THE ROTTEN-KID THEOREM MEETS THE SAMARITAN'S DILEMMA

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ABSTRACT

In his work on the family Becker considers a one period setting and derives what is referred to as the Rotten-Kid Theorem, i.e., a child will not behave in a manner which lowers the parent's income more than it raises the child's. Not captured in Becker's analysis of a single period model, however, is that the family environment can exhibit what others have referred to as the Samaritan's Dilemma. That is, children may consume too much in early periods because by doing so they can increase the income transfers they receive in later periods. In this paper we formally consider the role played by the Samaritan's Dilemma in the family environment and, in particular, how it interacts with the Rotten-Kid Theorem in a two period version of Becker's model.

I. Introduction

In recent years economists have paid considerable attention to the "economics of the family". Interactions among family members differ from that among unrelated individuals in that familial interactions are mediated by some "bond of caring" on the part of at least one member – denoted the head or parent – rather than by the impersonal marketplace. The main assertion in this literature is that the family frequently acts as if it were a single rational individual. That is, it acts as if it chooses among the alternatives available in the family budget set according to a consistent and transitive set of preferences. In this paper we consider the validity of this assertion when one moves only slightly away from the environment typically considered in this literature.

The result that the family frequently acts as if it were a single rational individual comes out of the work of Becker (1974, 1976). Becker addresses the issue of how family members behave when they can undertake actions that alter the wealth levels of other members of the family. He considers a world where, consistent with the above, parents care about the welfare of their children, and hence, might best be described as altruists. Children, on the other hand, are assumed to be egoists who care only about their own welfare. Further, in the world Becker analyzes parents make income transfers to their children. Becker demonstrates that, in a one period setting, if the income transfers are made after the children have already chosen their actions, then, as stated above, the family frequently acts as if it were a single rational individual.[1] That is, it acts as if choices were made according to a consistent and transitive set of preferences. A particular aspect of this result is known as the Rotten-Kid Theorem – a child does not behave in a manner which lowers the parents' income more than it raises the child's. In a later paper, Becker (1977) also suggests that if parents are assumed to behave in a retaliatory fashion, e.g., a tit for tat strategy, then the Rotten-Kid Theorem holds for all periods of a multi-period world.[2]

In this paper we formally consider a two period version of Becker's world. It is clear Becker is correct in his assertion that, if parents are allowed to behave in a retaliatory fashion, then the Rotten-Kid Theorem will hold for both periods. However, since this is a finite period game, the last period problem suggests that retaliatory behavior will not be an equilibrium strategy. That is, when making his last transfer to the child, the rational parent should seemingly let bygones be bygones. Thus, the first issue we explore is, if parents are <u>not</u> able to behave in a retaliatory fashion, to what extent does the Rotten-Kid Theorem still hold? The second issue we explore concerns the fact that, once we move to a two period setting, the family environment considered by Becker is quite similar to the type of environment referred to in discussions of the Samaritan's Dilemma.[3] To understand the latter, consider a world with two individuals – a giver and a receiver. Assume the giver is willing to help out the receiver if the receiver comes upon hard times. The Samaritan's Dilemma is simply that, if the receiver anticipates the giver will act in this way, then the receiver will behave in a manner which makes his probability of becoming impoverished "too high"; for example, he will overconsume in his early years. The same problem can arise in a two period version of Becker's world. A child will realize that by consuming more in the first period he can increase the income transfer he will receive from the parent in the subsequent period. The logic of the Samaritan's Dilemma suggests that, from the standpoint of the family as a whole, children will consume too much in the first period. Hence, in this paper we also explore the role played by the Samaritan's Dilemma and, in particular, how it interacts with the Rotten-Kid Theorem in a two period version of Becker's world.

Our analysis demonstrates that when members of a family overlap for two periods, then two sets of decisions must be considered. The first set concerns actions which determine the level of family wealth, while the second set relates to the timing of consumption by family members. Our main conclusion is that, in general, transfers by altruistic parents to family members result in one set of decisions being made efficiently, but not both. If a parent makes a positive income transfer at the end of the second period, then the actions undertaken by the child in <u>both</u> periods will typically be efficient in the sense of maximizing family income. That is, Becker's Rotten-Kid Theorem will hold for actions in both periods without the assumption that parents behave in a retaliatory fashion. However, the existence of an operative second period transfer results in the child choosing a first period consumption level that is too high from the point of view of family efficiency. This inefficiency reflects what we have referred to above as the Samaritan's Dilemma.

The existence of this consumption inefficiency opens the possibility that the parent may wish to make the second period transfer non-operative by pre-committing his gift in the form of a sufficiently large first period transfer. In this case the child would no longer consume too much in the first period, but the absence of an operative second period transfer implies that the child will choose a second period action which maximizes his own, rather than family income. In other words, with a non-operative second period transfer Becker's Rotten-Kid Theorem does not hold, but in return there is no longer inefficiency of the Samaritan's Dilemma kind.

Ultimately, then, the parent must choose between an operative second period transfer with its associated Samaritan's Dilemma type inefficiency, and a non-operative second period transfer in which case a Rotten-Kid type inefficiency is present. Hence, in a two period setting, when parents cannot behave in a retaliatory fashion, the family does not act like a single rational individual. Rather, rotten kids actually act rotten in at least one dimension, with the result being that the family unit does not achieve the Pareto frontier.

II. The One Transfer Case

In our model there is a single parent and a single child, and we allow for only two periods. Extending the model to more families or more children per family is straightforward, but would complicate the exposition. An analysis which allows for more than two periods is mathematically much more challenging, and is beyond the scope of the present paper. Our conjecture, however, is that the qualitative nature of the results would remain unchanged.

We begin by describing the child. The child is an egoist who cares only about his own consumption. His utility function is given by

$$U(C_1^c,W_2^c)=\mu_1(C_1^c)+\mu_2(W_2^c), \tag{1}$$

where C_1^c is the child's first period consumption and W_2^c is the wealth level of the child after the income transfer in the second period. It is additionally assumed that $\mu_j'>0$ and $\mu_j''<0$ for $j=1,2$. Note that by writing utility as a function of W_2^c rather than C_2^c we are trying to make clear this is a two period model because the parent lives for two periods, and it is quite possible that the child lives beyond the two periods we are considering. It is assumed the child has three sources for obtaining wealth. He is endowed with an initial wealth level, denoted W_o^c. He is also able to generate income in each period, where the income generated is a function of the action the child takes. That is, $I_j^c(A_j)$ is the income generated in period j, where A_j, $A_j \epsilon [0,\infty)$, is the action taken in period j, j=1,2. It is assumed that for each period I_j^c reaches its maximum at some action, denoted A_j^+, which is finite. Finally, as indicated earlier the child can also obtain wealth through income transfers from his parent.

The parent is an altruist who cares about the welfare of his child. His utility function is given by

$$V(C_1^p,C_2^p,U)=v_1(C_1^p)+v_2(C_2^p)+wU, \tag{2}$$

where C_j^p denotes the parent's consumption in period j and U simply denotes the utility level of the child. It is additionally assumed that $v_j'>0$ and $v_j''<0$ for j=1,2, and that w>0. The parent has two ways of obtaining wealth. He is endowed with an initial wealth level, denoted W_o^p. He also generates income in each period where the income generated is a function of the action the <u>child</u> takes. In particular, $I_j^p(A_j)$ is the income generated for the parent in period j, j=1,2. It is assumed there exists a value $\tilde{A}$, $\tilde{A}<\infty$, such that $I_j^{c\prime}(A)+I_j^{p\prime}(A)<0$ for all $A>\tilde{A}$, j=1,2. This assumption guarantees that, from the standpoint of the family unit, the efficient action for the child to take is finite. Finally, it is assumed that the parent transfers income to the child. This income transfer is denoted T, and it takes place after the child has chosen A_2. Note, another way to specify the model would be to allow an income transfer at the end of each period. In the following section we will consider what happens when this alternative specification is employed.

The final assumption of the model to be specified concerns the capital market. Our assumption is that both agents have access to perfect capital markets. This means that in the first period both the child and the parent can borrow or lend all they want at an interest rate r.

We now proceed to the analysis. The Rotten-Kid Theorem and the Samaritan's Dilemma refer to possible inefficiencies which arise in the choices of A_1, A_2 and C_1^c. We begin by establishing benchmarks for these three choice variables with which later results can be compared. Let A_1^*, A_2^* and C_1^{c*} be defined by the following.[4]

$$A_1^* = \arg\max_{A_1} I_1^p(A_1) + I_1^c(A_1) \tag{3a}$$

$$A_2^* = \arg\max_{A_2} I_2^p(A_2) + I_2^c(A_2) \tag{3b}$$

$$C_1^{c*}(\bar{W}) = \arg\max_{C_1^c} U(C_1^c, \bar{W} - (1+r)C_1^c) \tag{3c}$$

(3a) and (3b) state that our benchmarks for A_1 and A_2 are simply the actions which maximize family income in periods 1 and 2, where to keep the model interesting it is assumed $A_1^* \neq A_1^+$ and $A_2^* \neq A_2^+$. The benchmark for C_1^c is only slightly more complex. This benchmark is not a constant, but rather depends on the total wealth the child has available to him, denoted $\bar{W}$. Specifically, (3c) states that for a given $\bar{W}$, our benchmark for C_1^c is the consumption in the first period which maximizes utility given $\bar{W}$ as fixed.[5]

The next step is to demonstrate that this model can be reduced to capture Becker's original Rotten-Kid Theorem. Suppose the first period has elapsed, and in the first period the child consumed an amount $\bar{C}_1^c$, chose an action $\bar{A}$, and the parent consumed an amount $\bar{C}_1^p$. In the second period the child now faces the following maximization problem.

$$(4) \quad \max_{W_2^c, A_2, T} U(\bar{C}_1^c, W_2^c)$$

$$\text{s.t. } (1+r)\bar{C}_1^c + W_2^c \leq (1+r)[W_o^c + I_1^c(\bar{A})] + I_2^c(A_2) + T$$

$$T = \arg\max_{\hat{T}} V(\bar{C}_1^p, \hat{C}_2^p, U(\bar{C}_1^c, \hat{W}_2^c)),$$

$$\text{where } \hat{C}_2^p = (1+r)[W_o^p + I_1^p(\bar{A}) - \bar{C}_1^p] + I_2^p(A_2) - \hat{T}$$

$$\text{and}$$

$$\hat{W}_2^c = (1+r)[W_o^c + I_1^c(\bar{A}) - C_1^c] + I_2^c(A_2) + \hat{T}$$

Equation (4) states that the child will maximize his utility subject to a budget constraint, and an incentive compatibility constraint on the choice of T. Notice that in writing out the incentive compatibility constraint we have used the parent's budget constraint to substitute in for C_2^p, and the child's budget constraint to substitute in for W_2^c. Proposition 1 shows that by reducing the problem in this way we are able to capture Becker's original Rotten-Kid Theorem. Note, all proofs are relegated to an Appendix.

Proposition 1: The solution to (4) is such that if $T>0$, then $A_2 = A_2^*$.

Proposition 1 tells us that, as originally pointed out by Becker, if in a one period setting the equilibrium is such that the parent transfers income to his child, then the child's choice of action will maximize the joint income of the family. The intuition is straightforward. If the child chose an action which yielded himself a higher income, but a lower joint income for the family, the transfer would be reduced more than the increase in the child's income. Hence, the child maximizes his own utility by doing what is best for the family.

We now set up the maximization problem faced by the child at the beginning of the first period.

$$(5) \quad \max_{\substack{C_1^c, W_2^c, C_1^p \\ A_1, A_2, T}} U(C_1^c, W_2^c)$$

$$\text{s.t. } (1+r)C_1^c+W_2^c \leq (1+r)[W_o^c+I_1^c(A_1)]+I_2^c(A_2)+T$$

$$T = \arg\max_{\hat{T}} V(C_1^p, \hat{C}_2^p, U(C_1^c, \hat{W}_2^c)),$$

$$\text{where } \hat{C}_2^p = (1+r)[W_o^p+I_1^p(A_1)-C_1^p]+I_2^p(A_2)-\hat{T}$$

$$\text{and}$$

$$\hat{W}_2^c = (1+r)[W_o^c+I_1^c(A_1)-C_1^c]+I_2^c(A_2)+\hat{T}$$

$$\text{ICCs for } C_1^c, C_1^p, A_2$$

There are two major differences between equations (4) and (5). First, the child now also gets to choose first period consumption levels and a first period action. Second, there are now incentive compatibility constraints for these consumption levels, and for the child's second period action.[6]

Before proceeding to analyze equation (5) one point needs to be addressed. We know that the Rotten-Kid Theorem will only have a chance of holding if the transfer is operative, i.e., if the implicit constraint $T \geq 0$ is not a binding constraint. Hence, for the following proposition we assume this to be the case.[7]

Proposition 2: The solution to (5) is such that $A_2 = A_2^*$ and $C_1^c > C_1^{c*}(W^+)$, where $W^+ = (1+r)[W_o^c+I_1^c(A_1)]+I_2^c(A_2)+T$. Further, if $\mu_2''' \leq 0$ and $v_2''' \geq 0$, then $A_1 = A_1^*$.

Proposition 2 tells us that the equilibrium in our two period model exhibits a version of the Samaritan's Dilemma, i.e., the child consumes more in the first period than is efficient given the total wealth he receives over the two periods. In addition, even without assuming that the parent behaves in a retaliatory fashion, the Rotten-Kid Theorem frequently holds for actions in both periods. That is, there is a tendency for the child to choose a first period action, as well as a second period action, which maximizes the joint income of the family.

The intuition for these results is as follows. That the child overconsumes in the first period is, as already indicated, simply an application of the Samaritan's Dilemma. To understand it intuitively consider equation (6) which is the first order condition that is satisfied for the child's maximizing choice of C_1^c.

(6) $$\mu_1'(C_1^c) - \left\{\frac{1}{1+r} - \frac{\partial T}{\partial C_1^c}\right\}\mu_2'(W_2^c) = 0$$

Because $\frac{\partial T}{\partial C_1^c} > 0$, i.e., the parent transfers more to the child if the child consumes more in the first period, the child will overconsume in the first period relative to his final total wealth. The reason the transfer is related to first period consumption in this way is simple. Hold the transfer fixed, and let the child raise his first period consumption. This will lower the marginal utility for consumption in the first period, and raise it in the second. We know, however, that when the parent chooses the size of the transfer he has no effect on the first period consumption level, but rather affects only the second. Hence, since the marginal utility of second period consumption has increased, the parent has an incentive to increase his transfer.

We now consider why the Rotten-Kid Theorem tends to hold for the first period action. Given that the child has perfect access to capital markets, the child's consumption choices can be made independently from the temporal sequence in which he receives his total wealth. In turn, this suggests that choosing A_1 is analogous to choosing A_2, and hence, that A_1 tends to be chosen efficiently follows from the intuition presented for Proposition 1.

One question that might be asked is what is the role played by the assumptions $\mu_2''' \leq 0$ and $v_2''' \geq 0$? Basically, these assumptions guarantee that the model exhibits the following property. If the child's actions are held fixed and the wealth of the parent is increased, then the child will necessarily wind up being better off. This would seem to be a very plausible property for the model to exhibit. Further, it is not surprising that, unless the model exhibits this property, the first period action chosen by the child will not necessarily be efficient.[8]

III. <u>The Two Transfer Case</u>

In the previous section we considered what happens when the parent can make an income transfer only at the end of the second period. We demonstrated that actions tend to be chosen efficiently, but there is inefficiency because the child overconsumes in the first period. We now consider the model under the assumption that the parent has two opportunities to transfer income — once at the end of each period. In particular, the parent will be able to make a first period transfer after the child has chosen a first period action, but before first period consumption levels are chosen. We demonstrate that when parents have this extra opportunity to transfer income, they then face a choice as to whether they would prefer efficient actions or efficient consumption choices.

The first result of the two transfer case is simply that Proposition 1 still holds. That is, the nature of the problem in the second period has not changed, and hence, the Rotten-Kid Theorem holds for the second period action. The next step is to proceed to the full two period problem.

The only difference between the problem faced by the child at this point and equation (5) is that there are now two transfers, and there is a separate incentive compatibility condition on each transfer. Let T_1 denote the transfer made at the end of the first period, and T_2 denote the transfer made at the end of the second. Analysis of this new maximization problem yields the following proposition.

Proposition 3: Suppose the parent has the option of making a transfer at the end of each period. If $T_2>0$, then $A_2=A_2^*$ and $C_1^c>C_1^{c*}$. If $T_2=0$, then $A_2=A_2^+$ and $C_1^c=C_1^{c*}$.

Proposition 3 tells us that the equilibrium can now take either of two forms. On the one hand, the second period transfer may be positive. If this is the case, then the equilibrium is qualitatively similar to the equilibrium found in the previous section. That is, the second period action is efficient, but the child overconsumes in the first period. On the other hand, the second period transfer may be zero. When this occurs there are important differences between the one transfer and two transfer cases. In particular, consumption choices will now be efficient, but the child will choose an inefficient second period action.

The intuition lying behind Proposition 3 is as follows. Consider the environment after the child has chosen a first period action. The parent at that stage has two options. If he transfers a relatively small amount in the first period, then the wealth he carries into the second period will be large enough to keep the second period transfer operative. This means the child faces the same incentives he faced in the one transfer case. Hence, he chooses an efficient second period action but an inefficient first period consumption level. On the other hand, the parent may make a relatively large first period transfer such that the second period transfer becomes non-operative. This has the advantage of eliminating the problem of the Samaritan's Dilemma in that, if the second period transfer is non-operative, there is no longer any reason for the child to overconsume in the first period. However, a non-operative second period transfer also means that in choosing a second period action the child will maximize his own private income, rather than the joint income of the family. Hence, in choosing the first period transfer the parent faces a trade-off. He can make a small first period tranfer in which case there will be efficient actions, but the first period consumption choice will be inefficient. Or, he can make a large first period transfer and guarantee an efficient consumption choice, but then the second period action will be inefficient.

This result is related to Hirshleifer's comments concerning the Rotten-Kid Theorem (see Hirshleifer (1977, 1985)). Hirshleifer considers a one period setting and makes the point that the child will only choose an efficient action if the parent gets to move last. Our point above is that in a two period world, the parent may make a large transfer in the first period and in this way deliberately force the child into a position where the child moves last. The reason the parent might do this is that by having the child move last, the parent avoids the inefficient consumption choice which arises when the parent moves last.

The final issue to be addressed concerns the child's choice of a first period action in the two transfer case. We did not characterize this choice in Proposition 3 because of a problem concerning multiple equilibria. That is, we have not been able to rule out the possibility that, for some choices of a first period action by the child, the resulting subgame will have both an equilibrium with a positive second period transfer as well as one with a zero second period transfer. In the following propostion we show that, given a particular way of resolving this multiple equilibria problem, there is again a tendency for the first period action to be chosen efficiently.

Proposition 4: Suppose the parent has the option of making a transfer at the end of each period and that, when multiple equilibria exist, the equilibrium which is realized is the one which gives the child higher utility. Given this, if $\mu_2''' \leq 0$ and $v_2''' \geq 0$, then $A_1 = A_1^*$.

Proposition 4 states that, if multiple equilibria are always resolved in favor of the child, then the conditions which guarantee an efficient first period action in the one transfer case also guarantee it for the two transfer case. In other words, we have a suggestion that in the two transfer case the Rotten-Kid Theorem will frequently hold for first period actions.[9]

The intuition behind this result is as follows. Given $\mu_2''' \leq 0$ and $v_2''' \geq 0$, there is necessarily an equilibrium at A_1^* which yields the child a higher level of utility than any equilibrium associated with any other first period action. However, if A_1^* is associated with multiple equilibria and there is uncertainty concerning which of the multiple equilibria will be realized, then even though a better equilibrium exists at A_1^* the child might decide to choose a different first period action. This problem disappears if multiple equilibria are always resolved in favor of the child.[10]

IV. Conclusion

It is not really surprising that altruism by one member of the family may induce some form of inefficient behavior by another egotistical member. After all, there is little, at least in Becker's model, which distinguishes the relationship between the parent and the rotten kid from the relationship which exists between a more general altruist and the community at large. Certainly the idea that altruistically motivated transfers across society members can have deleterious effects on economic efficiency is not particularly novel. Why should it be any different among family members?

What we find most interesting about the presence of altruistic transfers is the conflict between efficiency in terms of contemporaneous decisions and efficiency in terms of prior decisions. The essence of the Rotten-Kid Theorem is that a selfish individual will frequently make contemporaneous choices efficiently even when these choices impose a negative externality on another individual. This occurs as long as the individuals are linked through an operative set of transfers. In other words, the presence of altruism internalizes an externality without the need to bargain over Coase style side-payments. As Becker (1974, p. 267 fn 26) puts it, altruism makes the required side-payments "automatic".

Our paper has stressed that, through the use of prior decisions, these automatic side-payments can be manipulated to the advantage of the recipient. That is, altruism introduces an externality of the Samaritan's Dilemma type which did not exist without altruism. Our analysis suggests that, unless altruists have the ability to bind themselves so that they behave in a retaliatory fashion, altruistic transfers will not be made in a manner which results in efficiency in both contemporaneous and prior decisions. For example, in our model the child chose either an inefficient second period action (the contemporaneous decision), or an inefficient first period consumption level (the prior decision). Once again the old adage "there is no such thing as a free lunch" seems appropriate.

Appendix

Proof of Proposition 1: The incentive compatibility constraint yields the following first order condition.

$$-v_2'(C_2^p)+w\mu_2'(C_2^c)=(\leq)0, \tag{A1}$$

if $T>(=)0$. Suppose $T>0$ and $A_2 \neq A_2^*$. The child could have instead chosen the second period action A_2^*. Let $\tilde{C}_2^p$, $\tilde{C}_2^c$ denote the resulting consumption levels. Given the definition of A_2^* and that the budget constraints must hold as strict equalities, we have that if $C_2^c \geq \tilde{C}_2^c$, then $C_2^p < \tilde{C}_2^p$. In turn, given $T>0$, $v_2''<0$, and $\mu_2''<0$, $C_2^c \geq \tilde{C}_2^c$ now yields a contradiction with equation (A1). Hence, $C_2^c < \tilde{C}_2^c$. However, this contradicts $A_2 \neq A_2^*$.

Proof of Proposition 2: $A_2 = A_2^*$ follows from Proposition 1. Now consider the child's choice of C_1^c. C_1^c is chosen after A_1 and simultaneously with C_1^p. Hence, given this and the preceding result, in characterizing C_1^c we take as fixed A_1, A_2, and C_1^p. Suppose $C_1^c < C_1^{c*}(W^+)$. The child could have instead chose $C_1^c > C_1^{c*}(W^+)$ such that if T is held constant the child's lifetime utility remains unchanged. However, (A1) yields that T would increase given this alternative choice. Hence, $C_1^c \geq C_1^{c*}(W^+)$.

Now suppose $C_1^c = C_1^{c*}(W^+)$. Equation (A1) yields

$$\frac{\partial T}{\partial C_1^c} = \frac{w\mu_2''(W_2^c)}{v_2''(C_2^p)+w\mu_2''(W_2^c)} > 0. \tag{A2}$$

Now consider the first order condition for C_1^{c*}.

$$\mu_1'(C_1^{c*})-\left(\frac{1}{1+r}\right)\mu_2'(W_2^c)=0 \tag{A3}$$

Given $\frac{\partial T}{\partial C_1^c}>0$ and $C_1^c \geq C_1^{c*}$, a comparison of (A3) and (6) yields $C_1^c > C_1^{c*}$.

We now consider the child's choice of A_1. Suppose $A_1 \neq A_1^*$. We will demonstrate this implies a contradiction in that, if the child were to choose $A_1 = A_1^*$, then he would necessarily be better off. Let $\tilde{C}_1^c$, $\tilde{C}_1^p$, $\tilde{W}_2^c$ and $\tilde{C}_2^p$ denote the consumption levels which would result if the child were to choose A_1^*. Combining (6), (A1) and that the parent's consumption choices must satisfy $v_1'(C_1^p) = (\frac{1}{1+r})v_2'(C_2^p)$, yields

$$v_1'(C_1^p) = \frac{v_2'(C_2^p)}{1+r} = \frac{w\mu_2'(W_2^c)}{1+r} = \frac{w\mu_1'(C_1^c)}{1-(1+r)\frac{\partial T}{\partial C_1^c}} \qquad \text{(A4)}$$

If $\tilde{C}_1^p > C_1^p$, then (A4) immediately implies $\tilde{C}_2^p > C_2^p$ and $\tilde{W}_2^c > W_2^c$. If $\mu_2''' \leq 0$ and $v_2''' \geq 0$, this result combined with (A2) and (A4) yields $\tilde{C}_1^c > C_1^c$. Hence, if $\tilde{C}_1^p > C_1^p$, then the child is necessarily better off. On the other hand, $\tilde{C}_1^p$ cannot be less than or equal to C_1^p because in equilibrium the budget constraints must hold as strict equalities.

Proof of Proposition 3: The case $T_2>0$ follows from the proof of Proposition 2. If $T_2=0$, the child will not take into account the second period transfer in choosing C_1^c and A_2. Hence, $C_1^c = C_1^{c*}$ and $A_2 = A_2^+$.

Proof of Proposition 4: The proof of Proposition 4 is somewhat tedious so we will just outline it here. Suppose $A_1 \neq A_1^*$ and $T_2>0$. Let the child's utility associated with this action be denoted $\bar{U}^c$. The child could have instead chose A_1^*. Given $\mu_2''' \leq 0$ and $v_2''' \geq 0$, when $A_1 = A_1^*$ there is a unique set of consumption levels and W_2^c consistent with equation (A4). Denote the associated utility of the child as $\hat{U}^c$, and note that $\hat{U}^c > \bar{U}^c$. Suppose that after the parent chooses a first period transfer, any subsequent multiple equilibria problems are resolved in favor of the child.[11] There are now three possibilities for what could occur if the child were to choose A_1^*. One possibility is that T_2 remains greater than zero. $\hat{U}^c > \bar{U}^c$ tells us that the child would now have to be better off than when he chose the action A_1. Another possibility is that T_2 now equals zero, and that after the first period transfer the consumption levels and W_2^c which satisfy equation (A4) are also an equilibrium. Given $\hat{U}^c > \bar{U}^c$ and our supposition that multiple equilibria at this stage are always resolved in favor of the child, it is again the case that the child would now have to be better off than when he chose A_1. The third possibility is that T_2 equals zero, and after the first period transfer the consumption levels and W_2^c which satisfy equation (A4) are not an equilibrium. For equation (A4) not to be an equilibrium after the first period transfer, the child must be able to do better than $\hat{U}^c$ by getting a zero second period transfer, efficiently smoothing his consumption stream and setting $A_2 = A_2^+$. We now have that there necessarily exists an equilibrium to the game after the child chooses A_1^* such that the child's utility is better than $\bar{U}^c$. Given our assumption that multiple equilibria after the child chooses an action are resolved in favor of the child, this implies that if $T_2 > 0$, then $A_1 = A_1^*$.

Suppose $A_1 \neq A_1^*$ and $T_2 = 0$. Let $\tilde{U}^c$ now denote the child's utility associated with this situation. It is easy to demonstrate that if the equilibrium is such that $T_2 = 0$, the parent will want to choose the smallest first period transfer which results in $T_2 = 0$. Call this value T_1^*. For any $T_1 < T_1^*$ the child gets the utility associated with action A_1 and equation (A4), i.e., what we referred to as $\bar{U}^c$ above. In addition, for any first period transfer, the utility the child receives in an equilibrium cannot be less than if he efficiently smoothes his consumption stream, sets $A_2 = A_2^+$, and $T_2 = 0$. Since this constraint is continuous in T_1, we have that at T_1^* the utility of the child must be less than or equal to $\bar{U}^c$, i.e., $\tilde{U}^c \leq \bar{U}^c$. Using the logic in the above paragraph, we now also have that if $T_2=0$, then $A_1=A_1^*$.

Footnotes

[1] See Hirshleifer (1977, 1985) for the importance of the sequence in which players move in this environment. Also, see the discussion at the end of Section III of the present paper.

[2] Bernheim et al (1985) consider a one period world where parents and children derive utility directly from actions taken, as well as from monetary outcomes. They show that in such a world actions will not be chosen efficiently unless parents make strategic bequests, i.e., behave in what we have referred to as a retaliatory fashion. We will abstract away from this aspect of the problem, and instead analyze what occurs in a two period setting when agents only care about monetary outcomes and strategic or retaliatory bequests are not possible.

[3] See Buchanan (1975) and Thompson (1980) for discussions of the Samaritan's Dilemma.

[4] It is assumed that the income functions are such that both A_1^* and A_2^* are uniquely defined. The separability assumption on $U(.,.)$ guarantees that, for any $\bar{W}$, C_1^{c*} is uniquely defined.

[5] Another way of thinking about our benchmarks is in terms of the assumption that the parent behaves as a dictator. That is, if the parent could choose the child's actions and consumption levels, then we would have $A_1 = A_1^*$, $A_2 = A_2^*$, and $C_1^c = C_1^{c*}((1+r)C_1^c + C_2^c)$.

[6] Formally writing down the incentive compatibility constraints for C_1^c, C_1^p, and A_2 is difficult, however, the analysis of (5) follows without the formalization. Note, there are incentive compatibility constraints on C_1^c and A_2 even though it is the child who is directly making these choices. The

reason is that each choice is constrained to be optimal, given the situation the child faces at the time the choice is being made.

[7]Suppose the child chose actions A_1^+ and A_2^+, and let $C_1^c=C_1^{c*}((1+r)(W_o^c+I_1^c(A_1^+))+I_2^c(A_2^+))$. A sufficient condition to guarantee that the transfer is operative is that, even given these choices, the parent would want to transfer income to the child.

[8]These assumptions also guarantee that, given a choice of a first period action by the child, there is a unique equilibrium to the subgame which follows

[9]Another interesting point is that, given the assumptions in Proposition 4, if $T_2>0$ then the equilibrium in the two transfer case is identical to the equilibrium in the one transfer case. That is, all actions and consumption levels are unchanged, as is the total transfer $((1+r)T_1+T_2=T)$.

[10]One interesting question is whether a proposition similar to Proposition 4 could be derived in which it is assumed that multiple equilibria are always resolved in favor of the parent. To this point we have been unable to answer this question.

[11]There are two subgames in which multiple equilibria can arise in this model. Multiple equilibria can arise after the child chooses a first period action, and multiple equilibria can arise after the parent makes a first period transfer. Our assumption in the proposition is that, after the child chooses an action, multiple equilibria are resolved in favor of the child. We can also prove the proposition under the related assumption that multiple equilibria which arise after the first period transfer are resolved in favor of the child.

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