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INSURANCE ASPECTS OF LABOR MARKET CONTRACTING:

AN OVERVIEW

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I. Introduction

Since the work of Azariadis (1975) and Baily (1974), it has been widely recognized that part of the role of labor market contracting is for firms to provide insurance to workers. For example, in the initial papers cited, firms provided workers a constant wage and in this way insured workers against random fluctuations in output. More recent developments have shown that there are insurance aspects of labor market contracts in an even wider set of circumstances. Harris and Holmstrom (1982) demonstrate how the labor market contract can insure workers against the uncertainty they face concerning their own ability, while Weiss (1984) concentrates on insurance in a world where worker output grows stochastically.

The present paper overviews this literature through an analysis which proceeds in three parts. First, we differentiate into categories the various labor market situations wherein insurance is an issue. In particular, we characterize the form of insurance provided in the labor market contract as a function of three key variables: (i) what access do workers have to capital markets; (ii) what are the mobility costs of workers; and (iii) what is the nature of the uncertainty. The motivation here is that by categorizing in terms of these three variables, one can get a clear understanding of why in each particular situation the labor market contract takes the specific form it does. Second, we investigate a capital market assumption intermediate between the ones which have previously been employed in the literature. In the past, it has either been assumed that workers face perfect capital markets, or that workers have no access to capital markets. As will be clear from our characterization of the literature, under certain situations neither assumption provides a particularly appealing result as regards the type of insurance provided. We demonstrate that by allowing workers access to capital markets, although not perfect capital markets, in these situations there is a major effect on the resultant labor market contract which corresponds to a much more intuitive outcome as regards the type of insurance provided. Third, we discuss the implications of our analysis for the shape of age earnings profiles. Specifically, our analysis suggests that, given an intermediate capital market assumption, upward sloping age earnings profiles will be more prevalent than has previously been realized.

II. A Taxonomy

In our taxonomy we restrict attention to situations where the ability of firms to insure workers is the primary focus. This means we will only be concerned with situations which display the following three properties. First, firms must be risk neutral, while workers risk averse.[1] Second, third party insurance must not be available. Third, firms must either be able to costlessly monitor workers, or workers must receive no disutility for effort. That is, we will not be concerned with situations where incentive effects play an important role.[2]

There are three key variables to consider: (i) what access do workers have to capital markets; (ii) what are the mobility costs of workers; and (iii) what is the nature of the uncertainty. Before proceeding to the actual taxonomy, we will describe the different assumptions which will be allowed for each of the variables. In general, we will try to follow the distinctions already prevalent in the literature.

As mentioned earlier, the literature has concentrated on two assumptions concerning the capital market. First, most papers assume that workers have no access to capital markets, which basically means that workers have no ability to borrow. Second, some of the more recent papers, i.e., Topel and Welch

(1983) and Weiss (1984), have considered the implications of a perfect capital market assumption. In both cases this has meant that workers can borrow and save all they like at a rate of interest equal to the rate at which both workers and firms discount the future.

We can similarly identify two assumptions concerning mobility costs. Starting with the initial work of Azariadis and Baily, many papers have assumed that mobility costs are prohibitive. That is, once a worker agrees to a contract with a firm, the worker is locked into employment with only that firm. More recently, however, a set of papers have turned to the assumption that mobility costs are low or zero (see e.g., Freeman 1977, Harris and Holmstrom 1982, Holmstrom 1983, Waldman 1984, and Weiss 1984). The basic implication of this alternative assumption is that in a long term contract setting, the actual contract offered is constrained by future spot market wages the worker might command.

The third variable is what is the nature of the uncertainty. The key distinction here concerns whether output fluctuations are correlated or uncorrelated. By correlated fluctuations we mean that one period's fluctuation provides information concerning subsequent fluctuations. An example of this would be a world where a worker's output in each period provided information concerning the worker's ability, and thus information about his likely output in subsequent periods (see Harris and Holmstrom 1982). By uncorrelated fluctuations we simply mean that one period's fluctuation does not provide information concerning subsequent fluctuations.[3]

We begin by considering cases where mobility costs are prohibitive. As is clear from Table 1, under this assumption workers always wind up being completely insured. The logic here is simply that, given prohibitive mobility costs, there are no effective constraints on the form of the contracts which

	No Access to Capital Markets	Access to Perfect Capital Markets
Correlated Fluctuations	Complete Insurance through the Firm (see Weiss 1984)	Complete Insurance through the Firm (see Weiss 1984)
Uncorrelated Fluctuations	Complete Insurance through the Firm (see Azariadis 1975 and Baily 1974)	Complete Insurance either through the Firm or through Self-Insurance (see Topel and Welch 1983)

Table 1: Prohibitive Mobility Costs

	No Access to Capital Markets	Access to Perfect Capital Markets
Correlated Fluctuations	Incomplete Insurance (see Freeman 1977, Harris and Holmstrom 1982, Holmstrom 1983, and Weiss 1984)	Complete Insurance through a Combination of the Firm and the Capital Market (see Weiss 1984)
Uncorrelated Fluctuations	Complete Insurance through the Firm	Complete Insurance either through the Firm or through Self-Insurance

Table 2: Low Mobility Costs

can be offered. Therefore, the only issue which remains for this case is who provides the insurance. If either fluctuations are correlated or workers have no access to capital markets, then insurance is provided through the firm by a wage stream which does not depend on realized productivity.[4] If, however, workers have access to perfect capital markets and fluctuations are uncorrelated, then it is possible for workers to provide self-insurance.[5] The logic here is that even in the face of an income which varies over time, workers can obtain an optimal consumption stream by intertemporally shifting income through the capital market.

The more interesting set of cases is where mobility costs are low. This set is depicted in Table 2. Let us begin by considering the two lower boxes. As is clear from a comparison of Tables 1 and 2, if the fluctuations are uncorrelated, then it does not matter what the mobility costs are. The intuition here is straightforward. With uncorrelated fluctuations a high output in one period does not provide any indication of high output in subsequent periods. Thus, if the initial employer offers a full insurance contract, other firms will never have an incentive to bid workers away.

Now consider the top right box, i.e., correlated fluctuations under a perfect capital market assumption. Here the low mobility cost assumption has an effect, although it is relatively minor. To see the effect consider the model of Weiss (1984) where worker output grows in a stochastic fashion. Because of low mobility costs the firm can no longer offer complete insurance by simply offering a wage which does not vary over time or with worker output. The reason is that if there is a positive realization of a worker's output in an early period, then other firms will have an incentive to bid that worker away in subsequent periods. What the firm does instead is offer a sharply upward sloping age earnings profile, but one which still does not vary with worker output. The logic is that by tilting the profile sufficiently, later wages can be made high enough such that other firms will never have an incentive to bid a worker away. Complete insurance then results because access to a perfect capital market means that workers can obtain an optimal consumption stream by intertemporally shifting income through the capital market.

Thus, the only major effect of the low mobility cost assumption occurs when there are correlated fluctuations and workers have no access to capital markets. In this case complete insurance is no longer provided. The logic is that the low mobility cost assumption again causes firms to have an incentive to offer upward sloping age earnings profiles, however, because workers have no access to capital markets this is not consistent with complete insurance.

What we find quite interesting is the specific form of the contract which typically results in this case. Consider the models of Freeman (1977), Harris and Holmstrom (1982), and Weiss (1984).[6] In each model, if a sufficiently positive fluctuation occurs, then the worker's wage is increased just enough to stop the worker from being bid away by another firm. For any other realization of the worker's output, however, there is no change in the wage. To us this seems counter to the idea that a major role of the labor market contract is the provision of insurance to workers. It is as if workers faced a lottery, and the firm provided the following prizes. If a worker wins the lottery, then the firm promises to increase his wage. If, however, a worker loses the lottery, then the firm's promise is only to not cut the worker's wage. If any, this is a very weak type of insurance.

Notice, if we simply employed the other standard capital market assumption, i.e., a perfect capital market assumption, we still don't get a result which seems a particularly plausible description of reality. Under that assumption everyone gets a wage increase, but the problem is that everyone gets the same wage increase. In real world settings some attention is usually paid to realized output.[7]

What we demonstrate in the following section is that if workers are allowed access to capital markets, but not perfect capital markets, then there will be incomplete insurance, however, it will now be of a much more plausible type. It is still the case that if a sufficiently positive fluctuation occurs, then the worker's wage is increased just enough to stop the worker from being bid away by another firm. For other realizations of the worker's output, however, there will now be a wage increase, although one smaller than the increases provided under the very positive fluctuation. That is, the insurance takes the form of a wage increase even for the case where the worker loses the lottery.

III. Analysis

The model we use to demonstrate our results is a simplified version of Harris and Holmstrom (1982). To begin we state the assumptions that constitute our model.

Assumptions

1) Within the economy there is only one good produced and the price of this good is normalized to one.
2) Workers live for two periods, and in each period labor supply is perfectly inelastic and fixed at one unit for each worker.
3) Workers display no disutility for effort. However, each worker has associated with him or her a value for a variable which will be called ability, and which will be denoted by A.

4) A worker's output at a firm simply equals the value of his ability.

5) Previous to his first period of employment a worker's ability is unknown both to the worker and to all the firms in the economy. However, a worker's output in every period is public information, which in turn yields that after a single period of employment a worker's ability becomes public knowledge.

6) Each worker's value for A is a draw from a random variable which equals A^H with probability p, and equals A^L with probability $(1-p)$, where $A^H > A^L$.

7) A worker's preferences over the consumption stream (c_1,c_2) are given by

$$U(c_1,c_2) = \mu(c_1) + \beta\mu(c_2),$$

where $\mu' > 0$, $\mu'' < 0$, and $\beta < 1$. This simply states that workers are risk averse with a discount factor equal to β.

8) Firms are risk neutral, where a firm's valuation over the profit stream (c_1,c_2) is given by

$$\Pi(c_1,c_2) = c_1 + \beta c_2.$$

9) In agreeing to a contract a worker cannot irrevocably bind himself to a firm.

10) A worker can change firms after his first period of employment without incurring any costs. However, for expositional simplicity it is assumed that, given equal wage offers prior to his second period of employment, a worker will choose to remain with his first period employer.

11) There is free entry.

Before proceeding to analyze the model, it is necessary to stipulate a contracting environment. It is assumed that firms offer young workers long-term or implicit contracts which specify three wage rates, denoted W_1, W_2^L and W_2^H. These contracts bind the firm in the following ways. First, the firm is obligated to pay a worker accepting the contract the wage W_1 during the worker's first period of employment. Second, the firm is restricted from firing such a worker after the worker's first period of employment. Third, if the worker is revealed to be of low (high) ability, then the firm is obligated to offer the worker the wage $W_2^L(W_2^H)$. Finally, the contract must also satisfy the restriction on wages, $W_2^j > A^j$ for $j = L,H$. This restriction guarantees that second period wages are high enough to stop the worker from being bid away by another firm.[8]

We first analyze our model under the assumption that workers can lend any amount they choose at an interest rate equal to $(1-\beta)/\beta$, but that they are completely restricted from borrowing. Under this capital market assumption, equilibrium is characterized by the wages and consumption levels which solve the following maximization problem. Note, below $c_2^L(c_2^H)$ denotes the second period consumption of a worker who is revealed to be of low (high) ability.

(1) $$\max_{\substack{W_1, W_2^L, W_2^H \\ c_1, c_2^L, c_2^H}} \mu(c_1) + \beta[p\mu(c_2^H) + (1-p)\mu(c_2^L)]$$

$$\text{s.t. } p[A^H - W_1 + \beta(A^H - W_2^H)] + (1-p)[A^L - W_1 + \beta(A^L - W_2^L)] > 0$$

$$W_2^j > A^j \quad \text{for} \quad j = L,H$$

$$(1/\beta)\ (W_1 - c_1) + W_2^j - c_2^j > 0 \quad \text{for} \quad j = L,H$$

$$c_1 < W_1$$

Equation (1) is explained as follows. The objective function simply states that the equilibrium contract will maximize a worker's discounted expected lifetime utility. The first constraint ensures that the discounted expected profits for the firm offering the contract are non-negative. The second constraint is simply our earlier mentioned restriction on wages which guarantees that, after his first period of employment, a worker accepting the contract is not bid away by another firm. The third constraint states that the consumption stream can never exceed what is affordable given the wage stream. The fourth constraint rules out borrowing. The following proposition characterizes the solution to (1). Note, to keep the exposition from becoming bogged down in detail, we have relegated all proofs to an Appendix.

Proposition 1: When workers are completely restricted from borrowing, then

i) $A^L < W_1 = W_2^L < W_2^H = A^H$

ii) $c_1 = W_1$, $c_2^L = W_2^L$, $c_2^H = W_2^H$.

The results in Proposition 1 are consistent with the upper left hand box of Table 2. That is, low mobility costs, no access to capital markets and correlated fluctuations in combination yield that the resulting contract will only provide incomplete insurance. Additionally, the specifics of the contract are exactly as in Freeman (1977), Harris and Holmstrom (1982), and Weiss (1984). If a worker is revealed to be of high ability, then he receives a raise just sufficient to stop the worker form being bid away by another firm. If, however, he is revealed to be of low ability, then his subsequent wage is equal to what he received in the previous period.

We now analyze the model under a perfect capital market assumption. Specifically, workers are allowed to lend and borrow any amount they choose at an interest rate equal to $(1-\beta)/\beta$. Under this capital market assumption equilibrium is characterized by the same maximization problem as previously, except now the last constraint no longer applies. The following proposition characterizes the solution to this new maximization problem.[9]

Proposition 2: When workers have access to a perfect capital market, then

i) $W_1 < W_2^L = W_2^H > A^H$

ii) $c_1 > W_1$, $c_2^L < W_2^L$, $c_2^H < W_2^H$

iii) $c_1 = c_2^L = c_2^H$.

Proposition 2 tells us that, when workers face a perfect capital market, then the outcome is a first best result. That is, workers face no risk because the second period wage received is independent of the ability revealed, while borrowing allows workers to smooth out their consumption stream. This full insurance result is consistent with the top right box of Table 2.

Propositions 1 and 2 illustrate our claim of the previous section. That is, given either no access to capital markets or perfect capital markets, the resulting contract does not seem particularly plausible. In the one case, a very weak form of insurance is observed, while in the other workers revealed to be of low ability receive the same raises as those revealed to be of high ability. For these reasons, we now consider an intermediate capital market assumption. This intermediate assumption is intended to reflect the casual observation that workers face a higher interest rate when they borrow than when they lend. Formally, we assume that workers can lend as much as they choose at the interest rate $(1-\beta)/\beta$. However, if workers choose to borrow then they face the interest rate $((1-\beta)/\beta) + I(c_1-W_1)$, where $I(.)$ is twice continuously differentiable and satisfies the following restrictions: $I(0) = 0$, $I'(0) = 0$ and $I'(x) > 0$ for all $x > 0$. Under this capital market assumption, equilibrium is characterized by the following maximization problem.

$$\max_{\substack{W_1, W_2^L, W_2^H \\ c_1, c_2^L, c_2^H}} \mu(c_1) + \beta[p\mu(c_2^H) + (1-p)\mu(c_2^L)] \tag{2}$$

$$\text{s.t. } p[A^H - W_1 + \beta(A^H - W_2^H)] + (1-p)[A^L - W_1 + \beta(A^L - W_2^L)] \geq 0$$

$$W_2^j \geq A^j \quad \text{for } j = L,H$$

$$[(1/\beta) + I(c_1 - W_1)](W_1 - c_1) + W_2^j - c_2^j \geq 0 \quad \text{for } j = L,H$$

The following proposition characterizes the solution to (2).

Proposition 3: The solution to (2) is characterized by,

i) $W_1 < W_2^L < W_2^H = A^H$

ii) $c_1 > W_1$, $c_2^L < W_2^L$, $c_2^H < W_2^H$

iii) $c_1 = c_2^L < c_2^H$.

Proposition 3 tells us that under our intermediate capital market assumption there is incomplete insurance, but it is of a much more plausible type than what occurs when workers have no access to capital markets. Specifically, it is still the case that if a worker is revealed to be of high ability, then he receives a raise just sufficient to stop the worker from being bid away by another firm. However, if he is revealed to be of low ability, he now receives a wage increase, but one smaller than the increase received by those revealed to be of high ability. That is, the insurance takes the form of a wage increase even for the case where the worker loses the lottery.

The intuition behind these results is as follows. Suppose the worker is revealed to be of high ability. Given no access to capital markets, this immediately translates into a fixed level of utility in period 2. This combined with risk aversion then implies that the worker is best off if he has a flat age earnings profile for the case where he is revealed to be of low ability. Now suppose he has access to capital markets, although not perfect capital markets. By borrowing in period 1 he can now affect the utility he receives in period 2 if he is revealed to be of high ability. He is therefore best off by having an upward sloping age earnings profile even for the case where he is revealed to be of low ability, because in conjunction with borrowing it allows him to shift utility from high ability states of the world to low ability states of the world. Finally, because the imperfect capital market assumption means the worker pays a penalty when he borrows, the wage increase under a realization of low ability winds up being less than the wage increase which results under a realization of high ability.

One interesting perspective which follows from our analysis concerns the specific role of capital market imperfections in the type of insurance that firms provide to workers. When fluctuations are uncorrelated, capital market imperfections serve as an incentive for firms to provide insurance, with the result being that firms always provide complete insurance. Now consider what happens when fluctuations are correlated and mobility costs are low. Here there is an incentive for firms to insure workers even in the absence of capital market imperfections. The result is that rather than serving as an incentive for insurance, capital market imperfections in this case serve as a barrier to insurance. That is, the greater the extent of the capital market imperfection, the less complete will be the insurance that firms provide to workers.

This concludes our analysis. In the following section we relate our results to some of the recent literature on age earnings profiles.

IV. Implications for Age Earnings Profiles

In a widely cited article, Medoff and Abraham (1980) presented evidence concerning the relationship between experience, compensation and productivity among managerial employees. Their conclusion was that for workers in the same job category there seems to be first, a strong positive correlation between experience and compensation, and second, no correlation or a negative correlation between experience and productivity. Similar results have been found by, among others, Dalton and Thompson (1971) and Pascal and Rapping (1972). In particular, Dalton and Thompson found that engineers over the age of thirty five were in general below average in terms of productivity, while at the same time being above average in terms of compensation. On the other hand, Pascal and Rapping found that, even after controlling for productivity differences, there seems to be a positive correlation between experience and compensation for major league baseball players.

The above somewhat paradoxical results have brought forth a host of competing explanations. Examples of explanations which have been put forth to explain these results are those of Salop and Salop (1976), Grossman (1977), and Lazear (1979, 1981). In Salop and Salop workers vary in terms of an innate quit propensity, and firms in turn employ upward sloping age earnings profiles to screen out potential employees who are 'quitters'. Grossman's argument also relies on quitting behavior, but there the crucial factor is that young workers have on average a higher probability of quitting. This tends to lower wages for young workers, because it increases the probability that in the better states of nature the worker will leave the firm. Lazear's argument is one concerned with shirking. That is, by deferring payments firms can increase the penalty associated with being fired, and in this way deter employees from shirking.[10]

A different explanation for the paradox comes out of some of the papers mentioned earlier. Suppose productivity does not depend on experience, and there are low mobility costs. Freeman (1977) and Harris and Holmstrom (1982) demonstrate that, if workers have no access to capital markets, then compensation will be positively related to experience because workers revealed to be of high ability will have their wages bid up over time. More recently Weiss (1984) considered a similar model in the presence of a perfect capital market. The implication of his analysis is that the paradox might occur not only because workers revealed to be of high ability receive raises, but also because workers revealed to be of low ability receive raises. The logic is that the raises for the workers revealed to be of low ability serve as a form of insurance against the uncertainty workers face concerning their own ability.[11] As mentioned earlier, however, the Weiss analysis yields the unappealing property that the raises for the two types of workers are identical.

The analysis of this paper tells us two things concerning wage increases in the absence of productivity increases. First, for workers revealed to be of low ability to receive raises in the absence of productivity increases, it is not necessary that workers face perfect capital markets. Rather, this will be the case as long as workers are not completely restricted from borrowing. Second, the property that workers revealed to be of low ability receive raises can be obtained without having the raises be of the same magnitude as those of workers revealed to be of high ability. Overall, then, consider the paradox that wages grow with experience in the absence of productivity increases, and the explanation that it is somewhat due to the idea that workers are receiving insurance against the uncertainty they face concerning their own ability. The analysis of this paper suggests that this explanation is much more plausible than has previously been thought.

V. Conclusion

The insurance aspects of employer-employee attachments has been one of the major focuses of the burgeoning literature on labor market contracts. In this literature it has, in general, either been assumed that workers have no access to capital markets or that workers face perfect capital markets. In this paper we have argued that if productivity fluctuations are correlated over time, then both capital market assumptions yield implausible results as regards the type of insurance provided. With workers having no access to capital markets, a very weak form of insurance is provided. In this case it is as though workers faced a lottery in which the winners are promised wage increases, while losers are only promised not to have their wages cut. Alternatively, with perfect capital markets, the insurance provided is implausible in that it involves bad workers receiving the same wage increases as good workers.

In our analysis we demonstrated that if workers are allowed access to capital markets, but not perfect capital markets, then the insurance provided is much more plausible. As before, in response to positive news regarding a worker's future productivity, the worker's wage will be increased. In contrast, however, other realizations of the worker's expected future output now also result in a wage increase, but one smaller than the increase provided under a more positive realization. That is, the insurance now takes the form of a wage increase even for the case where the worker loses the lottery.

Given these results, a natural question is why did the early contract theorists impose such strict capital market assumptions. One potential reason is that the initial contract papers were concerned with situations in which fluctuations were uncorrelated, and in this case the degree of the capital market imperfection is irrelevant. That is, any imperfection in the capital market results in firms providing complete insurance to workers. In contrast, when there are correlated fluctuations, the severity of the capital market imperfection does have an effect. As discussed above, if mobility costs are low, then the more severe the imperfection, the less complete will be the insurance that firms provide to workers. This in turn suggests that in the more recent literature concerning uncorrelated fluctuations, there may now be a role for an intermediate capital market assumption. In particular, the recent literature has allowed asymmetric information, with the result being that the contracts offered no longer yield a first best result. Similar to how the extent of the capital market imperfection affected the divergence from full insurance in the correlated fluctuation case, the extent of the capital market imperfection may very well affect the divergence from a first best result in this recent asymmetric information literature.

Footnotes

[1]This restriction rules out the recent literature concerning implicit contracts under asymmetric information, because that literature assumes firms are somewhat risk averse. See, for example, Azariadis (1983), Chari (1983), Grossman and Hart (1981, 1983b), and Green and Kahn (1983).

[2]There is a vast literature where insurance effects and incentive effects interact. Some of the more important references in this literature include Green and Stokey (1983), Grossman and Hart (1983a), Lazear and Rosen (1981), and Stiglitz (1974).

[3]If mobility costs are low, then with uncorrelated fluctuations it is also important to know whether any information concerning the fluctuation is revealed prior to the fluctuation, or is all information revealed ex post. In the taxonomy we restrict ourselves to the case where all information is revealed ex post. If some information is revealed ex ante, then the results are very similar to what occurs when there are correlated fluctuations.

[4]Some of our discussion is only precise for the case where workers' utility functions display separability between labor and leisure. Assuming the alternative, however, has no bearing on whether there is complete or incomplete insurance, and who provides the insurance.

[5]This statement is only precise for the case where workers are infinitely lived. Otherwise, even uncorrelated fluctuations can have an effect on a worker's 'permanent' income.

[6]One paper which best fits into this category, but which is not consistent with the following discussion is Waldman (1984). The reason the results of that paper differ is that, as opposed to the other papers, that paper has an asymmetry between firms. Specifically, after a period of

employment the initial employer gets to observe ability, while other firms only get to observe the subsequent job assignment.

[7]It is obvious that one way to get a more plausible contract is by adding incentive effects. However, we want to demonstrate that incentive effects are not necessary to get a more plausible contract.

[8]If for some realization of the worker's ability the second period wage did not satisfy the restriction, then the worker would be bid away and in terms of worker utility and firm profits it would be as if the restriction was satisfied as an equality. Thus, following Harris and Holmstrom, we simply assume that the contract always satisfies the restriction.

[9]There are multiple wage profiles which solve this new maximization problem. In Proposition 2 we simply present properties which all such wage profiles exhibit.

[10]In an earlier paper on the economics of law enforcement, Becker and Stigler (1974) make a similar point.

[11]Medoff and Abraham (1980) had previously suggested this explanation, but their discussion contained no reference to the relevance of the capital market.

Appendix

Proof of Proposition 1: The optimality conditions from (1) reduce to:

(A1) $$\mu'(c_1) = p\mu'(c_2^H) + (1-p)\ \mu'(c_2^L) + \lambda_1$$

(A2) $$\lambda_2 = \mu'(c_1) - \mu'(c_2^H)$$

(A3) $$\lambda_3 = \mu'(c_1) - \mu'(c_2^L)$$

(A4) $$\lambda_2[\beta p(W_2^H - A^H)] = 0, \quad \lambda_2 \geq 0$$

(A5) $$\lambda_3[\beta(1-p)\ (W_2^L - A^L)] = 0, \quad \lambda_3 \geq 0$$

(A6) $$\lambda_1\ [W_1 - c_1] = 0, \quad \lambda_1 \geq 0$$

and the constraints in (1), (where λ_1, λ_2 and λ_3 are the Kuhn-Tucker multipliers associated with the constraints $W_1 \geq c_1$, $W_2^H \geq A^H$, and $W_2^L \geq A^L$ respectively).

To prove the proposition, first note that at the optimum both the worker's budget constraint and the firm's non-negative profit constraint must hold as equalities, while (A2) and (A3) imply $c_2^H \geq c_1$ and $c_2^L \geq c_1$. Suppose $W_1 > c_1$. By (A6), this implies $\lambda_1 = 0$, which given (A1), (A2) and (A3) implies $c_1 = c_2^H = c_2^L$. By the budget constraint and (A4) this implies $W_2^H = W_2^L \geq A^H$. By the expected profit constraint this implies $W_1 \leq W_2^H = W_2^L$, which in turn yields the contradiction $c_1 > W_1$. Hence $W_1 = c_1$. Given this, the budget constraint yields $W_2^H = c_2^H$ and $W_2^L = c_2^L$.

Now suppose that $c_2^L > c_1$. The above and (A3) now imply $c_2^L = W_2^L = A^L > W_1$. By the expected profit constraint this implies $W_2^H > A^H$, which given (A2) and (A4) yields $c_1 = c_2^H > c_2^L$. This involves a contradiction, and hence we have $W_1 = W_2^L = c_1 = c_2^L$.

Finally, suppose that $W_1 = W_2^L = A^L$. The expected profit constraint now implies $W_2^H > A^H$, which given the arguments above yields the contradiction $c_1 = c_2^H > c_2^L$. Hence, we have $W_1 = W_2^L > A^L$, which given the expected profit constraint yields $W_2^H = A^H > W_1 = W_2^L > A^L$. This completes the proof of the proposition.

Proof of Proposition 2: The optimality conditions in this case reduce to (A1)-(A5) (where $\lambda_1 = 0$) and the constraints in (1) (excluding $W_1 \geq c_1$). As previously, the worker's budget constraint and the firm's non-negative profit constraint must hold as equalities. Now, to prove the proposition, we will first prove that $c_1 = c_2^H = c_2^L$. By (A1), observe that if $c_2^H \neq c_2^L$, then either $c_2^H > c_1 > c_2^L$ or $c_2^L > c_1 > c_2^H$. Using (A2) and (A3) both of these yield contractions. Hence, $c_2^H = c_2^L$. By (A1), this implies $c_1 = c_2^H = c_2^L$, which, given (A4), (A5) and the budget constraints, yields $W_2^H = W_2^L \geq A^H$. Given this, the zero expected profit constraint implies $W_1 < A^H \leq W_2^H = W_2^L$. Since $c_1 = c_2^H = c_2^L$ and $W_1 < W_2^H = W_2^L$, this implies by the budget constraints that $c_1 > W_1$, $c_2^H < W_2^H$ and $c_2^L < W_2^L$. This completes the proof of the proposition.

Proof of Proposition 3: The optimality conditions from (2) reduce to (A2)-(A5),

$$\text{(A6)} \qquad \mu'(c_1) = [p\mu'(c_2^H) + (1-p)\,\mu'(c_2^L)]\;[1 + \beta[I + I'\;(c_1-W_1)]],$$

and the constraints in (2). As previously, the worker's budget constraint and the firm's non-negative expected profit constraint must hold as equalities. We first prove that $W_2^H > W_2^L > W_1$. Suppose $W_2^L \geq W_2^H$. Given (A3) and

$W_2^H > A^H > A^L$, this implies $c_1 = c_2^L$. Note, as well, that $W_2^L > W_2^H$ implies $c_2^L > c_2^H$ by the budget constraints. Since $c_1 = c_2^L > c_2^H$ and $c_2^H > c_1$, this implies $c_1 = c_2^H = c_2^L$. The zero expected profit constraint implies $W_1 < A^H$ when $W_2^L > W_2^H$. Hence, with $c_1 = c_2^H = c_2^L$ and $W_2^L > W_2^H > W_1$, by the budget constraints, $c_1 > W_1$. Yet, if $c_1 = c_2^H = c_2^L$ and $c_1 > W_1$, then the optimality condition (A6) is violated. Thus, $W_2^L > W_2^H$ yields a contradiction. It can similarly be shown that $W_2^L < W_1$ yields a contradiction. Hence, we have $W_2^H > W_2^L > W_1$.

Next, we prove that $c_1 = c_2^L < c_2^H$. First, since $W_2^H > W_2^L$, (A2), (A3) and the budget constraints yield $c_2^H > c_2^L > c_1$. Suppose $c_2^L > c_1$. Then by (A3) and (A5), $W_2^L = A^L$. Since $c_2^H > c_1$, we have by (A2) and (A4), $W_2^H = A^H$. By the zero expected profit constraint this implies $W_1 = pA^H + (1-p)A^L > A^L = W_2^L$. This contradicts $W_2^L > W_1$. Hence, $c_2^L = c_1$. Note, as well, that $c_2^H > c_1$ implies by (A2) and (A4) that $W_2^H = A^H$. Given this, by the arguments above, $W_2^L > A^L$, since otherwise we have a contradiction. Taken together, we have $W_1 < W_2^L < W_2^H = A^H$ and $c_1 = c_2^L < c_2^H$. By the budget constraint, this implies $c_1 > W_1$, $c_2^L < W_2^L$ and $c_2^H < W_2^H$. This completes the proof of the proposition.

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