INVENTORIES, MULTIPERIOD IMPLICIT CONTRACTS, AND THE DYNAMIC BEHAVIOR OF THE FIRM UNDER UNCERTAINTY

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1. Introduction

This paper develops an intertemporal model of a firm that makes price, output, inventory and labor input decisions under uncertainty. The key features of the model are that the firm holds inventories to satisfy buffer stock motives and it engages in implicit multiperiod contracts with its workers that govern both temporary and permanent adjustments of the workforce. Our purpose is to explore the interaction of the firm's decisions in responding to cyclical fluctuations in demand and changes in costs.

The model is an attempt to integrate the main features of a variety of models of firm behavior in the existing literature. To place the analysis in perspective, it is useful to undertake a brief survey of the relevant literature. We begin with intertemporal models of inventory behavior which have been studied by a number of authors, including Blinder (1982), Maccini (1984), and Zabel (1972), among others. In these models, the firm makes price and output decisions before demand is revealed, and inventories are used as a buffer stock to absorb random shocks to demand. In these models, however, temporary layoffs are ignored as mechanism for absorbing random shocks to demand, and the factors that govern permanent adjustments in the workforce are not considered.

Analyses of the determination of the optimal workforce have been done by Mortensen (1970), Salop (1973), and Sargent (1978). The inertia that firms may experience in adjusting their workforce in these models is due to adjustment costs or to the nature of the monopsony power that firms possess in the labor market. While these models provide considerable insight into the determination of the optimal work force, they are models in which neither temporary layoffs nor inventory adjustments are made.

The interaction of inventories and employment decisions has been studied by Eichenbaum (1984), Nadiri and Rosen (1973) and Rossana (1984). These studies, however, suffer from the limitation that labor input decisions are summarized in an employment variable. Changes in employment are net of layoffs -- permanent and temporary -- and new hires. Focusing on changes in employment precludes an analysis of the different types of labor input decisions. This is especially important for the distinction between temporary and permanent layoffs, which have different motives, may respond differently to exogenous variables, and may interact differently with inventories.

Temporary layoffs have of course been analyzed in the implicit contract literature -- see, for example, Azariadis (1975), Baily (1977), Burdett and Mortensen (1980) etc. In these models, temporary layoffs serve as a mechanism for the firm to absorb random shocks to demand. These models, however, are one-period models in which intertemporal adjustments to the firm's workforce through permanent layoffs and new hires are abstracted from. Extensions of the implicit contract literature to allow for permanent as well as temporary adjustments to the firm's workforce have been made by Haltiwanger (1984) and Holmstrom (1983). In all these studies, however, inventories are ignored as a mechanism for absorbing demand shocks. Further, the analysis is done with models of individual competitive firms so that the implications of the structure of contracts for the price of output are not undertaken.

An analysis of the interaction between temporary layoffs and inventories has been done by ourselves in a recent paper, Haltiwanger and Maccini (1984), and by Topel (1982). Both papers, however, ignore the forces (multiperiod contracts, costs of adjusting the workforce, etc.) that give rise to the dynamics of permanent adjustments in the labor force so that a meaningful distinction betwen permanent and temporary layoffs cannot be made. Further, the two models focus on different forces governing inventory and temporary layoff behavior. In our model, inventories are held to satisfy buffer stock motives and inventory adjustments and layoffs are due to unanticipated (i.e., random) changes in demand. In Topel's model, on the other hand, inventories are held to satisfy speculative motives and both inventory adjustments and layoffs arise from perfectly anticipated fluctuations in demand. This work needs to be extended to allow both for an analysis of permanent as well as temporary layoffs and for an integration of the different forces causing inventory adjustments and temporary layoffs.

In this paper, we develop a model of a firm that makes decisions on a rich set of variables, including price, finished goods inventories, permanent and temporary layoffs, new hires, and output.[1] Further, the model permits an analysis of the response of these decisions to changes in the structure of demand, production conditions, inventory and turnover costs, and the characteristics of implicit contracts that govern employment arrangements between the firm and its workers. An analysis of a model of a firm with this breadth of decision-making has two advantages: First, it permits a study of the interaction of decision variables in responding to changes in exogenous forces that cannot be derived from the relatively narrow models of the existing literature. Secondly, it yields a wealth of predictions that can ultimately be subjected to an empirical analysis.

The paper proceeds as follows. In Section II, the optimization problem the firm faces is set down. We distinguish between variables that must be fixed prior to the realization of demand in each period (denoted ex ante variables) and those that are chosen contingent on the realization of demand (ex post variables). The former include the price, new hires and permanent

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[1]In developing the model, we abstract from an hours decision, but the model can easily be extended to allow for one. This is discussed below.

layoffs whereas the latter include inventories and temporary layoffs.

In Section III, we undertake an analysis of the interaction of the decision variables of the firm. We begin the analysis with the ex post decisions. We establish the conditions under which the firm will use temporary layoffs or inventories to buffer demand shocks, and analyze how this choice is affected by changes in exogenous and predetermined variables.

We then take up an analysis of the ex ante decisions. These consist of two types. The first is the employment decision. In this regard, we first study the conditions under which the firm will expand or contract its workforce in accordance with the structure of the implicit contracts it engages in with its workers. We then examine how the firm's ex ante decision on permanent layoffs is influenced by the fact that it can make temporary layoffs or hold inventories ex post.

The other type of ex ante decision we examine is the interaction between price and output. Since the firm sets its labor force ex ante but can make temporary layoffs ex post, output has both ex ante and ex post dimensions. We therefore look at the firm's decision on expected output which is the level of output the firm plans to produce ex ante, taking into account the production revisions it can make ex post through temporary layoffs. We then relate this decision to the firm's price decision, and compare our results with those that emerge from existing inventory models.

In a final section, we explore the implications of the model for inventory investment. In particular, we demonstrate that explicitly modelling the ability of the firm to make ex post production revisions through temporary layoffs helps justify and provide a new interpretation for existing empirical inventory investment equations.

II. The Model

A. Assumptions

We begin with a list of notation:

p_i = Price of output

q_i = Output

ε_i = Random component of demand

L_i = End of period experienced worker force

R_i = Permanent layoffs

N_i = Hires

u_i = Utilization rate of the attached work force

X_i = Temporary layoffs

ℓ_i = Total layoffs

C_i = Turnover costs

A_i = Hiring costs

W_i^n = Wage rate paid to new workers

W_i^o = Wage rate paid to experienced workers

${}_iY_{i+1}$ = Expected income promised for $i+1$, made at time i

ρ_i = Discount rate

K_i = *Ex post* opportunity cost of time

V_i = Market determined discounted income

Ω_i = Expected discounted income available elsewhere

S_i = Real sales

H_i = Inventory holding costs

Z_i = End-of-period inventories

The firm is assumed to formulate a plan for all i where $i = t, t+1, \ldots, t+T$ and where t is the initial date of the plan and T is the length of the horizon. In formulating the firm's decision problem it is helpful to distinguish between those variables chosen in each period that are fixed prior to the realization of demand in that period and those chosen contingent on realized demand. The former will be denoted as ex ante decision variables while the latter will be denoted as ex post decision variables. This differentiation implies a qualitative difference in the flexibility of alternative decisions. Following the buffer stock models of inventory behavior as well as the implicit contract models of labor behavior, it is assumed that both price and productive capacity for each period must be chosen and fixed ex ante. In contrast, inventories and temporary layoffs are chosen contingent on the ex post realization of demand and thus are the short run response mechanisms for the firm to absorb random fluctuations in demand.

We consider a firm that possesses monopoly power in the market for output. Its demand curve is

(1) $$m(p_i) + \varepsilon_i = \bar{m}_i - mp_i + \varepsilon_i, \quad \bar{m}_i > 0, \quad m > 0$$

where $m(p_i)$, defined over $\bar{p}_i > p_i > 0$, is riskless demand and ε_i is a random variable with probability density function, $f(\varepsilon_i)$, defined over $\underline{\varepsilon}_i < \varepsilon_i < \infty$ (with $-m(\bar{p}_i) < \underline{\varepsilon}_i < 0$) and $E[\varepsilon_i] = 0$.

The ex ante choice of productive capacity involves adjustments to the stock of attached workers. At the beginning of period i, the firm has inherited a stock of experienced workers from the previous period, L_{i-1}. The firm may increase or decrease this attached work force ex ante through permanent layoffs, R_i, or new hires, N_i, respectively. The stock adjustment process of the attached work force is described by:

(2) $$L_i = L_{i-1} + N_i - R_i$$

Temporary layoffs, as opposed to permanent layoffs and new hires, are chosen contingent on the ex post realization of ε_i. The probability of temporary layoffs, or the layoff rate, is $1-u_i$ where u_i is the ex post utilization rate of the attached work force $(u_i \leq 1)$. The number of temporary layoffs X_i, is the product of the layoff rate and the attached work force, i.e., $X_i = (1-u_i)L_i$. By definition, workers who are permanently laid off are unavailable for recall whereas temporarily laid off workers may be recalled. This distinction represents a difference in commitment on the firm's part between telling a laid off worker that the firm has no future plans that include the worker and telling a laid off worker that the firm expects to be able to recall the worker within a reasonable period of time. This distinction will be more apparent following the specification of the implicit contract constraints below.

The firm's production of output is governed by the following strictly concave production function:

$$q_i = g(u_i L_i), \quad g' > 0, \quad g'' < 0. \tag{3}$$

That is, output depends on labor services which are the product of the utilization rate and the attached work force in period i.

When the firm lays off workers it incurs turnover costs, which include unemployment insurance taxes and other separation costs. These are defined by

$$C_i = c_i \ell_i, \quad c_i > 0, \tag{4}$$

where ℓ_i is total layoffs including both permanent and temporary layoffs. Using previously stated definitions for permanent and temporary layoffs, total layoffs are given by $\ell_i = R_i + X_i$.

On the other hand, when new workers are hired, hiring and training costs are incurred. These are defined by

$$A_i = a_i N_i, \quad a_i > 0 \tag{5}$$

The training new workers receive is undertaken immediately when they are hired, and this training is assumed to raise the new workers to a level of productivity equal to that of workers retained from the previous period. Hence, newly trained workers and experienced workers are perfect substitutes in production.

The firm retains a workforce by engaging in multiperiod contracts with its workers.[2] It is assumed that the firm can contractually bind itself to its workers but that the workers cannot be so bound. Workers are assumed to be risk neutral. Hence, the contracts require that in each period the expected discounted income available from the contract with the firm equal or exceed the expected discounted income available elsewhere. These constraints imply that attached workers never have an incentive to quit.

For new workers in period i, the contract is assumed to take the following form:

$$E_i[u_i W_i^n + (1-u_i)K_i + {}_iY_{i+1}\,\rho_i] \geq V_i \tag{6}$$

where E_i is the expectational operator based on information known at the beginning of time i, W_i^n is the wage rate paid to new workers at time i, K_i is the ex post opportunity cost of a worker's time (which includes unemployment compensation together with the value of leisure time), ${}_iY_{i+1}$ is the expected income promised to workers in the future (where the promise is made

[2]The formulation of multiperiod contracts set down here is similar to that developed in Haltiwanger (1984). The motivation for the long term attachments and thus the long term contracts in this setting is based on the hiring, training and turnover costs facing firms as well as the mobility costs facing workers. Thus, the existence of long term contracts is not motivated by risk shifting considerations which are the focus of much of the implicit contract literature. It should be emphasized that the risk shifting motive and the motive for contracts considered here are not mutually exclusive. Further, it should be noted that there is a growing contract literature that emphasizes this latter motive (e.g., Baily (1977), Burdett and Mortensen (1980), Haltiwanger (1984), Hall and Lazear (1984)).

at time i) ρ_i is a discount factor and V_i is the market determined discounted income available elsewhere.[3]

Following this line of reasoning, a contract for experienced workers may be specified. At time i, the expected discounted income an experienced worker can expect to receive, ${}_{i-1}Y_i$, is predetermined, having been promised at time i-1. Note that in order to induce the worker to remain attached to the firm, ${}_{i-1}Y_i$ must have been chosen so that ${}_{i-1}Y_i \geq \Omega_i$. The contract constraint for experienced workers is thus given by:

$$(7) \qquad (1 - \frac{R_i}{L_{i-1}}) E_i[W_i^o u_i + (1-u_i)K_i + {}_iY_{i+1}\,\rho_i] + \frac{R_i}{L_{i-1}}\Omega_i] \geq {}_{i-1}Y_i, \quad {}_{i-1}Y_i \geq \Omega_i$$

where $\frac{R_i}{L_{i-1}}$ is the probability of being laid off permanently and Ω_i is the expected discounted income available elsewhere, net of search and mobility costs. Given that V_i is the market determined expected discounted income available elsewhere, this implies $\Omega_i < V_i$.

In this model, the firm need not sell its entire output ex post; it may hold inventories of finished goods. To capture this, we assume that the firm faces the following constraints on its behavior:

$$(8a) \qquad S_i \leq m(p_i) + \varepsilon_i$$

$$(8b) \qquad S_i \leq Z_{i-1} + g(u_i L_i)$$

The constraint, (8a), states that realized sales, S_i, can be no greater than demand, while (8b) states that realized sales can be no greater than "starting

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[3]To formulate this contract, we have used a convenient analytical device. In particular, instead of explicitly stipulating future wage rates and future temporary and permanent probabilities of layoffs to new workers, the firm simply promises that expected discounted income in the future will be at least equal to ${}_iY_{i+1}$.

stock", i.e., initial inventories, Z_{i-1}, plus production.

If the firm holds inventories, it will of course incur holding costs. These include both the financial costs, namely, the real interest charges, and the storage and insurance costs of holding inventories. The holding costs at time i, H_i, depend on end-of-period inventories, Z_i, and are defined by

(9) $$H_i = h_i Z_i, \quad h_i > 0$$

where $Z_i = Z_{i-1} + g(u_i L_i) - S_i \geq 0$.

Given the above assumptions it is useful to outline explicitly the firm's decision process. At time t, the firm has inherited from the past a stock of experienced workers, L_{t-1}, and a stock in inventories of finished goods, Z_{t-1}, and it has committed itself to an expected level of income for experienced workers, ${}_{t-1}Y_t$. Given these magnitudes, the firm makes decisions on the ex ante variables p_t, R_t and N_t so that these variables are fixed prior to the realization of ε_t. Simultaneously, the firm chooses u_t, S_t, W_t^o, W_t^n and ${}_tY_{t+1}$ contingent on ε_t. In addition, at time t, the firm must make decisions on future values of these same variables with the same ex ante, ex post distinction holding in each period.

To make these decisions, the firm is assumed to maximize the present value of expected profits, defined by

(10) $$E_t[\sum_{i=t}^{t+T} [p_i S_i - u_i(W_i^o(L_{i-1}-R_i) + W_i^n N_i) - c_i \ell_i - h_i Z_i - a_i N_i]\rho_t^{i-t}]$$

subject to (2), (6), (7), (8), and the definitions of ℓ_i and Z_i.[4] Profit in

[4]Note that as of the beginning of time t the firm (and the workers) do not know ε_i for $i = t,\ldots,t+T$. Hence, the maximization of expected profits is based on E_t. This implies that the firm and the workers at time t take expectations of equations (6) and (7) for $i = t+1,\ldots,t+T$. In other words, the relevant constraints for the maximization of (10) are (6) and (7) with expectations as of time t (E_t) taken on both sides of the equations.

each period is simply revenue less the wage bill, turnover costs, inventory-holding costs, and hiring costs. In stating (10), we have assumed for simplicity that real interest rates are expected to remain constant at current levels so that $\rho_i = \rho_t$.

To solve the model, it is useful to reduce its dimensions. In particular, it is easy to show that the contract constraints, (6) and (7), will be binding at an optimum. Hence, these constraints can then be used to eliminate W_i^o W_i^n, and ${}_iY_{i+1}$ from (10). The firm then maximizes

$$E_t[\sum_{i=t}^{t+T}[p_iS_i - c_i\ell_i - h_iZ_i - a_iN_i + L_i(1-u_i)K_i + R_i\Omega_i - N_iV_i]\rho^{i-t}] - L_{t-1\ t-1}Y_t \tag{11}$$

subject to (2), (8) and the definitions of ℓ_i and Z_i.

B. Optimality Conditions and Interpretation

After some work, the optimality conditions for this problem reduce to (for $t \leq i < t+T$):[5]

$$p_i + h_i - \alpha_i - \theta_i - \Gamma_{i+1}\rho_t = 0 \tag{12}$$

$$(\alpha_i - h_i + \Gamma_{i+1}\rho_t)\ g'(u_iL_i) = K_i - c_i + \lambda_i \tag{13}$$

$$E_i[\sum_{k=i}^{t+T}(\alpha_k - h_k + \Gamma_{k+1}\rho_t)g_k'\rho_t^{k-i}] = \Omega_i - c_i + \delta_{1i} \tag{14}$$

[5]Note that for period i, the ex ante variables are chosen contingent on all ε_{i-k}, $k = 1,\ldots,i-t$, and the ex post variables are chosen contingent on ε_{i-j}, $j = 0,\ldots,j-t$. The optimality conditions for period $t+T$ are identical to (12)-(20) except for the omission of Γ_{i+1} from all of the equations in which it appears.

(15) $$E_i[\sum_{k=i}^{t+T} (\alpha_k - h_k + \Gamma_{k+1}\rho_t)\, g'_k \rho_t^{k-i}] = V_i + a_i - \delta_{2i}$$

(16) $$E_i[S_i - \theta_i m] = 0$$

(17) $$\alpha_i(Z_{i-1} + g_i - S_i) = 0, \quad \alpha_i \geq 0$$

(18) $$\theta_i(m(p_i) + \varepsilon_i - S_i) = 0, \quad \theta_i \geq 0$$

(19) $$\lambda_i(1-u_i) = 0, \quad \lambda_i \geq 0$$

(20a) $$\delta_{1i}R_i = 0, \quad \delta_i \geq 0$$

(20b) $$\delta_{2i}N_i = 0, \quad \delta_{2i} \geq 0$$

where $\Gamma_{i+1} = E_{i+1}[\sum_{k=i+1}^{t+T} (\alpha_k - h_k)\rho_t^{k-i-1}]$, α_i, θ_i, λ_i, δ_{1i} and δ_{2i} are the Kuhn-Tucker multipliers associated with the inequality constraints, and for ease of notation g'_k denotes $g'(u_k L_k)$. The multipliers have the following interpretation: α_i is the shadow value of an additional unit of starting stock; θ_i is the shadow value of an additional unit of demand; λ_i is the shadow price of having an additional worker available ex post; $\delta_{1i}(\delta_{2i})$ is the shadow value of having one more (one less) experienced worker available ex ante. Given the interpretation of these multipliers, observe that the term $\rho_t\Gamma_{i+1}$ is the future discounted marginal value of holding an additional unit of inventories. An interpretation of the optimality conditions will be given below as the analysis proceeds.

III. An Analysis of the Model

A. Some Basic Results

The model of the firm we have set down is quite complex. It is an intertemporal model with uncertainty and two state variables (inventories and the attached work force). To make the model analytically tractable, we assume that the firm possesses a two-period planning horizon.[6] This substantially simplifies the mathematics, and enables us to capture virtually all the interesting implications of the model. Furthermore, in the analysis we will focus on the decisions for the initial date of the plan, $i = t$, since these are the decisions the firm will actually implement.

At the same time, the model is really quite rich. The firm makes decisions on new hires, permanent layoffs, the rate it utilizes its workforce and thus temporary layoffs, inventories, price, and output. It has inherited an attached workforce and an initial stock of inventories, and it faces a variety of exogenous variables, including current and future anticipated levels of demand, real interest rates, turnover costs, inventory holding costs, unemployment insurance benefits, market-determined contract values, etc. To avoid a lengthy and tedious taxonomy of results, we will be selective in presenting results. In each subsequent section, we will present only the results that are new or extend or alter those in the existing literature on inventories and labor input decisions. Other results will be referred to briefly in footnotes or left to the reader.

We begin with a definition. Define $\varepsilon_t > b_t$ where $b_t = Z_{t-1} + g(L_t) - m(p_t)$ to be a state of "excess demand". To understand this definition, observe that, using the definition of b_t, $\varepsilon_t > b_t$ implies $m(p_t) + \varepsilon_t > Z_{t-1} + g(L_t)$. Hence, in this case, the firm faces a state of excess demand in the sense that the realized level of demand exceeds its potential starting stock, i.e., the maximum amount of goods the firm will have for sale this

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[6]We note that other methods for making multiperiod models under uncertainty tractable do not work well here. For example, because of the nature of the constraints, especially the contract constraints, quadratic criterion-linear constraint procedures cannot be used.

period. Similarly, $\varepsilon_t < b_t$ denotes a state of "excess supply".

The following proposition begins a characterization of the firm's ex post decisions on inventories and temporary layoffs in the current period (note that proofs of all propositions are provided in the Appendix).

Proposition 1: Assume $\varepsilon_t \geq b_t$. Then,

(i) if $\Omega_t - K_t - \rho_t(V_{t+1}+a_{t+1}) > 0$, then $X_t = 0$

(ii) if $p_t > \rho_t \Gamma_{t+1} - h_t$, then $Z_t = 0$

Alternatively, assume $\varepsilon_t < b_t$. Then, either $X_t > 0$ or $Z_t > 0$, or both.

This proposition essentially states that, if the firm finds itself in a state of excess demand $(\varepsilon_t \geq b_t)$, then, under the assumed conditions, optimal behavior dictates that it fully utilize its labor force so that no temporary layoffs are made $(X_t = 0)$ and deplete its holdings of inventories $(Z_t = 0)$. Alternatively, when faced with a state of excess supply $(\varepsilon_t < b_t)$, the firm will find it optimal to either make temporary layoffs $(X_t > 0)$ or to hold inventories $(Z_t > 0)$, or both.

A word is in order regarding the conditions (i) and (ii) of Proposition 1. These conditions essentially preclude the firm from using temporary layoffs to hoard labor and from engaging in pure speculation on inventories.

Consider first condition (i) of Proposition 1. The term $\rho_t(V_{t+1}+a_{t+1})$ represents the present value of the cost of hiring a worker next period, including both the contractual and training costs. The term $\Omega_t - K_t$ is the income available elsewhere to a worker in this period. When condition (i) is reversed so that $\rho_t(V_{t+1}+a_{t+1}) > \Omega_t - K_t$, the firm may have an incentive to retain workers to avoid the expected future cost of adding to its workforce, and workers have relatively little incentive to desert the firm. Consequently, the firm may make temporary layoffs this period no matter how high the level of demand. In this sense, it is engaging in a form of labor hoarding.[7] In what follows, we rule out this form of labor hoarding, because we want to focus on conditions where the firm has an incentive to fully utilize its workforce at least for some states of demand.

Next, define pure speculative inventory accumulation to be a situation in which the firm holds inventories, $Z_t > 0$, even though demand is relatively high, $\varepsilon_t > b_t$. For this to occur in our model, the discounted future value of an additional unit of inventories, net of the holding costs, must at least equal the current price, i.e., $\rho_t \Gamma_{t+1} - h_t \geq p_t$, which is the reversal of condition (ii) of Proposition 1. In addition, in our model it can be demonstrated that pure speculative inventory accumulation requires that no temporary layoffs be made this period, i.e., $X_t = 0$. In effect, if it is profitable to speculate on inventory accumulation, it is profitable for the firm to fully utilize its labor force this period to produce output and add to inventories. These results, together with Proposition 1, suggest that inventory accumulation and temporary layoffs interact or are "substitutes" only when inventories are being used to buffer low states of demand. Hence, in a slump when temporary layoffs are being made, any inventory accumulation that takes place must be for buffer stock rather than speculative purposes. In what follows, we are primarily interested in situations where inventory accumulation and temporary layoffs can occur simultaneously. Accordingly, we assume that condition (ii) of Proposition 1 prevails and assume away pure

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[7]This form of labor hoarding differs from that in the conventional literature, e.g., Solow (1968). In the latter, labor hoarding consists of the excess employment of labor to have them available in the future. In our model, if condition (i) is reversed, labor hoarding consists of the excess attachment of workers to the firm for future availability and the attachment may take the form of employment or temporary layoffs.

speculative inventory accumulation.[8]

The next task is to look more closely at a situation where the firm is faced with a state of excess supply. Proposition 1 states that under these circumstances the firm will either hold inventories or make temporary layoffs or both. To determine just what it will do, it is necessary to consider different types of firms.

Proposition 2: Assume $\varepsilon_t < b_t$ and consider a small neighborhood of b_t. Then,

(a) if $(\rho_t \Gamma^o_{t+1} - h_t)g'(L_t) > K_t - c_t$, then $Z_t > 0$ but $X_t = 0$, and the firm is inventory-biased;

(b) if $(\rho_t \Gamma^o_{t+1} - h_t)g'(L_t) < K_t - c_t$, then $X_t > 0$ but $Z_t = 0$, and the firm is layoff-biased;

(c) if $(\rho_t \Gamma^o_{t+1} - h_t)g'(L_t) = K_t - c_t$, then both $X_t > 0$ and $Z_t > 0$, and the firm is unbiased

where the superscript, "o", denotes that Γ_{t+1} is evaluated at the point where $\varepsilon_t = b_t$ or $Z_t = 0$ and $X_t = 0$.

The term $(\rho_t \Gamma^o_{t+1} - h_t)g'(L_t)$ is the net discounted marginal value of using an additional unit of labor services to produce output and add to inventories, evaluated at the point where realized demand is just about to dip below potential starting stock. The term $K_t - c_t$ is the net subsidy to layoffs. This proposition indicates that if the former exceeds (is less than) the latter, then the firm is inventory-biased (layoff-biased) in that its response to a "small" decline in demand is to use inventories (temporary

[8]This contrasts with Topel (1982), who emphasizes speculative motives for holding inventories in his model. As a result, in his model, the firm will only begin making temporary layoffs when inventory stocks are completely exhausted.

layoffs) alone to absorb the shock. On the other hand, when $(\rho_t \Gamma^o_{t+1} - h_t) g'(L_t) = K_t - c_t$, then the firm is unbiased in that it will use both inventories and layoffs to absorb the shock even for small declines in demand.

Our previous paper, Haltiwanger and Maccini [1984], was devoted to a detailed analysis of the differences between inventory-biased and layoff-biased firms. There we showed that under general conditions each type of firm will eventually use both inventories and temporary layoffs to buffer demand shocks, provided the decline in demand is large enough. In this paper, we assume that the firm is unbiased which means that both inventory accumulation and temporary layoffs are used to buffer even small declines in demand. This assumption greatly simplifies and streamlines the analysis without jeopardizing the main results we intend to establish in this paper.

B. Ex Post Decisions

At this point, it is useful to explore more fully the firm's ex post decisions. In any period, once the random variable is revealed, the firm must make decisions on u_t, given the decisions it has made ex ante on p_t, R_t and N_t. Further, once u_t is set, X_t and Z_t are determined. Our objective here is to characterize the interaction between temporary layoffs and inventory accumulation as the firm responds to the revelation of demand.

To do this, we consider an unbiased firm that is faced with excess supply ($\varepsilon_t < b_t$) so that $u_t < 1$, $X_t > 0$ and $Z_t > 0$. In this case, (13) reduces to

(21) $$(\rho_t \Gamma_{t+1} - h_t) g'(u_t L_t) = K_t - c_t$$

where

(22a) $$\Gamma_{t+1} = \Gamma(Z_t, \bar{m}_{t+1}, \ldots)$$

(22b) $$Z_t = Z_{t-1} + g(u_t L_t) - \bar{m}_t + mp_t - \varepsilon_t$$

Observe that $\rho_t \Gamma_{t+1} - h_t$ is the marginal value, net of holding cost, to carrying a unit of inventories into the next period. Hence (21) requires that the firm set u_t, its employment rate, at the point at which the marginal value of using an additional worker to produce output to add to inventories is equated to the net subsidy to making layoffs. Given u_t, Z_t is determined by (22b) and $X_t = (1-u_t)L_t$. The basic responses of the firm are contained in the following proposition.

Proposition 3: Suppose $\varepsilon_t < b_t$, and the firm is unbiased. Then $X_t > 0$ and $Z_t > 0$ and for given p_t, R_t, and N_t we have:

(i)	$\partial X_t/\partial \varepsilon_t < 0$	(ii)	$-1 < \partial Z_t/\partial \varepsilon_t < 0$
(iii)	$\partial X_t/\partial Z_{t-1} > 0$	(iv)	$1 > \partial Z_t/\partial Z_{t-1} > 0$
(v)	$\partial X_t/\partial \bar{m}_t < 0$	(vi)	$-1 < \partial Z_t/\partial \bar{m}_t < 0$
(vii)	$\partial X_t/\partial \bar{m}_{t+1} < 0$	(viii)	$\partial Z_t/\partial \bar{m}_{t+1} > 0$
(ix)	$\partial X_t/\partial h_t > 0$	(x)	$\partial Z_t/\partial h_t < 0$
(xi)	$\partial X_t/\partial K_t > 0$	(xii)	$\partial Z_t/\partial K_t < 0$
(xiii)	$\partial X_t/\partial c_t < 0$	(xiv)	$\partial Z_t/\partial c_t > 0$

Consider first the response of temporary layoffs and inventories to the size of the demand shock. The more severe the decline in demand, ε_t, the greater the degree of inventory accumulation and temporary layoffs. However, in contrast to standard inventory models which do not allow for temporary layoffs, a unit decline in demand in general gives rise to an increase in inventories of less than a unit. Further, higher initial inventories, or a downward shift in the firm's current demand curve (i.e., a lower $\bar{m}_t$), will raise both temporary layoffs and end-of-period inventories. Higher future anticipated demand (represented by an increase in $\bar{m}_{t+1}$), on the other hand, will reduce temporary layoffs but increase inventories. Since the latter relationship is the opposite of the relationship between inventories and current anticipated demand, studies of inventory behavior that lump together current and future demand effects are subject to misspecification.

Perhaps the most interesting results from Proposition 3 concern the effects of changes in the cost structure. An increase in marginal inventory holding costs leads to a decline in inventory accumulation and an increase in temporary layoffs. In contrast, a decline in the net subsidy to layoffs (either a decrease in K_t or an increase in c_t) leads to a decrease in temporary layoffs and an increase in inventories. These results capture substitution possibilities in response to parametric changes in costs that are completely ignored in standard inventory and layoff models. They indicate that a proper characterization of inventory and temporary layoff behavior involves relative costs rather than just absolute direct costs. Firms, for example, with low storage costs may not be prone to accumulating inventories if they face high net subsidies to making layoffs. To the extent that inventories and temporary layoffs are close substitutes in the manner characterized here, empirical specifications of inventory (temporary layoff) behavior that do not include costs of temporary layoffs (inventories) involve the omission of important explanatory variables.

Further insight into ex post decisions is gained by investigating the response of ex post production to changes in unanticipated demand. Using (21) and differentiating q_t appropriately, we have that

$$(23)\quad \frac{\partial q_t}{\partial \varepsilon_t} = g_t' L_t \frac{\partial u_t}{\partial \varepsilon_t} = -g_t' \frac{\partial X_t}{\partial \varepsilon_t} = \frac{\rho_t (g_t')^2 \Gamma_Z}{(\rho_t \Gamma_{t+1} - h_t) g''_t + \rho_t (g_t')^2 \Gamma_Z} = \omega_t \qquad 0 \leq \omega_t \leq 1$$

The term, ω_t, thus measures the fraction of a unit decline in demand that is absorbed by reducing output through making temporary layoffs (i.e., reducing u_t). Further, using (23) it follows that:

$$\frac{\partial Z_t}{\partial \varepsilon_t} = g_t' L_t \frac{\partial u_t}{\partial \varepsilon_t} - 1 = \omega_t - 1 \tag{24}$$

Hence, $1 - \omega_t$ is the fraction of an unanticipated decline in demand absorbed by inventory accumulation.

Using (21), ω_t may be rewritten as:

$$\omega_t = \frac{1}{1 + \dfrac{g_t''}{\rho_t \Gamma_Z} \dfrac{(\rho_t \Gamma_{t+1} - h_t)^3}{(K_t - c_t)^2}}$$

This expression reveals that ω_t will be closer to unity, so that the firm will use temporary layoffs and production rather than inventories to absorb demand shocks, (a) the smaller is the incentive to smooth production (the smaller is g_t''), (b) the smaller is the net marginal value to accumulating inventories $(\rho_t \Gamma_{t+1} - h_t)$, (c) the larger is the net subsidy to making layoffs $(K_t - c_t)$, and (d) the faster the marginal value to accumulating inventories (Γ_Z) declines. The magnitude of ω_t will play a prominent role in the subsequent analysis.

An intertemporal model under uncertainty allows us to consider the interaction of inventories and layoffs in light of the signal extraction problem faced by the firm of deciding whether a given change in demand is temporary or permanent. In order to distinguish between temporary and permanent demand shocks, we now consider the following simple AR(1) process for the stochastic structure:

$$\varepsilon_{t+1} = \mu \varepsilon_t + \eta_{t+1}; \quad 0 \leq \mu \leq 1$$

where η_{t+1} is a white noise disturbance term. With this notation, we can now state the following proposition.

<u>Proposition 4</u>: For negative demand shocks (i.e., $\varepsilon_t < 0$), the more persistent the shocks are expected to be (i.e., as μ rises), the more likely the firm will use temporary layoffs and the less likely the firm will accumulate inventories in response to these shocks (for given p_t, R_t and N_t).

The underlying intuition for this proposition is as follows. The more persistent a negative demand shock is expected to be, the lower expected future demand. The lower expected future demand, the lower will be the future net value of carrying over an additional unit of inventories into next period. This implies that the greater the persistence of a negative demand shock, the higher the net cost of using inventories to respond to the shock. Thus, the greater the persistence, the more likely the firm will use temporary layoffs instead of inventories in response to demand shock.

This proposition in combination with the previous analysis helps explain the following stylized pattern of behavior by firms over the cycle. Given the signal extraction problem facing the firm with regard to whether shocks to demand are temporary or permanent, at the onset of a prolonged slump the firm is likely to misperceive that the fall in demand is widespread. This suggests that the firm's perception of μ is likely to be low in the initial stages of the slump. Hence, according to Proposition 4, the firm will initially use inventories to absorb negative shocks to demand. However, in subsequent periods as the firm becomes aware of the extent of the slump μ will rise and the firm will switch to temporary layoffs to absorb shocks to demand. Further, it can be shown that there is a tendency for ω_t to increase as Z_{t-1} increases.[9]

[9]For given p_t and L_t, $\partial\omega_t/\partial Z_{t-1} > 0$ as long as Γ_Z and g_t'' are approximately constants.

This implies that intensive inventory accumulation in one period will lead to more intensive temporary layoffs in subsequent periods. Thus, the analysis suggests that we are likely to see firms switch from inventories to temporary layoffs as a business cycle slump proceeds both because awareness of the extent of the slump is likely to increase over time and higher initial inventories leads to the firm becoming more temporary layoff intensive.

This result is apt to be of considerable importance for empirical work. Specifically, this proposition suggests that we ought to observe inventories being relatively more responsive to temporary shocks and temporary layoffs more responsive to permanent shocks.

C. Ex Ante Employment Decisions

We now turn to an analysis of the firm's ex ante employment decisions in terms of permanent layoffs and new hires. We are particularly interested in how these decisions interact with the ex post decision variables temporary layoffs and inventories. Ex ante, there are three key variables determined, p_t, N_t and R_t. One can solve the problem backwards recursively yielding the following three equations determining p_t, R_t and N_t respectively:

$$m(p_t) - D(b_t) - mp_tF(b_t) + m\int_{\underline{\varepsilon}_t}^{b_t} [\rho_t\Gamma^*_{t+1}-h_t]dF_t = 0 \tag{25}$$

$$p_tg'(L_t)(1-F(b_t)) + \int_{\underline{\varepsilon}_t}^{b_t} (\Gamma^*_{t+1}\rho_t-h_t)g'(u^*_tL_t)dF_t + \int_{\underline{\varepsilon}_t}^{b_t} \Sigma^*_{t+1}\rho_tdF_t = \Omega_t-c_t \tag{26}$$

if $R_t > 0$.

$$p_tg'(L_t)(1-F(b_t)) + \int_{\underline{\varepsilon}_t}^{b_t}(\Gamma^*_{t+1}\rho_t-h_t)g'(u^*_tL_t)dF_t + \int_{\underline{\varepsilon}_t}^{b_t} \Sigma^*_{t+1}\rho_tdF_t = V_t + a_t \tag{27}$$

if $N_t > 0$

where $\Sigma^*_{t+1} = p^*_{t+1}g'(L^*_{t+1})(1-F(b^*_{t+1})) + (K_{t+1}-c_{t+1})F(b^*_{t+1})$, $D(b_t) = \int_{b_t}^{\infty} [\varepsilon_t-b_t]dF_t$ is expected stockouts and the asterisk (*) denotes the optimized value contingent on p_t, R_t and N_t. Condition (25) requires that the expected marginal revenue from selling a unit of output this period must be equated to the expected marginal net revenue of holding it in inventory and selling it next period. Observe that the LHS of (26) and (27) are identical. The left-hand side of each of these conditions is the expected marginal revenue product of an additional attached worker. Each equation thus equates this expected marginal revenue product to the opportunity cost of an additional attached worker. When the firm is making permanent layoffs, $\Omega_t - c_t$ is the relevant opportunity cost, and when the firm is hiring new workers $V_t + a_t$ is the relevant opportunity cost.

A diagrammatic analysis of the conditions governing ex ante employment decisions yields considerable insight. We denote the expected marginal revenue product of adding another worker by Σ_t. Taking price, p_t, as given for the moment, Σ_t can be expressed as a function of L_t. Differentiating Σ_t appropriately, we have that

$$(28)\qquad \frac{\partial \Sigma_t}{\partial L_t} = p_t g''(L_t)(1-F(b_t)) - f(b_t)(p_t - \Gamma^*_{t+1}\rho_t + h_t)(g'(L_t))^2$$

$$+ \rho_t \int_{\underline{\varepsilon}_t}^{\infty} \left(\frac{\partial \Sigma^*_{t+1}}{\partial Z_t} \frac{\partial Z^*_t}{\partial L_t} + \frac{\partial \Sigma^*_{t+1}}{\partial L_t}\right) dF_t < 0$$

Hence, Σ_t is a decreasing function of L_t; it is plotted in Figure I. Figure I also pictures the relevant opportunity costs, which under the assumptions we have made thus far, are horizontal lines. Further, since $V_t + a_t > \Omega_t - c$, the former lies above the latter.

We see from Figure I that the firm's ex ante employment decision is determined by the position of Σ_t. If Σ_t passes through $\Omega_t - c_t$ to the left of L_{t-1}, as Σ^1_t does, then the firm will make permanent layoffs in period t. Alternatively, if Σ_t passes through $V_t + a_t$ to the right of

L_{t-1}, as Σ_t^2 does, then the firm will make new hires in period t. Thus, one obvious insight from Figure I is that the firm will not simultaneously hire or permanently layoff workers. Of more interest, however, is that there is a range over which the firm will make neither permanent layoffs nor new hires. That is, if Σ_t passes through the vertical dotted line between $V_t + a_t$ and $\Omega_t - c_t$, as Σ_t^3 does, then the attached workforce will not be changed in period t. Further, variables which shift Σ_t, e.g., changes in current and future levels of anticipated demand, initial inventories, etc., will have no effect on the attached workforce as long as Σ_t continues to cut opportunity costs in the range between $\Omega_t - c_t$ and $V_t + a_t$. The key insight is then that the higher are hiring costs $(V_t + a_t)$, and turnover costs (c_t), and the lower is the opportunity cost of experienced workers (Ω_t), the more "sticky" will be the firm's attached workforce.

These results extend those of conventional models of the dynamics of the firm's workforce — e.g., Sargent (1978). First, a stickiness in the firm's workforce emerges naturally in our model through the structure of implicit contracts which create a discontinuity in the opportunity cost of an additional worker rather than through the more common arbitrary assumption of convex adjustment costs. Second, asymmetries in the response of permanent layoffs and new hires to changes in exogenous variables arise readily in our framework. Consider a straightforward generalization of our model in which marginal hiring costs, a_t, rise with N_t but marginal turnover costs remain constant.[10] In this case, the upper branch of the opportunity cost schedule

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[10]Alternatively, suppose c_t rises with R_t but a_t rises faster than c_t. This would give the same asymmetry. Further, differences of this sort in the rates at which marginal hiring and turnover costs rise are quite plausible in that it is difficult to rationalize rising marginal turnover costs. The latter are mainly unemployment insurance taxes which are essentially constant at the margin.

will be upward-sloping, as the dashed line in Figure I indicates. Consequently, as is readily apparent from the diagram, permanent layoffs will be more sensitive to forces, e.g., shifts in anticipated demand, that shift Σ_t than new hires.

We now draw particular attention to differences between permanent layoffs, temporary layoffs and total layoffs. Total layoffs, looked at from an ex ante point of view, is formalized as expected total layoffs, defined by $\bar{\ell}_t = E_t[\ell_t] = R_t + E_t(X_t)$. Expected total layoffs is thus permanent layoffs plus the temporary layoffs the firm can expect to make once demand is revealed. We collect the main results in a proposition.

Proposition 5: Assume that expected profits are strictly concave in p_t and R_t. Assume further that the firm is contracting its workforce so that $R_t > 0$ and $R_{t+1} > 0$.[11] Then

$$\frac{\partial R_t}{\partial Z_{t-1}} > 0 \qquad \frac{\partial R_t}{\partial \bar{m}_t} < 0 \qquad \frac{\partial R_t}{\partial K_t} < 0 \qquad \frac{\partial R_t}{\partial \Omega_t} > 0 \qquad \frac{\partial R_t}{\partial c_t} < 0$$

$$\frac{\partial \bar{\ell}_t}{\partial Z_{t-1}} > 0 \qquad \frac{\partial \bar{\ell}_t}{\partial \bar{m}_t} < 0 \qquad \frac{\partial \bar{\ell}_t}{\partial K_t} \gtreqless 0 \qquad \frac{\partial \bar{\ell}_t}{\partial \Omega_t} > 0 \qquad \frac{\partial \bar{\ell}_t}{\partial c_t} > 0$$

Qualitatively, some of these results are intuitively straightforward. Both permanent layoffs and total layoffs are positively related to initial inventories, inversely related to the anticipated current level of demand,

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[11]A similar analysis may be conducted for an expanding firm in which $N_t > 0$ and $N_{t+1} > 0$. The responses for N_t are precisely the opposite as those for R_t, the only exception being $\frac{\partial N_t}{\partial c_t}$ which is negative. Furthermore, $E(X_t) > 0$ even if $N_t > 0$ since even an expanding firm may experience unanticipated low realizations of demand. Hence, the predictions for expected layoffs are similar to those above.

positively related to the expected discounted income available elsewhere.

Several of the qualitative results, however, are worth highlighting. First, a surprising result is that permanent layoffs are inversely related to the ex post opportunity cost of a worker's time. This result is of particular interest in light of the predictions from implicit contract models that do not distinguish between temporary and permanent layoffs. The latter predict a positive relationship between the ex post opportunity cost of a worker's time (in particular, unemployment benefits) and layoffs (imprecisely defined). In Proposition 3, we demonstrated that such a positive relationship exists between unemployment benefits and temporary layoffs. Proposition 5 suggests that the opposite relationship may hold for permanent layoffs. The intuition is that an increase in K_t makes temporary layoffs less costly. As a result, the firm can attach a larger workforce (by reducing R_t) so that it is able to satisfy high realizations of demand.[12] The conflicting results on tmporary and permanent layoffs imply that total layoffs are ambiguous with respect to changes in unemployment benefits. These results highlight the importance of distinguishing between temporary and permanent layoffs for both theoretical and empirical purposes.

A second result of interest in Proposition 5 is that permanent layoffs are inversely related to marginal turnover costs, c_t. This is of interest because, as we point out in footnote 10, new hires are also inversely related to c_t. These results together of course mean that higher marginal turnover costs, e.g., unemployment insurance taxes, tend to give rise to a "sticky"

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[12] This result holds for a given ex ante opportunity cost of a worker's time and thus may be affected if the ex ante market value of a worker's time is affected (which it most likely will be) by unemployment benefits. Nevertheless, this result suggests that unemployment benefits are much more important for temporary layoffs.

attached labor force.

Finally, let us return for a moment to the case of serially correlated shocks. Then, the result that a decrease in anticipated demand results in an increase in permanent layoffs can be used to shed light on the effect of serially correlated shocks to demand on permanent layoffs. Recall that in Proposition 4 we demonstrated that in response to a shock in demand that is expected to persist, the firm is likely to use temporary layoffs as opposed to inventories to buffer the shock initially. With serially correlated shocks, last period's negative shock translates into a lower anticipated demand in the current period. That is, with serially correlated shocks, expected demand in the current period (using the notation from Section III.B) is given by $\bar{m}_t - mp_t + \mu\varepsilon_{t-1}$. This indicates that a change in $\mu\varepsilon_{t-1}$ is analytically like a change in $\bar{m}_t$ in terms of the effects on permanent layoffs. Hence, last period's negative shock to demand will result in greater permanent layoffs in the current period the greater is μ. Overall, we see that a shock to demand that is expected to persist will initially be absorbed by temporary layoffs (since they can be adjusted quickly) but will eventually result in greater permanent layoffs.

To understand the interaction of permanent layoffs, temporary layoffs, and inventories, however, it is helpful to look in more detail at the quantitative responses. Consider first the response of expected total layoffs to changes in demand conditions, in particular, a change in current anticipated demand. This is given by:

$$\frac{\partial \bar{\ell}_t}{\partial \bar{m}_t} = (1 - F(b_t)) \frac{\partial R_t}{\partial \bar{m}_t} + \int_{\underline{\varepsilon}_t}^{b_t} \frac{\omega_t}{g_t'} [\frac{m \partial p_t}{\partial \bar{m}_t} - 1] dF_t < 0 \tag{29}$$

The term $(\frac{m\partial p_t}{\partial \bar{m}_t} - 1)$ represents the response of expected sales to a change in $\bar{m}_t$ and is negative. Hence, (29) implies that the smaller is ω_t, the greater is the tendency of the firm to use inventories rather than temporary layoffs to absorb demand shocks, the smaller in magnitude will be the response of total layoffs to a change in demand conditions. This indicates that considering the possibility of inventory holding behavior by the firm weakens the response of layoffs to changes in demand conditions in comparison with models that focus solely on layoffs (e.g., Haltiwanger (1984)) or employment (e.g., Mortensen (1970), Salop (1973), Sargent (1978)). Empirically, industries characterized by small ω_t's, e.g., due to a high ratio of the value of accumulating inventories to the net subsidy to making layoffs, can be expected to be prone to experiencing fewer layoffs.

Alternatively, consider the impact of inventories on the response of expected total layoffs to a change in cost conditions; for instance, consider a change in turnover costs, c_t. This is given by:

$$\frac{\partial \bar{\ell}_t}{\partial c_t} = (1-F(b_t))\frac{\partial R_t}{\partial c_t} + \int_{\underline{\varepsilon}_t}^{b_t} \frac{\omega_t}{g_t'}\left[\frac{\partial p_t}{\partial c_t} m + \frac{1}{\Gamma_Z \rho_t (g_t')}\right] dF_t \tag{30}$$

It can be shown that $\left(\frac{\partial p_t}{\partial c_t} m + \frac{1}{\Gamma_Z \rho_t g_t'}\right) < 0$. This implies that the lower is ω_t, the smaller will be the magnitude of the response of total layoffs to a change in turnover costs. Thus, there is a tendency for the response of total layoffs to be less responsive to changes in cost conditions than would be expected in a typical layoff model that neglects inventories.

The overall tendency that emerges is that the ability to accumulate inventories dampens the initial impact of changes in economic conditions on layoffs. This makes intuitive sense because inventories absorb some of the changes in economic conditions that would otherwise be borne by layoffs. However, an increase in inventories in a particular period induces higher initial inventories in subsequent periods. Further, an increase in initial inventories in a particular period results in greater layoffs. That is,[13]

$$(31) \qquad \frac{\partial \bar{\ell}_t}{\partial Z_{t-1}} = (1-F(b_t))\frac{\partial R_t}{\partial Z_{t-1}} + \int_{\underline{\varepsilon}_t}^{b_t} \frac{\omega_t}{g_t'}\left[m\frac{\partial p_t}{\partial Z_{t-1}} + 1\right]dF_t > 0$$

Thus, a more complete description of the influence of inventories might be that there is a tendency for the effect on layoffs from changes in economic conditions to be dampened initially but to be spread out over time. This interpretation is particularly interesting given the recent empirical findings that persistence in aggregate cyclical unemployment appears to be partially associated with inventory behavior (see Darby, Haltiwanger and Plant (1985)). Our model provides solid theoretical underpinnings for these findings.

D. Price and Output Decisions

In work with intertemporal inventory models, a common objective is to investigate the responses of price and output to changes in predetermined and exogenous variables. See in particular the work by Blinder [1982], Maccini [1984], and Zabel [1972]. In the standard model, price and output in each period are set ex ante, i.e., before the random variable for that period is realized. In our model, on the other hand, although price is set ex ante each period, output has both ex ante and ex post dimensions.

In particular, output is determined by labor services which are the product of the attached labor force and the utilization rate for the period. The firm commits itself to an attached labor force ex ante, but because

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[13]Note that since ω_t tends to be an increasing function of Z_{t-1} (see Section III.B), this implies that the magnitude of (31) tends to be greater as Z_{t-1} increases.

temporary layoffs are permitted the firm may vary its utilization rate ex post. This mixture of ex ante and ex post factors in the output decision gives rise to interesting implications for price and output responses. In this section, we proceed to draw out these implications, and compare them with those in the existing inventory literature.

To conduct this analysis, it is useful to define expected output to the firm, $\bar{q}_t$, as

(32)
$$\bar{q}_t = \int_{\underline{\varepsilon}_t}^{\infty} q_t f(\varepsilon_t) d\varepsilon_t = \int_{\underline{\varepsilon}_t}^{\infty} g(u_t L_t)\ f(\varepsilon_t) d\varepsilon_t$$

In effect, $\bar{q}_t$ is the level of output the firm plans to produce ex ante, taking into account the possible ex post realizations of demand.

The following proposition collects the basic price and output responses for the firm. The results are a consequence of an analysis of the optimality conditions (25) or (26) and (28) together with (32).

<u>Proposition 6</u>: Assume that expected profits are concave in p_t and R_t, and that the firm is contracting its workforce so that $R_t > 0$ and $R_{t+1} > 0$.[14] Then

$$\frac{\partial p_t}{\partial Z_{t-1}} < 0 \qquad \frac{\partial p_t}{\partial \bar{m}_t} > 0 \qquad \frac{\partial p_t}{\partial \rho_t} \gtrless 0 \qquad \frac{\partial p_t}{\partial \Omega_t} > 0 \qquad \frac{\partial p_t}{\partial K_t} \gtreqless 0$$

$$\frac{\partial \bar{q}_t}{\partial Z_{t-1}} < 0 \qquad \frac{\partial \bar{q}_t}{\partial \bar{m}_t} > 0 \qquad \frac{\partial \bar{q}_t}{\partial \rho_t} > 0 \qquad \frac{\partial \bar{q}_t}{\partial \Omega_t} < 0 \qquad \frac{\partial \bar{q}_t}{\partial K_t} \lesseqgtr 0$$

[14] Analogous results can be obtained for an expanding firm where $N_t > 0$ and $N_{t+1} > 0$.

Qualitatively, some of these results correspond to those that exist in the inventory literature. Specifically lower initial inventories or a higher current level of anticipated demand (measured by $\bar{m}_t$) both raise price and expected output. A higher real interest rate (i.e., a lower ρ_t) lowers expected output, but has an ambiguous effect on price. New results do emerge, however, for the opportunity cost of making temporary and permanent layoffs.

Particularly interesting results attach to the quantitative responses. These are influenced by the ability of the firm to make temporary layoffs. Consider, for example, the response of output to a change in current anticipated demand; this is given by:

$$\frac{\partial \bar{q}_t}{\partial \bar{m}_t} = -g_t'(1-F_t)\frac{\partial R_t}{\partial \bar{m}_t} + \left(1-m\frac{\partial p_t}{\partial \bar{m}_t}\right)\int_{\underline{\varepsilon}_t}^{b_t} \omega_t dF_t$$

The first term of this expression picks up the effect on output arising as a result of the firm adjusting its attached labor force; it is the ex ante effect that is present in standard inventory models. The second term, however, essentially captures the effects on output of the ability of the firm ex post to alter the utilization of its workforce through temporary layoffs. As long as $\omega_t > 0$, the firm, in response to a change in $\bar{m}_t$, plans to make ex post adjustments in u_t, the rate it utilizes the attached labor force, which in turn alters the inventory position it will carry over into the next period. A particularly interesting implication of this result is that, even if $\frac{\partial p_t}{\partial \bar{m}_t}$ is small so that changes in demand have little effect on price, a positive ω_t may generate a substantial response of output to changes in demand. This contrasts with the result in Blinder (1982) that both price and output will exhibit small responses to changes in demand if the firm has the ability to hold output inventories. The difference in results arises because in our model even though the firm has the ability to accumulate inventories when anticipated demand falls it may find making temporary layoffs relatively more attractive. In this case, output will exhibit a relatively large response even though price does not.

To take up the case of a change in a cost variable, consider next the response of output to a change in the discount factor, which of course is the inverse of a change in the real interest rate. Differentiating (32), the response of expected output is

$$\frac{\partial \bar{q}_t}{\partial \rho_t} = -g'(L_t)\ (1-F(b_t))\ \frac{\partial R_t}{\partial \rho_t} + \frac{1}{\rho_t} \int_{\underline{\varepsilon}}^{b_t} \frac{\omega_t}{\gamma_t} Z_t dF - m \frac{\partial p_t}{\partial \rho_t} \int_{\underline{\varepsilon}_t}^{b_t} \omega_t dF_t$$

where $\gamma_t = \Gamma_Z \frac{Z_t}{\Gamma_{t+1}} > 0$ measures the responsiveness of Γ_{t+1}, the value of carrying inventories over into next period, to a change in inventories. The first term captures the effects of a change in ρ_t on expected output operating through permanent layoffs. A reduction in ρ_t, i.e., an increase in the real interest rate, raises R_t and thus reduces expected output as the firm reduces its attached labor force. This enables the firm to avoid the imputed costs to holding workers and to reduce its inventory stocks and thereby to avoid the financial costs of holding inventories as well. This is essentially the effect that emerges from conventional inventory models, as can be seen by observing that it is the only effect when $\omega_t = 0$ in which case temporary layoffs and ex post output adjustments are never made.

When ex post adjustment to output are allowed, the second term is also positive; given ω_t, the smaller is γ_t, i.e., the smaller is the value to accumulating inventories when inventories rise, the greater is the incentive to cut output today. This reinforces the conventional effect. At the same time, assuming $\frac{\partial p_t}{\partial \rho_t} > 0$, which is the traditional effect, a lower ρ_t reduces current price which raises expected sales, so that expected sales are reduced rather than output to avoid inventory accumulation. If the latter effect is small, which it may very well be since $\frac{\partial p_t}{\partial \rho_t}$ is ambiguous in our model, then output will be more sensitive to changes in real interest rates than conventional inventory models predict.

E. Implications for Inventory Investment

Inventory investment equations that are used in empirical work frequently take the following flexible accelerator form:

$$Z_t - Z_{t-1} = \pi_1(Z_t^* - Z_{t-1}) + \pi_2\varepsilon_t$$

$$0 \leq \pi_1 \leq 1, \quad -1 \leq \pi_2 \leq 0$$

where Z_t^* is the desired stock of inventories, based on information available at time t, π_1 is an "adjustment" coefficient, and π_2 is a coefficient that captures the extent to which "sales surprises" are absorbed by production revisions or inventory adjustments. If $\pi_2 = 0$ sales surprises are absorbed entirely by output adjustments, while if $\pi_2 = -1$, they are absorbed entirely by inventory adjustments. Clearly, since Z_t^* is defined on the basis of long-term considerations and is thus independent of Z_{t-1} and ε_t,

$$\frac{\partial Z_t}{\partial Z_{t-1}} = 1 - \pi_1 \qquad \frac{\partial Z_t}{\partial \varepsilon_t} = \pi_2$$

Inventory investment equations of this type have typically been rationalized on the basis of standard buffer stock models of inventory behavior. But, as noted above, the standard buffer stock model presumes that output is set ex ante so that inventories alone absorb ex post random shocks to demand. This implies that $\pi_2 = -1$. Beginning with Lovell [1961], numerous authors of empirical studies of inventories have recognized that some ex post production revision may be possible and have therefore allowed for the possibility that $\pi_2 > -1$. The difficulty with this is that the argument is an ad hoc amendment to the theoretical model, and, although appeals to production-smoothing considerations are made, a clear-cut theoretical analysis of what determines the size of π_2 and its relationship to π_1 is lacking. An advantage of the model developed here is that ex post production revisions through temporary layoffs as well as inventory adjustments emerge naturally from the structure of the model. This permits an analysis of the forces at a theoretical level that determine π_2 and its relationship to π_1.

To draw out the implications of our model for inventory behavior, suppose the firm is neither suffering a stockout nor operating at full capacity. Its actual inventory investment at time t is then

$$Z_t - Z_{t-1} = g(u_t L_t) - \bar{m}_t + mp_t - \varepsilon_t \tag{33}$$

$$= I(Z_{t-1}, \varepsilon_t, \bar{m}_t, \bar{m}_{t+1}, \rho_t, \ldots)$$

Inventory investment thus depends on initial inventories, the "sales surprise", as well as current and future levels of anticipated demand, real interest rates, etc.

Our interest here is primarily in the response of inventories to Z_{t-1} and ε_t. Differentiating (33) appropriately yields:

$$\frac{\partial Z_t}{\partial Z_{t-1}} = (1-\omega_t)(1 + m \frac{\partial p_t}{\partial Z_{t-1}})$$

$$\frac{\partial Z_t}{\partial \varepsilon_t} = \omega_t - 1$$

Hence, in terms of our model, the coefficients of empirical inventory investment equations may be expressed as

$$\pi_1 = 1 - (1-\omega_t)(1 + m \frac{\partial p_t}{\partial Z_{t-1}})$$

$$\pi_2 = -(1-\omega_t)$$

Once again, the term ω_t captures the tendency of the firm to use temporary layoffs and therefore ex post production revisions to absorb demand shocks. When such ex post production revisions are disallowed or uneconomical so that $\omega_t \to 0$, then $\pi_1 \to -m \frac{\partial p_t}{\partial Z_{t-1}}$ and $\pi_2 \to -1$, which essentially are the predictions of the standard buffer stock inventory model. On the other hand, when it is profitable to readily make ex post production revisions so that $\omega_t \to 1$, then $\pi_1 \to 1$ and $\pi_2 \to 0$. In this case, inventories are easily kept at "desired" levels.

The size of π_2 depends solely on ω_t, the degree to which output changes ex post from temporary layoffs. The model thus provides a precise theory of the determinants of π_2. These include not only the degree to which incentives to smooth production exist, which the empirical literature has emphasized, but other factors as well. In particular, the model predicts that π_2 will be larger in absolute value the smaller is the net subsidy to making layoffs, the larger is the net value to accumulating inventories, etc.

As a final matter, observe that π_1 and π_2 are closely related since they are influenced by common factors. For example, the greater is the incentive to smooth production, i.e., the larger is g_t'', the smaller should be π_1 and the closer to unity in absolute value should be π_2. This is because the larger is g_t'' the smaller should be <u>both</u> ω_t and $\frac{\partial p_t}{\partial Z_{t-1}}$.[15] This is of course consistent with conjectures in the empirical literature.

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[15]Predicting the response of π_1 and π_2 to changes in other factors, however, is quite difficult because investigating how the expression $\frac{\partial p_t}{\partial Z_{t-1}}$ changes with exogenous variables is quite complex even under simplifying assumptions. This is unfortunate since it would be of interest to try to use the model to explain the puzzling empirical finding that both π_1 and π_2 are small.

IV. Conclusions and Extensions

In this paper, we have developed an intertemporal model of a firm that makes decisions on price, output, inventories, and labor inputs. We have focussed in particular on the interaction of these variables as the firm responds to fluctuations in demand and cost factors.

Our findings support the following description of a firm during a recession. In the initial stages of a slump, the firm may be uncertain as to whether the decline in demand will be of short or long duration. Since we demonstrate that inventories as opposed to temporary layoffs are more likely to be used in response to shocks in demand that are perceived to be temporary, this suggests that the firm will have a tendency to use inventories in the initial stages of the slump. As the firm becomes aware that the downfall in demand is widespread, it substitutes temporary layoffs and production cutbacks for inventories. As we demonstrate, this is because temporary layoffs are more likely to be used the more persistent a negative demand shock is expected to be and because using inventory accumulation in the initial stages of a slump leads to higher initial inventories in succeeding periods which in turn implies a shift towards temporary layoffs and away from further inventory accumulation. In especially deep and persistent recessions, the firm may view the decline in demand as being of sufficiently long duration so that it ultimately substitutes permanent layoffs rather than temporary layoffs, deeper cuts in production and reductions in price as well. The precise nature of this described interaction depends upon the demand conditions, the production conditions, the structure of contracts and the cost conditions facing the firm, as we show in various propositions in the paper. Nevertheless, this brief description of the implications of our model indicates that our model provides a basis for understanding the observed fluctuations and interaction of inventories, layoffs, output and prices over the business cycle.

To conclude the paper, we indicate a straightforward extension of the model and discuss the implications of the model for empirical work. The straightforward extension is to relax the assumption that hours per worker are fixed. Allowing for hours variation would merely add another dimension to the firm's ex post decisions in addition to temporary layoffs and accumulating inventories. Using procedures analogous to those used above, it can be shown that the firm will be less likely to use hours reductions as opposed to temporary layoffs or inventory accumulation in response to a random shock to demand, the larger is the net value to holding inventories and the net subsidy to making temporary layoffs, the smaller is the increase in utility associated with the reduction in hours, and the greater is the degree of increasing returns in hours per worker.

As our analysis indicates, the model developed above yields a rich menu of testable hypotheses. These include predictions on price output, inventories, permanent and temporary layoffs, and new hires. Perhaps the most interesting of these are the predictions on the interaction of inventories and temporary and permanent layoffs on which no empirical work has been done. To accomplish this series on temporary and permanent layoffs will need to be constructed; this can be done following the work of Lilien [1980]. This is planned for future research.

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APPENDIX

Proof of Proposition 1: (i) Suppose not, i.e., suppose $\varepsilon_t \geq b_t$, $X_t > 0$ $(u_t < 1)$ and $\Omega_t - K_t - \rho_t(V_{t+1}+a_{t+1}) > 0$. Since $\varepsilon_t \geq b_t$ and $u_t < 1$, by (18) $\theta_t = 0$. Since $u_t < 1$, by (19), $\lambda_t = 0$. Combining (12) and (13) yields in this case:

(A1) $$p_t g'(u_t L_t) = K_t - c_t$$

Since $g'' < 0$, this in turn implies:

(A2) $$p_t g'(L_t) < K_t - c_t.$$

Taking expectations of (13) based on information known at time t and combining this with (14) and (15) yields:

(A3) $$E_t(\lambda_t) \geq \Omega_t - K_t - (V_{t+1}+a_{t+1})\rho_t + \delta_t$$

By assumption (A3) implies that $\lambda_t > 0$ for at least some ε_t. Consider such an ε_t for which $\lambda_t > 0$. By (12) and (13) we must have:

(A4) $$\lambda_t = (p_t - \theta_t)g'(L_t) - K_t + c_t > 0$$

Yet (A2) and (A4) yield a contradiction. Hence, $u_t = 1$ and $X_t = 0$.

(ii) Suppose $\varepsilon_t = b_t$ and $Z_t > 0$. Then by (17) $\alpha_t = 0$ which in turn by (12) implies $\theta_t = p_t + h_t - \rho_t \Gamma_{t+1} > 0$. Since $\theta_t > 0$, $S_t = m(p_t) + \varepsilon_t$. Since $\varepsilon_t = b_t$, $Z_{t-1} + g(L_t) = m(p_t) + \varepsilon_t$. This in turn implies $S_t = Z_{t-1} + g(L_t)$ which implies $Z_t = 0$. Hence, we have a contradiction. Now suppose $\varepsilon_t > b_t$ and $Z_t > 0$. Since $\varepsilon_t > b_t$, $\theta_t = 0$ by (18). By (12) this implies $\alpha_t = p_t + h_t - \rho_t \Gamma_{t+1} > 0$ which by (17) implies $Z_t = 0$. Again, we have a contradiction and hence, $Z_t = 0$.

Finally, suppose $\varepsilon_t < b_t$. This implies $X_{t-1} + g(L_t) > m(p_t) + \varepsilon_t$. Suppose $u_t = 1$. Then since $S_t \leq m(p_t) + \varepsilon_t$, this implies $Z_{t-1} + g(L_t) >$

S_t and thus $Z_t > 0$. Suppose $Z_t = 0$. Then $Z_{t-1} + g(u_t L_t) = S_t$. This requires $u_t < 1$ since $S_t = Z_{t-1} + g(u_t L_t) \leq m(p_t) + \varepsilon_t < Z_{t-1} + g(L_t)$. Since $u_t < 1$, $X_t > 0$.

<u>Proof of Proposition 2</u>: (i) Given the assumptions in (i), and given that $\Gamma_Z < 0$ there must exist an $\varepsilon_t^* < b_t$ such that

(A5) $$(\Gamma_{t+1}^* - h_t)\, g'(L_t) > K_t - c_t$$

where Γ_{t+1}^* is evaluated at $Z_t^* = Z_{t-1} + g(L_t) - m(p_t) - \varepsilon_t^*$. Given (A5) combined with (12) and (13) yields that $\lambda_t > 0$ for $\varepsilon_t = \varepsilon_t^*$. Hence, $X_t = 0$ for $\varepsilon_t = \varepsilon_t^*$. Since $\Gamma_Z < 0$, this implies that for $b_t > \varepsilon \geq \varepsilon_t^*$, $X_t = 0$. By Proposition 1 this implies $Z_t > 0$ for $b_t > \varepsilon_t \geq \varepsilon_t^*$. This proves (i).
(ii) The proof of the layoff biased case follows similar arguments.
(iii) Suppose not, i.e., suppose $X_t = 0$. By Proposition 1, this implies $Z_t > 0$. In this case, by (17) $\alpha_t = 0$ and by (12) and (13) we have:

(A6) $$(\Gamma_{t+1} - h_t)\, g'(L_t) = K_t - c_t + \lambda_t$$

Since $\Gamma_Z < 0$ and $(\Gamma_{t+1}^o - h_t)\, g'(L_t) = K_t - c_t$, (A6) yields a contradiction. The alternative case, $Z_t = 0$ yields a contradiction following similar arguments. Hence, $Z_t > 0$ and $X_t > 0$.

<u>Proof of Proposition 3</u>: Since $X_t > 0$ and $Z_t > 0$, the relevant optimality condition determining u_t is (21). Given u_t, Z_t is determined by (22b) and X_t is determined by $X_t = (1-u_t)L_t$. To avoid a long taxonomy of comparative static results, we provide details of the results for $\partial X_t/\partial\varepsilon_t$ and $\partial Z_t/\partial\varepsilon_t$ only. Using (21) and (22b) yields:

(A7) $$\frac{\partial u_t}{\partial \varepsilon_t} = \frac{\rho_t \Gamma_Z\, g_t'}{\rho_t \Gamma_Z (g_t')^2\, L_t + (\rho_t \Gamma_{t+1} - h_t) g_t'' L_t} > 0$$

By (22b) we have (for given p_t, R_t and N_t):

$$\text{(A8)} \qquad \frac{\partial Z_t}{\partial \varepsilon_t} = g_t' \frac{\partial u_t}{\partial \varepsilon_t} L_t - 1$$

Given (A7) and (A8), $-1 < \frac{\partial Z_t}{\partial \varepsilon_t} < 0$. Using $X_t = (1-u_t)L_t$ (for given p_t, N_t and R_t) and (A7) yields:

$$\text{(A9)} \qquad \frac{\partial X_t}{\partial \varepsilon_t} = - \frac{\partial u_t}{\partial \varepsilon_t} \cdot L_t < 0.$$

The remainder of the results are derived in a similar fashion.

<u>Proof of Proposition 4</u>: Using (21) and (22b) yields (for fixed p_t, N_t and R_t)

$$\text{(A10)} \qquad \frac{\partial u_t}{\partial \mu} = \frac{-\rho_t g_t' \Gamma_{\bar{m}_{t+1}} \varepsilon_t}{\rho_t \Gamma_Z (g_t')^2 + (\rho_t \Gamma_{t+1} - h_t) g_t'' L_t}$$

For $\varepsilon_t < 0$, (A6) implies $\partial u_t / \partial \mu < 0$. Since $\frac{\partial Z_t}{\partial \mu} = g_t' \frac{\partial u_t}{\partial \mu} \cdot L_t$ and $\partial X_t / \partial \mu = - \frac{\partial u_t}{\partial \mu} L_t$, we have for $\varepsilon_t < 0$, $\partial Z_t / \partial \mu < 0$ and $\partial X_t / \partial \mu > 0$.

<u>Proof of Proposition 5</u>: To avoid a long taxonomy of comparative static results, we provide details for only $\partial R_t / \partial Z_{t-1}$ and $\partial \bar{\ell}_t / \partial Z_{t-1}$. The remainder of the results follow in a similar manner. Having solved the problem in a backward recessive manner (i.e., first solving for period $t+1$ decisions conditional on period t decisions and then solving for ex post period t decisions conditional on the ex ante period t decisions), this results in there being two key ex ante conditions in two unknowns, p_t and R_t. These conditions are given by (25) and (26) (where the u_t, Σ_{t+1}, Γ_{t+1} refer to the optimized values conditional on p_t and R_t). Taking the total differential of (25) and (26) and applying Cramer's rule yields:

(A11) $$\frac{\partial R_t}{\partial Z_{t-1}} = \frac{1}{\Delta t}\,[2mf(b_t)g_t'\,\beta_t - g_t'(1-F(b_t))^2 - \rho_t m g_t'(1-F(b_t))\int_{\underline{\varepsilon}_t}^{b_t}\Gamma_Z(1-\omega_t)dF_t]$$

where $\Delta_t = p_t g_t''(-2F(b_t)m - m^2 f(b_t)\beta_t + \rho_t m\int_{\underline{\varepsilon}_t}^{b_t}\Gamma_Z(1-\omega_t)dF_t)$ $- \rho_t(mg_t')^2 f(b_t)\beta_t\int_{\underline{\varepsilon}_t}^{b_t}\Gamma_Z(1-\omega_t)dF_t - 2mf(b_t)(g_t')^2\,\beta_t - (g_t'(1-F(b_t)))^2$ and $\beta_t = p_t + h_t - \rho_t\Gamma_{t+1}$. By the assumed concavity restrictions, $\Delta_t > 0$. Similarly, the first two terms in the numerator of (A11) sum to be positive because of the assumed concavity restrictions. The third term in the numerator of (A11) is positive since $\Gamma_Z < 0$ and $(1-\omega_t) > 0$. Hence, $\partial R_t/\partial Z_{t-1} > 0$. Using the definition of expected total layoffs yields:

(A12) $$\frac{\partial\bar{\ell}_t}{\partial Z_{t-1}} = (1-F(b_t))\frac{\partial R_t}{\partial Z_{t-1}} + \int_{\underline{\varepsilon}_t}^{b_t}\frac{\omega_t}{g_t'}\,[m\frac{\partial p_t}{\partial Z_{t-1}} + 1]dF_t$$

Cramer's rule yields:

(A13) $$\frac{\partial p_t}{\partial Z_{t-1}} = -\frac{1}{m} - \frac{1}{\Delta_t}\,[(1+F(b_t))(p_t g_t''(1-F(b_t)) - f(b_t)(g_t')^2\beta_t) + \frac{g_t'}{m}(1-F(b_t)(mf(b_t)g_t'\beta_t - g_t'(1-F(b_t))] < 0.$$

Combining (A12) and (A13) yields that the second term on the RHS of (A12) is positive. Since the first term on the RHS of (A12) is positive by (A11), this implies $\partial\bar{\ell}_t/\partial Z_{t-1} > 0$.

Proof of Proposition 6: To avoid a long taxonomy of results, we provide details for only $\partial p_t/\partial\bar{m}_t$ and $\partial\bar{q}_t/\partial\bar{m}_t$. The remainder of the results follow in a similar manner. Total differentiating of (25), (26) and (32) and applying Cramer's rule yields:

(A14) $$\partial p_t/\partial\bar{m}_t = \frac{1}{\Delta_t}\,[p_t g_t''\,(-(1+F(b_t)) - 2f(b_t)\beta_t + \rho_t m \int_{\underline{\varepsilon}_t}^{b_t} \Gamma_Z(1-\omega_t)dF_t)$$

$$+ f(b_t)(g_t')^2\beta_t\,(1-\rho_t m \int_{\underline{\varepsilon}_t}^{b_t} (1-\omega_t)\Gamma_Z dF_t)]$$

where Δ_t is given in (A11). Since $g_t'' < 0$, $\beta_t > 0$, $\Gamma_Z < 0$ and $\omega_t < 1$, (A15) yields $\partial p_t/\partial\bar{m}_t > 0$. Using the definition of $\bar{q}_t$ we have:

(A15) $$\partial\bar{q}_t/\partial\bar{m}_t = -g_t'(1-F_t(b_t))\;\partial R_t/\partial\bar{m}_t + (1-m\;\partial p_t/\partial\bar{m}_t)\int_{\underline{\varepsilon}_t}^{b_t} \omega_t dF_t$$

By Proposition 5, $\partial R_t/\partial\bar{m}_t < 0$. This combined with (A14) and (A15) yields $\partial\bar{q}_t/\partial\bar{m}_t > 0$.