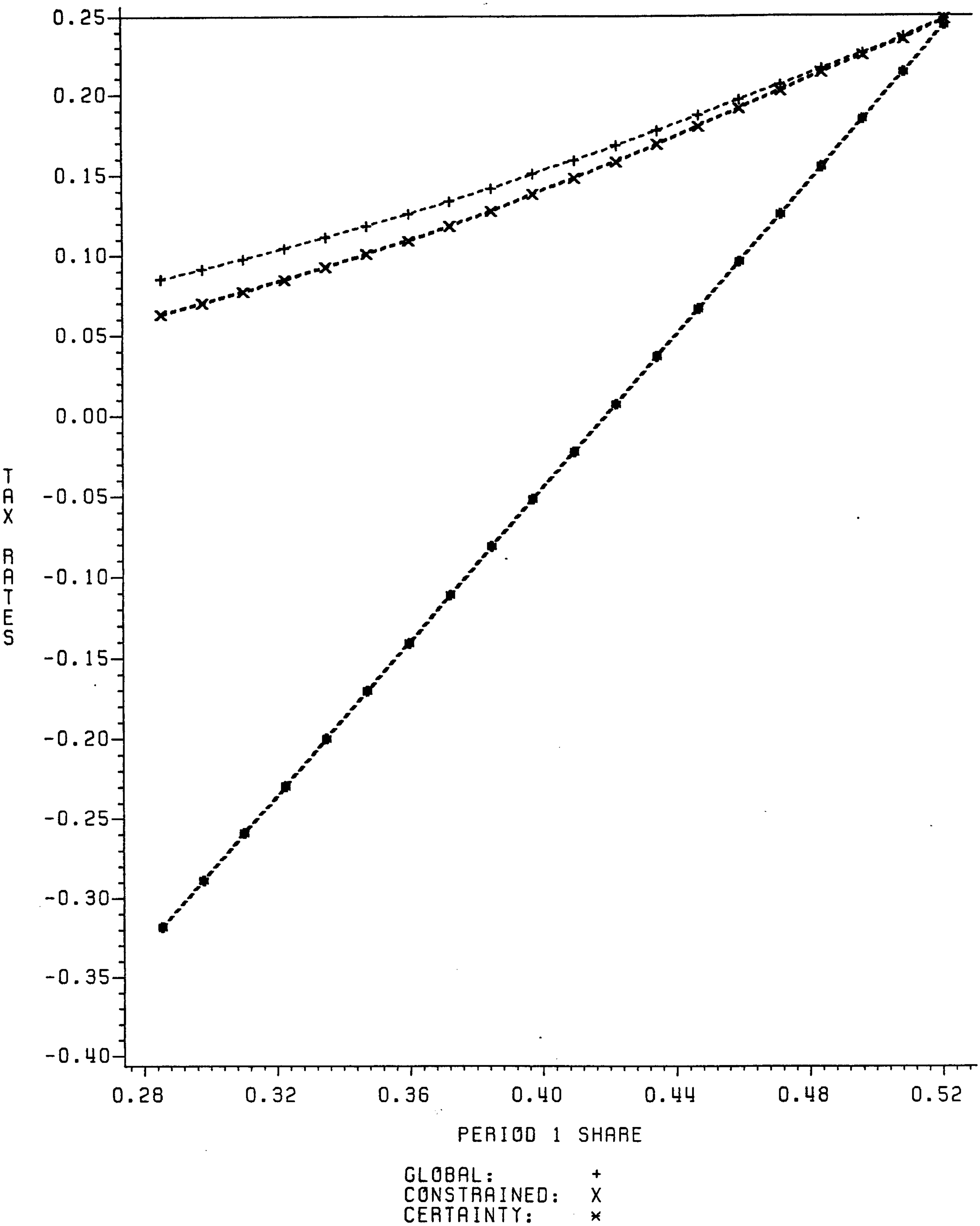
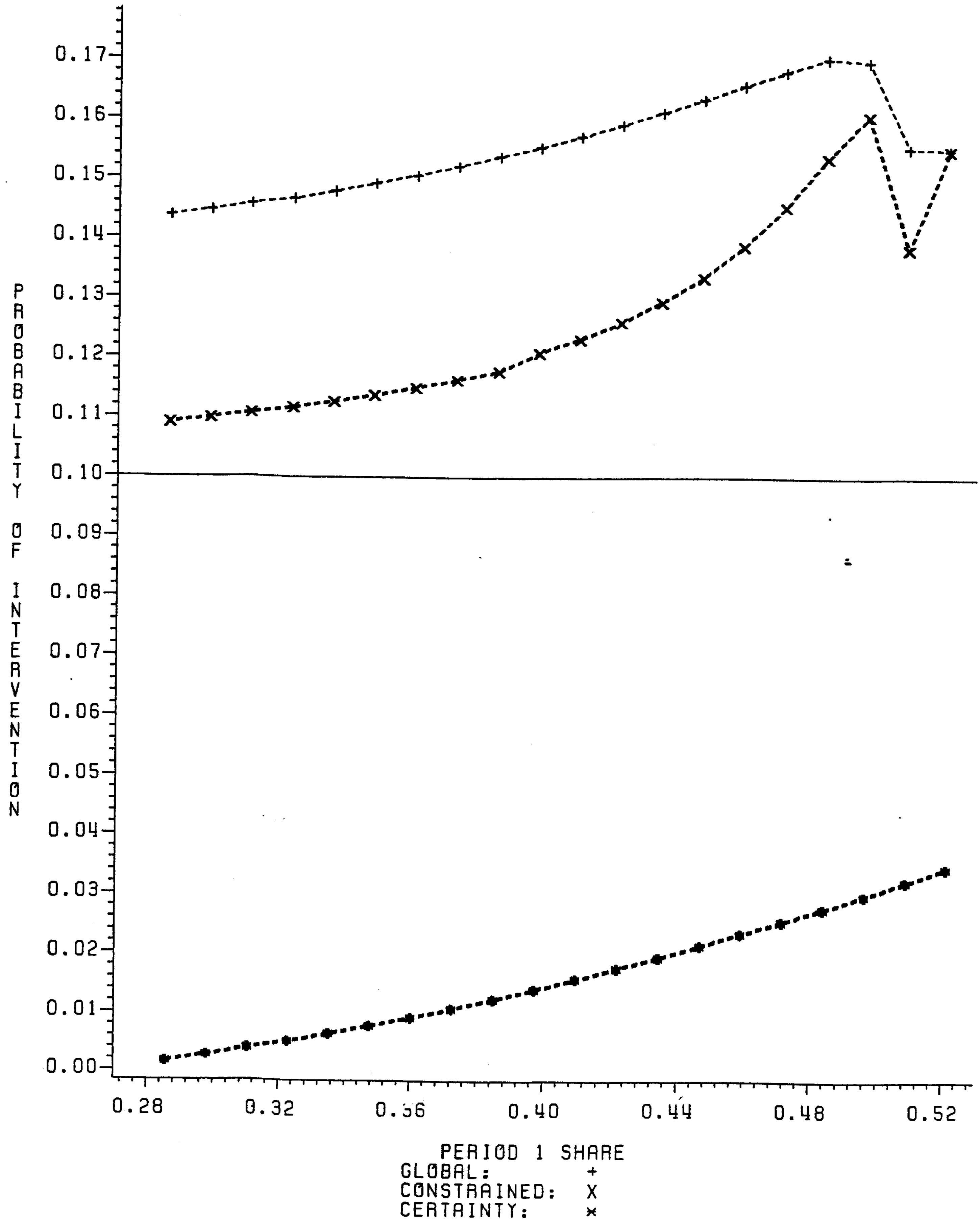

0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
TAX RATES
0.28
0.32
0.36
0.40
0.44
0.48
0.52
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: *

FIGURE 2B



PROBABILITY OF INTERVENTION
0.17
0.16
0.15
0.14
0.13
0.12
0.11
0.10
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0.00
0.28
0.32
0.36
0.40
0.44
0.48
0.52
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: *

FIGURE 3A

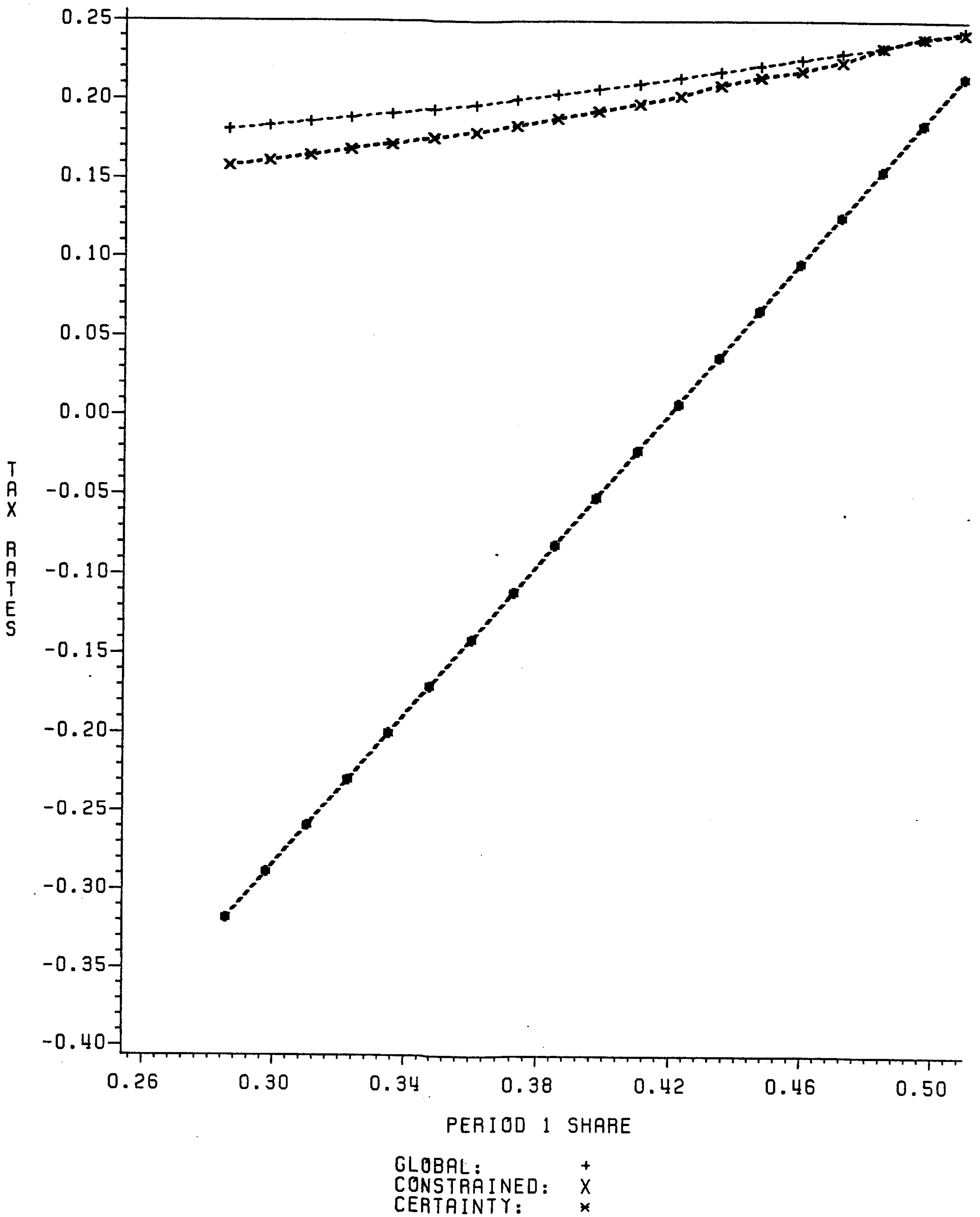
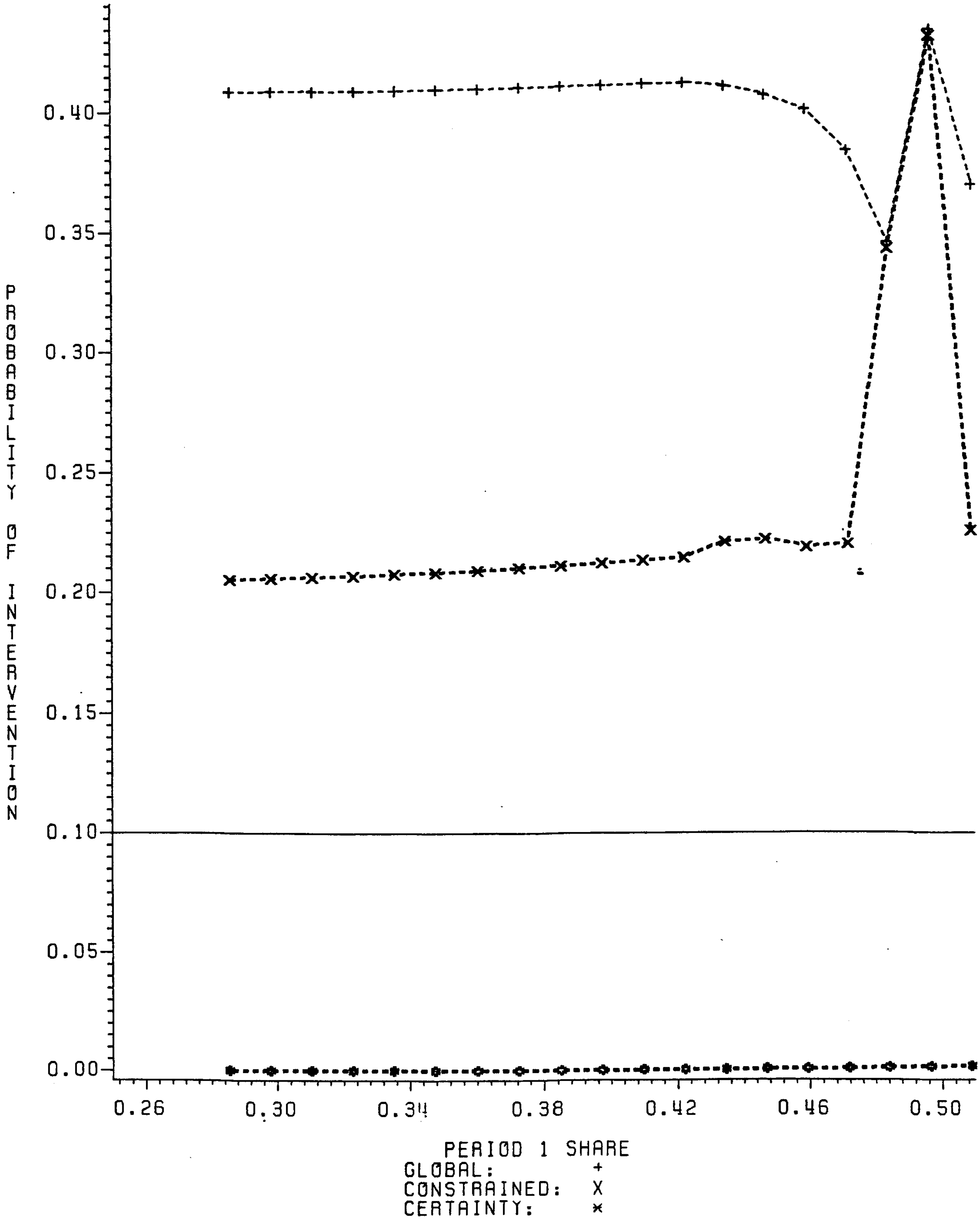

0.25
0.20
0.15
0.10
0.05
0.00
-0.05
-0.10
-0.15
-0.20
-0.25
-0.30
-0.35
-0.40
TAX RATES
0.26
0.30
0.34
0.38
0.42
0.46
0.50
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: *

FIGURE 3B



0.40
0.35
0.30
0.25
0.20
0.15
0.10
0.05
0.00
PROBABILITY OF INTERVENTION
0.26
0.30
0.34
0.38
0.42
0.46
0.50
PERIOD 1 SHARE
GLOBAL: +
CONSTRAINED: X
CERTAINTY: *

In Figure 3, the government faces much greater uncertainty — eighty percent of the variability of the credit market indicator is due to noise. As a result, the constrained and globally optimal policies call for setting θ_{1C} fairly close to θ_{1N}. In essence, the two regimes begin to collapse into one. Consequently, the probability of choosing θ_{1C} no longer has much impact on expected welfare. This fact is reflected by the erratic plots in Figure 3(b). The numerical optimization procedure fails to discriminate very well between alternative values for this probability. Note that under the certainty policy, the probability of imposing θ_{1C} falls close to zero.

V. Conclusion

If consumers face intermittent financial constraints, then an activist countercyclical policy may increase social welfare. That optimal policy might call for varying the tax rate between normal and constrained periods is not surprising. What makes this analysis of interest is that we consider the optimal tax policy under the assumption that the government is uncertain about the state of credit markets. This leads to an interesting tradeoff between the frequency and magnitude of implementing countercyclical policy.

Our primary results regarding this tradeoff are derived from a numerical sensitivity analysis of alternative tax policies to key parameters of the model. We find that if uncertainty about the state of credit markets is minimal, then a fine tuning policy which virtually undoes the effects of credit markets is optimal. However, even with only a small amount of uncertainty, we find that such a fine tuning policy will be imposed less frequently than credit constraints actually occur. Thus, we see that a small amount of uncertainty induces the tradeoff to take the form of a powerful policy imposed relatively infrequently. At the other extreme, if uncertainty is overwhelming, then we find that the optimal policy converges to a completely non-contingent tax rate for all periods. This case is consistent with the view that countercyclical policy is counterproductive.

Finally, we consider an intermediate case that yields the most interesting findings. We find that if uncertainty plays a significant but not dominant role, then a moderate limited countercyclical policy is called for. This policy involves one fixed tax rate for normal periods and another approximately fixed and slightly lower tax rate for constrained periods. As one would expect, we find that the magnitude of the countercyclical policy in this case is reduced relative to the policy under certainty. It now becomes optimal to impose this countercyclical policy more frequently than credit constraints actually occur. Thus, a moderate amount of uncertainty induces a moderate countercyclical policy imposed relatively frequently. In addition, this moderate countercyclical policy roughly fits our notion of a limited countercyclical policy, that is, two discretely different fixed rules for the two regimes. These results provide support for the idea that countercyclical policy may be feasible even if policymakers face significant uncertainty if simple limited countercyclical policies are used.

FOOTNOTES

*The authors would like to thank Michael Darby, Bryan Ellickson, David Levine, Mark Plant, Marc Robinson and participants of the Money Workshop and the Public Finance Workshop at UCLA for many helpful comments. A chance remark by Finis Welch also contributed substantially to the direction of this research. Very able research assistance was supplied by Kishore Gawande. The authors are responsible for any remaining errors.

[1]Hall's seminal paper (1978) has generated an avalanche of ever more sophisticated tests of the permanent income hypothesis. Since we use this model primarily to illustrate our ideas of limited countercyclical policies, Hall's simple model suits our purposes. Readers interested in state of the art treatments of this theory should consult Hansen and Singleton (1983), Hayashi (1982, 1984), and Mankiw, et. al. (1982).

Note that many other models could have been used to illustrate our policy proposals; for example, some versions of the labor contracting model could have been used. Again, we felt that the relative simplicity of the Hall consumption model made it more suitable for our expositional purposes.

[2]It might be argued that this modified permanent income hypothesis adjusted for imperfect capital markets is what Keynes had in mind for the consumption function.

[3]For microeconomic oriented explanations for the existence of credit rationing see Jaffee and Modigliani (1969) and Stiglitz and Weiss (1981). For discussions of why credit rationing may be more prevalent during recessions see Mishkin (1976) and (1977).

[4]To simplify the notation, we assume that the time path of Y_t and the periods in which γ_t is non-zero are known with certainty at time t. Our results are essentially unchanged in the case where the consumer maximizes

expected welfare under uncertainty about future income and credit constraints.

[5]As Hall noted, the random walk hypothesis of consumption requires that no variables have any significant ability to predict future consumption (after the effect of current consumption is taken into account). In Hall's regressions, only stock prices violated this prediction. In our corridor theoretic approach, the predictive ability of stock prices is consistent with the general correctness of the permanent income hypothesis.

[6]If, in fact, credit constraints play an important, though intermittent, role in altering the time path of consumption, then aggregate consumption data is probably not the best data for considering the permanent income hypothesis. Credit constraints operate on a household (and firm) level. Even during a vigorous expansion, some households are unable to borrow in amounts that are consistent with their permanent wealth. Even in the depths of a recession, some borrowing takes place. In aggregate data, the value of a proxy for γ_t is a measure both of the general tightness of credit markets and of the proportion of households that are actually constrained.

[7]Note that the standard error of the regression is minimized in regression 5 even though regression 5 is nested in both regressions 3 and 4. However, regression 5 is estimated by ordinary least squares while regressions 3 and 4 are estimated by instrumental variables. The instrumental variables method does not guarantee that the standard error of the regression will be reduced when additional regressors are added. (Note that the coefficient of $CONSTR_t$ differs substantially across these regressions.)

[8]Hall and Mishkin (1982) do test the rational expectations version of the permanent income hypothesis using data from the Panel Study of Income Dynamics. They find substantial excess sensitivity of expenditures on food to current income.

[9]We are implicitly assuming that the government's potential deficits during credit crunches will not further exacerbate matters by "crowding out" the consumers. This reflects an implicit assumption that the government has access to capital markets (perhaps world capital markets) that are unavailable to the consumer during these periods.

[10]In the standard optimal taxation problem, it is rarely possible to find a closed form solution for the optimal tax rates. Additionally, the first-order conditions, that is, the tangency of the indifference curve and the government's budget line, may not be sufficient conditions for a maximum of welfare. We have constructed our model to deliberately sidestep these problems. In our case, the first-order conditions determine the unique values of the optimal tax rates. Indeed, we have set up one of those special cases where uniform taxation is optimal.

[11]While it is straightforward to calculate the constrained and globally optimal policies in our simple model, it would be very difficult to do so for the real economy. The point here is to see how well these optimal policies can be approximated by analytically more tractable candidates.

[12]Imagine that the government chooses θ_{1C} as given by equation (3.13) and that

$$\frac{A_o+X_1}{Y} < \left(\frac{1+\delta}{2+\delta}\right)^2.$$

In this case, θ_{1C} is a subsidy to, rather than a tax on, period 1 consumption. If the government is mistaken, if credit markets are actually open, then it can be shown that it will be impossible for the government to satisfy the revenue constraint in this case. Note that this bound for $(A_o+X_1)/Y$ is a number that is slightly larger than 1/4. In the context of our model, the mean for $(A_o+X_1)/Y$ should probably be a little larger than 1/2. Thus, if

$(A_o+X_1)/Y$ falls below 1/4, it falls to less than half its average value. For the purposes of this paper, we feel comfortable assigning a probability of zero to such an event.

[13]In every case, the value of θ_N under the globally optimal policy is very close to 0.25, its value under the other two policies. (The minimum value for θ_{1N} is 0.22. This occurs when $\sigma_\varepsilon^2 = 4$ and the period 1 share is 0.274.) As a consequence,, the constrained optimal and globally optimal policies are always very similar.

[14]Note that, in the case depicted in Figure 2, the potential correlation between z and the period 1 share is largely irrelevant.

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Appendix A: Probabilities and Derivatives In The Two Period Model

Derivation of $\frac{\partial P_{CC}}{\partial y^*} = -\frac{\partial P_{NC}}{\partial y^*}$ and $\frac{\partial P_{CN}}{\partial y^*} = -\frac{\partial P_{NN}}{\partial y^*}$:

Recall that the true state in period 1 is C if $z < z^*$ and N if $z > z^*$. Also the government chooses θ_{1C} if $y < y^*$ and θ_{1N} if $y > y^*$. The observable indicator y is related to z by the formula

$$y = z + \varepsilon$$

where ε is independent of z. We assume that $z \sim N(0,1)$ and $\varepsilon \sim N(0,\sigma_\varepsilon^2)$, thus $y \sim N(0,1+\sigma_\varepsilon^2)$. We begin by considering P_{CC}, the probability that the government imposes θ_{1C} when the true state of the economy is C. We can write this probability as

$$P_{CC} \equiv P[y < y^*,\ z < z^*] = P[\varepsilon < y^* - z,\ z < z^*] \tag{A.1}$$

$$= \frac{1}{2\pi\sigma_\varepsilon} \int_{-\infty}^{z^*} \int_{-\infty}^{y^*-z} \exp\left\{-\frac{1}{2}\left(z^2 + \frac{\varepsilon^2}{\sigma_\varepsilon^2}\right)\right\} d\varepsilon\, dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z^*} \exp\left(-\frac{z^2}{2}\right) \left[\frac{1}{\sigma_\varepsilon\sqrt{2\pi}} \int_{-\infty}^{y^*-z} \exp\left(\frac{\varepsilon^2}{2\sigma_\varepsilon^2}\right) d\varepsilon\right] dz$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z^*} \Phi\left(\frac{y^*-z}{\sigma_\varepsilon}\right) e^{-\frac{1}{2}z^2}\, dz \tag{A.2}$$

where $\Phi(.)$ is the standard Normal distribution function. If we let $\phi(.)$ denote the standard Normal density function, then

$$\frac{\partial P_{CC}}{\partial y^*} = \frac{1}{\sqrt{2\pi}} \frac{1}{\sigma_\varepsilon} \int_{-\infty}^{z^*} \phi\left(\frac{y^*-z}{\sigma_\varepsilon}\right) e^{-\frac{1}{2}z^2}\, dz$$

$$= \frac{1}{2\pi} \frac{1}{\sigma_\varepsilon} \int_{-\infty}^{z^*} \exp\left\{-\frac{1}{2}\left[\left(\frac{y^*-z}{\sigma_\varepsilon}\right)^2 + z^2\right]\right\} dz \tag{A.3}$$

$$= \frac{1}{2\pi} \frac{1}{\sigma_\varepsilon} \int_{-\infty}^{z^*} \exp \left\{ - \frac{1}{2} \frac{\sigma_y^2}{\sigma_\varepsilon^2} (z - \frac{1}{\sigma_y^2} y^*)^2 - \frac{1}{2} \frac{1}{\sigma_y^2} y^{*2} \right\} dz$$

$$= \phi (\frac{y^*}{\sigma_y}) \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z^*} \exp \left\{ - \frac{1}{2} \frac{\sigma_y^2}{\sigma_\varepsilon^2} (z - \frac{1}{\sigma_y^2} y^*)^2 \right\} dz$$

$$= \frac{\sigma_\varepsilon^2}{\sigma_y} \phi (\frac{y^*}{\sigma_y}) \Phi [\frac{\sigma_y}{\sigma_\varepsilon^2} (z^* - \frac{1}{\sigma_y} y^*)] \tag{A.4}$$

By similar arguments, we have

$$P_{CN} = \frac{1}{\sqrt{2\pi}} \int_{z^*}^{\infty} \Phi (\frac{y^*-z}{\sigma_\varepsilon}) e^{-\frac{1}{2}z^2} dz \tag{A.5}$$

$$\frac{\partial P_{CN}}{\partial y^*} = \frac{\sigma_\varepsilon}{\sigma_y} \phi (\frac{y^*}{\sigma_y}) \left\{1 - \Phi [\frac{\sigma_y}{\sigma_\varepsilon^2} (z^* - \frac{1}{\sigma_y} y^*)]\right\} \tag{A.6}$$

$$P_{NC} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z^*} [1 - \Phi (\frac{y^*-z}{\sigma_\varepsilon})] e^{-\frac{1}{2}z^2} dz \tag{A.7}$$

$$P_{NN} = \frac{1}{\sqrt{2\pi}} \int_{z^*}^{\infty} [1 - \Phi (\frac{y^*-z}{\sigma_\varepsilon})] e^{-\frac{1}{2}z^2} dz \tag{A.8}$$

$$\frac{\partial P_{NC}}{\partial y^*} = - \frac{\partial P_{CC}}{\partial y^*}, \quad \frac{\partial P_{NN}}{\partial y^*} = - \frac{\partial P_{CN}}{\partial y^*} \tag{A.9}$$

Derivation of equation (3.8): Note that by combining (A.4) and (A.6) we have

$$\frac{\partial P_{CN}/\partial y^*}{\partial P_{CC}/\partial y^*} = \frac{1 - \Phi[g(y^*)]}{\Phi[g(y^*)]}$$

where $g(y^*) = \frac{\sqrt{1+\sigma_\varepsilon^2}}{\sigma_\varepsilon} (z^* - \frac{y^*}{(1+\sigma_\varepsilon^2)})$.

This establishes equation (3.8).

Proof of Proposition 1: Substituting (3.8) into (3.7) and rearranging yields

$$\Phi[g(y^*)] = 1 - w \tag{A.10}$$

where

$$w \equiv \frac{W_{CC}-W_{NC}}{(W_{CC}-W_{NC}) + (W_{NN}-W_{CN})} \tag{A.11}$$

w is the welfare cost of a Type II error as a percentage of the sum of both types of potential welfare losses. Combining (3.9) and (A.10) yields

$$\frac{y^*}{\sqrt{1+\sigma_\varepsilon^2}} = \sqrt{1+\sigma_\varepsilon^2}\, z^* - \sigma_\varepsilon\, \Phi^{-1}(1-w) \tag{A.11}$$

Note that, for a given value of y^*, the probability that the government chooses θ_{1C} is $\Phi(\frac{y^*}{\sqrt{1+\sigma_\varepsilon^2}})$. The probability that the credit market is actually closed in period 1 is $\Phi(z^*)$. We wish to consider whether the government should choose θ_{1C} more or less frequently than the credit market is actually closed. First consider the case where the welfare cost of both types of error is the same. Then $w = \frac{1}{2}$, $\Phi^{-1}(1-w) = 0$, and

$$\frac{y^*}{\sqrt{1+\sigma_\varepsilon^2}} = \sqrt{1+\sigma_\varepsilon^2}\, z^*. \tag{A.12}$$

The factor multiplying z^* on the right-hand side of (A.12) is the multiplicative inverse of the correlation between y and z and is therefore guaranteed to be greater than one. Thus if z^* is a positive number, then $\frac{y^*}{\sqrt{1+\sigma_\varepsilon^2}}$ is a larger positive number, and θ_{1C} will be chosen more frequently than the credit market is actually constrained. Conversely, if z^* is a negative number, then θ_{1C} will be chosen less frequently than the credit market is actually constrained.

Appendix B: Proof of Proposition 2

Before proceeding to the proof of the proposition we establish the following steps:

Derivation of θ_{1C} and θ_{2C}: The slope of the consumer's (and hence the government's) indifference curves in tax space is given by:

$$\left.\frac{\partial\theta_{2C}}{\partial\theta_{1C}}\right|_{W=\overline{W}} = -(1+\delta)\,\frac{(1+\theta_{2C})}{(1+\theta_{1C})}$$

while the slope of the government's budget constraint is given by:

$$\left.\frac{\partial\theta_{2C}}{\partial\theta_{1C}}\right|_{R=\overline{R}} = -(1+r)\,\frac{A_o+X_1}{X_2}\left(\frac{1+\theta_{2C}}{1+\theta_{1C}}\right)^2$$

Thus, the tangency occurs when:

$$(1+r)\,\frac{A_o+X_1}{X_2}\left(\frac{1+\theta_{2C}}{1+\theta_{1C}}\right)^2 = (1+\delta)\left(\frac{1+\theta_{2C}}{1+\theta_{1C}}\right). \tag{B.1}$$

Combining (B.1) with the government's budget constraint and rearranging terms yields

$$\theta_{1C} = \theta_N - \frac{[Y - (\frac{2+\delta}{1+\delta})\,(A_o+X_1)]}{Y-R}$$

$$\theta_{2C} = \theta_N + (1+\delta)\,\frac{[Y - (\frac{2+\delta}{1+\delta})\,(A_o+X_1)]}{Y-R}$$

Derivation of $W_{NN} = W_{CC}$: Substituting calculated values of θ_{1N}, θ_{1C}, θ_{2NN} and θ_{2CC} into C_{1N}, C_{1C}, C_{2NN} and C_{2CC} respectively yields:

$$\begin{aligned} W_{NN} &= \log(C_{1N}) + \frac{1}{1+\delta}\log(C_{2NN}) \\ &= \log\left[\frac{1+\delta}{2+\delta}\,\frac{1}{1+\theta_{1N}}\,Y\right] + \frac{1}{1+\delta}\log\left[\frac{1+r}{2+\delta}\,\frac{1}{1+\theta_{2NN}}\,Y\right] \\ &= \log\left(\frac{1+\delta}{2+\delta}\right) + \frac{1}{1+\delta}\log\left(\frac{1+r}{2+\delta}\right) + \frac{2+\delta}{1+\delta}\log(Y-R) \end{aligned}$$

$$W_{CC} = \log\ (C_{1C}) + \frac{1}{1+\delta} \log(C_{2CC})$$

$$= \log\ (\frac{1}{1+\theta_{1C}}\ (A_o+X_1)) + \frac{1}{1+\delta} \log(\frac{1}{1+\theta_{2CC}}\ X_2)$$

$$= \log(\frac{1+\delta}{2+\delta}) + \frac{1}{1+\delta} \log(\frac{1+r}{2+\delta}) + \frac{2+\delta}{1+\delta} \log(Y-R)$$

Hence, $W_{NN} = W_{CC}$.

Derivation of $(\frac{1+\delta}{2+\delta})^2 < \frac{A_o+X_1}{Y} < \frac{1+\delta}{2+\delta} \implies W_{CN} < W_{NC}$:

To determine $(W_{CN} - W_{NC})$, we need to know θ_{2NC} and θ_{2CN}, the second period tax rates associated with those situations in which the government has made an error with regard to the state of the credit markets. The solutions for θ_{2NC} and θ_{2CN} come from the government's revenue constraint and the consumer's optimal consumption plans. For θ_{2NC}, we have:

$$\frac{\theta_{1N}}{1+\theta_{1N}}\ (A_o+X_1) + \frac{X_2}{1+r}\ \frac{\theta_{2NC}}{1+\theta_{2NC}} = R \tag{B.2}$$

Using the expression for θ_{1N} above, (B.2) yields $\theta_{2NC} = \frac{R}{Y-R}$. Note that this implies that if the government incorrectly perceives that period 1 is normal, the second period tax is the same as it would have been if the government had been correct. Alternatively, for θ_{2CN}, we have:

$$\frac{\theta_{1C}}{1+\theta_{1C}}\ \frac{(1+\delta)}{(2+\delta)}\ Y + \frac{\theta_{2CN}}{1+\theta_{2CN}}\ \frac{1}{2+\delta}\ Y = R \tag{B.3}$$

Using the expression for θ_{1C} above, (B.3) yields

$$\theta_{2CN} = \frac{-(1+\delta) + (2+\delta)\frac{R}{Y} + \frac{(1+\delta)^2}{2+\delta}\frac{(Y-R)}{(A_o+X_1)}}{2+\delta - (2+\delta)\frac{R}{Y} - \frac{(1+\delta)^2}{2+\delta}\frac{(Y-R)}{(A_o+X_1)}}$$

Substituting this expression for θ_{2CN} into the expression for C_{2CN} yields:

$$C_{2CN} = (1+r)\ [1 - (\frac{1+\delta}{2+\delta})^2 \frac{Y}{A_o+X_1}]\ (Y-R) \tag{B.4}$$

Observe that (B.4) implies that $C_{2CN} > 0$ only if $\frac{Y}{A_o+X_1} < (\frac{2+\delta}{1+\delta})^2$. The interpretation of this result is that only if $\frac{Y}{A_o+X_1} < (\frac{2+\delta}{1+\delta})^2$ is it possible to satisfy the government's budget constraint when the government incorrectly perceives that period 1 is constrained. For otherwise, the government's imposition of θ_{2CN} in period 2 in such instances would drive $C_{2CN} = 0$. It is because we only want to consider situations for which the government's budget constraint is feasible that we have assumed $\frac{Y}{A_o+X_1} < (\frac{2+\delta}{1+\delta})^2$.

Given the expressions above for θ_{1C}, θ_{1N}, θ_{2NC} and θ_{2CN}, we can evaluate $(W_{CN} - W_{NC})$. Substituting the expressions for θ_{1N}, θ_{1C}, θ_{2CN} and θ_{2NC} into C_{1C}, C_{1N}, C_{2CN} and C_{2NC} respectively yields:

$$W_{CN} - W_{NC} = -2 \log [\frac{2+\delta}{1+\delta}\frac{A_o+X_1}{Y}] + \frac{1}{1+\delta} \log [1 - \frac{A_o+X_1}{Y}]$$

$$+ \frac{1}{1+\delta} \log [1 - (\frac{1+\delta}{2+\delta})^2 \frac{Y}{A_o+X_1}] \tag{B.5}$$

First, suppose $\frac{A_o+X_1}{Y} = \frac{1+\delta}{2+\delta}$. Substituting this into $W_{CN} - W_{NC}$ yields $W_{CN} - W_{NC} = 0$. Now consider the derivative of $W_{CN} - W_{NC}$ with respect to $\frac{A_o+X_1}{Y}$ (holding Y constant):

$$\frac{\partial[W_{CN} - W_{NC}]}{\partial[\frac{A_o+X_1}{Y}]} = (1+\frac{2+\delta}{1+\delta})\frac{Y}{A_o+X_1}\left\{\frac{[(\frac{Y}{A_o+X_1}) - (\frac{2+\delta}{1+\delta})]^2}{\frac{1}{(1+\delta)^2}\frac{Y}{A_o+X_1} - [\frac{Y}{A_o+X_1} - \frac{2+\delta}{1+\delta}]^2}\right\} \quad (B.6)$$

The numerator of the term in brackets in (B.6) is clearly always positive. Now consider the denominator of the term in brackets in (B.6). It can be rewritten as:

$$(\frac{Y}{A_o+X_1})\ [((\frac{2+\delta}{1+\delta})^2 - \frac{Y}{A_o+X_1})\ (1 - \frac{A_o+X_1}{Y})] \quad (B.7)$$

Since $1 > \frac{1+\delta}{2+\delta} > \frac{A_o+X_1}{Y}$ and $(\frac{2+\delta}{1+\delta})^2 > \frac{Y}{A_o+X_1}$ by assumption, the expression in (B.7) is greater than zero. Hence,

$$\frac{\partial[W_{CN} - W_{NC}]}{\partial[\frac{A_o+X_1}{Y}]} > 0 \quad \text{for} \quad \frac{A_o+X_1}{Y} \in ((\frac{1+\delta}{2+\delta})^2\ ,\ (\frac{1+\delta}{2+\delta}))\ .$$

Given that $W_{CN} - W_{NC} = 0$ for $\frac{A_o+X_1}{Y} = \frac{1+\delta}{2+\delta}$, this implies that

$$W_{CN} - W_{NC} < 0 \quad \text{for} \quad \frac{A_o+X_1}{Y} \in ((\frac{1+\delta}{2+\delta})^2\ ,\ (\frac{1+\delta}{2+\delta}))\ .$$

We now proceed to the proof of the proposition. For Proposition 2 to be true, it is sufficient (but not necessary) that $w < 1/2$. w is determined by the realized welfares in all four potential states of the world. We need therefore to evaluate those realized welfares.

The value of w, and hence of y^*, depends not only on welfare when the government correctly perceives the state of the credit market, but also on the realized welfare of the consumer when the government is mistaken about credit conditions. Of course, if the credit constraint is not binding, then $W_{CC} = W_{CN} = W_{NN} = W_{NC}$. As a result, $w = 1/2$ and, by Proposition 1, θ_{1C} will be chosen less frequently than credit constraints actually occur.

Note that

$$w < 1/2 \text{ iff } (W_{CC} - W_{NC}) < (W_{NN} - W_{CN}).$$

As shown above, even when the credit constraint is binding, W_{CC} is identically equal to W_{NN} -- the government's choice of θ_{1C} and θ_{2C} precisely undoes the effects of the credit contraint. Thus,

$$w < 1/2 \text{ iff } W_{CN} < W_{NC}.$$

As shown above,

$$\left(\frac{1+\delta}{2+\delta}\right)^2 < \frac{A_o+X_1}{Y} < \frac{1+\delta}{2+\delta} \implies W_{CN} < W_{NC}.$$

This establishes Proposition 2.