COMPLEMENTARITY AND PARTIAL NONEXCLUDABILITY:

AN ANALYSIS OF THE SOFTWARE/COMPUTER MARKET

by

Ian E. Novos
Department of Economics
University of Southern California

and

Michael Waldman
Department of Economics
University of California, Los Angeles
Los Angeles, CA   90024

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ABSTRACT

Previous authors have considered markets for partially nonexcludable commodities in a variety of contexts. It is well known that two social welfare losses, the underproduction and underutilization losses, arise in such markets. This paper considers such a market in the context of a partially nonexcludable commodity — computer software — being complementary to a second good, i.e., a computer. Our results indicate that the underproduction and underutilization losses that occur in software markets depend both on the market structure for computers and on whether computer manufacturers are permitted to enter the software market.

I. Introduction

The concept of nonexcludability is generally associated with public goods. Many privately produced commodities, however, also display aspects of nonexcludability. For example, any recording is partially nonexcludable if copies of the recording can be made by individuals other than the original producer. A number of recent papers have considered markets of this type. For example, Ordover and Willig (1978) and Liebowitz (1985) both consider the implications of journal publishers discriminating in price between libraries and personal subscribers, Novos and Waldman (1984) consider the effects of government restrictions on copying, while Johnson (1985) and Besen (1986) are generally concerned with the supply of partially nonexcludable commodities. The present paper also considers such markets, but in the context of a different issue. In particular, this paper considers the role of complementary goods when partial nonexcludability is present.

There are two social welfare losses associated with the private marketing of partially nonexcludable goods. The first is known as the social welfare loss due to underproduction. The logic here is that some individuals get access to the good without paying the original producer, and hence, from a societal standpoint either quality or variety will be too low because the incentive to produce is inadequate. This loss is analogous to the loss that occurs in the classic public goods case (see e.g., Samuelson (1954)). The second loss is known as the social welfare loss due to underutilization. This loss stems from the fact that the producer's price will typically exceed his marginal cost of production because of the market power he possesses. We can separate this social

welfare loss into two components. The first component is the loss due to consumers who would be willing to pay the marginal cost of production, but who do not consume the good. The second is the loss due to consumers who expend more real resources in copying than would be incurred if these consumers purchased from the producer.[1]

Consider the computer software market. Computer software fits the description of a partially nonexcludable good, while also possessing another important characteristic not previously mentioned. The producers of computer software are also frequently the producers of computers.[2] This is due to the complementary nature of the products. That is, when two separate firms produce complementary products, each firm does not internalize how its own price and quality decisions affect the profitability of the other firm. Hence, there is an incentive for a single firm to produce both goods since this will increase aggregate profits.[3]

We can now consider what happens to the underproduction and underutilization losses when a single firm produces both computers and software. There are two important factors affecting these losses. The underproduction loss arises because those who copy do not pay the actual producer of the software. However, since in this case copiers must buy the computer from the same firm, there exists an avenue by which the producer of software can internalize at least some of the societal benefits enjoyed by copiers. This is the first factor present, and it suggests that having the same firm produce both goods should result in a reduction in the size of the underproduction loss. On the other hand, when a single firm produces both goods, the firm will likely have an

incentive to increase the price of the computer and decrease the price of software. Such behavior would tend to increase profits because of the subsequent decrease in the number of copiers. This is the second factor, and it suggests that having the same firm produce both goods should result in a reduction in both the underproduction and underutilization losses.

In this paper we formally investigate the role played by complementarity when partial nonexcludability is present. Our analysis demonstrates that the above reasoning is correct. That is, the presence of a complementary good can help eliminate the social welfare losses due to both underproduction and underutilization. Our analysis also indicates one of the factors which help determine what proportion of these losses will be eliminated. Specifically, the analysis suggests that for the existence of a complementary good to help eliminate these losses, it is important that the sellers of the complementary good have some market power. The logic here is that in the absence of market power any change in either the production or pricing of the nonexcludable good will be reflected in the industry demand for the complementary good, rather than the demand for the product of any particular firm. Hence, if a manufacturer produced both goods, but had no market power in the market for the complementary good, neither of the previously mentioned factors which help reduce the underproduction and underutilization losses would be present.

The outline for the paper is as follows. Section II sets forth a model of an economy containing a single partially nonexcludable good, where there exists a second good which is complementary to the partially

nonexcludable good. Section III analyzes the model under the two polar assumptions that this second good is produced by a monopolist, and that this second good is produced in a perfectly competitive environment. Section IV extends some of our results to the case where there are many partially nonexcludable goods, all complementary to the same second good. An appropriate interpretation here is that the partially nonexcludable goods represent a set of programs all written for the same computer system, where the complementary good is simply the computer system for which the programs are written. Section V contains some concluding remarks.

II. The Model

In our world there are two relevant markets. In the first market, which will be referred to as the market for a particular type of computer software, a partially nonexcludable good is produced by a monopolist. This good is indivisible - in particular, each individual will consume either zero units or one unit of the software. Equation (1) describes the monopolist's total costs, denoted $TC(x,Q)$, of producing x units of software of quality Q.

(1) $$TC(x,Q) = F(Q) + cx,$$

where $F(Q)$ denotes the fixed costs of production, and where cx denotes the variable costs. The function $F(Q)$ is also assumed to satisfy the following restrictions: $F(0)=0$, $F'(.)>0$, and $F'(\infty)=\infty$.

The idea that the good is partially nonexcludable means that there are two ways a consumer can acquire a unit of software. The first method is for the consumer to purchase a unit directly from the

monopolist. The second method is for the consumer to borrow a unit from some other agent who has in one way or another acquired it, and then make a reproduction. We refer to the monopolist as the primary source for obtaining software, and to agents who lend out software as secondary sources. Agents who lend out software incur no costs in the lending process. If consumer i acquires a unit of software by borrowing a unit from a secondary source and copying it - a process we will refer to as going through the secondary market - then he incurs reproduction costs $c+z_i$. One interpretation for the term z_i is that it is the cost the consumer incurs which is due to the time he expends in making a reproduction. Note that this specification ensures that the marginal cost of production for the monopolist is less than any consumer's private cost for reproducing the monopolist's output. That is, to produce an extra unit of output the monopolist incurs a cost c, while to make a reproduction each consumer i incurs a cost $c+z_i$.[4]

We assume that individuals differ in terms of their costs of obtaining a reproduction. In terms of our model, this means that the z_i's are not constant across individuals. Rather, z_i's are distributed between the extreme values 0 and Z, where this distribution is described by a density function h(.) that is continuously differentiable and nonzero in the specified interval. We also assume that each consumer i's value for z is unobservable to everyone but himself.

The second market consists of a good which is complementary to the partially nonexcludable good. This market will be referred to as the market for computers. It is assumed that computers can only be produced at a single quality, where production costs are a constant b per unit.

In the analysis we consider two polar assumptions concerning the market structure for computers. The first assumption is that computers are produced by a monopolist, while the second is that computers are produced in a perfectly competitive environment.

To stay consistent with the notion that the partially nonexcludable good is a particular type of computer software, it is assumed that a consumer derives no return from the consumption of software unless he also purchases a computer. Let Q_i be the quality of the unit of software purchased by consumer i (note: $Q_i=0$ if individual i does not consume a unit of software), and let e_i be the cost consumer i incurs in acquiring the two goods. If consumer i does purchase a computer, then he receives profits denoted π_i. π_i is given by equation (2).

$$(2) \qquad\qquad \pi_i=y_i+v(Q_i)-e_i,$$

where $v(0)=0$, $v'>0$ and $v''<0$. Equation (2) states that each consumer has the same valuation for increases in the quality of software, but that consumers do differ in terms of their valuation for the computer alone. In particular, y_i's are distributed between the extreme values 0 and Y, where this distribution is described by a density function g(.) that is continuously differentiable and nonzero in the specified interval. It is further assumed that each consumer i's value for y is unobservable to everyone but himself, and that the values for y_i and z_i are independently distributed in the population.

III. Analysis

In this section we analyze the model presented in the previous section. As indicated earlier, the focus of the analysis concerns how

the underproduction and underutilization losses are affected by the possibility that the same firm may produce both the partially nonexcludable good and the complementary good.

Before proceeding two points need to be made. First, only a single quality of software will be offered for sale. This is a consequence of the assumption that all consumers have the same valuation for quality. That is, because of this assumption, the software producer will not be able to price discriminate by offering a variety of qualities for sale.[5] Second, because secondary sources incur no costs in the lending process, in equilibrium they will lend the same quality good the software producer is selling, but at no charge to the consumer.[6] This in turn tells us which consumers will prefer to borrow rather than purchase. Let P_S denote the price being charged by the software producer. Given that the borrowing price equals zero, if consumer i decides to acquire a unit of software, he will borrow rather than purchase when $c+z_i<P_S$.

The first step of the analysis is to establish a benchmark which will allow us to identify the social welfare loss due to underproduction. Our benchmark is a second best notion for quality which first appeared in Spence (1975). That is, hold fixed the total number of consumers who either purchase a unit of software from the software producer or acquire a unit by going through the secondary market, and define the second best quality of software as the quality which maximizes social welfare given this fixed number of consumers. Let N denote this number of consumers, and $Q^*(N)$ denote the second best quality given that N consumers are acquiring a unit of software. $Q^*(N)$ is defined by equation (3).

(3)
$$Q^*(N) = \arg\max_{Q'} Nv(Q') - F(Q')$$

We will now compare this benchmark with the market outcome when software is excludable. Note, one of the cases considered in Proposition 1 is where there is a computer monopolist, and the firm is not integrated into software production. In dealing with this case we utilize either a Nash assumption on prices, or the assumption that the computer firm is a Stackelberg leader. All results for this case hold under both assumptions. Further, so that the exposition does not become bogged down in detail we have relegated all proofs to an Appendix.

Proposition 1: If software is excludable, then $Q=Q^*(N)$, where Q now denotes the market outcome.

Proposition 1 tells us that, when software is excludable, the quality of software produced satisfies our second best notion for quality. This is true independent of the assumed market structure for computers, and independent of whether a single firm produces both computers and software.[7] What this means is that any underproduction loss which arises under partial nonexcludability can be identified as the deviation from this second best notion for quality.

We will now go back to the assumption that software is partially nonexcludable, and consider the market outcome when the computer is monopolistically provided and the firm is not integrated into software production. Note, $N_2(N_1)$ will denote the total number of consumers who acquire a unit of software when the computer monopolist is not (is)

integrated into software production, while $Q_2(Q_1)$ will denote the software quality produced.

Proposition 2: Suppose the computer is monopolistically provided and $Q^*(N_2)>0$. Then $Q_2<Q^*(N_2)$.

Proposition 2 tells us that, when the computer is monopolistically provided and two different firms produce the computer and the software, the quality of software provided is less than the socially optimal quality, i.e., there is an underproduction loss. The intuition for this result is well known from the discussions of previous authors who have dealt with partially nonexcludable goods. Because of the nonexcludable nature of the software, some consumers get access to the software without paying the original producer. Hence, from a societal standpoint quality will be too low because the incentive to produce is inadequate.

In the Introduction we suggested that the presence of a good which is complementary to the partially nonexcludable good may play an important role. In particular, the situation analyzed in Proposition 2 is characterized by an incentive to have a single firm produce both goods. The logic is that, when two firms are producing, each firm does not internalize how its own choice of price and quality affects the profit of the other firm. Hence, profit for an integrated manufacturer is higher than the combined profits of two separate firms.

In the following we investigate how having a single firm produce both goods affects the size of the underproduction loss.

Proposition 3: Suppose the computer is monopolistically provided. Then $Q_1=Q^*(N_1)$.

Proposition 3 tells us that, when the computer is monopolistically provided and a single firm produces both goods, the quality of software produced equals the second best quality level, i.e., there is no underproduction loss. As evidenced by the proof contained in the Appendix, this result illustrates the first factor mentioned in the Introduction. That is, the underproduction loss arose previously because those who copy did not pay the actual producer of the software. Now, however, a single firm produces both the computer and the software, and hence, through his sales of the computer the producer of the software will internalize the benefits that copiers enjoy from increased quality. The result is that the firm provides software which is of the socially optimal quality.

In the Introduction we discussed a second factor which we claimed would reduce both the underproduction and underutilization losses. However, since Proposition 3 states that the underproduction loss is completely eliminated by the first factor identified in the Introduction, for this model the second factor will obviously not play a role in reducing the underproduction loss. Proposition 4 deals with a case where this second factor does reduce the underutilization loss.[8]

Proposition 4: Suppose y_i can only take on a single value and the computer is monopolistically provided. Then allowing integration reduces the underutilization loss to zero.

The intuition for Proposition 4 is as follows. When a single firm produces both computers and software, the firm has an incentive to increase the price for computers and decrease the price for software.

The reason is that through such a change in prices the firm can reduce the number of copiers. In turn, this reduction in the number of copiers implies a reduction in the loss due to underutilization.

One interesting question is, how much do our results depend on the assumed market structure for computers? We will investigate this question by considering what occurs when computers are produced in a perfectly competitive environment.[9]

Proposition 5: Suppose that computers are produced in a perfectly competitive environment. Then $Q_1 = Q_2$, $N_1 = N_2$, $Q_2 < Q^*(N_2)$ (as long as $Q^*(N_2) > 0$), and allowing integration has no effect on the underutilization loss.

Proposition 5 tells us two things. On the one hand, when there is no firm which produces both computers and software, the result here is qualitatively similar to the monopoly case considered in Proposition 2. For example, there is an underproduction loss both when computers are provided monopolistically, and when they are produced in a perfectly competitive environment. On the other hand, when there is a firm which produces both goods, the result is quite different from the monopoly case. Previously the underproduction loss disappeared when we moved from the non-integrated to the integrated case, and there was a suggestion that the underutilization loss would also decrease. Here, however, moving from the non-integrated to the integrated case has no effect on the market outcome. In other words, there is no effect on either the underproduction or underutilization loss.

The logic for the above result is as follows. Previously, when the

same firm produced both computers and software there was an incentive for the firm to provide higher quality software, because higher quality in that case caused an increase in the demand for the firm's computers. Similarly, there was also an incentive for the firm to lower the software price, since this lowered the number of copiers while at the same time again increasing the demand for computers. Now consider what happens when the computer industry is perfectly competitive. If in this situation an integrated firm decided to either increase software quality or decrease software price, there would be an increase in the <u>industry</u> demand curve for computers. However, given that the computer industry is perfectly competitive, there would be no increase in the demand curve faced by the integrated firm. The result is that integration has no effect on the incentive for the firm to either raise software quality or lower the software price, and hence, no effect on either the underproduction or underutilization loss. Or to summarize, for the existence of a complementary good to help eliminate the underproduction and underutilization losses, the sellers of the complementary good must have some market power.

A final word concerns our result that, when the computer is monopolistically provided, allowing a single firm to produce both software and computers completely eliminates the underproduction loss. The intuition we present suggests that the underproduction loss should be reduced in this case, but not that it should be completely eliminated. The reason the underproduction loss is completely eliminated in our analysis is because we employ the assumption that all consumers place the same valuation on increases in the quality of

software. Our conjecture is that, if one could analyze our model without this assumption, one would find that allowing a single firm to produce both goods tends to reduce the underproduction loss, but does not in general eliminate it. Unfortunately, such an analysis is quite difficult. The reason is that when one moves from the non-integrated to the integrated case, the total number of consumers who get access to software may very well change. Hence, it is difficult to identify whether changes in quality are due to changes in the total number of software users, or to changes in the underproduction loss. By considering a case where the underproduction loss is completely eliminated we are able to avoid this problem.

Note, further, there is a similar rationale for why in Proposition 4 we consider a case where allowing integration completely eliminates the underutilization loss. When one moves from the non-integrated to the integrated case, it is still true that the total number of consumers who get access to software may very well change. Allowing integration, therefore, may change the underutilization loss not only because of the second factor identified in the Introduction, but also because of changes in the total number of consumers who get access to software. Hence, it is difficult to identify the affect of this second factor unless, as in Proposition 4, we consider a case where allowing integration reduces the underutilization loss to zero.

IV. An Analysis of Variety

The underproduction loss can manifest itself either as insufficient quality, as in the previous section, or as too little variety. In this section we consider a model where the underproduction loss is exhibited

as too little variety, and demonstrate that the presence of a complementary good can still help eliminate the loss. The model we employ is similar to the model considered previously.

We again consider a single complementary good referred to as a computer. As before, computers can only be produced at a single quality level, where production costs are a constant b per unit. In addition, for this model we only consider the case of computers being produced by a monopolist.

There will now be J different types of computer software, denoted $S_1, \ldots S_J$. Further, each type can only be produced at a single quality level. Equation (4) describes the total costs, denoted $TC_j(x)$, of producing x units of S_j.

$$(4) \qquad TC_j(x) = \begin{cases} F_j + cx & \text{if } x > 0 \\ 0 & \text{if } x = 0. \end{cases}$$

where F_j denotes the fixed costs of production which vary across types, and where cx denotes the variable costs of production which do not vary across types.

As in the previous section, we consider two different assumptions concerning who produces the software. First, we assume the computer monopolist is also a monopolist in each of the J software markets. Second, we assume the computer monopolist only produces computers, and correspondingly that there is a separate software monopolist for each of the J types of software.

There are J different groups of consumers, each group being of

equal size. If consumer i in group j puchases a computer, then he receives profits denoted $\pi_{i,j}$. $\pi_{i,j}$ is given by equation (5).

(5)
$$\pi_{i,j} = L_{i,j} V_j - e_{i,j},$$

where $L_{i,j} = 1(0)$ if consumer i has (has not) purchased a unit of S_j, and $e_{i,j}$ is the expenditure of the consumer on the two goods. Equation (5) states that consumers in group j place a zero valuation on any software type S_k, $k \neq j$. In addition, a consumer in group j only has a positive valuation on a computer if he purchases software type S_j, while he has a zero valuation for S_j alone.

The key aspect of the model, of course, is that software is again partially nonexcludable. The specification here follows closely that of Section II. In particular, for each of the J groups of consumers z_i's are distributed according to a density function h(.)/J, where h(.) is as defined in Section II.

We begin the analysis by considering what happens under excludability.[10] Below $M_1^E (M_2^E)$ will denote the number of software types produced when the computer monopolist is (is not) integrated into software production, and software is excludable. Note, in dealing with the case where the computer monopolist is not integrated into software production, we assume that the computer monopolist behaves like a Stackelberg leader in terms of prices.[11,12]

Proposition 6: $M_1^E \geq M_2^E$

Proposition 6 tells us that, even under excludability, there is a return in terms of variety of having a single firm produce both

computers and software. The result stems from the complementary nature of the goods. Even under excludability, when an additional type of software is produced, one of the effects is an increase in the demand for computers. Further, when a single firm produces both goods this effect is internalized, and thus, more types of software are made available when a single firm produces both goods than when there are two separate producers.

We can now consider what occurs when software is partially nonexcludable. In the following $M_1^N (M_2^N)$ will denote the number of software types produced when the computer producer is (is not) integrated into software production, and software is partially nonexcludable.[13]

Proposition 7: If $V_j - V_k \leq c$ for all j, k pairs, then $M_1^N \geq M_2^N$ and allowing integration reduces the underutilization loss to zero.

Proposition 7 states that, under partial nonexcludability, there is again a tendency for more types of software to be produced when a single firm produces both computers and software than when there are separate producers. In addition, allowing this type of integration reduces the underutilization loss to zero. There are now two reasons why more types of software are produced under integration. As above, one factor is simply the complementary nature of the products. That is, when a single firm produces both computers and software, more types of software are made available because the firm internalizes how the availability of an additional type affects the demand curve for computers. Second, as in the previous section, the partially nonexcludable nature of the software

is important. This importance stems from the second factor identified
in the Introduction. When there are two separate firms, the number of
software types tends to be too small because some consumers who get
access to the software do not pay the original producer, i.e., there
tends to be an underproduction loss. However, when the same firm
produces both computers and software, there is an incentive for the firm
to increase the computer price, while at the same time decreasing the
prices for software. Such an action would reduce the amount of copying,
and in turn, this should result in more types of software being made
available. In addition, since the amount of copying is reduced, this
also explains why there is a reduction in the loss due to
underutilization.

One might argue that in Proposition 7 we have not really
demonstrated that integration tends to reduce the underproduction loss.
The logic is as follows. Propositions 6 and 7 tell us that integration
tends to increase variety both under excludability and under partial
nonexcludability. Hence, one might reasonably argue that the increase
in variety found in Proposition 7 is due strictly to complementarity,
rather than anything to do with partial nonexcludability. In
Proposition 8 we consider a special case of the model being analyzed,
and show that at least for this special case integration does reduce the
underproduction loss.

Proposition 8: If $V_j = \bar{V}$ for all j, then $M_1^E = M_1^N \geq M_2^E \geq M_2^N$.

Proposition 8 considers the case where the valuation a consumer in

group j places on software S_j is independent of the group. It shows
that for this case integration does reduce the underproduction loss.
The interpretation is as follows. Integration under partial
nonexcludability causes variety to move from M_2^N to M_1^E. Further, only
the movement from M_2^E to M_1^E can be explained solely through
complementarity, and thus, the movement from M_2^N to M_2^E must constitute a
decrease in the underproduction loss.

V. Conclusion

The literature has identified two social welfare losses associated
with the private marketing of partially nonexcludable goods. The first
is known as the social welfare loss due to underproduction. This loss
refers to the idea that some consumers get access to the product without
paying the original producer, and hence, the incentive to produce either
quality or variety will be inadequate. The second loss is known as the
social welfare loss due to underutilization. This loss refers to the
inefficiencies which arise because the producer's price is typically
above his marginal cost of production.

In this paper we have considered the market for a particular type
of partially nonexcludable good — computer software. This is an
interesting market because it displays a characteristic not previously
considered in analyses of partially nonexcludable goods. That is,
computer software is frequently sold by the same firm which markets a
complementary product, i.e., computers. This is important in that
having the same firm market this complementary product opens up a number
of avenues through which the losses referred to above can be avoided.
In particular, our analysis suggests that as long as the integrated

manufacturer has some market power in the computer market, then having the same firm produce both goods will tend to reduce both the loss due to underproduction and the loss due to underutilization.

As a final point, however, we would like to mention that one should be somewhat cautious in drawing strong public policy conclusions from our results. For example, as indicated earlier our analysis suggests that having a single firm produce both goods lowers the underproduction and underutilization losses associated with the marketing of computer software. Some might interpret this as saying that government policy should therefore be to encourage this type of integration in production. We feel that such a conclusion would be premature. If computer manufacturers have market power, as is shown to be necessary for our results to hold, then there will be an additional social welfare loss in that too few computers will be sold. Preliminary analysis suggests that having a single firm produce both computers and computer software can move this third social welfare loss in either direction. Hence, even given our results concerning the reduction of inefficiencies in the software market, it is not necessarily clear that this type of integration in production will always be socially beneficial.

Appendix

Proof of Proposition 1: Proposition 1 follows immediately from Spence (1975). The logic is straightforward. Independent of the assumed market structure for computers and independent of whether the software producer is integrated into computer production, one can think of the software producer's problem as choosing a price and quality given that the demand curve is positively related to the quality chosen. As derived in Spence, this will result in socially optimal quality given our assumption that all consumers place the same valuation on increases in the quality of software.

Proof of Proposition 2: Let $\int_0^Y g(y_i)dy_i = \int_0^Z h(z_i)dz_i = \bar{N}$, and let P_C denote the price charged by the computer monopolist. Given this, the maximization problem faced by the software producer is given by

$$(A1) \quad \max_{P_S,Q} \quad (1/\bar{N}) \int_{P_S+P_C-v(Q)}^Y \int_{P_S-c}^Z (P_S-c)h(z_i)dz_i g(y_i)dy_i - F(Q).$$

In equilibrium $P_S-v(Q)$ equals some value δ. We can take a first order condition with respect to Q such that $P_S-v(Q)$ always equals δ. This is given by

$$(A2) \quad (1/\bar{N}) \Big[\int_{P_S+P_C-v(Q_2)}^Y \int_{P_S-c}^Z v'(Q_2)h(z_i)dz_i g(y_i)dy_i$$

$$-\int_{P_S+P_C-v(Q_2)}^Y v'(Q_2)(P_S-c)h(P_S-c)g(y_i)dy_i \Big] - F'(Q_2) = 0.$$

In addition, equation (3) now yields the following first order condition

(A3) $\quad N_2 v'(Q^*(N_2)) - F'(Q^*(N_2)) = 0,$

$$\text{where } N_2 > (1/\bar{N}) \int_{P_S+P_C-v(Q_2)}^{Y} \int_{P_S-c}^{Z} h(z_i) dz_i g(y_i) dy_i.$$

A comparison of (A2) and (A3) immediately yields $Q_2 \neq Q^*(N_2)$. Suppose $Q_2 > Q^*(N_2)$. Combining the preceding result with (3) and (A1) now yields

(A4) $\qquad N_2 v(Q^*(N_2)) - F(Q^*(N_2)) > N_2 v(Q_2) - F(Q_2),$

and

$$(A5) \quad (1/\bar{N}) \int_{P_C+\delta}^{Y} \int_{\delta+v(Q_2)-c}^{Z} (\delta + v(Q_2) - c) h(z_i) dz_i g(y_i) dy_i - F(Q_2)$$

$$> (1/\bar{N}) \int_{P_C+\delta}^{Y} \int_{\delta+v(Q_2)-c}^{Z} (\delta + v(Q^*(N_2)) - c) h(z_i) dz_i g(y_i) dy_i - F(Q^*(N_2)).$$

Equations (A4) and (A5), in turn, imply

$$(A6) \quad v(Q^*(N_2)) \left[N_2 - (1/\bar{N}) \int_{P_C+\delta}^{Y} \int_{\delta+v(Q_2)-c}^{Z} h(z_i) dz_i g(y_i) dy_i \right]$$

$$> v(Q_2) \left[N_2 - (1/\bar{N}) \int_{P_C+\delta}^{Y} \int_{\delta+v(Q_2)-c}^{Z} h(z_i) dz_i g(y_i) dy_i \right].$$

Further, (A6) yields $Q_2 \leq Q^*(N_2)$, i.e., a contradiction. Q.E.D.

Proof of Proposition 3: Because all consumers place the same valuation on software quality, either all consumers who purchase a computer will acquire a unit of software or all consumers won't. Let $(\hat{P}_S, \hat{P}_C, Q_1)$ denote the prices and quality chosen by the integrated manufacturer. Further, consider the following problem. That is, find the profit maximizing triplet such that: (i) $P_S = \hat{P}_S$, and (ii) the number of

consumers who acquire a unit of software is equal to the number who acquire a unit given $(\hat{P}_S, \hat{P}_C, Q_1)$. By definition the triplet $(\hat{P}_S, \hat{P}_C, Q_1)$ must be a solution to this problem.

As indicated earlier, the number of consumers who acquire a unit of software given $(\hat{P}_S, \hat{P}_C, Q_1)$ equals either zero or all computer purchasers. Further, because P_S and the number of consumers who acquire software are being held fixed, the revenue associated with software sales minus the variable costs of software production is also fixed.[14] Together these two facts imply that the constrained choice of P_C and Q discussed above is analogous to the problem investigated in Spence (1975), where because all consumers place the same valuation on software quality, the marginal consumer's valuation for quality equals the average consumer's. Hence, $Q_1 = Q^*(N_1)$. Q.E.D.

Proof of Proposition 4: Let $(\hat{P}_S, \hat{P}_C, Q_1)$ again denote the prices and software quality chosen by an integrated firm. Suppose $\hat{P}_C > c$ and consider the alternaive triplet (P'_S, P'_C, Q'_1), such that $P'_S = c$, $P'_C = \hat{P}_C + \hat{P}_S - c$, and $Q'_1 = Q_1$. Under this alternaive triplet profit will be higher because all surplus is now being captured by the firm, and the total surplus is now no longer reduced by excessive copying. Hence, $\hat{P}_S$ cannot exceed marginal cost under integration, and this implies that the underutilization loss is reduced to zero. Q.E.D.

Proof of Proposition 5: Suppose first that the software producer is not integrated into computer production. Then $P_C = b$ and the same logic as in the proof of Proposition 2 yields $Q_2 < Q^*(N_2)$. Suppose now the software

producer is integrated into computer production. By varying either software quality or the software price, the firm cannot affect the demand curve it faces in the computer market. Hence, $Q_1 = Q_2$, $N_1 = N_2$, and allowing integration has no effect on the software price. Q.E.D.

Proof of Proposition 6: Let $\int_0^Z h(z_i)dz_i = \bar{N}$, P_{S_j} denote the price of a unit of software type j, and P_C again denote the price of a computer. Suppose first the software producer is not integrated into computer production. Any equilibrium will be such that, for any software type j produced, $P_C + P_{S_j} = V_j$, where $P_C \geq b$. Otherwise the software producer could increase profits by increasing P_{S_j}. This yields that S_j will be produced if and only if

(A7) $$(\bar{N}/J)(V_j - P_C - c) \geq F_j.$$

Now suppose the software producer is integrated into computer production. Using the same logic as before we again get that, for any software type j produced, $P_C + P_{S_j} = V_j$. This now yields that software type j will be produced if and only if

(A8) $$(\bar{N}/J)(V_j - b - c) \geq F_j.$$

Since in the non-integrated case $P_C \geq b$, a comparison of (A7) and (A8) yields that if type j is produced in the non-integrated case, then it will necessarily also be produced in the integrated case. Q.E.D.

Proof of Proposition 7: Suppose first the software producer is not integrated into computer production. Then software type j will be

produced if there exists a P_{S_j}, $P_{S_j} \leq V_j - P_C$, such that

$$(A9) \qquad \int_{P_{S_j}-c}^{Z} (P_{S_j}-c)(h(z_i)/J)dz_i - F_j \geq 0,$$

and the actual price chosen will be the price which maximizes the left hand side of (A9). In turn, (A9) implies that for each type j produced, $P_{S_j} \geq c$, and we also know $P_C \geq b$.

Now consider the integrated case. Here software type j will be produced if there exists a P_{S_j}, $0 \leq P_{S_j} \leq V_j - P_C$, such that

$$(A10) \qquad \int_{0}^{Z} (P_C-b)(h(z_i)/J)dz_i + \int_{P_{S_j}-c}^{Z} (P_{S_j}-c)(h(z_i)/J)dz_i - F_j \geq 0,$$

(A10) states that for each type j produced, the optimal strategy for the firm is, if feasible, to set P_C and P_{S_j} such that $P_C + P_{S_j} = V_j$ and $0 \leq P_{S_j} \leq c$. Because of the restriction $V_j - V_k \leq c$ for all j, k pairs, there is nothing to stop the firm from doing this for all software types (note: without the restriction the firm may run up against the non-negativity constraint on software prices). In turn, given that this is the optimal strategy for the firm, a comparison of (A9) and (A10) yields that if type j is produced in the non-integrated case, then it will necessarily also be produced in the integrated case. Further, given that under integration all software types are priced at or below marginal cost, allowing integration must reduce the underutilization loss to zero. Q.E.D.

Proof of Proposition 8: Given the optimal strategy derived for the

integrated manufacturer in the proof of Proposition 7, a comparison of (A8) and (A10) yields $M_1^E = M_1^N$. Hence, given Propositions 6 and 7, all we need demonstrate is $M_2^E \geq M_2^N$.

Suppose $M_2^N \geq M_2^E$. We can order the J types such that $F_1, \ldots F_J$ are in ascending order. Also, let $M_2^N = R$ and $M_2^E = T$. Given $V_j = \bar{V}$ for all j, the net return under excludability for the computer monopolist to produce variety R rather than variety T is given by

$$(A11) \qquad (\hat{P}-c)(\bar{N}/J)(R-T)-(F_R-F_T)T,$$

where $\hat{P}$ is the computer price at which software type R can just break even.

The gross return under partial nonexcludability for the computer monopolist to induce variety R rather than variety T is given by

$$(A12) \qquad (P^+-c)(\bar{N}/J)(R-T),$$

where P^+ is the computer price at which software type R can just break even. We know $P^+ < \hat{P}$. On the other hand, for both variety R and variety T the computer price plus the price of a produced software type must equal $\bar{V}$. Otherwise the computer firm could raise P_C and still get the same software production. Further, because the software price will be higher under variety R, the software producers under variety R will be selling to less consumers in each market than under variety T. Hence, the cost of moving from T to R must exceed $(F_R-F_T)T$. In turn, this in combination with (A11) and (A12) yields $M_2^N \leq M_2^E$. Q.E.D.

Footnotes

[1] See Arrow (1962), Hirshleifer and Riley (1979), Ploman and Hamilton (1980), and Novos and Waldman (1984) for a further discussion of these losses.

[2] For example, IBM and Apple both market software for their home computers.

[3] For a discussion of the pricing of complementary goods see Telser (1979).

[4] Our assumption that the marginal cost of production for the monopolist is less than any consumers's private cost for reproducing the monopolist's output follows the specifications contained in Novos and Waldman (1984) and Johnson (1985). Liebowitz (1985) criticizes this assumption, citing journal articles as an example where the private cost of reproduction is frequently less than the monopolist's marginal cost of production (see also Besen (1986)). We agree that the assumption may be poor for the case of journal articles, since individuals typically copy an article in a journal rather than the whole journal. However, since this type of partial copying is rare in the case of computer software, we feel that our assumption is much more accurate for this case. Note, further, for most of our results this assumption is not critical (see footnote 13). Rather, all that is required is that consumers are heterogeneous in terms of their costs of reproduction - and "for some" consumers this cost exceeds the monopolist's marginal cost of production.

[5] A formal proof of this point is available from the authors upon request.

[6]More formally, what we are assuming here is that the lending market is perfectly competitive, and that there are zero costs involved in lending. Hence, in equilibrium the lending price equals zero. This specification again follows Novos and Waldman (1984) and Johnson (1985). See Besen (1986) and Liebowitz (1985) for interesting analyses which move away from this specification. In particular, Besen allows consumers to form groups when purchasing from the original producer, while Liebowitz allows some agents (libraries) to have lower costs of lending and then explores the implications if the original producer can price discriminate in such a setting.

[7]If it was not assumed that consumers all have the same valuation on increases in the quality of software, then the excludable case would not in general satisfy our second best notion for quality (see Spence (1975)). For a discussion concerning why we impose this assumption see the end of this section.

[8]When we refer to the underutilization loss, we mean the loss due to the _software_ price being above marginal cost.

[9]In dealing with this case we need to make explicit assumptions concerning the compatibility of the software produced across different computers, and the legality of tie-in sales. Our assumption is that the software produced can be used on any firm's computer, and that tie-in sales are not legal.

[10]One point to note is that, in contrast to the result in the previous section that the excludable case always results in the socially optimal quality, here the excludable case will not in general result in the socially optimal variety. Hence, as opposed to what we did in the

previous section, in the following we will not define the underproduction loss as the deviation from what is socially optimal.

[11] We don't allow a Nash assumption on prices because a severe multiple equilibria problem arises.

[12] Because of the discrete nature of the problem, Propositions 6, 7, and 8 are all stated as weak inequalities. For all these propositions it is easy to construct examples where the inequalities are strict.

[13] If we did not assume that each agent's private cost of reproduction exceeds the monopolist's marginal cost of production, then in Proposition 7 allowing integration would not reduce the underutilization loss to zero. This is the only result in the paper which depends on that assumption.

[14] What we are implicitly assuming here is that either there is perfect competition in the secondary market, or the firm sells zero units of software in the primary market, in which case there is no secondary market.

References

Arrow, Kenneth J., "Economic Welfare and the Allocation of Resources for Invention," in The Rate and Direction of Inventive Activity: Economic and Social Factors, Universities-National Bureau Conference Series. Princeton, N.J.: Princeton University Press (for N.B.E.R.), 1962.

Besen, Stanley M., "Private Copying, Reproduction Costs, and the Supply of Intellectual Property," Information Economics and Policy, 5, 1986.

Hirshleifer, Jack, and John G. Riley, "The Analytics of Uncertainty and Information-an Expository Survey," Journal of Economic Literature, 17, December 1979, 1375-1421.

Johnson, William R., "The Economics of Copying," Journal of Political Economy. 93, February 1985, 158-74.

Liebowitz, S.J., "Copying and Indirect Appropriability," Journal of Political Economy. 93, October 1985, 945-957.

Novos, Ian E., and Michael Waldman, "The Effects of Increased Copyright Protection: An Analytic Approach," Journal of Political Economy, 92, April 1984, 236-246.

Ordover, Janusz A., and Robert D. Willig, "On the Optimal Provision of Journals qua Sometimes Shared Goods," American Economic Review, 68, June 1978, 324-338.

Ploman, Edward W., and L. Clark Hamilton, Copyright: Intellectual Property in the Information Age, Boston: Routledge & Kegan Paul, 1980.

Samuelson, Paul A., "The Pure Theory of Public Expenditure," Review of Economics and Statistics, 36, November 1954, 387-389.

Spence, A. Michael, "Monopoly, Quality, and Regulation," Bell Journal

    of Economics, 6, Autumn 1975, 417-429.

Telser, Lester G., "A Theory of Monopoly of Complementary Goods,"

    Journal of Business, 52, April 1979, 211-230.