INFORMATION ON WORKER ABILITY:

AN ANALYSIS OF INVESTMENT WITHIN THE FIRM

by

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ABSTRACT

This paper considers a world in which firms invest in the production of information concerning workers, where this information is used to help allocate workers among tasks. The question addressed is, does the firm have an incentive to make a socially efficient investment? The major results of the analysis are two. If information produced is not observed by other firms and individual task assignments are also not publicly observed, then the firm has an incentive to invest optimally in information production. If, however, either the information produced or the task assignment is public knowledge, then there is a tendency for the firm to underinvest.

There are two reasons why a worker's productivity might be positively correlated with age. The standard rationale is that it is due to the accumulation of human capital. That is, skills tend to improve both with schooling and experience, and this subsequently increases productivity. More recently, however, a number of papers have considered an alternative explanation. Over time information is accumulated concerning workers, and this improves the match between workers and firms, and between workers and tasks within firms. This paper falls into the latter category, and in particular concerns the accumulation of such information when the key element is the assignment of workers to tasks.[1]

Many papers have concentrated on the issue of the assignment of workers to tasks, where information concerning the optimal assignment is either costly or incomplete. For example, MacDonald (1980) and Burdett and Mortensen (1981) both consider models where individuals invest in such information prior to beginning their work lives. Further, the case where information is accumulated after the worker is already at a firm has also been considered (see e.g., Ross et al 1981, MacDonald 1982, and Waldman 1984). In each of these latter papers, however, the firm has no control over the quantity of information produced. In this paper I consider a world where information is accumulated within the firm, but where the firm has control over the amount of information produced. The main issues I want to address in this context concern the optimality of the investment in information production. First, in such an environment does the firm have an incentive to make a socially efficient investment in information production, or is there a divergence between the actual investment and the socially optimal investment. Second, if such a divergence exists, what factors affect the size of the divergence?[2]

One previous paper which does allow accumulation of information inside the firm, and for the firm to have control over the amount of information produced is Prescott and Visscher (1980). They consider a model wherein workers can be assigned to any of three tasks, but only one task provides information concerning worker attributes. By varying the length of time a worker is assigned to the "screening" task, the firm can vary the amount of information produced concerning that worker. There are two major differences between the present paper and the Prescott and Visscher analysis. The first concerns the focus of the two papers. Prescott and Visscher were interested in demonstrating that this type of information accumulation process can lead to constraints on a firm's growth rate. As mentioned earlier, the present paper is concerned with the social optimality of the firm's investment in information production. Second, Prescott and Visscher are somewhat ad hoc in terms of the market contract which gets offered to workers in that they do not explicitly state their assumptions concerning such issues as mobility costs and publicness of information. This paper will be much more explicit concerning these aspects of the problem, and in particular will demonstrate that what is assumed concerning publicness of information is quite crucial.[3]

To analyze the issues discussed above I construct a simple model in the spirit of Harris and Holmstrom (1982). The key property of the model is that there is an asymmetry in the ability of agents to bind themselves to a transaction. That is, though contracting firms can bind their future behavior, however, individuals in agreeing to a contract cannot irrevocably bind themselves to firms. The analysis yields that in this type of environment the less public is the information produced, the closer will be the investment in information production to the social optimum. That is, when the information produced is not observed by other firms and individual task assignments are also not publicly observed, then a first best result emerges. However, if either the information produced or the task assignment is public knowledge, then the firm has a tendency to underinvest in information production.

The above result contrasts sharply with the conclusions of Burdett and Mortensen. They found that the more public the information produced, the closer is the economy to a first best optimum. Upon close inspection, however, the different conclusions are easily understandable. As mentioned earlier, Burdett and Mortensen analyze a model wherein individuals invest in information production prior to beginning their work lives. To give individuals an incentive to invest optimally, the private benefit to investing and the social benefit must coincide. This will occur when information is public, because then the competition among firms will result in the social return from the information being completely incorporated into the wage. This paper considers a model where investment takes place after workers are already at firms. Because firms have superior access to capital markets, it is more efficient for firms to finance the investment. Thus, to now get an optimal investment, the firm must reap all the returns from the information produced. This won't be the case when the information produced is public, because then competition from other firms will shift some of the return to the workers. The result then being that when the information is public, there tends to be an underinvestment in information production.

It bears repeating that what is driving the results is not a simple application of the public goods problem. It is not that an underinvestment occurs when information is public because the agent making the investment does not internalize all the returns from the investment. Rather, the results follow from the fact that the publicness of the information produced determines who must finance the investment. When information produced is private the firm finances the investment, and given the firm's superior access to capital markets this results in an efficient investment. On the other hand, when information produced is public each worker must finance his own investment, and this tends to yield an underinvestment because of the limited access workers have to capital markets.

The outline for the paper is as follows. Section I presents a model wherein firms choose an investment in information production. Section II analyzes the model under the two polar assumptions that the information produced is completely private information, and that it is completely public. Section III analyzes the model under an intermediate information assumption. Specifically, the information produced is only directly revealed to the firm making the investment, however, other firms infer some information about a worker's ability by considering his task assignment. Section IV presents some concluding remarks.

I. The Model

The model presented in this section is most closely related to Harris and Holmstrom (1982) and Waldman (1984). As indicated earlier, its main distinguishing characteristic is that it allows firms to have control over the amount of information produced concerning worker abilities.

Within the economy there is only one good produced and the price of this good is normalized to one. Further, output is produced by identical risk neutral firms, and there is free entry into production.

Workers live for two periods, and in each period labor supply is perfectly inelastic and fixed at one unit for each worker. During their first period of employment workers will be referred to as young, while old will refer to workers who are in their second period of employment.

Workers do not have access to either capital markets or insurance markets, and therefore a worker's utility in each period is simply a function of the wage received in that period.[4] Preferences are such that workers are risk averse over wages, i.e., if a worker receives W_t in period t, then utility equals $V(W_t)$, where $V' > 0$ and $V'' < 0$. Further, both firms and workers are assumed to have a zero rate of discount.

Workers display no disutility for effort. However, each worker has associated with him or her a value for a variable which will be called ability, and which will be denoted A_i for worker i. Each A_i is a draw from a random variable which equals A^H with probability p, and A^L with probability (1-p), where $A^H > A^L > 0$.

Output can be produced by the performance of either of two tasks, where workers are indifferent between the two tasks. The first task is referred to as the skilled task and is denoted S, while the second is referred to as the unskilled task and is denoted U. If worker i is assigned to the skilled task, then he produces an amunt A_i. On the other hand, if he is assigned to the unskilled task then his output is independent of ability, and equals x. It is also assumed that $A^H > x > A^L$, and $pA^H + (1-p)A^L < x$. The first assumption states that high ability workers are better suited to the skilled task, while low ability workers are better suited to the unskilled task. The second assumption states that <u>on average</u> workers are better suited to the unskilled task. Note that the second assumption is not at all crucial for the results, but rather serves to simplify the exposition.

Before his first period of employment a worker's ability is unknown both to the worker and to all the firms in the economy. While the worker is young the first period employer can invest in the production of information concerning the worker's ability. Specifically, if the firm invests z in the production of information concerning worker i, then the firm receives an indicator of ability θ. θ is a draw from a random variable which has a probability density function $g(\theta,z,A_i)$. That is, the probability that any particular value for θ will actually be realized depends on both the investment in information production, and the true ability of the worker about whom the information is being produced.

Rather than speaking of the information produced as being the realized value of the indicator θ, however, it will in general be convenient to suppress θ and speak of the information produced as being the inference that can be made from this realization. That is, given θ and an investment in information production of size z, it is possible to derive a value for expected ability, denoted $A_E(\theta,z)$. $A_E(\theta,z)$ is given by equation (1).

$$(1) \qquad A_E(\theta,z) = \frac{pg(\theta,z,A^H)A^H + (1-p)g(\theta,z,A^L)A^L}{pg(\theta,z,A^H) + (1-p)g(\theta,z,A^L)}$$

Obviously, $A_E(\theta,z)$ will always fall somewhere in the interval $[A^L,A^H]$.

Suppose the worker is actually of ability $A^H(A^L)$. Given the above, A_E can be thought of as a draw from a random variable whose probability density function depends on the investment in information production. Let $f_H(A_E,z)$ $(f_L(A_E,z))$ denote this probability density function, and let $F_H(A_E,z)$ $(F_L(A_E,z))$ denote the corresponding cumulative distribution function.[5] It is further assumed that for any A_E, both $F_H(A_E,z)$ and $F_L(A_E,z)$ are continuously differentiable in z.

The final aspect of the model to be described is the contracting environment. It is assumed that in agreeing to a contract a worker cannot irrevocably bind himself to a firm, and that a worker can change firms after his first period of employment without incurring any costs. Additionally, given equal wage offers prior to his second period of employment, a worker will choose to remain with his first period employer. This last assumption is simply made for expositional convenience.

For the other side of the market it is assumed that firms offer young workers long-term or implicit contracts which specify four things. First, the contract specifies the wage the worker will receive while young, denoted W^Y. Second, it specifies what the firm will invest in the production of information concerning the worker, denoted z. Third, it specifies the worker's second period wage as a function of the realized value of A_E, denoted $W^O(.)$. Fourth, it specifies the worker's second period task assignment as a function of the realized value of A_E, denoted $T^O(.)$. Finally, one additional restriction is imposed on the contracting process. Specifically, firms will be restricted from offering contracts where wages are contingent on output. This is already implicit in the long-term contracts described above in that the second period wage is only a function of the realized value of A_E. Thus, I am here simply extending the restriction to the contracts offered to old workers by firms other than the first period employer. The restriction can be justified by assuming that only aggregate output can be observed, and that there are economies of scale, although not modeled, such that firms hire many workers.[6]

II. Analysis

In this section the model developed in the previous section is analyzed under two polar assumptions: (1) that information produced and task assignments are both not publicly observable; and (2) that information produced is publicly observable.[7]

Before proceeding to these two polar assumptions I will establish some benchmarks with which later results can be compared. In particular, following Burdett and Mortensen (1981) I will consider the behavior of a social planner whose goal is to maximize expected net output.

Given that the social planner's goal is to maximize expected net output, the social planner will assign an old worker to task S(U) if $A_E >(\leq)\ x$.[8] The more interesting question is, how much will the social planner invest in the production of information? The preceding implies that, given an investment of size z, the expected output of a representative old worker, denoted $Q^E(z)$, is given by (2).

$$(2) \qquad Q^E(z) = \int_{A^L}^{x} x[pf_H(A_E,z) + (1-p)f_L(A_E,z)]dA_E + p\int_{x}^{A^H} A^H f_H(A_E,z)dA_E + (1-p)\int_{x}^{A^H} A^L f_L(A_E,z)dA_E$$

Let z^* be the investment in information production made by the social planner. (2) yields that z^* is defined by (3). Note, to make the model interesting it is of course assumed that $z^* > 0$.[9]

$$(3) \qquad z^* = \arg\max_{z} \int_{A^L}^{x} x[pf_H(A_E,z) + (1-p)f_L(A_E,z)]dA_E + p\int_{x}^{A^H} A^H f_H(A_E,z)dA_E + (1-p)\int_{x}^{A^H} A^L f_L(A_E,z)dA_E - z$$

Equation (3) completes the analysis of how a social planner would behave. I will now proceed to compare and contrast this behavior with how the actual market behaves.

The first case to be considered is when information produced and task assignments are both not publicly observable. Consider a firm other than the first period employer and a representative old worker. Given that information produced is purely private, all such a firm knows about the worker is that he has ability A^H with probability p, and ability A^L with probability $(1-p)$. Thus, prior to his second period of employment such a firm will only be willing to offer the worker the wage x.[10] Now consider the long-term contract that firms offer to young workers in equilibrium. The preceding implies that the terms specified in the contract must solve the following maximization problem. Note, in the following let $Q(A_i,S) = A_i$, $Q(A_i,U) = x$, and $f(A_e,z) = pf_H(A_E,z) + (1-p)f_L(A_E,z)$ for all A_E, z pairs.[11]

$$\max_{\substack{W^Y, W^O(\cdot) \\ z, T^O(\cdot)}} V(W^Y) + \int_{A^L}^{A^H} V(W^O(A_E))f(A_E,z)dA_E \tag{4}$$

$$\text{s.t. } x - z + \int_{A^L}^{A^H} Q(A_E,T^O(A_E))f(A_E,z)dA_E \geq W^Y + \int_{A^L}^{A^H} W^O(A_E)f(A_E,z)dA_E$$

$$W^O(A_E) \geq x \quad \text{for all } A_E$$

$$Q(A_E,T^O(A_E)) - W^O(A_E)$$

$$\geq Q(A_E,T^O(\tilde{A}_E)) - W^O(\tilde{A}_E) \quad \text{for all } A_E,\tilde{A}_E \text{ pairs}$$

The above objective function simply states that the contract will maximize a worker's expected lifetime utility. The first constraint ensures that the firm's expected profitability is not negative. The second constraint ensures that the wage always exceeds what other firms are willing to bid for the worker. Finally, because information produced is not publicly observable, there is an incentive compatibility constraint. It guarantees that, for every realization of A_E, the firm has an incentive to behave in accordance with what is specified in the contract. Analysis of equation (4) yields the following proposition. Note, all proofs are relegated to an Appendix.

<u>Proposition 1</u>: When information produced and task assignments are both not publicly observable, then

i) $W^Y = W^O(A_E)$ for all A_E

ii) $z = z^*$

iii) $T^O(A_E) = S(U)$ if $A_E >(<) x$.

Proposition 1 demonstrates that when information produced and task assignments are both not publicly observable, then the model yields a first best result. This is evidenced in three ways. First, the investment in information production and the assignment of old workers to tasks follows exactly the behavior which would be exhibited by a social planner.[12] Second, workers are completely insured against the uncertainty they face concerning their own ability. Third, the flat age earnings profile means that workers are not hurt by their inability to borrow.

The next step of the analysis is to consider the other polar case, i.e., information produced is publicly observable. Again consider a firm other than the first period employer, and a representative old worker. Given that information produced is purely public, this firm knows as much about the worker as the worker's first period employer. Thus, prior to his second period of employment such a firm will be willing to offer the worker a wage equal to $\max\{x, A_E\}$. That is, the firm will be willing to offer the worker the expected output of the worker, given that he is assigned to the task which maximizes this expectation. Now consider the long-term contract that firms offer to young workers in equilibrium. The above implies that the terms in the contract must solve the following maximization problem.

$$(5) \qquad \max_{\substack{W^Y, W^O(.) \\ z, T^O(.)}} \quad V(W^Y) + \int_{A^L}^{A^H} V(W^O(A_E)) f(A_E, z) dA_E$$

$$\text{s.t. } x - z + \int_{A^L}^{A^H} Q(A_E, T^O(A_E)) f(A_E, z) dA_E \geq W^Y + \int_{A^L}^{A^H} W^O(A_E) f(A_E, z) dA_E$$

$$W^O(A_E) \geq \max\{x, A_E\} \quad \text{for all } A_E$$

There are two differences between equations (4) and (5). The first is the constraint on the old worker wage function. This change reflects the earlier mentioned idea that now all firms receive the information produced, and this extra information is incorporated into the wage offers an old worker receives from the other firms in the market.[13] The second difference is that because information produced is publicly observable, there is no incentive compatibility constraint.

Before proceeding to the analysis of equation (5) it is necessary to introduce an additional assumption.

<u>Assumption 1</u>: For every z', $z' > z^*$, $f(A_E,z')$ is a mean preserving spread of $f(A_E,z^*)$.

By definition $f(A_E,z')$ must have the same mean as $f(A_E,z^*)$. Thus, the only restriction in Assumption 1 is that $f(A_E,z')$ must be a "spread" of $f(A_E,z^*)$. The role of this assumption will be made clear further on. We can now proceed to the analysis of equation (5).

<u>Proposition 2</u>: When information produced is publicly observable, then

i) $W^Y = x - z$

ii) $W^O(A_E) = \max\{x,A_E\}$ for all A_E

iii) $z < z^*$ if Assumption 1 holds

iv) $T^O(A_E) = S(U)$ if $A_E >(<) x$.

Proposition 2 tells us that when information produced is publicly observable, then the model no longer yields a first best result. This is evidenced in Proposition 2 in three ways. First, workers face an upward sloping age earnings profile, which given their inability to borrow is not optimal. Second, workers are no longer insured against the uncertainty they face concerning their own ability. Third, there is an underinvestment in information production as long as Assumption 1 holds.[14] Note, however, there is one aspect of the contract which is optimal. That is, given the investment in information production, there is an optimal assignment of old workers to tasks.

At this point it is worthwhile providing some intuition for Propositions 1 and 2. When nothing is publicly observable, then the firm receives all the returns from investments in information production. This means it has an incentive to invest in information production and assign workers to tasks in a manner which maximizes net output. Further, again given that information produced is not publicly observable, there is nothing to stop the firm from providing complete insurance to workers, and from offering an optimal age earnings profile. Now consider the case where information produced is publicly observable. Here the workers receive all the returns from investments in information production, which means that workers must provide the financing. Given that workers cannot borrow, this leads to the age earnings profile being non-optimal, and to there being a tendency towards underinvestment in information production. Further, the wage offers other firms make to old workers also stop the firm from providing any insurance.

Another way of looking at the results is in terms of general versus firm specific human capital. When nothing is publicly observable, then investing in information production is like investing in firm specific human capital. Further, in a Harris and Holmstrom type world investments in firm specific human capital will be financed solely by the firm, which yields that there will be an efficient level of investment.[15] On the other hand, when information produced is publicly observable, then investing in information production is like investing in general human capital. This means that the worker does the financing, which given the capital market constraint implies a tendency toward underinvestment.

One question remains, i.e., what role does Assumption 1 play? Consider the following example. Let $A^L = 1$, $A^H = 4$, $x = 3$ and $p = 1/2$. Suppose z can only take on the values 0, 1/8, and 1/5, and assume that θ can only take on the values 0 and 1. Let $g(0,A^j,0) = 1$ for $j = L,H$, $g(0,A^L,1/8) = 1$, $g(0,A^H,1/8) = 1/4$, $g(0,A^L,1/5) = 9/10$, and $g(0,A^H,1/5) = 0$. Finally, let $V(W) = W$ for $W < \frac{41}{11}$, and $V(W) = \frac{41}{11} + \frac{1}{5}(W - \frac{41}{11})$ for $W > \frac{41}{11}$. Analysis of this example yields first that $z^* = 1/8$, and second that under public information $z = 1/5$.

The role of Assumption 1 is easily understood through the workings of this example. When $z = 1/8$ there is a 7/8 probability that $A_E < x$ in which case the second period wage equals 3, and a 1/8 probability that $A_E = A_H$ in which case the second period wage equals 4. On the other hand, when $z = 1/5$ there is a 9/20 probability that $A_E < x$ in which case the second period wage again equals 3, and an 11/20 probability that $A_E = 41/11$ in which case the second period wage equals 41/11. The above implies that the investment level $z = 1/8$ is in some sense "riskier" than the level $z = 1/5$, which, given workers are risk averse, makes $z = 1/8$ less attractive. Further, in the above example this aspect of the problem dominates, with the result being that the public information investment level is actually greater than the investment level which characterized the first best social optimum.[16]

The role of Assumption 1 is now clear. It guarantees that the riskiness of the investment does not fall as the investment rises. The result is that the riskiness of the investment can be ignored, and in turn that the dominant aspect of the problem is that workers have limited access to capital markets.

Notice, finally, rather than throwing doubt on the analogy between public information and general human capital, this qualification to the central result of the paper actually strengthens it. Consider again a Harris and Holmstrom type world where workers invest in general human capital. If the return to the investment is certain, then there is sure to be an underinvestment. Suppose, however, the return to the investment is uncertain. If it is further true that increasing the investment lowers the riskiness of the investment, then you could get a result completely analogous to what was found in the example. That is, because of the reduction in the riskiness of the investment, each worker's investment in general human capital could actually be larger than the investment which characterized the first best social optimum.

III. Task Assignment Signalling

In this section I employ the information assumption first investigated in Waldman (1984). Specifically, information produced about workers will not be publicly observable, but task assignments will be.

Before proceeding to the analysis it is necessary to establish another social welfare benchmark. In my earlier paper it was shown that when only task assignments are publicly observable, it is frequently the case that old workers are inefficiently assigned to tasks. Given this, it would be incorrect to use z^* as the social welfare benchmark because that was constructed given an efficient assignment of old workers to tasks. In other words, because of this other inefficiency, a second best notion for z is appropriate.

As previously, let $T^0(.)$ describe the manner in which old workers are assigned to tasks. Also, let $\hat{z}(T^0(.))$ be the investment in information production made by a social planner, given the assignment function $T^0(.)$. Because the social planner is interested in maximizing expected net output, $\hat{z}(T^0(.))$ is given by (6).

$$\hat{z}(T^0(.)) = \arg\max_{z} \int_{A^L}^{A^H} Q(A_E, T^0(A_E))f(A_E, z)dA_E - z \tag{6}$$

Note, if $T^O(A_E) = S(U)$ when $A_E >(<) x$, then $\hat{z}(T^O(.)) = z*$.

We can now proceed to the analysis of how the actual market behaves. Let $\hat{A}_E(t,z,T^O(.))$ be the expected ability of an old worker assigned to task t, when the investment in information production is z and $T^O(.)$ determines how the first period employer assigns old workers to tasks. Now consider a firm other than the first period employer, and an old worker assigned to task t. Given that only the task assignment is publicly observable, prior to the worker's second period of employment such a firm will be willing to offer the worker a wage equal to $\max\{x,\hat{A}_E(t,z,T^O(.))\}$.[17] That is, just as for the case where information produced is publicly observable, the firm will be willing to offer the worker the expected output of the worker, given that he is assigned to the task which maximizes this expectation. We can now set up a maximization problem similar to equations (4) and (5) of the previous section.

$$(7) \qquad \max_{\substack{W^Y, W^O(.) \\ z, T^O(.)}} \quad V(W^Y) + \int_{A^L}^{A^H} V(W^O(A_E))f(A_E,z)dA_E$$

$$\text{s.t.} \quad x - z + \int_{A^L}^{A^H} Q(A_E,T^O(A_E))f(A_E,z)dA_E \geq W^Y + \int_{A^L}^{A^H} W^O(A_E)f(A_E,z)dA_E$$

$$W^O(A_E) \geq \max\{x,\hat{A}_E(T^O(A_E),z,T^O(.))\} \quad \text{for all } A_E$$

$$Q(A_E,T^O(A_E)) - W^O(A_E)$$

$$\geq Q(A_E,T^O(\tilde{A}_E)) - W^O(\tilde{A}_E) \quad \text{for all } A_E,\ \tilde{A}_E \text{ pairs}$$

Equation (7) can be thought of as being intermediate between (4) and (5). It is similar to (4) in the sense that information produced is not publicly observable, and therefore there is an incentive compatibility constraint concerning the realization of A_E. As with (4), the constraint simply states that the firm has an incentive to behave in accordance with what is specified in the contract. On the other hand, (7) is similar to (5) in terms of the constraint on the old worker wage function. That is, firms other than the first period employer get some information about the ability of workers, and this is reflected in the constraint on the old worker wage function. As indicated earlier, the difference between (5) and (7) as regards this constraint is that in (5) other firms got to observe the actual realization of A_E, while here they only get to observe the worker's task assignment.

In proceeding there are two different assumptions one could make concerning how firms behave. Let $\bar{W}^U$ be the wage firms other than the first period employer offer to old workers assigned to the unskilled task, and let $\bar{W}^S$ be the wage firms other than the first period employer offer to old workers assigned to the skilled task. The first assumption is that in offering long-term contracts firms behave strategically. That is, in specifying the terms in the contract, firms take into account how $\bar{W}^U$ and $\bar{W}^S$ will be affected. Deriving the market contract under this assumption corresponds to solving (7) in the exact manner in which it is written.

It might, however, be argued that firms do not behave with such a high degree of sophistication. That is, a firm is likely to take into account that the wage offers a worker receives from other firms will depend on the worker's task assignment, but it is unlikely to take into account how $\bar{W}^U$ and $\bar{W}^S$ themselves can be manipulated by the firm's own actions. This assumption will be referred to as the firms behaving non-strategically. Deriving the market contract under this assumption corresponds to solving (7) with the following two changes. First, the second constraint must be rewritten in the following form, i.e.,

$$(8) \qquad W^0(A_E) > \begin{cases} \bar{W}^U & \text{if } T^0(A_E) = U \\ \bar{W}^S & \text{if } T^0(A_E) = S, \end{cases}$$

where $\bar{W}^U = \max\{x, \hat{A}_E(U,z,T^0(.))\}$ and $\bar{W}^S = \max\{x, \hat{A}_E(S,z,T^0(.))\}$. Second, in solving (7) one should take $\bar{W}^U$ and $\bar{W}^S$ as fixed, rather than as functions of z and $T^0(.)$.

I will consider the non-strategic case first. As was true when information produced was publicly observable, it is necessary to introduce an additional assumption.

Assumption 2: For every z', z'' pair, $z' > z''$, and $\tilde{A}$, $x < \tilde{A} < A^H$, $F(\tilde{A},z') < F(\tilde{A},z'')$.

Assumption 2 states that for every ability level greater than x, an increase in the investment will not decrease the probability that the realization of A_E is greater than or equal to that level.[18] The role of this assumption will be made clear further on. We can now proceed to the proposition.[19]

Proposition 3: Suppose information produced is not publicly observable, but task assignments are. If firms behave non-strategically, then

i) $W^Y < x - z$

ii) $W^0(A_E) = \bar{W}^S$ if $T^0(A_E) = S$

iii) $W^0(A_E) = W^{0,U} < \bar{W}^S$ if $T^0(A_E) = U$

iv) $z < \hat{z}(T^0(.))$ if $\hat{z}(T^0(.)) > 0$ and Assumption 2 holds

v) $T^0(A_E) = S(U)$ if $A_E > (\leq)\ x + \bar{W}^S - W^{0,U}$.

Proposition 3 tells us the following about what happens when only task assignments are publicly observable, and firms behave non-strategically. First, for old workers there is a single wage associated with the skilled task and a single wage associated with the unskilled task, where the skilled wage exceeds the unskilled wage. Second, as found in my previous paper, there is an inefficient assignment of old workers to tasks. That is, because of the higher wage associated with the skilled task, too few workers are assigned to that task. Third, similar to the case when information produced was publicly observable, there is a tendency for the firm to underinvest in information production.

The obvious question which arises is what role does Assumption 2 play. The basic intuition behind the underinvestment result is that as the investment increases the workers' first period wage decreases, which given the workers' limited access to capital markets is very costly. Given this, suppose one started with a contract where $z > \hat{z}(T^O(\cdot))$. One might think this contract could be ruled out by comparing it with a contract where $z = \hat{z}(T^O(\cdot)) - \varepsilon$. This, however is not always the case. It is possible for such a decrease in the investment to cause an increase in the probabiity of a worker being assigned to the skilled task, and hence an increase in the probability that in the second period the worker receives the higher skilled wage. In turn, this means that such a decrease in the investment could actually result in a decrease in W^Y, rather than the expected increase. If this were the case, then it is clear that the actual investment level could exceed the second best investment level. Assumption 2 rules out this possibility. It guarantees that a decrease in z necessarily causes a decrease in the probability of being assigned to the skilled task, and hence an increase in W^Y.

The next step of the analysis is to consider the strategic case. This case is much more difficult to analyze, and I have been unable to derive any general results. I have, however, worked out a specific example.[20] Let $p = 1/2$ and let $g(\cdot,\cdot,\cdot)$ have the following specific form.

$$g(\theta, A^H(A^L), z) = \begin{cases} r+(-)q(z)(\theta-\hat{\theta}) & \text{for all } \theta \in [\underline{\theta}, \bar{\theta}] \\ 0 & \text{otherwise,} \end{cases} \tag{9}$$

where $r = \frac{1}{\bar{\theta}-\underline{\theta}}$ and $\hat{\theta} = \frac{\bar{\theta}+\underline{\theta}}{2}$. Additionally, $q(0) > 0$, $q' > 0$, and $r+q(x)\ (\underline{\theta}-\hat{\theta}) > 0$.

Equation (9) states that the indicator of ability is a value between $\underline{\theta}$ and $\bar{\theta}$, where the density function is linear and is such that higher values for θ are more likely when $A_i = A^H$, while lower values are more likely when $A_i = A^L$. The specific form of the density function employed guarantees that the information accumulation process has the following desirable property. At any investment level, higher values for the indicator translate into higher values for expected ability. Note further, the last assumption on $q(\cdot)$ guarantees that the specified form for $g(\cdot,\cdot,\cdot)$ is feasible in the sense that for all relevant investment levels $g(\cdot,\cdot,\cdot)$ is never negative.

Analysis of this example yields that all the properties listed in Proposition 3 continue to be satisfied. This suggests that the underinvestment result may frequently hold even in the strategic case, but the evidence is far from conclusive.

IV. Conclusion

A number of recent papers have been concerned with the asssignment of workers to tasks, where information concerning the optimal assignment is either costly or incomplete. This paper looks at a model where this type of information is accumulated within the firm, and where the firm has control over the amount of information produced. The question addressed is, does the firm have an incentive to make a socially efficient investment in information production? The answer is that it depends on how much other firms can either observe or infer concerning the information produced. If information produced is not observed by other firms and individual task assignments are also not publicly observed, then the firm has an incentive to invest optimally in information production. If, however, either the information produced or the task assignment is public knowledge, then there is a tendency for the firm to underinvest.

One obvious direction in which the analysis in this paper could be extended would be to further analyze the case where only task assignments are publicly observable, and firms behave strategically. The question of interest is, how robust is the underinvestment result in this case?

A second direction in which the analysis could be extended concerns the idea of turnover. That is, the present paper has been modeled such that no actual turnover ever occurs, and it might be interesting to see what additional results follow when turnover is allowed. My conjecture is that, once turnover is allowed, there will then be a trade-off between the investment in information production and the utilization of the information produced. The argument follows. Consider the case where information produced is publicly observable and turnover actually occurs. Consistent with what was found in this paper, that case will likely result in an underinvestment in information production. The reason being that the worker will likely continue to finance the investment. On the other hand, there should be no underutilization of the information actually produced. That is, when workers actually switch employers the new employers will have access to the information previously produced, and there will thus be no efficiency loss due to a mismatching of these workers to tasks. Now consider the case where information produced is private and turnover actually occurs. That case will likely result in less of an underinvestment in information production than the public information case. The reason being that the firm should be willing to finance at least part of the investment (see footnote 15). Hence, consistent with the present paper, there is a suggestion that social welfare increases as one moves from the public information case to the private information case. As opposed to the present paper, however, there is now an additional consideration. Under private information there will be an underutilization of the information actually produced. When workers now switch employers the new employers won't have access to information previously produced, and thus these workers will not be matched to tasks as efficiently as if that information were available. Hence, as one moves from public information to private information in a world with turnover, any increase in social welfare due to a lessening of the under-investment in information production will likely be at least partially offset by a decrease in social welfare due to a worse utilization of the information actually produced.

Appendix

Proof of Proposition 1: The proof is fairly straightforward, so I will just outline it here. Consider equation (4), but with only the first constraint. It is clear that this constraint must hold as an equality. Further, by holding z and $T^0(.)$ fixed, it is also clear that risk aversion implies $W^Y = W^0(A_E)$ for all A_E. Call this wage $\hat{W}$. One question which remains is what is z and what is $T^0(.)$. The answer is that they will be chosen to maximize $\hat{W}$. We know that given z fixed, $\hat{W}$ will be maximized by setting $T^0(A_E) = S(U)$ when $A_E >(<) x$. In addition, equation (3) yields that given this form for $T^0(.)$, z^* will be the value for z which maximizes $\hat{W}$.

To complete the proof it is only necessary to show that this solution satisfies the additional constraints in (4). If $z = 0$, then the above solution yields $W^0(A_E) = x$ for all A_E. Further, given the logic above, when $z = z^*$ it must be that $W^0(A_E) > x$ for all A_E. Thus, the second constraint is satisfied. Now consider the incentive compatibility constraint. Because $W^0(.)$ is a constant, the firm cannot affect its second period wage bill. Therefore it will always want to set $T^0(.)$ such that expected net output is maximized, which is exactly how $T^0(.)$ was constructed above. Thus, the incentive compatibility constraint is also satisfied. Q.E.D.

Proof of Proposition 2: It is clear the first constraint must hold as an equality. Further, holding z fixed, $T^0(A_E) = S(U)$ when $A_E >(<) x$ maximizes the left hand side of the first constraint and therefore must be part of the solution to (5).

Again holding z fixed, suppose there is an interval in a relevant range for A_E, denoted $[\underline{A}_E,\bar{A}_E]$, and a value δ, $\delta > 0$, such that $W^0(A_E) > \max\{x,A_E\} + \delta$ for all A_E in the interval. Further, let β be defined by the following.

(A1) $$\beta = \int_{\underline{A}_E}^{\bar{A}_E} (W^O(A_E) - \max\{x,A_E\})f(A_E,z)dA_E$$

Now consider the alternative contract where z and $T^O(.)$ are as before, but wages equal $\hat{W}^Y$ and $\hat{W}^O(.)$. Let $\hat{W}^Y = W^Y + \beta$ and $\hat{W}^O(A_E) = \max\{x,A_E\}$ for all A_E in the interval $[\underline{A}_E,\bar{A}_E]$, while $\hat{W}^O(A_E) = W^O(A_E)$ elsewhere. The constraints will still be satisfied. Let λ denote the change in the value of the objective function. λ is given by (A2).

(A2) $$\lambda = V(\hat{W}^Y) - V(W^Y) - \int_{\underline{A}_E}^{\bar{A}_E} [V(W^O(A_E)) - V(\max\{x,A_E\})]f(A_E,z)dA_E$$

Because $V'' < 0$ and $\hat{W}^Y < x - z$, $V(\hat{W}^Y) - V(W^Y) > \beta V'(x-z)$. Utilizing this and the fact that $\max\{x,A_E\} > x$ for all A_E yields

(A3) $$\lambda > \beta V'(x-z) - \int_{\underline{A}_E}^{\bar{A}_E} V'(x)[W^O(A_E) - \max\{x,A_E\}]f(A_E,z)dA_E,$$

or

(A4) $$\lambda > \beta[V'(x-z) - V'(x)].$$

(A4) yields $\lambda > 0$, which contradicts the idea that the original contract was optimal. Thus, $W^O(A_E) = \max\{x,A_E\}$ for all A_E. Further, plugging this into the first constraint yields $W^Y = x - z$.

It is now only necessary to prove iii). Given the definition of z^*, the following must be true for any $z > z^*$.

(A5) $$z - z^* > \int_x^{A^H} (A_E-x) \; [f(A_E,z) - f(A_E,z^*)]dA_E$$

Let $F(A_E,z) = pF_H(A_E,z) + (1-p)F_L(A_E,z)$ for all A_E,z pairs. Integrating (A5) by parts now yields

(A6) $$z - z^* > \int_x^{A^H} R(A_E)dA_E,$$

where $R(A_E) = F(A_E,z^*) - F(A_E,z)$.

Suppose that the solution to (5) has $z > z^*$, and let us compare that solution to the solution where the investment in information production equals z^*. Let λ again denote the change in the value of the objective function. λ is given by

(A7) $$\lambda = V(x-z^*) - V(x-z) + \int_{A^L}^{A^H} V(\max\{x,A_E\})[f(A_E,z^*) - f(A_E,z)]dA_E,$$

which in turn yields

(A8) $$\lambda > (z-z^*)V'(x-z^*) - \int_x^{A^H} V'(A_E)R(A_E)dA_E.$$

(A6) yields

(A9) $$(z-z^*)V'(x-z^*) > \int_x^{A_1} V'(A_1)R(A_E)dA_E$$
$$+ \int_{A_1}^{A_2} V'(A_1)R(A_E)dA_E + \dots \int_{A_n}^{A^H} V'(A_1)R(A_E)dA_E,$$

where $R(A_E) \geq 0$ for $A_n < A_E < A^H$, $R(A_E) \leq 0$ for $A_{n-1} < A_E < A_n$, etc. (note it is possible that $A_n = x$). Assumption 1 tells us that $\int_{\tilde{A}}^{A^H} R(A_E) > 0$ for all $A^L < \tilde{A} < A^H$ (see Rothschild and Stiglitz 1970 for the definition of a mean preserving spread). Suppose $R(A_E) \leq 0$ for all $x < A_E < A_1$, and thus $R(A_E) \geq 0$ for all $A_1 < A_E < A_2$ (the other case follows similarly). Given what was just stated concerning Assumption 1, this supposition implies that (A9) can be rewritten as

(A10) $$(z-z^*)\ V'(x-z^*) > \int_x^{A_2} V'(A_E)R(A_E)dA_E$$

$$+ \int_{A_2}^{A_3} V'(A_3)R(A_E)dA_E + \ldots \int_{A_n}^{A^H} V'(A_3)R(A_E)dA_E.$$

Continuously repeating this argument, in turn, yields

(A11) $$(z-z^*)\ V'(x-z^*) > \int_x^{A^H} V'(A_E)R(A_E)dA_E.$$

Comparing (A8) and (A11), we now have $\lambda > 0$. This contradicts the idea that the original contract was optimal and thus $z < z^*$.

Now we need only rule out $z = z^*$. Let 0 denote the value of the objective function. Given i), ii), and iv), $\frac{d0}{dz}$ can be written as

(A12) $$\frac{d0}{dz} = -V'(x-z) + \int_x^{A^H} V'(A_E)\,\frac{dF(A_E,z)}{dz}\,dA_E.$$

Given iv), we know the left hand side of the first constraint is maximized at $z = z^*$, which yields

(A13) $$-1 + \int_x^{A^H} \frac{dF(A_E,z^*)}{dz}\,dA_E < 0.$$

Using an argument similar to above, this, in turn, yields

(A14) $$-V'(x-z^*) + \int_x^{A^H} V'(A_E)\,\frac{dF(A_E,z^*)}{dz} < 0.$$

A comparison of (A12) and (A14) yields $\left.\frac{d0}{dz}\right|_{z=z^*} < 0$. Hence $z < z^*$. Q.E.D.

<u>Proof of Proposition 3</u>: Suppose there are two values for A_E, denoted A_E' and A_E'', such that $T^0(A_E') = T^0(A_E'')$ and $W^0(A_E') > W^0(A_E'')$. This violates the incentive compatibility constraint because $Q(A_E',T^0(A_E')) - W^0(A_E') <$

$Q(A_E', T^O(A_E'')) - W^O(A_E'')$. Thus, there must be a single wage associated with all values for A_E for which $T^O(A_E) = S$, and a single wage associated with all values for which $T^O(A_E) = U$. Call the first wage $W^{O,S}$ and the second wage $W^{O,U}$. Further, the above combined with the incentive compatibility constraint immediately yields $T^O(A_E) = S(U)$ if $A_E >(\leq)\ x + W^{O,S} - W^{O,U}$.

The second constraint guarantees that the expected second period wage is greater than or equal to the expected second period output. Thus, $W^Y < x - z$ follows from the zero expected profit constraint. This proves i).

Suppose $W^{O,S} > \bar{W}^S$ and $W^{O,U} > x$. Consider an alternative contract where z and $T^O(.)$ are as previously and wages equal $\hat{W}^Y$, $\hat{W}^{O,S}$, $\hat{W}^{O,U}$, where $\hat{W}^{O,S} = W^{O,S} - y$, $\hat{W}^{O,U} = W^{O,U} - y$, and $\hat{W}^Y$ is such that the first constraint holds as an equality. This alternative contract will satisfy all the constraints for small values of y. Now take the derivative of the objective function with respect to y (evaluated at $y = 0$), i.e.,

$$\text{(A15)} \qquad \left.\frac{dO}{dy}\right|_{y=0} = V'(W^Y) - F(\hat{A},z)V'(W^{O,U}) - (1-F(\hat{A},z))V'(W^{O,S}),$$

where $\hat{A} = x + W^{O,S} - W^{O,U}$. Given $W^Y < W^{O,U}$ and $W^Y < W^{O,S}$, (A15) yields $\left.\frac{dO}{dy}\right|_{y=0} > 0$. Therefore, y can be varied in a positive direction from $y = 0$, and the value of the objective function will rise while all the constraints will continue to be satisfied, i.e., the initial contract could not have been optimal.

Now suppose $W^{O,S} > \bar{W}^S$ and $W^{O,U} = x$. The incentive compatibility constraint now yields that no one is assigned to task S. This makes ii) irrelevant for this case, and means that we are free to set $\bar{W}^S$ equal to anything we like. v) is valid if for this case we let $\bar{W}^S = A^H$.

The above two arguments prove ii) and v). Now suppose $W^{O,U} > W^{O,S} = \bar{W}^S$. Consider an alternative contract where z is as previously and wages equal $\hat{W}^Y$, $\hat{W}^{O,U}$, $\hat{W}^{O,S}$, where $\hat{W}^{O,U} = \hat{W}^{O,S} = W^{O,S}$ and $\hat{W}^Y$ is such that the first constraint holds as an equality. Further, let old workers be assigned according to $\hat{T}^O(.)$, where $\hat{T}^O(.)$ is such that the third constraint continues to be satisfied. This alternative contract will satisfy all the constraints. Let λ denote the change in the value of the objective function in moving from the initial contract to this alternative contract, i.e.,

$$\begin{aligned}\text{(A16)}\qquad \lambda = {} & V(\hat{W}^Y) - V(W^Y) + V(W^{O,S}) \\ & - F(\hat{A},z)\ V(W^{O,U}) - (1-F(\hat{A},z))\ V(W^{O,S}),\end{aligned}$$

or

$$\text{(A17)}\qquad \lambda > (\hat{W}^Y - W^Y)\ V'(\hat{W}^Y) - F(\hat{A},z)(W^{O,U} - W^{O,S})\ V'(W^{O,S}).$$

Because $\hat{T}^O(.)$ is a more efficient assignment of workers than $T^O(.)$, it must be the case that $\hat{W}^Y - W^Y > F(\hat{A},z)(W^{O,U} - W^{O,S})$. Combining this with the fact that $\hat{W}^Y < W^{O,S}$ yields $\lambda > 0$. Hence, the initial contract could not have been optimal.

Suppose $W^{O,U} = W^{O,S} = \bar{W}^S$. Consider an alternative contract where z is as previously and wages equal $\hat{W}^Y$, $\hat{W}^{O,U}$, $\hat{W}^{O,S}$, where $\hat{W}^{O,S} = W^{O,S}$, $\hat{W}^{O,U} = W^{O,U} - y$ and $\hat{W}^Y$ is always such that the first constraint holds as an equality. Further, let old workers be assigned according to $\hat{T}^O(.)$, where $\hat{T}^O(.)$ is such that the third constraint continues to be satisfied. This alternative contract will satisfy all the constraints for small values of y. Now take the derivative of the objective function with respect to y (evaluated at $y = 0$), i.e.,

$$\text{(A18)}\qquad \left.\frac{dO}{dy}\right|_{y=0} = F(\hat{A})V'(W^Y) - F(\hat{A})V'(W^{O,U}).$$

Because $W^Y < W^{O,U}$, (A18) yields $\lambda > 0$. Hence, the initial contract could not have been optimal. This proves iii).

Given i), ii), iii), and v), suppose $z > \hat{z}(T^O(.)) > 0$. Consider an alternative contract where $T^O(.)$, $W^{O,U}$ and $W^{O,S}$ are as previously, while the investment in information production equals $\hat{z}$ and the first period wage equals $\hat{W}^Y$. Let $\hat{z} = \hat{z}(T^O(.))$ and $\hat{W}^Y$ be such that the first constraint holds as an equality. It is clear all the constraints will continue to be satisfied. Let λ again denote the change in the value of the objective function in moving from the initial contract to the alternative contract, i.e.,

$$\lambda = V(\hat{W}^Y) - V(W^Y) - [F(\tilde{A},\hat{z}) - F(\tilde{A},z)][V(W^{O,S}) - V(W^{O,U})]. \tag{A19}$$

Given Assumption 2, (A19) yields

$$\lambda > (\hat{W}^Y - W^Y)V'(\hat{W}^Y) - [F(\tilde{A},\hat{z}) - F(\tilde{A},z)](W^{O,S} - W^{O,U})V'(W^{O,U}). \tag{A20}$$

Given the definition of $\hat{z}$, it must be true that $\hat{W}^Y - W^Y >$ $[F(\tilde{A},\hat{z}) - F(\tilde{A},z)](W^{O,S} - W^{O,U})$. Hence, (A20) implies $\lambda > 0$, which means that the initial contract could not have been optimal.

Now suppose $z = \hat{z}(T^O(.)) > 0$. Consider an alternative contract where $T^O(.)$, $W^{O,U}$ and $W^{O,S}$ are as previously, while the investment in information production equals $\hat{W}^Y$. Let $\hat{z} = z - y$ and $\hat{W}^Y$ be such that the first constraint always holds as an equality. This alternative contract will satisfy all the constraints for small values of y. Now take the derivative of the objective function with respect to y (evaluated at $y = 0$), i.e.,

$$\left.\frac{dO}{dy}\right|_{y=0} = V'(W^Y)\left.\frac{d\hat{W}^Y}{dy}\right|_{y=0} - [V(W^{O,S}) - V(W^{O,U})]\left.\frac{dF(\tilde{A},\hat{z})}{dy}\right|_{y=0}. \tag{A21}$$

Given Assumption 2, (A21) yields

(A22) $$\left.\frac{dO}{dy}\right|_{y=0} > V'(W^Y)\left.\frac{d\hat{W}^Y}{dy}\right|_{y=0} - (W^{O,S}-W^{O,U})V'(W^{O,U})\left.\frac{dF(\hat{A},\hat{z})}{dy}\right|_{y=0}.$$

Given the definition of $\hat{z}(T^O(\cdot))$, it must be the case that $\left.\frac{d\hat{W}^Y}{dy}\right|_{y=0} = (W^{O,S}-W^{O,U})\left.\frac{dF(\hat{A},\hat{z})}{dy}\right|_{y=0}$. Hence, (A22) implies $\left.\frac{dO}{dy}\right|_{y=0} > 0$, which means that the initial contract could not have been optimal. Q.E.D.

Analysis of example on p. 19: There are a number of things which follow from the same arguments as in the proof of Proposition 3. First, there must be a single wage associated with all values for A_E for which $T^O(A_E) = S$, denoted $W^{O,S}$, and a single wage associated with all values for which $T^O(A_E) = U$, denoted $W^{O,U}$. Second, $T^O(A_E) = S(U)$ if $A_E >(\leq)\ x + W^{O,S} - W^{O,U}$. Third, $W^Y < x - z$ (this proves i)). Fourth, there is a contradiction if $W^{O,S} > \bar{W}^S$ and $W^{O,U} > x$. Fifth, if $W^{O,S} > \bar{W}^S$ and $W^{O,U} = x$, then ii) is irrelevant and v) is valid. The fourth and fifth points together prove ii) and v).

For this example any z will result in $f(A_E,z)$ being a uniform density function over an interval $[\underline{A},\bar{A}]$, where $\bar{A}$ is an increasing function of z and $\frac{\bar{A}+\underline{A}}{2}$ is independent of z and less than x. Further, this implies that for this example, $\bar{W}^S = \frac{x + W^{O,S} - W^{O,U} + \bar{A}}{2}$. Now suppose $W^{O,U} > W^{O,S} = \bar{W}^S$. Consider an altrnative contract where z is as previously and wages equal $\hat{W}^Y$, $\hat{W}^{O,U}$, $\hat{W}^{O,S}$, where $\hat{W}^{O,U} = W^{O,S}$, $\hat{W}^{O,S} = W^{O,U}$ and $\hat{W}^Y$ is such that the first constraint holds as an equality. Further, let old workers be assigned according to $\hat{T}^O(\cdot)$, where $\hat{T}^O(\cdot)$ is such that the third constraint continues to be satisfied. Given the equation for $\bar{W}^S$ for this example, all the constraints will continue to be satisfied. Let λ again denote the change in the value of the objective function in moving from the initial contract to the alternative contract, i.e.,

(A23) $$\lambda = V(\hat{W}^Y) - V(W^Y) + \beta(\tilde{A}'-\underline{A})V(W^{O,S}) - \beta(\tilde{A}-\underline{A})V(W^{O,U})$$
$$+ \beta(\bar{A}-\tilde{A}')V(W^{O,U}) - \beta(\bar{A}-\tilde{A})V(W^{O,S}),$$

where $\beta = \frac{1}{\bar{A}-\underline{A}}$ and $\tilde{A}' = x + W^{O,U} - W^{O,S}$. (A23) reduces to

(A24) $$\lambda > (\hat{W}^Y-W^Y)V'(\hat{W}^Y) - [2x - (\bar{A}+\underline{A})](W^{O,U}-W^{O,S})V'(W^{O,S}).$$

However, because expected second period production under the two contracts is exactly the same, it must be the case that $\hat{W}^Y - W^Y = [2x-(\bar{A}+\underline{A})](W^{O,U}-W^{O,S})$. Hence, (A24) yields $\lambda > 0$, which means that the initial contract could not have been optimal.

Suppose $W^{O,U} = W^{O,S} = \bar{W}^S$. Consider an alternative contract where z is as previously and wages equal $\hat{W}^Y$, $\hat{W}^{O,U}$, $\hat{W}^{O,S}$, where $\hat{W}^{O,U} = W^{O,U} - y$, $\hat{W}^{O,S} = W^{O,S} + y$, and $\hat{W}^Y$ is always such that the first constraint holds as an equality. Further, let old workers be assigned according to $\hat{T}^O(.)$, where $\hat{T}^O(.)$ is such that the third constraint continues to be satisfied. This alternative contract will satisfy all the constraints for small values of y. Now take the derivative of the objective function with respect to y (evaluated at $y = 0$), i.e.,

(A25) $$\left.\frac{dO}{dy}\right|_{y=0} = \beta[2x - (\bar{A}+\underline{A})][V'(W^Y) - V'(W^{O,U})].$$

(A25) yields $\left.\frac{dO}{dy}\right|_{y=0} > 0$, and hence the initial contract could not have been optimal. This proves iii).

Given i), ii), iii), and v), iv) follows from the same arguments as in the proof of Proposition 3.

Footnotes

[1]Papers which have been concerned with the match between workers and firms include Johnson (1978), Mortensen (1978), and Jovanovic (1979).

[2]The fact that there may be a divergence between the actual investment and the socially optimal investment does not necessarily imply a role for government intervention. Before coming to that conclusion one would have to consider the policy tools available to the government, and the inefficiencies associated with intervention. See Demsetz (1969) for a further discussion of these issues.

[3]Other related papers include Guasch and Weiss (1982), and Holmstrom and Ricart i Costa (1984). The differences in approach are as follows. Guasch and Weiss allow firms to accumulate information concerning workers, but the information is not used to better assign workers to tasks. Holmstrom and Ricard i Costa also allow firms to accumulate information concerning workers, but it is the workers who control how much information is produced.

[4]Since in the model analyzed it is never optimal for a worker to save, I am in actuality only restricting borrowing. For a related analysis which employs a less restrictive capital market assumption see Haltiwanger and Waldman (1985).

[5]I am assuming here that the information accumulation process results in a probability density function for A_E, rather than a probability mass function. This assumption basically serves to simplify the exposition.

[6]One paper which allows firms other than the first period employer to offer contingent contracts is Ricart i Costa (1984).

[7]For the case where information produced is publicly observable, it is not necessary to specify whether or not task assignments are also publicly observable. That is, the results of the analysis are the same regardless of

the assumption concerning task assignments.

[8]It is being assumed that if the agent making the assignment, i.e., the social planner or the firm, is indifferent concerning whether the worker should be assigned to task S or task U, then the worker is assigned to task U. This assumption is made strictly for expositional convenience.

[9]Equation (3) does not necessarily uniquely define z^*. The text will be written as if z^* were uniquely defined. In the footnotes I will make clear how the analysis changes if z^* can take on more than one value.

[10]One might think the above logic is too simplistic in that firms other than the first period employer might be able to infer some information about a worker's ability by either his decision to switch employers (as in Greenwald 1979), or through the worker's wage rate if wage rates are publicly observable. In the equilibrium which follows, however, there is no turnover, and there is no wage variation among old workers.

[11]In equation (4) I take advantage of the fact that $pA^H f_H(A_E,z) + (1-p)A^L f_L(A_E,z) = A_E f(A_E,z)$ for all A_E, z pairs.

[12]If z^* is not uniquely defined (see footnote 9), then z can equal any of these multiple values for z^*.

[13]If for some realization of the worker's ability the second period wage did not satisfy the restriction, then the worker would be bid away and in terms of worker utility and expected firm profits it would be as if the restriction was satisfied as an equality. Thus, following Harris and Holmstrom I simply assume that the contract always satisfies the restriction.

[14]If z^* is not uniquely defined, then z is less than the lowest of these multiple values for z^*.

[15]The typical argument is that when there is firm specific human capital the investment will be shared by the worker and the firm (see e.g., Becker

1962 and Oi 1962). This isn't the case here because in the model no actual turnover ever occurs. Note, this is also why Proposition 1 is characterized by a flat age earnings profile, rather than one which is upward sloping. For a further discussion of this point see the conclusion.

[16]The above example is not consistent with the specified model in a number of ways. However, the only crucial feature is that the investment $z = 1/8$ is "riskier" than the investment $z = 1/5$, and thus the example is inconsistent with the spirit of Assumption 1.

[17]As in footnote 10, it might again be argued that firms other than the first period employer might be able to infer some information about a worker's ability by either his decision to switch employers, or through the worker's wage rate if wage rates are publicly observable. In the results which follow, however, there is no turnover, and among old workers assigned to the same task there is no wage variation.

[18]Assumption 2 is obviously related to the idea that, as the investment increases, density weight is pushed into the tails of the distribution. The mean preserving spread assumption (Assumption 1) is also related to the idea that, as the investment increases, density weight is pushed into the tails of the distribution. However, the two assumptions are not equivalent.

[19]For the non-strategic case there is a question as to whether or not an equilibrium exists. In Proposition 3 I ignore this issue, and instead derive properties which must characterize an equilibrium if one exists.

[20]The following example satisfies both Assumptions 1 and 2.

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