ENDOGENOUS MONETARY AND FISCAL POLICIES UNDER ALTERNATIVE INSTITUTIONAL SETTINGS -- A GAME THEORETIC ANALYSIS

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Abstract

This paper analyzes a dynamic linear-quadratic game between the fiscal and the monetary authorities, the equilibrium outcome of which is the time path of public debt. A closed form solution for the game with infinite horizon is computed for the open-loop and the closed-loop Nash equilibrium, and for the cooperative equilibrium. The closed-loop time consistent Stackelberg equilibrium with the fiscal authority as the dominant player is simulated. These different solutions are compared and interpreted with reference to some new and old suggestions for a Monetary and Fiscal Constitution.

Some of the results are: (i) The time path of public debt over national income need not be explosive, even if the real interest rate is larger than the rate of growth of real output and in the absence of any debt monetization. (ii) Increasing the degree of central bank independence makes the time path of public debt more likely to be unstable in all the Nash equilibria, but not necessarily so in the Stackelberg Equilibrium. (iii) The rate of adjustment towards the steady state is fastest in the cooperative equilibrium and slowest in the closed-loop Nash equilibrium.

I. Introduction

In most industrial countries, neither the monetary nor the fiscal authorities can commit themselves to a specific policy rule. Some implications of this institutional feature have recently been studied in a number of models where the policymakers play a dynamic game against the private sector.[1] In all of these models, both policymakers are assumed to have the same objective function of the "representative" consumer in the economy; the lack of some fiscal instruments is then shown to give rise, under particular circumstances, to time inconsistencies in the design of optimal fiscal and monetary policies.

This paper analyzes the consequences of the institutional feature mentioned above, that arise when: (i) the monetary and the fiscal authorities have conflicting objectives; and (ii) the time path of public debt outstanding is non-neutral with respect to the ultimate goals pursued by the two policymaking authorities.

That the first assumption is a relevant feature of the real world is probably noncontroversial, given the differences in the decision processes of the two authorities. The second assumption presupposes that the behavior of the private sector is somehow going to be affected by the stock of public debt in circulation. Furthermore, it presupposes that the government cannot employ a combination of lump sum taxes and transfers with which to undo the real effects of public debt — in other words, as in the existing literature on the time consistency of optimal fiscal policy, it presumes the lack of some fiscal instruments. This paper does not derive points (i) and (ii) from more primitive behavioral assumptions.[2] It simply takes them as a starting point, and investigates their implications for the behavior of the two policymakers and for the time path of public debt.

The paper focuses on a situation particularly relevant for a number of industrial countries since the beginning of the 1980s: the stock of public debt as a proportion of national income is assumed to be above the time path optimal for the two authorities. In this scenario, both authorities face a grim dilemma: whether to adjust their policy instruments so as to slow down the rate of growth of public debt towards its optimal path; but in so doing they would be foregoing other (monetary or fiscal) objectives. Or not to adjust, in the hope that the burden of the adjustment will be borne mainly by the other policymaking authority; but in this case the adjustment of public debt may be slower or it may never come about.

If neither policymaker can commit itself to pursue a specific course of action, strategic issues play a crucial role. These issues are analyzed in the setting of a dynamic game between the fiscal and the monetary authority, the equilibrium outcome of which is the time path of public debt. The game is solved for different equilibrium concepts and under different hypotheses about the form of the objective functions of the two players. The equilibria are then compared and interpreted with reference to some new and old suggestions for institutional reforms. In order to be able to obtain a closed form solution, the model is formulated as a linear-quadratic dynamic game and the private sector is ignored.

The main findings are that: (i) the time path of public debt over national income need not be explosive, even if the real interest rate is larger than the rate of growth of real output and in the absence of any debt monetization, provided that the fiscal authorities care "sufficiently" about the deviations of the stock of debt outstanding from the optimal time path. This provision was ruled out by hypothesis in most previous theoretical analyses of the topic,[3] but seems to be roughly consistent with the empirical evidence of some industrial countries. (ii) Increasing the degree of central bank independence (appropriately defined) makes the time path of public debt more likely to be unstable in a Nash equilibrium, but not necessarily so in a Stackelberg equilibrium in which the fiscal authority acts as the Stackelberg leader. (iii) In a Nash equilibrium both players are better off when they play "naively" according to open-loop strategies, than if they choose closed-loop strategies. (iv) Even if one of the two players is playing a closed-loop strategy, the opponent may be better off in choosing a simple, open-loop, strategy. (v) The adjustment towards the steady state is fastest, and the model is most likely to be stable, in a cooperative equilibrium than in any of the other equilibria; and it is slowest in the closed-loop Nash equilibrium.

The outline of the paper is as follows. Section 2 sets up the basic model. Section 3 characterizes the general nature of the solution. Sections 4 through 6 compute the Nash equilibrium of the game under different hypothesis about the strategy spaces. Section 7 computes the cooperative equilibrium. Section 8 compares all previous equilibria. Section 9 takes up again the issue of central bank independence within the framework of a time consistent Stackelberg equilibrium, with the fiscal authority acting as the leader in the game. Section 10 contains the conclusions.

2. The Basic Model

In the following analysis it will be convenient to write the government budget constraint in continuous time and to scale all variables by nominal income:

$$\dot{d}(t) = rd(t) + f(t) - m(t) \tag{1}$$

where: $d(t)$ = stock of nominal outstanding public debt; $f(t)$ = fiscal deficits net of interest payments; $m(t)$ = creation of monetary base against liabilities of the Treasury,[4] all variables being scaled to nominal income; and where r can be shown to be the difference between the real rate of interest net of taxes and the rate of growth of real income.

Throughout the paper it will be assumed that the policymakers consider r as a parameter when solving their optimization problem. This assumption is needed in order to have a linear dynamics; however, it has two drawbacks. First of all, it implies that the government faces a flat demand for its debt. Secondly, it makes it impossible to model regulatory actions (such as portfolio constraints on financial intermediaries, controls on international capital movements, tax exemptions on public debt) undertaken by the central bank or by the fiscal authority with the aim of reducing the real interest rate. In order to reduce the damage inflicted by the first drawback, the model analyzed in this paper can be interpreted as referring to a small open economy facing perfect capital markets. The second drawback must be taken to imply that regulatory actions are chosen at longer intervals of time than the decisions of monetary and fiscal policy analyzed in this paper, and thus are considered as given when the latter are made.

The simplest possible way to describe the behavior of the fiscal and monetary authorities is then to suppose that they optimize the two following loss functions: the monetary authorities (M) choose the time path of $m(t)$ to minimize:

$$V^M(d(t)) = \frac{1}{2} \int_t^\infty ((m(s)-\bar{m})^2 + \tau d^2(s)] \, e^{-\beta(s-t)} ds \qquad \tau > 0 \qquad (2)$$

subject to the government budget constraint, equation (1), to some hypothesis — yet to be specified — about the process generating the time path of $f(t)$, and to some initial conditions on $d(t)$. And the fiscal authorities (F) choose the time path of $f(t)$ to minimize:

$$V^F(d(t)) = \frac{1}{2} \int_t^\infty [(f(s)-\bar{f})^2 + \lambda d^2(s)]\, e^{-\beta(s-t)} ds, \quad \lambda > 0 \qquad (3)$$

also subject to the government budget constraint and to some hypothesis about the time path of $m(t)$.

Equation (2) states that M attempts to minimize deviations of changes in the domestic component of base money from a given (constant) target, $\bar{m}$; and deviations of the stock of outstanding public debt from a desired value, for notational convenience taken to be zero. The parameter τ indicates the relative weight assigned by M to the two objectives. Similarly, equation (3) states that F attempts to minimize deviations of fiscal deficits net of interest payments from a given target, $\bar{f}$, and deviations of the stock of outstanding public debt from zero. The parameter λ indicates the relative weight assigned to the two objectives. Recall that all variables are expressed in proportion to nominal income.

The final objectives of monetary and fiscal policies can be thought of as being implicit in the desired targets for m, f, d. The target for m is presumably determined with reference to balance of payments or exchange rate objectives, or, in the case of a closed economy, to the desired rate of growth of nominal income or prices.

The preferences about f, the value of fiscal deficits net of interest payments, could reflect both standard macroeconomic objectives and "public choice" kind of considerations -- see, for instance, Buchanan & Wagner (1977), Mueller (1980): fiscal deficits are associated with wealth transfers to particular constituencies and interest groups; the political costs of such transfers are low since either they fall on future generations (if the deficit is financed by means of public debt); or they are hidden in the inflation tax (if the deficit is financed by monetary accommodation), for which the fiscal authorities can always blame someone else -- the trade unions, the retailers, the price of oil, and so on. Given the purpose of the paper, it seems unnecessary to try and gain realism by complicating the model in order to distinguish between preferences about expenditures and preferences about taxation. The key strategic variable in the determination of the time path of public debt is the size of fiscal deficits net of interest payments, and to simplify the computations this is also going to be the only relevant control variable for the fiscal authorities throughout the paper.[5]

The preferences about d, for both fiscal and monetary authorities, can be justified on a number of grounds: (i) issuing public debt to the private sector of the economy is formally equivalent to making a current lump sum transfer and paying for it with future lump sum taxes equal in present value to the transfer. Under appropriate hypothesis about the private sector, this combination of transfers and taxes can be shown to give rise to a smaller steady state capital stock (for a closed economy) or to a larger external debt in the steady state and during the dynamic adjustment (for a small open economy).[6] Hence, the existence of an optimal capital to labor ratio or of an optimal time path of external debt would imply an optimal rate of growth for public debt. In addition, intergenerational redistributions of risk or intragenerational redistributions of wealth could contribute to determine a desired time path of public debt.[7] (ii) If lump sum taxes are not available, a larger stock of public debt implies larger tax distortions in order to pay interest on the debt.[8] Moreover, whenever the interest rate paid on national debt fluctuates, the fluctuations of taxes needed to pay for the debt are larger the larger is the stock of public debt.[9] (iii) If interest bearing national debt is issued only in large denominations, and if it can be converted into perfectly divisible intermediary's liabilities only by bearing some real resource cost, then any positive amount of such debt is inefficient.[10]

Any of these reasons would, by itself, be sufficient to include the stock of public debt in the objective function of both players. The assumption that the desired value of d and the rate of time preference, β, are the same for both players simplifies notations and computations, with practically no loss of generality. Moreover, assuming that the desired time path for d is the same for both authorities allows τ to be interpreted as the extent to which M is independent from F. For $\tau \to \infty$, monetary policy has the only role of financing a budget deficit exogenously chosen by the fiscal authorities. With $\tau = 0$, we have an independent central bank absolutely committed to pursuing its intermediate monetary target. The present situation in most industrial countries belongs to the intermediate case, with $0 < \tau < \infty$. i.e., a relatively independent central bank, which nonetheless cannot completely disregard the size of the stock of public debt.

The model could be modified by adding some preferences for money growth in the objective function of fiscal authorities, or some preferences about budget deficits in the objective function of the central bank; provided there remain some differences between the two objective functions (i.e., provided there is some form of conflict), the nature of the results would not change.

3. The General Solution of the Dynamic Game Under Infinite Horizon and Complete Information

In the next four sections I will compute the solution of the dynamic game under different hypotheses about the strategy spaces and the rules of the game, but always maintaining the assumptions of infinite horizon and complete information. Since the game is linear-quadratic, all solutions will have the same form:

$$m(t) = \theta_o + \theta_1\, d(t) \qquad \text{(i)}$$

$$f(t) = \pi_o - \pi_1\, d(t) \qquad \text{(ii)} \qquad (4)$$

$$\dot{d}(t) = (r-\pi_1-\theta_1)\, d(t) + (\pi_o-\theta_o) \qquad \text{(iii)}$$

where θ_i, π_i are coefficients that will have to be determined and which depend on the original parameters of the model. The next four sections will solve explicitly for the θ_i and π_i when both players move simultaneously and: (a) They both take as given the future actions of the opponent — i.e., they both choose among open-loop strategies; (b) They both take as given the decision rule according to which the future actions of the opponent are set, i.e., they both choose among closed-loop strategies; (c) The fiscal authority is a "dominant" — or rather "sophisticated" -- player in that it chooses a closed-loop strategy, while the monetary authority is restricted to choosing an open-loop strategy; (d) The two authorities cooperate by setting m and f so as to minimize a weighted average of their objective functions.

In cases (a) and (d) the equilibrium, if stable, will be saddle-path stable. As it will be shown below, the transversality condition will then impose the choice of the negative root of the dynamic system. In the other two cases it is impossible to say whether there is more than one negative root. I will then appeal to McCallum (1983) criterion of "minimal set of state variables" to choose among the multiple roots of the dynamic system -- see Section 4 of the Apendix for details.

Indicating the stable root of the dynamic system by $-\gamma = r - \pi_1 - \theta_1$ and integrating (4.iii), we have:

$$d(t) = d_o e^{-\gamma t} + d^s \qquad (5)$$

where d_o is the initial condition (i.e., the stock of public debt outstanding when the game is started) and where d^s is the steady state value of public debt. Throughout the paper it will be assumed that $d_o > 0$ — i.e., that, when the game is started, d is above its desired path. Using (4) and (6), it follows that:

$$m(t) = m^s + m_o e^{-\gamma t} \tag{6}$$

$$f(t) = f^s - f_o e^{-\gamma t} \tag{7}$$

where m^s and f^s are the steady state values of m and f, and where $m_o = \theta_1 d_o$, $f_o = \pi_1 d_o$.

In the open-loop Nash equilibrium and in the cooperative equilibrium, the rate of growth of all variables will be shown to be real. This, together with the result on saddle path stability, insures that in these two cases, whenever the system is stable, the adjustment towards the steady state is monotonic for all variables. If one or both players can choose among closed-loop strategies, however, the rate of growth of the dynamic system could be complex for some parameter values, and monotonicity of the adjustment process is no longer insured. In all the cases considered, furthermore, it will be shown that θ_1, $\pi_1 > 0$. Thus, a sudden increase in the stock of public debt outstanding, due for instance to an unexpected cyclical deficit, will lead immediately to some debt monetization and to a reduction of fiscal deficits net of interest payments.[11] If the system is stable, over time monetary policy will become more restrictive and fiscal policy more expansionary while the stock of public debt will converge towards its steady state value. In the closed-loop equilibria, fluctuations in m and f during the adjustment towards the steady state cannot be ruled out. Naturally, this qualitative description of the adjustment process hinges crucially on the postulated absence of adjustment costs for both authorities and on having neglected serially correlated stochastic shocks.

4. Open-Loop Nash Equilibrium

This is the simplest case: both players move simultaneously taking as given the current and future actions of the opponent. The current value Hamiltonian for the monetary authority is — the time variable will be omitted when not indispensible:

$$H_M = \frac{1}{2}(m-\bar{m})^2 + \frac{1}{2}\tau d^2 + \mu_1(rd+f-m) \tag{8}$$

where μ_1 is the costate variable associated with (1); the first order conditions, in addition to (1), are:

$$\begin{aligned} m &= \bar{m} + \mu_1 \\ \dot{\mu}_1 &= (\beta-r)\,\mu_1 - \tau d \end{aligned} \tag{9}$$

Similarly, the current value Hamiltonian for the fiscal authority is:

$$H_F = \frac{1}{2}(f-\bar{f})^2 + \frac{1}{2}\lambda d^2 + \mu_2(rd+f-m) \tag{10}$$

where μ_2 is the costate variable associated with (1); the first order conditions yield:

$$\begin{aligned} f &= \bar{f} - \mu_2 \\ \dot{\mu}_2 &= (\beta-r)\,\mu_2 - \lambda d \end{aligned} \tag{11}$$

In addition, the transversality conditions give the following sufficient conditions (see Arrow and Kurz (1970), Cohen and Michel (1984)):

$$\lim_{t\to\infty} \mu_i(t)\, d(t) = 0, \quad i = 1,2 \tag{12}$$

Since the optimization problem of both players is linear-quadratic, condition (12) enables us to conclude that, if there exists a stable solution to the dynamic system made up of (1), (9) and (11), it is the unique equilibrium to the dynamic game.

The closed form solution can be computed by means of the method of undetermined coefficients, as follows:

From (4):

$$\begin{aligned} \dot{m} &= \theta_1 \dot{d} \\ -\dot{f} &= \pi_1 \dot{d} \end{aligned} \tag{13}$$

From (9) and (11), using (4):

$$\begin{aligned} \dot{m} &= \dot{\mu}_1 = (\beta-r)(\theta_o+\theta_1 d-\bar{m}) - \tau d \\ -\dot{f} &= \dot{\mu}_2 = (\beta-r)(\bar{f}-\pi_o+\pi_1 d) - \lambda d \end{aligned} \tag{14}$$

Putting together (13) and (14):

$$\begin{aligned} \dot{d} &= \frac{1}{\theta_1}(\beta-r)(\theta_o+\theta_1 d-\bar{m}) - \frac{\tau}{\theta_1} d \\ \dot{d} &= \frac{1}{\pi_1}(\beta-r)(\bar{f}-\pi_o+\pi_1 d) - \frac{\lambda}{\pi_1} d \end{aligned} \tag{15}$$

Equating coefficients of (15) and (1), using (4) again and solving, yields (see Section 1 of the Appendix):

$$\begin{aligned} \theta_1 &= \frac{\tau}{\beta+\gamma^o-r} && \text{(i)} \\ \pi_1 &= \frac{\lambda}{\beta+\gamma^o-r} && \text{(ii)} \\ \gamma^o &= \frac{-\beta + \sqrt{(\beta-2r)^2 + 4(\tau+\lambda)}}{2} && \text{(iii)} \end{aligned} \tag{16}$$

Notice that the rate of growth, γ^o, is always a real number. Moreover, using (1), (9) and (11), one obtains the steady state values:

$$d^{so} = \frac{(\bar{f}-\bar{m})\ (\beta-r)}{\lambda+\tau-r(\beta-r)} \qquad \text{(i)}$$

$$m^{so} = \bar{m} + \frac{(\bar{f}-\bar{m})\tau}{\lambda+\tau-r(\beta-r)} \qquad \text{(ii)} \qquad (17)$$

$$f^{so} = \bar{f} - \frac{(\bar{f}-\bar{m})\lambda}{\lambda+\tau-r(\beta-r)} \qquad \text{(iii)}$$

where the o superscript in (16) and (17) reminds us that this is the solution to the open-loop equilibrium.

Equations (16) and (17) contain the following relevant information:

(i) A sufficient condition for the model to be stable — i.e., for $\gamma^o > 0$, is that

$$\lambda + \tau > r(\beta-r) \qquad (18)$$

This condition is always met if $r < 0$ (that is, if the rate of growth of real output is larger than the real interest rate net of taxes), which is the familiar condition discussed for instance in McCallum (1981), Sargent and Wallace (1981), Darby (1984). However, even for $r > 0$, the condition is met if both players care "enough" about the stock of debt outstanding. In the "Unpleasant Monetarist Arithmetics" of Sargent and Wallace (1981), it was assumed that $\lambda = 0$, $r > 0$. Their conclusion, consistent with (18), was that $\tau = 0$ was incompatible with stability of the time path of public debt over national income. Some of the existing literature -- cf., for instance Darby (1984), has questioned the realism of the assumption that $r > 0$ forever. Condition (17) highlights the restrictiveness of the other assumption made in Sargent and Wallace (1981), namely that fiscal policy be completely unresponsive to the size of the stock of public debt outstanding.[12,13]

(ii) Changes in the original parameters of the model have the following impact on the equilibrium solution:

$$\frac{\partial\gamma^o}{\partial\lambda},\ \frac{\partial\gamma^o}{\partial\tau} > 0;\quad \frac{\partial\gamma^o}{\partial r},\ \frac{\partial\gamma^o}{\partial\beta} < 0$$

Thus, for instance, a more independent central bank (that is, a lower τ) or a higher real interest rate cause a slower adjustment of public debt to its steady state value. Moreover:

$$\frac{\partial\theta_1}{\partial\tau} > 0, \quad \frac{\partial\pi_1}{\partial\tau} < 0$$

and symmetrically for changes in λ. Thus, as one would expect, a more independent central bank will be less responsive to public debt, but will force the fiscal authority to be more responsive.

Finally:

$$\frac{\partial d^{so}}{\partial\tau}, \frac{\partial d^{so}}{\partial\lambda} < 0, \quad \frac{\partial f^{so}}{\partial\tau} > 0, \quad \frac{\partial m^{so}}{\partial\tau} \gtrless 0 \quad \text{as} \quad \lambda + \tau \lessgtr 1 + r(\beta-r)$$

and similarly for $\frac{\partial f^{so}}{\partial\lambda}$, $\frac{\partial m^{so}}{\partial\lambda}$;

$$\frac{\partial m^{so}}{\partial r} > 0 \quad \text{and} \quad \frac{\partial f^{so}}{\partial r} < 0 \quad \text{for} \quad \beta > r, \quad \frac{\partial d^{so}}{\partial r} \gtrless 0$$

$$\frac{\partial d^{so}}{\partial(\bar{f}-\bar{m})} > 0, \quad \frac{\partial m^{so}}{\partial\bar{m}} > 0 \quad \text{for} \quad \lambda > r(\beta-r), \quad \frac{\partial f^{so}}{\partial\bar{m}} > 0 \quad \text{for} \quad \lambda + \tau > r(\beta-r).$$

Thus, for instance, a more independent central bank leads to a larger steady state stock of public debt outstanding and to a smaller steady state deficit, but it can either increase or reduce the degree of debt monetization, m^{so}. A more restictive monetary policy -- that is a lower value of $\bar{m}$ would have effects working in the same direction.

Notice that an increase in r takes both authorities further away from their desired targets (assuming that $\bar{f} > \bar{m}$). This result can explain why monetary and fiscal authorities can be induced to collude and to impose regulations on the credit system aimed at reducing the real interest rate (such as portfolio constraints or international capital controls). Naturally, regulatory constraints are only one — possibly the worst — device by which the authorities can attempt to lower the real interest rate. An alternative device consists in issuing public debt in a large menu of forms: with variable interest rate, indexed to the price level, denominated in foreign currencies, and so on. To the extent that these devices succeed in lowering r, they bring about a faster adjustment of public debt to its steady state value, a monetary policy closer to the desired target, but also larger deficits net of interest payments.

5. Closed-Loop Nash Equilibrium

The hypothesis that both players take the future actions of the opponent as given when selecting their own strategies is unsatisfactory: as argued in the introductory section, neither player can commit itself to a future course of action. Hence, even if both players move simultaneously within each single period, they ought to take into account the fact that their current actions will influence the future actions of the opponent through their effect on the time path of public debt. In other words, they ought to take as given the opponent's decision rule rather than its actions. This more sophisticated behavior on the part of both players leads to what is known as a closed-loop Nash equilibrium. In order to avoid the multiplicity of equilibria that emerges in games with infinite horizons, I will restrict the strategy spaces of both players to be closed-loop but memoryless.[14]

Consider the optimization problem of the monetary authorities first. They know that future deficits will be set according to (4.ii); thus, substituting (4.ii) into (2), we obtain the following current value Hamiltonian:

$$H_M = \frac{1}{2}(m-\bar{m})^2 + \frac{1}{2}\tau d^2 + \mu_1(\bar{\pi}_o + (r-\pi_1)d-m) \tag{19}$$

The first order conditions are:

$$m = \bar{m} + \mu_1$$
$$\dot{\mu}_1 = (\beta+\pi_1-r)\ \mu_1 - \tau d \tag{20}$$

Repeating the same procedure for the fiscal authority, we get:

$$f = \bar{f} - \mu_2$$
$$\dot{\mu}_\tau = (\beta+\theta_1-r)\ \mu_2 - \lambda d \tag{21}$$

In addition, the transversality condition (12) still holds, forcing us to choose the stable root of the dynamic system, whenever it exists.

The equilibrium solution can be computed by means of the method of undetermined coefficients, as in Section 4. Section 2 of the Appendix shows that:

$$\theta_1 = \frac{\tau}{\beta+\gamma^c+\pi_1-r}, \quad \pi_1 = \frac{\lambda}{\beta+\gamma^c+\theta_1-r} \tag{22}$$

where $\gamma^c = \theta_1 + \pi_1 - r$ is the rate of adjustment of the dynamic system to its steady state in this closed-loop Nash equilibrium. Using (22) and the definition of γ^c, it can be shown that, as in the open-loop Nash equilibrium, $\frac{\partial\theta_1}{\partial\tau} > 0$, $\frac{\partial\pi_1}{\partial\tau} < 0$, $\frac{\partial\gamma^c}{\partial\tau} > 0$: that is, a more independent central bank monetizes less and forces the fiscal authority to bear a larger share of the adjustment; the first effect prevails, so that the adjustment of public debt is slower the more independent is the central bank. See Section 2 of the Appendix for the formal details and for a computation of the steady state values of d, m and f. As shown in the Appendix, for certain parameter values γ^c could be complex even if stable, so that monotonicity of the adjustment process is no longer guaranteed.

6. "Asymmetric" Closed-Loop Nash Equilibrium

Suppose that the fiscal authority is a "sophisticated" player choosing a closed-loop strategy. Will the central bank be better off if it plays "naively" by choosing an open-loop strategy, or if it too chooses a closed-loop strategy? The answer to this question is of interest not so much because the monetary authorities would deliberately choose to behave "naively", but because open-loop strategies could be interpreted as constitutional rules chosen at the starting date and then imposed on the monetary authority in the form of binding committments. In other words, an open-loop strategy can be interpreted as a "simple" constitutional rule, a closed-loop strategy would be a more flexible and complicated rule.[15]

This issue is analyzed more thoroughly in Section 8 below, where all the different equilibria are compared to each other. Here I briefly describe how to obtain a closed form solution for a game in which the monetary authority is restricted to choosing among open-loop strategies, whereas the fiscal authority chooses a closed-loop strategy. For want of a better name, I call this equilibrium an "asymmetric closed-loop Nash equilibrium."[16] The opposite case, in which the fiscal authority chooses among open-loop strategies and the central bank is allowed to implement a closed-loop strategy, is symmetric to this one.

The procedure to compute the closed form solution for this game should by now be familiar. The monetary authority policy rule is obtained exactly as in Section 4, equations (8) and (9). The fiscal authority policy rule, instead, is obtained as in Section 5, equation (21). The resulting expressions for θ_1 and π_1 are (16.1) and (22) respectively. Section 3 of the Appendix shows that, as before, $\frac{\partial\theta_1}{\partial\tau} > 0$, $\frac{\partial\pi_1}{\partial\tau} < 0$ and $\frac{\partial\gamma^A}{\partial\tau} > 0$, where γ^A is the rate of adjustment to the steady state in this particular equilibrium. Thus, even if the fiscal authority is sophisticated and takes fully into account the future actions of the central bank when setting current fiscal deficits, a more independent central bank (whether "naive" as here or "sophisticated" as in a closed-loop Nash equilibrium) leads to a slower adjustment towards the steady state. Section 3 of the Appendix computes the steady state values of d, m and f, and characterizes γ^A more precisely. Again, γ^A could take up complex values even if stable.

7. Cooperative Equilibrium

What would be the time path of public debt, deficits and revenue from money creation if the decisions of monetary and fiscal policy could be coordinated by a single decision unit? The issue is very important, for this cooperative equilibrium has a very natural interpretation in terms of institutional settings: Congress could be in charge of both fiscal and monetary policy decisions, for instance by setting operative guidelines for the time path of fiscal deficits, revenue from money creation and public debt for a prolonged interval of time.[17]

The answer is quite simple; from an analytical point of view, the dynamic game is transformed into an optimal control problem, by merging the two objective functions into a single one:

$$V(d(t)) = \min_{m(t),f(t)} \frac{1}{2} \int_t^{\infty} [(f(s)-\bar{f})^2 + \omega(m(s)-\bar{m})^2 + (\lambda+\tau\omega)d^2(s)] \cdot$$

$$\cdot\, e^{-(s-t)\beta}\, ds \tag{23}$$

where ω is the weight given to the original central bank objectives relative to those of the fiscal authority. The optimization problem is now solved by a single agent who controls both $m(t)$ and $f(t)$ and is subject to the government budget constraint, equation (1). The first order conditions are:[18]

$$
\begin{aligned}
f - \bar{f} &= \omega(\bar{m}-m) = -\mu && \text{(i)} \\
\dot{\mu} &= (\beta-r) - (\lambda+\omega\tau)d && \text{(ii)}
\end{aligned}
\qquad (24)
$$

where μ is the costate variable associated with equation (1). In addition, the transversality condition (12) and the initial condition on d still hold as before.

Equation (24.i) implies that the deviations of fiscal and monetary policy from their respective targets are always going to be proportional to each other, the constant of proportionality being ω, the weight assigned to the original central bank objectives. This makes intuitive sense; in a cooperative equilibrium both players internalize the costs born by the opponent; thus the costs of deviations of f and m from their desired targets, weighted by ω, are equated at the margin.

The closed form solution can be easily calculated either with the method of undetermined coefficients, or by solving the characteristic equation of the dynamic system consisting of (1) and (24). Section 4 of the Appendix contains the computations and shows that:

$$
\begin{aligned}
\gamma^P &= \frac{-\beta + \sqrt{(\beta-2r)^2 + 4(\lambda+\omega\tau)\left(\frac{1+\omega}{\omega}\right)}}{2} \\
\pi_1 &= \frac{\lambda+\omega\tau}{\gamma^P+\beta-r} \\
\theta_1 &= \frac{\tau+\lambda/\omega}{\gamma^P+\beta-r}
\end{aligned}
\qquad (25)
$$

where the p superscript denotes the cooperative equilibrium. As in all previous sections, $\frac{\partial\gamma^P}{\partial\tau} > 0$. As in the open-loop Nash equilibrium, γ^P is real.

The steady state values of m, f and d are shown in Section 4 of the Appendix. As in the open-loop Nash equilibrium:

$$\frac{\partial d^{sp}}{\partial \tau} < 0, \quad \frac{\partial f^{sp}}{\partial \tau} > 0;$$

However here, unlike in the non-cooperative equilibrium, $\frac{\partial m^{sp}}{\partial \tau} < 0$. That is, an increase in the weight assigned by the monetary authorities to the stock of public debt outstanding leads to a more rapid adjustment towards the steady state, and to smaller deviations of <u>both</u> f^{sp} and m^{sp} from their respective targets. Intuitively, the burden of the faster adjustment is now shared evenly by the two players. Naturally, here the parameter τ can no longer be interpreted as the degree of central bank independence.

Notice that for $\omega \to 0$ we get the same results that we would obtain in the open-loop Nash equilibrium for $\tau \to \infty$. In other words, assigning zero weight to the monetary objectives in the cooperative equilibrium is equivalent to having a central bank which has completely lost its independence from the Treasury and whose only role is to provide sufficient revenue from money creation.

8. Comparision of the Different Equilibria

The four cases considered thus far differ along two relevant dimensions: the steady state values of debt, deficits and monetization, and the speed of adjustment towards the steady state. As already remarked in Section 4 above, in all four cases the steady state is unique. However, whereas the adjustment is always monotonic in the open-loop Nash equilibrium and in the cooperative equilibrium, it could be oscillatory when one or both players choose closed-loop strategies. The conditions insuring stability are obviously least restrictive for the equilibria that, if stable, exhibit fastest adjustment.

Comparing the expressions that characterize the speed of adjustment, γ, in Sections 1-4 of the Appendix, it is easily established that:

$$\gamma^{p} > \gamma^{o} > \gamma^{A} > \gamma^{c} \tag{26}$$

where the superscripts are: p for cooperative, o for open-loop Nash, A for asymmetric closed-loop Nash, c for closed-loop Nash.

The intuition behind (26) can best be grasped by means of the following analogy.[19] Think of the model as a game between two agents producing the same public good. The public good is the reduction of the time path of public debt towards the optimal value; the cost of producing it consists in the deviations of the policy instruments from their desired targets, $\bar{f}$ and $\bar{m}$. The parameter γ is then the rate at which the public good is produced. If the two agents cooperate, they internalize the benefit of the public good to the opponent, hence the quantity produced is larger than in any noncooperative equilibrium (i.e., γ^{p} is the largest of all γ's). If they do not cooperate, they are going to produce the least amount of the public good in the case in which they are both "sophisticated" players; for in this case, they both realize that, whenever one of them reduces its production of the good, its opponent will find it optimal to step up its own production to some extent; if both take advantage of this fact, the equilibrium production will be smaller than in any of the other Nash equilibria (i.e., γ^{c} is the smallest of all the γ's). If only one of them takes advantage of it (that is, if only one of them chooses a closed-loop strategy), the production of the public good is larger than in the previous case, but smaller than in the case in which both act "naively" (i.e.: $\gamma^{o} > \gamma^{A} > \gamma^{c}$).

Exactly the same intuition explains the rankings of the steady state value of public debt over national income in the four different cases (the ranking is established in Sections 1-4 of the Appendix):

$$d^{sp} < d^{so} < d^{sA} < d^{sc} \tag{27}$$

where the s superscript stands for "steady state" and the second superscript is as in (26).

The comparison of m^s and f^s in the four equilibria leads to ambiguous results, except for the following two inequalities that hold for any parameter value:

$$m^{sA} > m^{so}, \quad f^{sA} > f^{sc} \tag{28}$$

Equations (27), (28) enable us to conclude that, in the steady state: (i) a central bank restricted to choosing open-loop strategies is better off if its opponent is also restricted to choose among open-loop strategies than if the opponent is unrestricted; (ii) the fiscal authority is better off if it is the only one to play closed-loop strategies than if both players are allowed to do so.

In the other cases, some simple calculations establish that:

$$\begin{array}{llr}
m^{sA} \lesseqgtr m^{sc} \quad \text{as} \quad \lambda \gtreqless r(\beta+\theta_1-r) & & \text{(i)} \\
f^{sA} \gtreqless f^{so} \quad \text{as} \quad \tau \gtreqless r(\beta-r) & & \text{(ii)} \\
m^{sc} > m^{so} \quad \text{if} \quad \lambda < \tau & & \\
\quad < \quad \text{if } \lambda > \tau \text{ and } \lambda > r(\beta+\theta_1)\dfrac{\pi_1}{\pi_1-\theta_1} & & \text{(iii)} \qquad (29) \\
f^{sc} < f^{so} \quad \text{if} \quad \lambda > \tau & & \\
\quad > \quad \text{if} \quad \lambda < \tau \text{ and } \tau > r(\beta+\pi_1-r)\dfrac{\theta_1}{\theta_1-\pi_1} & & \text{(iv)}
\end{array}$$

The rankings involving the steady state values of the cooperative equilibrium, m^{sp} and f^{sp}, are also ambiguous and depend on the relative weight parameter ω.

There is a common intuition behind the conditions in (29); consider for instance (29.ii). Recalling (17) in Section 4 above, if $\tau < r(\beta-r)$ then the central bank by itself is unable to stabilize the time path of public debt (i.e., if $\lambda = 0$ the system would be unstable). Condition (29.ii) tells us that in this case $f^{sA} < f^{so}$, that is, in the steady state the fiscal authority would be better off by playing open-loop rather than closed-loop, given that the opponent is playing open-loop. In other words, if the central bank is unable to stabilize the time path of public debt by itself, a closed-loop strategy, in the long run, retorts itself against the fiscal authority. The same happens in (29.i): if the central bank faces a "tough" opponent (tough in the sense that it does not care enough about the stock of public debt outstanding to be able to stabilize it), the central bank would do better, in the long run, by playing according to a simple open-loop rule than according to a more sophisticated closed-loop strategy, even if the fiscal authority keeps playing its closed-loop strategy.

9. Closed-Loop Stackelberg Equilibrium With the Fiscal Authority As the Leader

As argued elsewhere (Tabellini (1983)), the hypothesis that both players move simultaneously is inadequate to describe the strategic problem faced by the two authorities: under the institutional setting presently existing in all countries, monetary policy is implemented sequentially, as an ongoing process. At each stage of the process the central bank has the opportunity of deviating from the course of action previously announced. The decision process behind the determination and implementation of fiscal deficits, instead, is much longer; decisions already taken are practically irreversible within the current period. In the terminology of game theory: the fiscal authority moves first, and the central bank is forced to play as the Stackelberg follower in the game, taking the size of current fiscal deficits as given.

Since the game is dynamic, however, the monetary authority will still move before the fiscal authority has committed itself to a specific course of action for the future. Thus, in a closed-loop Stackelberg equilibrium the central bank will take as given the current but not the future actions of the fiscal authority — see also p. 24 below.

It is well known[20] that the optimal policy for the Stackelberg leader in a dynamic game is time inconsistent. Intuitively: when the game is started the fiscal authority would find it optimal to announce that it will have large fiscal deficits in the future, so as to induce the central bank to monetize a large portion of the debt right away. Once the central bank has done so, however, the fiscal authority has no incentive to set large fiscal deficits, and hence it will deviate from the announcement.[21]

The time consistent optimal fiscal policy is found by imposing the condition that the Stackelberg leader neglects the effect of its future actions on the current behavior of the follower. Naturally, in a closed-loop Stackelberg equilibrium both players track down the effect of their current actions on the time path of the state variable and through this channel on the future behavior of the opponent — see also the discussion in footnote 16, p. 16 above.

In order to compute the time consistent solution, it is most convenient to rewrite the model in discrete time and with a finite time horizon. Thus, the budget constraint becomes:

$$d_{t+1} = rd_t + f_t - m_t \tag{1'}$$

where now $r = \frac{1+i}{1+g}$, i being the real interest rate and g being the rate of growth of real output; and the objective functions of the two players are:

$$V_0^m(d_o) = \min_{\{m_t\}} \frac{1}{2} \sum_{t=o}^{T-1} [(m_t-\bar{m})^2 + \tau d_t^2]\rho^t + \frac{1}{2} \tau d^2(T)\rho^T \tag{2'}$$

$$V_0^F(d_o) = \min_{\{f_t\}} \frac{1}{2} \sum_{t=o}^{T-1} [(f_t-\bar{f})^2 + \lambda d_t^2]\rho^t + \frac{1}{2} \lambda d^r(T)\rho^T \tag{3'}$$

with d_0 given and with both players setting their policy instruments in periods $t = 0,\ldots,T-1$. That is: the last period of the game is at $t = T-1$.

In this section I characterize the closed-loop Stackelberg equilibrium for this game, with the fiscal authority acting as the dominant player. I then simulate the solution for different parameter values.

The procedure for finding the equilibrium is as follows. First the central bank's optimization problem is solved. Since by assumption the central bank is the Stackelberg follower, in each period it takes current fiscal deficits as given. In a closed-loop equilibrium, however, future deficits are not taken as given: as in the closed-loop Nash equilibrium, what is taken as given is the rule by which future deficits are generated in equilibrium. The solution to this optimization problem is computed recursively, appealing to Bellman's optimality principle and working backwards from the last period. Its general form is that of a linear equation expressing m_t as a function of d_t and f_t.

Next, the same procedure is repeated to solve the fiscal authority optimization problem; but now one additional requirement is imposed: in each period the fiscal authority takes into account the effect that current fiscal deficits have on current as well as on future actions of the central bank (on current monetary policy actions since the fiscal authority is the Stackelberg leader; and on future monetary policy actions since we want a closed-loop equilibrium). The solution to this optimization problem is a linear equation expressing f_t as a function of d_t. Since the strategies of both players have been computed by means of the Bellman's optimality principle, they are time consistent.[22]

(i) The Monetary authority optimization problem: Appealing to Bellman's optimality principle, rewrite (2') for the general period t as:

$$V_t^M(d_t) = \underset{m_t}{\text{Min}} \left\{\frac{1}{2}(m_t-\bar{m})^2 + \frac{1}{2}\tau d_t^2 + \rho V_{t+1}^M(d_{t+1})\right\} \tag{30}$$

subject to the government budget constraint, (1'), for given f_t and given that f_s, $s > t$, is set according to the fiscal policy reaction function (equation (34) below).

The first order condition for the optimum is:

$$m_t = \bar{m} + \rho V_{t+1}^{M'}(d_{t+1}) \tag{31}$$

where, appealing to (1') and to the hypothesis that f_t is taken as given in period t, we have implicitly set $\frac{\partial d_{t+1}}{\partial m_t} = -1$.

In the last period of the game,

$$V_T^{M'}(d_T) = \tau d_T \tag{32}$$

In any other period:

$$V_{t+1}^{M'}(d_{t+1}) = \tau d_{t+1} + \rho V_{t+2}^{M'}(d_{t+2}) \frac{\partial d_{t+2}}{\partial d_{t+1}} \tag{33}$$

where, since the monetary authority is playing a closed-loop strategy, $\frac{\partial d_{t+2}}{\partial d_{t+1}} = r + \frac{\partial f_{t+1}}{\partial d_{t+1}}$. (In an open-loop equilibrium, instead, $\frac{\partial f_{t+1}}{\partial d_{t+1}} = 0$.)

Conjecture that the policy rule selected by the fiscal authority in equilibrium is:

$$f_\tau^* = \alpha_t^F + \beta_t^F d_t \tag{34}$$

where α_t^F and β_t^F are parameters yet to be derived. Then, from (33), (34) and the previous expression for $\frac{\partial d_{t+2}}{\partial d_{t+1}}$:

$$V_{t+1}^{M'}(d_{t+1}) = \tau d_{t+1} + \rho(r+\beta_{t+1}^F)\, V_{t+2}^{M'}(d_{t+2}) \tag{35}$$

Starting from the last period of the game, we now have a recursive system of equations to solve, consisting of (1'), (34), (31), (32), (35). Section 5 of the Appendix shows that the solution of this system can be rewritten as:

$$m_t^* = \alpha_t^M + \beta_t^M d_t + \gamma_t^M f_t \tag{36}$$

where α_t^M, β_t^M, γ_t^M in turn are the solution to a recursive system illustrated in Section 5 of the Appendix.

(ii) The fiscal authority optimization problem: By Bellman's principle, rewrite (3') as:

$$V_t^F(d_t) = \min_{f_t} \{\frac{1}{2}(f_t-\bar{f})^2 + \frac{1}{2}\lambda d_t^2 + \rho V_{t+1}^F(d_{t+1})\} \tag{37}$$

subject to the government budget constraint, (1'), and to the policy rule followed by the central bank, (36). The first order conditions yield:

$$f_t = \bar{f} - \rho(1-\gamma_t^M)\, V_{t+1}^{F'}(d_{t+1}) \tag{38}$$

where, appealing to (1') and to (36), we have implicitly set $\frac{\partial d_{t+1}}{\partial f_t} = 1 - \gamma_t^M$. Notice that this is the only respect in which the optimization problem of the two players differ (recall that for the central bank we had $\frac{\partial d_{t+1}}{\partial m_t} = -1$): the central bank takes f_t as given when choosing m_t, whereas the fiscal authority knows that in each period the central bank is going to respond to f_t according to the reaction function (36). Since both players choose among closed-loop strategies, they both take into account that future actions of the opponent are going to depend on the current value of the state variable, as specified in (34) and (36) respectively. The remainder of the solution to the fiscal authority optimization problem can be computed by repeating the same steps illustrated above for the monetary authority. In the last period of the game:

$$V_T^{F'}(d_T) = \lambda d_T \tag{39}$$

In any other period:

$$V_{t+1}^{F'}(d_{t+1}) = \lambda d_{t+1} + \rho(r-\beta_{t+1}^M)\, V_{t+2}^{F'}(d_{t+2}), \tag{40}$$

where, using (36), and (1') it has been implicitly set $\frac{\partial d_{t+2}}{\partial d_{t+1}} = r - \beta^M_{t+1}$.

Section 5 of the Appendix shows that the solution to this recursive system of equations yields the fiscal authority reaction function that was conjectured above, equation (34). The coefficients α^F_τ and β^F_t that appear in it are the solution of a recursive system also shown in Section 5 of the Appendix.

Figures 1-3 below report the results of a numerical simulation of this recursive system, for a time horizon of 31 periods. The parameters have been set at the following values: $d_o = 1$, $\bar{f} = .15$, $\bar{m} = 0$, $\lambda = .2$, $r = 1.05$, $\rho = .95$. Moreover, τ takes on the values 0, .1,.4.

The first diagram reveals immediately that, unlike in the Nash equilibria, the time path of public debt is not monotonic in τ. When τ goes from .1 to .4 in Fig. 1, the rate of adjustment of public debt towards the desired value increases, and the time path of public debt is always lower than with $\tau = .1$. But the same happens as τ goes from .1 to 0; in this case public debt remains very close to zero for 10 periods. Thus, unlike in the Nash equilibria, enhancing central bank independence (i.e., reducing τ) can lead to a <u>lower</u> time path of public debt and to a faster adjustment, as long as the initial value of τ is sufficiently small (in this numerical example, not larger than .2). The intuition behind this result is easy to grasp. A more independent central bank will monetize a smaller portion of public debt outstanding, and thus will force the fiscal authority to bear a larger portion of the adjustment: as revealed by Figures 2 and 3, both m and f are monotonically increasing in τ. For small values of τ, the reduction of fiscal deficits brought about by having a more independent central bank prevails over the reduction in the degree of monetization, and public debt adjusts faster to the optimal value.

The numerical simulations also reveal that the size of fiscal deficits is relatively insensitive to changes in $\bar{m}$. This can be explained by noting that the tradeoff between debt and deficits faced in each period by the fiscal authority depends crucially on τ, but is unaffected by $\bar{m}$. This information has important implications for how to design an institutional arrangement leading to monetary stability. Suppose that it is desirable to maintain a low rate of growth of the monetary base. There are two ways to achieve this goal: by choosing a low value of τ or by choosing a low value of $\bar{m}$. In the first case, fiscal deficits are also going to be low; but in the second case they will be much larger, and the stock of public debt will soar. Fig. 1 and 4 below illustrate this point, under the assumption that the goal is to maintain a money growth close to zero. If this desired time path of money growth is achieved by setting $\tau = 0$ and $\bar{m} = 0$, the stock of public debt falls rapidly over time and remains in the range of 0 for a prolonged number of periods (see Fig. 1). As indicated in Fig. 2, here fiscal deficits are taking up the whole burden of the adjustment. But if the desired time path of money growth is achieved by the combination $\tau = .1$, $\bar{m} = -.09$, the stock of public debt remains always higher (see Fig. 4). Notice that in this case m_t is slightly negative for most of the time. The contrast is even sharper in the case in which $\tau = .4$. Here it is impossible to maintain m_t on a steady path close to 0. For $\bar{m} = -3.5$, money growth remains very close to zero for 17 time periods, and then, towards the end of the game, it drops sharply. Yet, the stock of public debt never falls below .7, and it reaches a value of 4.1 at the end of the game.

Thus, in the setting of this Stackelberg equilibrium, there are two ways of achieving monetary stability: by having an independent central bank; or by having a central bank which sets low monetary targets but whose incentive

$\rho = .95$ $\bar{m} = 0$

$A = .2$ $\bar{\xi} = .145$

$r = 1.05$

Fig. 1

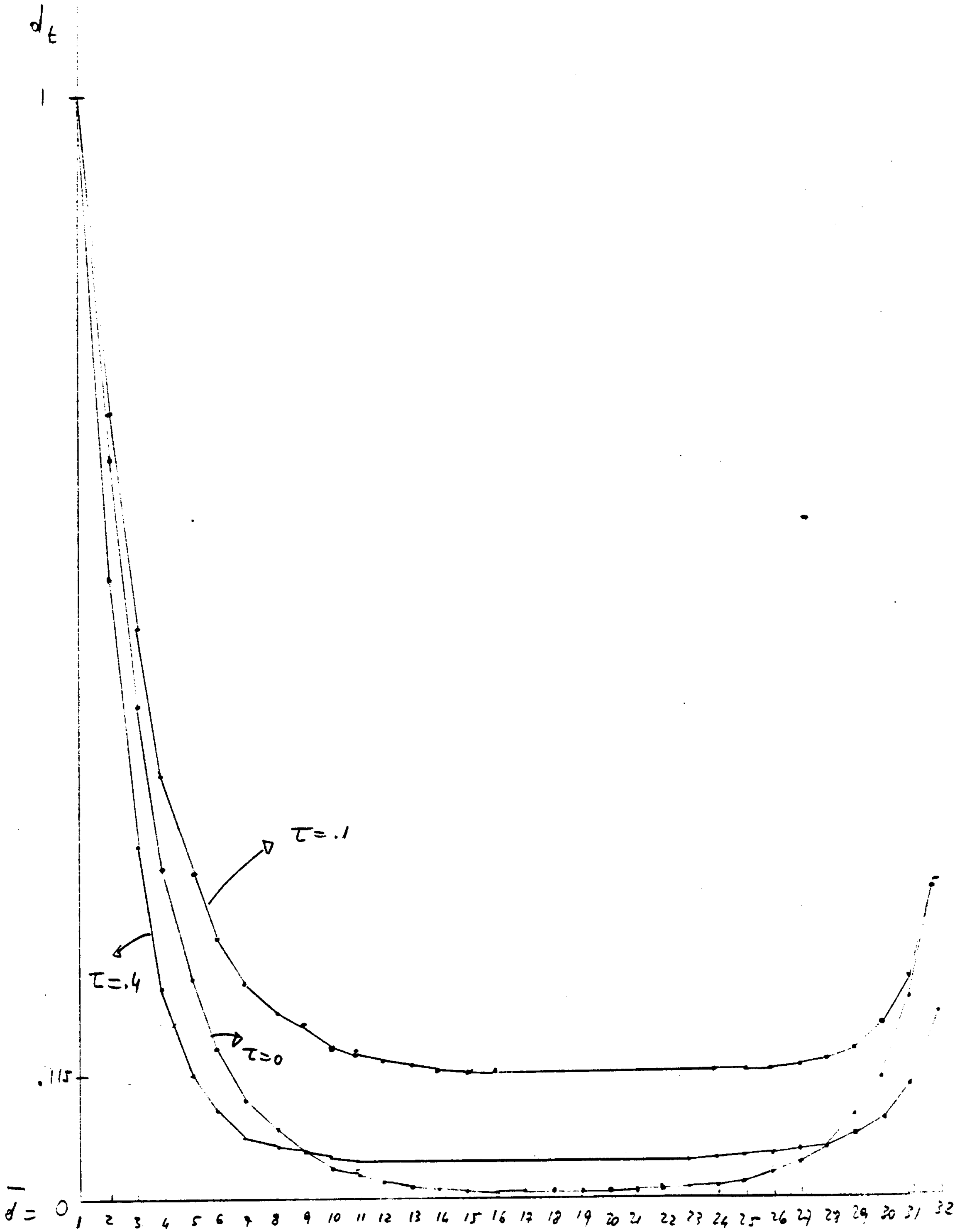
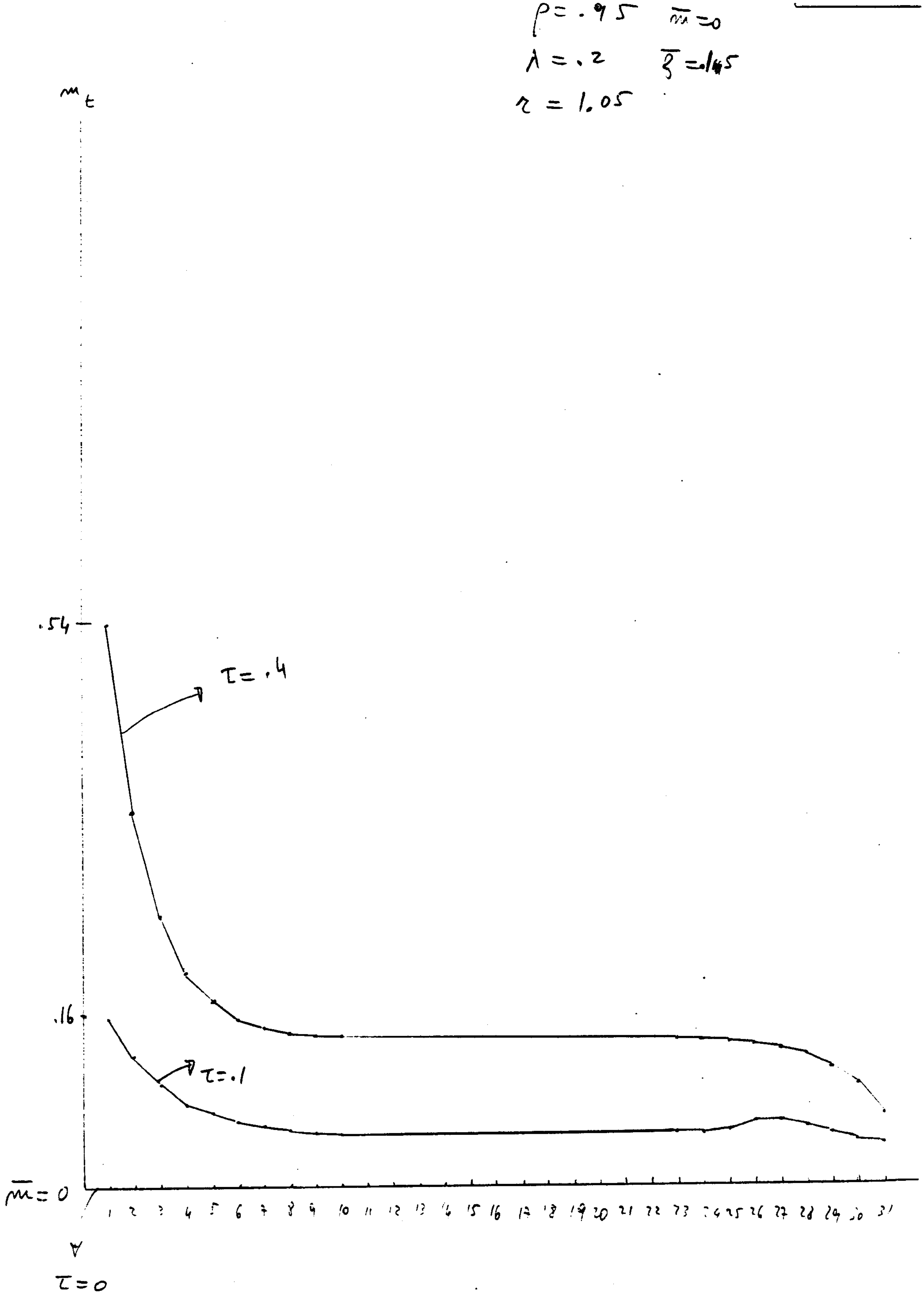


Fig. 2



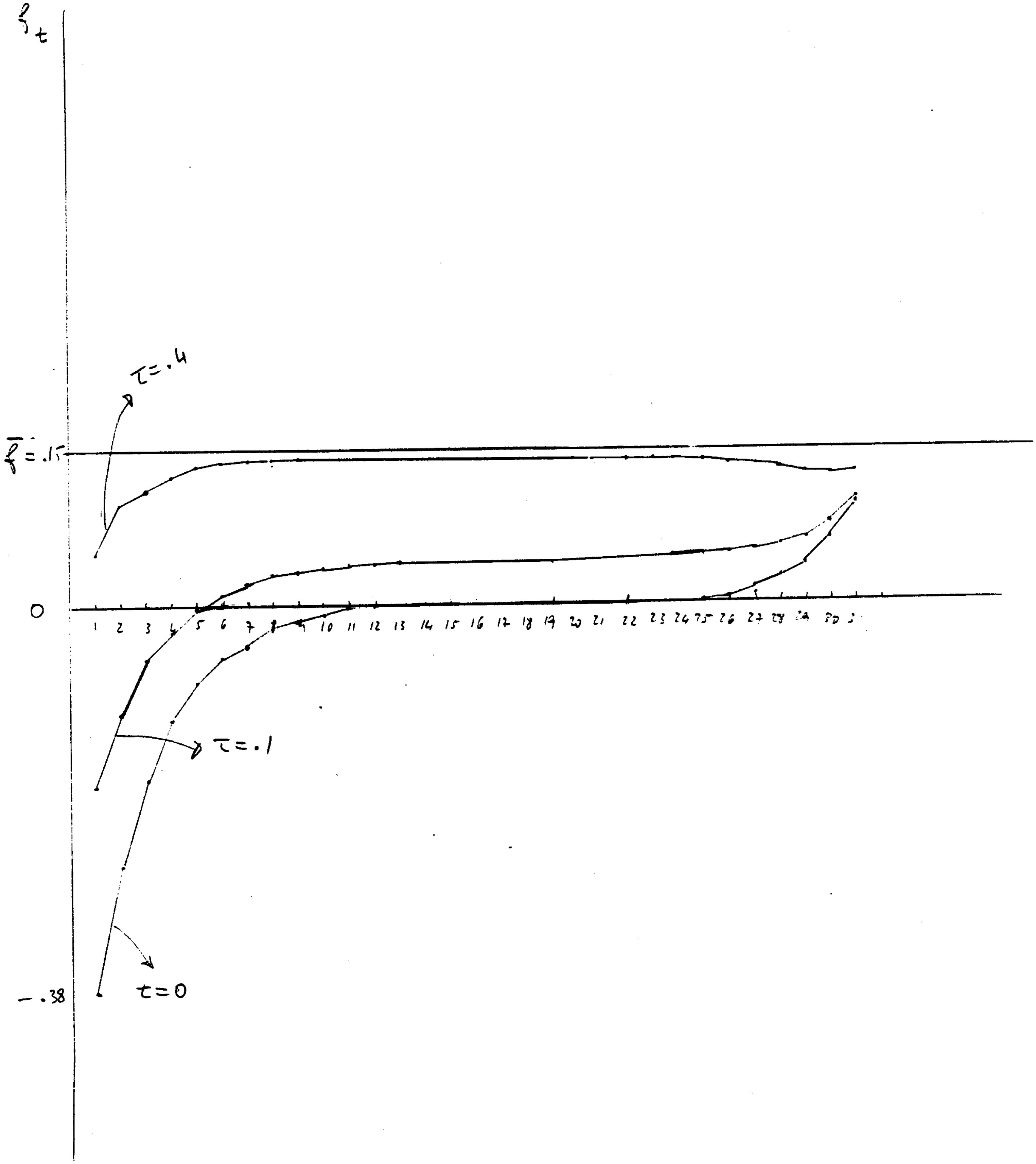
ρ = .95 $\bar{m}$ = 0
λ = .2 $\bar{\xi}$ = .145
r = 1.05
m_t
.54
.16
τ = .4
τ = .1
$\bar{m}$ = 0
τ = 0
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31

$\rho = .95$ $\quad \bar{m} = 0$

$\lambda = .2$ $\quad \bar{g} = .15$

$r = 1.05$

Fig. 3



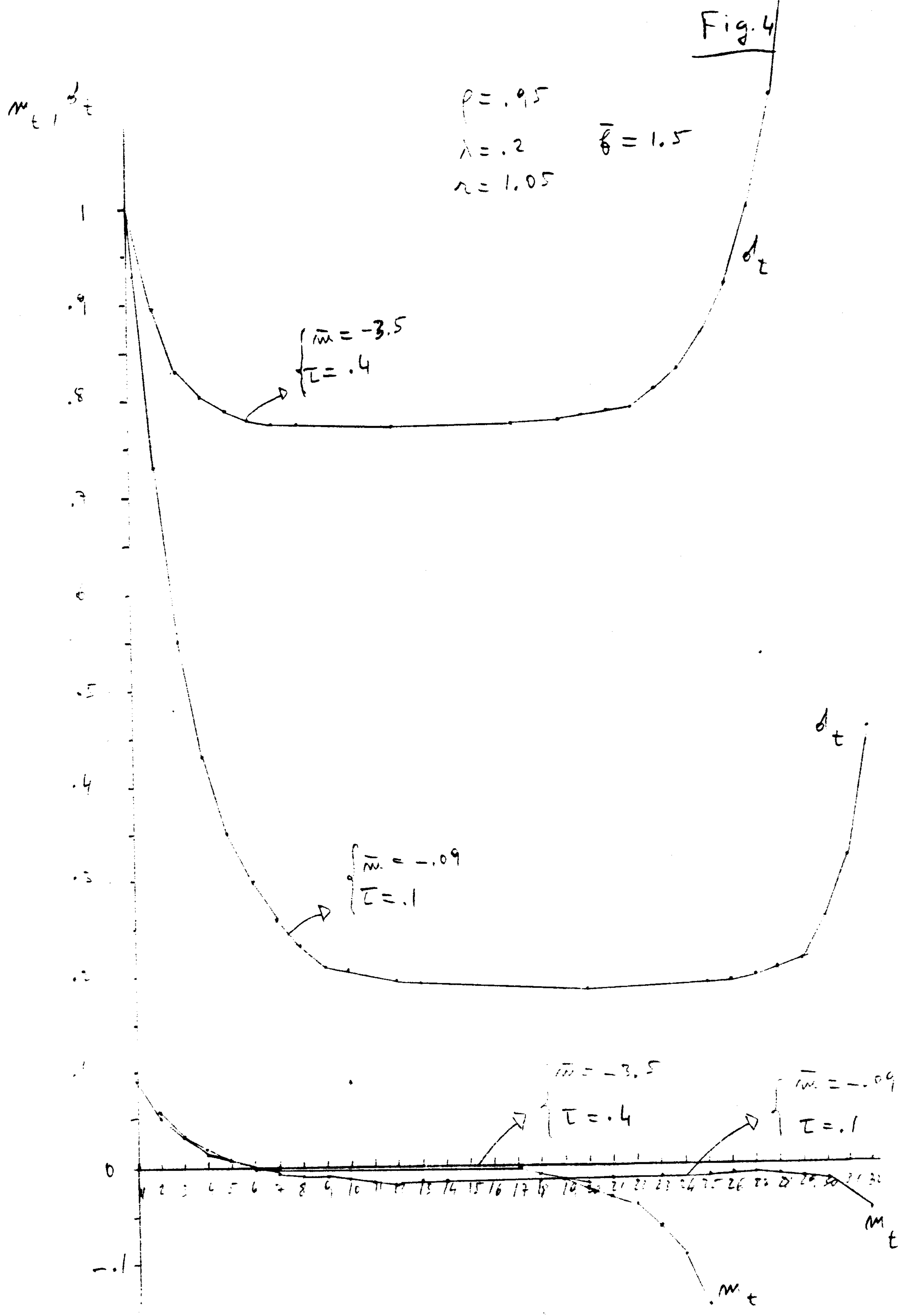


Fig. 4
$\rho = .95$
$\lambda = .2$
$r = 1.05$
$\bar{f} = 1.5$
m_t, d_t
d_t
$\bar{m} = -3.5$
$\tau = .4$
d_t
$\bar{m} = -.09$
$\tau = .1$
$\bar{m} = -3.5$
$\tau = .4$
$\bar{m} = -.09$
$\tau = .1$
m_t
m_t

structure ties it closely to the Treasury. The time path of public debt and of fiscal deficits are both going to be much higher in the second case than in the first one.

10. Conclusions

The government budget constraint provides a dynamic link between fiscal deficits, the creation of monetary base and the time path of public debt. In most industrial countries, the size of fiscal deficits and the growth of monetary base are selected by independent authorities with potentially conflicting objectives. Thus, strategic considerations are bound to play a major role in shaping monetary and fiscal policy. This paper has analyzed the strategic interaction between the central bank and the fiscal authority within a dynamic linear-quadratic game with complete information. The equilibrium outcome of this game determines the time path of public debt.

The model has the following positive implications:

(i) Monetary and fiscal policies respond to exogenous shocks affecting the time path of public debt (such as cyclical deficits or changes in the real interest rate). A positive shock to the stock of public debt held by the private sector, for instance, induces the Central Bank to increase its revenue from money creation and the Treasury to reduce fiscal deficits net of interest payments, in an effort to bring the debt back towards its desired value.

(ii) The reactions of the two policymakers to any shock affecting the time path of public debt depend crucially on the expected behavior of the other player in the game. Specifically, the more the fiscal authority cares about the stock of debt (i.e., the larger is λ), the smaller the reaction of monetary policy. And vice versa, the more independent the central bank is (i.e., the smaller is τ), the larger the reaction of fiscal policy. This result has some testable implications about the pattern of fiscal and monetary policies in countries with different institutional settings. Moreover, it casts doubts on the validity of the existing empirical literature which seeks to estimate policy reaction functions for the two policymakers in isolation.

(iii) The time path of public debt over national income can be stable even if the real interest rate exceeds the rate of growth of real income, provided that one or both authorities care "enough" about the stock of public debt outstanding. Hence, the observation of a rising time path of public debt over national income together with an interest rate larger than the rate of growth of output should not by itself be taken as an indication that the monetary-fiscal policy mix is not sustainable: as argued in some of the existing literature on the topic,[23] this rising time path could have been caused by a sequence of cyclical fiscal deficits hiding large adjustments in the instruments of fiscal policy.

In addition, some of the results derived in the paper can be interpreted as normative suggestions for institutional reforms. Three kinds of reforms have been discussed in the previous sections:

(i) Changes in the degree of central bank independence (measured by the τ parameter in the central bank's objective function). The consequences of having a more independent central bank are smaller deficits and smaller revenue from money creation. The net effects on the time path of public debt are ambiguous and depend on the nature of the equilibrium: in all Nash equilibria, public debt is larger the more independent is the central bank; in the Stackelberg equilibrium with the fiscal authority as the dominant player, it could be either larger or smaller. By contrast, in the Stackelberg equilibrium fiscal deficits are insensitive to changes in the monetary target, $\bar{m}$. This suggests that, in countries where the time paths of public debt and of fiscal deficits are above their optimal values, monetary stability is achieved more efficiently by enhancing Central Bank independence than by reducing its monetary targets; for in the second case a reduction in monetary growth leaves deficits unaffected and leads to an even larger public debt.

(ii) Coordination of monetary and fiscal policies by a single decision unit (say, Congress). By transforming the noncooperative equilibrium into a cooperative equilibrium, this reform brings about a more rapid adjustment of all variables to their steady state values and a smaller steady state stock of public debt. In practice, such an institutional arrangement can only come about if Congress is in charge of closely monitoring the behavior of monetary and fiscal authorities. But proposals of institutional reforms going in this direction have invariably drawn a main objection: that they would destroy central bank independence and that consequently major monetary policy decisions would be left to the vagaries of the political market.

The objection can be made more precise with the help of the model analyzed in Section 7. As remarked on p. 18 above, the first order conditions holding in the cooperative equilibrium (equation (24)) imply that the deviations of monetary policy from its target are proportional to the deviations of fiscal policy from its own target. Within the framework of this model, thus, the aforementioned objection must be taken to mean either: (i) that any realistic implementation of this proposal would give too little weight to monetary policy objectives -- that is, ω in equation (24) would be too small; or, (ii) that enactment of the proposal would lead to inappropriate monetary policy targets, $\bar{m}$, or, (iii): that it would have deleterious effects on the parameters $\bar{f}$ and λ characterizing the preferences of the fiscal authorities.

Point (i) is probably what the critics have in mind; for the appropriate monetary targets could still be chosen by the central bank even if Congress was formally responsible of approving them; and there is no reason to believe that point (iii) is valid. But even point (i) is not fully convincing: an institutional setting in which Congress is the only body primarily responsible for the deviations of both f and m from the "technically" optimal targets would provide large political incentives to limit the size of the revenue from money creation, and it would prevent the dilution of responsibilities for which many economists[24] have blamed the decentralized system currently prevailing in practically all industrial countries.

(iii) Restrictions on the admissible policy rules. These can take at least two forms: (i) requiring one or both players to select open-loop strategies; (ii) requiring one or both players to set strategies non-contingent on the stock of public debt outstanding (for instance, a k percent growth rule for the monetary base, or a requirement of balanced — or constant — budget net of interest payments). Restrictions of type (i) have the effect of moving the non-cooperative equilibrium closer to the cooperative benchmark and, under some circumstances, they may be advantageous for the player on which the restriction is imposed. Restrictions of type (ii) slow down the rate of adjustment of public debt towards the steady state, and may even bring about an explosive time path for public debt. However, if imposed on the fiscal authority, they bring about a lower steady state value of public debt.

Finally, the model analyzed in this paper lends itself to a straightforward extension. By dropping the assumption that both players have complete information, one could study the consequences of reputational effects on the selection of monetary and fiscal policies. In any of the non-cooperative equilibria analyzed in this paper, both players have an incentive to announce that they will set their policy instruments non-contingent upon the stock of public debt outstanding, so as to leave the whole burden of the adjustment on the opponent. However, because of the assumption of complete information and of memoryless strategies, these announcements are not credible. Adding to the model an element of incomplete information would provide new insights on the issue of the credibility of policy announcements.

Footnotes

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[1]See Calvo (1978), Turnovsky and Brock (1980), Fisher (1980), Lucas and Stokey (1983), Person and Svensson (1984).

[2]However, see Section 2 below for some references in which assumption (ii) has been derived from more primitive hypotheses.

[3]See Sargent and Wallace (1981), McCallum (1981), Liviatan (1984), Drazen (1985).

[4]That is, $m(t) = \frac{dM(t)}{dt} / Y(t)$, where $M(t)$ is the stock of monetary base corresponding to liabilities of the Treasury and where $Y(t)$ is nominal income.

[5]In a previous version of the paper, the model was formulated in discrete time and the distinction between structural deficit and cyclical deficit was introduced; the latter was assumed to be an exogenous random variable. The same distinction could be easily introduced in the present version of the paper, at the price of some minor computational complications. However no extra insights would be gained by doing so.

[6]In an overlapping generations setting with no bequest motive, Diamond (1965) and Phelps and Shell (1969) have shown that issuing public debt reduces the steady state capital stock of the economy, and that this is welfare

decreasing if, in the steady state, the real interest rate is above the rate of growth of the population. Persson (1984) has recently extended the Diamond (1965) model to the case of a small open economy. Again, issuing public debt is welfare decreasing in the steady state, due to the future (lump sum) taxes that have to be raised in order to pay interest on the debt. Notice that this effect is relevant for public debt held by the private sector, but not for the public debt held by the central bank, only under the assumption that the interest paid to the central bank is remitted to the Treasury.

In the context of the Yaari (1965) model, Blanchard (1985,1984) has shown that issuing public debt has the effect of raising human wealth. As a consequence, in a closed economy issuing public debt decreases the steady state stock of capital, and in a small open economy it increases the external debt of the country. See also Buiter (1984) for an application of Blanchard's model to the open economy. Strictly speaking, given the assumption that r is exogenous in equation (1) in the text, only the results concerning the small open economy are relevant here.

[7]On the intergenerational redistribution of risk see for instance Stiglitz (1983), Buiter (1983), Fisher (1983). On the intragenerational redistributions of wealth see Atkinson and Stiglitz (1980).

[8]This argument was originally put forward by Lerner (1948) and by Meade (1958).

[9]This argument can be found, for instance, in Blanchard, Buiter and Dornbusch (1985).

[10]This argument is elaborated in Bryant and Wallace (1979, 1980).

[11]In a previous version of the model, where the distinction between structural and cyclical deficits was introduced and where only structural deficit were controllable by the fiscal authority, a sudden increase in public

debt led to a reduction of structural fiscal deficits net of interest payments but not necessarily to a reduction of overall fiscal deficits.

[12]Notice that the stability condition (18) would still hold if the fiscal authority was able to control only structural (or "full employment") deficits, and cyclical deficits followed an exogenous stochastic process. The OECD Economic Outlook (Dec. 1983) provides some evidence against the hypothesis that $\lambda = 0$: In the period 1981-83 public debt rose for all European countries, due to a sequence of cyclical deficits; in the same period, the structural budget deficit for the 10 EC countries moved from an average of zero in 1981 to an average surplus of 2.5% of GDP in 1983 -- see also Giavazzi (1984).

[13]Naturally, the hypothesis that $\lambda = 0$ would become appropriate whenever the fiscal authority faces a binding constraint which does not allow it (for political or technical reasons) to further reduce the size of structural deficits net of interest payments — see Blanchard (1984).

[14]See, for instance, Oudiz and Sachs (1984), Maskin and Tirole (1982), Pindyck (1976), Basar and Olsder (1982), Kydland (1975) for further methodological comments on the appropriate hypotheses about the strategy spaces in dynamic games.

[15]A second reason for being interested in games of this kind concerns the case in which the player choosing the closed-loop strategy is the only "big" player in the game -- e.g., the government, and the "player" choosing the open-loop strategy is an aggregate of smaller players acting noncooperatively, who individually have no influence on the time path of the state variable -- see, for instance Cohen and Michel (1984). However, see also footnote 16 below.

[16]The equilibrium analyzed in this section should not be confused with a Stackelberg equilibrium in which the fiscal authority acts as the Stackelberg

leader. (Cohen and Michel (1984) seem to have made this confusion in a recent paper studying a similar analytical problem.) What distinguishes a Stackelberg equilibrium from a Nash equilibrium is the order of play, and not whether the players choose open-loop or closed-loop strategies. In a correctly defined time consistent Stackelberg equilibrium, the Stackelberg leader moves first in each period, and thus it takes into account the effect of its actions on the current actions of the follower. Whether he will take into account also the effect on future actions, and how, will depend on whether he chooses open-loop or closed-loop strategies, and on whether the equilibrium is time consistent or not.

[17]Institutional reforms of this kind have been advocated in the past and in recent times by a large number of economists. See for instance Friedman (1961), Leijonhufvud (1984), Monti (1984).

[18]Since the setting is non-stochastic, here open-loop and closed-loop strategies are identical. See Basar and Olsder (1982).

[19]This analogy was suggested to me by Jean Tirole.

[20]See Kydland and Prescott (1977), Lucas and Sargent (1981), Cohen and Michel (1984).

[21]See Cohen and Michel (1984) for an explicit computation of this time inconsistent policy in a similar analytical framework.

[22]Oudiz and Sachs (1984) use a similar procedure to solve a closed-loop Nash equilibrium.

[23]Buiter (1982), Giavazzi (1984), Blanchard, Buiter and Dornbusch (1985).

[24]For instance, see Friedman (1961), Monti (1984).

Appendix

1. Derivation of (16)

Equating coefficients of (15) and (1), using (4), we get:

$$\beta - r - \frac{\tau}{\theta_1} = \beta - r - \frac{\lambda}{\pi_1} = r - \theta_1 - \pi, \tag{1.1}$$

$$(\beta-r)\left(\frac{\theta_o-\bar{m}}{\theta_1}\right) = (\beta-r)\left(\frac{\bar{f}-\pi_o}{\pi_1}\right) = \pi_o - \theta_o \tag{1.2}$$

Recalling that $r - \theta_1 - \pi_1 = -\gamma$, (16.i) and (16.ii) immediately follow. Furthermore, substituting (16.i) and (16.ii) in $\gamma = \theta_1 + \pi_1 - r$, gives us:

$$\gamma^2 + \beta\gamma + (r(\beta-r) - (\tau+\lambda)) = 0$$

which has at least one unstable (negative) root.

Solving for γ and choosing the stable root — cf., the discussion in the text, p. 11 we obtain (16.iii).

The same solution could have been obtained, alternatively, by solving the characteristic equation of the dynamic system made up of (1), (9) and (11) -- see Cohen and Michel (1984).

2. Solution of the Closed-Loop Nash Equilibrium

From (4):

$$\begin{aligned} \dot{m} &= \theta_1 \dot{d} \\ -\dot{f} &= \pi_1 \dot{d} \end{aligned} \tag{2.1}$$

From (20) and (21), using (4):

$$\begin{aligned} \dot{m} &= \dot{\mu}_1 = (\beta+\pi_1-r)(\theta_o+\theta_1 d-\bar{m}) - \tau d \\ -\dot{f} &= \dot{\mu}_2 = (\beta+\theta_1-r)(\bar{f}-\pi_o+\pi_1 d) - \lambda d \end{aligned} \tag{2.2}$$

Putting together (2.1) and (2.2):

$$\begin{aligned} \dot{d} &= \frac{1}{\theta_1}(\beta+\pi_1-r)(\theta_o+\theta_1 d-\bar{m}) - \tau d \\ \dot{d} &= \frac{1}{\pi_1}(\beta+\theta_1-r)(\bar{f}-\pi_o+\pi_1 d) - \lambda d \end{aligned} \tag{2.3}$$

Equating coefficients of d in (2.3) with those of d in (1), using (4), we get:

$$(\beta+\pi_1-r) - \frac{\tau}{\theta_1} = r - \theta_1 - \pi_1 = (\beta+\theta_1-r) - \frac{\lambda}{\pi_1} \tag{2.4}$$

which immediately yields (22) in the text.

Defining $\gamma^c = \theta_1 + \pi_1 - r$ and solving (2.4) we obtain:

$$\begin{aligned} \theta_1 &= \frac{\beta + 2\gamma^c - \sqrt{(\beta+2\gamma^c)^2 - 4\tau}}{2} > 0 \\ \pi_1 &= \frac{\beta + 2\gamma^c - \sqrt{(\beta+2\gamma^c)^2 - 4\lambda}}{2} > 0 \end{aligned} \tag{2.5}$$

where the roots are real only for $\tau,\ \lambda < \frac{(\beta+2\gamma^c)^2}{4}$, and where the negative square roots have been chosen in conformity with McCallum (1983) criterion of "minimal set of state variables": when $\lambda = 0$ we want $\pi_1 = 0$, when $\tau = 0$, we want $\theta_1 = 0$.

In order to compare γ^c with γ^o, rewrite (2.4) as follows:

$$\begin{aligned} \theta_1^2 + 2\theta_1\pi_1 + (\beta-2r)\theta_1 - \tau &= 0 \\ \pi_1^2 + 2\theta_1\pi_1 + (\beta-2r)\pi_1 - \lambda &= 0 \end{aligned} \tag{2.6}$$

Summing both terms and letting $x = \theta_1 + \pi_1$, we obtain:

$$x^2 + (\beta-2r)x - (\tau+\lambda) = -2\theta_1\pi_1 < 0 \tag{2.7}$$

In order to compare the solution of (2.7) with the open-loop Nash-equilibrium, notice that (1.1) in Section 1 of the Appendix yields:

$$y^2 + (\beta-2r)y - (\tau+\lambda) = 0 \tag{2.7'}$$

where $y = (\theta_1^o + \pi_1^o)$, and θ_1^o, π_1^o, are the parameters of the open-loop Nash equilibrium given in (16) of the text. Since the right hand side of (2.7) is negative, it follows that $x < y$. Recalling that $\gamma^o = y - r$ and that $\gamma^c = x - r$, we can conclude that $\gamma^o > \gamma^c$.

It remains to be shown that $\frac{\partial \gamma^c}{\partial \tau} > 0$. Substitute (2.5) in $\gamma^c = r - \theta_1 - \pi_1$, to obtain:

$$g(\gamma^c,\tau) = 2(\gamma^c+\beta-r) - \sqrt{(\beta+2\gamma^c)^2-4\tau} - \sqrt{(\beta+2\gamma^c)^2-4\lambda} = 0 \tag{2.8}$$

then apply the implicit funciton theorem to (2.8):

$$\frac{\partial \gamma^c}{\partial \tau} = - \frac{\partial g/\partial \tau}{\partial g/\partial \gamma^c}.$$

Since $\frac{\partial g}{\partial \tau} > 0$, sign $\{\frac{\partial \gamma^c}{\partial \tau}\} = -$ sign $\{\frac{\partial g}{\partial \gamma^c}\}$. Moreover,

$$\frac{\partial g}{\partial \gamma^c} = 1 - \frac{(\beta+2\gamma^c)}{\sqrt{(\beta+2\gamma^c)^2-4\tau}} - \frac{(\beta+2\gamma^2)}{\sqrt{(\beta+2\gamma^c)^2-4\lambda}} < 0$$

So that $\frac{\partial \gamma^c}{\partial \tau} > 0$.

Applying the same procedure to (2.6) yields that

$$\frac{\partial \theta_1}{\partial \tau} > 0, \quad \frac{\partial \pi_1}{\partial \tau} < 0.$$

Finally, in order to compute the steady state equilibrium corresponding to the closed-loop Nash equilibrium, use (1), (20), (21) to obtain:

$$d^{sc} = \frac{(\bar{f} - \bar{m})}{\frac{\lambda}{\beta+\theta_1-r} + \frac{\tau}{\beta+\pi_1-r} - r}$$

$$m^{sc} = \bar{m} + \frac{(\bar{f}-\bar{m})\tau}{\tau+\lambda \frac{(\beta+\pi_1-r)}{(\beta+\theta_1-r)} - r(\beta+\pi_1-r)}$$

$$f^{sc} = \bar{f} - \frac{(\bar{f}-\bar{m})\,\lambda}{\lambda + \tau\,\frac{(\beta+\theta_1-r)}{(\lambda+\pi_1-r)} - r(\beta+\theta_1-r)}$$

3. Nash Equilibrium With F Playing Closed-Loop and M Playing Open-Loop Strategies

As stated in the text, θ_1 is given by (16.i) and π_1 is given by (22). Solving (22) for π_1, using (16.i), gives

$$\pi_1 = \frac{\beta+2\gamma - \sqrt{(\beta+2\gamma)^2 - 4\gamma}}{2} \tag{3.1}$$

which not surprisingly is exactly as in Section 2 of the Appendix.

Let γ^A be the rate of adjustment of public debt in this particular equilibrium. From $\gamma^A = \theta_1 + \pi_1 - r$, using (16.i) and (3.1) to substitute away π_1, θ_1, we get:

$$g(\gamma^A,\tau) = \gamma^A - \frac{\tau}{\beta+\gamma^A-r} - \frac{\beta+2\gamma^A - \sqrt{(\beta+2\gamma^A)^2 - 4\lambda}}{2} + r = 0$$

Now, as before, appeal to the implicit function theorem:

$$\frac{\partial\gamma^A}{\partial\tau} = -\frac{\partial g/\partial\tau}{\partial g/\partial\gamma^A}$$

where $\partial g/\partial\tau = -1$ and $\partial g/\partial\gamma^A > 0$, so that $\frac{\partial\gamma^A}{\partial\tau} > 0$.

In order to compare γ^A with γ^c and γ^o, rewrite (16.i) and (3.1) as:

$$\begin{aligned} &\theta_1^2 + \theta_1\pi_1 + (\beta-2r)\,\theta_1 - \tau = 0, \\ &\pi_1^2 + 2\theta_1\pi_1 + (\beta-2r)\,\pi_1 - \lambda = 0 \end{aligned} \tag{3.2}$$

Let $z = \theta_1 + \pi_1$ and sum both equations together, to get:

$$z^2 + (\beta-2r)z - (\lambda+\tau) = -\theta_1\pi_1 \tag{3.3}$$

Repeating the same argument used in Section 2 of the Appendix and comparing (3.3) with (2.7) and (2.7'), we can conclude that $\gamma^{o} > \gamma^{A} > \gamma^{c}$.

The steady state corresponding to this equilibrium can be computed from (1), (13) and (21); denoting it with an sA superscript, some simple algebra yields:

$$d^{sA} = \frac{(\bar{f}-\bar{m})}{\frac{\tau}{\beta-r} + \frac{\lambda}{\beta+\theta_1-r} - r} \tag{3.4}$$

$$m^{sA} = \bar{m} + \frac{(\bar{f}-\bar{m})\ \tau}{\tau + \frac{\lambda(\beta-r)}{\beta+\theta_1-r} - r(\beta-r)} \tag{3.5}$$

$$f^{sA} = \bar{f} - \frac{(\bar{f}-\bar{m})\ \lambda}{\lambda + \tau\,\frac{(\beta+\theta_1-r)}{\beta-r} - r(\beta+\theta_1-r)} \tag{3.6}$$

Comparing these expressions with the corresponding ones in Section 2 of the Appendix and in (17) of the text, we obtain (27)-(29) in Section 8 of the text.

4. Computation of Cooperative Equilibrium

The characteristic equation of the dynamic system made up of (24), (1) is:

$$Z^2 - \beta Z + (r(\beta-r) - (\lambda+\omega\tau)\ (\frac{1+\omega}{\omega})) = 0$$

whose negative solution is the negative of γ^{P} in (25).

The initial conditions at $t = 0$, together with (24.i), yield the expressions for θ_1 and π_1 stated in (25).

Equations (24) and (1) can also be solved to find the steady state values — denoted with the sp superscript:

$$d^{sp} = \frac{(\bar{f}-\bar{m})\ (\beta-r)}{(\lambda+\omega\tau)\ (\frac{1+\omega}{\omega}) - r(\beta-r)} \tag{4.1}$$

$$f^{sp} = \bar{f} - \frac{(\bar{f}-\bar{m})}{(\lambda+\tau\omega) - r(\beta-r)\frac{\omega}{1+\omega}} \tag{4.2}$$

$$m^{sp} = \bar{m} + \frac{(\bar{f}-\bar{m})}{\omega(\lambda+\tau\omega) - r(\beta-r)\frac{\omega^2}{1+\omega}} \tag{4.3}$$

5. Computation of the Closed-Loop Stackelberg Equilibrium

(i) The central bank policy rule:

The solution can be computed recursively from (1'), (34), (36), (32), (35).

From (1') and (32):

$$V_T^{M'}(d_T) = \tau r d_{t-1} + \tau f_{t-1} - \tau m_{T-1} \tag{5.1}$$

From (36):

$$V_T^{M'}(d_T) = \tau(r-\beta_{T-1}^M)d_{T-1} + \tau(1-\gamma_{T-1}^M)f_{T-1} - \tau\alpha_{T-1}^M \tag{5.2}$$

which can be rewritten as:

$$V_T^{M'}(d_T) = A_{T-1} + B_{T-1}\, d_{T-1} + C_{T-1}\, f_{T-1} \tag{5.3}$$

Now substitute (5.3) in (35), with $t+1 = T-1$:

$$\begin{aligned} V_{T-1}^{M'}(d_{T-1}) = {} & (\tau + \rho(r+\beta_{T-1}^F)B_{T-1})d_{T-1} + \\ & + \rho(r+\beta_{T-1}^F)C_{T-1}\, f_{T-1} + \rho(r+\beta_{T-1}^F)A_{T-1} \end{aligned} \tag{5.4}$$

which, using (1'), (34) and (36), becomes:

$$\begin{aligned} V_{T-1}^{M'}(d_{T-1}) = {} & (\tau + \rho(r+\beta_{T-1}^F)B_{T-1})((r-\beta_{T-2}^M)d_{T-2} + \\ & + (1-\gamma_{T-2}^M)f_{T-2} - \alpha_{T-2}^M) + \rho(r+\beta_{T-1}^F)C_{T-1}\,\cdot \\ & \cdot\,(\alpha_{T-1}^F + \beta_{T-1}^F\,((r-\beta_{T-2}^M)d_{T-2} + (1-\gamma_{T-2}^M)f_{T-2} - \alpha_{T-2}^M)) + \\ & + \rho(r+\beta_{T-1}^F)A_{T-1} \end{aligned}$$

Collecting terms, we have:

$$V_{T-1}^{M'}(d_{T-1}) = A_{T-2} + B_{T-2}d_{T-2} + C_{T-2}f_{T-2} \tag{5.5}$$

where:

$$A_{T-2} = \rho(r+\beta_{T-1}^{F})(C_{T-1}\alpha_{T-1}^{F} - (C_{T-1}\beta_{T-1}^{F}+B_{T-1})\alpha_{T-2}^{M}+A_{T-1}) - \tau\alpha_{T-2}^{M}$$

$$B_{T-2} = (r-\beta_{T-2}^{M})\ [\tau + \rho(r+\beta_{T-1}^{F})(C_{T-1}\beta_{T-1}^{F}+B_{T-1})]$$

$$C_{T-2} = (1-\gamma_{T-2}^{M})\ [\tau + \rho(r+\beta_{T-1}^{F})(C_{T-1}\beta_{T-1}^{F}+B_{T-1})]$$

Repeating the same procedures for $\tau = T-2, T-3, \ldots,$ we get the following general solution for $t < T-1$:

$$V_{t}^{M'}(d_{t}) = A_{t-1} + B_{t-1}d_{t-1} + C_{t-1}f_{t-1} \tag{5.6}$$

$$A_{t-1} = \rho(r+\beta_{t}^{F})(A_{t}+C_{t}\alpha_{t}^{F}) - K_{t}\alpha_{t-1}^{M}$$

$$B_{t-1} = K_{t}(r-\beta_{t-1}^{M}) \tag{5.7}$$

$$C_{t-1} = K_{t}(1-\gamma_{t-1}^{M})$$

with $K_t = [\tau + \rho(r+\beta_{t}^{F})(B_{t}+C_{t}\beta_{t}^{F})]$.

Now, use (5.6) in (31), to obtain the following expressions for the coefficients in (36)

$$\alpha_{t}^{M} = \bar{m} + \rho A_{t}$$

$$\beta_{t}^{M} = \rho B_{t} \tag{5.8}$$

$$\gamma_{t}^{M} = \rho C_{t}$$

so that the final recursive solution to the A-C coefficient is, for $t < T - 1$

$$A_{t-1} = (1+\rho K_{t})^{-1}\ [\rho(r+\beta_{t}^{F})(A_{t}+C_{t}\alpha_{t}^{F}) - K_{t}\bar{m}$$

$$B_{t-1} = (1+\rho K_{t})^{-1}\ rK_{t} \tag{5.9}$$

$$C_{t-1} = (1+\rho K_t)^{-1} K_t$$

with the solution for $t = T-1$ being given in (5.2).

(ii) The fiscal authority policy rule:

From (1') and (39):

$$V_T^{F'}(d_T) = \lambda r d_{T-1} + \lambda f_{T-1} - \lambda m_{T-1} \tag{5.10}$$

From (34) and (36):

$$V_T^{F'}(d_T) = \lambda r d_{T-1} + \lambda\alpha_{T-1}^F + \lambda\beta_{T-1}^F d_{T-1} - \lambda\alpha_{T-1}^M - \lambda\beta_{T-1}^M d_{T-1} - - \lambda\gamma_{T-1}^M \alpha_{T-1}^F - \lambda\gamma_{T-1}^M \beta_{T-1}^F d_{T-1}$$

which becomes:

$$V_T^{F'}(d_T) = D_{T-1} + E_{T-1} d_{T-1}, \tag{5.11}$$

where

$$D_{T-1} = \lambda((1-\gamma_{T-1}^M)\alpha_{T-1}^F - \alpha_{T-1}^M)$$

$$E_{T-1} = \lambda(r-\beta_{T-1}^M + (1-\gamma_{T-1}^M)\beta_{T-1}^F)$$

Now substitute (5.11) in (40) with $t+1 = T-1$:

$$V_{T-1}^{F'}(d_{T-1}) = \lambda d_{T-1} + \rho(r-\beta_{T-1}^M)(D_{T-1}+E_{T-1}d_{T-1}) \tag{5.12}$$

Using (1'), (34) and (36) in (5.12):

$$V_{T-1}^{F'}(d_{T-1}) = \rho(r-\beta_{T-1}^M)D_{T-1} + (\lambda+\rho(r-\beta_{T-1}^M)E_{T-1}) \cdot \cdot (rd_{T-2} + \alpha_{T-2}^F(1-\gamma_{T-2}^M) + (1-\gamma_{T-2}^M)\beta_{T-2}^F d_{T-2} - - \alpha_{T-2}^M - \beta_{T-2}^M d_{T-2}).$$

Collecting Terms:

$$V_{T-1}^{F'}(d_{T-1}) = D_{T-2} + E_{T-2} d_{T-2} \tag{5.13}$$

where:

$$D_{T-2} = \rho(r-\beta_{T-1}^M)D_{T-1} + (\lambda+\rho(r-\beta_{T-1}^M)E_{T-1})\ ((1-\gamma_{T-2}^M)\alpha_{T-2}^F - \alpha_{T-2}^M)$$

$$E_{T-2} = (\lambda+\rho(r-\beta^M_{T-1})E_{T-1})\ (r + (1-\gamma^M_{T-2})\beta^F_{T-2} - \beta^M_{T-2}).$$

Repeating for $t = T-2, T-3, \ldots,$ we get the following general solution for $t < T-1$:

$$V^{F'}_t(d_t) = D_{t-1} + E_{t-1}d_{t-1} \tag{5.14}$$

with

$$D_{t-1} = N_t L_{t-1} + \rho(r-\beta^M_t)D_t$$

$$E_{t-1} = N_t H_{t-1}$$

and

$$L_{t-1} = (1-\gamma^M_{t-1})\alpha^F_{t-1} - \alpha^M_{t-1}$$

$$H_{t-1} = r - \beta^M_{t-1} + (1-\gamma^M_{t-1})\beta^F_{t-1}$$

$$N_t = \lambda + \rho(r-\beta^M_t)E_t$$

Using (5.14) in (40), we immediately obtain:

$$\begin{aligned} \alpha^F_t &= \bar{f} - \rho(1-\gamma^M_t)D_t \\ \beta^F_t &= -\rho(1-\gamma^M_t)E_t \end{aligned} \tag{5.15}$$

The equilibrium time path of debt is obtained by plugging (34), (36) into (1') and using the previously derived expressions for α^i_t, β^i_t, $\gamma^M{}_t$, $i = M,F$:

$$d_{t+1} = H_t d_t + L_t \tag{5.16}$$

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