INDIVIDUAL AND COLLECTIVE WAGE BARGAINING

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1. Introduction

The traditional competitive model predicts that workers will be paid their marginal productivity, but in practice wage variations are often encountered that do not correspond directly to differences in productivity. Existing theoretical explanations take into account factors like e.g., turnover costs, search costs, and employees risk aversion. However, these explanations are mainly concerned with the technological or organizational context and tend to ignore the fact that wages (if not imposed from above by the government) are directly or indirectly the outcome of negotiated contracts between the workers (individually or collectively) and the employers -- and that, except for situations where one side or the other has monopolistic control, there is significant bargaining power on both sides which ought to be analyzed and taken into account.

The objective of this study is to develop a game theoretical framework for wage determination that explicitly considers the bargaining positions that the workers and employers enjoy under a variety of broadly-defined institutional structures that might be expected to influence the structure and outcome of wage negotiations. Our primary focus in this paper is on modeling the labor side, specifically, the nature of the outside opportunities available for those "inside" workers who might (as a bargaining tactic) threaten to leave the occupation,[1] the nature of the "outside" work force that is available to the employer (if such a threat were carried out), and, most important, the extent to which the workers, both insiders and outsiders, may be organized into a union or unions. In future work we would like to investigate in more detail the various institutional structures that might be adopted on the

1. It will be observed that in considering differentiated outside opportunities we depart from the usual assumptions of the competitive model by allowing workers and employers to be *heterogeneous:* the workers in terms of personal traits possessed in addition to those skills required for a particular job, and the employers in terms of the specific subsets of personal traits that they do require.

employers' side, there being a somewhat delicate balance to be struck between cooperation in wage negotiations and competition in the product market.

The theory of oceanic games (Milnor and Shapley [1961,1978], Hart [1973], Guesnerie [1977]) is applied in solving for the negotiated settlement. The virtue of this theory is that it deals effectively in the same model with two very different types of players. On the one hand there are the large players - to wit, employers and unions, which by their individual decisions can have substantial impact on the overall outcome of the game. On the other hand, there is a continuum of individual players,[2] namely the pool of unorganized workers, who influence the "big picture" only in so far as they may join together in coalitions having measure greater than zero. The present investigation is perhaps the first substantial application of oceanic game theory outside the field of voting games.[3]

Our model obtains a wage solution whose major feature is that the variations in the workers' wages for the job under consideration depend on variations in the relationship between their productivity on the job and their outside opportunities. Specifically, it is shown that when we control for employers' structure and on-the-job productivity, the worker's wage is positively affected by his outside opportunities and is inversely related to the size of the total labor force. In this case, wage variations will *overstate* productivity variations. As a generalization of the theory, workers may differ from each other in their on-the-job productivity. If variations in on-the-job productivity are larger than variations in the outside opportunities our theory will predict that variations in earning among individuals will *understate* productivities variations. As payoffs under different institutional structures are derived, it is also shown that under certain conditions the members of a partial union can do

2. "...referred to collectively as an 'ocean', to suggest [their] total lack of order or cohesion..." (Milnor and Shapley [1978]).

3. Some simple oceanic production models were examined in Shapley and Shubik [1967] and Shapley [1967]. Recent application of value theory can be found in Levy [1986]. For a voting application see Shapley [1961] or the appendix to Milnor and Shapley [1978].

better by bargaining collectively than by bargaining as individuals. In addition, when *all* workers are unionized and bargain as a unit, their total payoff is under quite general conditions higher than what they get if they bargain as individuals.

The paper is organized as follows: Section 2 describes the approach and the theory of oceanic games. In Section 3, model 1 which derives the wage solution in the presence of unorganized labor is presented. This is followed by an example, given in Section 4. Section 5 contains model 2 and deals with the cases of partially organized and fully organized labor force. A union is introduced and the wage solution is derived. In Section 6 we illustrate the solution of model 2 by using demand and supply functions of a simple linear form. Section 7 contains a discussion, and concluding remarks are given in Section 8.

2. Approach

To the extent that occupations differ in their requirements for personal traits such as ability, physical strength, health and occupational training, there will be a difference for a given worker between productivity value at his "best" job and productivity value at his "next best" job. Likewise to the extent that workers have different endowments of traits, they will differ in their productivity value in performing a given job for a given employer. In addition, workers may differ in demographic characteristics that do not affect productivity at any job, but affect the set of feasible jobs available to them (e.g., marital status). The existence of such differences makes it costly for a given worker to substitute jobs or employers and makes it costly for a given employer to substitute workers. These costs of substitution create a potential for strategies to be used in wage bargaining, both on the part of the employer and the part of the workers. Workers have power to bargain as individuals or alternatively to organize and bargain as a unit.

To develop the key ideas as clearly and simply as possible without distorting the essential issues, and to highlight the role of outside alternatives, we would like to consider the restricted type of heterogeneity in which workers have the same productivity in the job under consideration and differ only in their outside opportunities.[4] This is not so drastic an assumption as it might appear, since we are in any case using a continuum to approximate the population of workers. Indeed, if the workers have different productivity on the job under consideration, we can "normalize" by representing them by infinitesimals of different size. In effect, this reformulates the supply and demand functions for labor in terms of productivity units rather than people units. Thus, all the results will apply, after proper normalization, to the more general case in which workers productivity may vary both on-the-job and outside.

On the demand side, we let the employers negotiate collectively with the union, i.e., bargaining occurs at the occupation level.[5] Let the labor demand be given by $f(x)$, which is the dollar value of the marginal product of the job under consideration as a function of the number of employed workers. Let $g(x)$ be the labor supply curve, which is the dollar value of the xth worker's best outside opportunity. Variables that affect labor supply and demand are assumed to be constant. In the absence of bargaining, some workers would be employed at the occupation under consideration (call it "the employer") at the equilibrium wage associated with the intersection of $f(x)$ and $g(x)$,[6] whereas others would be working in a different

4. For example, consider two workers who differ only in their physical strength. Suppose that the "best" job does not require any physical strength whereas the second best job does. In this case the workers are equivalent with respect to one employer but not equivalent with respect to the other employer. The worker with the greater physical strength will have a lower potential wage differential between the jobs.

5. Although the institutional structure of the employers clearly plays a major role in wage outcome, investigation in detail of that issue is beyond the scope of this paper. Here we view the employers as forming an employers' association which negotiates as a unit with labor.

6. As a reference point, the simple competitive solution is considered. Nonsalvageable capital investment or alternatively different costs functions among employers is sufficient to create specific rent in the occupation.

occupation. The latter are willing to switch to the employer only at a wage at least as high as the wage in their present job. Let the workers who would be employed by the employer at equilibrium be called *insiders* and the remainder *outsiders*.[7]

Insiders clearly have some bargaining power, for they can always choose to quit. The employer then has to hire a more expensive workers (i.e., ones with better outside alternatives), thereby cutting the total surplus available. The significance of this effect depends on the alternative wages of the outsiders. The higher their alternative wages the more power the insiders will have. In the extreme case, there are no outsiders (i.e., their alternative wages approach infinity) and the insiders enjoy maximum power, since no substitution is available for the employer.

On the other hand, the employer has obvious bargaining power over the inside workers, since he can threaten to fire them. The imposed costs are equal to the difference between the current wage and the insider's outside opportunities. The employer has most power over those workers with the most inferior opportunities, since they are the ones who would lose most by being fired. Finally, it is interesting to remark that the outsider workers also have something to bargain about, simply because the extent of their availability affects the relative bargaining positions of the employer and the insiders. This last item is a relatively minor effect, but our game theory model will be able to detect it and quantify it.

All these bargaining powers, variously possessed by employer and workers, are ignored by the classical, marginal productivity solution. But they are evidently relevant to the analysis of wage determination and should be taken into account if possible. In the present, game-theoretic approach, a central role is played by the so called "characteristic function" of the game, a function $v(S)$ which reflects the bargaining power of the players by specifying how much each subset S of them (a "coalition") could achieve separately if there is no general,

7. Since workers have equal productivity from the point of view of this employer, the insiders will be the ones with the lowest market opportunities.

overall agreement. More precisely, $v(S)$ denotes the total dollar value that S can obtain for its members *at least*, regardless of the actions of those outside the coalition[8].

The specific solution concept we shall employ is the Shapley value [1953], as extended to apply to the "oceanic" type of model in which a finite set of *major* players (called "atoms"), each capable of influencing the outcome, co-exist with a continuum of *minor* players (the "ocean"), whose influence is felt only when integrated over a set of positive measure. The Shapley value of such a game affords an unbiased appraisal of the contributions of all players, major and minor, toward the grand agreement on how the fruits of full cooperation should be divided among the participants[9]. A more precise development of these ideas will be given in context, during consideration of the models in the sequel.

3. Model 1 - Fully Unorganized Labor

Consider a market consisting of an employer E and an ocean $[0,n]$ of workers. Let f denote the labor demand function and g the labor supply function. With no essential loss of generality, we may assume that $g(n)=f(0)$. The "worth" $v(S)$ of a coalition S is the total

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8. This is a "one-dimensional" characteristic function; it is justifiable here because the issues at stake (wages, shares of the surplus) are naturally measured in terms of a money, which, for our purpose, can be assumed to be freely transferable among the players. When other kinds of issues are involved (e.g., working conditions, job security, union recognition, etc.), serious modeling problems arise in *defining* such a one-dimensional utility function for coalitions. (cf. Blair and Crawford [1984]), and a more general game theoretical apparatus should be employed.

9. Thus, this theory of multi-lateral bargaining is a "Pareto optimal" theory, in which an efficient settlement is always attained. In effect, we assume that the costs of reaching agreement (including lost wages, boycotts, etc.), though not necessarily negligible, do not interfere with the terms of the agreement.

surplus it can achieve by itself; thus for the grand coalition $S=E\bigcup[0,n]$ it is the shaded area in Figure 3.1.

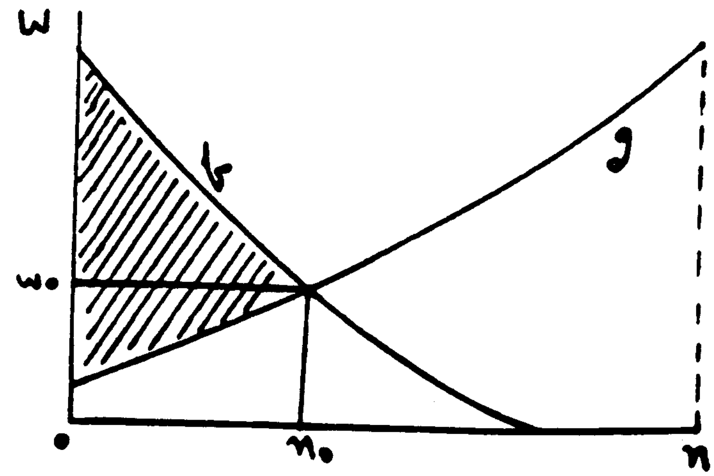

Figure 3.1

Consider now a randomly chosen subset of $[0,n]$ of size tn, $0\leq t\leq 1$. Since $[0,n]$ is a continuum, the latter will be a faithful sample of $[0,n]$, so we merely replace $g(x)$ by $g_t(x)$ where $g_t(x)$ is defined as $g(x/t)$, as shown in Figure 3.2.

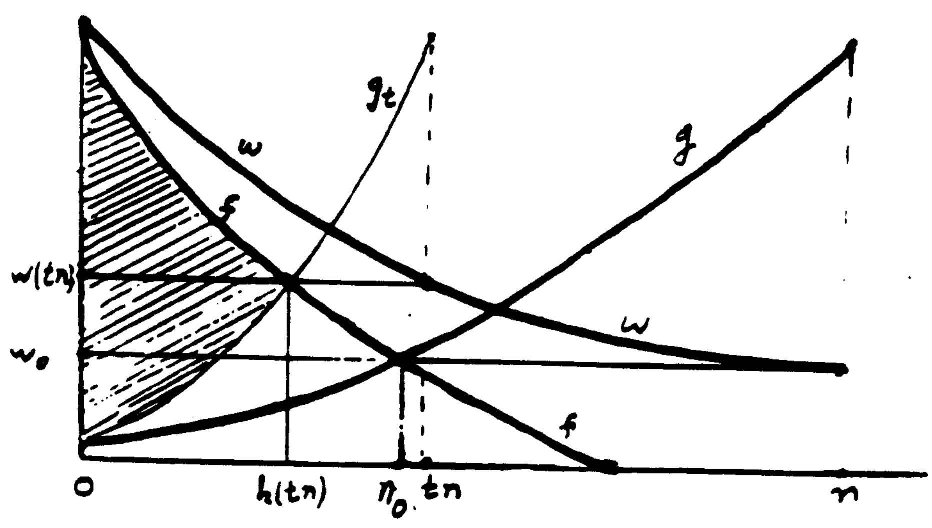

Figure 3.2

Figure 3.2 also shows the curve $w(tn)$, the equilibrium wage associated with the intersection of f and g_t; $k(tn)$ is the corresponding number of workers employed. Note that

$w(x) > f(x)$ over the domain if f is defined, and that $k(x) < x$.

We can now express the worths of some of the other coalitions. Specifically, the surplus available to a coalition consisting of E and a random "oceanic" subset of size tn is given by

$$S(t) = \int_{x=0}^{k(m)} [f(x) - g_t(x)]dx = \int_{x=0}^{k(m)} [f(x) - g(x/t)]dx \tag{3.1}$$

as indicated by the shaded area in Figure 3.2.

This turns out to be all we need, since work forces that are not faithful samples of the whole will play no part in the solution (they have probability zero), and any coalition that does not include E has zero worth.

To compute the Shapley value of a game, we imagine the players being brought into the general, overall settlement in a random order, each one receiving his incremental contribution to the worth of the growing coalition[10].

Thus, for the Shapley value Φ_E of the atom E, we select at random a number $t \in [0,1]$ representing the "time" at which E enters the coalition, and award to E the entire surplus $S(t)$ -- this being E's incremental contribution. Then we have

$$\Phi_E = \int_0^1 S(t)dt = \int_0^1 \int_0^{k(m)} [f(x) - g(x/t)]dxdt. \tag{3.2}$$

This double integral may be visualized (Figure 3.3) as the volume of a solid whose base is the region bounded by f and g and the vertical axis, and whose height varies from 1 (on the axis) to 0 (on g), the level sets being given by the curves g_t. In this representation, the height of a given point is the *probability* (under the random order model) that E will obtain the bit of revenue represented by the point.

For the Shapley values of the oceanic players, a distribution function on $[0,n]$ must be

10. The Shapley value rests on an axiomatic foundation; the random-order "story" is merely a useful technical device for carrying out calculations. It does not purport to be a model of an actual negotiation process.

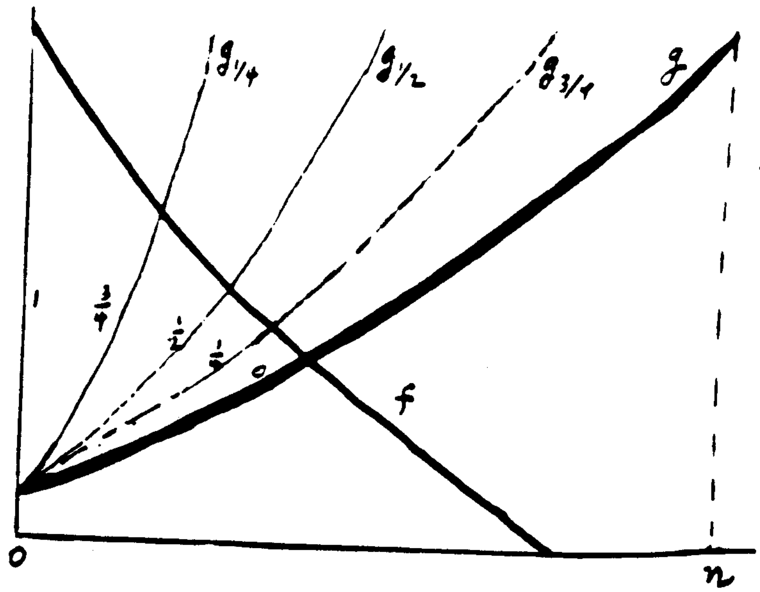

Figure 3.3

used, since any individual worker's value is infinitesimal. Let $\Phi(x)$ be the total value payoff to the interval $[0,x]$ in the domain of g. The derivative $\phi(x)=\frac{d\Phi(x)}{dx}$ is then the "payoff density" to a worker whose alternative wage is $g(x)$.

Let t be fixed and let $\hat{y}$ be a fixed alternative wage. We must determine the increment to the surplus when we add a small, hypothetical set $\Delta_{\hat{y}}$ of additional workers having alternative wage $\hat{y}$ to the set of workers who are "on hand" at the time t, i.e., the workers represented by the curve g_t. Let $\delta>0$ denote the size of $\Delta_{\hat{y}}$. There are two possible configurations. The first is illustrated in Figure 3.4; it assumes $g(0)\leq\hat{y}<w(tn)$. The increment due to $\Delta_{\hat{y}}$ is the shaded area on the left. Its area, however, is equal by translation to the simpler shaded area on the right, or $[w(tn)-\hat{y}]\delta+o(\delta)$. The second configuration assumes $w(tn)\leq\hat{y}\leq g(n)$; in this case, the contribution of $\Delta_{\hat{y}}$ to the surplus is nil.

In order to obtain the value-density ϕ, let $\hat{x}$ be the point in $[0,n]$ at which we wish to evaluate ϕ, and let t_E be the time of E's joining the grand coalition. Then we have[11]

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11. The variable of integration x identifies the time at which $\Delta_{g(t)}$ joins the grand coalition, i.e., $t=x/n$. The condition $x\geq t_E n$ ensures that E is already on hand when this occurs.

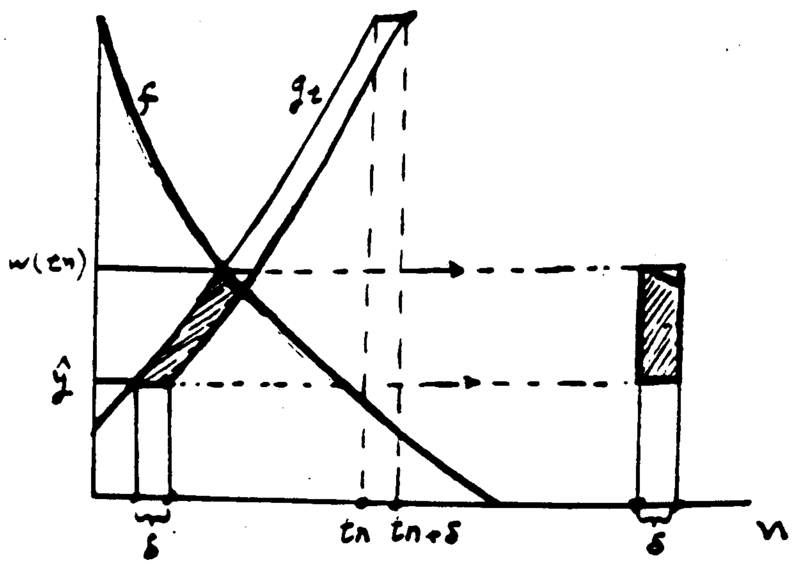

Figure 3.4

$$\Phi(\hat{x}) = \int_{t_E=0}^{1} \int_{x=t_E n}^{\bar{u}} \frac{1}{n}[w(x)-g(\hat{x})]dxdt \tag{3.3}$$

where the upper limit $\bar{u}$ breaks into three cases. To present this, let $\hat{t}$ be given by $w(\hat{t}n)=g(\hat{x})$, i.e., $\hat{t}=w^{-1}(g(\hat{x}))/n$. (If $g(\hat{x})$ is less than $w(n)$ we formally set $\hat{t}=+\infty$.) Case I: If $g(\hat{x})\leq w(n)$, then $\bar{u}=n$. Case II: If $w(n)\leq g(\hat{x})\leq w(t_E n)$, then $\bar{u}=\hat{m}$. Case III: If $g(\hat{x})\geq w(t_E n)$, then $\bar{u}=t_E n$ (so $\delta=0$). The three cases are shown in Figure 3.5.

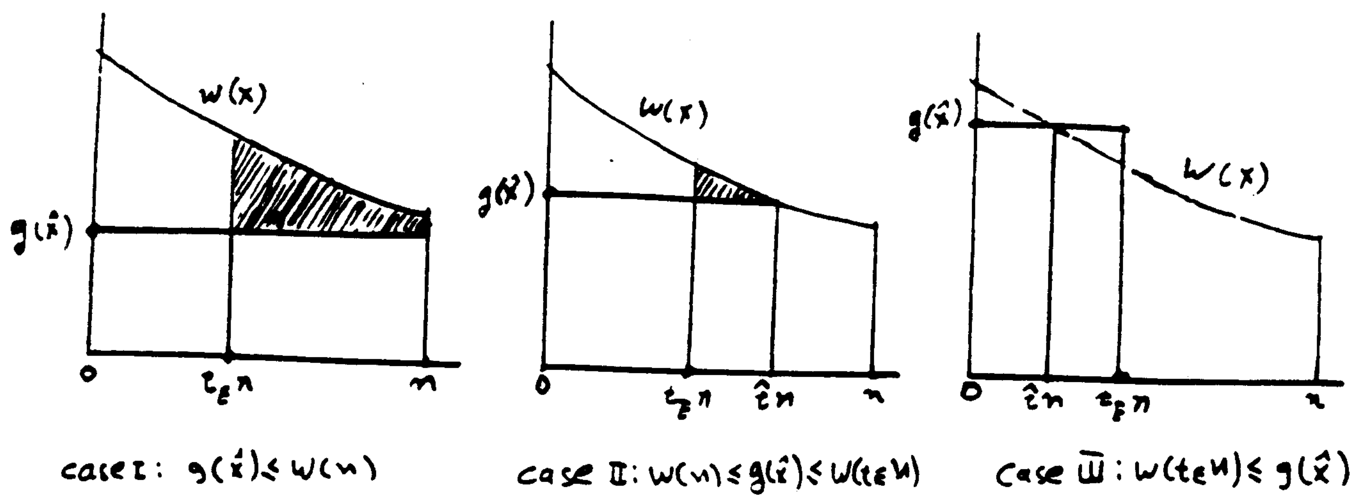

Figure 3.5

We summarize all this by writing $\tilde{u}=med\{n,\hat{m},t_E n\}$, where $med\{a,b,c\}$ denotes the median of the three numbers a, b and c. Thus, the general formula is

$$\phi(\hat{x}) = \int_0^1 \int_{t_E n}^{med\{n,\hat{m},t_E n\}} \frac{1}{n}[w(x)-g(\hat{x})]dx dt_E$$

$$= \int_0^1 \int_{t_E n}^{med\{n,\hat{m},t_E n\}} \frac{1}{n}w(x)dx dt_E \;-\; \frac{1}{n}g(\hat{x})\int_0^1 (med\{n,\hat{m},t_E n\}-t_E n)dt_E. \tag{3.4}$$

In case I, the "insider" case, (3.4) simplifies considerably.

$$\phi(\hat{x}) = \int_0^1 \int_{t_E n}^{med\{n,\hat{m},t_E n\}} \frac{1}{n}w(x)dx dt_E \;-\; g(\hat{x})\int_0^1 (1-t_E)dt_E$$

$$=\frac{1}{n}\int_0^1 [W(n)-W(t_E n)]dt_E - \frac{1}{2}g(\hat{x}), \tag{3.5}$$

where $W(x)$ is the integral of w from 0 to x. Note that the first term is independent of $\hat{x}$. The second term shows that the Shapley values of the workers decrease as their alternative wages increase, as one would expect.

Cases II and III we handle together, since the case distinction depends on t_E, which is a variable. We have:

$$\phi(\hat{x})=\frac{1}{n}\int_0^1 \int_{t_E n}^{\max\{t_E n,\hat{m}\}} w(x)dx dt_E - g(\hat{x})\int_0^1 (\max\{t_E,\hat{t}\}-t_E)dt_E$$

$$=\frac{1}{n}\int_0^1 (W(\max\{t_E,\hat{t}\})-W(t_E n))dt_E - g(\hat{x})\int_0^{\hat{t}} (\hat{t}-t_E)dt_E$$

$$=\frac{1}{n}\int_0^{\hat{t}} (W(\hat{m})-W(t_E n))dt_E - \frac{1}{2}g(\hat{x})\hat{t}^2. \tag{3.6}$$

4. Model 1 - Examples

In the first example the workers are fully heterogeneous in opportunities, with g as well as f linear. The objective here is merely to demonstrate the actual calculation of the Shapley values, using the general expressions derived in the previous section.

In the second example there are only two types of workers: the insiders having one alternative wage and the outsiders another, higher alternative wage. The objective here is to make some qualitative observations on the relationship between the economic variables (such as the alternative wage and the size of the external labor pool) and the Shapley-value payoffs to the employer and the workers.

EXAMPLE 1: Let $f(x)=1-x$ $(0<x<1)$, $g(x)=x/2$ $(0<x<n)$ and $n=2$. Recall that $g_t(x)\equiv g(x/t)$. Also $w(x)$, $k(x)$ are the equilibrium wage and the number of workers employed, respectively, associated with the intersection of f and g_t where $t=x/n$. Thus:

$$g_t(x)=\frac{x}{2t}\ , \quad w(x)=\frac{1}{1+x}\ , \quad k(x)=\frac{x}{1+x}.$$

From equation (3.1) the surplus is:

$$S(t) = \int_0^{k(2t)} [1-x-x/2t]dx = \left[x-\frac{x^2}{2}-\frac{x^2}{4t}\right]_0^{\frac{2t}{1+2t}} = \frac{t}{1+2t},$$

and so the value payoff for the employer is:

$$\Phi_E = \int_0^1 S(t)dt = \int_0^1 \frac{t}{1+2t}dt = \frac{1}{2}-\frac{1}{4}\ln 3 = 0.22535.$$

Note that $\Phi_{\text{grand coalition}}=S(1)=1/3$, so E gets about 2/3 of the surplus. Also, note that if all workers were to unionize, and behave as the 'classic' single monopolist, E gets 0.08, which is much less than Φ_E. Under a competitive solution E gets 0.222, slightly less than Φ_E.

Now we compute the workers' values. We have

$$W(x) = \int_0^x w(u)du = \ln(1+x)$$

Also,

$$\hat{t} = h^{-1}(g(\hat{x}))/n = h^{-1}(\frac{\hat{x}}{2})\cdot\frac{1}{2} = \frac{1}{\hat{x}} - \frac{1}{2} \quad , \qquad \hat{x} = \frac{2}{2\hat{t}+1}$$

In case I we have $g(\hat{x}) \leq w(n)$, that is, $\hat{x} \leq 2/3$. Using equation (3.5):

$$\phi(\hat{x}) = \frac{1}{n}\int_0^1 \Big(W(n) - W(t_E n)\Big)dt_E - \frac{g(\hat{x})}{2}$$

$$= \frac{1}{2}\int_0^1 \Big(\ln 3 - \ln(1+2t_E)\Big)dt_E - \frac{\hat{x}}{4}$$

$$= \frac{1}{2} - \frac{1}{4}\ln 3 - \frac{\hat{x}}{4}.$$

In the remaining case (combining the previous II and III) we have $2/3 \leq \hat{x} \leq 2$. So, from equation (3.6)

$$\phi(\hat{x}) = \int_0^{\hat{t}} \Big(W(\hat{t}n) - W(t_E n)\Big)dt_E - \frac{n}{2}g(\hat{x})\hat{t}^2 = \int_0^{\hat{t}} \Big(\ln(1+2\hat{t}) - \ln(1+2t_E)\Big)dt_E - \frac{\hat{x}}{2}\hat{t}^2$$

$$= \frac{1}{2\hat{x}} - \frac{\hat{x}}{8} - \frac{1}{2}\ln(2/\hat{x}).$$

In Figure 4.1, we plot both the supply curve $g(x)$ and the total wage $\phi(x)+g(x)$. The difference between the two curves is the oceanic value $\phi(x)$, which is a decreasing but positive function of x. In addition, the competitive wage is plotted, which in this case is w_0 for $x \leq 2/3$ and $g(x)$ for $2/3 < x$. Also is plotted the monopolistic solution. In this case, the wage is w_M for $0 < x < 0.4$ and $g(x)$ for $0.4 < x$. Further discussion of these results is given at the end of this section.

EXAMPLE 2: Let $f(x) = 1 - x$ $(0 \leq x \leq 1)$ and let $z < .6$, let $n \geq .5$, and let

$$g(x) = \begin{cases} z & \text{if} \quad 0 < x < 1/2 \\ .6 & \text{if} \quad 1/2 < x < n \\ \infty & \text{if} \quad n < x \end{cases}$$

First, calculate the employer's payoff. There are three types of configuration that enter the calculation; let the incremental surplus due to E be A_I, A_{II} and A_{III} respectively. Then we have (see the shaded areas in Figure 4.2):

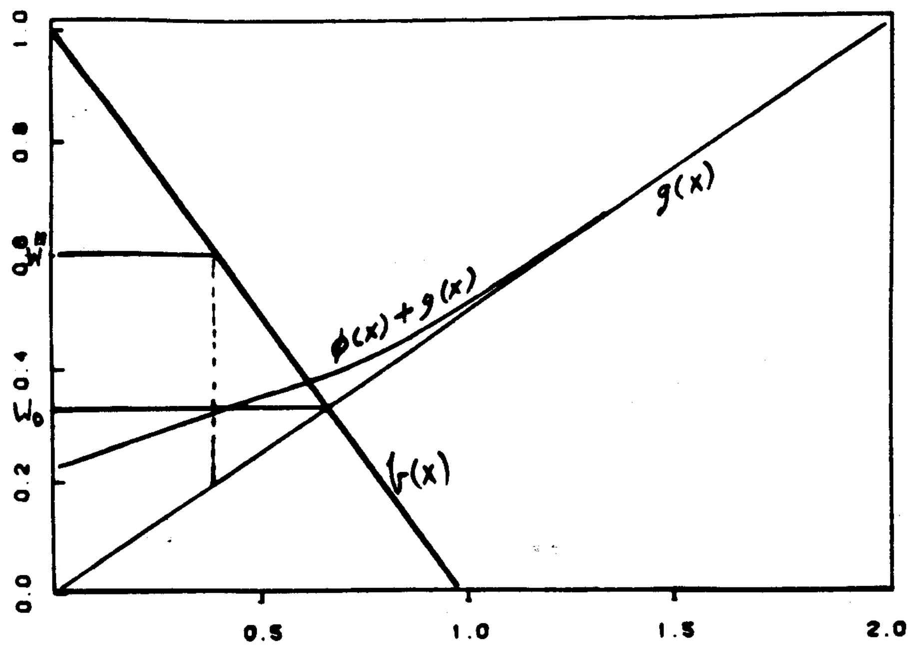

Figure 4.1

CASE I: $t<.4/n$ and $A_I = \frac{(tn)^2}{2}+(1-tn-z)\frac{t}{2}+(1-tn-.6)(tn-\frac{t}{2})$.

CASE II: $.4/n<t<.8$ and $A_{II} = .08+\frac{t}{2}(.6-z)$.

CASE III: $.8<t<1$ and $A_{III} = \frac{(t/2)^2}{2}-(1-t/2-z)\frac{t}{2}$.

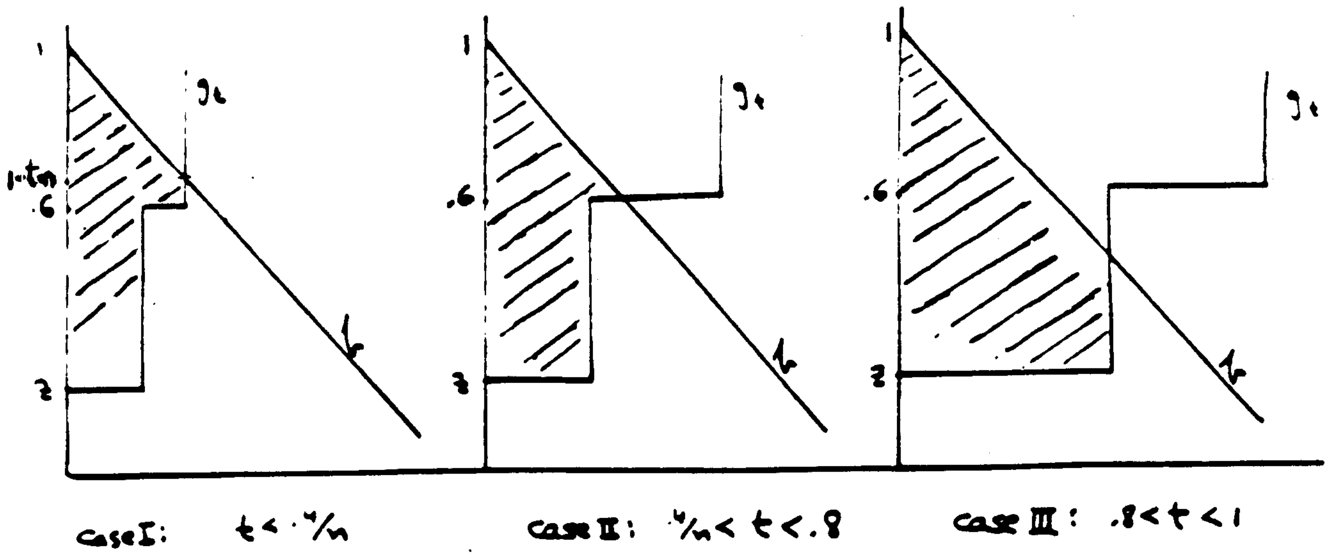

Figure 4.2

Thus,

$$\Phi_E = \int_0^{.4/n} A_I dt + \int_{.4/n}^{.8} A_{II} dt + \int_{.8}^{1} A_{III} dt$$

Substitution for A_I, A_{II}, A_{III} and integration yields the employer's payoff:

$$\Phi_E = 0.23 - 0.25z - \frac{.01}{n}.$$

Now, all the insiders fall into the category of case I and the value can be calculated from (3.5). First, $w(x)$ and $W(x)$ are calculated (see Figure 4.2):

$$w(x) = \begin{cases} 1-x & 0<x<.4 \\ .6 & .4<x<.8n \\ 1-\frac{x}{2n} & .8n<x<n \end{cases}$$

and

$$W(x) = \begin{cases} x-\frac{x^2}{2} & 0<x<.4 \\ .08+.6x & .4<x<.8n \\ .08+.6x-\frac{(x-.8n)^2}{4}n & .8n<x<n \end{cases}$$

Thus:

$$\int_0^1 W(t_E n) dt_E = \frac{1}{n}[-.01+.08n+.3n^2]$$

and

$$W(n) = .08+.59n.$$

Substituting these expressions into (3.5), we get the payoff density to the insiders:

$$\phi_{\text{insiders}}(\hat{x}) = .29 - \frac{z}{2} + \frac{.01}{n^2}.$$

Next, from (3.4), the payoff density to the outsiders is

$$\phi_{\text{outsiders}}(\hat{x}) = \int_{t_E=0}^{1} \int_{x=nt_E}^{.4} \frac{1}{n}[w(x)-g(\hat{x})] dx dt_E$$

Clearly, outsiders will not enter unless $x<.4$. Thus, $t_E<.4/n$. So,

$$\phi_{\text{outsiders}}(\hat{x}) = \int_{t_E=0}^{.4/n} \int_{z=nt_E}^{.4} \frac{1}{n}[w(x)-g(\hat{x})]dxdt_E = \frac{.01}{n^2}$$

As a check, we note that the sum of the employer's, insiders' and outsiders' payoff is

$$(0.23-0.25z-\frac{.01}{n})+\frac{1}{2}(0.29-\frac{z}{2}+\frac{.01}{n^2})+(n-\frac{1}{2})(\frac{.01}{n^2}) = 0.375-\frac{1}{2}z,$$

which is equal to the total surplus $S(1)$.

4.1 On the Results

These calculations reveal that the individual worker has bargaining power, which despite being infinitesimal is not negligible. Though workers are unorganized, the results still indicate variations in wages which do not correspond to variations in the workers' productivity on their current job.

Specifically, example I demonstrates the positive relationships between the value of the workers' outside opportunities and their wage (although the Shapley values of the workers decrease as their alternative wage increase, the wage, which includes the value of the opportunities, increases with opportunities). We analyzed a case where workers are relatively more heterogeneous in outside opportunities than on the present job. For such instances, the theory predicts that the wage variations will overstate variations in the marginal productivity (which in this case are zero). For the more general case where workers may have different productivity on the current job, if the variations among workers in on-the-job productivities are larger than the variations in outside opportunities, wage variations will understate productivity variations.

From Example II we learn that the relationships between the employer's payoff and the institutional structure ("oceanic" game vs. competitive), depend on the size of the labor force n and on the distribution of the workers' opportunities z (where the employer's structure is taken as given and the workers are assumed to have homogeneous productivity on their current "best" job). In particular, if n is small and z large, the worse is the employer's bargaining payoff relative to the competitive solution. Thus, the stronger is the employer's incentive to keep a competitive structure. This is clearly reversed as we examine the workers' incentives. The workers' wage is higher under the bargaining structure if n is small and z large. Moreover, actual wages were shown to be a direct function of outside opportunities (for a given employer and a given on-the-job productivity). Note that although our model recognizes the bargaining power of the workers (in contrast to the competitive model), the wages are not necessarily higher than in the competitive model; indeed they will be lower when the labor pool is large and the alternative wage is low.

In both examples the outsiders are able to extract some benefit from the situation because their existence keeps the actual wage below what it would otherwise be. We shall return to this point in section 7.

Finally, note that this is not a competitive structure. The wage solution obtained is not stable in the competitive sense: some workers are getting more than their physical marginal productivity. What drives the employer to agree to sign such a binding contract (which awards the worker a weighted average of his marginal contribution to every subset of other individual workers) is the implicit threat of potential organization of the individual workers.

5. Model 2: Labor Partially Organized

Model 2 is an extension of Model 1. We assume that a certain subset of workers have already formed a union U in order to bargain with the employer. Since some but not all of the workers are organized, we still have an oceanic game. But now there are two atoms: E and U.

For simplicity, we assume that U consists of just the workers who occupy a certain interval $[a,b]$ of the alternative-wage scale. This is admittedly a special assumption, but it is not entirely unreasonable, since union formation is likely to be a highly selective process. We shall also assume (most of the time) that

$$a < b < w_0 \tag{5.1}$$

where $w_0 = g(n_0)$ is the equilibrium wage rate. Thus U consists entirely of "insiders" -- people who would be employed at equilibrium if there were no union.

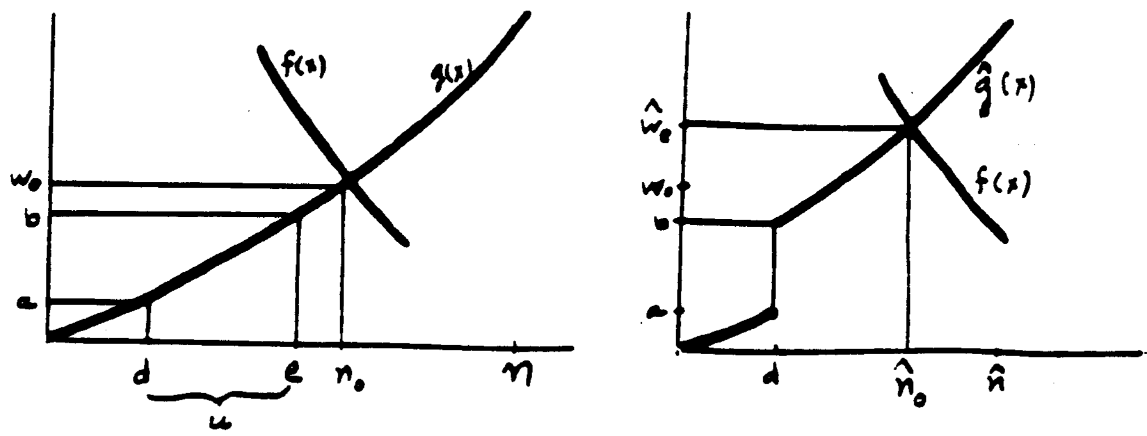

Figure 5.1:

Figure 5.1 displays some further notation. Thus, u is the size of the union; $\hat{g}$ is the supply function that characterizes the unorganized workers -- i.e.,

$$\hat{g}(x)=\begin{cases} g(x) & \text{if } 0\leq x\leq d \\ g(x+u) & \text{if } d<x\leq\hat{n} \end{cases} \tag{5.2}$$

and $\hat{w}_0$ is the corresponding equilibrium wage rate.

To determine the Shapley value of this two-atom oceanic game by the "random order" method, we shall require two independent uniformly-distributed random variables, say t_E and t_U, representing the "times of entry" of the atoms E and U into the ordered continuum of unorganized workers. Our probability space is therefore a unit square, as shown in Figure 5.2. The quantity of unorganized ("oceanic") workers who are present when E arrives on the scene is $t_E\hat{n}$ and their alternative-wage distribution is given by the compressed curve $\hat{g}_{t_E}$, defined like the g_t of Section 3 (Figure 3.2). Note the discontinuity along the diagonal. If $t_U>t_E$ the union's entry is responsible for a substantial increase in the surplus, but if $t_U<t_E$ U brings in nothing. The boundary case $t_U=t_E$ can be ignored, since it has probability 0.

Let us now develop a formula for the union's value Φ_U, extending to the two-atom case the geometrical techniques we used in Model 1. To reduce notational clutter we shall write t for t_U until further notice.

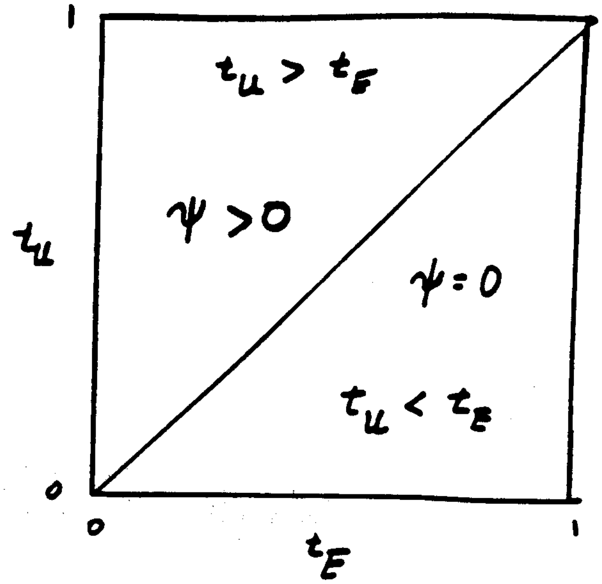

Figure 5.2:

Let $\Psi(t)$ denote U's contribution to the surplus if it enters at time t *and E is already present,* i.e., $t = t_U > t_E$. (Note that $\Psi(t)$ in this case is independent of t_E.) Then U's value is given by

$$\Phi_U = \int_{t=0}^{1} \int_{t_E=0}^{t} \Psi(t) dt_E dt = \int_{t=0}^{1} \Psi(t) t dt. \tag{5.3}$$

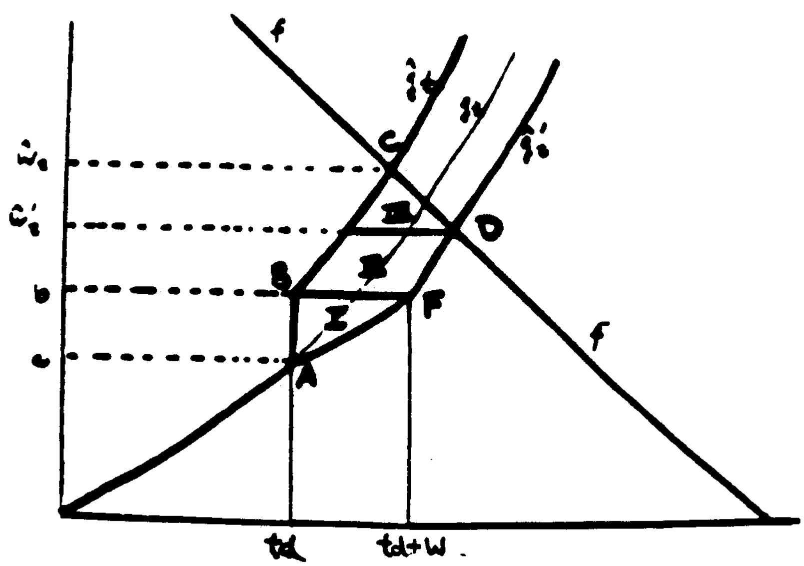

Figure 5.3:

Figure 5.3 provides a geometric representation of the function $\Psi(t)$, namely, the area bounded by $\overline{ABCDFA}$. Here, $\overline{ABC}$ is a portion of the compressed version of $\hat{g}$, defined by

$\hat{g}_t(x)=\hat{g}(x/t)$ (cf. (5.2)); $\overline{CD}$ is a portion of the graph of f; and $\overline{AFD}$ is a portion of the graph of a function we shall call $\hat{g}'_t$, defined by

$$\hat{g}'_t = \begin{cases} g_t(x) & \text{if } 0 \leq x \leq td \\ g(x+(1-t)d) & \text{if } td \leq x \leq td+u \\ \hat{g}_t(x-u) & \text{if } td+u \leq x \leq t\hat{n}+u \end{cases} \tag{5.4}$$

which represents the labor supply at time t with *all* the members of U included. (Thus, the segment $\overline{AF}$ is *not* compressed.) From the wage level b on up, the graphs of $\hat{g}_t$ and $\hat{g}'_t$ are parallel, with a horizontal separation of u.[12] In order to obtain an analytical expression, we have divided the area representing $\Psi(t)$ into three parts, as shown, whose separate areas are easily written down:

$$\begin{aligned} \Psi_I &= \int_{td}^{td+u} [b-\hat{g}'_t(x)]dx = bu - \int_{d}^{d+u} g(x)dx, \\ \Psi_{II} &= u(\hat{w}'_t - b), \\ \Psi_{III} &= \int_{\hat{w}'_t}^{\hat{w}_t} [f^{-1}(y)-\hat{g}_t^{-1}(y)]dy. \end{aligned} \tag{5.5}$$

Here $\hat{w}_t$ is the equilibrium wage for $\hat{g}_t$ and $\hat{w}'_t$ is the equilibrium wage for $\hat{g}'_t$. Combining (5.3) and (5.5), we obtain

$$\begin{aligned} \Phi_U &= \int_0^1 \Psi(t_U)dt_U = \int_0^1 (I+II+III)t_U dt_U \\ &= \int_0^1 \left\{ u\hat{w}'_{t_U} - \int_d^e g(x)dx + \int_{w'_{t_U}}^{\hat{w}_{t_U}} [f^{-1}(y)-\hat{g}_{t_U}^{-1}(y)]dy \right\} t_U dt_U. \end{aligned}$$

where we now restore the subscript to "t_U".

By a similar calculation, which we omit, it can be shown that the corresponding expression for the employer is

12. By our "insiders only" assumption (5.1) we ensure that F lies below D in the figure, whatever the value of t. Without this assumption, additional case distinctions would appear as t approaches 1.

$$\Phi_E = \int_{t_U=0}^{1} \Big\{ \int_{t_E=0}^{t_U} \int_{0}^{\hat{n}_{t_E}} [f(x)-\hat{g}_{t_E}(x)]dx dt_E + \int_{t_E=t_U}^{1} \int_{0}^{\hat{n}'_{t_E}} [f(x)-\hat{g}'_{t_E}(x)]dx dt_E \Big\} dt_U,$$

where $\hat{n}_t = f^{-1}(\hat{w}_t)$ and $\hat{n}'_t = f^{-1}(\hat{w}'_t)$.

Finally we calculate $\phi(x)$, the value-density function for the unorganized workers (compare section 3, pp. 8-9). The probability space is now a cube, but we can represent it easily in two dimensions by treating t_x, the arrival time of a typical infinitesimal oceanic player x, as a variable marker on the t_E and t_U scales, as shown in Figure 5.4.

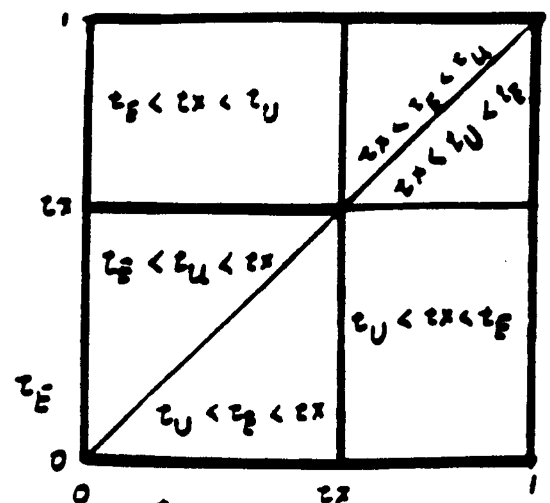

Figure 5.4

The six possible order of entry of E, U and the infinitesimal player x are conveniently grouped into three cases (heavy lines in the figure). If $t_x < t_E$ (at the right -- probability $1-t_x$), the oceanic player contributes nothing. If $t_x > t_E$ but $t_x < t_U$ (upper left -- probability $t_x(1-t_x)$) he contributes $\max\{0, \hat{w}_{t_x} - g(x)\}dt_x$. If $t_x > t_E$ and $t_x > t_U$ (lower left -- probability t_x^2) he contributes $\max\{0, \hat{w}'_{t_x} - g(x)\}dt_x$. So we obtain (writing "t" for "t_x")

$$\phi(x) = \int_0^1 [(t-t^2)\max\{0, \hat{w}_t - g(x)\} + t^2 \max\{0, \hat{w}'_t - g(x)\}]dt.$$

5.1 Collective vs. Individual Bargaining -- I

As an application of this analysis we shall show that it is better for the members of U to bargain as a union than as individuals – at least if the functions f and g are linear.

Thus, we shall be comparing Φ_U (above) with

$$\int_{x \in U} \tilde{\Phi}(x)dx, \tag{5.6}$$

where $\tilde{\Phi}$ is the value-density function for the "ocean" of Model 1.

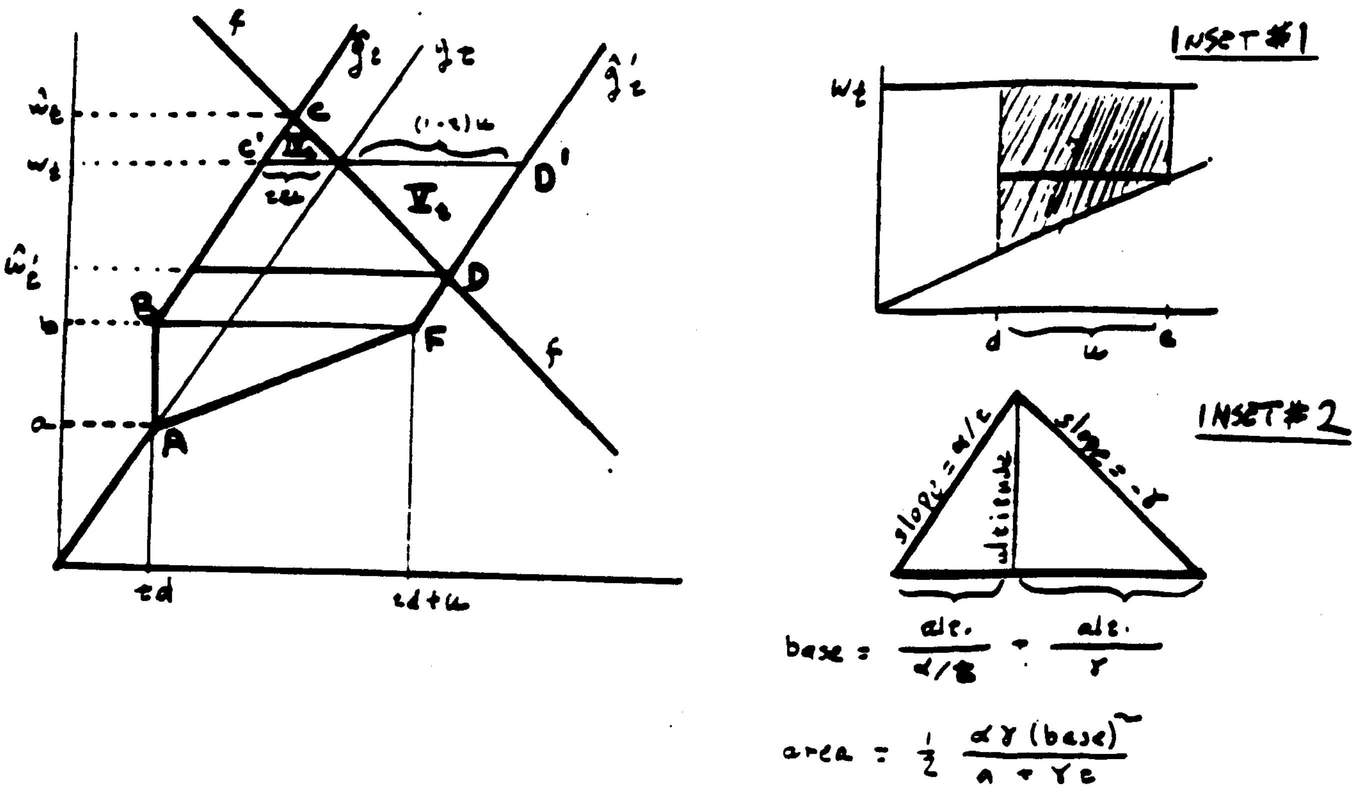

Figure 5.5:

Figure 5.5 shows the comparison. As previously, the integrand $\Psi(t)$ in the unionized case (Model 2) is given by the area of $\overline{ABCDFA}$. The corresponding integrand for an infinitesimal set "dx" of unorganized workers in Model 1 is given by a narrow strip along the g_t curve (see Figure 3.4). Its vertical extent is from $g_t(tx)$ to w_t while its horizontal extent is everywhere dx, so the area (disregarding second-order infinitesimals) is given by $(w_t - g_t(tx))dx$. Since $g_t(tx)$ is just $g(x)$, the combined contributions of all the members of U is

$$\int_d^e (w_t - g(x))dx,$$

as shown in inset #1, which has the same area as $\overline{ABC'D'FA}$ in the main diagram of Figure 5.5; call this area $\bar{\Psi}(t)$. We must therefore examine the difference[13] $\Psi(t) - \bar{\Psi}(t)$. But this is just the difference between the two triangles IV_t and V_t. Our claim is that IV_t is *larger on average* than V_t when all values of t_E and t with $t_E \leq t$ are taken into account.

Note first that IV_t and V_t are similar triangles, with bases tu and $(1-t)u$ respectively. Let

$$g(x) = \alpha x \ , \qquad f(x) = \beta - \gamma x,$$

where α, β, γ are positive constants. Then, as shown in inset #2,

$$IV_t = \frac{\alpha\gamma t^2 u^2}{2(\alpha+\gamma t)} \ , \qquad V_t = \frac{\alpha\gamma(1-t)^2 u^2}{2(\alpha+\gamma t)}. \tag{5.7}$$

Our claim is that

$$\int_{t=0}^{1} \int_{t_E=0}^{t} [IV_t - V_t] dt_E dt > 0.$$

From (5.7) we have

$$IV_t - V_t = \frac{\alpha\gamma u^2(2t-1)}{2(\alpha+\gamma t)}$$

In particular, for t between 0 and 1/2 we have,

$$\int_0^t [IV_t - V_t] dt_E = \frac{\alpha\gamma u^2(2t-1)t}{2(\alpha+\gamma t)},$$
$$\int_0^{1-t} [IV_{1-t} - V_{1-t}] dt_E = \frac{\alpha\gamma u^2(2t-1)(1-t)}{2(\alpha+\gamma(1-t))}.$$

The sum of these two expressions is

$$\frac{\alpha\gamma u^2(1-2t)}{2}\left[\frac{t}{\alpha+\gamma t} - \frac{1-t}{\alpha+\gamma(1-t)}\right] = \frac{\alpha^2\gamma u^2(2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))},$$

which we see is always nonnegative, and in fact is positive everywhere except at $t = 1/2$. So we conclude:

$$\Phi_U - \int_U \tilde{\phi}(x)dx = \left(\int_{t=0}^{1/2} + \int_{1/2}^{1}\right) \int_{t_E=0}^{t} [IV_t - V_t] dt_E dt = \int_{t=0}^{1/2} \int_{t_E=0}^{t} [IV_t - V_t + IV_{1-t} - V_{1-t}] dt_E dt$$

---

13. In order to set up this comparison we have changed the order of integration, bringing the $\int dx$ integral inside the double integral $\int\int dt_U dt_E$.

$$= \int_0^{1/2} \frac{\alpha^2\gamma u^2(2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))} dt > 0.$$

This completes the proof that in the linear case the members of U are better off organized than unorganized.[14]

5.2 Collective vs. Individual Bargaining -- II

We shall now prove that when *all* workers (insiders and outsiders) are unionized and bargain as a unit, their total payoff is higher than what they get in Model 1, where none are unionized and the employer E is the only atom. This result does not require that g and f be linear functions.

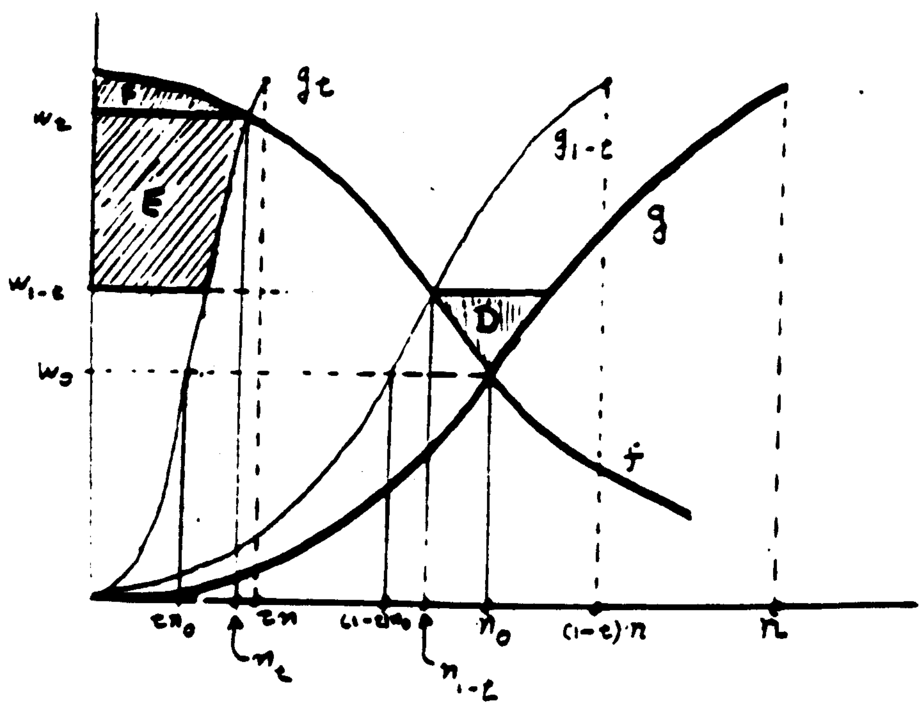

Figure 5.6:

14. On the basis of several examples we have calculated, we conjecture that this remains true for all monotonic functions g and f and for any measurable set U consisting of insiders. But we have found that it is not true in general if U contains outsiders, even if g and f are linear.

Consider Figure 5.6. The total surplus is

$$S = \int_0^{n_0} [f(x)-g(x)]dx, \tag{5.8}$$

and the value of the game for the employer (as derived in Section 3) is

$$\Phi_E = \int_{t=0}^{1} \int_{x=0}^{n_t} [f(x)-g_t(x)]dxdt = \int_0^{1/2} \left\{ \int_0^{n_t} [f(x)-g_t(x)]dx + \int_0^{n_{1-t}} [f(x)-g_{1-t}(x)]dx \right\} dt$$

Taking y in place of x as the independent variable, we can rewrite this as

$$\Phi_E = \int_0^{1/2} \left\{ \int_{y=0}^{f(0)} [A_1+A_2]dy \right\} dt$$

where

$$A_1 = \min\{g_t^{-1}(y), f^{-1}(y)\}$$
$$A_2 = \min\{g_{1-t}^{-1}(y), f^{-1}(y)\}.$$

There are then three cases:

$$\begin{array}{lll} \text{for } 0 \le y \le w_{1-t}: & A_1 = g_t^{-1}(y), & A_2 = g_{1-t}^{-1}(y), \\ \text{for } w_{1-t} \le y \le w_t: & A_1 = g_t^{-1}(y), & A_2 = f^{-1}(y), \\ \text{for } w_t \le y \le f(0): & A_1 = f^{-1}(y), & A_2 = f^{-1}(y), \end{array}$$

and we observe that

$$\int_0^{w_{1-t}} g_t^{-1}(y)dy = \int_0^{w_{1-t}} (g^{-1}(y)-g_{1-t}^{-1}(y))dy,$$

as indicated by the horizontal dotted lines in Figure 5.5. So the employer's value payoff may be calculated as follows:

$$\Phi_E = \int_{t=0}^{1/2} \left\{ S + \int_{w_0}^{w_{1-t}} [g^{-1}(y)-f^{-1}(y)]dy + \int_{w_{1-t}}^{w_t} g_t^{-1}(y)dy + \int_{w_t}^{f(0)} f^{-1}(y)dy \right\} dt$$
$$= \int_0^{1/2} Sdt + \int_0^{1/2} Kdt = \frac{S}{2} + \frac{K}{2},$$

where K denotes the sum of the areas of D, E and F in Figure 5.5. Since the total value to all players is the surplus S, the value to the workers must be

$$S - \Phi_E = S/2 - K/2.$$

On the other hand, if all the workers get together and bargain as a unit, then it is just a two-player simple bargaining game, and the value payoff to each side is just $S/2$. So if the entire labor force is unionized, their total wage is $K/2$ greater than if they had no union at all.

6. Model 2: Examples

Here the objective is to demonstrate the computation and behavior of oceanic game solutions in specific examples. In Section 6.1 we apply the general solution to the case where outsiders are homogeneous with respect to their alternative wage. This is followed in section 6.2 by an example of specific functions where the payoffs are computed and compared under several institutional structures: perfect competition, a union of insiders with an "ocean" of outsiders, a union of insiders with a union of outsiders, and a union of all workers.

6.1 General Results for Homogeneous Outsiders

Let $g(x)$ be monotonically increasing until the equilibrium employment level, and constant thereafter; thus, the insiders are heterogeneous and the outsiders are homogeneous.

Consider Figure 6.1. The value of the outsiders' opportunities is w^*. Assume that the union consists of all the inside workers: $U=[0,n_0]$. The number of workers employed when the union members quit the industry is denoted by n_2, and n^* will denote the total number of outsiders; we assume that $n^* \geq n_2$. It will be seen that this is a specialization of the situation depicted in Figure 5.1. The players in this oceanic game are E, U, and the continuum $[0,n^*]$. In view of the homogeneity of the outsiders, the possible coalitions reduce essentially to the following four cases, where t represents the fraction of outsiders present:

$S_1(t)$ - A coalition consisting only of the set $[0,tn^*]$ of outsiders.

$S_2(t)$ - A coalition consisting of E and $[0,tn^*]$.

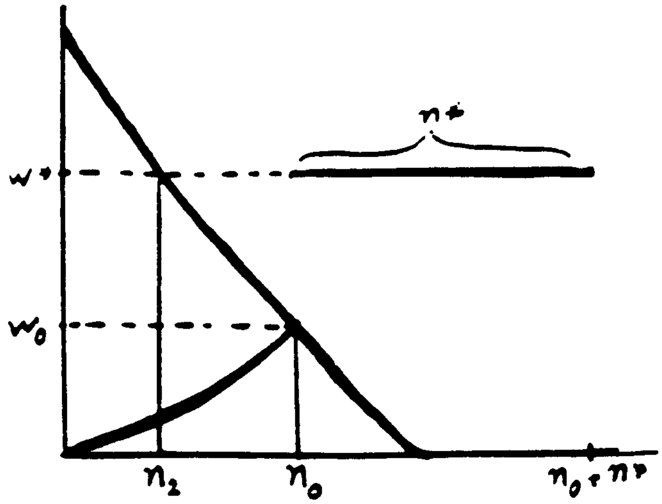

Figure 6.1

$S_3(t)$ - A coalition consisting of U and $[0,tn^*]$.

$S_4(t)$ - A coalition consisting of E and U and $[0,tn^*]$.

The characteristic function v is then given by

$$v(S_1(t)) = 0\ , \qquad v(S_2(t)) = \int_0^{\min(tn^*,n_2)} [f(x)-w^*]dx\ , \qquad v(S_3(t)) = 0\ , \qquad v(S_4(t)) = \int_0^{n_0}[f(x)-g(x)]dx \tag{6.1}$$

The value payoff to the union, Φ_U, is derived as follows: With probability 1/2, U enters before E; in this case the marginal contribution of U is zero. With the remaining probability 1/2, U enters after E; in this case U's marginal contribution is $v(S_4(t_U))-v(S_2(t_U)) = S-v(S_2(t_U))$, where S denotes the total surplus (see equation 5.8). Thus, the value payoff to the union is

$$\Phi_U = \int_{t_U=0}^{1} \int_{t_E=0}^{t_U} [v(S_4(t_U))-v(S_2(t_U))]dt_E dt_U = \int_0^1 [v(S_4(t_U))-v(S_2(t_U))]t_U dt_U$$

$$= \int_0^1 v(S_4(t_U))t_U dt_U - \int_0^{n_2/n^*} v(S_2(t_U))t_U dt_U - \int_{n_2/n^*}^{1} v(S_2(\frac{n_2}{n^*}))t_U dt_U$$

$$= \frac{1}{2}S - \int_0^{n_2/n^*} v(S_2(t_U))t_U dt_U - \int_{n_2/n^*}^{1} v(S_2(\frac{n_2}{n^*}))t_U dt_U \qquad (6.2)$$

The value payoff to the employer is derived as follows: When the employer enters before the union, his marginal contribution is $v(S_2(t_E))$. When he enters after the union, it is $v(S_4(t_E))$. Thus,

$$\Phi_E = \int_{t_E=0}^{1}\int_{t_U=t_E}^{1} v(S_2(t_E))dt_U dt_E + \int_{t_E=0}^{1}\int_{t_U=0}^{t_E} v(S_4(t_E))dt_U dt_E]$$

$$= \int_0^1 v(S_2(t_E))(1 - t_E)dt_E + \int_0^1 v(S_4(t_E))t_E dt_E$$

$$= \frac{1}{2}S + \int_0^1 v(S_2(t_E))(1 - t_E)dt_E \qquad (6.3)$$

Finally, since the combined value of all the players is S, the total value to the oceanic players (outsiders) is

$$\Phi_{\text{outsiders}} = S - \Phi_U - \Phi_E = \int_0^{n_2/n^*} v(S_2(t))tdt + \int_{n_2/n^*}^{1} v(S_2(n_2/n^*))tdt - \int_0^1 v(S_2(t))(1-t)dt.$$

6.2 Comparison of Different Institutional Structures

Let n_0, n_2, n^*, w_0, and w^* be as in Section 6.1, and for ease of calculation let us linearize f and g as follows:

$$f(x) = \alpha - \beta x$$

$$g(x) = \begin{cases} \gamma x & \text{for } 0 \le x \le n_0 \\ w^* & \text{for } n_0 < x \le n_0 + n^* \\ \infty & \text{for } x > n_0 + n^* \end{cases}$$

where α, β and γ are positive numbers. Note the identities $\alpha - \beta n_0 = \gamma n_0$ and $\alpha - \beta n_2 = w^*$. The total available surplus in this case is easily seen to be $S = \alpha n_0/2$.

STRUCTURE I: PARTIAL UNION

Let the union U consist of the inside workers, while the outsiders form an unorganized "ocean". From (6.1), the characteristic function is given by

$$v(S_1(t)) = 0,$$
$$v(S_2(t)) = \alpha M_t - \beta M_t^2/2 - w^* M_t = \beta n_2 M_t - \beta M_t^2/2,$$
$$v(S_3(t)) = 0,$$
$$v(S_4(t)) = \alpha n_0 - \beta n_0^2/2 - \gamma n_0^2 = \beta n_0^2/2,$$

where M_t denotes $\min(tn^*, n_2)$ and we have made use of the identities mentioned above.

From (6.2) the value payoff to U is

$$\Phi_U = \frac{S}{2} - \int_0^{n_2/n^*} (\beta n_2 tn^* - \beta(tn^*)^2/2)t\,dt - \int_{n_2/n^*}^{1} (\beta n_2^2/2)t\,dt$$

$$= \frac{S}{2} - \frac{\beta n_2^2}{4} + \frac{\beta n_2^4}{24 n^{*2}}$$

by straightforward calculus. From (6.3), the value payoff to E is

$$\Phi_E = \frac{S}{2} + \int_0^{n_2/n^*} (\beta n_2 tn^* - \beta(tn^*)^2/2)(1-t)\,dt + \int_{n_2/n^*}^{1} (\beta n_2^2/2)(1-t)\,dt$$

$$= \frac{S}{2} + \frac{\beta n_2^2}{4} - \frac{\beta n_2^3}{6n^*} + \frac{\beta n_2^4}{24 n^{*2}}.$$

Finally, the total value payoff to the oceanic players is

$$\Phi_{\text{outsiders}} = S - \Phi_U - \Phi_E = \frac{\beta n_2^3}{6n^*} - \frac{\beta n_2^4}{12 n^{*2}};$$

this would of course have to be realized by a side payment, not through wages (see the discussion of this point in Section 7). We see that if $n^* \to \infty$ this payment goes to zero, leaving us in the limit with

$$\Phi_E = \frac{S}{2} + \frac{\beta n_2^2}{4}, \quad \Phi_U = \frac{S}{2} - \frac{\beta n_2^2}{4}.$$

The term $\frac{\beta n_2^2}{4}$ is half of the area in Figure 6.1 that lies above the wage level w^*. Thus, in the presence of an infinite pool of outsiders willing to work for w^*, the value solution for the negotiation game awards all the "high" surplus to E and divides the rest equally between E and U.

It is interesting (but not necessary) to express the union value in terms of an equivalent wage rate w_U, on the assumption that all union members will be paid an equal amount regardless of their alternative wages. Thus:

$$w_U = \frac{\Phi_U + \gamma n_0^2/2}{n_0} = \frac{w_0}{2} + \frac{\alpha}{4} - \frac{\beta n_2^2}{4n_0} + O\left(\frac{1}{n^{*2}}\right).$$

Note that it is entirely possible that $w_U < w_0$. In other words, the insiders may be worse off than if they did not bargain at all. This could happen, for example, if α were low (i.e., near w_0) or if n_2 were high (i.e., near n_0); in the former case there would not be much employer surplus for the union to attack while in the latter case w^* is only slightly above w_0 so the union's bargaining threats are weak.

STRUCTURE II: FULL UNION

Now let $\bar{U}$ consist of all available workers, inside and outside. The bargaining game is reduced to just two players E and $\bar{U}$, and the Shapley value divides the surplus equally:

$$\Phi_E = \Phi_{\bar{U}} = \frac{S}{2} = \frac{\alpha n_0}{4}.$$

Comparing with the previous case, we see that the full union $\bar{U}$ is better for all workers combined than the partial union U with outsiders acting independently:

$$\Phi_{\bar{U}} \geq \Phi_U + \Phi_{\text{outsiders}}.$$

In fact, this holds more generally for the example in its unlinearized form, since from (6.3) we have

$$\Phi_E = \frac{S}{2} + \int_0^1 \int_0^{M_t} [f(x) - w^*](1-t)\,dx\,dt,$$

which shows that the employer does better in the presence of U than $\bar{U}$ -- and hence the workers collectively do worse (since the total "pie" to be divided is the same). However, this does not mean that an insiders' union would necessarily find it advantageous to admit outsiders if its objective is to maximize *per capita* gain to its members.

STRUCTURE III: TWO UNIONS

Finally, let there be two unions: U_1 consisting of the n_0 insiders and U_2 consisting of the n^* outsiders. The game now have three atoms and no ocean; its characteristic function is

$$v(\varnothing) = 0 \ , \quad v(\{E\}) = 0 \ , \quad v(\{U_1\}) = 0 \ , \quad v(\{U_2\}) = 0 \ , \quad v(\{U_1,U_2\}) = 0$$

$$v(\{E,U_1\}) = \int_0^{n_0}[f(x)-g(x)]dx = S \ , \quad v(\{E,U_2\}) = \int_0^{n_2}[f(x)-w^*]dx \ ,$$

$$v(\{E,U_1,U_2\}) = \int_0^{n_0}[f(x)-g(x)]dx = S \ ,$$

assuming that $n_2 \leq n^*$. In a finite game, the Shapley value to a player P may be determined by taking a random permutation of the players and calculating the expected value of P's contribution to the all-player coalition, formed in that order. For player U_1 only the orders EU_1U_2, EU_2U_1 and U_2EU_1 matter, because of all the null coalitions in the characteristic function, and we have

$$\Phi_{U_1} = \frac{1}{6}[v(\{E,U_1\})-v(\{E\})] + \frac{1}{3}[v(\{E,U_1,U_2\})-v(\{E,U_2\})]$$

$$= \frac{S}{6} + \frac{1}{3}\left[S-\int_0^{n_2}[f(x)-w^*]dx\right] = \frac{S}{2} - \frac{1}{3}\int_0^{n_2}[f(x)-w^*]dx.$$

Similarly,

$$\Phi_{U_2} = \frac{1}{6}[v(\{E,U_2\})-v(\{E\})] + \frac{1}{3}[v(\{E,U_1,U_2\})-v(\{E,U_1\})]$$

$$= \frac{1}{6}\int_0^{n_2}(f(x)-w^*)dx+\frac{1}{3}[S-S] = \frac{1}{6}\int_0^{n_2}(f(x)-w^*)dx.$$

So

$$\Phi_E = S-\Phi_{U_1}-\Phi_{U_2} = \frac{S}{2}+\frac{1}{6}\int_0^{n_2}(f(x)-w^*)dx,$$

and we see immediately that "one union is better than two" for the workers, since in the present structure they are again getting less than $S/2$.

Plugging in our linear functions, we obtain without difficulty

$$\Phi_{U_1} = \frac{S}{2} - \frac{\beta n_2^2}{6}\,, \quad \Phi_{U_2} = \frac{\beta n_2^2}{12}\,, \quad \Phi_E = \frac{S}{2} + \frac{\beta n_2^2}{12}$$

This may be compared with the corresponding solution obtained for structure I. It is noteworthy that the size of n^* does not affect the present solution, except through the assumption that $n^* \geq n_2$.

7. Discussion

Several theoretical explanations of wage variations that do not correspond directly to differences in productivity have been proposed and treated extensively in the literature, using e.g. turnover costs, search costs, employees' risk aversion, and status considerations (see, for example, Frank [1984]). However, these explanations tend to ignore the fact that wages are directly or indirectly the outcome of negotiated contracts between the workers (individually or collectively) and the employers.

The structures of labor markets are not uniform: they vary from the extreme situation where a single employer faces a single worker to the other extreme where many workers face a single large firm or many small firms. Since most real structures are at neither extreme but rather something in between, we have developed a framework for wage determination which can deal effectively with many intermediate structures on a consistent basis. In particular, the use of "oceanic" games allows us to consider labor contract bargaining even when the setting includes a continuum of unorganized workers.

Our approach in this paper has stressed the role of the workers' alternative opportunities; as a result, our models are rather abstract. We do not attempt to represent the bargaining *process* as a game in strategic form, with proposals and counter proposals following according to some fixed protocol. Such process-specific models are suitable for formalized bargaining situations (e.g., auctions), but are too restrictive to do justice to the free-wheeling and essentially cooperative[15] nature of labor-management negotiations. Instead, we adopt the view point that the wage determined will be the result of the underlying bargaining powers of the participants, irrespective of tactical considerations, and use a cooperative game in characteristic function form as the basic model and the Shapley value as the solution concept. Specifically, we predict conditions under which wage differentials may be expected to overstate (or understate) productivity differentials as a result of varying outside opportunities of the workers.

Let us mention two examples in which it seems appropriate to consider this factor explicitly. (1) Consider the market for legal workers and their illegal counterparts. In a recent empirical study, Kossaudji and Ranney [1985], examined the role of legal status in determining the wage rate. The study found that for every skill level and occupation considered there were significantly large variations between workers with and without legal status. Although other factors (like different probability of continuing employment, differences in other rewards to work such as room and board, and obvious data problems) may have contributed to such variations, we speculate that the role of outside opportunities in this market is important. The legal immigrants are likely to have better outside opportunities than their illegal counterparts, simply because the illegal entrants are confined to a more limited class of jobs and are in a bad bargaining position. How strong this effect is is clearly an empirical issue. (2) Consider the wage schedule for the members of technical staff in the large semi-research corporations. Researchers with degree in mathematics are hired at wages which may be 10-15 percent less than computer science graduates, hired in the same job classification to do the same type of work. (In the margin, it is not unreasonable to think of the two as perfect one-to-one substitutes.) We conjecture that a detailed study on wage determination for these workers will find the role of outside opportunities to be rather important.

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15. "Cooperative" in the game theory sense; i.e., when the contenders come to terms, they can write any contract they please, and then are bound by it.

Another proposition derived from our model is that the outsiders may also get paid. The game theoretic explanation is clear: their existence and availability keeps the wage of insiders below what it would otherwise be. The economic observation related to this proposition need a more careful examination. Although direct payments to outsiders are unlikely to occur, situations where outsiders receive other benefits are not uncommon. Not having sufficiently detailed data, let us speculate here on one such situation, the interesting phenomenon of charitable contributions by corporations.[16] If one views the corporation as a rational economic entity devoted to earning profits for the stockholders, then the charitable contributions should have the property that they provide returns greater than costs. A corporation may have several incentives to contribute, based on product market considerations (community goodwill), property value of the corporation (improving the quality of the community), or factor market considerations.

We shall elaborate here on the latter. The development of the local environment of the corporation improves the quality of life and thus attracts potential employees (among others) to the area. That benefits the corporation because the availability of additional workers strengthens its power in negotiations with its current employees. The potential employees are getting paid, although indirectly, by receiving goods or services, like better public education, which they would have to pay for in a different location.

8. Summary

A model of wage determination which considers only the physical technology and fails to account for bargaining between employers and workers is an insufficient representation of the factors affecting wage. As one recognizes the potential losses in terminating employment

16. This is the fastest growing segment of private philanthropy. Since 1970, annual giving by U.S corporations has increased 220 percent to a total of 2.7 billion in 1980 (C.C.Garvin [1982]).

relationships, it becomes clear that the bargaining power generated by the ability of each side to inflict costs on the other should be explicitly considered in the analysis of wage determination.

This study used cooperative game theory to formulate a theoretical analysis of wage determination, under different institutional market structures. It explicitly integrated both the actual on-the-job productivity and the costs of separation into a bargaining model.

In the first model we dealt with unorganized labor. It was shown that for a given employers' structure and a given on-the-job productivity, the workers wage is inversely related to the size of the labor force and is positively affected by the workers' outside opportunities. The model is capable of predicting wage variations among workers that are independent of productivity variations, even when workers bargain multilaterally as individuals without a formal union. In the second model such a formal union was introduced, acting as a single agent to represent at least some of the workers. The Shapley value payoff to all the participants was derived, and the question of the wage to the union members under different institutional structures was addressed. The major predictions of model I hold in presence of a union U as well. In addition it was shown, among other results, that when *all* workers (including outsiders) negotiate as a unit (bilateral monopoly), their total payoff is higher than what they get unorganized. ¶ In a subsequent research, we wish to extend the present model in two directions: to allow for more than one type of labor, and to consider explicitly the interplay between several employers, who may (or may not) cooperate in wage negotiations while competing in the product market.

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