INVESTMENT DECISION CRITERIA*

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Investment is present sacrifice for future benefit. Individuals, firms, and governments all are regularly in the position of deciding whether or not to invest, and how to choose among the options available. An individual might have to decide whether to buy a bond, plant a seed, or undertake a course of training; a firm whether to purchase a machine or construct a building; a government whether or not to erect a dam. Under the heading of investment decision criteria, economists have addressed the problem of how to rationally choose in such situations involving a tradeoff between present and future.

The Economic Theory of Intertemporal Choice

The object of investment is to optimize one's pattern of consumption over time. The elements needed to determine an individual's investment decision are: (a) his endowment, in the form of a given existing income stream over time; (b) his preference function, which orders in desirability all possible time-combinations of consumption; and (c) his opportunity set, which specifies the possibilities for transforming the original endowment into other time-combinations of consumption.

[Figure 1 about here.]

Figure 1 illustrates an artificially simple case of only two periods (say, this year and next) under conditions of certainty. Each point represents a combination of current consumption c_0 and future consumption c_1. The endowment combination Y has coordinates (y_0,y_1). Time-preferences are portrayed by the indifference curves $U_1,U_2,U_3,\ldots,$ each such curve connecting all combinations yielding equal satisfaction. The curve QQ' through the endowment position Y pictures the intertemporal productive opportunities. By sowing seed, for example, a person can sacrifice current consumption for future consumption -- represented in the diagram by a movement from Y along QQ' to the northwest. (There may also be disinvestment opportunities, i.e., the individual might be able to draw upon the future so as to augment current consumption, which would be represented by a movement from Y along QQ' to the southeast.)

For a Robinson Crusoe, the optimum balance of present and future consumption -- which in his isolated state must necessarily be identical to his provision for present and future production -- occurs at point X* along QQ'. In the situation pictured he achieves this optimum by investing the quantity y_0-x_0 of current consumption claims. E.g., having at hand a current corn endowment of y_0, he retains x_0 for current consumption and plant the remainder as seed. Next year he will reap as his return from investment the amount x_1-y_1 to augment his endowed availability for future consumption.

If markets for trading between present and future income claims exist, however, in contrast with the Robinson Crusoe situation the individual will be able to disconnect the amount he invests from the amount he saves. These trading opportunities are shown in Figure 1 by the family of "market lines" whose general equation is:

(1) $$c_0 + c_1/(1+r_1) = W_0$$

Here r_1 is the interest rate that discounts one-year future claims c_1 into their equivalent value in terms of c_0 claims. Along each market line the parameter W_0 represents the associated level of wealth. Put another way, wealth in equation (1) measures the present worth of any specified (c_0,c_1)

vector — the future-dated element being "discounted" at the given market interest rate r_1. In the diagram two market lines are shown: MM' through the endowment vector $Y = (y_0, y_1)$ indicates the individual's endowed wealth $W_0^Y = y_0 + y_1/(1+r_1)$, while NN' represents the maximum attainable level of wealth $W_0^* = q_0^* + q_1^*/(1+r_1)$.

If an individual has both productive and market opportunities, his optimizing decision in Figure 1 can be thought of as taking place in two stages. First he locates his "productive solution" $Q^* = (q_0^*, q_1^*)$ by moving along QQ' so as to maximize attained wealth at the tangency with market line NN'. Second, he then transacts in the funds markets, by lending or borrowing (exchanging current for future claims or vice versa) along NN' to find his "consumptive solution" $C^* = (c_0^*, c_1^*)$ at the tangency of NN' with indifference curve U_2 in the diagram. Notice that his preferences do not at all affect the productive solution, but only how he chooses to "finance" the investments made. Specifically, in the diagram here the amount he invests $(y_0 - q_0^*)$ exceeds the amount he saves $(y_0 - c_0^*)$. By borrowing on the markets, in effect he has been able to get others to undertake part of the saving necessary to finance his projected investments.

This disconnection between the individual's productive and consumptive decisions in a regime of perfect markets is known as "Fisher's Separation Theorem". The essential implication is that individuals with diverging time-preferences can nevertheless come together and agree upon joint productive investments. Business firms and (to some extent) governments can be regarded as institutions designed for undertaking joint investments whose scale is too large for any single individual. The underlying principle is that those investment choices maximizing wealth value or present worth of the mutual undertaking will also maximize wealth for each and every participant therein.

The Present-Value Rule

The economic theory of intertemporal choice leads immediately to what is known as the Present-Value Rule for investment decision. This rule can be expressed in two essentially equivalent forms:

(i) Among the opportunities available, adopt the set of investments that maximizes investors' wealth W_0.

(ii) Adopt any single investment project if and only if its present value V_0 is positive. (Taking into account, of course, any repercussions of that project upon the returns yielded by other members of the adopted investment set.)

As an obvious corollary, if two available projects are mutually exclusive, the one with the larger present value V_0 should be chosen.

Generalizing to the multi-period context, wealth as maximand becomes:

$$(2) \quad W_0 = q_0 + q_1/(1+r_1) + q_2/[(1+r_2)(1+r_1)] + \ldots + q_T/[(1+r_T)\ldots(1+r_2)(1+r_1)]$$

Here the q_t are the coordinates of points along the T+1-dimensional productive opportunity surface $\phi(q_0,q_1,\ldots,q_T) = 0$, a generalization of curve QQ' in Figure 1. T is the "economic horizon," which may be infinite. And the r_t represent the successive short-term interest rates, each of which discounts prospective payments at any date into its wealth-equivalent at the next preceding date.

For a single project in the multi-date context, present value is defined as:

$$(3) \quad V_0 = z_0 + z_1/(1+r_1) + z_2/[(1+r_2)(1+r_1)] + \ldots + z_T/[(1+r_T)\ldots(1+r_2)(1+r_1)]$$

Here the z_t are the dated payments or "cash flows" associated incrementally with the project considered. Normally the z_t elements for earlier dates would include some with negative signs — else the project could not be described as an investment — while those for later dates would have predominantly positive signs. In the special case where $r_1 = r_2 = \ldots = r_T = r$ — that is, where interest rates are expected to remain constant at the level r over time — the Present-Value formulas reduce to the more familiar forms:

(2') $$W_0 = q_0 + q_1/(1+r) + q_2/(1+r)^2 + \ldots + q_T/(1+r)^T$$

(3') $$V_0 = z_0 + z_1/(1+r) + z_2/(1+r)^2 + \ldots + z_T/(1+r)^T$$

The Present-Value solutions can also be formally generalized to allow for continuous rather than discrete time. As an illustrative simplified example, consider a project whose scale of current input or investment sacrifice i_0 is fixed while the output date is subject to choice. (E.g., when to cut a growing tree.) In Figure 2, horizontal distances represent time t and vertical distances value V_t at each date. Present Value V_0 is indicated by height along the vertical axis. The curve GG' represents productive growth of the asset — in the case of a tree, market value of the standing timber at any date. The "discount curves" D, D', D",... are analogous to the "market lines" of Figure 1. Each such curve represents the growth of a specific sum of present dollars by continuous compounding at a constant market rate of interest r, or alternatively the Present Value of any future payment continuously discounted at r. The optimal investment period $t = t^*$ is then the one that maximizes Present Value V_0, subject to the constraint on the available V_t described by the curve GG', in the equation:

(4) $$V_0 = -i_0 + V_t e^{-rt}$$

Geometrically, t^* is determined by the tangency of GG' with the highest discount curve (constant-wealth curve) attainable. The solution condition is then:

(5) $$V_t'/V_t = r$$

[Figure 2 about here.]

Other Investment Criteria

Certain investment criteria employed in business practice are definitely erroneous. One such is rapidity of "payout" (the date when cash inflows first balance initial outlays), a formula that obviously fails to allow properly for time-discount. Controversy among theorists has centered upon a more interesting concept known variously as the "internal rate" or the "rate of return." The internal rate for a project (or set of projects) is defined as ρ in the discrete discounting equation:

$$0 = z_0 + z_1/(1+\rho) + z_2/(1+\rho)^2 + \ldots + z_T/(1+\rho)^T \tag{6}$$

As before the z_t here are the successive terms, positive or negative, of the payments-receipts sequence associated incrementally with a particular project. In the special "deepening" case illustrated in Figure 2, the corresponding concept under continuous compounding is defined implicitly in:

$$0 = -i_0 + V_t e^{-\rho t} \tag{7}$$

where once again the V_t at any date is described by the productive opportunity curve GG'. Under these conditions ρ represents an average compounded rate of growth.

There has been some confusion between two quite different investment decision rules that both employ the internal-rate measure ρ: (i) choose projects so as to maximize ρ, versus (ii) adopt projects incrementally so long as $\rho > r$.

Maximum-ρ Rule: If the internal rate ρ is interpreted as the average rate of growth, it may seem plausible that the investor should maximize ρ rather than wealth W_0. (Of course, maximizing a growth rate would scarcely make sense unless the initial outlay or scale of investment were held constant, which would not in general hold true.) The solution of (7) that maximizes ρ is shown in Figure 2 as $t = B$, notably earlier than the Present-Value solution $t = t^*$.

In favor of B over t^* it has been argued that, if the growth opportunity were to be replicated in perpetuity, returns from choosing the earlier "rotation period" B must ultimately dominate those associated with cutting on each cycle at t^*. That is certainly true. However, if the decision problem concerns infinite rotation rather than a one-time cutting, for a valid comparison the relevant Present-Value measure would have to be a generalized one that allows for the associated infinite sequence of discounted returns. It can be shown that this generalized Present Value does coincide with B if the growth opportunity can be reproduced on an ever-broadening scale (e.g., on new land) -- but only as funds are freed by cutting the tree or trees. This turns out to be an impossible or uninteresting case, because it implies that the productive opportunity must be of <u>infinite</u> market value if the maximized ρ exceeds the market interest rate r (and of zero value otherwise). In contrast, if the opportunity is a unique one which cannot be reproduced after cutting, as pictured in Figure 2, the simple $t = t^*$ solution remains correct. Another solution, $t = F$, found by the German forester Faustmann, is appropriate when the opportunity can be reproduced over time by cutting and replanting but cannot be broadened in scale. F would be found by maximizing the Present Value V_0 of an infinite sequence of rotations, each being a constant-scale replication of the original opportunity. Like all the correct solutions, it is equivalent to maximizing the present worth of the opportunity under the stated assumptions. (F is not shown in Figure 2 but would lie between B and t^*.)

ρ vs. r Comparison Rule: The Comparison Rule says to adopt any project whose internal rate ρ exceeds the market rate of interest r. This rule remains popular in business practice, in part because it offers a convenient division of labor: calculation of the ρ's on individual projects might be delegated to subordinates, while top decision-makers choose the cutoff rate r that corresponds to the relevant market interest rate faced by the firm. Unfortunately, however convenient such a decision of labor may be, once again this is not in general a correct method of project selection.

The difficulty with the Comparison Rule first came to be appreciated when it was discovered that a sequence of positive and negative cash flows could have more than one ρ serving as solution of equation (7) above. A project represented by the annual payments sequence $-1,5,-6$, for example, has two solutions: $\rho = 1$ and $\rho = 2$. (It can be shown that a project with $T+1$ dated elements may have as many as T solutions.) This of course destroys the idea that the internal rate can generally be identified with a growth rate; an outlay of one dollar cannot be said to grow at both 100% and 200%. Various answers have been offered to the puzzle of which ρ to use in such cases. But the difficulty is immediately explained and resolved if we think instead in terms of Present Value. It turns out that the sequence $-1,5,-6$ has positive V_0 (and is therefore worth adopting) for any constant market interest rate r between 100% and 200%, but at other values of r has negative Present Value (and should not be adopted). Perhaps even more illuminating is the project described by cash flows $-1,3,-2\frac{1}{2}$. This sequence has no real solution for ρ in equation (7), the reason in Present-Value terms being that V_0 is negative for any constant r. Yet this is a perfectly respectable investment opportunity. After all, there is no justification for postulating (as is implicitly done by the Comparison Rule) that the anticipated sequence of market interest rates $r_1, r_2, \ldots, r_T$ must be constant over time (always equal to a common r). It turns out that the cash-flow pattern $-1, 3, -2\frac{1}{2}$ has positive Present Value (i.e., the project would be worth adopting) for many possible non-constant interest-rate sequences -- for example, $r_1 = 100\%$ and $r_2 = 200\%$.

Summing up, therefore, the Present-Value Rule for investment decision -- corresponding as it does to the principle of maximizing wealth within the opportunities available -- is correct itself and also serves to define the range of validity of all the other rules considered.

Generalizations and Extensions

The preceding analysis needs to be extended in at least two important ways, so as to allow for: (1) uncertainty, and (2) imperfect and incomplete markets.

Uncertainty: Investment choices, involving as they do present sacrifice for future benefit, are peculiarly sensitive to uncertainty. However, so long as we can continue to assume a regime of complete and perfect markets, the Present-Value Rule is robust enough to retain validity even in a world of uncertainty. For, the proximate goal of any individual (or group of individuals organized in a firm or other joint enterprise) will still be to undertake productive activities so as to maximize wealth. Having achieved that goal, each and every individual investor will be in a position to distribute his attained wealth as desired over all possible dated contingencies in accordance with his time-preferences, degree of risk-aversion, and probability beliefs.

Economists use two main models for the analysis of uncertainty -- state-preference and mean-versus-variability analysis. Since the latter, under certain assumptions, can be regarded as a special case of the former, for our purposes attention can be limited to the state-preference model. If markets for state-claims are complete and perfect, any pattern of varying returns over states of the world at a given date has a certainty-equivalent in value terms as of that date. In equations (3) and (3'), the z_t for any project can now be interpreted as certainty-equivalents (rather than as simple cash flows) defined by:

(8) $$z_t = P_{t1}z_{t1} + P_{t2}z_{t2} + \cdots + P_{tS}z_{tS}$$

Here z_{ts} represents the cash flow at date t contingent upon state of the world s obtaining -- there being S distinguishable such states -- while P_{ts} is the price at which a unit claim to income in state s can be converted into (traded for) certainty income at date t.

Incomplete or imperfect markets: Markets are said to be incomplete if some objects of choice are non-tradable. For example, futures markets for some commodities at far-distant dates do not exist, nor is it possible to trade in claims contingent upon each and every conceivable future uncertain event. Markets are said to be imperfect if there are costs of trading -- e.g., brokerage fees, transaction taxes, or expenses in locating exchange partners. Any real-world regime of markets will necessarily be both incomplete and imperfect, but for some purposes the assumption of complete and perfect markets may be a usable idealization. Unfortunately, once we depart from this idealization the problem of investment decision criteria becomes very difficult. The reason is that the Separation Theorem fails. Only under complete and perfect markets is the concept of wealth or Present Value unambiguously defined, so that the choice of productive investments can be entirely disconnected from individuals' personal time-preferences, risk-preferences, beliefs, etc. Failure of the Separation Theorem particularly subverts the ability of investors to join together in undertaking large projects or groups of projects.

However, two different lines of analytical approach have yielded results of interest: (i) a number of techniques have been devised for locating "utility-free" or "efficient" investment choices. In general such techniques cannot determine an optimal project set, but they can serve to filter out options whose payoff patterns over dates and/or states are dominated by other available projects or project combinations. (ii) While investors' personal circumstances may diverge in innumerable ways, there should be some tendency for those similarly situated to group together. Thus, a firm whose investment opportunities yield far-future payoffs should tend to be owned by a "clientele" consisting of individuals with moderate time-preferences, willing to forego current dividends in the hope of large long-term gain. It follows that unanimity as to the investment choices to be made may after all govern <u>within</u> the firm, for example as to the discount rate to employ in calculating Present Value, even in the absence of perfect and complete markets.

NOTES ON THE LITERATURE

The modern theory of investment and intertemporal choice was set down in classic form by Irving Fisher as part of his great works on interest (1907, Ch. 8-9; 1930, Ch. 10-13). The seminal works on uncertainty theory include Arrow (1953) for the state-preference approach and Markowitz (1959) and Sharpe (1964) for the mean-versus-variability model. Choice over time and choice under conditions of uncertainty are integrated in the treatise by Hirshleifer (1970) that builds upon these foundations. All these topics have been followed up in an enormous literature, of which only a few illustrative instances can be cited here: on investment decision formulas, Samuelson (1976); on utility-free or dominant choices, Pye (1966), Hanoch and Levy (1969), and DeAngelo (1981). A survey of investment decision criteria used in current business practice appears in Schall, Sundem, and Geijsbeek (1978).

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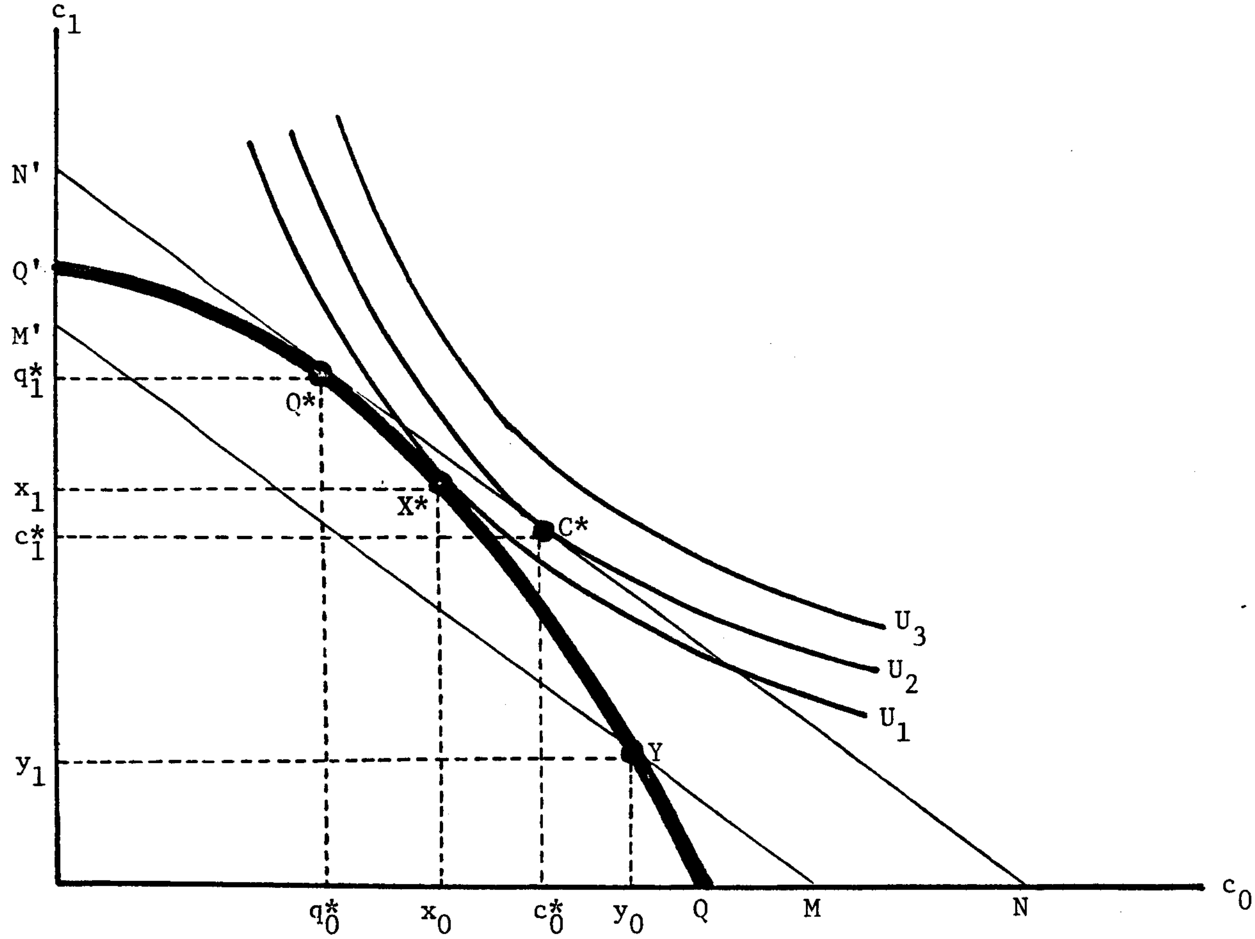

Fig. 1

Investment and saving in a 2-period model

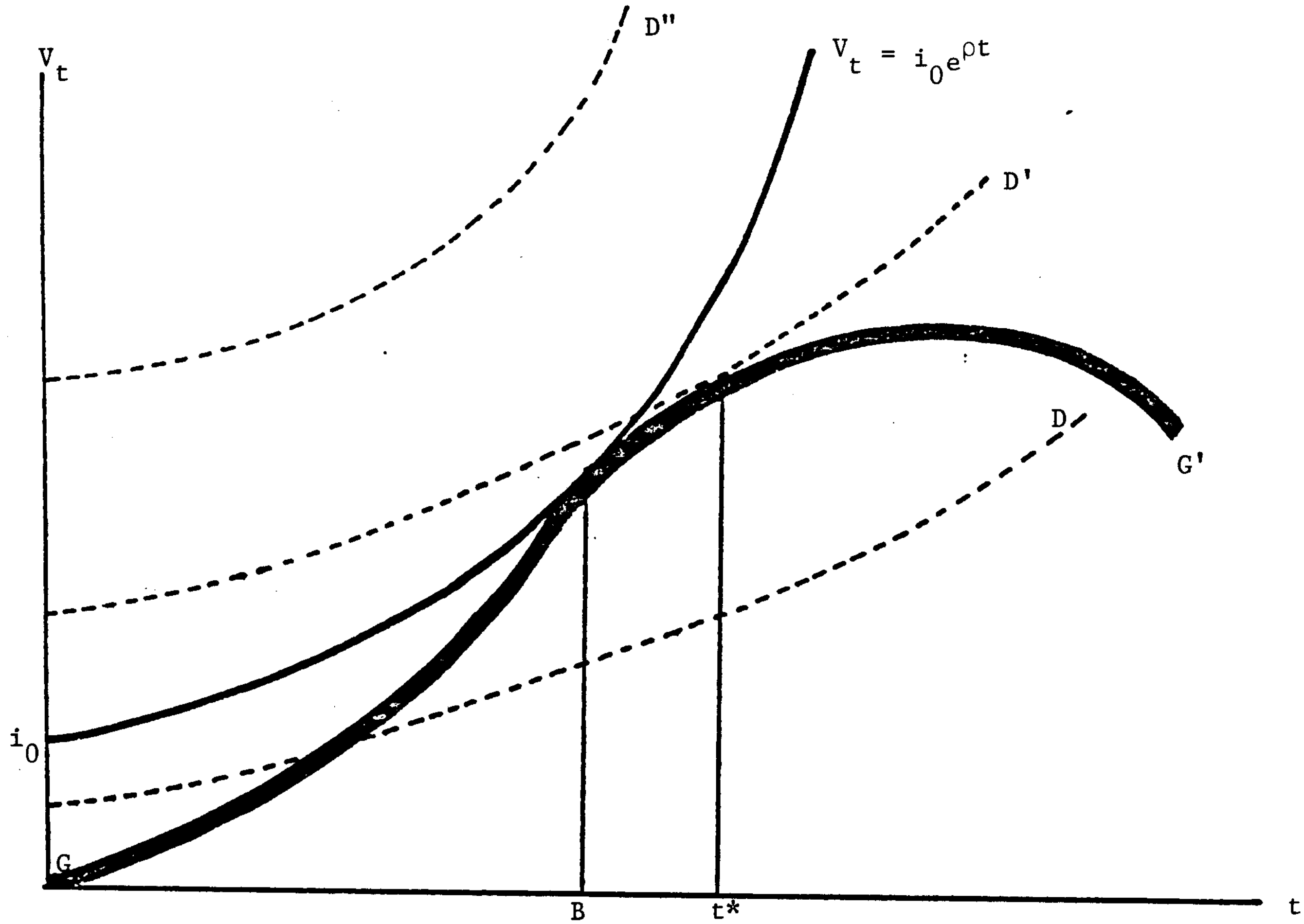

Fig. 2

Optimal duration of investment