

PATHS OF DEVELOPMENT IN THE  $3 \times n$  GENERAL EQUILIBRIUM MODEL

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If the Heckscher-Ohlin-Samuelson model with  $n$  commodities and  $n$  factors of production is used as a guide, the development process induced by capital accumulation has no dramatic effects on either the composition of output or the returns to factors, since factor prices are independent of factor supplies and since outputs of commodities are simple linear functions of factor supplies. The most frequently discussed alternative that allows graphical illustration of multiple cones of diversification is the uneven  $2 \times n$  model with two factors and  $n$  goods. Countries with endowment vectors in different cones of diversification are shown to have different factor prices and to produce different subsets of the goods. Thus capital accumulation leads to increases in wage rates and to an increasingly capital intensive mix of commodities produced. Though this is a great improvement over the even model which basically asserts that there is no such thing as a development process, the  $2 \times n$  model remains limited since it implies that there is only one path of development. All capital poor countries produce the same pair of commodities and have the same wages, all moderately capital rich countries produce the same pair of commodities and have the same wages, etc. The  $3 \times n$  model is the smallest model that is rich enough to allow more than one path of development. Larger models allow more diverse and more complex paths, but otherwise are essentially the same as the  $3 \times n$  model. Moreover, the  $3 \times n$  model is the smallest that an economist can comfortably maintain at an

empirical level since it is minimally necessary to distinguish the three factors: land, labor and capital, at least until it has been established that either labor or capital is sufficiently mobile that it is not a source of comparative advantage.

The  $2 \times 2$  model is commonly discussed in textbooks presumably because it lends itself to clear graphical presentation and because it allows neat conclusions such as the Stolper-Samuelson theorem: an increase in the price of a good leads to an increase in the real return to the factor used intensively in its production and to a reduction in the return to the other factor. The  $3 \times n$  is not thought to admit a clear graphical presentation and, if it is discussed at all, it is only to point out that the Stolper-Samuelson pairing of goods and factors does not easily extend to higher dimensions. The first function of this paper is to illustrate graphically the main features of the  $3 \times n$  model, including the determination of the signs of the Stolper-Samuelson effects (the derivatives of factor returns with respect to product prices).

Second, a cross-national data set is explored for evidence of the alternative paths of development suggested by the  $3 \times n$  model. In the  $2 \times n$  model, industry output divided by the total labor force depends only on the capital-labor endowment ratio and is positive only for those countries with capital-labor endowment ratios similar to the capital-labor input ratio of the industry. A third factor, land, opens up the possibility of many different paths of development. For example, countries scarce in land may become involved in manufacturing at much lower levels of capital per man than countries more abundant in land, and countries that are very abundant in land may never produce manufactures at all.

The data from the U.N. Growth of World Industry reveal clearly that the composition of output changes as capital accumulates relative to labor. The shift out of labor intensive manufactures and into capital intensive manufactures at higher levels of capital abundance is quite evident. It is more difficult to discern the influence of land on the development paths, which may not be surprising in light of the substantial conceptual and measurement difficulties that this type of study must surmount. Nonetheless, for several commodity groups the data do hint at the existence of the kind of development paths suggested by the  $3 \times n$  model.

#### 1. A $3 \times 4$ Model

A model with three factors and four goods is a useful starting point. The model is based on the following assumptions:

- 1) Commodities are produced with identical fixed-input technologies in all countries.
- 2) Factors of production move costlessly between industries within a country but are completely immobile internationally.
- 3) Commodities are freely traded with no transport costs or other trade impediments. Thus all countries face the same commodity prices.
- 4) There are two manufactured goods, both of which use capital and labor but no land as inputs.
- 5) One of the agricultural goods is very labor intensive, which is approximated by the assumption that it uses labor and land, but no capital as inputs.
- 6) The other agricultural good uses all three inputs. (Think of this as a capital intensive technique for agricultural production.)

### 1.1 Endowment Triangle

The allocation of factors to the production of commodities and the choice of factor returns in this three-factor four-good model seem mathematically complex but in fact can be well understood by appeal to a graph which is now to be discussed. Let the vectors of input combinations corresponding to the four commodities be represented by four activity vectors in a three dimensional figure with axes measuring the inputs of the three factors: land (T), labor (L) and capital (K). This three dimensional diagram can be collapsed into two dimensions by intersecting it with a plane defined by  $K + L + T = 1$ . The intersection of the positive orthant and this plane is an endowment triangle depicted in Figure 1 with the activity vectors in the three dimensional space represented by activity points in the interior of the endowment triangle. The corner labeled T in this triangle corresponds to the land axis and represents an activity vector with zero values for K and L. The straight line connecting T to L represents the plane in which K is zero. The point labeled  $A_1$  thus represents the input vector of the first agricultural product which uses land and labor but no capital. Similarly, the points labeled  $M_1$  and  $M_2$  on the line connecting L to K represent the input combinations of the two manufactured products. In the middle of this triangle is the point labeled  $A_2$  representing the second agricultural commodity which uses all three factors as inputs.

A fundamental feature of this endowment triangle is that the ratio of two factors is constant on a line through the vertex corresponding to the third factor. Therefore straight lines emanating from the vertexes order the input ratios. For example, the line segment connecting the activity point  $A_2$  to the land corner T represents linear combinations of the input vector corresponding to commodity  $A_2$  and the "dummy" input vector with positive T and

zero L and K. When this dummy vector is added to the  $A_2$  vector the input of T changes but both K and L are held fixed. Therefore the ratio of K to L is constant along this line. If extended, this line segment would intersect the line connecting L and K between the points  $M_1$  and  $M_2$ . This means that the capital per worker of the second agricultural good falls between the capital per worker of the two manufactured goods. Likewise, if the line segment connecting K to  $A_2$  is extended to the line segment connecting L to T, it intersects between the point  $A_1$  and T. This means that the second agricultural commodity has a higher ratio of land per worker than the first agricultural commodity, as would be expected if capital replaces workers.

### 1.2 Triangle of Diversification

The seven points in Figure 1 representing the four input vectors and the three axes are connected by line segments to divide the endowment triangle into triangles of diversification. Countries with endowment vectors in a given triangle of diversification have the same factor prices and produce the three commodities with input vectors represented by the vertices of the triangle. This division is logically founded on two results in the linear programming literature, e.g., Simonnard (1966). The programming problem in standard form implicitly considered here is to maximize the value of output  $p'x$  subject to the constraints  $Cx = v$ ,  $x > 0$ , where  $x$  is a  $7 \times 1$  vector of outputs,  $p$  is a  $7 \times 1$  vector of prices with zeroes in the last three elements, and where the first four columns of the  $(3 \times 7)$  matrix  $C$  are equal to the vectors of inputs required to produce a unit of output of the four commodities and the last three columns are the "dummy" input vectors representing activities that use one of the inputs but produce no outputs. The last three elements of  $x$  therefore select "slack" or "unemployed" factors.

A basic solution to this programming problem can be written as  $x^B = B^{-1}v$  where  $B$  is a  $3 \times 3$  matrix with columns extracted from  $C$  and where the other components of  $x$  are set to zero. In words, a basic solution expresses the endowment vector  $v$  as a linear combination of three of the columns of  $C$ . The division of Figure 1 into regions of specialization rests on the following results from the linear programming literature:

- 1) If a linear programming problem has at least one finite optimal solution, then it has at least one optimal basic solution.
- 2) If the endowment vector is perturbed from  $v$  to  $v^*$  and if  $x^B = B^{-1}v$  is a solution to the old problem then  $x^{*B} = B^{-1}v^*$  is a solution to the new problem, provided  $x^{*B} > 0$ .

These results imply that the endowment triangle depicted in Figure 1 can be divided into triangles of diversification with vertices at the activity points or at the dummy activity points (the vertices of the endowment triangle). This is the case because, first, a solution to the problem of allocating factors to activities divides the given factor supply among three of the activities or dummy activities (result (1)), and, second, given one such solution, the solution to a new problem implied by a new endowment vector involves the same three activities, provided the new endowment vector can be expressed as a positive combination of three activities, that is provided that the new endowment vector is in the same triangle of diversification (result (2)). The one other result in the linear programming literature that is useful here is the complementary slackness condition, which for our purposes can be said to imply that a factor rental rate is equal to zero if and only if the factor is in excess supply, that is if and only if the matrix  $B$  of the basic solution includes the dummy input corresponding to this factor. This implies that a country with an endowment vector in a triangle of

diversification with vertex equal to one of the vertices of the endowment triangle must have the corresponding factor return equal to zero.

These results are not sufficient to determine the exact division of the figure into regions of specialization and many alternatives will be discussed subsequently. One example is given in Figure 1. If a country has an endowment vector in the region labeled 1, she divides her factors between the three activities:  $A_1, M_1$  and the dummy activity corresponding to the labor axis  $L$ . Thus the country specializes in the production of the labor intensive agricultural good and the labor intensive manufacturing good. In this region, labor is a redundant factor with a zero wage.

### 1.3 Division of the Endowment Triangle

Figure 1 illustrates but one of several ways that the endowment triangle can be subdivided into regions of specialization by connecting the activity points and the dummy activity points with line segments. There are a considerable number of choices that have to be made concerning which line segments are drawn first and which are thereby excluded. The optimal choice of these regions of specialization depends on commodity prices to such a degree that regardless of the division that is hypothesized, there are commodity prices that would imply it. This is most easily discussed in the simpler case with two factors and two commodities. Unit value isoquants for the two commodities are depicted in Figure 2. As drawn both commodities are economically produced, provided the endowment vector is suitably selected. As drawn, there are three intervals of capital abundance ( $K/L$ ) selecting three different combinations of outputs. For middle values of  $K/L$  both commodities are produced, and for extreme values of  $K/L$  only one of the commodities is produced. A figure that is analogous to the endowment triangle in Figure 1 is an endowment line segment with end points representing the two axes (Figure



2b). Now suppose that the price of commodity one increases, thereby shifting inward the unit value isoquant towards the origin. Eventually, this isoquant gets so close to the origin that it is everywhere below the unit value isoquant for commodity two. Then it is uneconomical to produce commodity two regardless of the factor endowments. The corresponding adjustment of the endowment line segment is the complete elimination of the activity point representing commodity two, leaving only two intervals of diversification, both involving the production of commodity one.

Thus in the two factor case, the regions of specialization are divisions of the endowment line segment into intervals with endpoints representing the two commodities and the two axes. Which of these points are selected as endpoints for the intervals depends on commodity prices. Likewise, the endowment triangle for the three factor case is divided depending on commodity prices. Anthropomorphically speaking, commodities with relatively high prices insist on being connected by line segments. This can be made more explicit by considering an endowment vector  $v$  which lies in one of two different triangles, such as the one illustrated in Figure 3. As drawn,  $v_0$  can be expressed as a positive combination of the first three input vectors or the first two and the fourth. Thus if the first three input vectors form the columns of the matrix  $A$ , then the output vector  $x = A^{-1}v_0$  is a positive vector. And if the first two input vectors and the third form the matrix  $A^*$ , then the output vector  $x^* = A^{*-1}v_0$  is also a positive vector. The issue at hand is whether  $x$  or  $x^*$  is the more valuable output vector. Let  $p$  be the  $3 \times 1$  vector with prices of the first three commodities as elements and let  $p^*$  be a vector with the first two elements equal to the first two elements of  $p$  but with the last element equal to the price of commodity four. Then the first vector of outputs is more valuable than the second if  $p'x >$

$p^*x^*$ . The vector of outputs  $x$  can be written as  $x = A^{-1}A^*x^*$  since  $A^*x^* = v_0$ . The matrix  $A^{-1}A^*$  has  $(1,0,0)'$  and  $(0,1,0)'$  as the first two columns because  $A^{-1}A$  is the identity matrix. The third column of  $A^{-1}A^*$  is  $A^{-1}A_4$ . Therefore

$$\begin{aligned} p'x &= p_1x_1^* + p_2x_2^* + p'A^{-1}A_4x_3^* \\ &= p^*x^* + (p'A^{-1}A_4 - p_4)x_3^*. \end{aligned}$$

Thus the solution  $x$  is more valuable than the solution  $x^*$  if the term in parentheses is positive. This term is necessarily positive if  $p_4$  is zero and if the fourth input vector is in the triangle formed by the first three input vectors, that is if  $A^{-1}A_4$  is positive. But if  $A_4$  is not in this triangle, there will always be prices  $p$  at which it is better to allocate factors to the fourth activity than to the third, even if the fourth has a zero price. The prices that are required depend on the signs and magnitudes of  $A^{-1}A_4$ . The sign pattern of this vector is also illustrated in Figure 3 where the line segments connecting the first three input vectors have been extended to divide the space into the seven feasible orthants corresponding to the signs of the vector  $A^{-1}v$  with  $A$  composed of the first three activity vectors. The logic for this division rests on the observation that a vector lying on a line through two activity points can be expressed as a linear combination of the two input vectors that define the line, and the third with a zero coefficient. Thus the solution  $x$  to the equation  $Ax = v$  for  $v$  on such a line has a zero value for one component of  $x$ . If  $v$  is selected on one side of this line, that component of  $x$  must be positive, and if  $v$  is selected on the other side, that component must be negative. Any  $v$  in the triangle formed by the first three activity vector can be expressed as a positive combination of the three vectors, which means that  $A^{-1}v$  is a

positive vector. The rest of the figure can be filled in by suitably switching signs of components as lines are crossed.

The location of the fourth input vector in Figure 3 selects a vector with signs  $(-,+,+)$  for  $A^{-1}A_4$ . Thus the term in parentheses in the expression above has positive coefficients on  $p_2$  and  $p_3$ , and has negative coefficients on  $p_1$  and  $p_4$ . What this implies is that the division into regions of specialization will be as depicted if commodities 2 and 3 are high in price, but it is better to connect the first and fourth activity vectors with a line segment and always to produce these two commodities if their corresponding prices are high enough.

Three other ways of dividing the endowment triangle into regions of specialization are illustrated in Figure 4. In Figure 4a commodity  $M_2$  is very expensive and its activity point is connected to all the other possibilities. In this case, there would be one more step in the development process in which only  $M_2$  is produced and both  $K$  and  $T$  are redundant factors, as in region 6 in Figure 1. In Figure 1, commodity  $A_2$  is assumed to be very expensive and its activity point is connected to all other activity points. The two other cases correspond to situations in which  $M_1$  is very cheap but  $A_2$  is dear compared to  $M_2$  (Figure 4b) and in which  $A_1$  is cheap but  $M_2$  dear relative to  $A_2$  (Figure 4c).

If many countries are accumulating capital, the consequent changes in the supplies of commodities can be expected to cause price changes which may be sufficient to alter the division of the endowment triangle into triangles of diversification. Suppose for example that there are few countries that have much capital and as a result  $M_2$  is very dear. Then Figure 4a can be expected to apply. If several other countries accumulate enough capital to exit the triangle at the lower left, they begin to produce  $M_2$ . This may drive down

the price of  $M_2$  and Figure 4c may emerge with the same developing countries now better off producing the capital intensive agricultural commodity.

#### 1.4 Rybczynski and Stolper/Samuelson Effects

These same diagrams may be used to determine the signs of the derivatives of outputs with respect to the endowments, which because of the Samuelson reciprocity relationships are equal to the derivatives of factor returns with respect to product prices. A basic result of trade theory is that these derivatives are constant within a given region of specialization. Within a triangle of diversification the vector of derivatives of outputs with respect to factor supplies is  $A^{-1}(dv)$  where  $A$  is a square matrix with columns equal to the activity sectors that define the triangles. We have previously explained that Figure 3 indicates the sign of the vector  $A^{-1}v$ . The same figure can, of course, be used to determine the signs of  $A^{-1}(dv)$ . Suppose to  $v_0$  we add the endowment vector  $cv_1$  with  $c$  chosen small enough that  $v_0 + cv_1$  is in the same triangle of diversification as  $v_0$ . The resulting change in outputs,  $cA^{-1}v_1$ , has sign pattern  $(-,+,+)$  for reasons described above.

Likewise, to determine the effect of an increase in capital, we need only determine in which of the six orthants the capital dummy activity vector lies. Figure 3 is placed within an endowment triangle in Figure 5a. The capital vertex  $K$  falls in the region with signs  $(-,+,-)$ , which means that accumulation of capital causes increases in output of  $A_2$  and reductions of output of  $A_1$  and  $A_3$ . This seems quite sensible because the vector  $A_2$  is "closest" to the capital vertex and is therefore the most capital intensive product. In fact, in Figure 5a, there is a one-to-one association of factors and commodities:  $K$  with  $A_2$ ,  $L$  with  $A_1$  and  $T$  with  $A_3$ . But in Figure 5b,  $A_2$  has been moved on a line toward the land vertex  $T$ , and the  $K$  vertex selects a different sign pattern. Here, capital accumulation leads to

increases in both  $A_1$  and  $A_2$ . The economic interpretation is that the new capital has to be employed in such a way that the demands for land and labor are maintained. Because the land per man is higher in  $A_2$  than in  $A_3$  (can you see this in the Figure?), increases in  $A_2$  at the expense of  $A_3$  will leave unemployed laborers who are absorbed into the production of  $A_1$ , the labor intensive product.

The graphs therefore nicely illustrate the extension of the Stolper-Samuelson theorem and the Rybczynski theorem to the  $3 \times 3$  model, particularly the difficulty of associating a factor with a commodity. I leave to the reader graphical verification of the following results:

- 1) Each factor has at least one commodity friend and one commodity enemy in the sense that an increase in the product price either increases (friend) or decreases (enemy) the return to the factor (Ethier, 1974).
- 2) If a commodity is more intensive in a factor relative to both of the other factors, then an increase in the factor leads to an increase in the output of the commodity. (Note: one is not enough.)
- 3) A necessary and sufficient condition for the derivative of  $A_2$  with respect to capital to be positive is that  $A_2$  is more capital intensive than  $A_1$  and  $A_3$  jointly in the sense that the  $K$  vertex and  $A_2$  lie on the same side of the line connecting  $A_1$  and  $A_3$ .

This logic may be used to determine the signs of the Rybczynski effects in each of the regions in Figure 1. Consider, for example, region 3 and imagine that the line segments connecting  $A_2$ ,  $M_1$  and  $M_2$  are extended and the seven "orthants" that these lines delineate are labelled as in Figure 3. In this case the capital vertex lies in the orthant in which the  $M_1$  component is negative, the  $M_2$  component is positive, and the  $A_2$  component is zero. Therefore capital accumulation in this region induces no change in the

output of the agricultural good, increases output of the capital intensive manufacture and reduces output of the labor intensive manufacture. If this strikes you initially as a bit doubtful, remember that land is fully devoted to the agricultural commodity, and since land is held fixed, there can be no change in its output.

The same logic leads to the conclusion that an increase in land, beginning in region 3 in Figure 1, increases output of the agricultural good and reduces output of both manufactures. Note, however, that if the point  $M_2$  were to shift to the left beyond the extension of the line segment connecting T to  $A_2$ , that is if  $M_2$  were to become less capital intensive than the agricultural good, then increases in land would increase, not decrease,  $M_1$ . The reason for this is that the expansion of the agricultural good absorbs relatively more capital than labor in comparison with either of the manufactures; thus the extra labor that is released is employed in the labor-intensive manufacturing sector.

This logic leads to the signs of the Rybczynski derivatives reported in Table 2. Because of the Samuelson reciprocity relations these are the same as the derivatives of factor prices with respect to commodity prices. This implies, for example, that in region 2 a tax on  $A_1$  or  $M_1$  would raise the wage of labor, and a tax on  $A_2$  would lower the wage of labor. If the two agricultural activities represented two different techniques for producing the same commodity, the effect of raising the price of the agricultural good on the wage would be ambiguous because of the counteracting forces. Generally speaking, labor should favor measures to raise the price of the labor intensive agricultural and manufactured products. The effect of price increases of the other two commodities on the wage rate depend on the region of specialization. The interests of capitalists are somewhat less sensitive to

the region of specialization. The return on capital can never fall in response to a price increase of the capital intensive agricultural or manufactured good or in response to a price reduction of the labor intensive agricultural good. But the sign of the effect of a price increase of the labor-intensive manufacture on the return to capital is ambiguous -- positive when this is one of the more capital intensive goods produced but negative when  $M_2$  is produced. Land, however, has completely unambiguous interests -- favoring price increases for agricultural goods and opposing price increases for manufactures.

### 1.5 Paths of Development

The six regions of specialization implied by the division of the endowment triangle illustrated in Figure 1 are listed in Table 1 together with the commodities produced and the redundant factors. The three arrows in Figure 1 represent three different development paths through these regions corresponding to three different initial starting values for the relative supplies of land and labor. Countries which are very abundant in labor relative to land begin the journey up the ladder of development in region 1 with very low wage rates, and with production of the labor intensive agricultural good and the labor intensive manufactured good. With the accumulation of capital, such countries first produce the capital intensive agricultural good (region 2), then substitute the capital intensive manufactured good for the labor intensive agricultural good (region 3) and finally cease production of the labor intensive manufacture (region 4). This rather interesting sequence of events seems to portray the shift out of agriculture that actually occurs, with the least developed countries concentrating on the labor intensive products and drawing labor out of agriculture and eventually out of labor intensive manufactures as capital accumulates. The final rung on this ladder

of development has production of the capital intensive manufactured commodity and the capital intensive agricultural commodity. This seems to be a fairly accurate description of the United States. There are other paths of development implied by other starting values for the relative supplies of labor and land. Countries with intermediate endowments of land relative to labor begin in region 2 where both the agricultural products are produced. The first step in the development process is the production process is the production of the labor intensive manufacturing product. Thereafter, the process is the same as the first one described. Countries that are very well endowed in land never do produce the manufactured products and evidence development by complete specialization on the capital intensive agricultural product (Canada?).

These three development paths are illustrated in the three panels of Figure 6. Each panel illustrates the level of output of the four industries divided by the total labor force as a function of the level of the capital stock divided by the total labor force. Within a triangle of specialization, outputs are linear functions of the endowments such as

$$Q_j = \beta_K K + \beta_L L + \beta_T T$$

where  $Q_j$  refers to output of product  $j$ ,  $K$ ,  $L$ ,  $T$  refer to endowment supplies, and  $\beta_K$ ,  $\beta_L$ ,  $\beta_T$  are fixed parameters. Dividing this by the labor force produces the function which is graphed in Figure 6.

$$Q_j/L = \beta_K(K/L) + \beta_L + \beta_T(T/L).$$

In each of the three panels in Figure 6 it is assumed that land and labor are fixed, and only capital is varying. Thus  $T/L$  is fixed. The three panels correspond to the three different development paths in Figure 1. The top panel illustrates the effect of capital accumulation in land abundant countries, the second panel in intermediate countries, and the third panel in



land scarce countries.

The land abundant countries produce exclusively agricultural goods, shifting from labor intensive to capital-intensive techniques as capital accumulates. The development path of the land abundant countries are very different from the others. The paths of the land-scarce and intermediate countries are quite similar. The principal difference is that the manufactured goods,  $M_1$  and  $M_2$ , are produced by the land-scarce countries at much lower levels of capital abundance.

Another feature of each of the three panels in Figure 6 is that the sum of the output functions is the GNP per man, provided the outputs are measured in value terms. This per capita GNP function is convex with derivative equal to the marginal return to capital. By a standard construction, the wage rate can be found by extending the line tangent to the per capita GNP function to the vertical axis. The per capita GNP function for the land abundant countries exceeds the function for the intermediate countries which in turn exceeds the per capita GNP function for the land-scarce countries (because land contributes to GNP).

It is worth repeating that there are many other ways that the endowment triangle could be divided and there are consequently many other development paths. For example, if the manufactured products are very dear then both points  $M_1$  and  $M_2$  would be connected to the T axis. Then there would only be subtle differences between the development paths of land abundant and land scarce countries.

It should also be observed that if the technologies allow factor substitution then there would be regions separating the triangles of diversification in which only one or two commodities could be produced. The outputs would be non-linear functions of endowments in these regions.

## 2. Evidence

An implication of the  $3 \times n$  model is that the linear relationships expressing industry outputs as linear functions of the country's resource endowments are different in each triangle of diversification. If the clustering of countries into triangles of diversification were known, separate regressions of outputs on endowments could be estimated for each cluster. But it is clear from the previous section that the clustering of countries depends in a very complex way on factor input intensities, product prices and factor endowments. Even if this information were available, it would be too great a commitment to an abstract model to ignore data on output levels in forming the country clusters. But the use of data on output levels to form country clusters presents a formidable econometric problem which in principle can be solved by considering all possible clusters and by selecting the set of clusters that provide the best fit to the data. Instead, the countries are here merely split into two equal sized groups depending on land abundance and some effort is made to determine the effect of altering the clusters.

It is best to think of this empirical exercise as a test of the  $2 \times n$  model against the  $3 \times n$  model. In the  $2 \times n$  model, industry output divided by the total labor force depends on only the capital-labor endowment ratio and is positive only for those countries with capital-labor endowment ratios similar to the capital-labor input ratio of the industry. A third factor, land, opens up the possibility of many different paths of development. For example, as illustrated in Figure 6, countries scarce in land may become involved in manufacturing at much lower levels of capital per man than countries more abundant in land, and countries that are very abundant in land may never produce manufactures at all. Thus the procedure that will be followed is to inspect graphs in hopes of finding patterns of output like the

ones in Figure 6.

Data for thirty-eight countries on value of output and employment at the 3-digit ISIC level in 1978 were taken from the U.N. publication Yearbook of Industrial Statistics. The endowment data on land, labor and capital were taken from a data set prepared by Harry P. Bowen and myself in connection with research reported in Leamer (1984). There are a number of problems with both of these data sets that ought to be kept in mind. First, the theory discussed in the previous section is properly interpreted to refer to value added not to gross output. Although the U.N. does collect information on value added, the coverage of countries is too sparse to allow an interesting data analysis. The resort to gross output data means in effect that industries have been unfortunately aggregated. For example, the machinery sector includes iron-and-steel value added as well as machinery value added.

Aggregation in general is a problem with the U.N. three-digit ISIC data set. The textiles category (ISIC 321) includes specialty fabrics used for the suits of astronauts as well as coarse hand-made goods. We are hoping to discover some rather subtle features of the functional form of the relationship between outputs and endowments. When two commodities with very different functional forms are aggregated, subtle features will be blurred and may become quite undetectable.

The resource data set also has a number of problems. The land data excludes mountainous regions but otherwise infertile land is included. The labor variable is the total labor force including workers engaged in the service sector. The capital variable is formed by accumulating and discounting gross investment assuming an average asset life of fifteen years. This probably understates the average asset life and as a result overemphasizes recent investment. The figure includes residential capital and capital used

in the service sector. Furthermore, the figure is translated into dollars at the current exchange rate and is consequently excessively sensitive to exchange rate fluctuations. In addition, it includes no human capital, though this is probably unimportant since investments in physical and human capital may be almost proportional, and if they are, the total capital figure would be a constant multiple of the physical capital figure that is used. It is the case that the proportion of workers classified as professional/technical by the ILO is highly correlated with our measure of physical capital per man. Including estimates of the human capital embodied in these workers would therefore have little effect on the subsequent analysis.

The endowment triangle illustrating the resource data on thirty-eight countries is graphed in Figure 7. The ratios of the endowments can be read along the sides of the triangle, and any point on a line connecting the opposite vertex to one of these points has the same endowment ratio. Roughly in the middle of this figure is a point labelled TOT corresponding to the total endowments of the thirty-eight countries. By drawing lines from the vertices of the endowment triangle through the TOT point to the opposite sides, you can determine that the total endowments have a capital/labor ratio of \$12,000 per man, and land/labor ratio of 6 hectares per man and a capital/land ratio of \$2,000 per hectare. To determine how a country's endowment compares with this total endowment point, find a path from TOT to the point representing the country in question formed by vectors pointing in the directions of two of the vertices of the endowment triangle. Remember that as you move toward a vertex you are increasing the supply of the corresponding resource. For example, the point representing the United States can be reached from TOT by first moving toward the capital vertex and then toward the land vertex. Thus the U.S. has more land and more capital than TOT.

There are several interesting clusters of countries in the endowment triangle. Australia and Canada are extremely land abundant but also have a great deal of capital. The cluster of countries including Brazil, Peru, Chile, Colombia, and Panama appears to be close to Australia and Canada. But in fact these countries have much lower levels of capital per man. The scales in the endowment triangle are much more concentrated near the vertices than in the center. In order to determine if a group of countries is closely clustered, it is best to draw pairs of lines from each of the vertices of the endowment triangle to form a quadrangle that surrounds the countries. If this quadrangle is small, it means that the endowment ratios of all the countries are very similar. (Vocabulary note: a quadrangle need not have right angles.)

The most distinct cluster of countries in this sense is made up of the developed countries France, Denmark, Austria, Germany, Belgium/Luxembourg, Japan, Netherlands and Switzerland. The cluster of developed countries formed by Finland, Norway, Sweden and the United States also has high levels of capital per man, but much greater endowments of land. The countries that are scarce in capital relative to labor vary greatly in terms of land abundance with India and Korea very scarce in land relative to labor, and at the other end, Brazil and Peru very abundant.

If the  $3 \times n$  model is taken literally, this endowment triangle can be subdivided into triangles of diversification such that all countries within a triangle of diversification produce the same three commodities and only those three commodities. Moreover, among the manufactured commodities that use only two inputs (capital and labor) the  $m \times n$  model implies that only two are produced, even if  $m \neq 2$ . This seems to conflict sharply with the data which reveal at the three digit ISIC level that every commodity group is produced by

every country. But there are a number of ways in which this conflict can be resolved. First, the data are generated by a dynamic process with substantial adjustment lags. It may well be that output per man of clothing and footwear would peak at capital per man endowment ratios of \$15,000 per man and fall to zero by \$20,000 in the absence of adjustment costs. But the physical and human capital that was invested in these industries when the country in question had lower levels of capital or fewer competitors are not costlessly mobile and may hang on for many years after the full equilibrium level of output is zero. Postponement of adjustment is greatly encouraged by the very prevalent trade protection that is common in these labor intensive industries.

But even if there were no adjustment costs it is a fanciful notion that only two of the multitude of manufactured products would be produced. One reason for this within the context of the simple general equilibrium model is that labor is not a uniform input. By this I do not refer to human capital which in equilibrium can be included in the total capital figure provided only that savings is optimally divided between human and physical capital on the basis of productive value alone. Rather, for genetic reasons, a labor pool will vary greatly in its abilities to perform the many complex manufacturing tasks and in its complementarity with human capital (some people are hard to train). Provided that the labor pools of different countries have roughly the same composition of age-related and genetic abilities, this will imply that many commodities are produced (1 plus the number of labor categories) but otherwise the model is unaffected. Suppose, for example, that there are only two kinds of labor. If capital is the only other input we can form an endowment triangle with vertices representing capital and the two labor inputs. If, furthermore, the ratio of the two labor categories is the same in every country, then the endowment points of every country will fall on a straight

line emanating from the capital vertex. A point on this line can be fully identified from the capital per man endowment ratio, and the single possible development path is thus a function of the capital per man only. Output per man of each of the commodities is a piecewise linear function of the capital per man, just as it is in the  $2 \times n$  model. Each commodity will have two adjacent zones of production, one increasing and one decreasing as a function of capital per man, just as in the  $2 \times n$  model. The only difference is that at each point along this path, three (not two) commodities will be produced.

Yet another possible explanation for the production of more than two manufactures that would leave the foundation for this data analysis intact is the judicious selection of commodity prices leaving more than three products in each cone of diversification that can be economically produced. These prices are actually required for extreme commodities that cannot be formed into a diversification triangle which includes any countries. More generally, triangles of diversification that include countries with insufficient capacity to meet the demands of the corresponding commodities will be merged with adjacent triangles through product price adjustments. In that event, the level of production of each of the commodities by each of the countries is indeterminate. Diversification does seem likely either for reasons of price uncertainty or merely to avoid the costs of trade by making production match consumption as closely as possible.

The evidence in favor of the  $3 \times n$  model is formed by separating the countries by land abundance and by demonstrating that the development paths depend on land abundance. It is not enough merely for the levels of output per man to depend on land abundance, since that would be the case for a model that was everywhere linear in capital, labor and land. Rather, the slopes of the relationships between output per man and capital per man must be shown to

be different for countries with different levels of land abundance. This is a heavy burden for thirty-eight observations to carry, particularly when the capital variable is surely measured with a great deal of error.

I therefore interpret the theory loosely to imply that industry outputs divided by the total labor force are non-linear functions of the capital-per-man endowment ratio. A quadratic functional form is selected for reasons of convenience and also because a quadratic seems likely to be able to approximate the triangle shaped functions implied by the theory and plotted in Figure 6. The hypothesis of special interest is that these quadratics take significantly different forms depending on the availability of land. In order to study this, the data are divided into subsets of land abundant and land scarce countries.

Quadratic regressions that explain industry output divided by the total work force as a function of the capital-per-man endowment ratio are reported in Table 3. The  $R^2$ 's for these regressions are generally respectable, and when they are not it is due to the presence of one or more outliers (Hong-Kong and Malta for apparel, Singapore and Netherlands for petroleum refining). All of the commodities have positive values for the linear term which means that capital accumulation at low values of capital per man increases output per man. All but two of the commodities have negative estimated effects for the squared term. These estimated quadratics are suggestive of the  $2 \times n$  model which has peak values of output per man when the capital per man endowment ratio conforms to the capital per man input ratio for the commodity in question.

These estimated quadratics indicate substantial changes in the composition of output as capital accumulates. A ladder of development can be formed by ordering commodities by the modes in Table 3. At low levels of



capital per man, output is concentrated in those industries with small modes. As capital accumulates, output shifts in favor of those commodities with larger modes. The first commodity group to actually decline is apparel, which has a peak output per man at the endowment ratio of \$21,900 per man. The decline in apparel is followed by leather products, professional goods, and textiles, all commodities that we normally think of as labor intensive. In quite a few cases the estimated peak occurs at values of capital per man in excess of \$50,000 per man which is beyond the range of the data. (Norway is estimated to have the highest capital per man of \$44,000.) If the peak occurs much above \$50,000, the estimated quadratic function has very little curvature in the range of the data. In one case, non-ferrous metal products, the estimated coefficient of the squared term is positive, and the estimated parabola is U-shaped. Other than that, the commodities with the highest modes are, in order, furniture, machinery, food products, and paper products. Although these are the commodities on which output is concentrated in countries with high levels of capital per man, no conclusion regarding comparative advantage can be reached without information about the amount of trade that occurs. Food products, for example, may be effectively a non-traded commodity.

These results have been derived without controlling for the availability of land, and in particular without allowing for the fact that countries with different endowments of land may have very different development paths. In order to explore for this possibility, quadratic regressions estimated with the nineteen land abundant countries and the twenty land scarce countries are reported in Table 4. The third column in this table reports the F-statistic for testing if there is a difference between the land abundant countries and the land scarce countries. The conventional critical value (5%) of an F-statistic with 3 and 32 degrees of freedom is 2.9, and only seven of the

twenty-eight F-statistics exceed this number. (A Bayesian critical value in Leamer (1978) is even higher.) Overall, splitting of the data set does not produce a great deal of evidence that land abundance alters the development path. Nor should it necessarily, since there is no special reason for all countries comprising a data subset to have the same development path.

Though the evidence that land alters the development path is not overwhelming, what evidence is available is very suggestive. To make the effect of land as clear as possible, the commodities in Table 4 are arranged into clusters depending on features of the estimated functions. The commodities in the first two groups all have output per man declining at values of capital per man in the range of the data (up to 44). The effect of land for the first group is to shift the estimated quadratic to the right toward higher values of capital per man. For the second group of commodities, land abundance lowers the function overall, but does not substantially shift the modal capital/labor ratio. The estimated functions for the commodities comprising the third group have peaks in the range of the data for the land scarce but not the land abundant countries. In a sense the effect of land abundance is similar to aggregate 1 in shifting the function to the right. The commodities comprising the fourth cluster exhibit peaks for land abundant countries, but reveal no tendency for output to decline in land scarce countries. Aggregate 5 is composed of two forest related products with regressions that are greatly influenced by the three major producers: Canada, Finland and Sweden. This reveals the inadequacy of our three factor model since it does not allow land to be disaggregated into components that reflect among other things the supply of soft-wood forests. The petroleum products form a similar group, with Singapore and Netherlands as outliers. Food is the category that is most imperceptibly affected by land abundance. Finally, furniture, like

food, shows no tendency to peak at high levels of capital abundance, but land scarcity does raise the level of output per man.

The results reported in Table 4 ought to be viewed with suspicion, first, because the split into two equal size groups of countries is theoretically appropriate only under most unusual circumstances, second, because the functional form suggested by the theory is not quadratic, and, third, because the estimates and t-values may be unduly influenced by one or more outliers which might properly be attributed either to measurement errors or to special circumstances. An examination of scatter plots, Figures 8 to 17, can address each of these problems.

The estimated quadratic function for the land abundant countries is represented on these scatter plots by a solid curve, and the estimated function for the land scarce countries is represented by the dashed curves. Both curves are drawn on both graphs to facilitate comparison. The discussion of these figures focusses first on the sensitivity of the inferences and secondly on the identification of countries that are outliers in many of the figures. The conclusion that emerges is not much different from the conclusion that emerges from a review of Table 4. Namely, there does seem to be clear evidence in some but not all cases that land abundance alters the development path in the kind of subtle way suggested by the theory. These scatter plots do nothing to discourage the opinion that better measures of the resource endowments could substantially improve our understanding of these development paths.

#### Discussion of Scatters

Aggregate 1A: The conclusion that the land scarce countries have early peak values of apparel and ISIC390 is heavily influenced by the extreme countries, Hong Kong and Malta. The curvature of the land abundant function is largely

due to Norway, which has a very high estimated capital per man. Even without these extreme countries, there is pretty clear evidence that the land scarce countries have much higher levels of output per man at low levels of capital abundance, but not at high levels.

Aggregate 1B: The land abundant countries, except for the United States, fall into a fairly tight pattern with only slight evidence of curvature. The land scarce countries have much higher levels of output per man at all levels of capital abundance, and they evidence a clear tendency to have slower rates of output growth at higher levels of capital abundance.

Aggregate 2A: Both graphs suggest similar curvature. The land scarce countries have generally higher levels of output per man. Hong Kong is an extreme outlier with very high levels of output per man of this aggregate composed of textiles, leather products, rubber products and professional goods.

Aggregate 2B: If Norway were omitted, there would not be much evidence of curvature for the land abundant countries, but the curvature seems pretty clear for the land scarce countries. Again it is the case that the land scarce countries have a comparative advantage in this aggregate.

Aggregate 3: Except for the Austrian observation, there doesn't seem to be much difference between land scarce and land abundant countries.

Aggregate 4: The land scarce countries evidence much less curvature and the comparative advantage in favor of the land scarce countries seems clearly to increase with capital abundance.

Aggregate 5: The land abundant graph for this forest products aggregate is heavily influenced by the major producers: Finland, Sweden, Canada, Norway and the United States. Otherwise, all countries seem to be in pretty much the same path with output per man steadily increasing with capital abundance.

Aggregate 6: The petroleum products scatter suggests very little relationship between output per man and capital abundance. Singapore and the Netherlands are extreme outliers.

Aggregate 7: Food manufacturing seems surprisingly unrelated to land availability and grows steadily with capital abundance.

Aggregate 8: Furniture output, like food, is little affected by land abundance and seems to grow steadily with capital abundance.

### Outliers

When a country is an outlier in most of the scatters, the data are suggestive of either a mismeasurement of the capital abundance ratio or a mistaken clustering of countries. Inspection of the figures suggest that there are five major outliers of this form: (1) The United States has much higher levels of output per man than can be explained by capital abundance for almost all the commodities and would be better moved to the land scarce category. (2) Greece, on the other hand, has unexplainably low levels of output in all 10 of the aggregates which could be corrected by substantially reducing the estimate of the capital abundance. (3) Denmark has unexplainably low levels in all but two of the aggregates and the fit could be generally improved by moving Denmark to the land abundant countries. (4) Both Norway and Austria have very high levels of estimated capital abundance and these numbers could be reduced somewhat to improve generally the fit.

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**Table 1**  
**Production Implied By Figure 1**

<b>Region</b>	<b>Produce</b>	<b>Redundant Factor</b>
1	$A_1M_1$	L
2	$A_1M_1A_2$	-
3	$A_2M_1M_2$	-
4	$A_2M_2$	K
5	$A_1A_2$	T
6	$A_2$	T,K

Table 2

Rybczynski Derivatives Corresponding To Figure 1

Region	$\partial./\partial K$				$\partial./\partial L$				$\partial./\partial T$			
	A <sub>1</sub>	A <sub>2</sub>	M <sub>1</sub>	M <sub>2</sub>	A <sub>1</sub>	A <sub>2</sub>	M <sub>1</sub>	M <sub>2</sub>	A <sub>1</sub>	A <sub>2</sub>	M <sub>1</sub>	M <sub>2</sub>
1	0		+		0		0		+		0	
2	-	+	+		+	-	+		+	+	-	
3		0	-	+		0	+	-		+	-	-
4		0		0		+		+		+		-
5	-	+	0	0	+	0	0	0	0	0	0	0
6		0				+				0		



TABLE 3  
 Quadratic Regressions  
 $Q/L = b(0) + b(1)*(K/L) + b(2)*(K/L)**2$

ISIC	t-values		R-SQ	estimated	
	b(1)	b(2)		mode	zero
311 Food	3.2	-1.3	.60	47.6	95.3
313 Beverages	3.8	-2.2	.57	34.8	71.2
314 Tobacco	1.7	-.9	.26	40.0	85.1
321 Textiles	2.9	-2.3	.27	26.1	54.7
322 Apparel	1.5	-1.4	.06	21.9	46.8
323 Leather	4.7	-4.0	.43	23.8	48.9
324 Footwear	2.5	-1.9	.24	27.3	57.5
331 Wood	2.3	-.7	.54	70.8	139.6
332 Furniture	2.8	.0	.76	1857	3716
341 Paper	2.2	-1.0	.42	46.6	90.5
342 Printing	3.5	-.9	.75	77.7	153.7
351 Ind. Chem.	3.5	-1.7	.60	42.2	82.7
352 Other Chem.	5.0	-2.5	.75	40.7	80.7
353 Petro. Refin.	2.4	-1.8	.22	27.1	52.0
354 Petro, coal prod.	2.0	-1.6	.15	26.0	50.2
355 Rubber prod.	3.6	-2.8	.36	26.1	51.3
356 Plastics	3.2	-1.9	.45	33.2	65.9
361 Pottery	3.3	-2.4	.38	28.4	57.2
362 Glass	4.1	-2.1	.67	40.4	79.7
369 Non-metal prod.	5.1	-2.4	.78	43.1	86.6
371 Iron and Steel	3.5	-1.8	.57	38.7	75.6
372 Nonferr. mtls.	1.3	.5	.58	-54.7	-106.7
381 Metal. prod.	4.4	-2.5	.65	35.8	70.4
382 Machinery	4.0	-1.7	.71	48.3	93.4
383 Electrical Mach.	3.5	-2.1	.52	34.4	67.2
384 Transport Equip.	4.6	-2.8	.61	32.3	61.7
385 Prof. Goods	3.3	-2.6	.31	25.7	49.6
390 Other	3.2	-2.3	.35	28.0	55.8

mode is the value at which the max occurs

zero is the value at which the function becomes zero

Table 3

## Quadratic Regressions

$$Q/L = b(0) + b(1) * (K/L) + b(2) * (K/L) ** 2$$

ISIC	t-values			Estimated		
	<u>b(1)</u>	<u>b(2)</u>	<u>R<sup>2</sup></u>	<u>Mode</u>	<u>Zero</u>	
311	Food	3.2	-1.3	.60	47.6	95.3
313	Beverages	3.8	-2.2	.57	34.8	71.2
314	Tobacco	1.7	-.9	.26	40.0	85.1
321	Textiles	2.9	-2.3	.27	26.1	54.7
322	Apparel	1.5	-1.4	.06	21.9	46.8
323	Leather	4.7	-4.0	.43	23.8	48.8
324	Footwear	2.5	-1.9	.24	27.3	57.5
331	Wood	2.3	-.7	.54	70.8	139.6
332	Furniture	2.8	.0	.76	185.7	371.6
341	Paper	2.2	-1.0	.42	46.6	90.5
342	Printing	3.5	-.9	.75	77.7	153.7
351	Industrial Chemicals	3.5	-1.7	.60	42.2	82.7
352	Other Chemicals	5.0	-2.5	.75	40.7	80.7
353	Petroleum Refining	2.4	-1.8	.22	27.1	52.0
354	Petroleum, Coal Products	2.0	-1.6	.15	26.0	50.2
355	Rubber Products	3.6	-2.8	.36	26.1	51.3
356	Plastics	3.2	-1.9	.45	33.2	65.9
361	Pottery	3.3	-2.4	.38	27.4	57.2
362	Glass	4.1	-2.1	.67	40.4	79.7
369	Non-metal Products	5.1	-2.4	.78	43.1	86.6
371	Iron and Steel	3.5	-1.8	.57	38.7	75.6
372	Nonferrous Metals	1.3	.5	.58	-54.7	-106.7
381	Metal Products	4.4	-2.5	.65	35.8	70.4
382	Machinery	4.0	-1.7	.71	48.3	93.4
383	Electrical Machinery	3.5	-2.1	.52	34.4	67.2
384	Transport Equipment	4.6	-2.8	.61	32.3	61.7
385	Professional Goods	3.3	-2.6	.31	25.7	49.6
390	Other	3.2	-2.3	.35	28.0	55.8

Mode is the value at which the max occurs.

Zero is the value at which the function becomes zero.

TABLE 4  
 Quadratic Regressions with Data Subsets  
 and formation of aggregates

ISIC	F-stat	Land scarce countries				Land abundant countries				
		R-SQ	t-val	mode	zero	R-SQ	t-val	mode	zero	
AGGREGATE 1: Land scarce countries peak early and terminate early										
A: Commodities with peaks at low values of K/L										
322 Apparel	2.2	.05	-.8	17.6	43.2	.62	-3.3	26.9	52.9	
390 Other	2.7	.4	-2.2	24.2	48.5	.36	-1.2	34.4	68.4	
B: Commodities with peaks at relatively high values of K/L										
351 Ind. Che.	2.1	.64	-1.4	35.9	69.5	.6	-1.1	46.9	93.0	
356 Plast.	3.2	.39	-1.4	28.6	57.4	.75	-2.2	37.9	75.1	
383 Elect.	8.5	.59	-2.1	29.1	57.3	.68	-1.6	40.9	79.4	
AGGREGATE 2: Land affects the level of output, not modal K/L.										
A: Commodities with peaks at low values of K/L										
321 Text.	2.6	.15	-1.0	25.2	56.4	.59	-3.5	25.4	51.8	
323 Leath.	1.5	.42	-2.5	23.8	49.5	.45	-3.2	23.0	47.1	
355 Rubber	3.9	.38	-2.1	24.4	47.9	.57	-3.1	26.3	52.1	
385 Prof.	1.9	.26	-1.6	24.4	49.1	.36	-2.1	25.8	48.3	
B: Commodities with peaks at relatively high values of K/L										
361 Pottery	1.9	.65	-2.3	28.9	57.0	.2	-1.5	25.4	52.6	
381 Metal	1.6	.55	-1.2	35.1	70.8	.78	-2.6	35.8	69.3	
AGGREGATE 3: Land scarce countries peak, land abundant do not										
342 Print	.9	.61	-1.1	39.2	77.4	.89	.1	n.a.	n.a.	
372 N. Mtls	7.5	.55	-1.1	37.5	73.5	.79	1.6	n.a.	n.a.	
382 Mach.	1.2	.74	-1.9	35.2	67.1	.69	-.6	83.4	163.7	
384 Trans.	1.1	.64	-2.9	26.6	50.4	.64	-1.4	43.2	83.2	
AGGREGATE 4: Land scarce do not peak, abundant peak at high values										
313 Wood	2.0	.66	-1.3	38.7	77.5	.5	-2.1	29.4	61.7	
314 Tobacco	2.2	.33	-.3	70.8	149.1	.19	-1.2	27.2	59.3	
324 Shoes	1.4	.23	-.4	43.3	94.3	.32	-2.6	22.5	46.8	
352 O. Che.	5.7	.85	-1.5	46.9	92.9	.72	-2.6	33.3	66.6	
362 Glass	2.8	.77	-.8	62.4	124.4	.6	-2.3	31.3	61.6	
369 Non-mtl	1.7	.8	-.9	61.1	123.6	.77	-2.6	35.7	71.3	
371 Steel	.8	.54	-.6	49.4	97.6	.63	-2.2	32.7	63.3	
AGGREGATE 5: Forest products influenced by CAN, FIN and SWE										
331 Wood	13.6	.61	-2.4	27.7	54.3	.8	.4	n.a.	n.a.	
341 Paper	5.3	.78	-.8	63.2	126.7	.57	-.9	52.6	101.6	
AGGREGATE 6: Petroleum products, SNG and NET outliers										
353 Refin.	1.3	.22	-1.2	26.4	50.4	.52	-3.0	25.4	49.1	
354 Petro	2.1	.18	-1.3	24.2	46.3	.32	-1.9	26.1	51.5	
AGGREGATE 7: No land effect, slight curvature										
311 Food	.3	.66	-.6	62.0	123.2	.55	-1.3	40.4	81.6	
AGGREGATE 8: No peaks, land affects level, not curvature										
332 Furn.	1.0	.7	.1	<del>255.9</del> n.a.	<del>506.4</del> n.a.	.83	-.1	649.2	1298	

F-stat. is the F statistic for testing for differences in subsamples

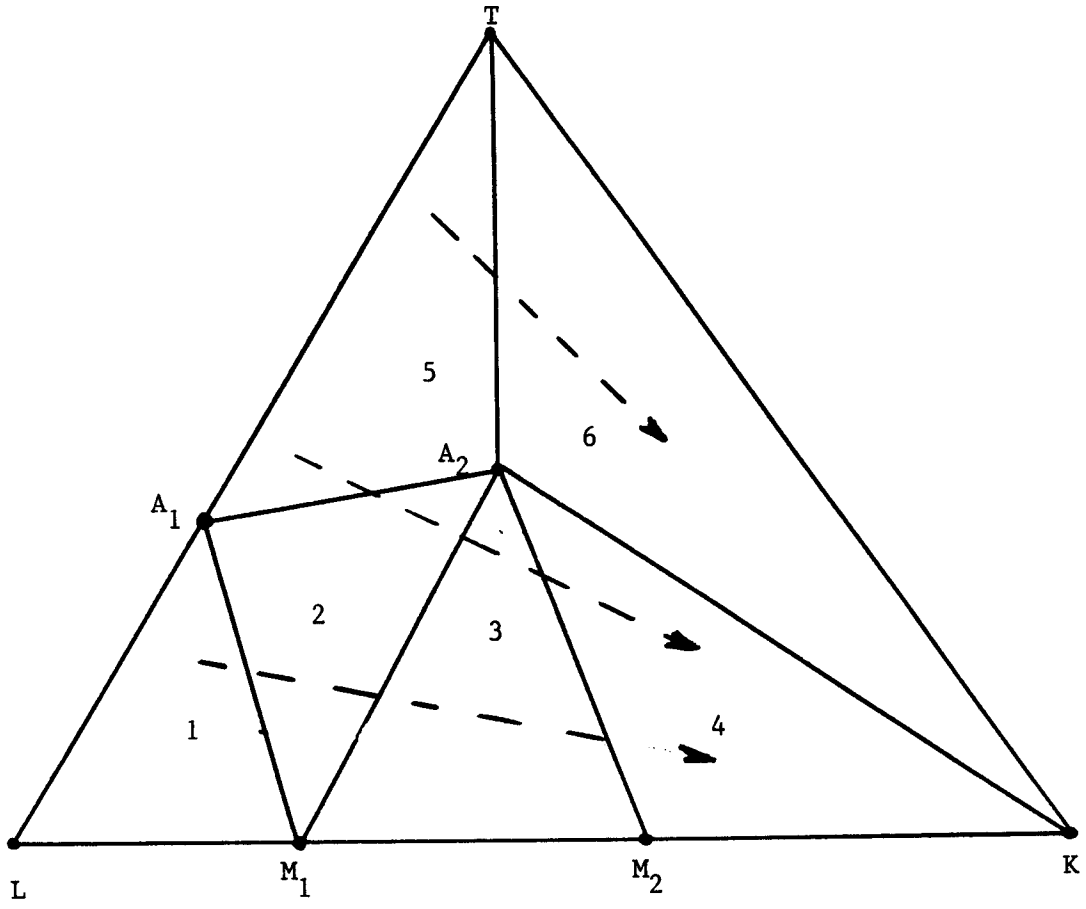


Figure 1

Triangles of Diversification

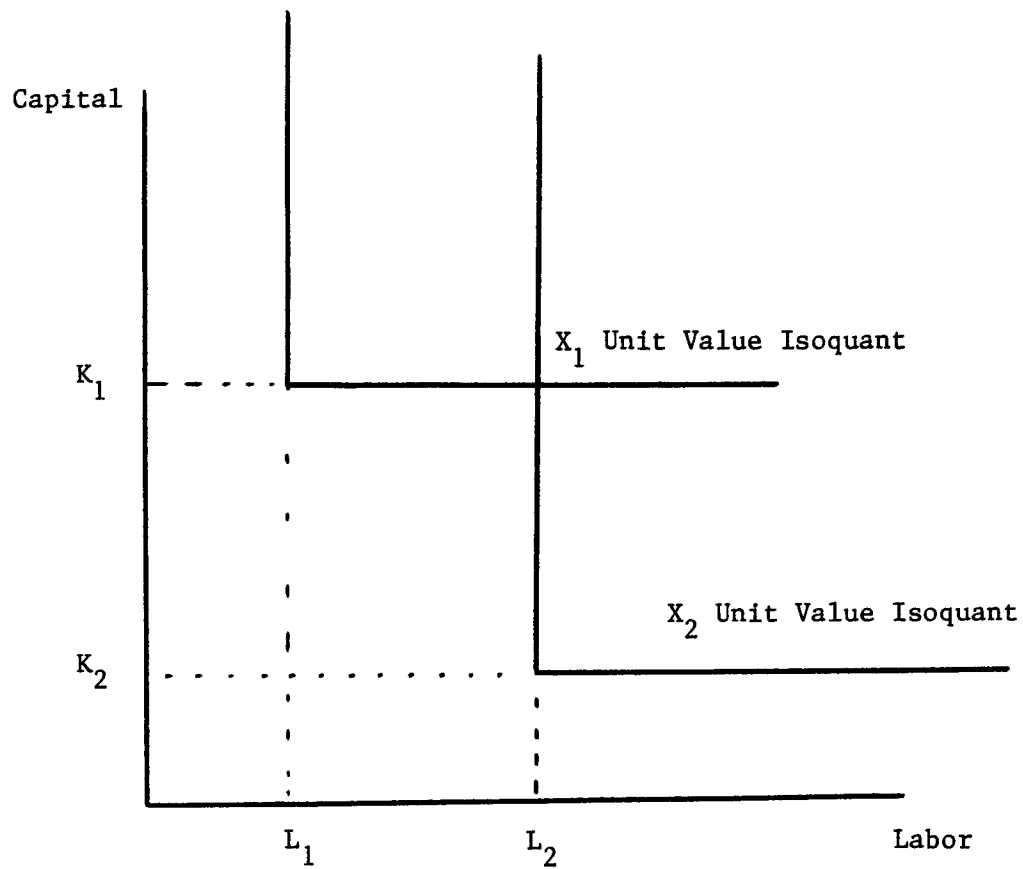


Figure 2a  
Unit Value Isoquants

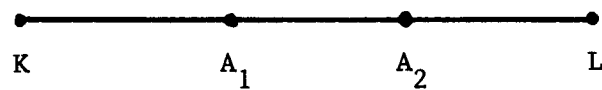
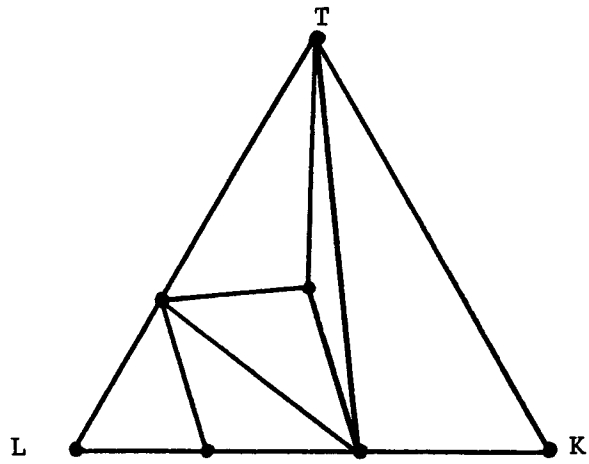
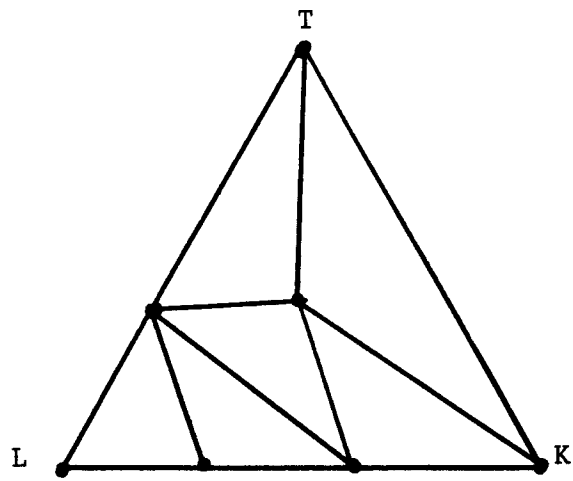


Figure 2b  
Endowment Line Segment

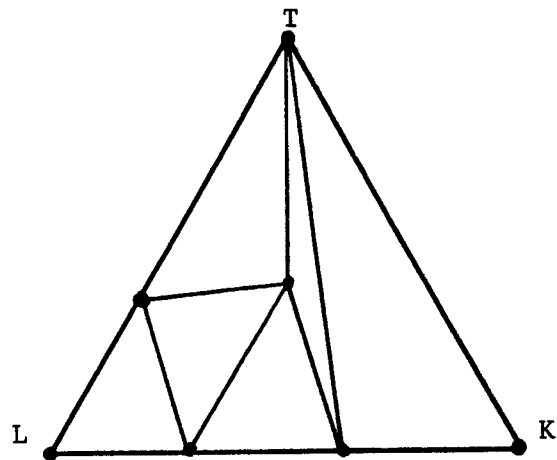




(a)



(b)



(c)

Figure 4  
Alternative Triangles of Diversification

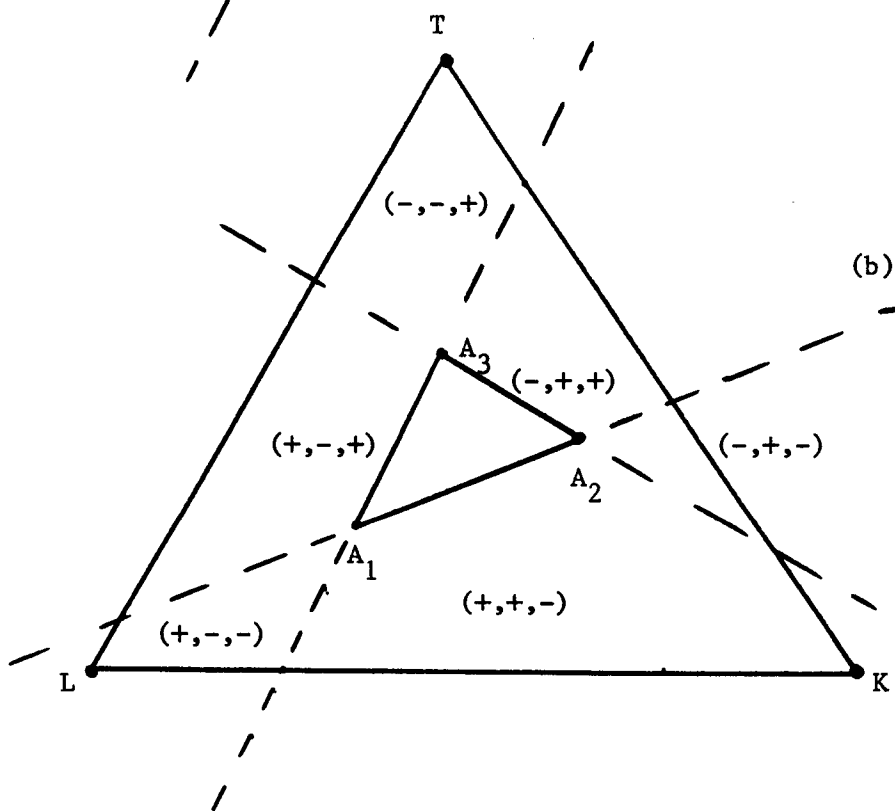
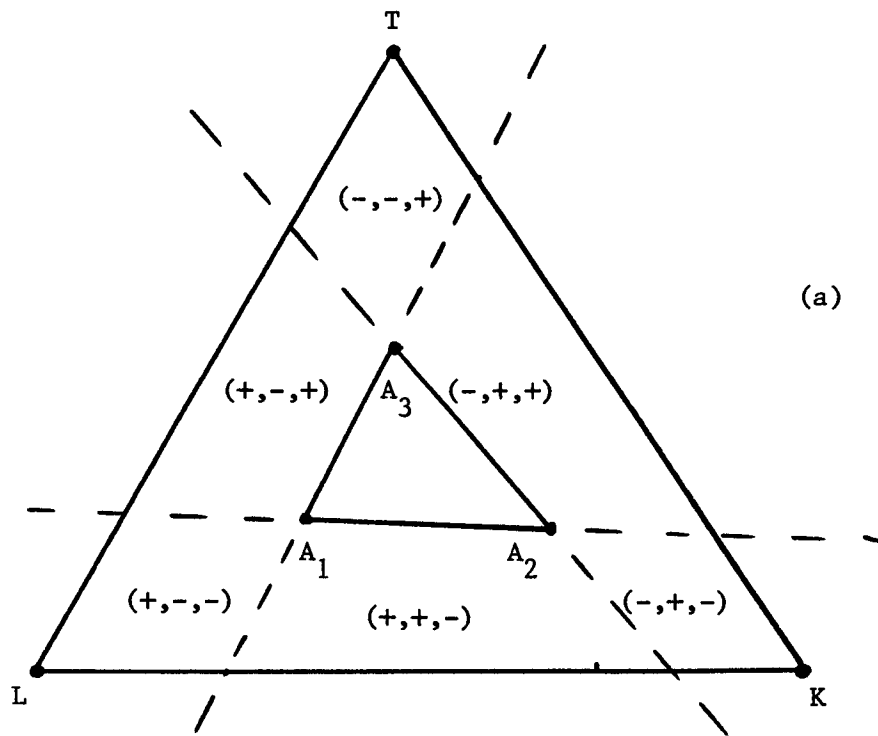


Figure 5  
Signs of Rybczynski Derivatives



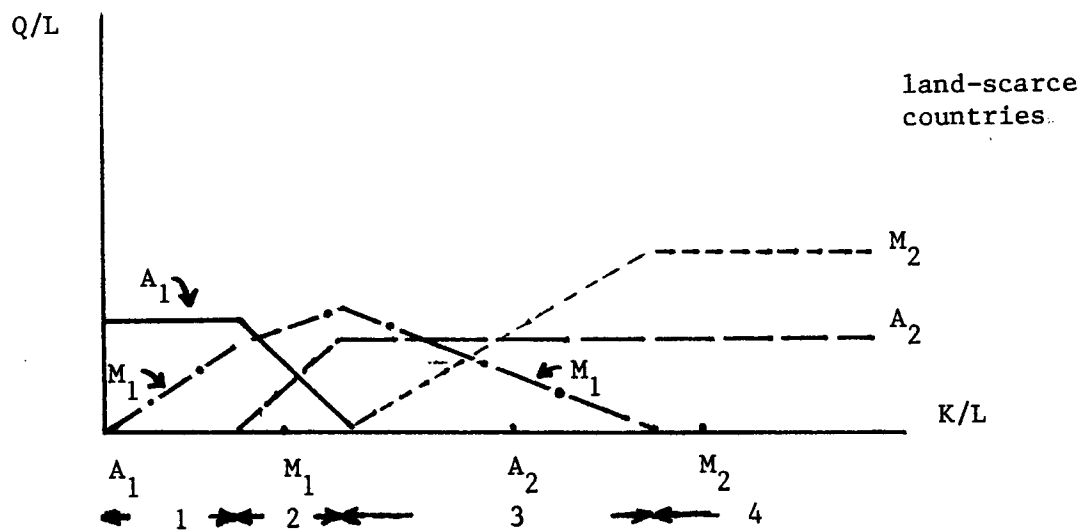
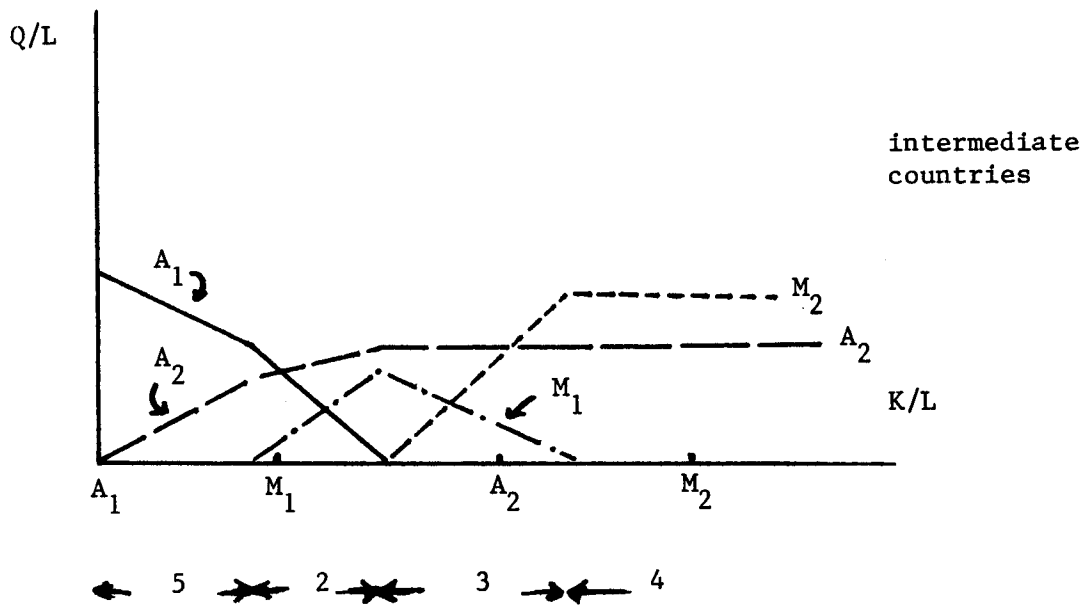
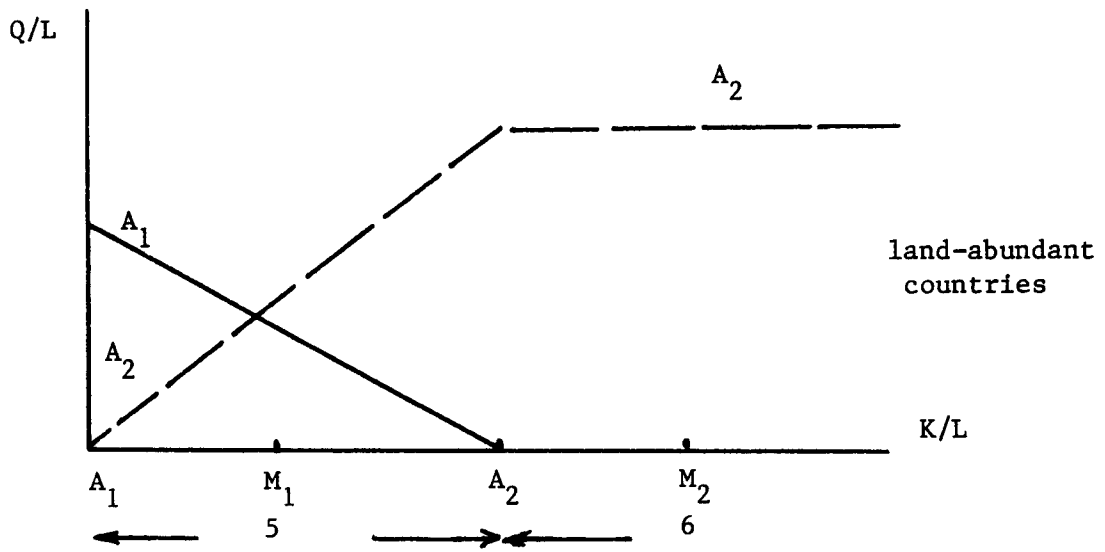


Figure 6  
Paths of Development

Figure 7

# ENDOWMENT TRIANGLE

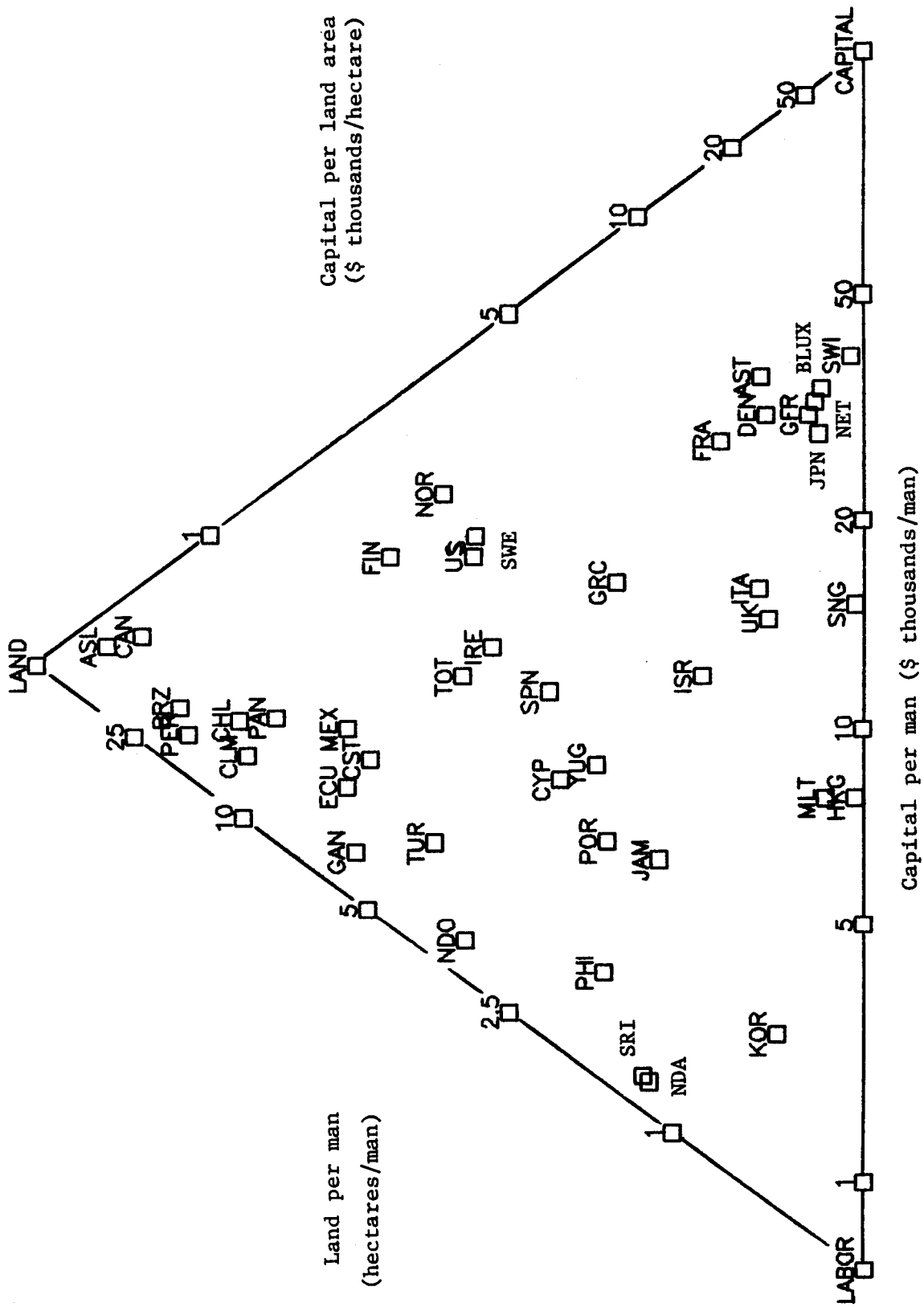
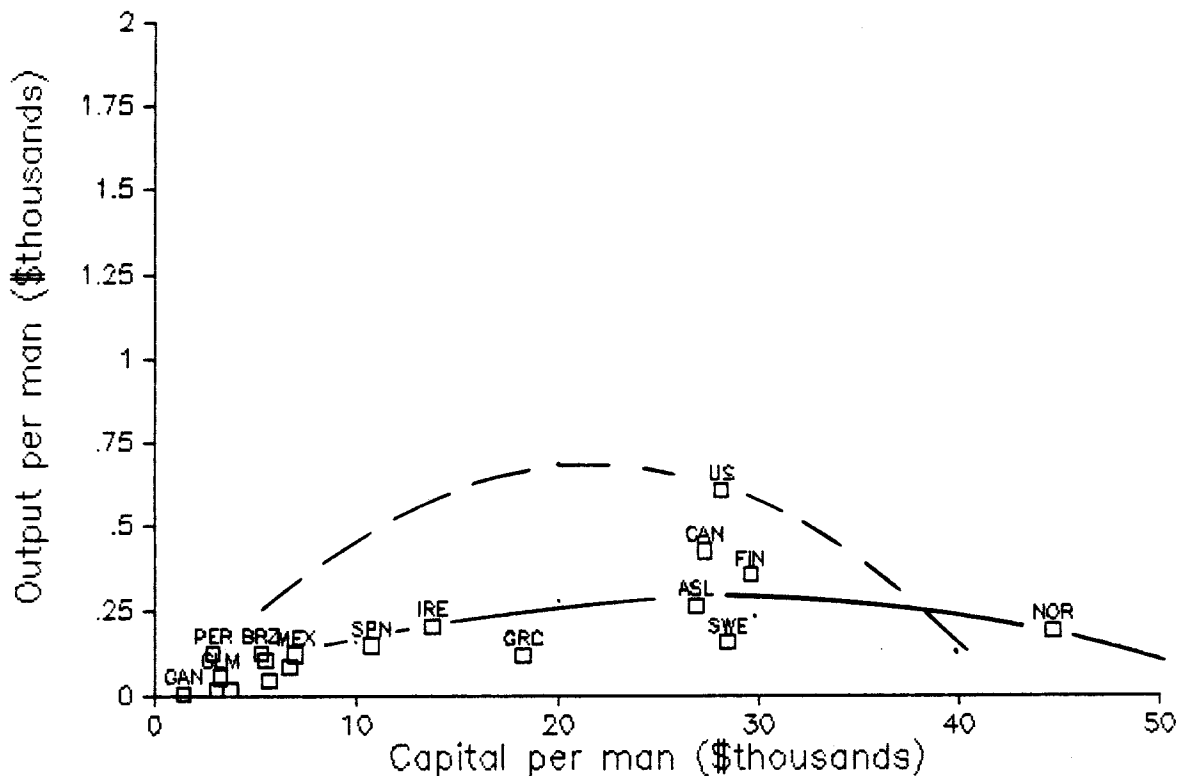


Figure 8

# AGGREGATE 1A

Land Abundant Countries ———



# AGGREGATE 1A

Land Scarce Countries - - - - -

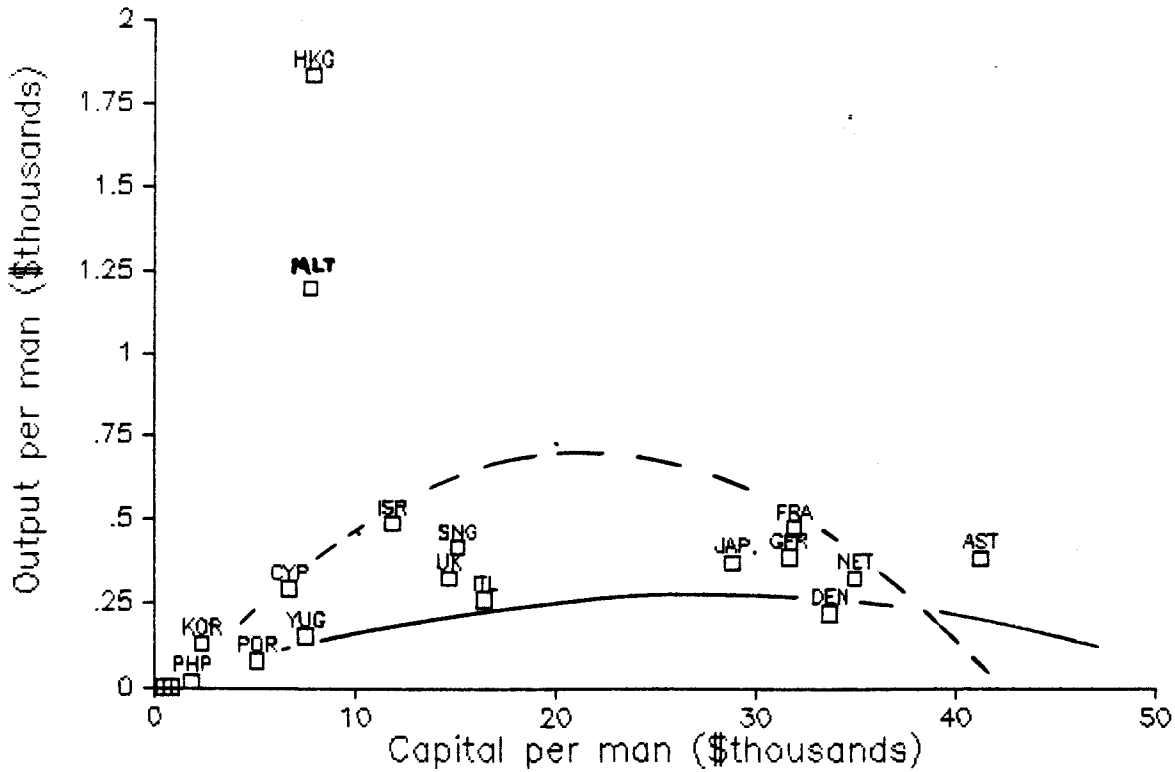
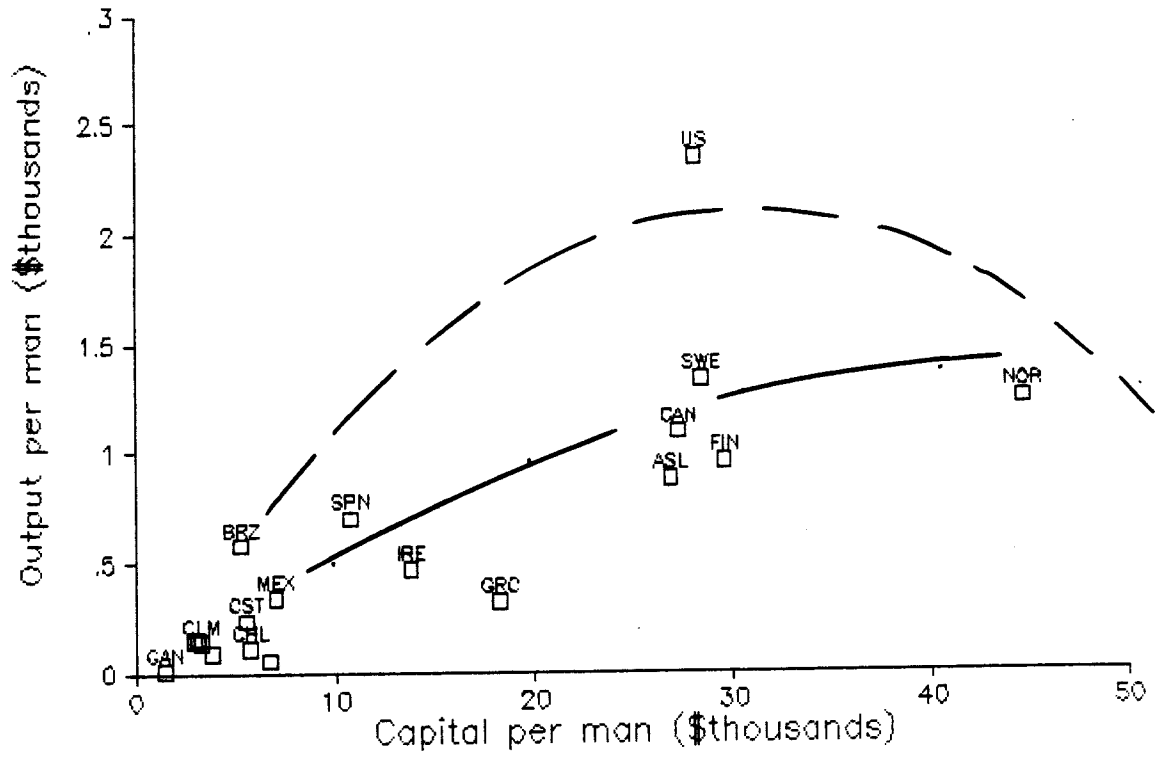


Figure 9

### AGGREGATE 1B Land Abundant Countries



### AGGREGATE 1B Land Scarce Countries

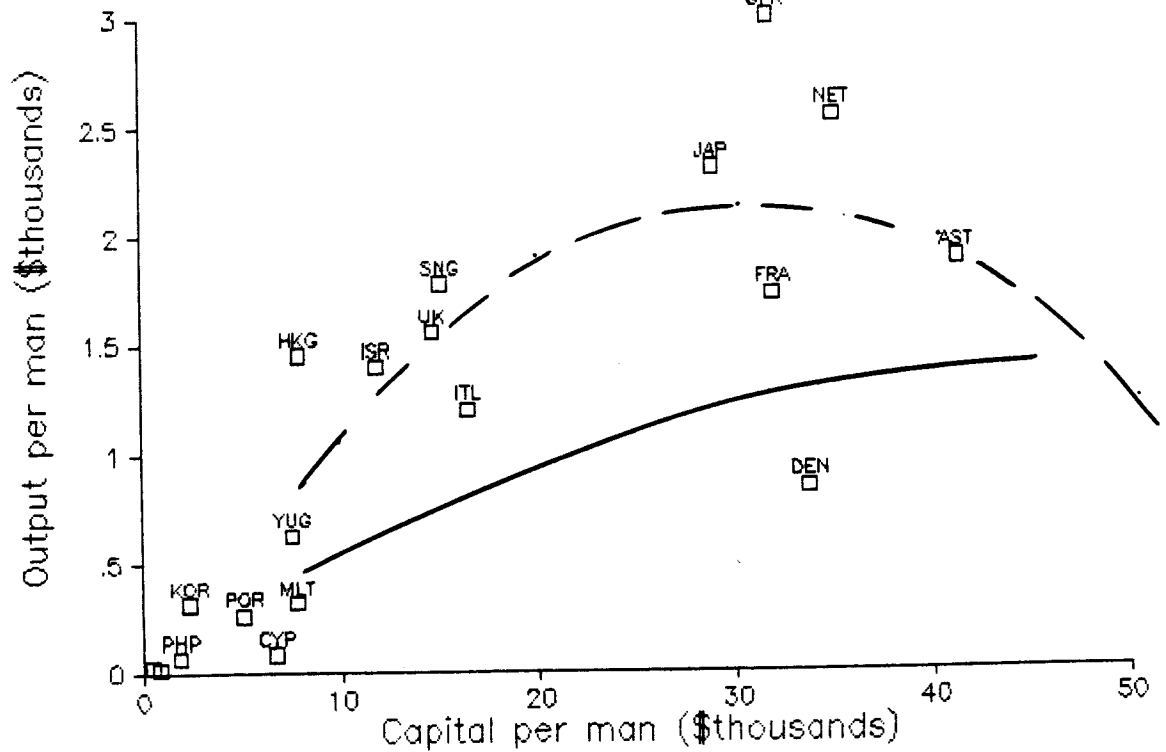
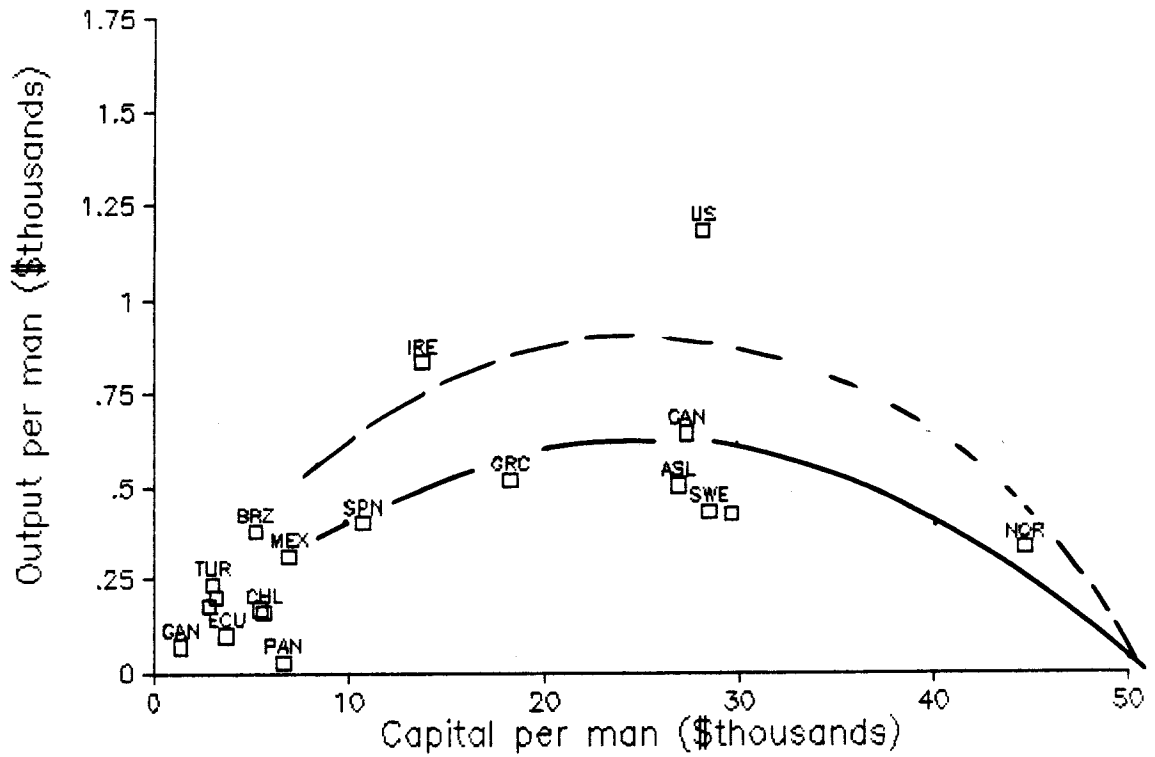


Figure 10

# AGGREGATE 2A

Land Abundant Countries



# AGGREGATE 2A

Land Scarce Countries

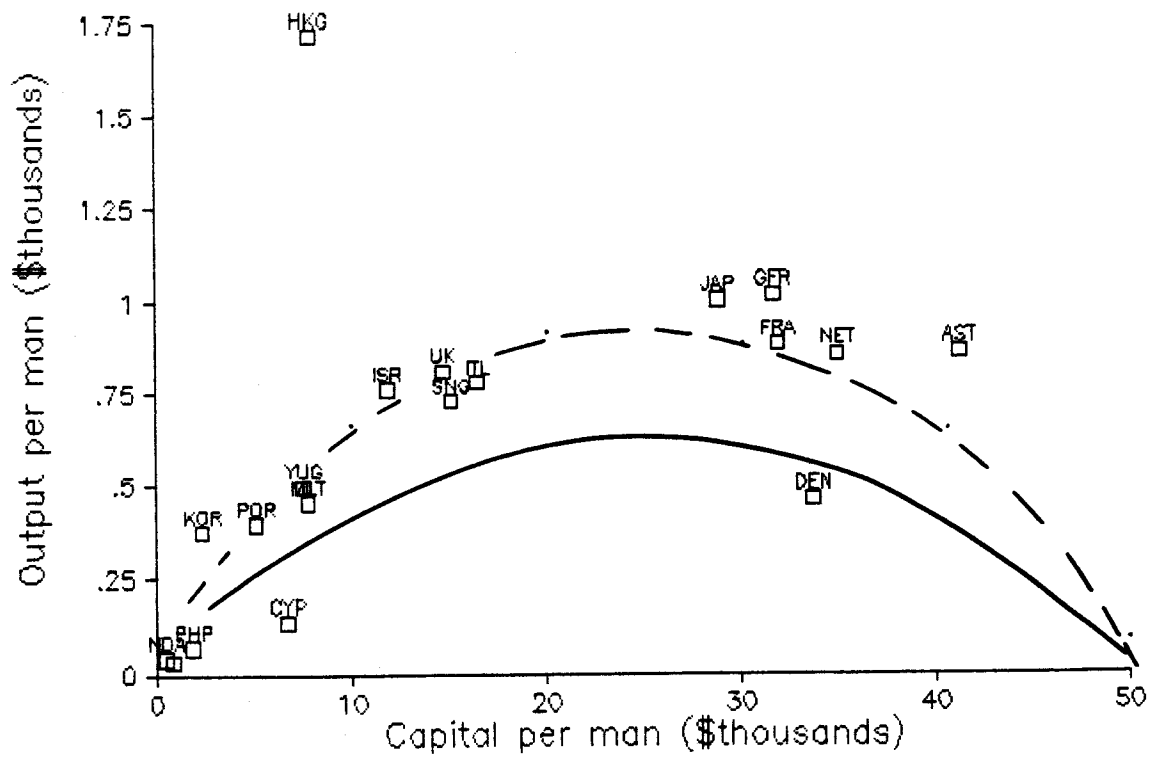


Figure 11

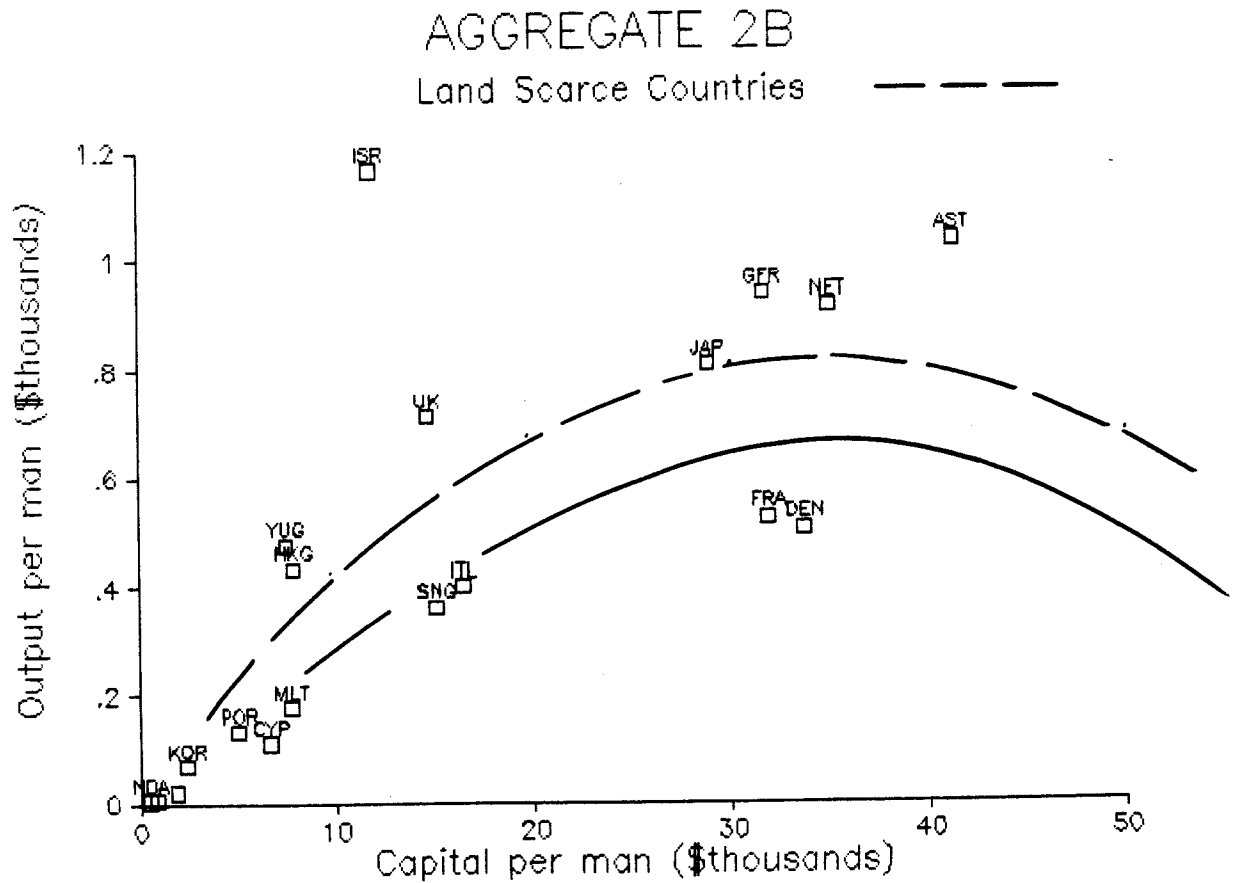
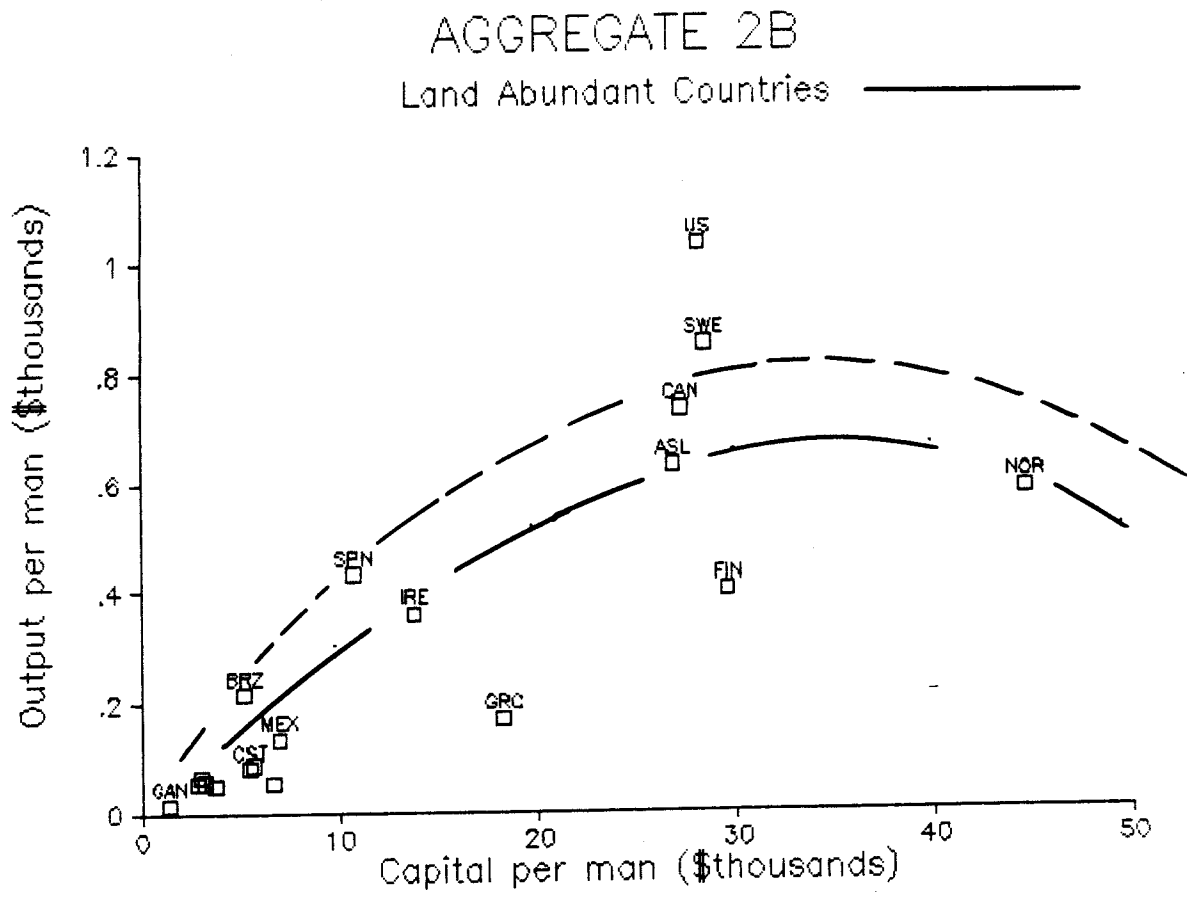


Figure 12

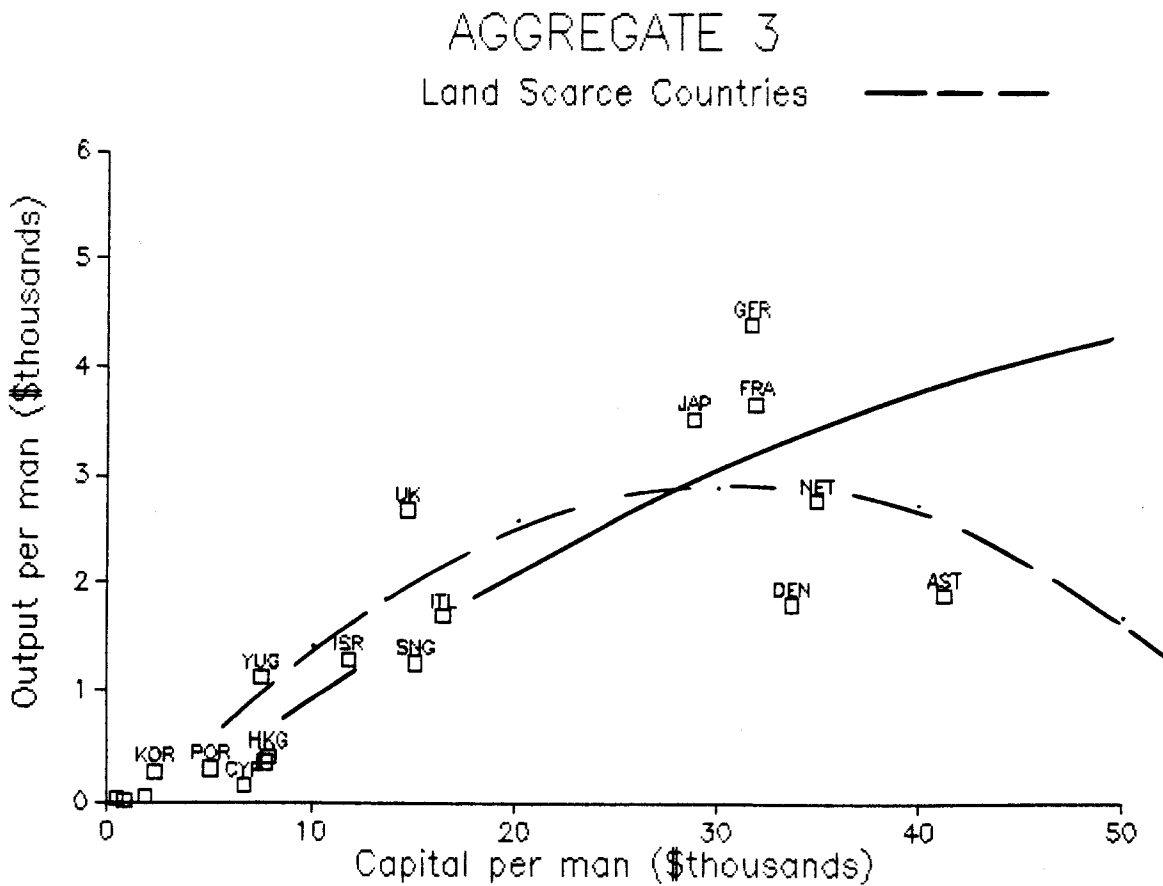
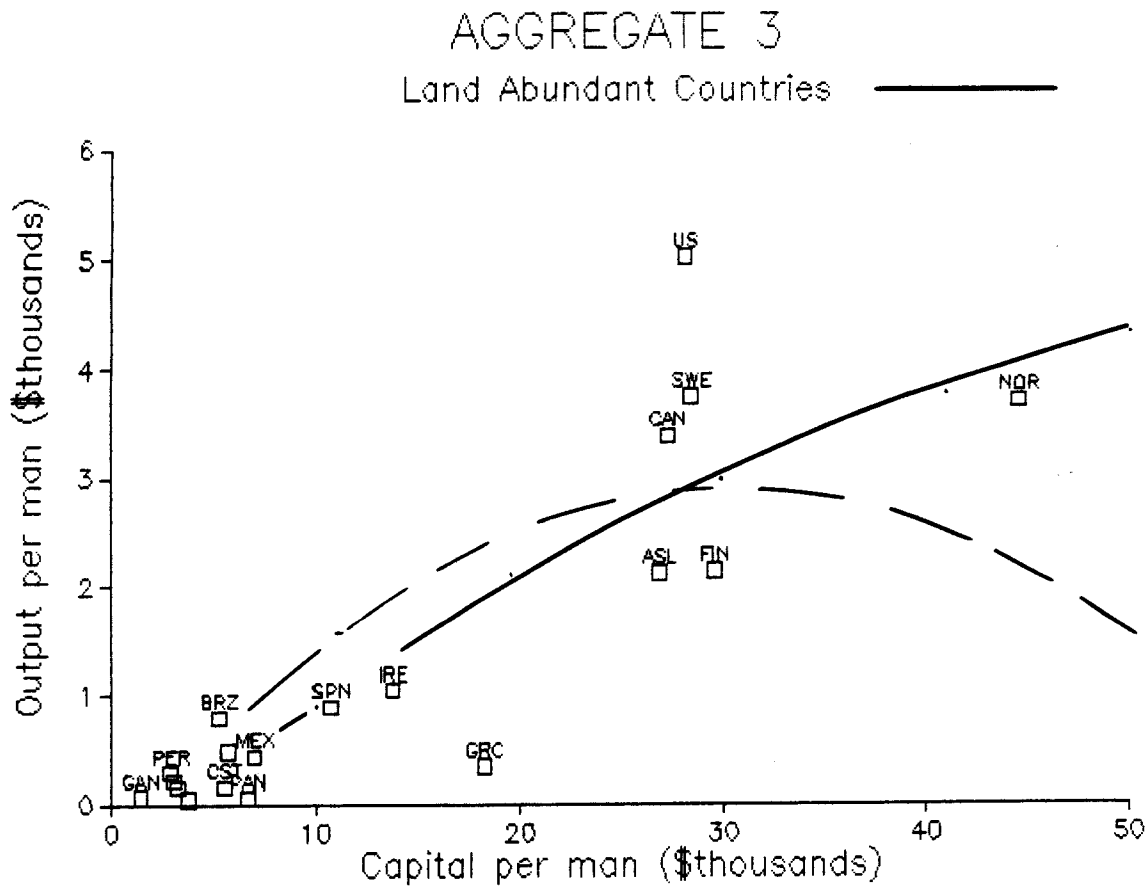


Figure 13

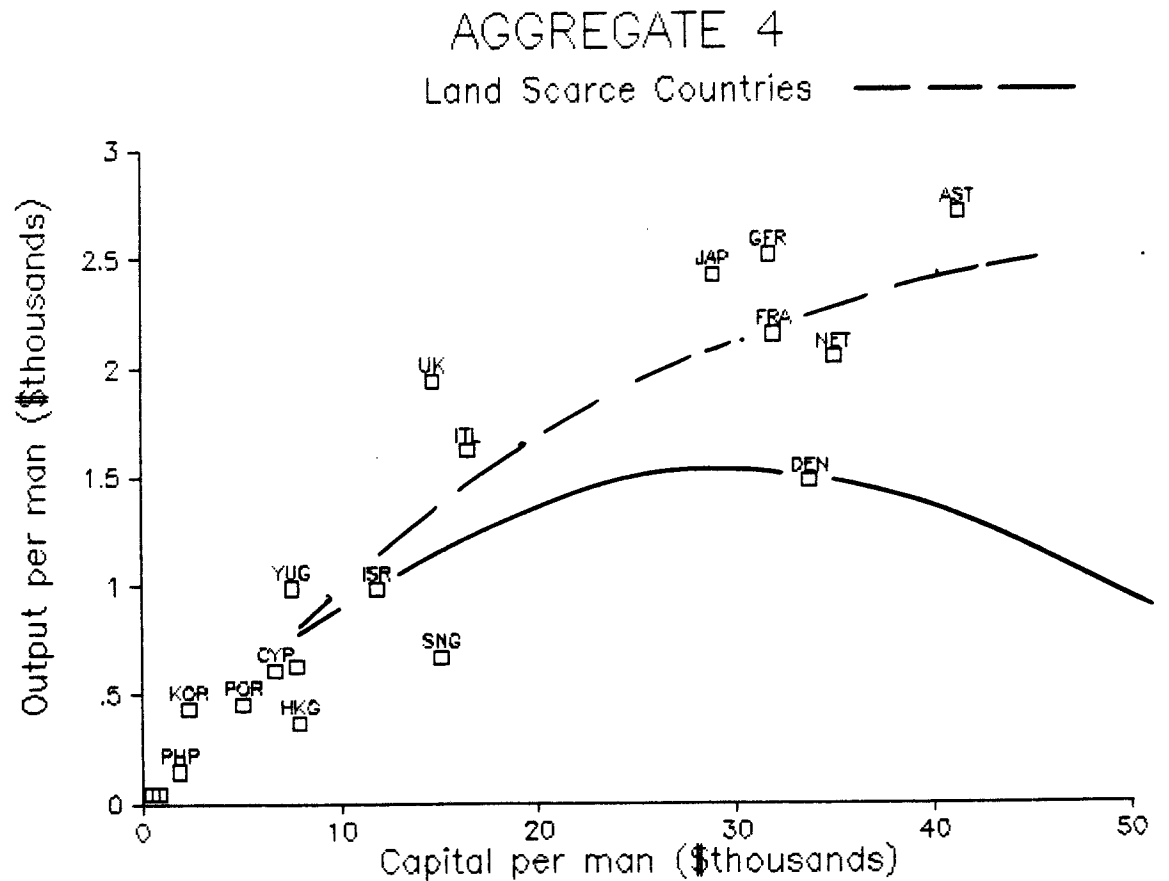
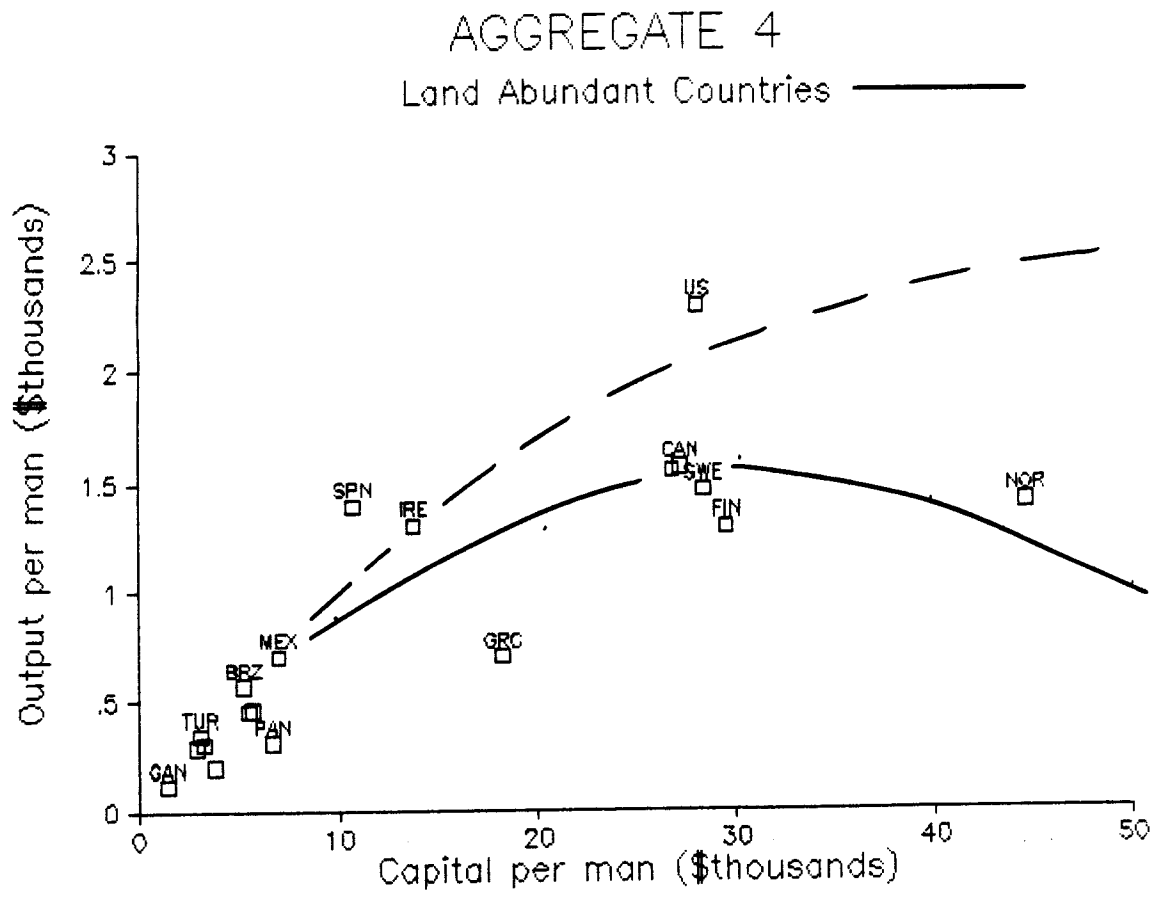




Figure 14

### AGGREGATE 5 Land Abundant Countries



### AGGREGATE 5 Land Scarce Countries

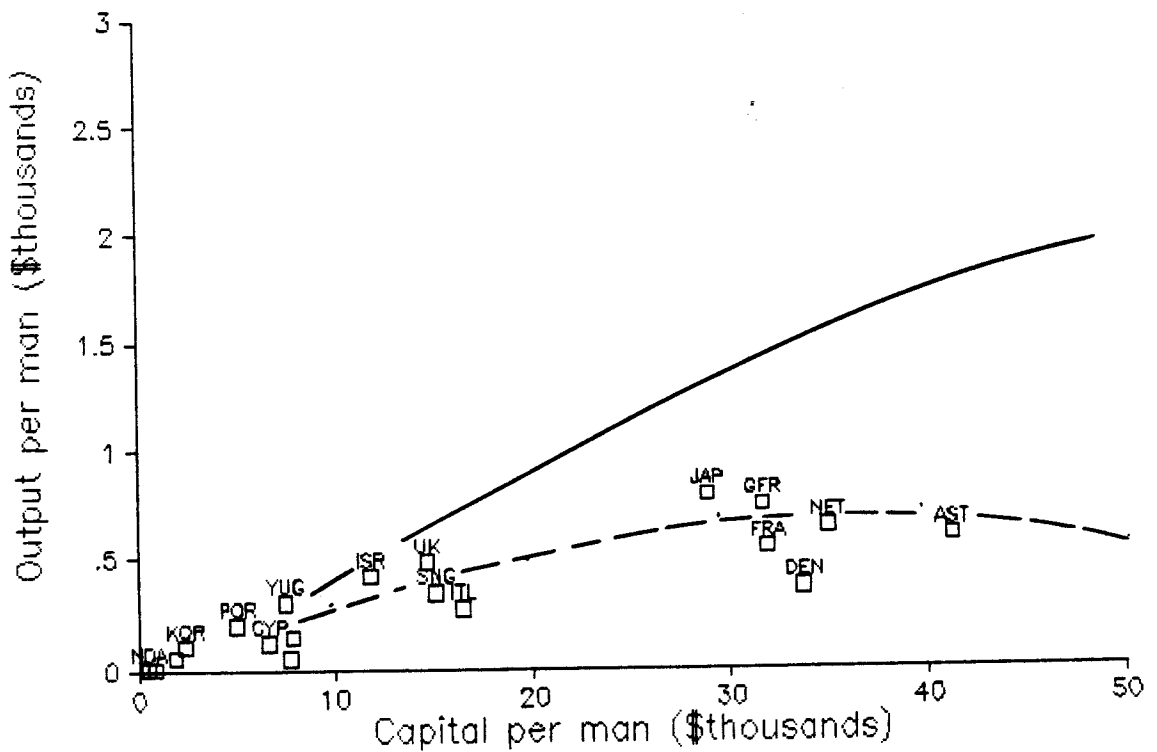
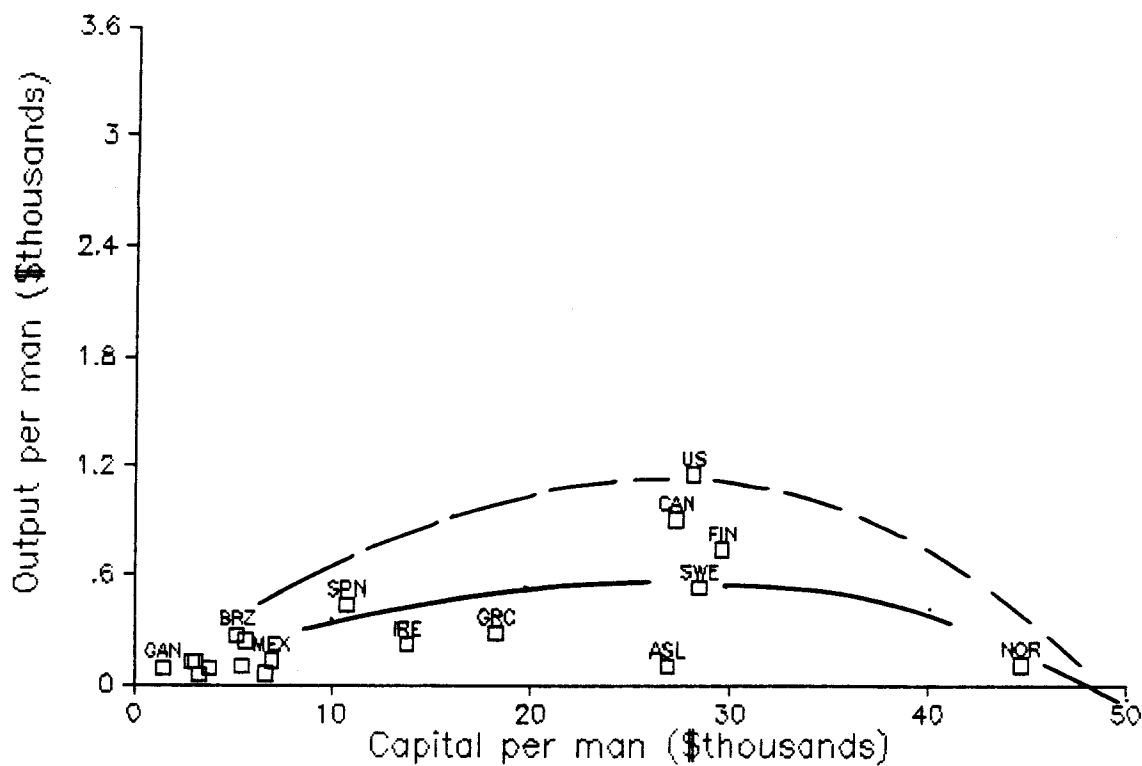


Figure 15

### AGGREGATE 6 Land Abundant Countries



### AGGREGATE 6 Land Scarce Countries

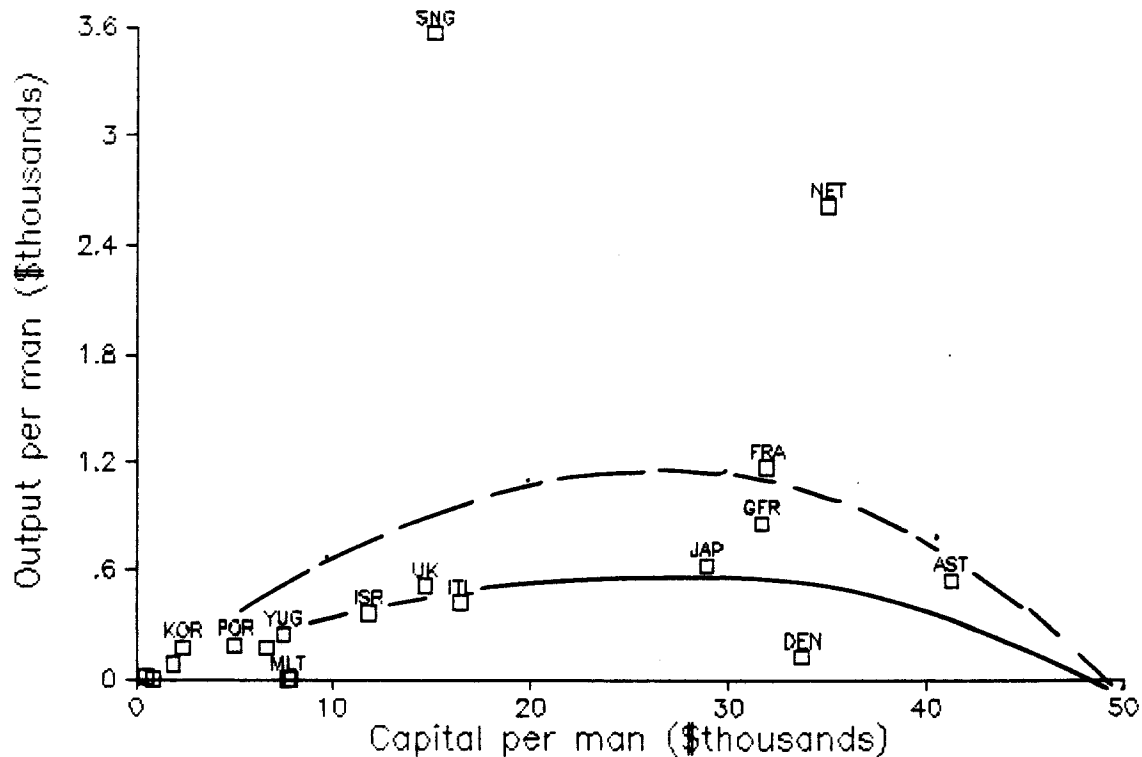


Figure 16

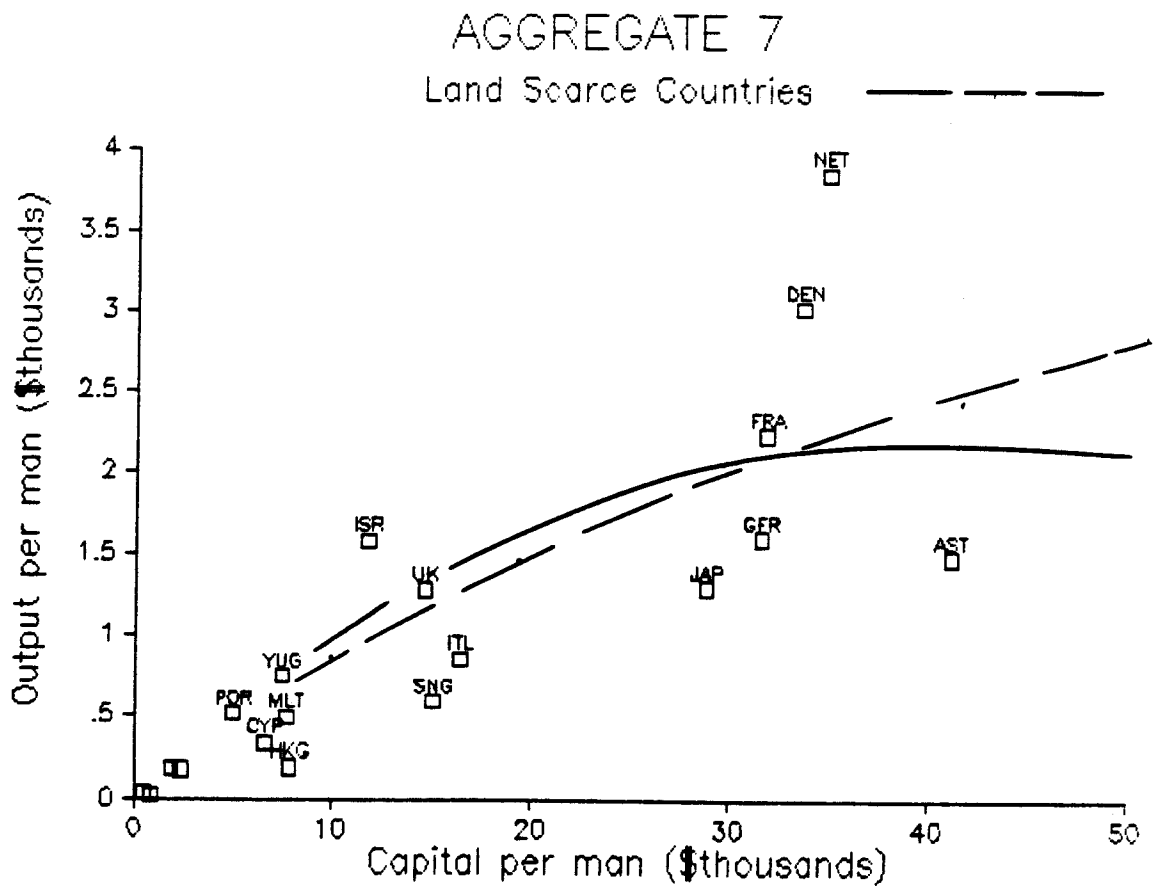
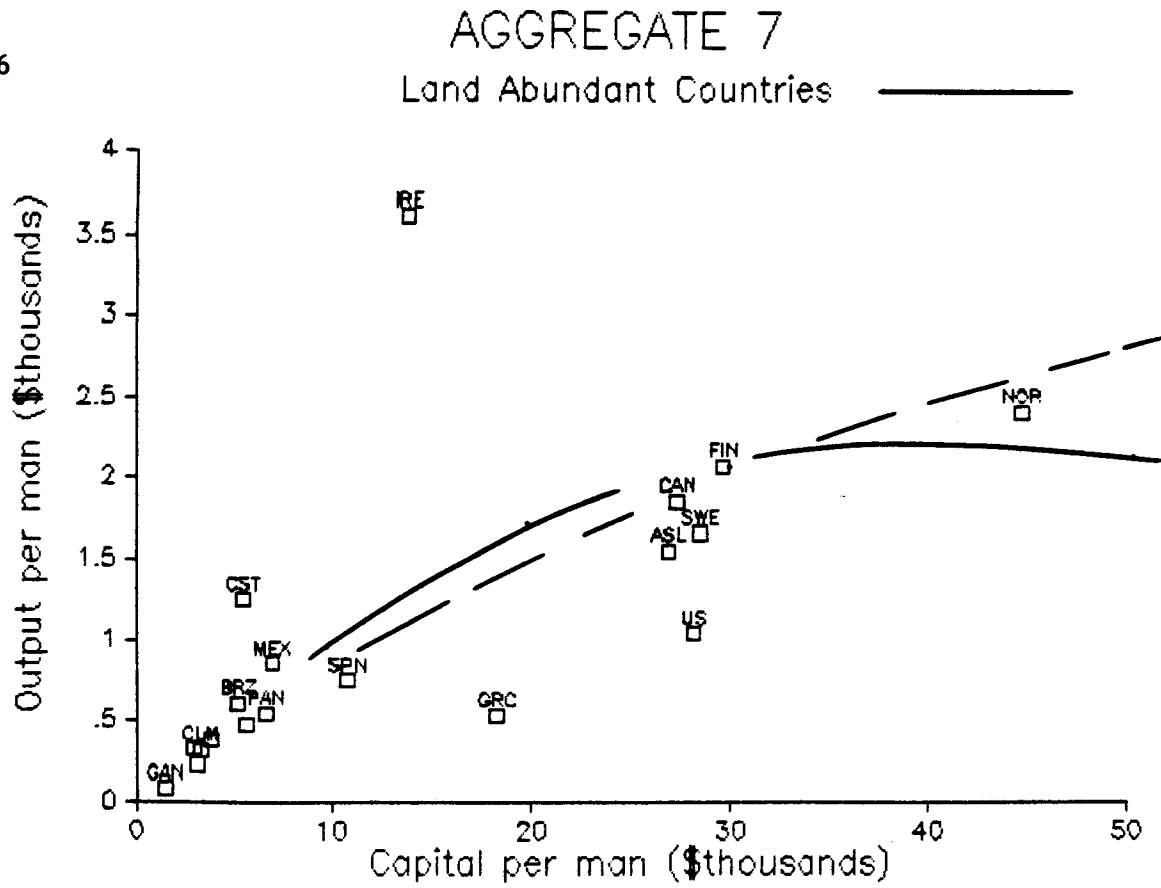
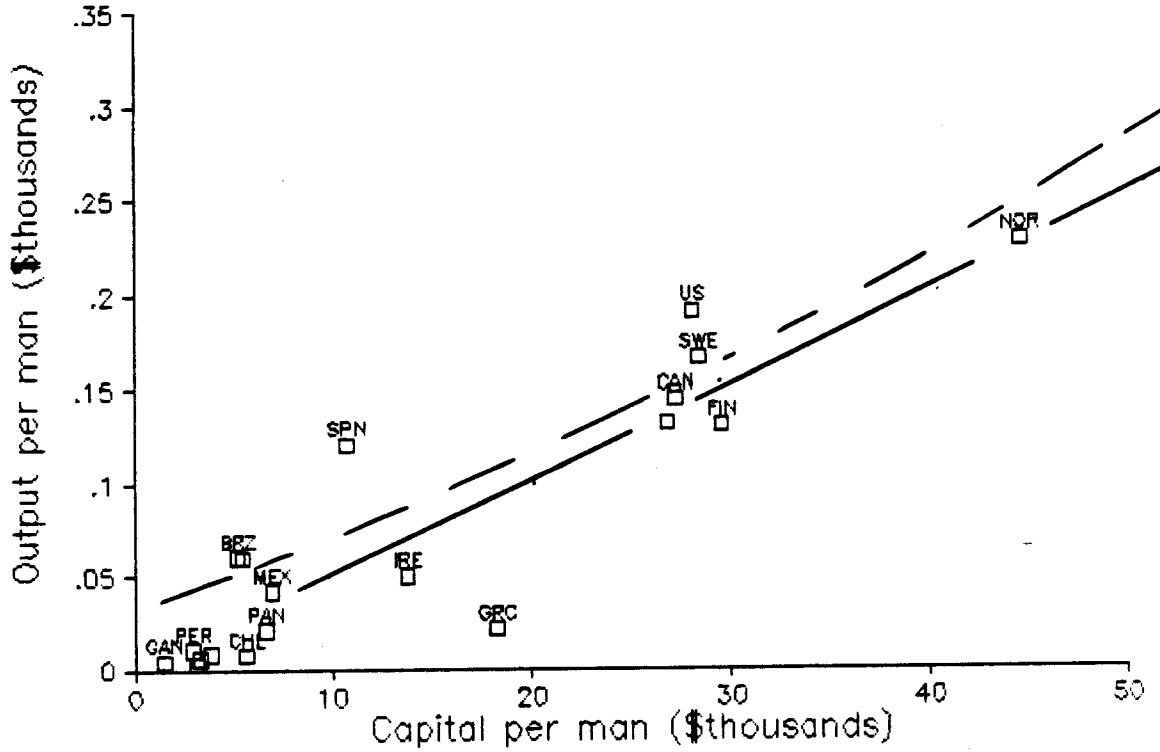


Figure 17

### AGGREGATE 8 Land Abundant Countries



### AGGREGATE 8 Land Scarce Countries

