CENTRALIZED WAGE SETTING AND MONETARY POLICY IN A REPUTATIONAL EQUILIBRIUM

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Abstract

This paper analyzes a repeated game between the central bank and a centralized trade union. The central bank would be better off if it could commit to a noninflationary strategy. When this commitment is not enforceable, a noninflationary equilibrium can still be sustained by a reputational mechanism, if the central bank has superior information about its own objective function. The qualitative properties of this reputational equilibrium are shown to differ from the cases considered in the existing literature, where the central bank was modeled as playing a game against competitive labor markets.

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1. Introduction

In all industrial countries macroeconomic policies are implemented sequentially, as an ongoing process. At each stage of the process, the policymakers can deviate at their discretion from previous announcements and take unexpected actions. Some implications of this institutional feature have been recently analyzed in a number of models in which the policymakers play a dynamic or a repeated game against the private sector or among themselves.[1]

This paper analyzes a repeated game between the central bank (CB) and a centralized trade union (TU). The real wage set by the TU is above what would be optimal for the CB. Thus, the CB has an incentive to lower the real wage by creating unexpected inflation. The fact that the CB cannot commit to a noninflationary strategy gives rise to a Nash equilibrium in which, from the point of view of the CB, the rate of inflation is too high and the growth of output is too low. If however the TU is incompletely informed about the nature of its opponent, reputational effects provide an incentive for the CB to choose a noninflationary monetary policy. For some parameter values, this incentive is shown to be large enough to sustain an equilibrium with no inflation until the last period of the game. For other parameter values, the equilibrium goes through a stage in which the CB chooses a randomized strategy; during this stage, nominal wage growth and output growth exhibit cyclical fluctuations.

Two features distinguish the present model from those already analyzed on this topic: The CB objective function is quadratic in output growth — in the existing literature it was taken to be linear. And the TU plays an active role in the game. In the existing literature the private sector is described by an expectation formation mechanism; this description may be appropriate for a setting of competitive labor markets (like the U.S.), but it certainly lacks realism for many European economies. These two features, and particularly the more explicit description of the private sector's behavior, turn out to make a substantial difference in the qualitative properties of the equilibrium.

The outline of the paper is as follows: Section 2 presents the basic model; Section 3 computes the cooperative equilibrium; Section 4 derives the Nash equilibrium under the hypothesis of complete information and finite horizon. Section 5 characterizes the Nash equilibrium under the hypothesis that the TU is incompletely informed about a parameter in the CB objective function (i.e., it characterizes the perfect Bayesian equilibrium of the game).[2] Section 6 contains a discussion of some results. Section 7 provides a summary and the conclusions.

2. The Model

The macroeconomy is described by two simple equations; an aggregate demand function:

$$m_t = p_t + x_t \tag{1}$$

where m_t = money supply growth; p_t = rate of inflation; x_t = real output growth. And an aggregate supply function:

$$x_t = \alpha(p_t - w_t), \qquad \alpha > 0 \tag{2}$$

where w_t = rate of growth of nominal wages.

The CB sets m_t so as to minimize:

$$V_t^M = \frac{1}{2} \sum_{k=t}^{T} [p_k^2 + \tau x_k^2] \, \beta^k, \qquad \tau > 0, \quad 1 > \beta > 0 \tag{3}$$

and the TU sets w_t so as to minimize:

$$V_t^U = \frac{1}{2} \sum_{k=t}^{T} [(w_k - p_k - v)^2 + \lambda x_k^2] \, \rho^k, \tag{4}$$

$$v > 0, \quad \lambda > 0, \quad 1 > \rho > 0$$

Equation (3) says that the CB wants to stabilize fluctuations of output growth and of inflation around some desired values, taken to be zero for notational convenience. The parameter τ indicates the relative weight assigned by the CB to the output objective. Equation (4) says that the TU wants to stabilize the rate of growth of output around zero and the rate of growth of real wages around some desired target, $v > 0$. The parameter λ indicates the relative weight assigned by TU to the output objective.

Implicitly, therefore, it is assumed that firms take nominal wages as given and that they set employment (and output) according to equation (2). This assumption, together with the specification of preferences for the two players, is standard in the literature.[3]

After some manipulations, equations (1) and (2) yield:

$$p_t = \frac{1}{1+\alpha}(m_t+\alpha w_t) \qquad \text{(i)}$$

$$x_t = \frac{\alpha}{1+\alpha}(m_t-w_t) \qquad \text{(ii)} \qquad (5)$$

$$w_t - p_t = \frac{1}{1+\alpha}(w_t-m_t) \qquad \text{(iii)}$$

Thus, for a given nominal wage, monetary policy is non-neutral, since it can affect real wages. Similarly, for a given rate of growth of money supply, nominal wages are non-neutral, since they affect real wages too.

The conflict between the two players is generated by the hypothesis that $v > 0$ in (4). For if $v = 0$, the objectives of the two players would be mutually compatible and there would be no game to be played. With $v > 0$, the TU will tolerate a loss in output in exchange for some positive growth in real wages. But this now gives the CB an incentive to inflate away the increase in real wages, so as to restore output growth at its optimal value of zero. If the TU realizes this, it will set nominal wages even higher. This strategic interaction between the two players is analyzed in Sections 3-5 below under different hypotheses about the information available to the two players.

3. The Cooperative Equilibrium

What would be the rate of growth of the macroeconomic variables if both players could make a binding commitment to set their policy variables so as to optimize a weighted average of their two objective functions? The answer to this question is interesting because the resulting cooperative equilibrium: (i) can serve as a benchmark against which to evaluate all the other noncooperative equilibria; and (ii) it can be interpreted as a "social contract" between the policymaker and the TU, of the kind advocated by many economists and policymakers in several European countries.

Let the common loss function in each period be - time subscripts will be omitted when superfluous:

$$H^c = \frac{1}{2}\left[(w-p-v)^2 + (\lambda+\gamma\tau)x^2 + \gamma p^2\right] \tag{6}$$

$\gamma > 0$ being the weight assigned to the CB objectives. Taking the first order conditions of (6) with respect to w and m, subject to (5), yields the cooperative equilibrium solution:

$$
\left.
\begin{array}{lr}
p^c = 0 & \text{(i)} \\[1ex]
x^c = \dfrac{-\alpha v}{1+\alpha^2(\lambda+\gamma\tau)} < 0 & \text{(ii)} \\[2ex]
w^c = w^c - p^c = \dfrac{v}{1+\alpha^2(\lambda+\gamma\tau)} > 0 & \text{(iii)} \\[2ex]
m^c = \dfrac{-\alpha v}{1+\alpha^2(\lambda+\gamma\tau)} < 0 & \text{(iv)}
\end{array}
\right\} \tag{7}
$$

the c superscript standing for cooperative equilibrium.

Thus, the TU sets a positive rate of growth of nominal wages and the CB a negative rate of growth of the money supply (recall that, for notational convenience, zero is assumed to be the optimal rate of growth of output for both players). This combination of w and m yields a zero rate of inflation and a negative growth of output.

Notice that as $\gamma \to \infty$ -- i.e., as the CB weight becomes predominant in the common loss function, all variables tend to zero. Vice versa, as $\gamma \to 0$ (i.e., as the TU weight becomes predominant in the common loss function), the rate of inflation remains at zero and the other variables reach the following limit values (denoted with a $-$):

$$\bar{\omega}^c = \bar{w}^c - \bar{p}^c = \frac{v}{1+\alpha^2\lambda} > w^c > 0 \qquad \text{(i)}$$

$$\bar{x}^c = \frac{-\alpha v}{1+\alpha^2\lambda} < x^c < 0 \qquad \text{(ii)} \qquad (8)$$

$$\bar{m}^c = \frac{-\alpha v}{1+\alpha^2\lambda} < m^c < 0 \qquad \text{(iii)}$$

The equilibrium summarized in (8) and with $p = 0$ can also be shown to be the Stackelberg equilibrium of a game in which the CB moves first and acts as the dominant player. Intuitively: in such an equilibrium the CB does not take nominal wages as given, but it takes into account how the TU will respond to its own actions. In this case monetary policy turns out to be neutral. Thus, the best thing the CB can do is to set $p = 0$ and let output be determined by the TU.

This interpretation of equation (8) highlights the fact that the cooperative equilibrium differs from the noncooperative equilibria analyzed below in two respects: by forcing the TU to moderate its real wage demands — whenever $\gamma > 0$; and by enabling the CB to enter a binding committment not to inflate away the real wage chosen by the TU. This second feature of the cooperative equilibrium holds irrespectively of the value taken by γ, and guarantees the achievement of the optimal rate of inflation. As argued in

Barro and Gordon (1983a,b), point (ii) can be brought about by the CB without entering into a "social contract", simply by unilaterally modifying its decision process.

4. The Nash Equilibrium

Under the current institutional setting currently prevailing in all industrial countries, neither the monetary authority nor the trade union can enter into binding commitments. Thus, the appropriate solution concept under the hypothesis that both players move simultaneously is that of Nash equilibrium. Since the game is repeated for a finite number of times, the only subgame perfect[4] Nash equilibrium for this game is obtained by restricting both players to choose among open-loop strategies. Given that the game is linear-quadratic, the Nash equilibrium computed below is thus the unique subgame perfect Nash equilibrium of the game.

Both players maximize their respective objective functions, subject to equations (5) and taking as given the current and future actions of their opponent (i.e., they are choosing open-loop strategies). The first order condition for the CB yields:

$$p = -\alpha\tau x \tag{9}$$

The first order condition for the TU yields:

$$(w-p) = v + \alpha\lambda x \tag{10}$$

Putting together (9), (10), (1) and (2) gives the Nash equilibrium (the N superscript standing for Nash):

$$p^N = \frac{\alpha^2\tau v}{1+\alpha^2\lambda} > 0 \qquad \text{(i)}$$

$$x^N = \frac{-\alpha v}{1+\alpha^2\lambda} < 0 \qquad \text{(ii)}$$

$$m^N = \frac{(\alpha\tau-1)\alpha v}{1+\alpha^2\lambda} \gtrless 0 \qquad \text{(iii)} \qquad (11)$$

$$w^N = \frac{v(1+\alpha^2\tau)}{(1+\alpha^2\lambda)} > 0 \qquad \text{(iv)}$$

$$w^N - p^N = \frac{v}{1+\alpha^2\lambda} \qquad \text{(v)}$$

Comparing the Nash equilibrium with the cooperative equilibrium computed in equation (8) of Section 3, under the hypothesis of zero weight on the CB objectives (i.e., with $\gamma = 0$), one sees immediately that the only differences concern the rate of inflation (which was zero in the cooperative equilibrium) and the other two nominal variables, w^N, m^N. Real output and real wages are both as in equation (8), where $\gamma = 0$. The following is then easily established, by comparing (7), (8) and (11):

<u>Proposition 1</u>: <u>The CB is better off in the cooperative equilibrium than in the Nash equilibrium, for any value of γ. Vice versa the TU is always better off in the Nash equilibrium than in the cooperative equilibrium, except if either $\gamma = 0$ or $\tau = 0$, in which cases it is indifferent</u>.

The result concerning the CB has already been discussed in Section 3 and in some of the existing literature; it arises from the institutional impossibility for the CB to make binding commitments in a Nash equilibrium. The result concerning the TU derives from the hypothesis that the TU is indifferent about the time path of inflation and of any other nominal variables. If this hypothesis was relaxed, the TU would be better off in a cooperative equilibrium for low values of γ, and it would be better off in the Nash equilibrium for high values of γ.

The assumption that the TU is indifferent about the rate of growth of nominal values is plausible. Its implication is that it will be very difficult (here, absolutely impossible) to entice the TU into a "social contract", since the TU would have nothing to gain from it (but it could lose if $\gamma > 0$).[5]

Finally, notice that in the Nash equilibrium, output and the real wage are unaffected by τ, the relative weight in the CB objective function.[6] The rate of inflation, instead, rises with τ, and it is zero if $\tau = 0$. Intuitively: if the CB assigns zero weight to the output objectives, it would not be tempted to inflate away any increase in real wages. The TU would realize this, and would set lower nominal wages. Since in a model with complete information the TU will never let the CB inflate away its real wage increases, a smaller τ leads to less inflation and leaves output unaffected.[7]

The fact that τ does not affect real variables also suggests that the TU would not benefit from acting as a Stackelberg leader in the game. In fact, some simple computations show that the equilibrium summarized in (11) coincides with the Stackelberg equilibrium in which the TU has the first move in the game.

5. Nash Equilibrium Under Incomplete Information

A crucial feature of the Nash equilibrium analyzed in Section 4, common to other papers on the same topic, is that the CB would gain by committing itself not to inflate away the higher real wages demanded by the TU. Even though this commitment cannot be undertaken in the current institutional setting, in many countries the monetary authorities announce in advance their intermediate monetary targets. This section analyzes the issue of the credibility of such announcements and the extent to which they influence the behavior of the TU and of the CB itself.

Since the Nash equilibrium computed in Section 4 is the unique subgame perfect equilibrium, under the assumptions of complete information and finite horizon no announcement will ever be believed unless it coincides with the CB first order condition given by equation (9) above. If either of the two assumptions is dropped, however, announcements may become an effective policy instrument.

Here I solve the game for a finite horizon and under the hypothesis that the TU has incomplete information about the parameter τ in the CB objective function. The setting is as in Kreps & Wilson (1982), Barro (1985), Backus & Driffill (1985a,b) and Stella (1983): this element of incomplete information now gives the CB an incentive to maintain its reputation in the early stages of the game by not deviating from the announcements. If this incentive is large enough, the announcements may become perfectly credible. Unlike in Backus and Driffill (1985a) or in Barro (1985), however, here the private sector has an active strategic role; and this turns out to make a difference in the qualitative features of the solution.

The game proceeds as follows: when the game is started the CB announces that it will follow to a noninflationary policy rule. Then both players choose their actions. In each period the TU sets nominal wage growth so as to minimize its expected loss, on the basis of its prior beliefs about the nature of its opponent. Then the CB moves and the TU, having observed the actual behavior of the CB, revises its beliefs according to Bayes rule. The resulting equilibrium is a perfect Bayesian equilibrium.[8]

5.1 Some Preliminary Results

For simplicity and without loss of generality, I assume that, when the game begins, the TU assigns a prior probability of $\bar{P}$ to the event that $\tau = 0$, and a probability of $(1-\bar{P})$ to the event that $\tau = \bar{\tau} > 0$. These prior beliefs are common knowledge. A CB with $\tau = 0$ will be called "callous" (in the sense that it does not care about output). Vice versa, a CB with $\tau = \bar{\tau} > 0$ will be called "responsive".[9]

If $\tau = 0$, then from the CB reaction function, equation (9):

$$p_t = 0 \tag{12}$$

If instead $\tau = \bar{\tau} = 0$, then from (9):

$$p_t = -\alpha\bar{\tau}x_t \tag{13}$$

The TU does not know the true value of τ, and thus is uncertain about whether (12) or (13) will hold. Let $P_t = \text{prob}(\tau=0)$ be the TU prior beliefs at time t, and let $Q_t = \text{prob}(p_t=0)$, $P_t^* = \text{prob}(p_t=0/\tau=\bar{\tau})$. Thus, Q_t is the unconditional probability that there will be no inflation at time t, and P_t^* is the conditional probability of zero inflation, given that the CB is responsive. As it will be shown below, P_t^* is chosen by a responsive CB. It then follows from these definitions that:

$$Q_t = P_t + (1-P_t)\, P_t^* \tag{14}$$

The hypothesis that P_t is revised according to Bayes rule implies that:[10]

$$\begin{aligned} &P_{t+1} = 0 && \text{if} \quad p_t \neq 0 \qquad \text{(i)} \\ &P_{t+1} = \frac{P_t}{P_t+(1-P_t)P_t^*} = \frac{P_t}{Q_t} && \text{if} \quad p_t = 0 \qquad \text{(ii)} \end{aligned} \tag{15}$$

P_t is a sufficient statistic for the history of the game up to time t, and is a natural measure of the CB reputation. If inflation is observed in period t, then the CB reputation of being callous is destroyed, and $P_{t+1} = 0$. If $p_t = 0$ is observed, the CB could be truely callous (i.e., $\tau = 0$); or it could simply pretend to be so, in order to maintain or enhance its reputation. The relevant posterior probability, then, depends on P_t^*, the probability that a responsive CB will tolerate zero inflation. In equilibrium, P_t^* must be consistent with the optimal behavior of a responsive

CB. In sections 5.3, 5.4 below, the optimal P_t^* will be deduced from the CB strategic problem. Before then, the TU optimization problem has to be solved.

5.2 The Trade Union's Optimization Problem

The TU sets nominal wages so as to minimize its expected loss, subject to (2), and subject to the conjecture that (12) holds with probability Q_t and that (13) holds with probability $(1-Q_t)$. After some substitutions, the TU's expected loss in each period is given by:

$$\hat{H}_t^U = \frac{1}{2} Q_t \; [(-\alpha w_t)^2\lambda + (w_t-v)^2] + \frac{1}{2}(1-Q_t) \cdot \cdot [(\frac{-\alpha}{1+\alpha^2\bar{\tau}} w_t)^2\lambda + (\frac{w_t}{1+\alpha^2\bar{\tau}} - v)^2] \tag{16}$$

The first order conditions with respect to w_t, taking Q_t as given, yield:

$$w_t^B = w^N \cdot \phi(Q_t) \tag{17}$$

where the B superscripts stand for Bayesian equilibrium, w^N is given in (11.iv), and $\phi(Q_t) = \frac{(1+\alpha^2\bar{\tau}Q_t)}{[1+\alpha^2\bar{\tau}(2+\alpha^2\bar{\tau})Q_t]} < 1$, with $\phi'(Q_t) < 0$, $\lim_{Q_t \to 0} \phi(Q_t) = 1$, and $\lim_{Q_t \to 1} \phi(Q_t) = \frac{1}{1+\alpha^2\bar{\tau}}$.

Comparing (17) with (11.iv) and (8.i), we obtain the following:

<u>Proposition 2</u>: <u>$\bar{w}^c < w_t^B < w^N$</u>, <u>with strict inequalities if</u> <u>$1 > Q_t > 0$</u>, <u>and with</u> <u>$w_t^B = \bar{w}^c$ if $Q_t = 1$, $w_t^B = w^N$ if $Q_t = 0$.</u>

That is, when a noninflationary monetary policy is seen as perfectly credible by the TU (i.e., when $Q_t = 1$), nominal wages in a Bayesian equilibrium are set at $\bar{w}^c$, as in the cooperative equilibrium with $\gamma = 0$. Vice versa, if the commitment to a noninflationary monetary policy is not credible at all (i.e., if $Q_t = 0$), then nominal wages are set at w^N, as in the Nash equilibrium. More generally, since w_t^B is decreasing in Q_t, the more credible is the commitment to a noninflationary monetary policy, the lower is the rate of growth of nominal wages and the closer is the TU behavior to the cooperative equilibrium.

5.3 The Central Bank's Optimization Problem

If the CB actually is callous (i.e., if $\tau = 0$), its optimal behavior is simply to set m_t so as to have $p_t = 0$ in any period. If the CB is responsive (i.e., if $\tau = \bar{\tau} > 0$), its optimal strategy is more sophisticated: the dependence of nominal wages on Q_t now gives the CB an incentive to maintain or enhance its reputation of being callous. This incentive could lead a responsive CB to choose a noninflationary monetary policy in the early stages of the game. The optimal strategy for a responsive CB is characterized in this and in the next subsection.

In the last period of the game, the CB will always inflate, since destroying its reputation can have no future adverse consequences. In the earlier periods, the CB will weight the short run costs of a noninflationary monetary policy against the long run gains in the form of a higher reputation for the remainder of the game. Let $\hat{H}_t^M$ be the indirect expected loss function of the CB in period t. With probability P_t^* the CB will set m_t so that $p_t = 0$; from (4) and (2), if $p_t = 0$ is realized, its loss will be $\frac{1}{2}\,\alpha^2\bar{\tau}(w_t^B)^2$. With probability $(1-P_t^*)$ the CB will play the optimal inflationary strategy given in (13); in this case, plugging (13) in (2) and then in (4), its loss is given by $\frac{\bar{\tau}\alpha^2}{1+\alpha^2\bar{\tau}}\,(w_t^B)^2$. Hence, the CB expected loss in period t is:

$$\hat{H}_t^M = \frac{1}{2}\,\alpha^2\bar{\tau}\,[P_t^*(w_t^B)^2 + (1-P_t^*)\,\frac{1}{1+\alpha^2\bar{\tau}}\,(w_t^B)^2] \tag{18}$$

If in period t the CB inflates, then, as stated in (15), $P_{t+i} = 0$, $i > 1$, so that from then on the outcome of the game is as in the Nash equilibrium:

$$\hat{H}^M_{t+1} = \frac{1}{2}(p^N)^2 + \frac{1}{2}(x^N)^2\bar{\tau} = \frac{\alpha^2\bar{\tau}}{1+\alpha^2\tau}\cdot(w^N)^2 \tag{19}$$

where the last equality follows from (11). If instead in period t the CB sticks to the noninflationary strategy, then P_{t+1} is formed according to (15.ii). Denoting by $V_t(P_t)$ the CB indirect loss function from period t until the end of the game, conditional on having played the noninflationary strategy up to period t, we then have, for $t < T$:

$$V_t(P_t) = \hat{H}^M_t + P^*_t\,\beta V_{t+1}(P_{t+1}) + (1-P^*_t)\,\frac{1}{2}\,\frac{\alpha^2\bar{\tau}}{1+\alpha^2\bar{\tau}}\,(w^N)^2\sum_{k=1}^{T-t}\beta^k \tag{20}$$

For given nominal wages (that is, for given Q_t), $V_t(P_t)$ is linear in P^*_t. But nominal wages (i.e., Q_t) have to be taken as given when computing the optimal CB strategy in equilibrium, since there is no mechanism forcing the CB to match the TU beliefs as implicit in Q_t and as incorporated in nominal wages.[11] In other words, imposing the condition that nominal wages are taken as given by the CB when solving its optimization problem is equivalent to imposing an incentive compatibility condition on the equilibrium behavior of the CB.

Since $1 > P^*_t > 0$, there are three cases to consider:

(i) $\dfrac{\partial V_t(P_t)}{\partial P^*_t} > 0$, which implies $P^*_t = 0$ (recall that $V_t(P_t)$ is a <u>loss</u> function). That is, the optimal CB strategy in period t is the pure strategy of no inflation. (ii) $\dfrac{\partial V_t(P_t)}{\partial P^*_t} < 0$, implying $P^*_t = 1$; here, the optimal strategy is the pure inflationary strategy of setting p_t as in (13).

(iii) $\dfrac{\partial V_t(P_t)}{\partial P^*_t} = 0$, in which case the CB chooses a mixed strategy (it plays $p_t = 0$ with probability $P^*_t > 0$, and p_t as in (13) with probability $(1-P^*_t) > 0$). Section 1 of the Appendix proves the following:

<u>Proposition 3</u>: $\frac{\partial V_t(P_t)}{\partial P_t^*} \gtrless 0$ as

$$(\frac{\bar{\tau}\alpha^2}{1+\bar{\tau}\alpha^2})\ \bar{\tau}\alpha^2(w_t^B)^2 \gtrless \beta\ (\frac{\bar{\tau}\alpha^2}{1+\bar{\tau}\alpha^2})\ [(w^N)^2 - (w_{t+1}^B)^2] \qquad (21)$$

Using (18) and (19), the left hand side of (21) can be shown to be equal to the net cost for the CB of <u>not</u> creating unexpected inflation today (i.e., it is the "temptation to cheat" of Barro & Gordon (1983a), Barro (1985)). The right hand side of (21) can be shown to be equal to the net opportunity cost for the CB of creating unexpected inflation <u>today rather than tomorrow</u> (since it is equal to the next period loss if the CB inflates today less next period loss if it unexpectedly inflates tomorrow). Thus, the right hand side of (21) is the incentive that sustains a noninflationary monetary policy today for a responsive CB. When the two sides of (21) are equal, the CB chooses a mixed strategy (i.e., $1 > P_t^* > 0$), since it is indifferent between creating unexpected inflation today rather than tomorrow. If (21) holds with a $>$ sign, then the net cost of waiting to inflate until tomorrow exceeds the corresponding net gain, and the CB chooses a pure inflationary strategy right away (i.e., $P_t^* = 0$). Conversely, if (21) holds with a $<$ sign, the net gain from preserving its reputation exceeds the cost of nonaccommodation, and the CB resists the temptation to inflate (i.e., $P_t^* = 1$).[12]

5.4 Characterization of the Perfect Bayesian Equilibrium

Since, by Proposition 2, w_t^B is constrained to lie between $\bar{w}^c$ and w^N, for some ranges of parameter values condition (21) will always hold with inequalities and the CB will select a pure strategy (i.e., it will set $P_t^* = 0$ or $P_t^* = 1$). These ranges are identified in the following:

<u>Proposition 4</u>: (i) <u>If</u> $\phi^2(\bar{P}) < \frac{\beta}{\beta+\bar{\tau}\alpha^2}$, <u>then</u> $P_t^* = 1$ <u>for all $t < T$.</u>

(ii) <u>If</u> $\beta < \frac{1}{2+\bar{\tau}\alpha^2}$, <u>then</u> $P_t^* = 0$ <u>for all $t < T$</u>.

<u>Proof</u>: Using (17), rewrite (21) as:

$$\bar{\tau}\alpha^2 \ \phi^2(Q_t) + \beta\phi^2(Q_{t+1}) \gtrless \beta \tag{22}$$

Recalling the discussion in p. 11 above, $\phi^2(\bar{P}) > \phi^2(Q_t) > \frac{1}{(1+\bar{\tau}\alpha^2)^2}$. Thus, if $(\bar{\tau}\alpha^2+\beta)\ \phi^2(\bar{P}) < \beta$, then (22) holds with a $<$ sign for all Q_t. From Proposition 3 and the discussion preceding it, then, $P_t^* = 1$ for all $t < T$. Similarly, if $\frac{(\bar{\tau}\alpha^2+\beta)}{(1+\bar{\tau}\alpha^2)^2} > \beta$, then (22) holds with a $>$ sign for all Q_t; Proposition 3 then implies that $P_t^* = 0$ for all $t < T$. The two inequalities stated in the text of Proposition 4 are simple transformations of these two inequalities. Q.E.D.

This result can be interpreted as follows: if the CB discount rate, β, is "too low", then the long run gains from reputation will always be outweighted by the short run losses of choosing a noninflationary monetary policy. Hence, a responsive CB will immediately inflate and destroy its reputation (i.e., $P_t^* = 0$). Conversely, if the CB reputation is "sufficiently high" when the game is started,[13] then the long run gains from maintaining that reputation always exceed the short run costs; even a responsive CB, in this case, will never inflate, except in the very last period of the game.

If any of the two conditions stated in Proposition 4 is met, the perfect Bayesian equilibrium is easy to describe: in the last period of the game the CB will always inflate. In case (i), the CB will not inflate in any of the previous periods, so that $Q_t = 1$ and nominal wages are as in the cooperative equilibrium (i.e., $w_t^B = \bar{w}^c$), except in the last period, when $Q_T = \bar{P}$ and nominal wages grow at the higher rate $w^N\phi(\bar{P})$.[14] In case (ii), the CB will inflate right away, at the beginning of the game. Nominal wages will grow at

$w^N\phi(\bar{P})$ in the first period, and from then on they will grow as in the Nash equilibrium (i.e., $w_t^B = w^N$, $t > 1$).[15]

If both conditions stated in Proposition 4 are violated, then $1 > P_t^* > 0$ is possible over some time interval. Let us assume that this occurs during the interval $k,k+1,\ldots,T-1$. In this case, the CB must be indifferent between its two options, so that (21) and (22) hold with equalities. Equations (22), (14) and (15) together then determine the time path of Q_t, P_t and P_t^*, subject to the initial condition $P_k = \bar{P}$ (since for $t < k$ no randomization occurs, so that (15) implies $P_{t+1} = \bar{P}$), and to the terminal condition $P_T^* = 0$ (or, equivalently, $Q_T = P_T$). This system of equations is highly nonlinear, and can be solved only by numerical simulations. However, some of its crucial qualitative features can be described analytically; this will be done next.

Let P_T be the value taken by P_t in the last period of the game. P_T is (and will remain) unknown. However, since by hypothesis $1 > P_t^* > 0$, it follows from (15) that $1 > P_T > \bar{P}$. As remarked above, $Q_T = P_T$ (since $P_T^* = 0$). It is then possible to solve (22) forward, as a function of $\phi^2(P_T)$. This yields, for $k \leq t < T$:

$$\phi^2(Q_t) = \frac{\beta}{\bar{\tau}\alpha^2+\beta} + \left(\frac{-\beta}{\bar{\tau}\alpha^2}\right)^{T-t}\left(\phi^2(P_T) - \frac{\beta}{\beta+\bar{\tau}\alpha^2}\right) \tag{23}$$

Several interesting features of this solution are worth noting:

(i) As long as the CB does not inflate, $1 \geq Q_t \geq \bar{P}$ (see (14) and (15)); it then follows from the discussion in p. 11 above that $\phi^2(\bar{P}) \geq \phi^2(Q_t) \geq \frac{1}{(1+\bar{\tau}\alpha^2)^2}$ for all $t \leq T$.

(ii) If $\phi^2(P_T)$ happens to be equal to $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$, then $\phi^2(Q_t) = \frac{\beta}{\beta+\bar{\tau}\alpha^2}$ throughout the game. This is analogous to the case investigated in Barro (1985) and in Backus & Driffill (1985a): nominal wages are constant, and P_t^*

declines through time while P_t grows at a constant rate.

(iii) If $\phi^2(P_T) \neq \frac{\beta}{\beta+\bar{\tau}\alpha^2}$, then $\phi^2(Q_t)$ oscillates symmetrically around $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$. Correspondingly, the rate of growth of nominal wages oscillates symmetrically around $\frac{\beta}{\beta+\bar{\tau}\alpha^2}(w^N)^2$. This implies that P_t^* also oscillates in the interval $(0,1)$, and that P_t grows over time, but at periodically different rates of growth.

(iv) If $\beta/\bar{\tau}\alpha^2 < 1$, the fluctuations of $\phi^2(Q_t)$ dampen as one goes from the end towards the beginning of the game. If the starting date is pushed sufficiently backwards (or, equivalently, if $T \to \infty$), $\phi^2(Q_t)$ converges to $\frac{\beta}{\bar{\tau}\alpha^2+\beta}$. Vice versa, if $\beta/\bar{\tau}\alpha^2 > 1$, the oscillations increase in amplitude as one goes backwards towards the starting date. Eventually, for T sufficiently large, $\phi^2(Q_t)$ hits one of the two boundaries stated in point (i) above. When this occurs, the mixed strategy can no longer be sustained, and the CB selects a pure strategy (i.e., $P_t^* = 0$ or $P_t^* = 1$), depending on which boundary is hit first. Figures 1 and 2 below illustrate these two cases.

(v) Which of the two boundaries is hit first (when $\beta/\bar{\tau}\alpha^2 > 1$) depends exclusively on which one of them is closest to $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$. Some tedious computations reveal that this in turn depends on the value taken by $\bar{P}$. If $\bar{P} > \hat{P}$, where an expression for $\hat{P}$ is given in Section 2 of the Appendix, then the upper boundary is hit first. And vice versa if $\bar{P} < \hat{P}$.

The intuition behind some of these results is as follows: while (23) holds, the CB must be indifferent between creating unexpected inflation today or tomorrow. Suppose that nominal wages today grow at a "low" rate (i.e., $\phi^2(Q_t)$ is relatively small). In this case, the net gain from creating unexpected inflation is also small -- since the output distortion is small.

Fig. 1 $(\beta/\bar{\tau}\alpha^2 < 1)$

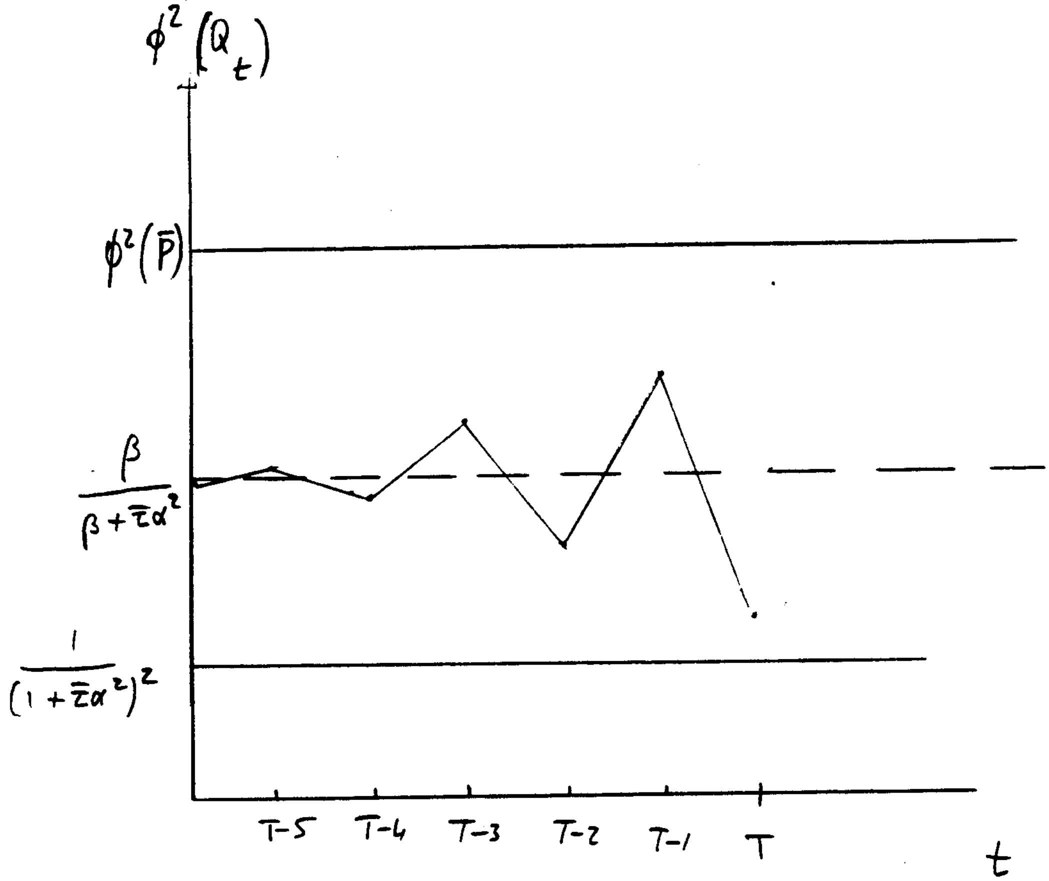
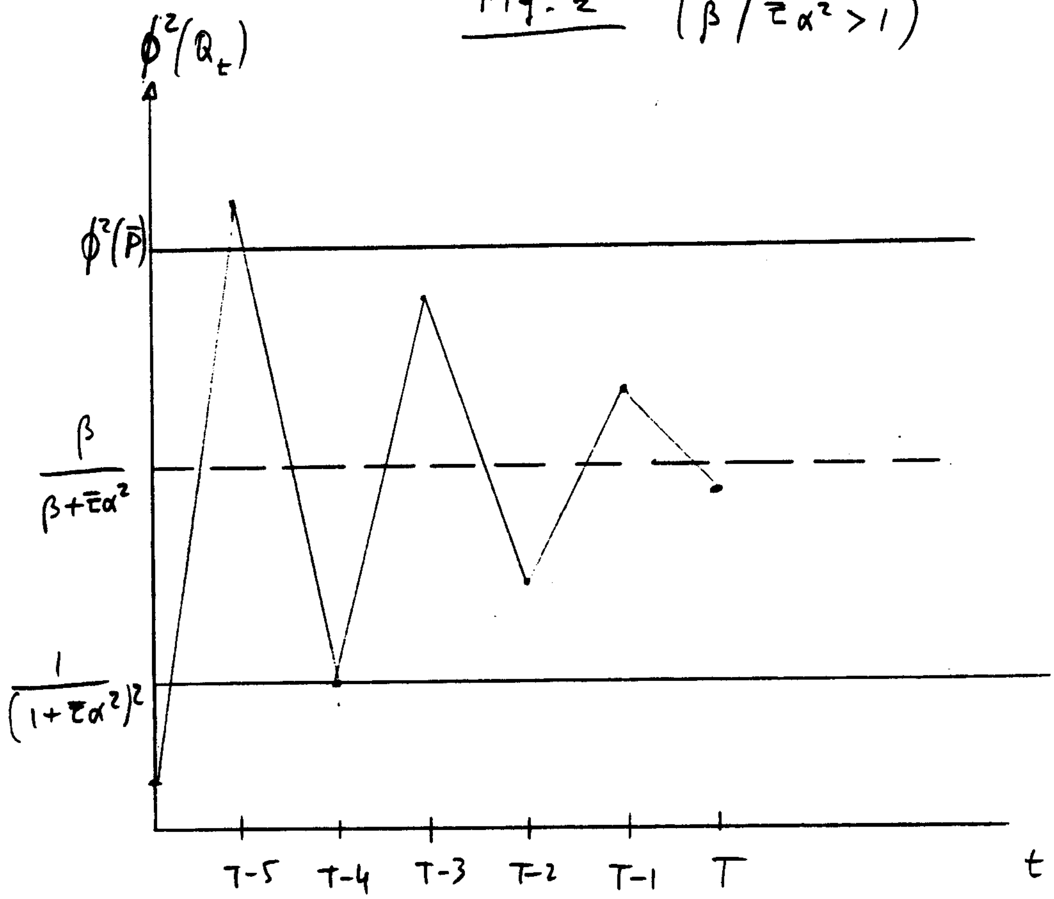


Fig. 2 $(\beta/\bar{\tau}\alpha^2 > 1)$



For the CB to be indifferent between inflating today or tomorrow, the opportunity cost of inflating today must be small. But this can be so only if nominal wage growth tomorrow, in the case of zero inflation today, is relatively high; for in this case the net gain from not inflating today (in the form of a higher reputation tomorrow) is also small. More generally, for the CB to be indifferent, future wage growth must be high whenever current wage growth is low, and vice versa. This gives rise to the oscillatory behavior of nominal wages.

It is now possible to give a characterization of the perfect Bayesian equilibrium of the game, for the case in which the two conditions stated in Proposition 4 are violated. There are several alternatives to consider:

(a) If $P_T = \frac{\beta}{\bar{\tau}\alpha^2+\beta}$, then the solution is as in Barro (1985), Backus & Driffill (1985a): From the start of the game up to $t = k-1$, the CB chooses the pure strategy of no inflation and the TU sets nominal wages as in the cooperative equilibrium, at $\bar{w}^c$. At $t = k$ the CB begins to randomize. P_t^* decreases towards zero as the game is played, and P_t rises towards $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$, until an inflationary policy is realized. Once this has occurred, the game is played as in the Nash equilibrium. For as long as the CB has not inflated, the drop in P_t^* and the rise in P_t completely offset each other, so that $Q_t = \frac{\beta}{\beta+\bar{\tau}\alpha^2}$ and nominal wages grow at the constant rate $w^N \phi(\frac{\beta}{\beta+\bar{\tau}\alpha^2})$ (see (17) and point (ii) in p. 16 above). The date at which randomization begins, k, is computed from (15) as the largest integer for which $P_{k-1} < \bar{P}$, given that $P_T = \frac{\beta}{\beta+\bar{\tau}\alpha^2}$ and given the equilibrium time path of P_t^*. The reader is referred to Barro (1985a) and to Backus & Driffill (1985a) for further details.

(b) If $P_T \neq \frac{\beta}{\bar{\tau}\alpha^2+\beta}$ and $\beta/\bar{\tau}\alpha^2 < 1$, the equilibrium is as in the previous case, with the difference that here, while the CB is randomizing, nominal wages oscillate around $w^N \phi(\frac{\beta}{\bar{\tau}\alpha^2+\beta})$, as dictated by (17) and (25). The oscillation of nominal wages becomes of larger amplitude as the game is played — see point (ii) and Figure 1 above. Also, notice that unlike in case (a), here P_t^* also oscillates, and P_t does not grow at a constant rate.

(c) If $P_T \neq \frac{\beta}{\beta+\bar{\tau}\alpha^2}$, $\beta/\bar{\tau}\alpha^2 > 1$, and $\bar{P} < \hat{P}$ (where $\hat{P}$ is given in Section 2 of the Appendix), the solution is as in case (b), with the following differences: that the oscillation of nominal wages becomes smaller in amplitude as the game is played; and that the date at which randomization begins, k, is determined as the largest integer for which <u>either</u> $P_{k-1} < \bar{P}$, <u>or</u> $\phi(Q_{k-1}) < \frac{1}{(1+\bar{\tau}\alpha^2)^2}$. (This second condition is needed to insure that $P_t^* < 1$) — see point (ii) above and Figure 2. The condition that $\bar{P} < \hat{P}$ insures that the lower bound in Figure 2 is hit first.

(d) If $P_T \neq \frac{\beta}{\beta+\bar{\tau}\alpha^2}$, $\beta/\bar{\tau}\alpha^2 > 1$, and $\bar{P} > \hat{P}$, then, the solution will be as in case (c) if $t' > t''$, where t' is the largest integer for which $P_{t'} < \bar{P}$, and t'' is the largest integer for which $\phi(Q_{t''}) > \phi(\bar{P})$ in (25). In other words, the solution will be as in case (c) if, in going backwards from the last stage of the game, the period in which P_t falls below $\bar{P}$ is met before the period in which $\phi(Q_t)$ hits the upper boundary in Figure 2. Vice versa, if $t' < t''$ (i.e., if the period in which $\phi(Q_t)$ hits the upper boundary is met first, in going from period T backwards), then the mixed strategy can no longer be sustained (i.e., in period t'' equation (25) is satisfied only for $P^*_{t''} < 0$). In this case, the CB will immediately choose the inflationary strategy and from then on the game is played as in the Nash equilibrium. In other words, if $t' < t''$, the equilibrium is as if condition (ii) in Proposition 4 holds.

6. Discussion

The characterization of the perfect Bayesian equilibrium given in the previous section provides the following information:

(i) The equilibrium strategies chosen by the CB depend only on the following structural parameters of the model: β, $\bar{P}$ and $\bar{\tau}\alpha^2$.[16] Changes in the parameters entering the TU objective function, λ and v, have no effect on the CB choices (even though they obviously affect the equilibrium rate of growth of all macroeconomic variables). This suggests that, within the framework of this model, the credibility of monetary policy announcements is independent of the attitude of the TU (and hence is independent of whether, for instance, the announcements are accompanied by income policies or by other policies changing the TU incentives or constraints).

(ii) Unlike in previous models on this topic, for some parameter values the equilibrium is always defined on pure strategies for both players. Equilibria sustained by mixed strategies are somewhat unattractive, since it is not clear what incentive mechanism is inducing the player who randomizes to choose probability assignments consistent with equilibrium. For the ranges of parameter values identified in Proposition 4, this unattractive feature of the perfect Bayesian equilibrium does not emerge in this model. Specifically, for "low" values of the CB rate of time preference, β, the CB will always inflate. And for "high" values of its reputation at the beginning of the game, (i.e., for a high $\bar{P}$), the CB will never inflate, except in the very last stage of this game. Moreover, the condition that gives rise to this second equilibrium does not involve the length of the horizon.[17]

(iii) While the CB is choosing a mixed strategy, the equilibrium rate of growth of nominal wages, and consequently also the rate of growth of output, exhibit an oscillatory pattern. Again, this contradicts earlier results on the same topic (by Barro (1985), Stella (1983), Backus & Driffill (1985a)). Moreover, it illustrates how uncertainty about the policymaker behavior can lead to apparently anomalous patterns of prices and quantities in equilibrium. An external observer unaware of the underlying learning process of the TU would be led to interpret these fluctuations in output and wages as "sunspot" equilibria.[18]

(iv) Changes in the structural parameters of the model have ambiguous effects on the nature of equilibrium. For instance, an increase in $\bar{P}$, the CB reputation at the start of the game, can lead the CB to switch to the non-inflationary pure strategy (if the increase in $\bar{P}$ is such that condition (i) in Proposition 4 now holds); but it could also lead the CB to choose the pure inflationary strategy (if $\bar{P}$ is raised above $\hat{P}$ and the conditions stated in case (c), p. 19, Section 5.4, hold). The same applies to changes in the other relevant parameters, β, T and $\bar{\tau}\alpha^2$. Again, this contradicts earlier models in which increases in β, T and $\bar{P}$ always induced the CB to choose the non-inflationary strategy for a longer period of time.

(v) The rate of growth of output in this equilibrium is as follows: It equals $x^N = \bar{x}^c < 0$ whenever the CB chooses a pure strategy (recall that output growth in the Nash equilibrium, x^N, is equal to output growth in the cooperative equilibrium with $\gamma = 0$, $\bar{x}^c$). It lies above x^N when unexpected inflation occurs (since unexpected inflation lowers the real wage). And it lies below x^N when the CB chooses a mixed strategy and no inflation occurs (since here unexpected deflation occurs, which raises the real wage). Notice that in these last two cases, the equilibrium rate of growth of output depends also on the value taken by $\bar{\tau}$ (the higher is $\bar{\tau}$, the higher is the rate of growth of output when unexpected inflation occurs, but the lower it is when unexpected deflation occurs).

7. Concluding Remarks

This paper has analyzed a repeated game between a centralized trade union and the monetary authorities. The hypothesis that the trade union cares about output and real wages, whereas the central bank cares about output and inflation, gives rise to a conflict of interests between the two players. This conflict cannot be resolved in a cooperative equilibrium since, for the trade union, the Nash equilibrium outcome always dominates the cooperative outcome. In the Nash equilibrium, the rate of inflation is always larger and the growth of output is either equal or strictly smaller than in the cooperative equilibrium. Hence, the central bank is always better off in the cooperative than in the Nash equilibrium. Moreover, as in recent papers by Barro & Gordon (1983a,b) and Barro (1985), the central bank would be better off if it could unilaterally commit itself to play the cooperative strategy of zero inflation.

Under the institutional setting currently prevailing in most industrial countries, such commitments are not feasible. The second part of the paper has investigated the issue of whether reputational effects can sustain a non-inflationary monetary policy in such an institutional setting. This same issue was investigated in a number of recent papers by Barro, by Backus & Driffill and by Stella. In those papers the private sector was described by a mechanism of expectations formation. Here instead the private sector has a more active strategic role, more similar to the active role played by a centralized trade union in many European countries. This different specification of the private sector turns out to be crucial for the qualitative features of the equilibrium. Depending on the values taken by the structural parameters, three kinds of equilibria can exist: (i) A pure strategy equilibrium in which the central bank inflates only in the last period of the game; independently of the length of the time horizon, in this particular equilibrium neither player ever chooses a mixed strategy. (ii) A mixed strategy equilibrium in which the central bank in each period inflates with some positive probability; in this equilibrium, output, nominal wages and expected inflation all exhibit an oscillatory pattern. (iii) Another pure strategy equilibrium in which the central bank inflates in each period of the game, right from the beginning.

Footnotes

*I wish to thank Jeffrey Sachs for some suggestions on how to simplify a previous version of this paper and Giovanna Mossetti for many helpful comments. The responsibility for any errors is my own.

[1]The cases considered in the existing literature are those of: (a) The central bank playing a game against competitive markets in a closed economy (Barro (1985), Barro & Gordon (1983a,b), Stella (1983), Alesina (1985), Rogoff (1983), Backus & Driffill (1985a,b), Canzoneri (1985)). (b) The central bank playing a game against a centralized trade union in a closed economy (Tabellini (1983)) and in an open economy, the policy instrument here being the exchange rate (Horn & Persson (1984)). (c) The fiscal authority playing a game against a centralized trade union (Driffill (1984), Calmfors & Horn (1983)). (d) The fiscal and the monetary authorities playing a game among themselves (Loewy (1983), Tabellini (1985)) and against a centralized trade union (Alesina & Tabellini (1985)).

[2]See for instance Kreps & Wilson (1982), Fudenberg & Tirole (1983).

[3]See, for instance Calmfors (1984), McDonald & Solow (1981), Oswald (1985, 1982), Rogoff (1983).

[4]See Kreps & Wilson (1982).

[5]Naturally, some gains from cooperation could emerge for the TU if the contract involved the fiscal authority as well as the central bank. This issue is explored more in detail in Alesina & Tabellini (1985).

[6]This result will no longer hold in the reputational equilibrium examined in Section 1.

[7]This point is elaborated more at length in Rogoff (1983) in a similar model.

[8]See Kreps & Wilson (1982), Fudenberg & Tirole (1983).

[9]These two CB types could also be interpreted with reference to political connotations — see Alesina (1985).

[10]In deriving (15.ii) from Bayes rule the following facts have been used: $\text{prob}(p_t \neq 0 \mid \tau = 0) = 0$; $\text{prob}(p_t = 0 \mid \tau = 0) = 1$. Cf., Kreps and Wilson (1982).

[11]On this point, see Barro (1985), or Barro & Gordon (1983a,b).

[12]See also the discussion in Barro (1985).

[13]Recall that $\phi^2(\bar{P})$ is decreasing in $\bar{P}$.

[14]These results follow from (14), (15) and (17), by noting that: (a) if $P_t^* = 1$, then $Q_t = 1$ and $P_{t+1} = \bar{P}$; (b) in the last period, $P_T^* = 0$, so that $Q_T = \bar{P}$.

[15]These results too follow from (14), (15) and (17), by noting that: (a) in the first period, $P_t^* = 0$, so that $Q_t = \bar{P}$; (b) from then on, since the CB has inflated, $P_{t+1} = Q_{t+1} = 0$.

[16]In the existing literature, instead, CB choices in equilibrium are independent of the relative weight assigned by the CB to its output objectives, $\bar{\tau}$.

[17]In Barro (1985), for $T \to \infty$ a responsive CB would never inflate, since the gains from reputation always exceed the costs of losing it.

[18]The idea that learning about the central bank objectives can explain otherwise paradoxical volatility of market prices is investigated more in detail in Tabellini (1984) with reference to U.S. financial markets.

Appendix

1. Proof of Proposition 3

Consider first the case in which $\frac{\partial V_t(P_t)}{\partial P_t^*} = 0$. Setting the right hand side of (20) equal to zero implies:

$$\beta V_{t+1}(P_{t+1}) = \frac{1}{2} \frac{\alpha^2 \bar{\tau}}{1+\alpha^2 \bar{\tau}} (w^N)^2 \sum_{k=1}^{T-t} \beta^k - \frac{\partial \hat{H}^M}{\partial P_t^*} \tag{1.1}$$

Plugging (1.1) back in (20) yields:

$$V_t(P_t) = \hat{H}_t^M - P_t^* \frac{\partial H^M}{\partial P_t^*} + \frac{1}{2} \frac{\alpha^2 \bar{\tau}}{1+\alpha^2 \bar{\tau}} (w^N)^2 \sum_{k=1}^{T-t} \beta^k \tag{1.2}$$

Now advance (1.2) by one period to substitute away the left hand side of (1.1), and simplify:

$$\beta(H_{t+1}^M - P_{t+1}^* \frac{\partial \hat{H}_{t+1}^M}{\partial P_{t+1}^*}) = \frac{1}{2} \frac{\alpha^2 \bar{\tau}}{1+\alpha^2 \bar{\tau}} (w^N)^2 \beta - \frac{\partial \hat{H}_t^M}{\partial P_t^*} \tag{1.3}$$

Using (18) this can be further simplified as:

$$\frac{(\bar{\tau}\alpha^2)^2}{1+\bar{\tau}\alpha^2} (w_t^B)^2 = \beta[(w^N)^2 - (w_{t+1}^B)^2] \frac{\bar{\tau}\alpha^2}{1+\bar{\tau}\alpha^2} \tag{1.4}$$

Repeating the same steps for $\frac{\partial V_t(P_t)}{\partial P_t^*} \lesseqgtr 0$ yields Proposition 3.

2. Computation of $\hat{P}$

For notational convenience, define:

$$x_t = \phi^2(Q_t) - \frac{\beta}{\beta+\bar{\tau}\alpha^2}; \quad \bar{x} = \phi^2(\bar{P}) - \frac{\beta}{\beta+\bar{\tau}\alpha^2} > 0; \quad \underline{x} = \frac{1}{(1+\alpha^2\bar{\tau})^2} - \frac{\beta}{\beta+\bar{\tau}\alpha^2} < 0;$$

then, $\bar{x} > x_t > \underline{x}$, and (25) can be rewritten as:

$$x_t = (\frac{-\beta}{\bar{\tau}\alpha^2})^{T-t} x_T. \tag{2.1}$$

Under the hypothesis that $\frac{\beta}{\bar{\tau}\alpha^2} > 1$, there is going to be a $\bar{t}$ for which $x_{\bar{t}} = \bar{x}$, and a $\underline{t}$ for which $x_{\underline{t}} = \underline{x}$. We want to find out for which values of $\bar{P}$ is $\bar{t} \gtrless \underline{t}$. It is most convenient to proceed under the assumption that t is real. Then, solving for $\bar{t}$ the condition that $x_{\bar{t}} = (\frac{-\beta}{\bar{\tau}\alpha^2})^{T-\bar{t}} = \bar{x}$, yields:

$$\bar{t} = T - \frac{\ln \bar{x} - \ln x_T}{\ln \beta - \ln(\bar{\tau}\alpha^2)} \tag{2.3}$$

where for conveinence $\bar{t}$ has been assumed to be even. Similarly, solving for $\underline{t}$ the condition that $x_{\underline{t}} = (\frac{-\beta}{\bar{\tau}\alpha^2})^{T-\underline{t}} = \underline{x}$, yields:

$$\underline{t} = T - \frac{\ln(-\underline{x}) - \ln x_T}{\ln \beta - \ln(\bar{\tau}\alpha^2)} \tag{2.4}$$

where now for convenience $\underline{t}$ has been assumed to be odd. Subtracting (2.4) from (2.3) yields:

$$\bar{t} - \underline{t} = \frac{\ln(-\underline{x}) - \ln(\bar{x})}{\ln\beta - \ln(\bar{\tau}\alpha^2)} \tag{2.5}$$

so that

$$\bar{t} - \underline{t} \gtrless 0 \quad \text{as} \quad -\underline{x} \gtrless \bar{x}$$

which in turn implies:

$$\bar{t} - \underline{t} \gtrless 0 \quad \text{as} \quad \frac{\beta}{\beta+\bar{\tau}\alpha^2} - \frac{1}{(1+\alpha^2\bar{\tau})^2} \gtrless \phi^2(\bar{P}) - \frac{\beta}{\beta+\bar{\tau}\alpha^2} \tag{2.6}$$

Condition (2.6) confirms what has been claimed in the text, point (v), p. 17: if the lower boundary in Figure 2 is closer to $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$ than the upper boundary (i.e., if $-\underline{x} < \bar{x}$), then $\bar{t} < \underline{t}$ (i.e., then the lower boundary is hit first as the game unravel backwards from the last period). Notice that condition (2.6) is independent of the actual value taken by P_T; this makes intuitive sense, since the oscillations of x_t around $\frac{\beta}{\beta+\bar{\tau}\alpha^2}$ are symmetric.

Condition (2.6) can be rewritten as:

$$\bar{t} - \underline{t} \gtrless 0 \quad \text{as} \quad \frac{2\beta}{\beta+\bar{\tau}\alpha^2} \gtrless \phi^2(\bar{P}) + \frac{1}{(1+\alpha^2\bar{\tau})^2} \tag{2.7}$$

which can be solved for $\hat{P}$, the value of $\bar{P}$ such that (27) holds with equality. Making use of the expression for $\phi^2(\bar{P})$ in p. 11 of the text, it is possible to show that:

$$\hat{P} = \frac{\Delta - (1+\bar{\tau}\alpha^2)}{\bar{\tau}\alpha^2[(1+\bar{\tau}\alpha^2) - (2+\bar{\tau}\alpha^2)\Delta]} \tag{2.8}$$

where $\Delta = \sqrt{\frac{\beta(1+2(\bar{\tau}^2\alpha)^2 + 4\bar{\tau}^2\alpha) - \bar{\tau}\alpha^2}{\beta + \bar{\tau}\alpha^2}}$

Since $\phi^2(\bar{P})$ is decreasing in $\bar{P}$ (see p. 11 of the text), if $\bar{P} < \hat{P}$, then $\bar{t} < \underline{t}$ (i.e., the <u>lower</u> bound is hit first). And if $\bar{P} > \hat{P}$, then $\bar{t} > \underline{t}$ (i.e., the <u>upper</u> bound is hit first).

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