

Limited Rationality and Synergism:
The Implications for Macroeconomics*

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ABSTRACT

In this paper we consider the implications of heterogeneity in information processing abilities for macroeconomic models that exhibit what we refer to as synergism. Synergism is the same concept that has received much attention in the recent macro literature under the heading of either Keynesian coordination problems, positive trading externalities, or strategic complementarities. The paper is concerned with the following question. When agents vary in terms of their ability to form expectations, is it the agents who form expectations in a "sophisticated" manner who have a disproportionately large effect on macroeconomic equilibrium, or is it the "naive" who are disproportionately important? We find that if macroeconomic interaction exhibits synergism, then it is the naive agents who have a disproportionate impact on equilibrium. Further, we show that this disproportionate impact can be helpful in explaining significant hump shaped persistence in response to one period macroeconomic shocks.

I. Introduction

A critical issue in the modeling of economic behavior is how well can agents process information. The standard approach is to assume that agents are "rational," or equivalently, that agents have unlimited abilities to process information. This has led to a recurring controversy, however, because as has been pointed out by many previous authors, real world agents are obviously limited in these abilities. Recent refinements to the concept of rationality have brought this conflict into even sharper focus. That is, rationality no longer simply implies that behavior is determined by the maximization of a well ordered function. Rather, it now typically implies the expected utility hypothesis of behavior under uncertainty, and the rational expectations hypothesis for the formation of expectations.^{1,2}

As a result of the above described controversy, several alternatives to the rationality assumption have been suggested over time. A prominent example is the concept of satisficing developed by Simon and his followers.³ More recently, however, work in this area has shifted in a new direction. A number of authors have investigated the idea that agents tend to be heterogeneous in terms of information processing abilities (see e.g., Conlisk (1980), Akerlof and Yellen (1985a,b), Haltiwanger and Waldman (1985), and Russell and Thaler (1985)). In particular, these authors have looked at models where one group of agents processes information in a very sophisticated manner, while others are much more limited in their capabilities. In the present paper we draw upon our own earlier work in this area which was concerned with this type of heterogeneity in the context of the formation of expectations, and consider the implications of this approach for macroeconomic behavior.

In our earlier paper we considered a world in which a proportion of the

population satisfies a rational expectations assumption, i.e., the sophisticated agents, while the agents termed "naive" exhibit incorrect expectations. The focus of the analysis was, given a world in which both sophisticated and naive agents are present, under what situations will the sophisticated have a disproportionately large effect on equilibrium, and under what situations will it be the naive who are disproportionately important. What we found to be crucial in answering this question was the manner in which the agents interact. In particular, consider a world which exhibits what we will refer to as synergism. By synergism we mean that the higher is the total number of agents who choose a particular behavior, the higher is the return to agent i choosing that behavior. One of the main conclusions of our analysis was, given a world which exhibits synergism, it is the naive agents who are disproportionately important. That is, the equilibrium more closely resembles what occurs when all agents are naive than would be suggested by the relative number of sophisticated and naive agents in the population. One might conclude, therefore, that since naive agents are disproportionately important in a world which exhibits synergism, in such a world the practice of assuming all agents have rational expectations is not well justified.^{4,5}

The above result should be of special interest to macroeconomists in that synergism is the same concept that has recently received much attention in the macro literature under the heading of either Keynesian coordination problems, positive trading externalities, or strategic complementarities. Examples of papers which deal with what we refer to as synergism include Diamond (1982), Bryant (1983), Heller (1983), Howitt (1985), Cooper (1985), and Cooper and John (1985). These papers introduce synergism in a number of different ways. For example, Diamond introduces synergism by employing a search process in

conjunction with positive trading externalities, while Heller depends on demand linkages in a multisector model. Three basic results have come out of this body of literature. First, the presence of synergism can result in an economy being characterized by multiple equilibria, where it is sometimes possible to pareto rank the various equilibria. Second, from a societal standpoint equilibria tend to be characterized by too low a level of aggregate activity. Third, synergism causes an economy to be characterized by multipliers very similar to those contained in old style Keynesian analysis.

We find the above results very interesting. However, we feel that synergism has macroeconomic implications beyond those identified above. Consider the following.

Aggregate activity is typically characterized by serially correlated movements in output that fit the pattern of what we will refer to as hump shaped persistence.⁶ By hump shaped persistence we mean that output tends to decline sharply in the initial stages of a business cycle slump, and then gradually returns to normal in the latter stages.⁷ As we all know, however, this poses a problem for macroeconomists in that simple rational expectations models do not yield serially correlated movements in output.⁸ The profession has responded to this problem by embedding a variety of imperfections and rigidities into rational expectations models. For example, staggered long-term contracts appear in Taylor (1980), lags in the production process are introduced in Kydland and Prescott (1982), uncertainty with Bayesian learning appears in Frydman and Phelps (1983), while Darby, Haltiwanger and Plant (1985) rely on inventory and search dynamics. We feel the above explanations may very well play a role, however, the possibility that the macro economy exhibits synergism suggests another factor may be important.

Our earlier work tells us that a rational expectations assumption is not well justified in a world which exhibits synergism. Hence, there is a suggestion that persistence may be related to the presence of synergism in combination with the idea that agents tend to vary in terms of their ability to process information. And in fact, as we will demonstrate in this paper, once one introduces both properties into a macroeconomic model, then an alternative explanation for persistence does arise.

Consider a macroeconomic model with synergism, where there are two types of agents. Sophisticated agents have unlimited abilities to process information and thus satisfy a rational expectations assumption, while naive agents are limited in their abilities, and hence satisfy an adaptive expectations assumption. Such a model will in general exhibit three important characteristics. The first is that, because of the presence of at least some agents with adaptive expectations, the economy will be consistent with what we have referred to as hump shaped persistence. That is, even in simple models the response to a one time shock will typically involve a slow movement back to steady state behavior, rather than the immediate return which would occur if all agents were sophisticated. The second characteristic is that the amount of persistence generated will be related to the degree of synergism in the environment. In particular, increasing the degree of synergism increases the amount of persistence.

While the above two results are of interest, by themselves they do not demonstrate the importance of synergism in the explanation of persistence. The first characteristic states that the economy will display the desired type of persistence, but it does not guarantee that the persistence generated will be significant. The second property states that persistence increases as

synergism increases, but similarly it does not rule out the possibility that even high degrees of synergism may yield low levels of persistence. The interesting question therefore is, how much persistence will such an economy display? The surprising answer is that it will display a relatively large amount. That is, the third characteristic states that at least for the first few periods after a shock, the naive agents are disproportionately important, i.e., the deviation from steady state behavior will be more than that suggested by the relative number of sophisticated and naive agents in the population. Or in summary, significant persistence can be explained by the presence of synergism as long as one allows "some" agents in the population to have adaptive expectations.

To understand why naive agents are disproportionately important for at least the first few periods of the adjustment process, consider the following. After a one time shock, the adaptive nature of the expectations of the naive causes the naive to only slowly revert back to steady state behavior. Sophisticated agents, on the other hand, anticipate that naive agents will behave in this way and, because of the synergism, compensate by having their behavior being biased in a manner similar to the bias of the naive. The result is that aggregate private production is farther from steady state behavior than if one simply considered the direct actions of the naive. Or overall, naive agents are disproportionately important for at least the first few periods, because sophisticated agents anticipate the bias of the naive and, due to synergism, compensate in a manner which tends to reinforce this bias.

In this paper we demonstrate the above argument in the context of a number of the synergistic macroeconomic models which have already appeared in the literature. The outline for the paper follows. Sections II and III

present and analyze a dynamic version of one of the models analyzed in Diamond (1982). Section IV considers a variant of the static models contained in Heller (1985) and Cooper (1985). Section V considers a standard IS-LM model, where synergism is incorporated in a manner similar to Howitt (1985). Section VI presents some concluding remarks. Note that, since Sections IV and V consider static models, in those sections it is not possible to explicitly consider the issue of persistence. What we do in those sections instead is consider the static analogue to the persistence question, i.e., how large is the one period effect of a shock. In particular, both Sections IV and V concentrate on the size of the effect of anticipated money shocks.

II. Model 1: Synergism through Trading Externalities

As indicated earlier, the model presented in this section is a variant of one of the models presented in Diamond (see pp. 886-87). Following Diamond we make no distinction between workers and firms. Rather, there is a continuum of agents who in each period must decide whether or not to undertake a production project. If in period t agent i decides to undertake a project, then he produces y units of output at a cost $c_{i,t}$, where $c_{i,t} = \alpha_t b_i$. The heterogeneity in costs across agents captured by the term b_i can be thought of either as heterogeneity in "reservation wages," or just variance in costs across projects. The latter interpretation simply means that, prior to deciding whether or not to produce, each agent i draws a production project from the distribution of projects. The distribution of b_i 's in the population is described by a density function $h(\cdot)$ which is positive over the interval $[0, \infty)$, and equals zero elsewhere. The term α_t is a common cost parameter shared by all agents in the population. Shifts in α_t are simply economy wide

cost shocks, which might, for example, be caused by shifts in input prices.

The key restriction on behavior is that each individual cannot consume what he himself produces, but must rather trade his own output for that which is produced by others. This assumption reflects the advantage that specialized production and trade have over self-sufficiency. Further, storage is not possible, so that if in a particular period an individual is unable to trade his output, then the output is simply wasted.

Let Y_t be period t 's aggregate private production and G_t be government output in period t , where government output is financed by lump sum taxes. The probability of an agent making a trade in period t is denoted p_t , where $p_t = p(Y_t + G_t)$ and $p' > 0$. The assumption $p' > 0$ captures the trading externality or the idea that the world exhibits synergism, i.e., more traders raises the likelihood of any individual trader being able to successfully complete a trade.

Agents are assumed to be risk neutral. Hence, in period t agent i will (will not) undertake a production project if

$$(1) \quad p_{i,t}^E y > (<) c_{i,t},$$

where $p_{i,t}^E$ is agent i 's expectation concerning the probability of successfully completing a trade in period t . There are two types of agents in the population. A proportion q of the population is sophisticated while a proportion $(1-q)$ is naive, where the distribution of sophisticated and naive agents in the population is independent of the distribution of b_i 's in the population. Sophisticated agents have unlimited abilities to form expectations, and thus their expectation formation process satisfies a rational expectations assumption.⁹ On the other hand, naive agents are limited in their ability to form expectations, and they are thus assumed to satisfy a simple adaptive expectations assumption, i.e., for each naive agent i

$$(2) \quad P_{i,t}^E = \delta_1 P_{t-1} + \delta_2 P_{t-2} + \dots + \delta_n P_{t-n},$$

where $\delta_1 > \delta_2 > \dots > \delta_n$ and $\sum_{j=1}^n \delta_j = 1$.¹⁰

Finally, we want to abstract away from the possibility that, because of the synergism present, the model may display multiple steady state equilibria. We thus assume

$$(3) \quad p'(Y_t + G_t) (y^2 / \alpha_t) h(p(Y_t + G_t) y / \alpha_t) < 1 \quad \text{for all } Y_t, \alpha_t > 0.$$

That is, we impose an upper bound on the degree of synergism present which is actually the lower bound necessary for Diamond's multiple equilibria result.

III. Analysis of Model 1

The focus of the analysis concerns how the interaction of synergism and limited rationality can help explain why an economy might exhibit hump shaped persistence. The outline for the section is as follows. First, we consider how, starting from a steady state, the economy adjusts to a one time change in either government output or α given the two polar assumptions concerning the expectations formation process, i.e., all agents are sophisticated ($q=1$) and all agents are naive ($q=0$). Second, we consider the case where the economy consists of a mix of sophisticated and naive agents ($0 < q < 1$), and compare the qualitative nature of the adjustment process in this case to the two polar cases previously considered. Third, we consider the effect of varying the degree of synergism in the heterogeneous case. Fourth, we again compare the heterogeneous case to the two polar cases previously considered, but this time rather than being qualitative in nature, the comparison is in terms of the amount of persistence generated in the three cases.

It should be clear that the steady state value for Y is independent of the value for q . Given this, let $Y_Z(\alpha, G)$ be the steady state value for Y when $\alpha_t = \alpha$ in every period and $G_t = G$ in every period. The manner in which α enters the model guarantees $\partial Y_Z / \partial \alpha < 0$, while the synergistic nature of the model guarantees $\partial Y_Z / \partial G > 0$. Additionally, (3) guarantees $\partial Y_Z / \partial G < 1$.

We will first consider how the economy adjusts to a one time shock given that all agents are naive. Below when we state that in period k the economy experiences a one time shock to either α or G , we formally mean the following. In period $k-1$ the economy is in a steady state where $\alpha = \bar{\alpha}$ and $G = \bar{G}$. Further, in period k there is a change in either α or G , while $\alpha_t = \bar{\alpha}$ and $G_t = \bar{G}$ for all $t > k$. Note, all proofs are relegated to an Appendix.

Proposition 1: Suppose that in period k the economy experiences a one time shock to either α or G . If $q=0$ and it is a cost shock, then i) through iv) are satisfied, where $w \geq k+1$.

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|------|--|--|
| i) | $Y_k > (<) Y_Z(\bar{\alpha}, \bar{G})$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| ii) | $Y_{k+1} < (>) Y_{k+2} < (>) \dots Y_w$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| iii) | $Y_w \geq (\leq) Y_{w+1} > (<) Y_{w+2} > (<) \dots$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| iv) | $\lim_{j \rightarrow \infty} Y_{k+j} = Y_Z(\bar{\alpha}, \bar{G})$ | |

If $q=0$ and it is a government shock, then ii) through iv) are correct while

- v) $Y_k = Y_Z(\bar{\alpha}, \bar{G})$.

Proposition 1 states that, if all agents have adaptive expectations, then the adjustment to a one time shock will display the following characteristics. First, in the period of the shock there will be a change in aggregate private production if it is a cost shock, and no change if it is a government shock. Second, this will be followed by a divergence from the steady state value for

Y, where this divergence may at first follow an increasing trajectory, but it eventually decreases with time and then disappears in the limit. Or overall, if all agents are naive, then the adjustment to a one time shock is consistent with the notion of hump shaped persistence.

We can now consider the adjustment process when all agents are sophisticated. For this case we need to distinguish between two different types of shocks. On the one hand, the shock could be anticipated. This means that sophisticated agents have knowledge of the one time change in either α or G prior to its actual occurrence, and that the sophisticated agents adjust their production decisions accordingly. Formally this translates into the assumption that if an anticipated shock occurs in period k , then $p_{i,k}^E = p(Y_k + G_k)$. On the other hand, the shock could be unanticipated. This means that agents have no knowledge of the one time change prior to its occurrence. The translation here is that if an unanticipated shock occurs in period k , then $p_{i,k}^E$ equals the previous steady state value for p .^{11,12}

Proposition 2: Suppose that in period k the economy experiences a one time shock to either α or G . If $q=1$ and the shock is anticipated, then

- i) $Y_k = Y_Z(\alpha_k, G_k)$
- ii) $Y_t = Y_Z(\bar{\alpha}, \bar{G})$ for all $t > k$.

If $q=1$ and the shock is an unanticipated cost shock, then ii) holds while

- iii) $Y_k > (<) Y_Z(\bar{\alpha}, \bar{G})$ if $\alpha_k < (>) \bar{\alpha}$.

If $q=1$ and the shock is an unanticipated government shock, then

- iv) $Y_t = Y_Z(\bar{\alpha}, \bar{G})$ for all $t \geq k$.

Proposition 2 tells us the following about the adjustment process when all agents are sophisticated. First, if the shock is either an anticipated shock or a cost shock, then there is an immediate adjustment to aggregate private production followed by private production immediately returning to the original steady state position. Second, if the shock is an unanticipated government shock, then there is simply no movement in private production. Or overall, if all agents are sophisticated, then the adjustment to a one time shock is inconsistent with the notion of hump shaped persistence.

The next step is to consider what occurs when there is a mix of sophisticated and naive agents. Notice that in dealing with this case below we retain the distinction concerning whether or not the shock is anticipated. This is because, in describing the adjustment process when both types of agents are present, it is still necessary to know whether the sophisticated agents have prior knowledge concerning the one time change in either α or G .¹³

Proposition 3: Suppose that in period k the economy experiences a one time shock to either α or G . If $0 < q < 1$ and the shock is anticipated, then i) through iv) are satisfied, where $w \geq k+1$.

- | | | |
|------|--|--|
| i) | $Y_k > (<) Y_Z(\bar{\alpha}, \bar{G})$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| ii) | $Y_{k+1} < (>) Y_{k+2} < (>) \dots Y_w$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| iii) | $Y_w \geq (\leq) Y_{w+1} > (<) Y_{w+2} > (<) \dots$ | if $Y_Z(\alpha_k, G_k) > (<) Y_Z(\bar{\alpha}, \bar{G})$ |
| iv) | $\lim_{j \rightarrow \infty} Y_{k+j} = Y_Z(\bar{\alpha}, \bar{G})$ | |

If $0 < q < 1$ and the shock is an unanticipated cost shock, then i) through iv) are still correct. If $0 < q < 1$ and the shock is an unanticipated government shock, then ii) through iv) are still correct while

- v) $Y_k = Y_Z(\bar{\alpha}, \bar{G})$.

Proposition 3 states that, if there is a mix of sophisticated and naive agents, then the qualitative nature of the adjustment process is closer to the pure naive case than to the pure sophisticated case. We can see this clearly by comparing the three propositions. Given a cost shock or an unanticipated government shock, a comparison of Propositions 1 and 3 indicates that the qualitative nature of the adjustment process with a mix of agents is exactly the same as when all agents are naive. On the other hand, with an anticipated government shock the results are only slightly more ambiguous. For the period in which the shock occurs, the adjustment process with a mix of agents more closely resembles what occurs when all agents are sophisticated in that aggregate private production exhibits some movement. However, for all later periods the adjustment process more closely resembles the pure naive case. That is, aggregate private production only slowly returns back to the original steady state as in the pure naive case, rather than the immediate return which characterized the pure sophisticated case.

To this point the reader may not have found the results particularly surprising. The only thing that can cause links between the periods in this model is expectations. Given this, suppose all agents are sophisticated. It is clear that anticipated shocks will simply cause Y_t to track along the path defined by the steady state values, while unanticipated shocks will cause a one period deviation from this path and then an immediate return. On the other hand, suppose there are some naive agents. A shock causes p_t to deviate from its original steady state value. Further, given the adaptive nature of at least some of the agents' expectations, this results in persistence as these expectations only slowly adjust back to the original steady state value.

We now turn to more interesting questions which concern the amount of persistence generated given that both sophisticated and naive agents are present. The first issue we address is, how is the amount of persistence affected by an increase in the degree of synergism? To answer this question we define what will be referred to as an increasing synergistic transformation of $p(\cdot)$. Suppose $\hat{p}(x^*) = p(x^*)$. Then $\hat{p}(\cdot)$ is an increasing synergistic transformation of $p(\cdot)$ if $\hat{p}(x) > (<) p(x)$ whenever $x > (<) x^*$. In other words, an increasing synergistic transformation involves a rotation of $p(\cdot)$ around some fixed point.

Proposition 4: Suppose that in period k the economy experiences a one time shock to either α or G . If $0 < q < 1$, then an increasing synergistic transformation of $p(\cdot)$ which leaves $Y_Z(\bar{\alpha}, \bar{G})$ unchanged will cause $|Y_t - Y_Z(\bar{\alpha}, \bar{G})|$ to increase for every $t > k$. In addition, if the shock is anticipated, then $|Y_k - Y_Z(\bar{\alpha}, \bar{G})|$ also increases.

Proposition 4 states that increasing the degree of synergism increases the persistence generated. That is, for every period after a shock occurs, an increase in synergism causes an increase in the deviation from steady state behavior.

Although an interesting result, Proposition 4 does not rule out the possibility that even high degrees of synergism may yield low levels of persistence. To get more directly at the issue of how much persistence is generated, we will now address the following question. When both types of agents are present in the economy, does one of the groups tend to be disproportionately important? We know that when everyone is sophisticated there is no persistence, while when everyone is naive there is persistence.

Further, simulations of the model suggest that the persistence generated when everyone is naive can be quite significant. Hence, if the answer to the above question is that the naive agents tend to be disproportionately important, then we will have strong evidence that the presence of synergism in combination with limited rationality can be an important factor in the generation of persistence.

We address the above question in two steps. First, we derive an analytic result concerning the first period following the shock. We then present some simulations which provide further information concerning this issue. Note, below Y_t^S (Y_t^N) denotes aggregate private production in period t when all agents are sophisticated (naive), i.e., $q=1$ ($q=0$).

Proposition 5: Suppose that in period k the economy experiences a one time shock to either α or G . If $0 < q < 1$ and $Y_{k+1}^N < (>) Y_{k+1}^S$, then $Y_{k+1} < (>) q Y_{k+1}^S + (1-q) Y_{k+1}^N$. Or equivalently, $|Y_{k+1} - Y_Z(\bar{\alpha}, \bar{G})| > (1-q) |Y_{k+1}^N - Y_Z(\bar{\alpha}, \bar{G})|$.

Proposition 5 tells us that, when both sophisticated and naive agents are present, the naive agents are disproportionately important the first period after the shock occurs. That is, in this first period, the deviation from steady state behavior is larger than would be suggested by the relative number of sophisticated and naive agents in the population.

Analytic characterization of later periods is difficult. Hence, to gain insight into disproportionality more than one period after the shock, we have run simulations of the model under various parameter values. A description of the specification used for our simulations follows. We let $h(\cdot)$ be uniform over the interval $[0,1]$, where $h(b_i) = 200$ if $0 \leq b_i \leq 1$. Further, $y=1$, while the δ 's satisfy a geometrically declining weight distribution which is truncated

after four periods.

The probability of an agent making a trade is assumed to be a linear function, i.e., $p(Y+G) = p_0 + p_1(Y+G)$, where the parameter p_1 represents the degree of synergism. To guarantee uniqueness we only consider values for $p_1 < 1/200$. Remember that our upper bound on the degree of synergism is the lower bound necessary for Diamond's multiple equilibria result.

There are a number of ways in which we try to be roughly consistent with the stylized facts. First, the ratio of government spending to total aggregate activity is kept between 20 and 30 percent. Second, a survey of empirical autoregressive models of price expectations indicates that a weight on the first lagged quarter of between .3 and .6 is reasonable.¹⁴ While our agents are not forming price expectations, these same magnitudes seem sensible in our context as well. Thus, for our simulations we allow δ_1 to take on the values .3 and .6.

We will consider both the impact of a one period unanticipated cost shock in period k , and a one period unanticipated fiscal policy shock in period k .¹⁵ The steady state value for G is set at 40, while the steady state value for the cost parameter α is set at 1. The one period cost shock is captured by allowing α to rise to 1.1 in period k , while $\alpha_t = 1$ for all $t > k$. The one period fiscal policy shock is captured by allowing G to fall to 30 in period k , while $G_t = 40$ for all $t > k$.

Table 1 reports simulation results for the unanticipated cost shock, while Table 2 reports results for the fiscal policy shock. In these tables we list four measures of private sector output: (1) Y^S - the pure sophisticated case; (2) Y^N - the pure naive case; (3) Y - actual private sector output; and (4) Y^C - the convex combination of the two polar cases, i.e., $qY^S + (1-q)Y^N$.

Table 1

THE EFFECT OF A ONE PERIOD UNANTICIPATED COST SHOCK

Parameter	Measure	Period																							
		k-1	k	k+1	k+2	k+3	k+4	k+5	k+6	k+7	k+8	k+9	k+10												
$p^1=1/ZZ5$ $qF.6$ $\delta F.6$	Y^S	100	90.91	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	
	Y^N	100	90.91	95.15	95.38	95.6	95.82	96.17	96.43	96.68	96.91	97.13	97.35	97.57	97.79	98.01	98.23	98.45	98.67	98.89	99.11	99.33	99.55	99.77	99.99
	Y^C	100	90.91	95.84	96.36	96.81	97.21	97.68	98.03	98.33	98.58	98.83	99.08	99.33	99.58	99.83	100.08	100.33	100.58	100.83	101.08	101.33	101.58	101.83	102.08
	Y	100	90.91	98.06	98.15	98.24	98.33	98.47	98.57	98.67	98.77	98.85	98.93	99.01	99.09	99.17	99.25	99.33	99.41	99.49	99.57	99.65	99.73	99.81	99.89
$p^1=1/ZZ5$ $qF.8$ $\delta F.6$	Y^S	100	90.91	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	90.91	95.15	95.38	95.6	95.82	96.17	96.43	96.68	96.91	97.13	97.35	97.57	97.79	98.01	98.23	98.45	98.67	98.89	99.11	99.33	99.55	99.77	99.99
	Y^C	100	90.91	96.64	97.35	97.92	98.36	98.81	99.1	99.32	99.49	99.62	99.71	99.79	99.87	99.95	100.03	100.11	100.19	100.27	100.35	100.43	100.51	100.59	100.67
	Y	100	90.91	99.03	99.08	99.12	99.16	99.23	99.29	99.34	99.38	99.43	99.47	99.52	99.57	99.62	99.67	99.72	99.77	99.82	99.87	99.92	99.97	100.02	100.07
$p^1=1/ZZ5$ $qF.6$ $\delta F.3$	Y^S	100	90.91	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	90.91	96.36	97.02	97.56	98.01	98.48	98.8	99.06	99.26	99.42	99.54	99.68	99.81	99.92	100.01	100.09	100.17	100.25	100.33	100.41	100.49	100.57	100.65
	Y^C	100	90.91	97.57	98.34	98.86	99.22	99.54	99.71	99.81	99.88	99.92	99.95	99.97	99.99	100.01	100.03	100.05	100.07	100.09	100.11	100.13	100.15	100.17	100.19
	Y	100	90.91	98.54	98.81	99.03	99.2	99.39	99.52	99.62	99.7	99.77	99.82	99.87	99.92	99.97	100.02	100.07	100.12	100.17	100.22	100.27	100.32	100.37	100.42
$p^1=1/ZZ5$ $qF.8$ $\delta F.3$	Y^S	100	90.91	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	90.91	97.61	97.25	96.83	96.35	97.29	97.27	97.3	97.42	97.62	97.71	97.79	97.87	97.95	98.03	98.11	98.19	98.27	98.35	98.43	98.51	98.59	98.67
	Y^C	100	90.91	97.96	97.72	97.46	97.17	98.12	98.2	98.31	98.47	98.7	98.81	98.9	99.01	99.11	99.21	99.31	99.41	99.51	99.61	99.71	99.81	99.91	100
	Y	100	90.91	99.05	98.9	98.73	98.54	98.92	98.91	98.92	98.97	99.05	99.08	99.13	99.18	99.23	99.28	99.33	99.38	99.43	99.48	99.53	99.58	99.63	99.68
$p^1=1/ZZ5$ $qF.6$ $\delta F.3$	Y^S	100	90.91	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	90.91	98.21	98.06	97.89	97.7	98.63	98.73	98.86	99.02	99.22	99.32	99.41	99.49	99.57	99.65	99.73	99.81	99.89	99.97	100.05	100.13	100.21	100.29
	Y^C	100	90.91	98.81	98.78	98.76	98.73	99.45	99.54	99.63	99.72	99.82	99.91	100	100	100	100	100	100	100	100	100	100	100	100
	Y	100	90.91	99.28	99.22	99.15	99.08	99.45	99.49	99.54	99.61	99.69	99.75	99.81	99.87	99.93	99.99	100.05	100.11	100.17	100.23	100.29	100.35	100.41	100.47

Table 2

THE EFFECT OF A ONE PERICO UNANTICIPATED FISCAL SHOCK

Parameter	Measure	Period												
		k-1	k	k+1	k+2	k+3	k+4	k+5	k+6	k+7	k+8	k+9	k+10	
$p^j=1/225$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	94.66	94.92	95.17	95.4	95.79	96.08	96.35	96.6	96.84	97.07	97.38
	Y^C	100	100	95.42	95.99	96.49	96.93	97.45	97.83	98.16	98.44	98.68	98.88	99.14
	Y	100	100	97.86	97.97	98.07	98.16	98.31	98.43	98.54	98.64	98.74	98.83	99.01
$p^j=1/225$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	94.66	94.92	95.17	95.4	95.79	96.08	96.35	96.6	96.84	97.07	97.38
	Y^C	100	100	96.3	97.09	97.71	98.2	98.69	99.01	99.26	99.44	99.58	99.68	99.83
	Y	100	100	98.93	98.98	99.03	99.08	99.16	99.22	99.27	99.32	99.37	99.41	99.51
$p^j=1/300$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	95.99	96.72	97.32	97.81	98.33	98.68	98.96	99.18	99.36	99.5	99.65
	Y^C	100	100	97.33	98.17	98.75	99.14	99.49	99.68	99.79	99.87	99.92	99.95	100
	Y	100	100	98.4	98.69	98.95	99.12	99.33	99.47	99.58	99.67	99.74	99.8	99.9
$p^j=1/225$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	97.38	96.98	96.51	95.98	97.02	97	97.03	97.16	97.38	97.48	97.65
	Y^C	100	100	97.75	97.49	97.2	96.88	97.94	98.02	98.14	98.32	98.57	98.7	98.85
	Y	100	100	98.95	98.79	98.61	98.39	98.81	98.8	98.81	98.86	98.95	99.01	99.1
$p^j=1/225$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	97.38	96.98	96.51	95.98	97.02	97	97.03	97.16	97.38	97.48	97.65
	Y^C	100	100	98.18	98.05	97.91	97.76	98.74	98.86	99	99.17	99.36	99.46	99.61
	Y	100	100	99.48	99.4	99.3	99.2	99.4	99.4	99.41	99.43	99.48	99.5	99.6
$p^j=1/300$	Y^S	100	100	100	100	100	100	100	100	100	100	100	100	100
	Y^N	100	100	98.03	97.86	97.68	97.47	98.49	98.6	98.74	98.92	99.14	99.25	99.4
	Y^C	100	100	98.69	98.66	98.63	98.6	99.4	99.49	99.59	99.69	99.8	99.85	100
	Y	100	100	99.21	99.14	99.07	98.99	99.4	99.44	99.5	99.57	99.66	99.7	99.85

Comparing Y and Y^C is obviously the appropriate comparison for evidence concerning disproportionality. In our simulations we allow three parameters to vary. First, as indicated earlier, we allow δ_1 to take on the values .3 and .6. Second, we allow the proportion of sophisticated agents to vary between the values .6 and .8. Third, by varying the parameter p_1 we consider two different degrees of synergism. Note, however, when we vary p_1 we simultaneously vary p_0 such that the steady state value for private sector output always remains at 100. By varying synergism in this way we match exactly the manner in which synergism was varied in Proposition 4.

Some of the results reported in the tables are illustrated in Figures 1-3. Figure 1 depicts the consequences of a cost shock when the degree of synergism is relatively high, 80% of the population is sophisticated and $\delta_1 = .6$. It shows that in this case naive agents are disproportionately important for several periods after the shock occurs, and that the magnitude of this disproportionality can be quite large. Further, Table 1 indicates that this basic description is unchanged either if 60% of the population is sophisticated, or if $\delta_1 = .3$. Taken together these results tell us that synergism can lead to significant "hump shaped" persistence from an absolute standpoint, and a significant disproportionality effect for several periods after a shock occurs. Figure 2 depicts what happens if the degree of synergism is reduced. As should be expected from Proposition 4, decreasing the degree of synergism reduces the quantity of persistence generated. More importantly, however, there also seems to be a reduction in the amount of disproportionality. That is, a decrease in synergism seems to result in the naive agents being disproportionately important for less periods, and for the magnitude of this disproportionality to be smaller. Nevertheless, even with a

FIGURE 1
 $q=.8; p'=1/225; \delta_1=.6$

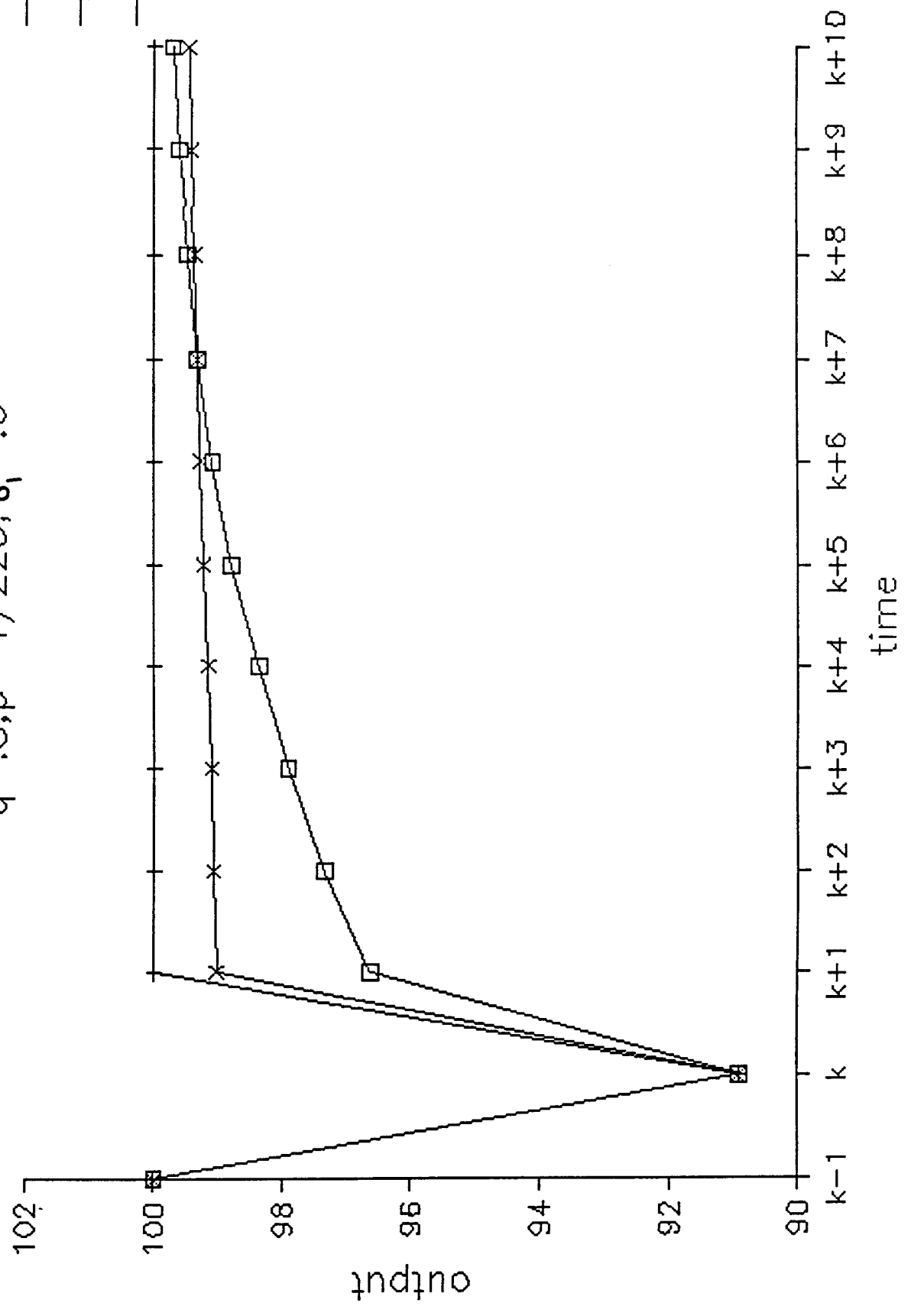


FIGURE 2

$q=.8; p'=1/300; \delta_1=.6$

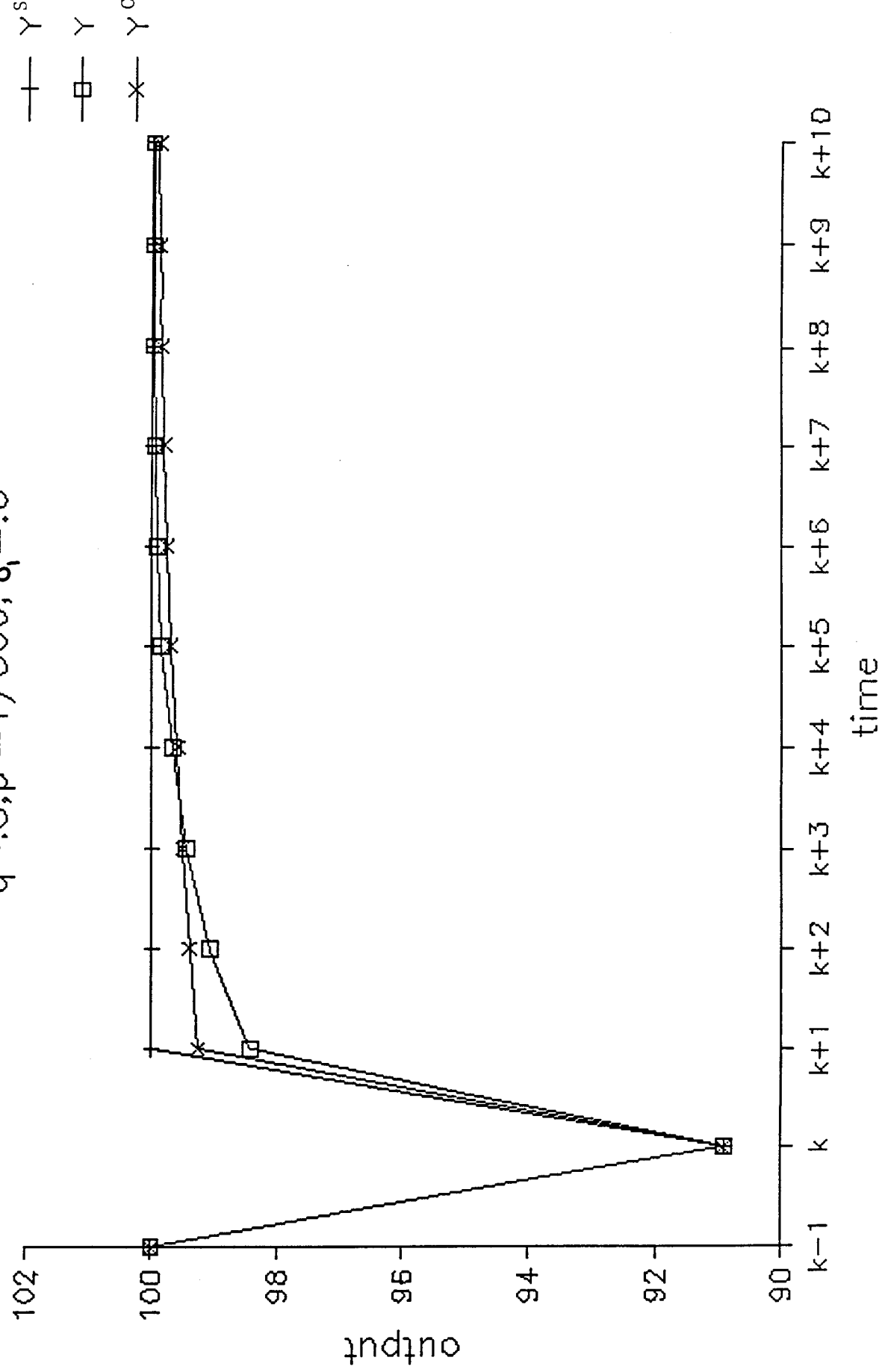
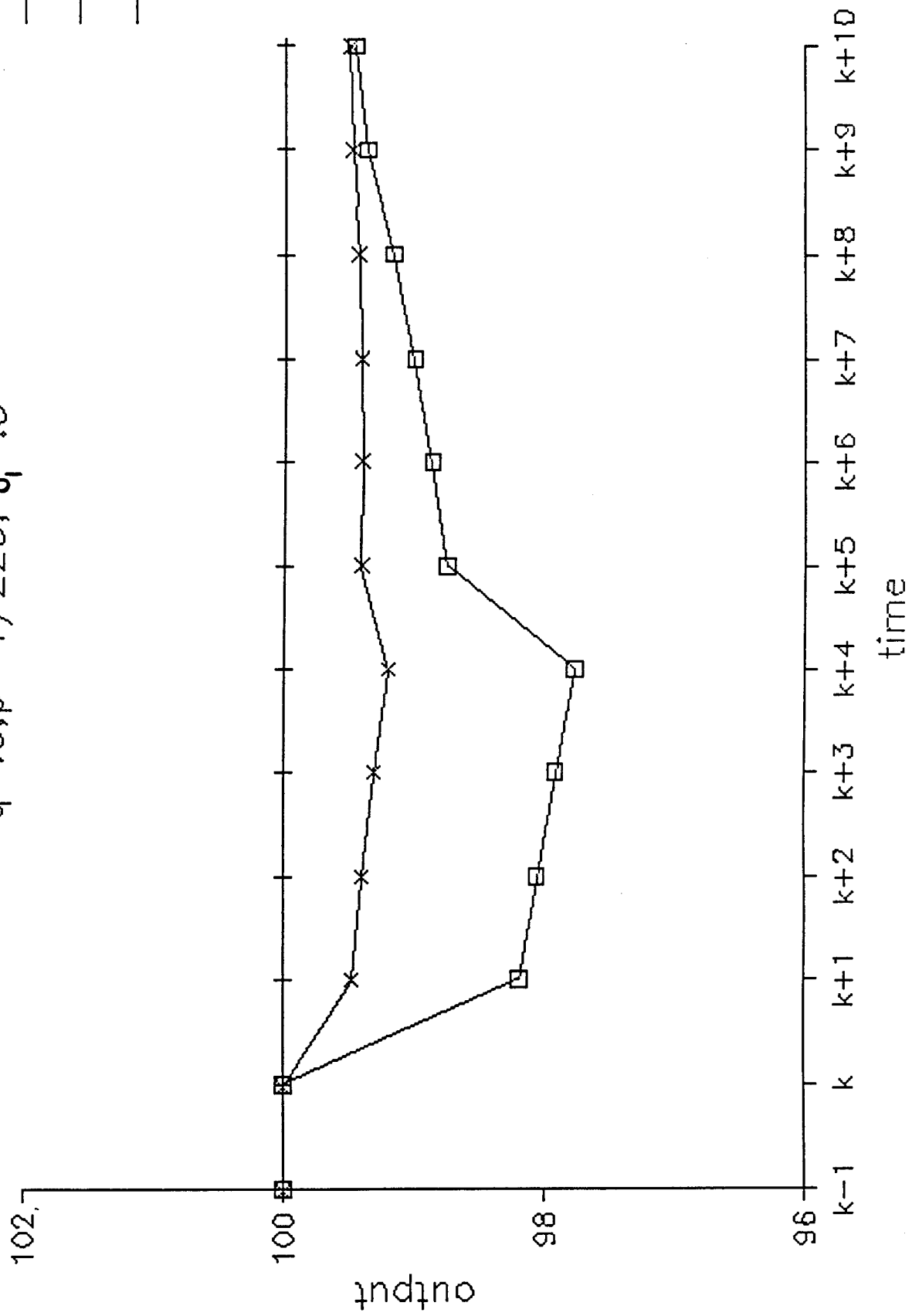


FIGURE 3

$q=.8; p'=1/225; \delta_1=.3$

—+— Y^s
—□— Y
—x— Y^c



relatively small degree of synergism, the naive are disproportionately important for the first few periods which follow the shock.

Figure 3 depicts the consequences of a government shock when the degree of synergism is relatively high, 80% of the population is sophisticated, and $\delta_1 = .3$. In addition to our results concerning disproportionality, we now have that the deviation from steady state behavior initially grows with time, and then after this initial growth the economy slowly reverts back to the steady state. Further, Table 2 indicates that this pattern holds as long as $\delta_1 = .3$. Or overall, in addition to the results concerning disproportionality, we also have that under some specifications this model is consistent with the stylized fact that the peak effect of a shock actually occurs a number of periods after the shock takes place.

We can now consider the intuition for our results concerning disproportionality. After a one time shock, the adaptive nature of the expectations of the naive causes the naive to only slowly revert back to steady state behavior. Sophisticated agents, on the other hand, anticipate this behavior and, because of the synergism, compensate by having their behavior being biased in a manner similar to the bias of the naive. The result is that aggregate private production is farther from steady state behavior than if one simply considered the deviation caused by the direct actions of the naive. This factor is present in all the periods which follow a shock, and it works in the direction of having the naive agents be disproportionately important.

In this dynamic setting, however, not only do the sophisticated react to the behavior of the naive, but the naive react to the behavior of the sophisticated (albeit with a lag). To understand the implications of this

latter factor, begin with a situation where all agents are naive, and consider how naive behavior is changed by the introduction of sophisticated agents. Suppose a shock occurs in period k . In any period t , $t > k$, sophisticated behavior will not be as far from steady state behavior as is the behavior exhibited by the naive. In turn, because the naive have adaptive expectations, in period $t+1$ this will result in a change in the behavior of the naive such that aggregate private production moves toward steady state behavior. Hence, there exists a second factor in the adjustment process, and this factor works in the direction of having the sophisticated agents be disproportionately important.

The second factor grows in importance the further in time one is from the initial shock, while, as we stated earlier, the first factor is present in all the periods which follow a shock. The result is that, as is evidenced by the simulations, the naive agents are disproportionately important at least for the first few periods after the shock occurs, while the sophisticated agents may be disproportionately important in later periods.

The above discussion also allows us to intuitively understand Proposition 4. We know that, after a one time shock, sophisticated agents anticipate the bias of the naive, and then respond in a manner which reinforces this bias. In addition, the higher is the degree of synergism, the greater will be the response. The result is then that the amount of persistence generated in response to a one time shock is positively related to the degree of synergism present in the economy.

A final word concerns the idea that in the above discussion we have stressed the importance of synergism. Given this, one might want to know how the results of the analysis would change if we were to instead assume that the

world exhibits what we have previously referred to as congestion (see footnote 4), i.e., $p' < 0$. We have considered this alternative specification and derived the following. On the one hand, if all agents are sophisticated, then there is no qualitative effect on how the economy adjusts to a one time shock. That is, for anticipated shocks Y_t will still track along the path defined by the steady state values, while unanticipated shocks will still cause a one period deviation from this path and then an immediate return. The other results, however, do not follow through to this case. First, the type of persistence generated when some or all of the agents are naive is much different under congestion than under synergism. For example, consider the special case of our model wherein, when forming expectations, naive agents place all the weight on what occurred in the previous period, i.e., $\delta_1 = 1$. Given the presence of naive agents, a one time shock with congestion will now cause aggregate private production to alternate between being above and then below the original steady state value, rather than the hump shaped persistence generated under synergism. Second, given congestion, a mix of agents will result in the sophisticated agents being disproportionately important in all periods. That is, in response to a one time shock, the deviation from steady state behavior would in each period be smaller than that suggested by the relative number of sophisticated and naive agents in the population.

Or overall, synergism serves three important and complementary roles in this model. First, it causes the model to exhibit the desired type of persistence. Second, it means that after a shock occurs, this persistence will be disproportionately large at least for the first few periods of the adjustment process. Third, from Proposition 4 we also know that an increase in synergism increases the amount of persistence generated.

IV. Model 2: Synergism through Demand Linkages

We now investigate a model wherein synergism arises because of demand linkages across sectors. The specific model considered is a variant of Heller (1984) and Cooper (1985). In our economy there are two consumption goods, two types of firms, and two types of workers. Firms in sector 1 (2) produce only good 1 (2), and use only type 1 (2) labor. On the other hand, type 1 (2) workers and the owners of type 1 (2) firms - hereafter referred to simply as firms - only desire good 2 (1). This specification is intended to capture the idea that agents are specialists in production, but generalists in consumption. Further, similar to Cooper (1985) we introduce a non-produced good into this simple economy. In particular, each agent is endowed with \bar{m} units of money, where money is the numeraire, and real balances enter explicitly into each agent's utility function.¹⁶

In each sector there are F firms. Each firm has a linear production technology, which, for simplicity, converts labor into output on a one-for-one basis. Hence, the profits of firm i in sector j are given by

$$(4) \quad \Pi_{i,j} = (P_j - W_j)y_{i,j},$$

where P_j is the price of sector j output, W_j is the sector j wage, and $y_{i,j}$ is the firm's output. Firm i in sector j spends its total income, $\Pi_{i,j} + \bar{m}$, on the output of the other sector and on real balances. The utility function for firm i in sector j is given by¹⁷

$$(5) \quad U_j((c_{i,-j})^\alpha (m_i/P_{-j})^{1-\alpha}),$$

where U_j is strictly increasing and concave and $0 < \alpha < 1$. In (5) $c_{i,j}$ is the firm's consumption of the other sector's output, while m_i/P_{-j} is the firm's holding of real balances. (5) yields that the firm's demands are given by

$$(6a) \quad c_{i,j} = (\alpha/P_{-j}) (\Pi_{i,j} + \bar{m})$$

and

$$(6b) \quad m_i = (1-\alpha) (\Pi_{i,j} + \bar{m}).$$

In each sector there are also T workers. Every worker is endowed with one unit of leisure time and \bar{m} units of money. Worker t in sector j has a utility function given by

$$(7) \quad V_j((c_{t,-j})^\alpha (m_t/P_{-j})^{1-\alpha} - d_t k_t),$$

where V_j is strictly increasing and concave. $d_t = 1(0)$ if the worker is (is not) employed. Hence, k_t can be loosely interpreted as the worker's disutility for working. Within a sector workers differ only in terms of this disutility. Specifically, in each sector the distribution of k_t 's is described by a density function $h(\cdot)$ which is assumed to be uniform over the interval $[0, K]$.

If a worker in sector j is employed, then he receives the wage W_j . Thus, the worker's total income equals $W_j + \bar{m}$ if he decides to work, while it simply equals \bar{m} if he decides not to. It is assumed that workers make their labor supply decisions before the market price of the other sector's output is revealed. Further, the expectation for P_{-j} held by worker t in sector j is denoted as $P_{t,-j}^E$. It is easily demonstrated that the worker will choose to work if $k_t < \alpha^* (W_j/P_{t,-j}^E)$, where $\alpha^* = 1/(\alpha^\alpha (1-\alpha)^{1-\alpha})$. Additionally, (7) yields that the worker's demands are given by

$$(8a) \quad c_{t,-j} = (\alpha/P_{-j}) (W_j d_t + \bar{m})$$

and

$$(8b) \quad m_t = (1-\alpha) (W_j d_t + \bar{m}).$$

Firms are assumed to behave as Cournot-Nash competitors within each sector, while across sectors both prices and quantities are taken as fixed. That is, firm i in sector j takes as given P_{-j} , Y_{-j} , and $Y_{-i,j}$, where Y_{-j} is the total production in the other sector and $Y_{-i,j}$ is the total production of the other $F-1$ firms in sector j . Hence, the firm faces a residual demand curve given by

$$(9) \quad P_j = R_{-j} / (y_{ij} + Y_{-i,j}),$$

where R_{-j} is the total amount spent on sector j goods by sector $-j$ agents.

Further, the preferences described above yield

$$(10) \quad R_{-j} = (T+F)\alpha \bar{m} + \alpha P_{-j} Y_{-j}.$$

The firm also faces a residual supply curve of labor, which depends upon the ability of workers in forming price expectations. As before, a proportion q of workers is assumed to be sophisticated, while a proportion $(1-q)$ is assumed to be naive. Sophisticated workers have rational expectations, and since we will only deal with anticipated money shocks this implies $P_{t,-j}^E = P_{-j}$. On the other hand, naive workers are assumed to all have the same incorrect expectations given exogenously by $P_{t,-j}^E = \bar{P}$. One interpretation for these exogenous expectations is that naive agents in actuality have adaptive expectations, where because we are dealing with a static framework, this translates into expectations being given exogenously. The above yields that the residual supply curve of labor is given by

$$(11) \quad W_j = (y_{ij} + Y_{-i,j})^\alpha \frac{K}{T} \left(\frac{q}{P_{-j}} + \frac{(1-q)}{\bar{P}} \right).$$

Finally, using (11) we can derive the first order condition for each firm's

choice of output, i.e.,

$$(12) \quad P_j - W_j + y_{ij} (\partial P_j / \partial y_{ij} - \partial W_j / \partial y_{ij}) = 0.$$

The above concludes the setup of the model. In the following we will restrict attention to the symmetric Nash equilibrium of the model. Let Y denote aggregate private production, while Y^S (Y^N) denotes aggregate private production for the special case where all agents are sophisticated (naive), i.e, $q=1$ ($q=0$). Analysis of the above model yields

$$(13a) \quad Y^S = 2(T/\alpha^* K) ((F-1)/(F+1)),$$

$$(13b) \quad Y^N = Y^S (P^N / \bar{P}), \text{ where } P^N = 2\alpha \bar{m} (T+F) / (1-\alpha) Y^N,$$

and

$$(13c) \quad Y = Y^S (q + (1-q)(P/\bar{P})), \text{ where } P = 2\alpha \bar{m} (T+F) / (1-\alpha) Y.$$

(13a) states that if all agents are sophisticated, then production is independent of the nominal price level. Hence, an anticipated change in the nominal money supply will have no effect on aggregate private production. Note, by an anticipated change in the nominal money supply we simply mean one which is known by sophisticated agents prior to firms making production decisions and workers making labor supply decisions. On the other hand, (13b) and (13c) consider the cases where there are at least some naive workers. The conclusion here is that when naive workers overestimate (underestimate) the nominal price level, aggregate private production is less than (greater than) the pure sophisticated case. Hence, even anticipated money is not neutral when $q < 1$.

Summarizing the above, we have that anticipated money is neutral in the pure sophisticated case, and non-neutral when either all agents are naive or

there is a mix of sophisticated and naive agents. In other words, just as in the previous section, qualitatively the equilibrium with a mix of agents more closely resembles the pure naive case than the pure sophisticated case. Similar to what was true earlier, however, the more interesting question is what is the magnitude of the non-neutrality when there is a mix of agents? In particular, is it the sophisticated agents or the naive agents who are disproportionately important? We address this issue in Proposition 6.¹⁸

Proposition 6: Suppose $0 < q < 1$ and initial conditions are such that $Y = Y^S = Y^N$. Call this initial value for Y , Y^0 . An anticipated change in the nominal money supply which results in $Y^N < (>) Y^S$, will also result in $Y < (>) qY^S + (1-q)Y^N$. Or equivalently, the result will be that $|Y - Y^0| > (1-q)|Y^N - Y^0|$.

Proposition 6 tells us that, when both sophisticated and naive agents are present, the naive agents are disproportionately important. That is, an anticipated change in the nominal money supply will result in a greater change in aggregate real activity than would be suggested by the relative number of sophisticated and naive agents in the population. The intuition for Proposition 6 is similar to that given for the previous model. The naive agents are fooled by the money shock, and this results in some real output effects. In turn, sophisticated agents anticipate this behavior and, because of the synergistic nature of the model, respond in a manner which reinforces the behavior of the naive. The result then being that naive agents are disproportionately important.

Notice that if we allowed a more dynamic analysis as in Section III, then a second factor would become important. That is, the presence of sophisticated agents would over time lessen the impact the shock has on the

behavior of the naive agents. Our conjecture is that in such a dynamic setting the outcome of the competing forces would be very similar to the result in the previous section. That is, the naive agents would tend to be disproportionately important in the first few periods which follow a shock, while in later periods the sophisticated agents may be disproportionately important.

Finally, the results of this section suggest a way of distinguishing between our explanation for persistence and other recent explanations (see Section I). A primary empirical puzzle faced by macroeconomists is that, in conflict with the predictions of a simple rational expectations analysis, evidence indicates that even relatively well anticipated changes in the money supply seem to have significant real effects. Most of the other recent explanations for persistence have difficulty explaining this evidence. On the other hand, the analysis in this section suggests that if one moves away from the rational expectations assumption by allowing just some agents in the population to have adaptive expectations, then this empirical evidence is no longer so puzzling. Hence, the fact that even relatively well anticipated changes in the money supply seem to have significant real effects provides support for our explanation of persistence.¹⁹

V. Model 3: Synergism in a Textbook Macroeconomic Model

We now consider the impact of explicitly introducing synergism and heterogeneity in information processing abilities into a standard textbook macroeconomic model. The manner in which we introduce synergism is taken from Howitt (1985).

Consider a typical aggregate macroeconomic model. Following Howitt

(1985) we assume that suppliers of output face transactions costs of selling output which depend on the aggregate level of economic activity. In particular, synergism is captured by assuming that an increase in aggregate activity decreases transactions costs per unit sold. The aggregate production function is given by $F(N,K)$, where N is labor input and K is the capital stock (assumed fixed for the horizon under consideration). Transactions costs are incorporated by assuming that for producers to sell Y they must produce $Y(1+\sigma(Y))$, where synergism implies $\sigma' < 0$.²⁰ With Y the aggregate level of output that is sold, we have $Y(1+\sigma(Y)) = F(N,K)$. Further, suppliers of output have an expectation about the aggregate level of activity, which because of transactions costs influences their individual decisions. Formally, this translates into labor demand for a typical firm being given by

$$(14) \quad n^d = n^d(W/P, Y^E), \quad \partial n^d / \partial (W/P) < 0, \quad \partial n^d / \partial Y^E > 0,$$

where W/P is the real wage and Y^E is the expected level of aggregate activity. The influence of synergism can be seen in (14) by the fact that $\partial n^d / \partial Y^E > 0$. Note that this formulation for labor demand is identical to that given in Howitt.

The rest of the macroeconomic model is similar to a standard textbook model, e.g., Gordon (1984) or Darby and Melvin (1986), with the exception being that we allow for heterogeneity in the expectations formation process. The model is given by

$$(15) \quad A(Y, i, G, t) = Y, \quad 0 < \partial A / \partial Y < 1, \quad \partial A / \partial i < 0, \quad \partial A / \partial G > 0, \quad \partial A / \partial t < 0,$$

$$(16) \quad M/P = L(i, Y), \quad \partial L / \partial i < 0, \quad \partial L / \partial Y > 0,$$

$$(17) \quad N^d = qN^d(W/P, E(Y|I)) + (1-q)N^d(W/P, \bar{Y}), \quad \partial N^d / \partial Y^E > 0, \quad \partial N^d / \partial (W/P) < 0,$$

$$(18) \quad N^S = qN^S(W/E(P|I)) + (1-q)N^S(W/\bar{P}), \quad N^S' > 0,$$

$$(19) \quad N^d = N^S,$$

and

$$(20) \quad Y(1+\sigma(Y)) = F(N, K),$$

where E is the expectation operator, A is the aggregate expenditure function, L is the demand for real balances, N^d is the aggregate demand for labor, N^S is the aggregate supply of labor, i is the interest rate, G is the real level of government spending, t is a parameter embodying taxes, M is the nominal money supply, q is the proportion of both producers and workers that are sophisticated, I is the information set of the sophisticated (which includes all parameters including q), \bar{P} is the expectation of the price level by the naive, and \bar{Y} is the expectation of the output level by the naive.²¹

As indicated earlier, other than the role played by transactions costs, the difference between this model and a standard textbook model is that we allow for heterogeneity in the expectations formation process. In particular, producers form expectations on the aggregate output level while workers form expectations on the aggregate price level, where for each type of agent the sophisticated have rational expectations and the naive have arbitrarily given exogenous expectations. Further, as previously, one interpretation of these exogenous expectations is that the naive agents in actuality have adaptive expectations, which in this static model translates into expectations being given exogenously.

In the model of (15)-(20), it is easy to demonstrate that if $q=1$, then an anticipated change in the nominal money supply will have no real effects. By an anticipated change in the nominal money supply we again simply mean one

which is known by sophisticated agents prior to firms making production decisions and workers making labor supply decisions. In contrast, for $q=0$ an anticipated change in the nominal money supply will have real effects since the expectations of the naive do not take into account current information. Thus far we have characterized what is already well known in the macroeconomics literature. That is, in a flexible price IS-LM type model, anticipated money shocks will have no real effects under rational expectations, while under adaptive expectations such shocks will have real effects. What is of interest is the nature of the effect such a shock will have when agents are heterogeneous in their ability to form expectations. The following propositions address this issue. Note, below Y^S denotes the aggregate output level when $q=1$, Y^N denotes the aggregate output level when $q=0$, and dM denotes anticipated changes in the nominal money supply.

Proposition 7: Suppose $0 < q < 1$ and initial conditions are such that $\bar{P} = E(P|I)$ and $\bar{Y} = E(Y|I)$, which in turn implies $Y = Y^S - Y^N$. Call this initial value for Y , Y^0 , and the initial value for M , M^0 . An increasing synergistic transformation of $\sigma(\cdot)$ which leaves $\sigma(Y^0)$ unchanged will cause $\left. \frac{dY}{dM} \right|_{M=M^0}$ to increase.²²

Proposition 8: Suppose $0 < q < 1$ and initial conditions are such that $\bar{P} = E(P|I)$ and $\bar{Y} = E(Y|I)$, which in turn implies $Y = Y^S - Y^N$. Then $\left. \frac{dY}{dM} \right|_{M=M^0} > q \left. \frac{dY^S}{dM} \right|_{M=M^0} + (1-q) \left. \frac{dY^N}{dM} \right|_{M=M^0}$.

Propositions 7 and 8 tell us the following about the effect of an anticipated money shock when there is a mix of agents. First, an increase in synergism increases the effect of such a shock (Proposition 7). Second, the effect of such a shock is larger than that suggested by the relative number of sophisticated and naive agents in the population, i.e., naive agents are

disproportionately important (Proposition 8). The intuition for these results is similar to that given for the other models in this paper. The money shock causes naive agents to make mistakes, and this results in real changes in aggregate activity. Further, sophisticated agents anticipate this behavior and, because of the synergistic nature of the model, respond in a manner which reinforces the bias of the naive. The result then being that naive agents are disproportionately important. In addition, since sophisticated agents respond more the higher is the degree of synergism, there is a positive relationship between the degree of synergism and the effect of an anticipated money shock.

VI. Conclusion

There is a rapidly growing body of work in the macroeconomics literature which investigates the role of synergism. Our own earlier work, however, suggests that when synergism is present in an environment, results are very sensitive to a relaxation of the rational expectations assumption. Hence, in the present paper we have considered a number of these synergistic macroeconomic models under the more realistic assumption that agents are heterogeneous in terms of their ability to process information.

In our analysis we assumed there were two types of agents. Sophisticated agents have unlimited abilities to process information and thus satisfy a rational expectations assumption, while naive agents are limited in these abilities and hence satisfy an adaptive expectations assumption. What our analysis uncovered was that synergism in combination with this type of heterogeneity provides an alternative explanation for the existence of persistence. That is, models with these two properties will in general exhibit three important characteristics. The first is that, because of the presence

of at least some agents with adaptive expectations, aggregate activity will respond to shocks in a manner consistent with observed hump shaped persistence. Second, the amount of persistence generated will be positively related to the degree of synergism in the environment. Third and most importantly, at least for the first few periods which follow a shock, the naive agents are disproportionately important. That is, the deviation from steady state behavior will be more than that suggested by the relative number of sophisticated and naive agents in the population. Or in summary, significant persistence can be explained by the presence of synergism as long as one allows just "some" agents in the population to have adaptive expectations.

In closing we will briefly consider the relationship between our results on synergism and persistence, and the existing literature's focus on synergism and multiple equilibria (see e.g., Diamond (1982), Heller (1984), and Howitt (1985)). One way to integrate these results may be along the lines of Leijonhufvud's (1981) corridor hypothesis. Leijonhufvud argues that there is a locally unique high output, or full employment equilibrium, towards which the economy moves when it is within the "corridor," while if the economy moves outside the corridor behavior becomes quite erratic. Our results could thus be interpreted as explaining what happens when shocks are relatively small, or in other words, what happens when the economy is within the corridor. The interpretation is that near the full employment equilibrium the degree of synergism is relatively small, and hence locally the full employment equilibrium is unique. This, in turn, implies the following. First, if the economy experiences a small shock, then it will return to the original equilibrium. Second, as stressed in the present paper, this return will be characterized by significant persistence as long as just some agents in the

population have adaptive expectations.

On the other hand, the multiple equilibria results of Diamond, Heller, and Howitt provide one possibility for what occurs when the economy moves outside the corridor. The interpretation here is that substantially below the full employment equilibrium the degree of synergism becomes relatively large, with the result being the existence of additional "low" employment equilibria. Hence, if the economy experiences a relatively large shock which moves it outside the corridor, then the economy won't necessarily move back to its original position.

This discussion suggests that the results concerning multiple equilibria may be better suited to explain cataclysmic economic episodes like the Great Depression. In contrast, our analysis with relatively less synergism may be better suited to explain the type of aggregate output fluctuations which have characterized the post WWII era in the United States. Of course, this discussion is only speculative at the present moment. Nevertheless, taken together, our results and those in the existing literature provide substantial evidence for the importance of synergism in the understanding of macroeconomic behavior.

Appendix

Proof of Proposition 1: Consider first the case of a change in α . Given that expectations for p_k are fixed, this will alter the critical value for b_i in the opposite direction. This proves i). Now consider period $k+1$. Because the realized value for p_k was different than the steady state value and because agents have adaptive expectations, there will again be a change in the critical value for b_i which is in the direction opposite of the change in α . In turn, using the same argument recursively we get that for all future periods the critical value for b_i will be biased in this same manner. Further, equation (3) guarantees that the deviation from steady state behavior may at first be increasing, but eventually it will be decreasing and then it will disappear in the limit. This proves ii) through iv).

For a government shock the argument works the same, except that in period k there is no change in the critical value for b_i , but rather simply the realized value for p_k is different than the steady state value.

Proof of Proposition 2: Given that the only thing linking the periods is expectations, and given our assumption that, for every period after a shock occurs, agents know that α and G have returned to their original steady state values (see footnote 9), it is clear that ii) must hold for all cases. Now suppose that the shock is anticipated. Aggregate private production must immediately adjust to the new steady state value. Hence, i). Suppose the shock is an unanticipated cost shock. Aggregate private production won't immediately adjust to the new steady state value, but because there will be a change in the critical value for b_i , there will be some change in aggregate

private production. This proves iii). Finally, suppose the shock is an unanticipated government shock. There will now be no change in the critical value for b_i . Hence, iv).

Proof of Proposition 3: For either type of unanticipated shock the logic works the same as in the proof of Proposition 1, except now only a subset of agents have adaptive expectations. Note, because $p' > 0$ the sophisticated always have their behavior being biased in a manner similar to the naive, with the result being that the deviation from steady state behavior is always in the same direction as in Proposition 1.

Consider now an anticipated cost shock. The only difference in period k between Propositions 1 and 3 is that the sophisticated agents correctly adjust their expectations for p_k . This, however, only reinforces the previously identified movement of the critical value for b_i , and hence, i) is correct. In turn, the rest of the proposition for this case follows from the proof of Proposition 1. Consider now an anticipated government shock. Qualitatively this works just like an anticipated cost shock, because sophisticated agents again correctly adjust their expectations for p_k , and this causes a movement in the critical value for b_i for these agents.

Proof of Proposition 4: Let \hat{Y}_t be the transformed value for Y_t following the increasing synergistic transformation from p to \hat{p} . Consider first the case where the shock in period k is such that $Y_Z(\alpha_k, G_k) < Y_Z(\bar{\alpha}, \bar{G})$. Our proof is by induction. Suppose that for each period j , $j < t$, $\hat{Y}_j \leq Y_j$. We will now show that for any period t , $t > k$, $\hat{Y}_t < Y_t$. Since the shock occurred in period k , we know $\alpha_t = \bar{\alpha}$ and $G_t = \bar{G}$. Hence, (1) implies

$$(A1) \quad Y_t - q \int_0^{p(Y_t + \bar{G})y/\bar{\alpha}} yh(b_i) db_i + (1-q) \int_0^{p^E_t y/\bar{\alpha}} yh(b_i) db_i$$

and

$$(A2) \quad \hat{Y}_t - q \int_0^{\hat{p}(\hat{Y}_t + \bar{G})y/\bar{\alpha}} yh(b_i) db_i + (1-q) \int_0^{\hat{p}^E_t y/\bar{\alpha}} yh(b_i) db_i.$$

Since $\hat{Y}_j \leq Y_j$ for all $j < t$, (2) and the definition of an increasing synergistic transformation yields $\hat{p}^E_t \leq p^E_t$. Let \tilde{Y}_t be defined by (A3).

$$(A3) \quad \tilde{Y}_t - q \int_0^{\hat{p}(\tilde{Y}_t + \bar{G})y/\bar{\alpha}} yh(b_i) db_i + (1-q) \int_0^{p^E_t y/\bar{\alpha}} yh(b_i) db_i$$

Since $\hat{p}^E_t \leq p^E_t$, we have $\tilde{Y}_t \geq \hat{Y}_t$. Let $F(X)$ and $\bar{F}(X)$ be defined by (A4) and (A5).

$$(A4) \quad F(X) - X - q \int_0^{p(X + \bar{G})y/\bar{\alpha}} yh(b_i) db_i - (1-q) \int_0^{p^E_t y/\bar{\alpha}} yh(b_i) db_i$$

$$(A5) \quad \bar{F}(X) - X - q \int_0^{\hat{p}(X + \bar{G})y/\bar{\alpha}} yh(b_i) db_i - (1-q) \int_0^{\hat{p}^E_t y/\bar{\alpha}} yh(b_i) db_i$$

Equation (3) yields $F' > 0$ and $\bar{F}' > 0$. Given (A1) and (A3), this in turn implies $F(X) \geq 0$ if $X \geq Y_t$ and $\bar{F}(X) \geq 0$ if $X \geq \tilde{Y}_t$. By definition $Y_Z(\bar{\alpha}, \bar{G}) = \hat{Y}_Z(\bar{\alpha}, \bar{G})$. Further, if $p^E_t - \hat{p}^E_t = p(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G})$, then (A1)-(A3) imply $Y_t - \hat{Y}_t - \tilde{Y}_t = Y_Z(\bar{\alpha}, \bar{G})$. Hence, if $p^E_t - \hat{p}^E_t = p(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G})$, (A4) and (A5) imply $F(Y_Z(\bar{\alpha}, \bar{G})) - \bar{F}(Y_Z(\bar{\alpha}, \bar{G})) = 0$. Since $\partial F / \partial p^E_t = \partial \bar{F} / \partial p^E_t$, this implies $F(Y_Z(\bar{\alpha}, \bar{G})) = \bar{F}(Y_Z(\bar{\alpha}, \bar{G}))$ for all values of p^E_t .

Combining this with the definition of an increasing synergistic transformation yields

$$(A6) \quad \bar{F}(X) \leq F(X) \quad \text{if } X \geq Y_Z(\bar{\alpha}, \bar{G}).$$

Now suppose that $\tilde{Y}_t \geq Y_t$. Since $Y_Z(\alpha_k, G_k) < Y_Z(\bar{\alpha}, \bar{G})$, Proposition 3 implies $Y_t < Y_Z(\alpha_k, G_k)$. Given (A6), this implies $\tilde{F}(Y_t) > F(Y_t)$. Since $F(Y_t) = 0$, $\tilde{F}' > 0$ and $\tilde{Y}_t \geq Y_t$, this implies $\tilde{F}(\tilde{Y}_t) > 0$ which is a contradiction. Hence, $\tilde{Y}_t < Y_t$. Combining this with $\tilde{Y}_t \geq \hat{Y}_t$, we now have $\hat{Y}_t < Y_t$. That is, we have established that for a shock in period k such that $Y_Z(\alpha_k, G_k) < Y_Z(\bar{\alpha}, \bar{G})$, if $\hat{Y}_j \leq Y_j$ for $j < t$ (where $t > k$), then $\hat{Y}_t < Y_t$.

Now consider $t = k + 1$. For $j < k$, we have $\hat{Y}_j = Y_j = Y_Z(\bar{\alpha}, \bar{G})$. Suppose the shock is unanticipated. We then have

$$(A7) \quad Y_k = q \int_0^{p(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G})y/\alpha_k} p^{y/\alpha_k}_{yh(b_i)} db_i + (1-q) \int_0^{p^E_k y/\alpha_k} p^E_{yh(b_i)} db_i$$

and

$$(A8) \quad \hat{Y}_k = q \int_0^{\hat{p}(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G})y/\alpha_k} \hat{p}^{y/\alpha_k}_{yh(b_i)} db_i + (1-q) \int_0^{\hat{p}^E_k y/\alpha_k} \hat{p}^E_{yh(b_i)} db_i.$$

Since $Y_Z(\bar{\alpha}, \bar{G}) = \hat{Y}_Z(\bar{\alpha}, \bar{G})$, we know $p(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G}) = \hat{p}(Y_Z(\bar{\alpha}, \bar{G}) + \bar{G})$. Further, since the economy is in a steady state for each period j , $j < k$, it must be that $p^E_k = \hat{p}^E_k$. Hence, for an unanticipated shock, (A7) and (A8) imply $Y_k = \hat{Y}_k$. Using arguments similar to those above, it can also be demonstrated that $\hat{Y}_k < Y_k$ for an anticipated shock. Hence, $\hat{Y}_k \leq Y_k$. Further, combining this with the result contained in the previous paragraph yields $\hat{Y}_t < Y_t$ for all $t > k$. Finally, the argument follows similarly if the shock is such that $Y_Z(\alpha_k, G_k) > Y_Z(\bar{\alpha}, \bar{G})$.

Proof of Proposition 5: Suppose first that $Y_{k+1}^N < Y_{k+1}^S$, and that there exists a \hat{q} , denoted \hat{q} , such that $Y_{k+1} > \hat{q} Y_{k+1}^S + (1 - \hat{q}) Y_{k+1}^N$. Let $p_{k+1}^{EN}(p_{k+1}^E)$ be the naive agents' expectation of the probability of making a trade when $q = 0$ ($q = \hat{q}$).

(1) and (2) imply

$$(A9) \quad Y_{k+1}^S - \int_0^{P(Y_{k+1}^S + \bar{G})} yh(b_i) db_i = Y_Z,$$

$$(A10) \quad Y_{k+1}^N - \int_0^{P_{k+1}^{EN} y/\bar{\alpha}} yh(b_i) db_i,$$

and

$$(A11) \quad Y_{k+1} - q \int_0^{P(Y_{k+1} + \bar{G}) y/\bar{\alpha}} yh(b_i) db_i + (1-q) \int_0^{P_{k+1}^E y/\bar{\alpha}} yh(b_i) db_i,$$

where

$$(A12) \quad P_{k+1}^{EN} - \delta_1 P(Y_k^N + G_k) + (1-\delta_1) Y_Z$$

and

$$(A13) \quad P_{k+1}^E - \delta_1 P(Y_k + G_k) + (1-\delta_1) Y_Z.$$

Propositions 1-3 tell us that, if the shock in period k is an unanticipated shock, then $Y_k^S - Y_k^N = Y_k$. For an anticipated shock the propositions state that, if $Y_k^S < (>) Y_k^N$, then $Y_k < (>) Y_k^N$. Since we are considering the case $Y_{k+1}^N < Y_{k+1}^S$, (A9)-(A13) imply $Y_k^S \leq Y_k^N$, and hence, $Y_k \leq Y_k^N$. (A12) and (A13) now imply $P_{k+1}^E \leq P_{k+1}^{EN}$. Given $P_{k+1}^{EN} \geq P_{k+1}^E$, (A9)-(A11), and $Y_{k+1} > q Y_{k+1}^S + (1-q) Y_{k+1}^N$, the above yields $p(Y_{k+1} + \bar{G}) > p(Y_Z + \bar{G})$. Since $p' > 0$, this in turn implies $Y_{k+1} > Y_Z$. Yet, since $P_{k+1}^E \leq P_{k+1}^{EN} < p(Y_Z + \bar{G})$, (A9), (A11) and (3) yield $Y_Z > Y_{k+1}$. This yields a contradiction. Hence, if $0 < q < 1$ and $Y_{k+1}^N < Y_{k+1}^S$, then $Y_{k+1} < q Y_{k+1}^S + (1-q) Y_{k+1}^N$. The other cases follow similarly.

Proof of Proposition 6: (13a)-(13c) imply

$$(A14) \quad Y - q Y^S - (1-q) Y^N = Y^S (1-q) (2\alpha \bar{m}(T+F) / \bar{P}(1-\alpha)) \left(\frac{1}{Y} - \frac{1}{Y^N} \right).$$

Utilizing (13a)-(13c) again, if $Y^S > Y^N$, then $Y^S > Y > Y^N$, and hence (A14) implies $Y < qY^S + (1-q)Y^N$. Similarly, if $Y^S < Y^N$, then $Y^S < Y < Y^N$, and (A14) now yields $Y > qY^S + (1-q)Y^N$.

Proof of Proposition 7: Since the increasing synergistic transformation of $\sigma(\cdot)$ leaves $\sigma(Y^0)$ unchanged, it is clear that at the initial conditions the transformation has no effect on the equilibrium values for Y , i , N , W and P . Further, since labor demand for profit maximizing firms is governed by $F'/(1+\sigma(Y^E))=W/P$, where $F'=\partial F/\partial N$, we have that $\partial N^d/\partial Y^E = -\sigma'/(1+\sigma(Y))F''$. Hence, at these initial conditions the increasing synergistic transformation causes $|\partial N^d/\partial Y^E|$ to increase. Taking the total differential of the model, applying Cramer's rule and evaluating the derivatives at $M=M^0$ yields

$$(A15) \quad \frac{dY}{dM} = (\partial A/\partial i)F'(1-q)(\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))W/P^3/\Delta,$$

where

$$(A16) \quad \Delta = (1+\sigma(Y)+Y\sigma')(-\partial A/\partial i)(L/P)(\partial N^S/\partial(W/P) - \partial N^d/\partial(W/P)) \\ + F'[(\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))(W/P^3)[(1-\partial A/\partial Y)P\partial L/\partial i + (\partial A/\partial i)P\partial L/\partial Y](1-q) \\ + q(\partial A/\partial i)(\partial N^d/\partial Y^E)(L/P)(\partial N^S/\partial(W/P))].$$

At the initial conditions the only terms in (A15) which change in response to the increasing synergistic transformation are σ' and $\partial N^d/\partial Y^E$. Hence, (A15) and the assumptions on the derivatives imply that the increasing synergistic transformation causes $\frac{dY}{dM}$ to decrease.

Proof of Proposition 8: Taking the total differential of the model, applying Cramer's rule, and evaluating the derivatives at $M=M^0$ yields

$$(A17) \quad q\frac{dY^S}{dM} + (1-q)\frac{dY^N}{dM} = (1-q)(\partial A/\partial i)F'(\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))(W/P^3)/\Delta^N$$

and

$$(A18) \quad \frac{dY}{dM} = (1-q)(\partial A/\partial i)F'(\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))(W/P^3)/\Delta,$$

where

$$(A19) \quad \Delta^N = (1+\sigma(Y)+Y\sigma')(-\partial A/\partial i)L(\partial N^S/\partial(W/P) - \partial N^d/\partial(W/P))/P \\ + F'[(\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))(W/P^3)((1-(\partial A/\partial Y))P\partial L/\partial i + (\partial A/\partial i)P\partial L/\partial Y)]$$

and

$$(A20) \quad \Delta = \Delta^N + qF'[(\partial A/\partial i)(\partial N^d/\partial Y^E)(L/P)\partial N^S/\partial(W/P) \\ - (\partial N^d/\partial(W/P))(\partial N^S/\partial(W/P))(W/P^3)((1-\partial A/\partial Y)P\partial L/\partial i + (\partial A/\partial i)P\partial L/\partial Y)].$$

The assumptions on the derivatives, in particular, $\partial N^d/\partial Y^E > 0$, imply $\Delta < \Delta^N$.

Given this, (A19) and (A20) yield $\frac{dY}{dM} > q\frac{dY^S}{dM} + (1-q)\frac{dY^N}{dM}$.

Footnotes

¹We are not suggesting that agents who have limited capacities to process information are not rational under some broad definition of the term, but rather that they are not rational under the literature's definition of the term "rationality." Note, this limited ability to process information is sometimes referred to as bounded rationality. However, because of its frequent association with the related concept termed satisficing, we will refrain from using the term bounded rationality in this paper.

²Evidence for the idea that real world agents are limited in their abilities to process information abounds in the literature of both cognitive psychologists and experimental economists. See, for example, Kahneman, Slovic, and Tversky (1982).

³References to the work of Simon and his followers include Simon (1959), Cyert and March (1963), Williamson (1975), and Nelson and Winter (1982). See also Radner (1975) for explicit modeling of Simon's ideas.

⁴In our earlier work we also considered what occurs when the environment displays congestion, i.e., the higher is the total number of agents who choose a particular behavior, the lower is the return to agent i choosing that behavior. The result here was that sophisticated agents are disproportionately important. That is, the equilibrium more closely resembles what occurs when all agents are sophisticated than would be suggested by the relative number of sophisticated and naive agents in the population. Hence, the conclusion one might draw here is that, since sophisticated agents are disproportionately important in a world which exhibits congestion, in such a world the practice of assuming all agents have rational expectations is relatively well justified.

⁵One might be interested in how our results on disproportionality relate

to the results of Akerlof and Yellen. Akerlof and Yellen also consider an environment where there are two types of agents – one type being similar to our sophisticated and the other type being similar to our naive. They show that in many interesting environments, if the naive make mistakes which are second order small in terms of the cost to the individual, there can still be a first order effect on the resulting equilibrium. Our results suggest when this first order effect is likely to be more significant. In particular, this first order effect is likely to be more significant when naive agents are disproportionately important, i.e., when the environment exhibits synergism.

⁶See Zarnowitz (1985) for a survey of the empirical evidence.

⁷Note that in using the term hump shaped persistence some authors add the restriction that the peak effect of a shock actually occurs a number of periods after the shock takes place. We will demonstrate that our model can yield hump shaped persistence of this latter type, however, this more restrictive type of hump shaped persistence is not a general characteristic of the model.

⁸See Poole (1976) for a discussion of this point.

⁹When we say that agents have rational expectations we mean they know the structure of the economy including the value of q .

¹⁰We do not explicitly consider how the naive agents determine the δ 's. Note that since the information set for the sophisticated agents includes current policy and cost information (except for unanticipated shocks), even for optimal choices of δ 's, the use of equation (2) to form expectations will typically lead to the naive having biased expectations. This is because the naive do not take into account relevant current information.

¹¹For an unanticipated cost shock, the interpretation is that each

sophisticated agent observes his own value for $c_{i,t}$, but does not realize that the economy has experienced an economy-wide shock.

¹²For both anticipated and unanticipated shocks we also assume that, for every period after the shock occurs, sophisticated agents know that α and G have returned to their original steady state values. That is, for every period t , $t > k$, $p_{i,t}^E = p(Y_t + G_t)$.

¹³Remember that we are not arguing that sophisticated and naive agents necessarily have different information sets. Rather, the crucial difference between sophisticated and naive agents lies in their respective abilities to process information.

¹⁴See, for example, Mullineaux (1980).

¹⁵We run simulations of unanticipated shocks because this is the worst case from the standpoint of persistence and disproportionality. That is, it is easy to demonstrate that, for any period t , $t > k$, moving from an unanticipated shock to an anticipated shock will always cause the deviation from steady state behavior to increase. In turn, this causes the number of periods for which our disproportionality result holds to never decrease.

¹⁶Heller (1984) and Cooper and John (1985) demonstrate multiple equilibria results in a model similar to the one which follows. As in Cooper (1985), by introducing a non-produced good into our model we eliminate the possibility of multiple equilibria. Note that, Cooper suggested that his non-produced good, called m , might be thought of as money. However, in his model utility depended on the consumption of m , not on real balances. Since we want to enter money into the utility function because of the transactions services it renders, we use the standard convention of entering real rather than nominal balances into the utility function.

¹⁷The demand for real balances is written in terms of the price level associated with the goods the agent is buying. However, since we only consider symmetric Nash equilibria in which $P_j = P_{-j}$, this is not at all critical to the results.

¹⁸As in Section III, one might also be interested in the effect of varying the degree of synergism. Unfortunately, because of the manner in which synergism enters the model, i.e., demand linkages across sectors, we have not found a tractable way to vary synergism.

¹⁹One of the other explanations for persistence mentioned in Section I, i.e., Taylor (1980), is consistent with anticipated money shocks having real effects. To be precise, therefore, evidence that anticipated money is non-neutral is not helpful for distinguishing between our explanation for persistence and that of Taylor.

²⁰As in Section II, to guarantee a unique solution an upper bound on the degree of synergism is required. In this case the condition is $1 + \sigma(Y) + \sigma'(Y)Y > 0$ for all Y .

²¹Note that we assume the same proportion of producers and workers is naive. Making this proportion the same for both producers and workers is not necessary for the results which follow. Rather, all that is required is that some firms be naive and some workers be naive. The precise meaning of disproportionality becomes complicated if the proportions differ across workers and firms, but the basic result will remain unchanged, i.e., the naive will be disproportionately important.

²²An increasing synergistic transformation of $\sigma(\cdot)$ is defined along the lines of a similar transformation in Section III. Specifically, $\hat{\sigma}(\cdot)$ is an increasing synergistic transformation of $\sigma(\cdot)$ if $\hat{\sigma}'(Y) < \sigma'(Y)$ for all Y .

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