UTILIZING "CLOSED-ENDED" CONTINGENT VALUATION SURVEY DATA FOR PRELIMINARY DEMAND ASSESSMENTS

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Working Paper #415

August, 1986

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Abstract

Recently, experimental economists have demonstrated the usefulness of "closed-ended contingent valuation" surveys for the assessment of demands in hypothetical markets. The literature has focused upon valuing (non-market) environmental resources. However, the interviewing strategy would seem equally appropriate for more-general market research applications. We have developed a new maximum likelihood estimation technique for use with this type of survey data which solves the problem of truncation bias and readily accommodates explanatory variables. Unlike earlier methods, the estimated models are as easy to interpret as ordinary least squares regression results.

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Market planners and product developers frequently need to assess the market potential for a product which is not yet available for actual trial marketing. Recent developments in techniques for assigning a social value to non-market environmental resources would seem ideally suited to market research tasks. The method is known as "contingent valuation."

In a contingent valuation survey directed at valuing an environmental resource, individual respondents are asked hypothetical questions about how much they would be willing to pay (*WTP*) for access or conversely, how much compensation they would demand (*CD*) to be induced to give up their access. There are three approaches to asking these questions: (i.) "open-ended", where the respondent is simply asked to name the sum, (ii.) "sequential bids", where respondents are asked whether or not they would pay or accept some specified sum (the question is then repeated using a higher or lower amount, depending on the initial response, and so on, until the true value is finally bracketed); and (iii.) "closed-ended", where the respondent is asked only whether or not they would pay or accept a single specific sum. In this third method, the arbitrary sum is varied across respondents.

It would seem that the last contingent valuation approach ought to be preferred, since it generates a scenario similar to that encountered by consumers in their usual market transactions. A hypothetical price is stated and the respondent merely decides whether to "take it or leave it," relieving him of the need to come up with a specific dollar value. It also avoids the pitfalls uncovered by Knetsch and Kahneman (1984) and Boyle *et al.* (1985), where the results from sequential bidding experiments are shown potentially to be strongly biased by the "starting point" (the initial amount quoted). There was no noticeable bias with closed-ended questions.

A comprehensive assessment of the contingent valuation approach has recently been offered in a volume edited by Cummings, Brookshire and Schulze (1986). In fact, one of the papers included in that volume (Randall 1986), anticipates the marketing application we propose here. In defense of the contingent valuation technique, Randall points out that

> "...the goods offered in contingent markets are not always familiar, and individuals may not associate these particular goods with trading possibilities. Nevertheless, unfamiliar goods are often introduced in 'real' markets and, especially, in market experiments. So the distinction between 'real' and contingent markets is, if anything, a matter of degree.

As long as the attributes of the proposed product can be described reasonably accurately, contingent valuation survey techniques may provide valuable information about the probable nature of demand.

An outline of the paper is as follows. In the next section, we review the estimation methods which have been used previously in the empirical literature on closed-ended contingent valuation. The following section contains details of our new maximum likelihood estimation technique for determining parameter values and mean conditional valuations for either marginal or total willingness-to-pay. We then suggest some candidate specifications for actual applications. We examine the viability of some flexible *ad hoc* models as well as conventional demand models which are loyal to the tenets of neoclassical microeconomic consumer theory.

EARLIER ESTIMATION STRATEGIES

Earlier empirical analyses of closed-ended contingent valuation data from field experiments include Bishop and Heberlein (1979), and Bishop, Heberlein, and Kealy (1983) and a pair of papers by Sellar, Chavas, and Stoll

(1985, 1986). Their analyses are based on traditional binary choice models for individual response data. (See Maddala, 1983, for detailed descriptions.) Assume that *Yi* is the individual's true underlying unobserved willingness-to-pay. Let t_i be the single randomly-assigned threshold value offered in the questionnaire, and let $y_i = 1$ signify a response which indicates the $Y_i > t_i$. Let $y_i = 0$ imply $Y_i < t_i$. Both of these earlier groups of researchers arbitrarily select "logit" (as opposed to "probit") analysis to derive fitted choice probabilities for the "yes" ($y_i = 1$) and "no" ($y_i = 0$) responses to the *WTP* questions.[1] In both logit and probit analysis, a linear "index," $\mathbf{x}_i'\beta^*$, captures the influence of the explanatory variables, $\mathbf{x}_i$, on choice probabilities. In the logit model, the choice probability is given by:

$$(1) \qquad \pi_i = \Pr(y_i = 1) = (1 + \exp(\mathbf{x}_i'\beta^*))^{-1}.$$

In the probit model, the choice probability is given by:

$$(2) \qquad \pi_i = \Pr(y_i = 1) = 1 - \Phi(-\mathbf{x}_i'\beta^*)$$

where Φ is the cumulative normal density function. In both of these cases, the underlying coefficients and the standard error, β and σ, cannot be separately identified. (The parameter β^* in (2) is actually β/σ.) Consequently, it is not possible to determine fitted values for the underlying implicit dependent variable, Y_i, because this requires $\mathbf{x}_i'\beta$. It is only possible to construct fitted values for the choice *probabilities*.

These earlier studies typically perform an initial estimation with the threshold value, t_i, as the sole explanatory variable for their binary choice model. They then interpret the fitted choice probabilities, π_i, as the probability that a randomly selected respondent would agree to pay the stated amount as a function of the dollar amount tendered--the "upper tail" of the discrete probability function for the distribution of *WTP* values. In a second step, these cumulative probabilities are used to estimate the "expected value" of *WTP*. Essentially, the process involves computing the value of $\mu = \Sigma Y_i$ $p(Y_i)$, but only values of Y_i between zero and t_i^{max} are included (where t_i^{max} is the largest of the randomly assigned thresholds in the sample). The authors are careful to acknowledge the potential for a serious problem with truncation bias because the offered amounts always have an upper limit (Bishop, Heberlein, and Kealy (1983), p. 623).

An alternative approach, adopted by Sellar, Chavas, and Stoll (1985, 1986), does not use the specific computed values of the fitted discrete probabilities, but instead leaves them in the form of continuous algebraic expressions. While the parameters are fitted on the basis of the discrete levels of Y_i (the t_i) proposed in the survey questions, these investigators prefer to determine the marginal expected value of Y essentially by numerical integration over a continuum of values from zero to the maximum level of t_i used on the questionnaires:

$$E(Y) = \int_0^{t_i^{max}} Y_i \, Pr(Y_i) \, dY_i \qquad (3)$$

However, the problem of truncation bias will still be present.

When other explanatory variables are available, they are included among the right-hand side variables, $\mathbf{x}_i$. However, it is still not possible to compute the *marginal* contribution of any of these other variables to the underlying valuation, $\partial Y_i / \partial \mathbf{x}_i$. Although these additional variables may improve the goodness-of-fit of the discrete choice model, this procedure is still only suited to determining the fitted choice probabilities, π_i, (or the fitted logit *formula* for these probabilities) and therefore permits one to compute only the overall approximate marginal distribution of valuations.

These two-step methods (logit, followed by discrete or continuous marginal mean computations) do not allow recovery of *conditional* distributions of valuation which control for heterogeneity among the respondents.

Important marketing decisions may depend on the extent to which certain *attributes* of a proposed product contribute to its overall value to consumers. The value to prospective buyers is certainly derived from the levels of--and interactions between--a wide variety of product characteristics (such as size, weight, color, fragrance, etc.) as well as the characteristics of the consumers themselves (i.e. income, age, sociodemographic group, and individual tastes), and probably even the local consumption environment (unemployment rates, interest rates, consumer optimism, or other general barometers of the health of the economy). Therefore, it is essential to know the marginal contributions to "unit" value of all these factors in order to be able to predict the impact on potential sales of (a.) minor adjustments to the attributes of the product itself, (b.) changes in the composition of the target consumer group, or (c.) changes in the local macroeconomic environment.

In contrast to the usual logit model, the maximum likelihood estimation method developed in the present paper is directly related to the probit model. At the end of the next section, after we have described this new approach, we will show explicitly how our method is related to a probit analog of the conventional procedure. In preview, these earlier applications could have used the parameter estimates from simple probit analysis to compute point estimates for the parameters of *any* arbitrarily specified valuation function.[2] Approximate standard error estimates, however, are somewhat tedious to compute, and will be different from the asymptotic standard errors produced directly by our new one-stage method.

NEW ESTIMATION PROCEDURE

Of course, if we knew the precise dollar figure each individual would be willing to pay for a particular hypothetical product, then any theoretically consistent model (or even a completely *ad hoc* specification) which yields inverse demand functions that are linear-in-parameters could be adopted, and straightforward ordinary least squares linear regression analysis would probably be quite satisfactory as an estimation technique. However, with the yes/no responses to "closed-ended" contingent valuation surveys, qualitative choice techniques are clearly necessary. Because the offered amounts are varied over individuals, the yes/no responses convey some diffuse information about the amount of dispersion in the presumed underlying continuous dependent variable, Y_i. Rather than using the familiar but limiting logit estimation methods described in the last section, we propose an innovative[3] "censored" dependent variable technique which exploits this information.

Assume that the unobserved continuous dependent variable is the respondent's true willingness-to-pay (*WTP*) for the product, Y_i. Each individual is confronted with a threshold value, t_i, and by his (yes/no) response, we conclude that his true *WTP* is either greater than or less than t_i. If we are willing to assume that the underlying distribution of Y_i, conditional on a vector of explanatory variables, $\mathbf{x}_i$, has some known distribution with a mean of $\mathbf{x}_i'\beta$, maximum likelihood techniques are appropriate. An individual will respond in a manner which suggests that his *WTP* is more than t_i if the difference between his true *WTP* and its conditional expected value, $u_i = (Y_i - \mathbf{x}_i'\beta)$, is greater than the difference $(t_i - \mathbf{x}_i'\beta)$. Let an acceptance of t_i be denoted by $y_i = 1$, and a rejection by $y_i = 0$. We can then write:

$$(4) \qquad \Pr(y_i=1 \mid \mathbf{x}_i) = \Pr(u_i > t_i - \mathbf{x}_i'\beta)$$

We know that the random error term u_i has a mean of zero and the same variance as the conditional distribution of Y given $\mathbf{x}$.[4]

We might choose to assume a normal distribution for this conditional density function,[5] which yields:

$$(5) \qquad \Pr(y_i = 1 \mid \mathbf{x}_i) = \Pr(u_i/\sigma > (t_i - \mathbf{x}_i'\beta)/\sigma) = \Pr(z_i > (t_i - \mathbf{x}_i'\beta)/\sigma)$$

where z_i is the standard normal random variable (in this context only). Hence

$$(6) \qquad \Pr(y_i = 1 \mid \mathbf{x}_i) = 1 - \Phi((t_i - \mathbf{x}_i'\beta)/\sigma)$$

$$\Pr(y_i = 0 \mid \mathbf{x}_i) = \Phi((t_i - \mathbf{x}_i'\beta)/\sigma)$$

For a given sample of n independent observations, the joint density function for the data, $f(y|t, x_1, \ldots, x_p, \beta, \sigma)$, can then be reinterpreted as the likelihood function:

$$(7) \qquad L = f(\beta, \sigma | y, t, x_1, \ldots, x_p)$$

$$= \prod_{i=1}^{n} \left[\left[1 - \Phi\left[\frac{t_i - \mathbf{x}_i'\beta}{\sigma}\right]\right]^{y_i} \left[\Phi\left[\frac{t_i - \mathbf{x}_i'\beta}{\sigma}\right]\right]^{1-y_i} \right]$$

Taking logs, we have

$$(8) \qquad \log L = \sum_{i=1}^{n} \left[y_i \log\left[1 - \Phi\left[\frac{t_i - \mathbf{x}_i'\beta}{\sigma}\right]\right] + (1-y_i)\log\left[\Phi\left[\frac{t_i - \mathbf{x}_i'\beta}{\sigma}\right]\right]\right]$$

Nonlinear optimization techniques may then be employed to maximize the value of this function with respect to the vector of coefficients, β, and the standard deviation of the conditional distribution of valuations, σ. For most such optimization algorithms, the estimation process can be facilitated by the provision of analytical first (and often second) derivatives. The formulas for these derivatives are provided in the Appendix.

Hypothesis testing in this framework is the same as in any maximum likelihood context. Asymptotic t-tests statistics can be used to assess individual parameters; Likelihood Ratio tests (or Wald, or Lagrange multiplier tests) can be used to test restrictions on subsets of the coefficients in the model).

Once the optimization process has yielded the required parameter estimates, an empirical investigator will usually be interested in determining the goodness-of-fit of the estimated model. Here, we can employ the same sorts of measures which are traditionally used with logit or probit models. Two standard measures are (a.) individual prediction success, and (b.) aggregate prediction success.

In computing individual prediction successes, one counts up the number of observations for which the model predicts a probability exceeding 0.5 that the respondent should be willing to pay the stipulated price when the individual is observed to respond that he *would* pay that amount. This is a "prediction success." A *conflict* between the individual's choice probability and the qualitative response indicates a prediction failure. The number of successes as a proportion of the total sample is one measure of the accuracy of the model in explaining individual choices. However, as with other discrete choice models, this can be an overly stringent criteria for judging goodness-of-fit because it ignores "near misses", attaching an equal degree of accuracy to a probability of 0.501 and to 0.999 (and likewise penalizing as harshly for a probability of 0.499 as for a probability of 0.001) when $y_i=1$ is actually observed.

An alternative measure of prediction success is "aggregate prediction success." In this case, each respondent is assumed to represent some large equal number of respondents with identical characteristics, so that the choice probability for this one individual can be viewed equivalently as the proportion of his identical cohort which would be willing to pay the stipulated price. The individual fitted probabilities for the "will pay/won't pay" responses can therefore be summed and compared to the actual frequencies of response in the data. Typically, this aggregate measure yields a prediction success rate much higher than in the individual case.[6]

We promised at the end of Section 2 that a simple relationship between our method and the results of simple probit estimation could be identified. This is easily seen by comparing the choice probability in equation (2) with that in equation (5). Clearly, the expression $(t_i - \mathbf{x}_i'\beta)/\sigma$ can be rewritten as the inner product:

$$(9) \qquad (t, \mathbf{x}') \, (-1/\sigma, \, \beta/\sigma) = -\mathbf{x}_a'\beta_a$$

and the augmented vectors of variables, $\mathbf{x}_a$, and coefficients, β_a, may be treated as one would treat the explanatory variables and coefficients in an ordinary probit estimation. In fact, this relationship between the conventional probit model and our new estimation method demonstrates that if previous researchers had utilized probit techniques, they would have had at their fingertips a set of point estimates of the underlying β parameters which (a) describe the incremental contribution to value of an extra unit of each explanatory variable and (b) are required for determining fitted values of the underlying willingness-to-pay. Specifically, if the threshold value t_i is included among the *explanatory* variables in an ordinary probit model, its coefficient yields an estimate of $\alpha = -1/\sigma$. The coefficients on the other variables are $\gamma_j = \beta_j/\sigma$, so the desired βs may be found by dividing the other coefficients by the coefficient on t_i, and changing the sign. The σ itself is of course easy to compute.[7]

Having identified this relationship between the two methods, it is evident that the elaborate second-stage numerical integration techniques employed by Sellar, Stoll, and Chavas (1985) are not required, even if simple probit estimation is used, since the point estimates (once they have been transformed to yield the β and σ parameters) can be interpreted in the same way as one would interpret the coefficients of a common multiple regression model.

The point estimates of the individual parameters should be identical by either the full maximum likelihood method or by the transformed probit technique, but for any exercise in hypothesis testing, one requires accurate standard error estimates. Ordinary probit analysis will generate asymptotic standard error estimates for the parameters α and γ_j. Once the probit model is estimated, it is straightforward to compute $\sigma = -1/\alpha$ and $\beta_j = -\gamma_j/\alpha$.[8] However, standard errors for these functions of the estimated parameters cannot be directly estimated. They must be derived, possibly by using Taylor series approximation formulas for their variances (Kmenta 1971, p. 444):

$$\mathrm{Var}(\sigma) = \mathrm{Var}(-1/\alpha) = [1/\alpha^2]^2\ \mathrm{Var}(\alpha)$$

$$\mathrm{Var}(\beta_j) = [\gamma_j/\alpha^2]^2\ \mathrm{Var}(\alpha) + [-1/\alpha]^2\ \mathrm{Var}(\gamma_j) + 2\ [\gamma_j/\alpha^2][-1/\alpha]\ \mathrm{Cov}(\alpha,\gamma_j)$$

In contrast, accurate asymptotic standard error estimates can be produced directly by our new estimation procedure. The negative of the inverse of the Hessian matrix (evaluated at the optimal parameter estimates) yields, asymptotically, the Cramer-Rao lower bound for the variance-covariance matrix for the estimated parameters. The square roots of the diagonal elements give the desired asymptotic standard errors. These facilitate hypothesis testing regarding the signs and sizes of individual β_j parameters, an important objective of the modeling process.

CANDIDATE SPECIFICATIONS FOR WILLINGNESS-TO-PAY MODELS

Researchers may find it appropriate to design the critical survey questions to address willingness-to-pay for a *single* unit with a particular configuration of characteristics (as in the case of a large consumer durable). Alternately, the proposed product may be a close substitute for a product that the consumer currently purchases regularly (such as public transportation, beer, long distance telephone services, or weekly newsmagazines, to cite a few examples). In this case, the survey questions should first establish the number of units of the good typically purchased during a given time interval. Willingness-to-pay in this context will be systematically related to the quantity of the product typically consumed. *Ceteris paribus*, we would expect willingness-to-pay for an extra unit to decline with the number of units consumed. However, it is important to infer correctly the information which is actually being elicited from the respondent. If the scenario requires the consumer to indicate (indirectly) the per-unit price willingly paid when their consumption flow is q_i units per period of time, then we must assume that this is their *average* willingness-to-pay. To uncover the inverse demand curve underlying this response, we must estimate the model using total value of the consumption stream, $T_i = q_i Y_i$. The *marginal* value to the consumer of the last unit consumed can then be determined from the fitted value of the corresponding expression for $\partial T_i / \partial q_i$.

For the following analysis, it will be useful to distinguish three types of variables: q_i, the number of units of the product being valued, $\mathbf{x}_i$, characteristics of the product being valued; $\mathbf{z}_i$, personal characteristics of the individual being asked to make the valuation; and $\mathbf{w}_i$, variables describing the current (local) macroeconomic environment (which will include the prices of other goods).

Valuing a Single Marginal Unit of a Product

With rich enough data on the circumstances under which the respondent is making the hypothetical purchase decision, one could adopt any functional form for the inverse demand relationship which was consistent with the microeconomic theory of consumer optimization. However, it is not straightforward simply to adapt one of the often-utilized theory-based demand formulas, such as the linear-in-parameters LES or AIDS models described in Phlips (1983).[9] The reason is that these models are couched in terms of quantities demanded (or desired expenditures), so that the demand functions are of the form $q = f(p,M,\ldots)$, where M is income or total expenditure. The usual problem addresses maximum quantity demanded as a function of exogenously determined prices and income. For our purposes, though, we require the *inverse* demand function: $p = g(q,M,\ldots)$. We are concerned with the maximum price willingly paid for a given consumption stream, q. The appropriate strategy would seem to be to solve the first order conditions for utility maximization for prices instead of quantities, and then to impose the necessary regularity conditions. Our suspicion is that the resulting system of inverse demands would be highly nonlinear,[10] since (for example) the homogeneity condition makes quantities demanded depend upon relative prices and real incomes. Supposing that an inverse demand function could be derived, incorporation of the formulas into our estimation estimation procedure would involve replacing $(t_i - \mathbf{x}_i'\beta)/\sigma$ in the log-likelihood function with $(t_i - g(q,M,\ldots))/\sigma$. While the option of using numeric derivatives will allow some very untidy log-likelihood functions to be optimized, the LES form loses its primary appeal, linearity, when it is converted to an inverse demand formula. This suggests that researchers may wish to delve into the possibility of deriving theoretically sound in~~direct~~verse demand systems which are linear-in-parameters.

In practice, therefore, most analyses will be constrained by deficient data to working with plausible *ad hoc* specifications for the *WTP* function. If we are addressing the case where a one-time purchase of a product is being considered, reasonable first-generation models might include variants of the following two basic forms:[11]

(10) *linear:* $Y_i = \beta_0 + \beta_2\mathbf{x}_i + \beta_3\mathbf{z}_i + \beta_4\mathbf{w}_i + \epsilon_i;$

(11) *log-linear:* $log(Y_i) = \beta_0 + \beta_2\mathbf{x}_i + \beta_3\mathbf{z}_i + \beta_4\mathbf{w}_i + \epsilon_i.$

Since the Y_i in these cases is considered to be the "height" of the demand curve, our microeconomic intuition applies very easily to the signs on the individual parameters.

Valuing Total Units of a Product

The conceptual problem is a little more complex when we are querying the consumer about what (average) price would be willingly paid to replace their current consumption stream with the new good. As indicated above, the implicit dependent variable will be $T_i = q_iY_i$, rather than just Y_i. Likewise, the threshold level of this variable must be similarly transformed: $\tau_i = q_it_i$. As with marginal *WTP*, it would be desirable to adhere to formal theoretical specifications whenever the data can support them, but we will typically have to be satisfied with sensible *ad hoc* specifications. To be consistent with our conviction that demand curves ought to slope downward, the first derivative (with respect to q_i) of any proposed total valuation function must vary with q_i. Some possibilities, with their associated marginal valuation functions, are as follows:

(12) *quadratic:*

$$T_i = q_i Y_i = \beta_0 + \beta_1 q_i + \alpha_1 q_i^2 + \beta_2 \mathbf{x}_i + \beta_3 \mathbf{z}_i + \beta_4 \mathbf{w}_i + \epsilon_i$$

$$\partial T_i / \partial q_i = \beta_i + \alpha_1 q_i$$

Fitted marginal *WTP* for an additional unit of the hypothetical product need not always be positive, but if β_1 is positive and α_1 is negative, it will be positive for at least some levels of q_i, and it will be linear and downward-sloping. This model would allow product attributes, consumer characteristics, and local macroeconomic conditions to affect total *WTP*, but not marginal *WTP*, which is somewhat restrictive. As an alternative, we might consider:

(13) *quadratic with interaction terms:*

$$T_i = q_i Y_i = \beta_0 + \beta_1 q_i + \alpha_1 q_i^2 + \beta_2 \mathbf{x}_i + a_2 \mathbf{x}_i q_i$$
$$+ \beta_3 \mathbf{z}_i + \alpha_3 \mathbf{z}_i q_i + \beta_4 \mathbf{w}_i + \alpha_4 \mathbf{w}_i q_i + \epsilon_i$$

$$\partial T_i / \partial q_i = \beta_1 + \alpha_1 q_i + \alpha_2 \mathbf{x}_i + \alpha_3 \mathbf{z}_i + \alpha_4 \mathbf{w}_i$$

This specification *will* allow product attributes, consumer characteristics, and local macroeconomic conditions to shift the marginal *WTP* curve in (Y,q)-space, where Y is in per-unit terms.

When using quadratic specifications, however, it is important to check the fitted marginal *WTP* function to ascertain whether the model predicts that particular respondents have negative marginal *WTP* at their current number of days. This is a distinct possibility, given the diffuse nature of the actual sample information on Y. If the investigator is unwilling to argue that negative marginal *WTP* is plausible, it will probably be advisable to consider alternative specifications which constrain the marginal *WTP* to be positive. One such alternative is:

(14) *linear in logarithms:*

$$\log T_i = \log q_i Y_i = \beta_0 + \beta_1 \log q_i + \beta_2 \mathbf{x}_i + \beta_3 \mathbf{z}_i + \beta_4 \mathbf{w}_i + \epsilon_i$$

$$\partial T_i / \partial q_i = (\beta_1 / q_i) \exp(\beta_0 + \beta_1 \log q_i + \beta_2 \mathbf{x}_i + \beta_3 \mathbf{z}_i + \beta_4 \mathbf{w}_i)$$

where we substitute the fitted value of T_i in the derivative because the actual total *WTP* is of course unobserved. This marginal *WTP* will be positive as long as $\beta_1 > 0$, and downward-sloping as long as $(\beta_0 + \beta_1 \log q_i + \beta_2 \mathbf{x}_i + \beta_3 \mathbf{z}_i + \beta_4 \mathbf{w}_i) > \beta_1$. Attribute effects, respondent characteristics, and macro conditions will shift the marginal *WTP* curve due to their presence in the fitted *WTP* function. However, virtually any transformation of T_i can be considered.[12]

This paper emphasizes our new methodology for estimating models using closed-ended contingent valuation data, highlighting its anticipated value in establishing the market potential for products which are not yet ready for actual test-marketing. Readers who may be interested specifically in an application of this technique to field data in a project directed at the valuation of a non-market resource are referred to our comprehensive study on a recreational salmon fishery, Cameron and James (1986b). A collateral paper is Cameron and James (1986a) which specifically addresses the accuracy of the probit approximation technique relative to the full maximum likelihood model. That paper focuses on the similarities between our new method and ordinary probit models. We demonstrate how estimates of the desired coefficients and approximate standard errors can be obtained even if a conventional probit algorithm is the only software available. The paper contains an example of a log-linear model for marginal *WTP* estimated by each of the two methods, plus a side-by-side comparison of the results.

Compared to conventional methods for estimating models using closed-ended contingent valuation data, the ability to determine the incremental contributions to total *WTP* of each explanatory variable is the clear advantage of this new approach. Nevertheless, market researchers will probably still be interested in the marginal mean of the distribution of individual *WTP* values in the population. (As outlined above, earlier models could determine *only* this quantity.) It is a simple matter to compute the weighted marginal distribution of *WTP* predicted by this model. Of course, the marginal mean depends entirely upon the distribution in the sample of the **x**, (or, more specifically, the **x**, **z**, and **w** variables utilized in the section on functional forms). If, for example, planned changes in product attributes affect the values of some of the **x** variables, one must know the incremental contribution of each variable to valuation before it is possible to simulate the effects of such a change on the marginal mean distribution of *WTP*s. An ability to do this makes our model superior to its predecessors.

SUMMARY

In this paper, we have described an efficient estimation method for fitting models which use "closed-ended" contingent valuation survey data. These survey instruments are becoming increasingly popular, and they show considerable promise for assessing the market potential for products which have not yet been developed, or are not yet being produced in quantities large enough to allow actual test marketing.

Any model of valuation must of course be based upon valid theoretical foundations. It is still up to the individual investigator to specify a theoretically plausible relationship between the underlying unobserved willingness-to-pay, *Y*, and the explanatory variables, **x**, appropriate to the application at hand. In the past, empirical work using contingent valuation data was been limited by the fact that we do not have an opportunity to observe *Y* directly. Use of inappropriate estimation methods limited the generality of the *WTP* functions unnecessarily. With the algorithms described in this paper, however, any specification which could be estimated by multiple regression techniques if *Y* were known can now be estimated with only contingent valuation responses, and the estimated coefficients can be interpreted in exactly the same way as in those regression models.

Analysts need no longer be limited merely to the estimation of approximate mean *WTP* over an entire sample under the implicit assumption that all consumers are identical and each is presented with the identical hypothetical product. Instead, it is possible to distinguish the incremental contributions to *WTP* made by product attributes, due to individual consumers' characteristics, and resulting from current local macroeconomic conditions.

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TECHNICAL APPENDIX

Using the notation established in the text, we first define the following simplifying abbreviations (z denotes the standard normal random variable in this appendix):

$$z_i = (t_i - x_i'\beta)/\sigma$$

$$\Phi_i = \Phi(z_i)$$

$$\phi_i = \phi(z_i)$$

$$\phi'_i = \phi'(z_i) = -z_i\, \phi(z_i)$$

$$R_i = x_{ir}x_{is}\, \phi'_i$$

$$S_i = x_{ir}x_{is}\, \phi^2_i$$

$$T_i = x_{ir}z_i\, \phi'_i$$

$$U_i = x_{ir}z_i\, \phi^2_i$$

$$V_i = z^2_i\, \phi'_i$$

$$W_i = z^2_i\, \phi^2_i$$

The gradient vector for this model is then given by:

$$\frac{\partial \log L}{\partial \beta_r} = \frac{1}{\sigma} \sum \left[\frac{y_i x_{ir}\, \phi_i}{1 - \Phi_i} - \frac{(1 - y_i)x_{ir}\, \phi_i}{\Phi_i} \right] \qquad r = 1,\ldots,p$$

$$\frac{\partial \log L}{\partial \sigma} = \frac{1}{\sigma} \sum \left[\frac{y_i z_i\, \phi_i}{1 - \Phi_i} - \frac{(1 - y_i)z_i\, \phi_i}{\Phi_i} \right] \qquad r = 1,\ldots,p$$

The elements of the Hessian matrix can be simplified if we define the function:

$$G(P,Q) = \sum \left[\frac{y_i(P_i[\Phi_i - 1] - Q_i)}{[\Phi_i - 1]^2} + \frac{(1 - y_i)(P_i\Phi_i - Q_i)}{\Phi^2_i} \right]$$

Then we can specify:

$$\frac{\partial^2 logL}{\partial\beta_r\partial\beta_s} = \frac{1}{\sigma} G(R,S) \qquad r,s = 1,\ldots,p$$

$$\frac{\partial^2 logL}{\partial\beta_r\sigma} = \frac{-1}{\sigma}\frac{\partial logL}{\partial\beta_r} + \frac{1}{\sigma^2} G(T,U) \qquad r = 1,\ldots,p$$

$$\frac{\partial^2 logL}{\partial\sigma^2} = \frac{-1}{\sigma}\frac{\partial logL}{\partial\sigma} + \frac{1}{\sigma^2} G(V,W)$$

Use of these analytic derivatives, instead of numerical approximations to the required derivatives, can reduce computational costs considerably.

FOOTNOTES

[1] There seems to be no compelling reason (other than computational convenience) for selecting logit over probit methods, since it is well known that the density functions involved are quite similar except in the tails.

[2] A further limitation of earlier empirical research is the authors' reliance on a specific functional form within the logit index. They utilize $\pi_i = (1 + \exp(-(\ln \alpha + \beta \ln t_i + \delta \ln q_i))$, where q_i is the number of usage days. The choice of logarithms seems quite arbitrary; this functional form simply offers algebraic convenience while simultaneously facilitating downward-sloping estimated underlying "demand" curves. These authors go to considerable lengths to derive the plausible range of fitted values for the coefficient δ, to ensure that the fitted model is consistent with downward-sloping demand, but surely, other functional relationships ought to be entertained.

[3] To our knowledge, the only paper adopting a strategy remotely similar to ours is Lerman and Kern (1983). In their model, however, the observable information on the dependent variable is in the form of a maximum bid. They argue that if the transaction price actually paid for a house is the maximum bid price in the population, and if the distribution of potential bid prices is Gumbel, the transaction price can be used to identify the shape parameter of that distribution.

[4] Homoscedasticity of the errors will be assumed in this development. In specific applications, a logarithmic (or possibly Box-Cox) transformation of the implicit dependent variable may accommodate a certain amount of systematic heteroscedasticity in the implicit dependent variable.

[5] The following formulation could be cast in terms of an analog to the familiar logit model with its hyperbolic secant-squared distributions. However, the normal density and cumulative probability density functions are more in keeping with the usual regression assumptions.

[6] As always, these "within sample" rates of prediction success should not be expected to apply to "out of sample" predictions.

[7] Ordinary probit analysis can therefore be employed to produce excellent starting values for the estimation process described in the previous section.

[8] Indeed, if the sample being utilized is representative of the population about which the investigator wishes to make inferences, the conventional probit algorithms in any one of a number of statistical packages may well be adequate. Problems arise, however, when it is necessary to devise weights for each observation so that it will more-accurately reflect the true frequency of each type of respondent in the population. (Exogenously determined weights are frequently required, for example, in surveys with systematic non-response.) If the packaged probit routine does not allow weights on the observations, it will be necessary either to modify the source code for the packaged routine, or to write new code which allows weights. If this much effort is to be invested, it is probably preferable to go directly to the algorithms proposed in this paper.

[9] These systems of demand equations satisfy the general restrictions required for the demand functions to be consistent with the received theory of constrained utility maximization: homogeneity of degree one in prices and income, "adding up", symmetry of the cross-substitution effects, and negativity of the own-price effect (see Phlips 1983, 32-56).

[10] For example, if we were to adhere to the functional form of a *single* equation in an LES demand system, (see Phlips 1983, 125), the corresponding "inverse demand function" would be:

$$p = g(q,M) = \beta \left[M - \sum_{j=i} p_j \gamma_j \right] / (q + (\beta - 1)\gamma)$$

where the p_j are the prices of all other relevant commodities, β is the marginal propensity to consume out of "discretionary income," and γ_j is interpreted as subsistence expenditure on the j^{th} good (or on this good, if no subscript). However, this neglects the presence of this good's price in the direct demand functions for each of the other goods in the system.

[11] In implementing the log-likelihood function, note that transformations of the unobservable Y_i variable appear in the formulas as the identical transformation applied to the threshold value, t_i. Fortunately, no Jacobian term is required to preserve the validity of the underlying density function, because the density pertains to the discrete variable, y_i, which is not transformed.

[12] In our application to the valuation of a recreational fishery, we utilize Box-Cox transformations of the implicit dependent variable (Cameron and James 1986b)