THE ROLE OF MARKET INTRANSPARENCY IN INSURANCE MARKET MODELS*

A Reconsideration of the Rothschild-Stiglitz Insurance Market Model

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Abstract

Rothschild and Stiglitz (1976) showed in their insurance market model (R-S model) that imperfect asymmetrically distributed information can imply non-existence of Nash equilibria. We supplement their model with market intransparency (i.e., customers do not know all offers). When no Nash equilibrium exists in the original R-S model then, under market intransparency, there emerges a market solution according to which many different contracts, arranged as to a certain distribution, are offered simultaneously. If there exists an equilibrium in the original R-S model, the two equilibria coincide. Customers' welfare is not affected by the degree of market intransparency. Instead of intransparency, alternative frictions ensure analogous solutions.

1. Introduction

Imperfect information is playing an increasingly important role in explaining economic phenomena (see Stigler 1961, Phelps 1970, Akerlof 1970 for early papers). Rothschild and Stiglitz (1976) and Wilson (1977) have shown in their insurance market model that imperfect information on one side of the market about the good to be exchanged may imply non-existence of market solutions.

In these models of Rothschild and Stiglitz and of Wilson (R-S model) customers with different probabilities of a loss look for insurance contracts. Insurance companies cannot distinguish between these customers, but customers with different risks have different preferences with respect to contracts: For an increase of the amount of coverage, high risk customers accept a higher increase of the premium than low risk customers. Rothschild and Stiglitz show that, for a high enough proportion of good risks in the whole population, to every set of cost-covering contracts offered in the market there can be offered an additional contract which makes positive profits while, due to the new contract, the old contracts either make losses or get no customers. In this case a Nash equilibrium does not exist.

In the analysis of a number of other markets these informational asymmetries raise analogous problems: On credit markets banks can judge the risks of their customers only imperfectly and the preferences of the borrowers with respect to the terms of the contract (credit amount and interest rate) differ with their riskiness (e.g., Jaffee and Russell 1976, Milde and Riley 1986). On labor markets the principals have the problem of filtering out the good workers among indistinguishable applicants (e.g., Miyazaki 1977, Rosenthal and Weiss 1984). In some cases these existence problems are formed to an argument for regulation of the concerned industry

(for references see Kunreuther, Kleindorfer and Pauly 1983, and Eisen 1984). In my view, these models make it evident that our conceptions of a market with imperfect asymmetrically distributed information are so far not subtle enough to describe markets consistently.

In this paper we show that a suitable supplementation of the R-S model by further informational frictions ensures the existence of a market solution. For this we take into account that for customers the market is intransparent: Every customer knows only a small random fraction of the contracts offered and chooses only within this subset. In view of the extremely small market transparency of real insurance markets this is a sensible abstraction.

If no Nash equilibrium exists in the original R-S model, then under market intransparency there exists an Epsilon-Nash equilibrium in pure strategies. That is, if the contracts offered by the firms are arranged corresponding to a certain canonical distribution then (1) all firms make zero expected profits and (2) no firm can increase its expected profit over a certain positive boundary ϵ by changing its offer while all competing firms stick to theirs. This upper boundary ϵ converges to zero if the size of the customer population grows to infinity while the publicity level of firms, that is the expected number of customers informed about a given offer, is kept constant. Thus, if firms incur small but positive switching costs each time they vary their offer and the customer population is large enough, then the canonical distribution forms a Nash equilibrium. If the customer population is not large enough then there always remain small incentives for firms to change their offers. However, large deviations of the actual distribution from the canonical distribution imply profit possibilities the exploitation of which reduces the deviation and, thus,

stabilizes the offer distribution. If there exists a Nash equilibrium in the original R-S model without market intransparency, it retains its equilibrium property when market intransparency is introduced.

The publicity level of firms has no effect on customers' welfare. If this level increases and each customer can choose among more firms, then the offer distribution of firms adapts such that the distribution of sales remains unchanged.

Instead of market intransparency, alternative frictions can be employed to ensure market solutions with analogous structures. We consider heterogeneity among firm-specific characteristics of offers -- e.g., with respect to the location of a firm's offices or special terms of the contract fitting to the needs of some consumer groups -- and preferences of customers which get influenced by these characteristics non-uniformly. If there is enough heterogeneity of this kind then the same canonical distribution leads to an Epsilon-Nash equilibrium, respectively to a Nash equilibrium in case of positive switching costs.

If none of these frictions is big enough to ensure the existence of a Nash equilibrium then the market will establish an alternative form in which firms allocate different offers to different customers randomly. In the extreme case, if firms can vary their offer without cost either over customers or quickly through time but cannot observe current offers made to a specific customer by competitors, then there exists a unique mixed-strategy pooling Nash equilibrium.

To ensure a market solution in the R-S model, alternative equilibrium concepts have been proposed, see Wilson (1977), Miyazaki (1977), Grossman (1979), and Riley (1979). However, as is shown in Zink (1985), each of these concepts becomes implausible if one takes into account that the market

share of each firm always remains a small fraction of the whole market. Another solution was proposed by Dasgupta and Maskin (1986) who proved the existence of mixed-strategy equilibria in games which contain the R-S model as a special case. Rosenthal and Weiss (1984) constructed such a mixed-strategy equilibrium for the case of linearized indifference curves. The structure of this equilibrium turns out to be similar to our mixed-strategy equilibrium, however, we offer an alternative interpretation of mixed-strategy equilibria.

In the present paper, for simplicity, we assume that every firm offers one contract only. In this case, for the existence of an Epsilon-Nash equilibrium, we have to assume a suitably bounded risk aversion of the customers. For the general case of arbitrary many different contracts per firm a similar market solution emerges but no such assumption on the risk aversion is required (Zink 1985).[1]

This paper is organized according to the following plan. In Section 2 we describe the assumptions of the model. Section 3 analyzes how the expected profit of a firm depends on the choice of its own contract and on the offers of competing firms. In Section 4 market solutions are constructed both for the case where the R-S model leads to an equilibrium and for the case where it predicts market solutions to fail. The Epsilon-Nash equilibrium property is proved in Section 5 and Appendices A-E. Subsequent sections deal with the stability of the offer distribution (Section 6), mixed-strategy equilibria (Section 7), heterogeneity of contracts and preferences (Section 8), welfare effects of market intransparency (Section 9), and real insurance markets (Section 10). Section (11) gives a summary. Footnotes are listed in the Appendix.

2. The Assumptions of the Model

We consider individuals whose wealth $W_1^o > 0$ reduces in case of accident to $W_2^o := W_1^o - d \geq 0$ and insurance companies who offer insurance contracts at a premium α_1 guaranteeing to pay the amount v if a loss occurs. A contract, therefore, is described as an ordered pair (W_1,W_2) where $W_1 = W_1^o - \alpha_1$ is the net wealth of the policyholder in case of no accident and $W_2 = W_1^o - d - \alpha_1 + v = W_2^o + \alpha_2$ is his net wealth in case of accident. Without a contract all customers face the same (W_1^o,W_2^o). (See Figure 1.)

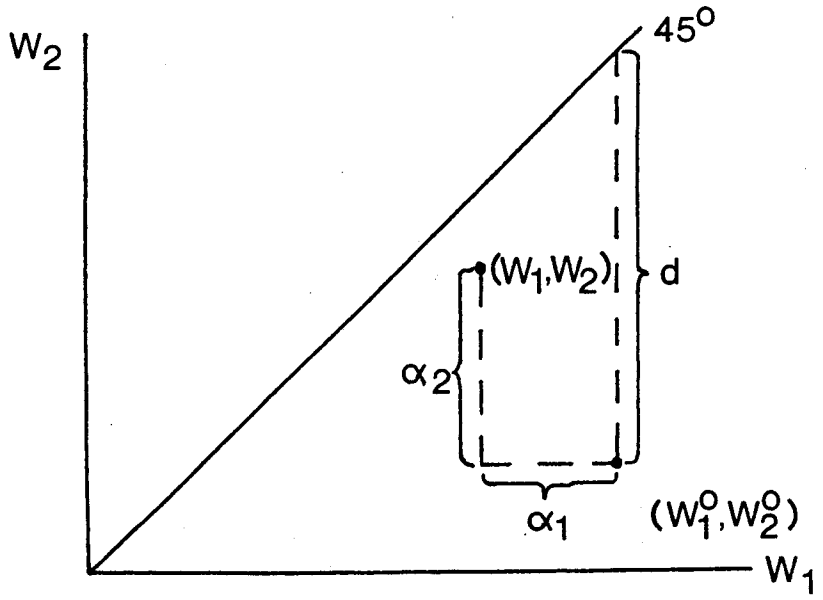

FIGURE 1

We assume that there are two groups of individuals (risk classes): low risk customers have a loss probability of p^g and high risks of p^s where $0 < p^g < p^s < 1$. Among the offered contracts known to him let every customer choose the contract which gives him highest expected utility,

$$U^x(W_1,W_2) := p^x\, V(W_2) + (1-p^x)V(W_1), \quad x \in \{g,s\}, \tag{1}$$

where x describes the risk class of the customer and V is a usual utility function with continuous derivatives $V' > 0$, $V'' < 0$, implying risk aversion. For simplicity, we complete this preference ordering by assuming that each customer prefers among two contracts with the same expected utility that one with the higher coverage v.[2] If a customer has to choose between contracts with the same parameters but of different companies, let each contract have the same probability of being bought. We assume that in each period t, $t \in N := \{1,2,\ldots\}$, k new customers buy contracts for their whole lifetime, $k \in N$, among them being an expected number of $\bar{Q}^g > 0$ low risks and an expected number of $\bar{Q}^s > 0$ high risks.[3] We denote $\bar{q} := \bar{Q}^g/\bar{Q}^s$ as market quality proportion.

Let there be infinitely (but countably) many firms i, $i \in N$, each of which offers one contract $P = (W_1,W_2)$ with $W_1^o \geq W_1 \geq W_2 \geq W_2^o$. At the beginning of each period each firm decides whether to offer the same contract as in the previous period or to change its offer. Its objective is to maximize its expected profit from the present period, $ER(P)$, in competition with other firms.[4] The occurrences of accidents are stochastically independent. If a firm offers a contract $P = (W_1,W_2)$ and sells to a mean number of EQ^g low risks and EQ^s high risks then its expected profit is given by

$$(2) \qquad ER(P) = EQ^g[(1-p^g)\alpha_1 - p^g\alpha_2] + EQ^s[(1-p^s)\alpha_1 - p^s\alpha_2]$$

where $\alpha_1 := W_1^o - W_1$ is the premium and $\alpha_2 := W_2 - W_2^o$ the net amount which the firm is paying to the policyholder in case of accident. We assume that firms are unable to distinguish between customers of different risk classes and, therefore, have to accept all risks which apply for their contract; in this respect information is asymmetrically distributed.

To incorporate market intransparency into this model, which is so far essentially identical to the R-S model, we assume that each customer knows the offer of a given firm with a probability π with $0 < \pi < 1$ where these informational relations are stochastically independent. We denote π as the degree of market transparency, and the expected number of customers informed about a given offer as publicity level of firms, $\mu = K\pi$. The degree of market transparency π we consider as exogenously given by the parameters K and μ. With growing market size K and constant publicity level μ the degree of transparency π converges to zero. However, for any market size, the customers informed about a given offer remain perfectly mixed among the whole population.

In reality, to know the offer of a given firm involves more than just being told that the policy terms of this firm are given by (W_1,W_2). As a preliminary, the customer has to acquire an understanding of the generally rather complicated clauses and to compare them with other offers. These obstacles are reasons for market intransparency.[5]

As a further friction we assume that each period a firm varies its offer, it incurs switching costs $\theta \geq 0$. However, unless stated otherwise, we assume $\theta = 0$.

3. The Determination of the Expected Profit of a Firm

In this section we determine how the expected profit ER of a firm depends on the position of its own offer P and on the position of the offers of the competing firms.

3.1 First we analyze the preferences of the customers. Although we have completed the preference ordering, we denote each set of contracts with equal expected utility as indifference curve. According to (1), indiffer-

ence curves have slopes

$$(3) \qquad \left.\frac{dW_2}{dW_1}\right|_{dU^x=0} = -\frac{1-p^x}{p^x}\,\frac{V'(W_1)}{V'(W_2)}, \qquad x \in \{g,s\}.$$

Thus, in every offer point (W_1,W_2) each indifference curve is downward sloping and, due to the risk aversion, it is the steeper, the higher W_2 and the smaller W_1. For low risks it is steeper than for high risks; for an increase of the indemnity v, high risks are willing to pay a higher price than low risks (see Figure 2).

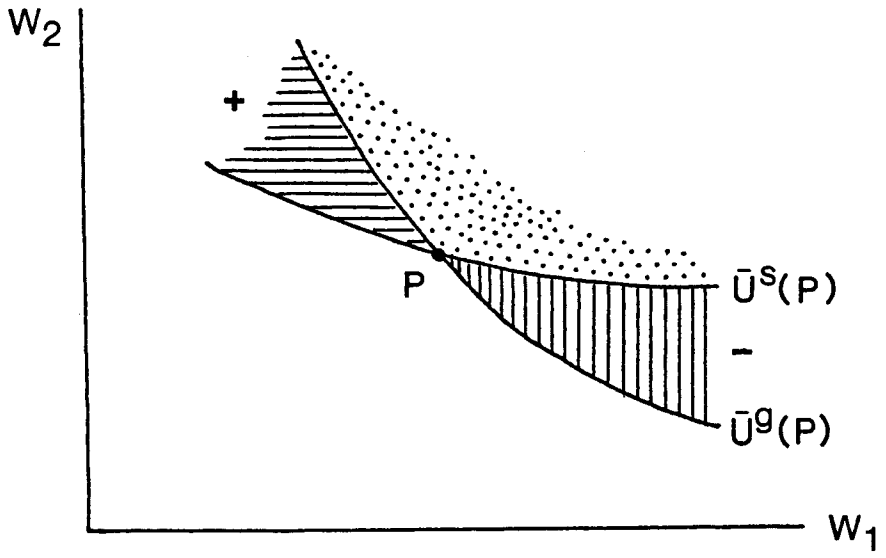

FIGURE 2

3.2 Now we calculate the risk composition of a firm's clientele.

A discrete offer distribution is defined as a mapping which assigns to each firm $i \in N$ an offer P_i in the offer space $\bar{A} := \{(W_1,W_2): W_1^o \geq W_1 \geq W_2 \geq W_2^o\}$. Alternatively, we describe a discrete offer distribution by the

set function H which assigns to each $\bar{A}' \subset \bar{A}$ the number of firms offering in $\bar{A}'$, $H(\bar{A}') := \#\{i \in N: P_i \in \bar{A}'\}$.

Firms, which offer in (W_1^o, W_2^o), always make zero profits. They are called inactive. All other firms are called active.

If a given firm j offers a contract $P \in \bar{A}$ and the distribution of all other active firms is equal to H, then the expected number of new type x customers, $x \in \{g,s\}$, buying from firm j is denoted by $E[Q^x(P)|H]$. We define the quality proportion of this firm as

$$q(P|H) := E[Q^g(P)|H]/E[Q^s(P)|H] \tag{4}$$

provided there is demand for firm j, i.e., $E[Q^g(P)|H] + E[Q^s(P)|H] > 0$. The quality proportion attains values in $[0,\infty]$.

Suppose, besides firm j there are m-1 firms, $m \in N$, offering in P from the distribution H. Then, a customer of risk class x, $x \in \{g,s\}$, buys the contract P if he knows at least one of the m offers in P but no offer which he would prefer to P. There are

$$\xi^x(P|H) := H(\{P' \in \bar{A}: [U^x(P') > U^x(P)] \text{ or } [U^x(P') = U^x(P) \text{ and } W_2(P') > W_2(P)]\}) \tag{5}$$

firms which he would prefer to P. Thus, since all informational relations are stochastically independent and each of the m firms in P has the same chance to be chosen, the expected number of type x customers of firm j is given by

$$E[Q^x(P)|H] = \bar{Q}^x \frac{1}{m} [1-(1-\pi)^m] (1-\pi)^{\xi^x(P|H)}. \tag{6}$$

Hence, with the short-hand

$$\rho := -\ln(1-\pi) > 0, \tag{7}$$

the quality proportion of firm j is

$$q(P|H) = \bar{q}e^{\rho[\xi^s(P|H)-\xi^g(P|H)]} \tag{8}$$

as long as there is demand for firm j so that (8) is well-defined.

Equation (8) can be interpreted graphically. The term $[\xi^s-\xi^g]$ describes the difference of the hatched areas in Figure 2 weighted accordingly to H. Firms in the vertically hatched area are preferred against P only by low risks and, thus, contribute to a worsening of the quality proportion in P. Firms in the horizontally hatched area are preferred against P only by high risks and, therefore, improve the quality proportion in P. Firms in the punctuated area are preferred against P by both risk groups and, hence, have no influence on the risk composition in P.

3.3 The expected profit of a firm also depends on the position of its own offer in the offer space. As fair odds value $\beta(P)$ of a contract P we define that quality proportion q which implies expected profits of zero for a firm offering in P (if such a quality proportion exists in $[0,\infty]$). According to (2), the fair odds value of any contract P is given by:

$$\beta(P) = -\frac{(1-p^s)\alpha_1-p^s\alpha_2}{(1-p^g)\alpha_1-p^g\alpha_2} \tag{9}$$

where (α_1,α_2) are the transformed coordinates of P according to Figure 1, provided $\beta(P) \in [0,\infty]$. All points with the same fair odds value β' form a straight and decreasing line in $\bar{A}$ running through (W_1^o,W_2^o). We call it the β'-fair odds line or $\{\beta=\beta'\}$. It has the slope

$$\left.\frac{dW_2}{dW_1}\right|_{d\beta=0} = -\frac{1-p^{\beta'}}{p^{\beta'}} \tag{10}$$

where $p^{\beta'} := p^g\beta'/(1+\beta') + p^s/(1+\beta')$ denotes the mean loss probability for

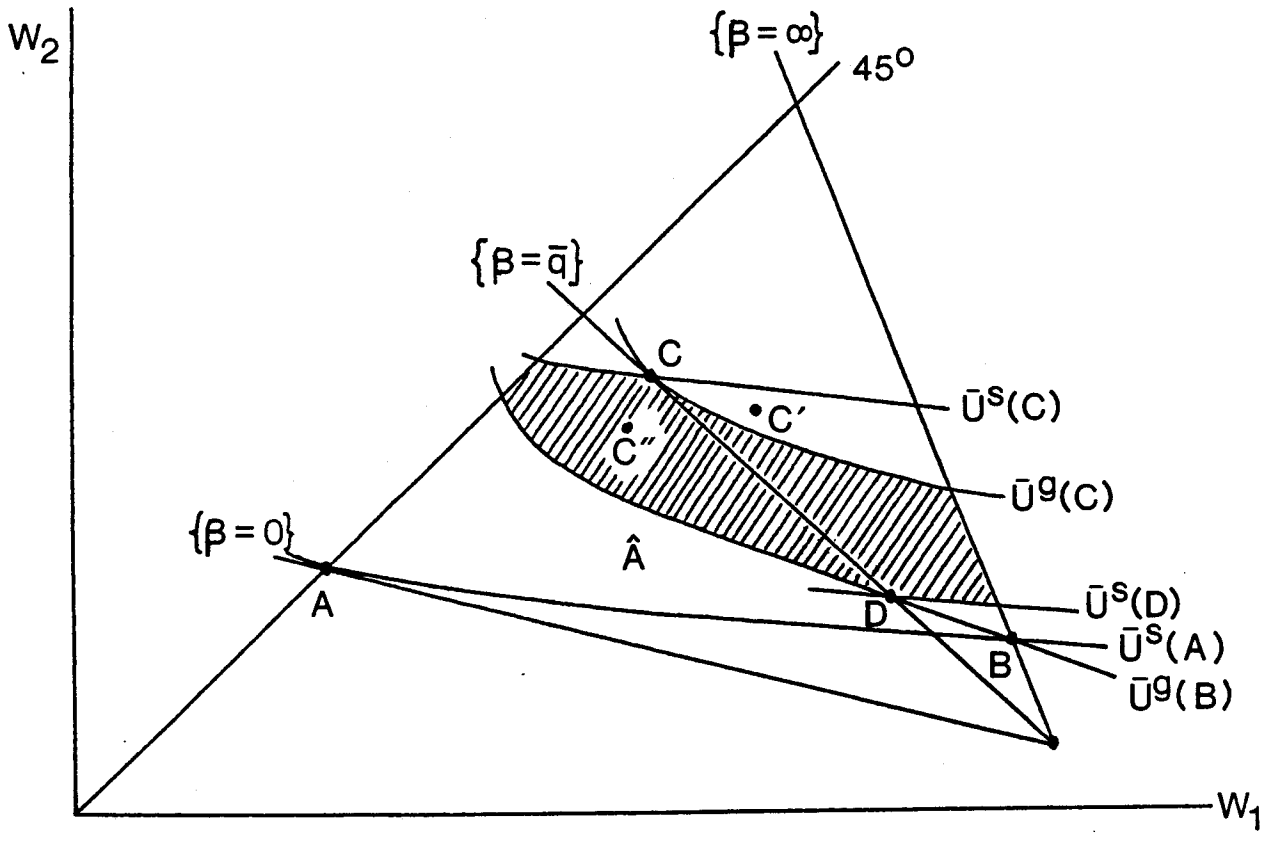

FIGURE 3

For all discrete distributions H the expected profit of a firm, which offers in P additionally to H, can be estimated using the quality proportion and the fair odds value,

$$(11) \qquad E[R(P)|H] \gtreqless 0 \iff q(P|H) \gtreqless \beta(P)$$

provided there is demand in P and $\beta(P) \in [0,\infty]$. Thus, a contract P, which is demanded with a quality proportion β', makes positive expected profits if P lies below the β'-fair odds line and negative expected

profits if P lies above.

4. The Construction of Market Solutions

4.1 To construct a market solution we first define some special contracts in the offer space in correspondence to Figure 3. Let A be that contract on the high risk fair odds line, $(\beta=0)$, which gives maximum expected utility to high risks. A comparison of (3) and (10) shows that A lies on the 45° line; at fair premiums customers prefer their wealth to be without uncertainty. Let B lie in the intersection of the low risk fair odds line, $(\beta=\infty)$, and the indifference curve of high risks through A, $\bar{U}^s(A)$. Let C be that contract on the market fair odds line, $(\beta=\bar{q})$, which gives maximum expected utility to low risks. Because of (3) and (10) C lies below the 45° line. If the market quality proportion $\bar{q}$ is so high that the indifference curve of low risks through B, $\bar{U}^g(B)$, intersects the market fair odds line, we denote this intersection by D. If it exists, D lies to the right of C.

If no intersection point D exists then, in the original R-S model without market intransparency, the contracts (A,B) form a Nash equilibrium (where it is assumed that at least two firms offer in B): Both firms make zero expected profits, since low risk customers buy contract B and high risks prefer contract A. No firm can gain positive profits by a variation of its offer, since any single deviating firm, which wanted to attract both risk types, had to place its offer above or on the low risk indifference curve $\bar{U}^g(B)$ and, thus, above or on the market fair odds line. But there, its quality proportion of $\bar{q}$ would imply losses.

In Appendix A we show that the introduction of market intransparency does not violate the equilibrium property of this separating market solution

{A,B}. For simplicity, though, we assume that both in A and B there are offers of infinitely many firms. This implies that each customer knows at least one offer in A and one in B. Hence, all high risks buy in A and all low risks in B as in the case of no intransparency. Again, no firm can gain positive profits by varying its offer, since any firm which could attract both risk types had to place its offer above or on the market fair odds line. Then it would receive a quality proportion of $\bar{q}$ which implies losses there.

In the remaining part of the paper we always assume $\bar{q}$ to be so high that an intersection point D does exist. In this case the R-S assumption of perfectly informed customers would prevent the existence of Nash or Epsilon-Nash equilibria: Pooling contracts like C" in Figure 3 (with $\beta(C") < \bar{q}$ and $U^g(C") > U^g(B)$) could profitably attract away all customers from {A,B}. Contracts like C' (with $U^g(C') > U^g(C)$ and $U^s(C') < U^s(C)$) could profitably attract away exclusively low risks from the pooling equilibrium candidate C. However, we will show that in this case the introduction of market intransparency ensures the existence of a market solution.

4.2 We give now a brief preview of this market solution. An endogenously determined finite number of firms will offer contracts simultaneously in the interval [C,D] while the remaining firms offer in {A,B}. In [C,D] each firm receives a positive expected number of customers and makes zero profits because its quality proportion equals $\bar{q}$. If for some $G \in [C,D]$ the offer of a firm moves to the right along the indifference curve $\tilde{U}^g(G)$ then its expected number of low risk customers remains constant. However, its expected number of high risks decreases, and it decreases the more, the larger the number of firms offering in the neighborhood of G.

Thus, a canonical continuous density of firms in [C,D] will be determined by the requirement that both the quality proportion and the fair odds value of a contract increase with the same rate if this contract moves away from G. We will show that each single firm can gain at most marginal profits by varying its offer if the contracts are arranged approximately to this density. Such discrete distributions, thus, will form Epsilon-Nash equilibria.

Before constructing these market solutions we make precise the notion of an Epsilon-Nash equilibrium.

Definition: A sequence of offer distributions $(H_K^+)_{K=1}^{\infty}$ is denoted as Epsilon-Nash equilibrium if for any profit boundary $\epsilon > 0$ there exists a market size $K^+(\epsilon,\mu) < \infty$ such that for any population of customers $K \geq K^+(\epsilon,\mu)$ and a distribution of active firms equal to H_K^+ we have:

(i) The expected profit of each active firm is zero.

(ii) If a firm i (previously active or inactive) varies its offer and all other firms stick to their contracts then the expected profit of firm i remains smaller than ϵ and its expected per customer profit remains below ϵ/μ. ***

We now construct discrete offer distributions H which we will prove to form Epsilon-Nash equilibria in the next section. For any two points G,S from the interval [C,D] on the market fair odds line let w_G, w_S be their W_1-coordinates. For any nonnegative, integrable function h on $[w_C,w_D]$ and any number $\ell \in [0,\infty]$ let $\bar{D}(h,\ell)$ be the set of all discrete distributions H on $\bar{A}$ with the following properties:

$$(12) \qquad \left|H([G,S]) - \int_{w_G}^{w_S} h(w)dw\right| \leq \ell$$

for all subintervals $[w_G,w_S] \subset [w_C,w_D]$, $H(\{A\}) = H(\{B\}) = \infty$, and $H(\{\tilde{A}'\}) = 0$ for all $\tilde{A}' \subset \tilde{A}$ which are disjunct from $\{A,B\} \cup [C,D]$. For any $\ell \in [1,\infty]$, each set $\tilde{D}(h,\ell)$ contains at least one discrete distribution, since the offers $P_1,P_2,\ldots$ of a discrete distribution H in $[C,D]$ can be chosen according to the formula $\int_{P_1}^{D} h\,dw = 1/2$, $\int_{P_{i+1}}^{Pi} h\,dw = 1$ for all $i = 1,2,\ldots$ (see Figure 4). If $H \in \tilde{D}(h,\ell)$, we call H a discrete approximation with respect to the density h and ℓ its approximation error.

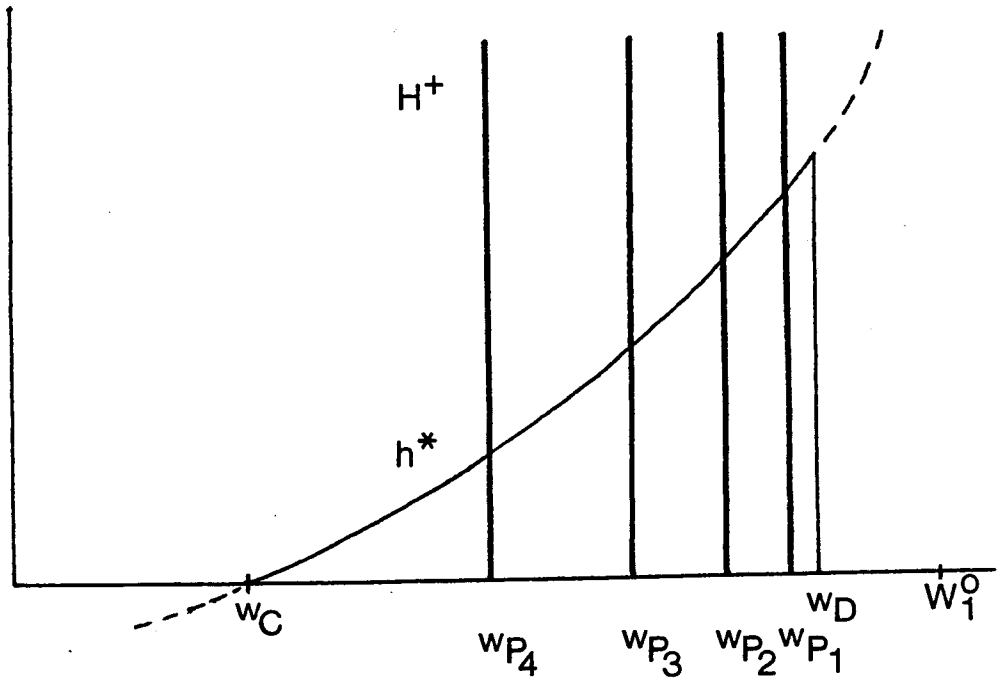

FIGURE 4

Now we can state the main result of this paper. It determines a unique function Γ such that discrete approximations with respect to Γ/ρ form an Epsilon-Nash equilibrium.

Proposition 1: Let the market quality proportion $\bar{q}$ be so high that there exists an intersection point D. Let the risk aversion of customers be bounded from above by inequality (C5) in Appendix C. Then there exists a nonnegative integrable function Γ on $[w_C, w_D]$ such that for any approximation error $\ell \geq 1$ each sequence of discrete distributions $(H_K^+)_{K=1}^{\infty}$ with $H_K^+ \in \bar{D}(\Gamma/\rho, \ell)$, $\rho = -\ln(1-\mu/K)$, $K \in N$, is an Epsilon-Nash equilibrium.[6] Except for sets of measure zero, the function Γ is uniquely determined by the above property.[7] Γ is presented in Lemma 1, equation (15). The set $\bar{D}(\Gamma/\rho, \ell)$ we denote as canonical set with respect to the approximation error ℓ; each element of it is called canonical discrete distribution. ***

If we take into account positive switching costs θ, then the above Epsilon-Nash equilibria become Nash equilibria.

Corollary 1: Let the assumptions of Proposition 1 hold true. If each firm incurs positive switching costs θ each time it changes its offer, then there exists a market size $K^{++}(\mu, \theta, \ell) < \infty$ and a publicity level of firms $\mu^{++}(K, \theta, \ell) > 0$ with the following property: if (i) $K \geq K^{++}$ or $\mu \leq \mu^{++}$ and (ii) all firms are distributed according to a canonical discrete distribution $H_K^+ \in \bar{D}(\Gamma/\rho, \ell)$, then all firms make zero profits and no firm has an incentive to change its offer. K^{++} increases in μ and decreases in θ, μ^{++} increases in K and θ. ***

Proof of Corollary 1: According to Proposition 1 we can choose $K^{++}(\mu, \theta, \ell)$ such that the maximal expected profit per period under any discrete distribution $H_K^+ \in \bar{D}(\Gamma/\rho, \ell)$ with $K \geq K^{++}$ is smaller than the switching costs θ. Thus, no firm has an incentive to change its offer.

From the estimation of the maximal expected profits in the proof of Proposition 1, (22), we will see that an analogous argument holds true for μ. From (22) we will also see how K^{++} is influenced by (μ,θ), and μ^{++} by (K,θ). ***

5. The Proof of Proposition 1

5.1 Let h be any nonnegative, integrable function on $[w_C,w_D]$, $\ell \in [1,\infty)$ and $H \in \bar{D}(h,\ell)$. Let all firms be distributed according to H. First, we prove that then each firm in $[C,D]$ receives a positive expected number of customers and all firms make zero expected profits. The proof uses Figure 3.

We have already seen that contracts in A are only demanded by high risks and contracts in B only by low risks. Thus, both of them make zero expected profits.

Contracts in $[C,D]$ are put in the same preference ordering by both risk classes because in all points of (C,D) the indifference curves of both risk classes are flatter than the market fair odds line. Thus, for any contract $P \in [C,D]$ the number of contracts from H with a higher expected utility level than P is finite and the same for both risk classes, $H([C,P))$. Any two customers of different risk classes buy contract P with the same positive probability. Hence, the expected number of customers in P is positive and the quality proportion equals that of the total population, $\bar{q}$. On the market fair odds line this implies zero expected profits. This result follows also from equation (8) since $\xi^s(P|H) = \xi^g(P|H)$.

5.2 To prove that for any market size K no firm from H_K^+ can gain more than marginal profits by choosing another offer, we utilize general offer distributions.

As general offer distribution H we denote any nonnegative σ-additive function on the set of Borel-measurable subsets $\tilde{A}'$ of $\tilde{A}$.[8] $H(\tilde{A}')$ can be interpreted as the number of firms offering in $\tilde{A}'$, however, it is not restricted to natural numbers as is the case for discrete distributions. Every discrete distribution is also a general distribution.

For any nonnegative, integrable function h on $[w_C, w_D]$, $\ell \in [1,\infty)$ let $\tilde{D}(h,\ell)$ be the set of all general distributions H on $\tilde{A}$ with the properties: (i) inequality (12) holds for all subsets $[w_G, w_S] \subset [w_C, w_D]$, (ii) $H(\{A\}) = H(\{B\}) = \infty$, (iii) $H(\{\tilde{A}'\}) = 0$ for all subsets $\tilde{A}' \subset \tilde{A}$ which are disjunct to $\{A,B\} \cup [C,D]$. $\tilde{D}(h,\ell)$ contains $\bar{D}(h,\ell)$. $\tilde{D}(0,\infty)$ contains all (general) distributions H with support $\{A,B\} \cup [C,D]$ and $H(\{A\}) = H(\{B\}) = \infty$. $\tilde{D}(h,0)$ contains exactly one distribution and on $[C,D]$ this has a density which coincides with h almost everywhere.

We will show in Lemma 1 that for each $K \in N$ there exists a unique general distribution $H_K^* \in \tilde{D}(0,\infty)$ such that, if all firms are distributed according to H_K^*, an additional firm would make zero expected profits on the support of H_K^* and negative profits with any other contract demanded by customers. The result of Proposition 2 then follows for discrete approximations of H_K^*.

First, we introduce another representation of offer points. Corresponding to Figure 3, let $\hat{A}$ be the set bounded by the indifference curves $\tilde{U}^g(C)$, $\tilde{U}^s(C)$, $\tilde{U}^g(B)$, and $\tilde{U}^s(D)$, the $\{\beta=\infty\}$-fair odds line and the bisector $\{W_1=W_2\}$, that is $\hat{A} := \{P \in \tilde{A}: U^g(C) \geq U^g(P) \geq U^g(D), U^s(C) \geq U^s(P) \geq U^s(D), 0 \leq \beta(P) \leq \infty\}$. All points $P \in \hat{A}$ we now describe by their indifference curves $\tilde{U}^g(P)$, $\tilde{U}^s(P)$: According to Figure 5, for each two points G and S of the interval $[C,D]$ with abscissa-values w_G and w_S we define point $P = P(G,S) = P(w_G, w_S)$ as the intersection of the indifference curves

$\bar{U}^g(G)$ and $\bar{U}^s(S)$. The first argument in $P(\cdot,\cdot)$, therefore, fixes the intersection of the $\bar{U}^g(P)$-curve with the market fair odds line and the second argument fixes the intersection of the $\bar{U}^s(P)$-curve with the market fair odds line. $P(G,S)$ lies to the righthand side of $\{\beta=\bar{q}\}$ if $w_G < w_S$, and to the lefthand side if $w_G > w_S$.

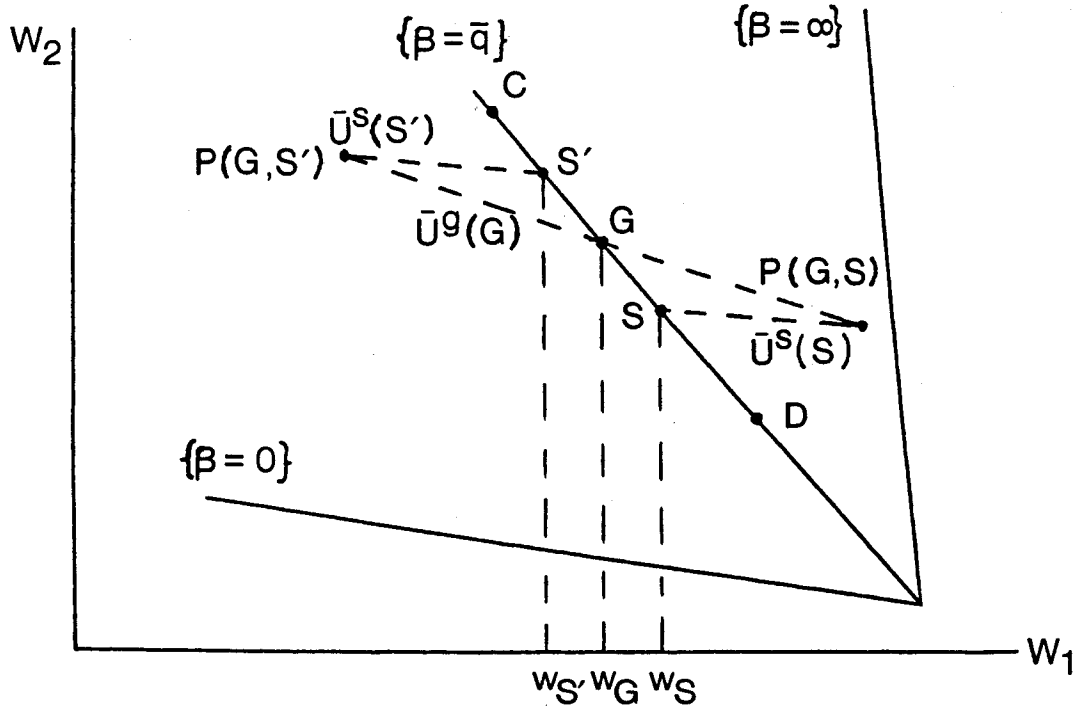

FIGURE 5

Now we extend our definition of the quality proportion to general distributions in $\tilde{D}(0,\infty)$. For any $H \in \tilde{D}(0,\infty)$ and any $P(G,S) \in \hat{A}$ we define

$$q(P(G,S)|H) := \begin{cases} \bar{q}e^{\rho\{H([C,S])-H([C,G])\}} & \text{if } \beta(P) \geq \bar{q} \\ \bar{q}e^{\rho\{H([C,S))-H([C,G))\}} & \text{if } \beta(P) \leq \bar{q}, \end{cases} \tag{13}$$

which coincides with (8) if H is discrete. If $H \in \tilde{D}(h,0)$ then we have

$$q(P(G,S)|H) = \bar{q}e^{\rho\int_G^S h\, dw}. \tag{14}$$

This definition can be interpreted in the following way. For points P on the righthand side of the market fair odds line, the integral $\int_G^S h\, dw$ describes the number of firms which are preferred against P by high risks, but not by low risks. Therefore, the quality proportion in P is greater than or equal to that of the population, $\bar{q}$, and it is the better the higher the density in the interval (G,S). For points on the lefthand side of the market fair odds line the integral $\int_G^S h\, dw$ is negative because of $w_S < w_G$. The absolute value of the integral describes the number of firms preferred against P by low, but not by high risks.

Now we can state our basic Lemma 1 which will also be used for the analysis of alternative models with other frictions than market intransparency (e.g., in Section 7.2).

Lemma 1: For all $w_G \in [w_C, w_D]$ let

$$\Gamma(w_G) := \frac{d}{dw_S}\Big|_{w_S=w_G} \ln \beta(P(w_G,w_S)) \tag{15}$$

and

$$h_K^*(w_G) := \frac{1}{\rho}\,\Gamma(w_G), \quad \rho = -\ln(1-\pi) = -\ln(1-\mu/K). \tag{16}$$

Let the risk aversion of customers be bounded from above by inequality (C5) in Appendix C. Then, for any general distribution $H \in \tilde{D}(0,\infty)$ the quality proportion $q(P(G,S)|H)$ fulfills

$$(17) \qquad q(P(G,S)|H) \begin{cases} = \beta(P(G,S)) & \text{for } G = S \\ < \beta(P(G,S)) & \text{for } G \neq S \end{cases}$$

for all points $P(G,S) \in \hat{A}$ if and only if $H \in \tilde{D}(h_K^*,0)$. The unique element in $\tilde{D}(h_K^*,0)$ we denote as the canonical continuous distribution H_K^*.[9] The quality proportion $q(P(G,S)|H_K^*)$ is independent of the market size K and of the publicity level of firms μ.[10] ***

Lemma 1 states that any distribution with support $\{A,B\} \cup [C,D]$ fulfills condition (17) if and only if on $[C,D]$ it has no atoms and is given there by the density h_K^*.

Before we prove Lemma 1 we make plausible that Γ is of the form depicted in Figure 4. We do not, however, use this argument in any of the following proofs. In C a marginal shift of a contract along the $\tilde{U}^g(C)$-curve, $dP(w_C,w_S)/dw_S$, results in a marginal fair odds value change of zero since the market fair odds line is tangent here. Hence, Γ is zero in C. In other points G of $[C,D]$ such a marginal shift causes a marginal β-shift of positive proportion. The higher the abscissa w_G of G, the more closely packed are the fair odds lines and the flatter are the indifference curves there. Thus, if G is approaching (W_1^o,W_2^o), the marginal shifts along $\tilde{U}^g(G)$ result in increasing and, in the limit, infinite increments of the logarithmed fair odds values.

Proof of Lemma 1:

1. In Appendix B we show that any $H \in \tilde{D}(0,\infty)$ which fulfills (17) is absolutely continuous on $[C,D]$ with a density given by (16). Here we show the same result in an easier way under the assumption that H has a continuous density on $[C,D]$.

Condition (17) requires for any point $G \in (C,D)$ that, if a contract P is moved along $\bar{U}^g(G)$ to the right, then the corresponding proportions $q(P|H)/\beta(P)$ reach a local maximum at G. Since we assumed the existence of a continuous density of H, both $q(P|H)$ and $\beta(P)$ are continuously differentiable in the interior of $\hat{A}$. Thus, the local maximum implies

$$\frac{d}{dw_S}\Big|_{w_S=w_G} \ln q(P(G,S)|H) = \frac{d}{dw_S}\Big|_{w_S=w_G} \ln \beta(P(G,S)) \tag{18}$$

for all $G \in [C,D]$. Straightforward differentiation of $\ln q(P(G,S)|H)$ according to (14) yields $\rho h = \Gamma$ and, hence, (16).

2. Now we show that $q(P(G,S)|H_K^*)$ fulfills (17) for all $P(G,S) \in \hat{A}$ and is independent of K and μ.

We know from (14) that

$$q(P(G,S)|H_K^*) = \bar{q}e^{\int_G^S \Gamma\, dw}. \tag{19}$$

Thus, it is independent of K and μ. For $G = S$ it is equal to $\bar{q} = \beta(P(G,S))$. The property $q(P(G,S)|H_K^*) < \beta(P(G,S))$ for $G \neq S$ is proven rigorously in Appendix C. However, in the following we make plausible by means of Figure 6 why this property holds true without requiring any restrictions on the risk aversion of customers.

For the purpose of this heuristic argument we assume that $h_K^* = \Gamma/\rho$ for all points on the market fair odds line in $\bar{A}$; this does not change the quality proportion for points P in $\hat{A}$. For each $G \in (C,D)$ we compare the growth rates of the quality proportions for movements along $\bar{U}^g(G)$, $d[\ln q(P(w_G,w_S)|H_K^*)]/dw_S$, with the corresponding growth rates of the fair odds values, $d[\ln \beta(P(w_G,w_S))]/dw_S$. The β-growth rates are monotonically increasing in w_S, in the tangency point T_G they are zero, and in P_G

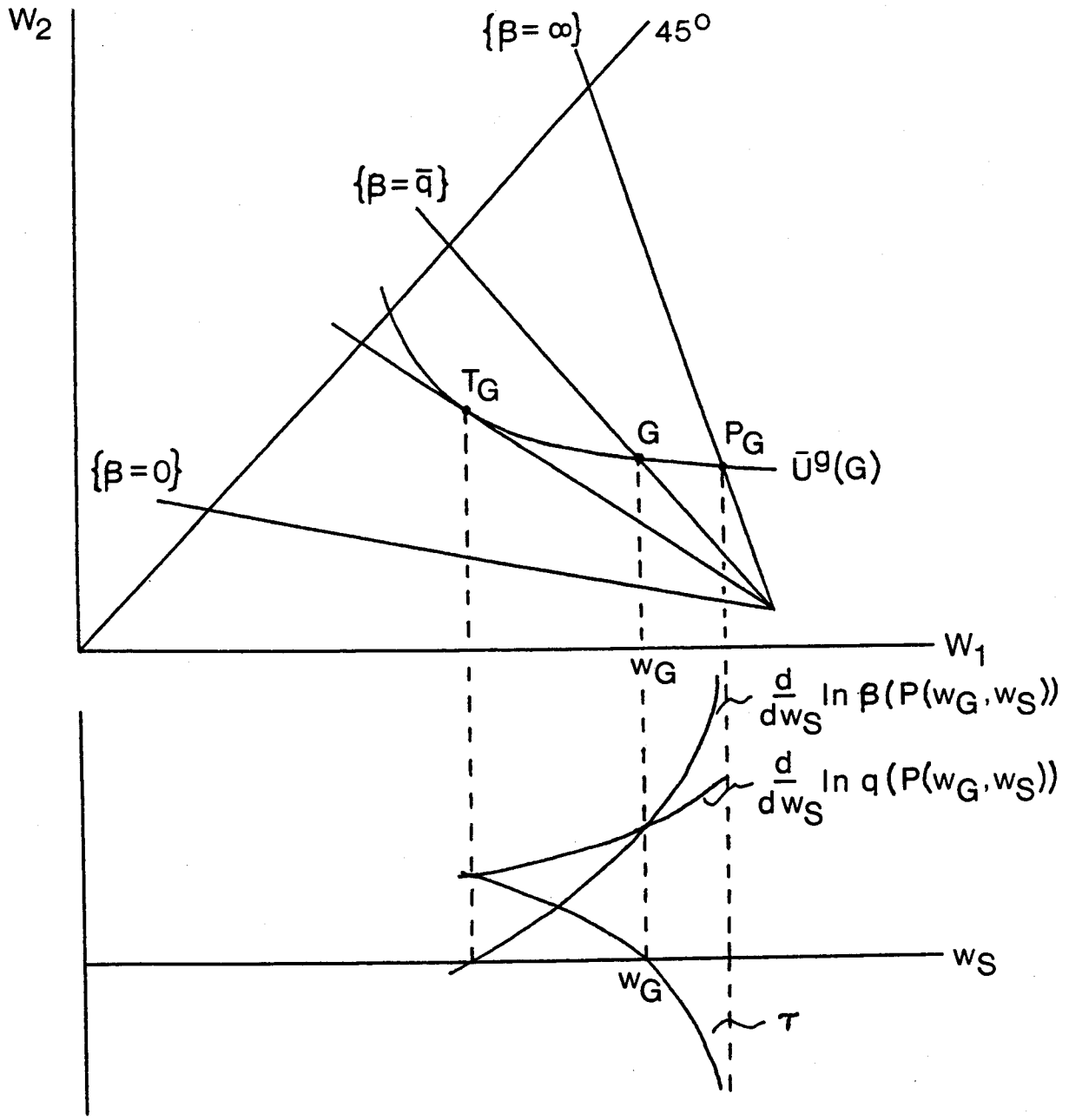

FIGURE 6

they reach infinity. The q-growth rates are also monotonically increasing. However, they remain positive and bounded. Therefore, the difference between both rates, τ :=d ln q/dw$_S$ - d ln β/dw$_S$, has to be positive in T_G, negative in P_G, and continuous everywhere. Thus, it must be zero in at least one point. We know already that, due to the definition of h_K^* and (17), the difference is zero in G. The existence of more than one zero

point is at least not immediately plausible. Under the assumption that there is only one zero point, the difference is positive on the lefthand side of G and negative on the righthand side. Hence, the fair odds value is larger than the quality proportion on both sides of G on $\bar{U}^g(G)$. ***

5.3 Now we proceed with the proof of Proposition 1. Using Lemma 1, we show that the maximal expected profits from changing an offer converge to zero if the market size K increases to infinity. Let H_K^+ be a (canonical discrete) distribution from the canonical set $\bar{D}(h_K^*,,\ell)$ for some $K \in N$, $\ell \geq 1$. Let all firms be distributed according to H_K^+.

First, we analyze the profit possibilities of a firm which in the previous period was inactive or had an offer in $\{A,B\}$. Suppose, in the current period it offers in $P(G,S) \in \hat{A}$. We make use of the following three relations: From (8), or (13), we know

$$q(P(G,S)|H_K^+) = \bar{q}e^{\rho H((G,S])} \tag{20}$$

where in case of $w_G > w_S$ we define $H((G,S]) := -H([S,G))$. From Lemma 1 we use the inequality

$$q(P(G,S)|H_K^*) = \bar{q}e^{\rho \int_G^S h_K^* dw} \leq \beta(P(G,S)). \tag{21}$$

From the definition of the fair odds value and the abbreviation $c^x := (1-p^x)\alpha_1 - p^x\alpha_2$, $x \in \{g,s\}$, we get $ER = EQ^g c^g + EQ^s c^s$, $\beta = -c^s/c^g$, and $0 \leq -c^s \leq p^s v$. Now, the expected profit can be estimated as

$$E[R(P(G,S))|H_K^+] = EQ^g\, c^g + EQ^s\, c^s \tag{22}$$

$$= -c^s\, EQ^s \left[\frac{q(P|H_K^+)}{\beta(P)} - 1\right] \leq p^s v\mu \left[\frac{q(P|H_K^+)}{q(P|H_K^*)} - 1\right]$$

$$\leq p^s v\mu \left[e^{\rho[H_K^+((G,S])-\int_G^S h_K^* dw]} - 1\right] \leq p^s v\mu \left[e^{\rho\ell}-1\right].$$

For the expected profit divided by the expected number of customers we get analogously

$$(23) \qquad \frac{E[R(P(G,S))\,|H_K^+]}{E[Q^g(P)\,|H_K^+] + E[Q^g(P)\,|H_K^+]} \leq -c^s\left[\frac{q(P|H_K^+)}{\beta(P)} - 1\right] \leq p^s v[e^{\rho\ell}-1].$$

If a firm i changes its offer from $[C,D]$ to some $P(G,S) \in \hat{A}$ then the distribution describing firm i's competitors is no longer H_K^+ but some other discrete distribution, $H_{K,-i}^+$, which now lies in $\tilde{D}(h_K^*, \ell+1)$. Thus, the same boundaries as in (22) and (23) remain valid if ℓ is replaced by $\ell+1$.

The expected profits from offer variations in $\hat{A}$, thus, converge to zero if $\rho = -\ln(1-\mu/K)$ is sent to zero. In Appendix D we show that the same boundaries are valid if a firm moves its offer to some $P \notin \hat{A}$.

The uniqueness of Γ we provide in Appendix E. ***

6. The Stability of the Offer Distribution

6.1 The profit possibilities which remain in each period with zero switching costs under a discrete distribution $H_K^+ \in \tilde{D}(\Gamma/\rho, \ell)$ are illustrated in Figure 7.

Let P_{n-1}, P_n, P_{n+1} be three neighboring offer points from H_K^+. The firm offering in P_n could increase its profits by increasing its premium. As long as its new offer remains in the triangle (P_{n-1}, P_{n+1}, P') its quality proportion remains constant and the decrease of its fair odds value results in positive expected profits.

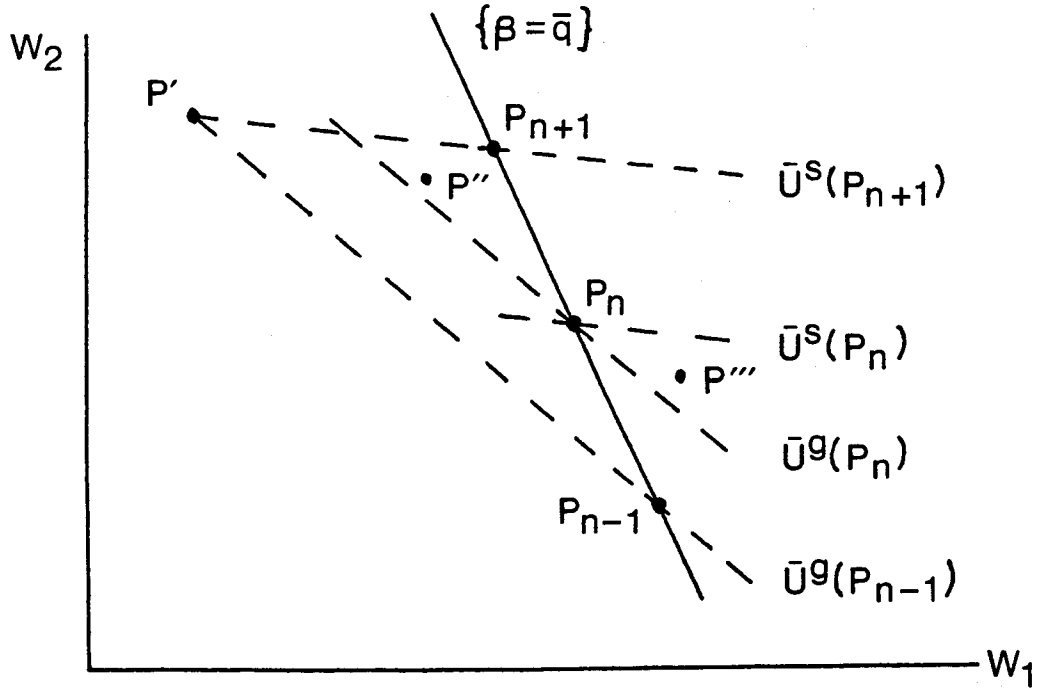

FIGURE 7

Any firm previously offering A or B could make profits with a contract like P" or P"'. In P" its quality proportion is $\bar{q}$ while its fair odds value is smaller than $\bar{q}$. In P"' the quality proportion is higher than $\bar{q}$ since high risks can get attracted away by one more firm than low risks. The fair odds value of P"', however, can be chosen arbitrary near to $\bar{q}$. This case corresponds to the instability argument of Rothschild and Stiglitz. But in contrast to their model, market intransparency ensures that the firm in P"' attracts some high risk customers as well as low risks.

In all three cases, the profit possibilities are the smaller the greater the market size K. An increase in K increases the canonical discrete densities and, therefore, the distance between neighboring firms

P_{n-1}, P_n, P_{n+1}. This pushes P' and P'' nearer to the market fair odds line. In the case of P''' a decrease of π reduces the filtering function of the offer P_n to attract away high risks from P'''.

6.2 Both, exogenous disturbances and the exploitation of the small profit possibilities which remain in canonical discrete distributions, could magnify deviations of the actual offer distribution from the canonical continuous distribution $H_K^* \in \tilde{D}(h_K^*,0)$. We will see that these deviations cause new profit possibilities. The exploitation of these possibilities by some firms can affect the quality proportions of other firms and, thus, result in further reactions. Although we do not analyze this adaptation process explicitly we show why large deviations of the actual distribution from H_K^* are corrected in the course of this process.

A deviation of the actual offer distribution which leads to an additional offer P sufficiently far away from the [C,D]-interval, but which leaves the distribution on [C,D] unchanged, will experience an immediate correction since P will suffer losses. A deviation which only affects the offer distribution within [C,D], however, will not receive a direct correction since the profits of all firms remain zero. For simplicity, in the following we only consider the impacts of such disturbances and we assume that the actual offer distribution $\hat{H}$ in [C,D] is absolutely continuous and has a piecewise continuous density $\hat{h}$ there, $\hat{H} \in \tilde{D}(\hat{h},0)$.

First we analyze the profit possibilities given under $\hat{H}$, $E[R(P)|\hat{H}]$.

Proposition 2.1: If for some interval $(w',w'') \subset [w_C,w_D]$ we have $\hat{h}(w) \gtrless h_K^*(w)$ for all $w \in (w',w'')$ then profit possibilities, $E[R(P(G,S)|\hat{H}] > 0$, exist for some points $P(G,S) \in \hat{A}$ with w_G and $w_S \in (w',w'')$ and $\beta(P(G,S)) \gtrless \bar{q}$. ***

Proof: Let G be any contract in $[C,D]$ with $w' < w_G < w''$. Then $q(G|\hat{H}) = \beta(G)$. From (14) and (15) we get

$$(24) \qquad \left.\frac{d}{dw_S}\right|_{w_S=w_G} \ln q(P(w_G,w_S)|\hat{H}) - \left.\frac{d}{dw_S}\right|_{w_S=w_G} \ln \beta(P(w_G,w_S))$$

$$= \rho[\hat{h}(w_G) - h_K^*(w_G)].$$

Thus, in case of $(\hat{h} - h_K^*)|_{(w',w'')} > 0$ a marginal shift of the contract from G along $\bar{U}^g(G)$ to the right side will increase the quality proportion more than the fair odds value and, therefore, result in positive expected profits. In case of $[\hat{h}(w) - h_K^*(w)]|_{(w',w'')} < 0$ the difference in (24) is negative and a marginal shift of the contract from G along $\bar{U}^g(G)$ to the left will increase its expected profits. ***

Now we show that the exploitation of profit possibilities at points $P(G,S) \in \hat{A}$ with $G \neq S$ will lead to a partial correction of the actual distribution $\hat{H}$.

Proposition 2.2: Suppose $E[R(P(G,S))|\hat{H}] > 0$ for some $P(G,S) \in \hat{A}$ with $G \neq S$. Then, in case of $\beta(P(G,S)) > \bar{q}$ the interval (G,S) is overcrowded, i.e., $\int_G^S (\hat{h}-h_K^*)dw > 0$, and additional offers in $P(G,S)$ imply losses for offers in $P' \in (G,S)$. In case of $\beta(P(G,S)) < \bar{q}$ the interval (S,G) is underloaded, i.e., $\int_S^G (\hat{h}-h_K^*)dw < 0$, and additional offers in $P(G,S)$ imply positive profits for offers in $P'' \in (S,G)$. ***

Proof: $E[R(P(G,S))|\hat{H}] > 0$ implies $q(P(G,S)|\hat{H}) > q(P(G,S)|H_K^*)$ and, thus, due to (14), $\int_G^S \hat{h}\, dw > \int_G^S h_K^*\, dw$. In case of $\beta(P(G,S)) > \bar{q}$ we have $w_G < w_S$ and, hence, the interval (G,S) is overcrowded. In case of $\beta(P(G,S)) < \bar{q}$ we have $w_S < w_G$ and the interval (S,G) is undercrowded.

Let m firms offer in $P(G,S)$ in addition to $\hat{H}$. If $\beta(P(G,S)) > \bar{q}$ then the quality proportion for any contract $P' \in (G,S)$ is given by $q(P'|\hat{H},P(G,S)) = \bar{q}e^{-\rho m} < \bar{q}$ implying losses there. If $\beta(P(G,S)) < \bar{q}$ then the quality proportion for any contract $P'' \in (S,G)$ is given by $q(P''|\hat{H},P(G,S)) = \bar{q}e^{m\rho} > \bar{q}$ implying profits. ***

Proposition 3 shows that overcrowded intervals $(w',w'') \subset [w_C,w_D]$ with $\hat{h}(w) > h_K^*(w)$ for all $w \in (w',w'')$ cause profit possibilities, the exploitation of which implies losses for all firms within some subinterval $(w_G,w_S) \subset (w',w'')$. Thus, these firms have an incentive to vary their offer. If some of them leave (w',w''), then the deviation measured as $|\int_{w'}^{w''}(\hat{h}-h_K^*)\,dw|$, decreases. Undercrowded intervals $(w',w'') \subset [w_C,w_D]$ with $\hat{h}(w) < h_K^*(w)$ for all $w \in (w',w'')$ cause profit possibilities, the exploitation of which implies profits for all firms located in some subinterval $(w_G,w_S) \subset (w',w'')$. Firms in A and B, and possibly others, have an incentive to move their contract to this subinterval which, again, would reduce the deviation.

6.3 We have not analyzed the adaptation process explicitly. In particular, this would require assumptions about the speed with which firms switch their offers in order to exploit profit possibilities, respectively to avoid losses. In reality, a calculation of the profitability of an offer variation has to take into account uncertainties concerning the behavior of competitors and customers, costs of acquiring information to reduce these uncertainties, and switching costs. Together with the analysis in Section 6.2 this suggests that the probability, respectively the expected speed with which firms react to profit differentials is the greater, the larger these differentials are. Thus, smaller deviations are not corrected quickly, but

with increasing deviations, correcting reactions will set in with increasing vigor.

Therefore, the performance of the market depends critically on the market size K, the publicity level of firms μ, and the switching costs θ which determine the magnitude of the profit possibilities remaining under canonical discrete distributions.

If the parameters K, $1/\mu$ and θ are high enough (for given approximation error $\ell \geq 1$), then according to Corollary 1, any canonical discrete distribution $H_K^+ \in \bar{D}(h_K^*, \ell)$ forms a Nash equilibrium in pure strategies.

If these parameters are small enough then firms will vary their contracts permanently both to exploit profit possibilities and to avoid a predictable behavior which would be exploited by competitors and, thus, cause losses. Ideally, in case of $\theta = 0$, a mixed strategy equilibrium will evolve as we shall discuss in Section 7.

If the parameters K, $1/\mu$ and θ lie between these two extremes then there always remain small though uncertain profit possibilities. The above arguments suggest that these profit possibilities are exploited reluctantly. If this results in larger deviations from the canonical continuous distribution, these deviations are gradually corrected as indicated by Propositions 3.1 and 3.2. The larger the deviations are, the higher is the expected adjustment speed.

7. Mixed Strategy Equilibria

In this section we present a variation of the market intransparency model. We take into account that, in some markets, firms change their offers frequently and unsystematically either over time or over customers,

and that firms, consequently, are not perfectly informed about the offers being made by competing firms to a specific customer. We will show that for every finite (not to small) number of firms there exists a unique mixed-strategy Nash pooling equilibrium even if customers have perfect information about all offers.

Let all assumptions of Section 2 be employed with the following supplementations. The publicity level of firms may be equal to the market size, i.e., $\mu \in (0,K]$ respectively $\pi \in (0,1]$. Each customer chooses among all contracts known to him according to his completed preference ordering, no customer can wait for the next period to get better offers. There are $M \geq 2$ firms i, $i \in I_M := \{1,2,\ldots,M\}$, $M < \infty$. In each period each firm chooses an offer in $\bar{A}$ without knowing which offers will be chosen by competing firms.

To keep the model simple, we further assume that there are three additional firms i, $i \notin I_M$, which are restricted to offer in $\{A,B\}$ and are known with certainty; two of them offer in B, the other in A. This is denoted as assumption (*). We will show that assumption (*) can be dropped in case of $\pi = 1$. These three additional firms make zero expected profits independently of the strategies of firms $i \in I_M$ since only low risks buy in B and only high risks buy in A. We will show that in equilibrium the maximal expected profit any one of these additional firms could gain from an unrestricted variation of its offer converges to zero if M increases to infinity.

Firm $i \in I_M$ is said to follow a pooling-strategy ϕ_i if ϕ_i is a nondecreasing function on $[w_C,w_D]$ with values in $[0,1]$ and firm i chooses its offer P_i randomly from the subset $[C,D] \cup \{A,B\}$ such that the probability of P_i lying in $[C,G] \subset [C,D]$ is given by $\phi_i(w_G)$. The

pooling-strategies $(\phi_i)_{i=1}^{M}$ form a (mixed-strategy) Nash equilibrium iff each firm $i \in I_M$ makes zero expected profits in the support of its strategy ϕ_i and negative expected profits with any other offer which receives customers with positive probability, provided all other firms $j \in I_M \backslash \{i\}$ offer according to ϕ_j.

Proposition 3: Let the market quality proportion $\bar{q}$ be so high that there exists an intersection point D and let the risk aversion of customers be bounded from above by inequality (C5) in Appendix C.
The pooling-strategies $(\phi_i)_{i=1}^{M}$ form a (mixed-strategy) Nash equilibrium if and only if for all $i \in I_M$

$$\phi_i(w_G) = \frac{1}{\pi}\left(1 - e^{-\frac{1}{M-1}\int_{w_C}^{w_G} \Gamma(w)dw}\right) \tag{25}$$

where Γ is defined in (15).[11]
Any single additional firm $i \notin I_M$ (including any one of those which were assumed to offer in $\{A,B\}$ only) could make positive expected profits if all firms $i \in I_M$ offer according to (25). However, the magnitude of these profit possibilities converges to zero as M grows to infinity.
In case $\pi = 1$ the above remains true if we drop assumption (*) but require instead that $M \geq 3$ and at least one firm $i \in I_M$ chooses its offer from $[C,D] \cup \{A\}$ and at least two firms $i \in I_M$ choose their offer from $[C,D] \cup \{B\}$. ***

Proof: First, we determine the quality proportions of firms' clienteles. If any firm $i \in I_M$ chooses an offer $P(G,S) \in \hat{A}$ with $G \neq S$ and the other firms $j \in I_M \backslash \{i\}$ offer according to the pooling-strategies ϕ_j, then firm i is the only firm offering in P and any given customer of

risk class x, $x \in \{g,s\}$, buys from firm i with probability

$$(26) \qquad \chi^x(P(G,S)) = \begin{cases} \prod_{j \in I_M \setminus \{i\}} [1 - \pi\phi_j(w^x)] & \text{if } \beta(P) > \bar{q} \\ \prod_{j \in I_M \setminus \{i\}} [1 - \lim_{w \uparrow w^x} \pi\phi_j(w)] & \text{if } \beta(P) < \bar{q} \end{cases}$$

where $w^g = w_G$ and $w^s = w_S$. Using the abbreviation

$$(27) \qquad \psi_j(w) := -\ln (1-\pi\phi_j(w)),$$

in case of $\beta(P) > \bar{q}$, firm i's quality proportion is given by

$$(28a) \qquad q(P(G,S)|\phi_j,\ j \in I_M \setminus \{i\}) = \bar{q} \prod_{j \in I_M \setminus \{i\}} \frac{1-\pi\phi_j(w_G)}{1-\pi\phi_j(w_S)}$$

$$= \bar{q} e^{\rho[\frac{1}{\rho} \sum_{j \in I_M \setminus \{i\}} \psi_j(w_S) - \frac{1}{\rho} \sum_{j \in I_M \setminus \{i\}} \psi_j(w_G)]}.$$

In case of $\beta(P) < \bar{q}$ it is equal to

$$(28b) \qquad \bar{q} \prod_{j \neq i} \frac{1-\lim_{w \uparrow w_G} \pi\phi_j(w)}{1-\lim_{w \uparrow w_S} \pi\phi_j(w)} = \bar{q} e^{\rho[\frac{1}{\rho} \lim_{w \uparrow w_S} \sum_{j \neq i} \psi_j(w) - \frac{1}{\rho} \lim_{w \uparrow w_G} \sum_{j \neq i} \psi_j(w)]}$$

In case of $G = S$ the quality proportion in $P(G,S)$ is equal to $\bar{q}$ since both risk classes apply the same preference ordering over contracts in $[C,D]$. Hence, in this case (28) is valid, too.

The pooling-strategies $(\phi_i)_{i=1}^M$ form a Nash equilibrium if and only if the quality proportions fulfill

$$(29) \qquad q(P(G,S)|\phi_j, j \in I_M \setminus \{i\}) \begin{cases} = \beta(P(G,S)) & \text{if } G = S \\ < \beta(P(G,S)) & \text{if } G \neq S \end{cases}$$

for all $G,S \in [C,D]$ and all $i \in I_M$, since contracts outside of $\hat{A}$ can be dealt with in analogy to Section 5, respectively Appendix D. Since the right side of (28) has the same form as the quality proportions defined in (13), Lemma 1 can be applied: The quality proportions $q(P|\phi_j, j \neq i)$ fulfill condition (29) for all $P \in \hat{A}$ and $i \in I_M$ if and only if for all $w_G \in [w_C, w_D]$

$$(30) \qquad \frac{1}{\rho} \sum_{j \in I_M \setminus \{i\}} \psi_j(w_G) = \int_{w_C}^{w_G} h_K^*(w)dw.$$

Thus, $(\phi)_{i=1}^M$ is a mixed-strategy Nash pooling equilibrium if and only if (30) holds true on $[w_C, w_D]$.

Now we show that (30) is equivalent to (25). From (30) and (16) we get

$$(31) \qquad M\int_{w_C}^{w_G} \Gamma(w)dw = \sum_{i=1}^{M} \sum_{j \in I_M \setminus \{i\}} \psi_j(w_G) = (M-1) \sum_{i=1}^{M} \psi_i(w_G),$$

and from (30) and (31) for all $i \in I_M$

$$(32) \qquad \psi_i(w_G) = \sum_{j=1}^{M} \psi_j(w_G) - \sum_{j \in I_M \setminus \{i\}} \psi_j(w_G) = \frac{1}{M-1} \int_{w_C}^{w_G} \Gamma(w)dw.$$

This implies (25). On the other hand, straightforward calculation shows that the strategies $(\phi_i)_{i=1}^M$ from (25) fulfill (30).

If all firms $i \in I_M$ offer according to (25), then the profit possibilities of an additional firm $i \notin I_M$ are given in analogy to (28) by

$$(33) \qquad q(P(G,S)|\phi_i, i \in I_M) = \bar{q} e^{\sum_{i=1}^{M} \psi_j(w_S) - \psi_j(w_G)} = \bar{q} e^{\frac{M}{M-1} \int_G^S \Gamma(w)dw}.$$

For movements of firm $M+1$ along $\tilde{U}^g(G)$ this implies

$$(34)\quad \left.\frac{d}{dw_S}\right|_{w_S=w_G} \ln q(P(G,S)\,|\,\phi_i, i\in I_M) = \frac{M}{M-1}\,\Gamma(w_G) > \left.\frac{d}{dw_S}\right|_{w_S=w_G} \ln \beta(P(G,S)).$$

Thus, firm $M+1$ can make positive profits with contracts, which have slightly reduced premiums compared with those of firms $i \in I_M$. However, these profit possibilities converge to zero if M grows to infinity since then $q(P|\phi_i,\ i \in I_M)$ converges to $q(P|H_K^*)$ according to (33).

In case $\pi = 1$ we can relax assumption (*) in the way described since then each customer still knows at least two firms with expected utility $U^S \geq U^S(A)$ and $U^g \geq U^g(B)$. Thus no firm can gain positive profits by increasing its premium in events in which it would otherwise offer in $\{A,B\}$. ***

Dasgupta and Maskin (1986) have shown the existence of mixed-strategy Nash equilibria for a wide class of games which covers in particular the R-S problem. In a recent paper Rosenthal and Weiss (1984) constructed such a mixed-strategy Nash equilibrium for a labor market model, which has a structure analogous to the R-S model but assumes linear indifference curves for workers. This result is similar to the mixed-strategy equilibrium derived in the present paper for the case of perfectly informed customers.[12]

Rosenthal and Weiss point out that their model does not deter entry. In our model, however, we do estimate the maximal gains from entry and show that these gains converge to zero if the number of firms increases to infinity.

Rosenthal and Weiss note that the interpretation of the mixed-strategy equilibrium as a one-shot game is not appropriate for labor and insurance markets since randomized strategies require each offer on the one hand side to be known to all customers and on the other hand side not to be known to any of the competing firms.

We offer an alternative interpretation of our mixed-strategy equilibrium. We identify each new customer with a new shot of the game, i.e., the number of new customers K per period is assumed to equal 1. This implies that each firm offers different contracts to different customers. In Section 10 we show that this presents a sensible description of many industry insurance markets. When classifying a given (industrial) insurance object into a rating system, each insurer relies so much on subjective judgment that competing firms have to view the outcome as random event. Since all customers get different offers from a given firm, it is quite difficult for competing firms to get valid information about a particular offer of the considered firm prior to the ratification of the contract.

8. Heterogeneity of Contracts and Preferences

In this subsection we describe a further variant of the model where customers have perfect information about all offers but products and preferences are heterogeneous. Suitably simplified, this mdoel has the same formal structure as the market intransparency model.

We apply the same assumptions as listed in Section 2, except that $\mu = K$, i.e., $\pi = 1$, and the utility maximization is modified in the following way. Each customer evaluates an offer not only according to the expected utility calculated from the premium and the amount of insurance. He also takes into account further firm-specific properties like special terms of the contract, fitting to the needs of some customer groups, distance to the insurer's office, the firm's reputation, etc. We assume that each customer j combines his subjective evaluation of each firm i's specific properties to a number $Z(i,j)$. The effective utility which customer j from risk

class x assigns to an offer P_i of firm i is then given by

$$U^{eff}(i,j) := U^x(P_i) + Z(i,j). \tag{35}$$

This variable contains both the heterogeneity of the insurance contracts and the heterogeneity of individual preferences. Let each customer j choose that offer i which gives him highest effective utility, $U^{eff}(i.j) \geq U^{eff}(i,k)$ for all firms k. Draws are decided by random.

On real markets, subjective evaluations vary considerably among customers and firms. Since we do not want to analyze individual evaluations, we assume that $(Z(i,j))_{i=1,2,\ldots;j=1,K}$ are stochastically independent and identically distributed random variables. In the present paper we do not analyze this variant in this generality. Instead, with the help of a crude simplification, we show the analogy to the market intransparency model. We assume that for each (i,j):

$$Z(i,j) = \begin{cases} 0 & \text{with probability } \pi \\ \bar{\alpha} & \text{with probability } (1-\pi) \end{cases} \tag{36}$$

where $\bar{\alpha}$ is prohibitively high, $\bar{\alpha} > V(W_1^o) - U^g(B)$. Then, each customer either accepts the offer of a firm or he rejects it completely. Replacing the events $\{(Z(i,j) = 0\}$ and $\{Z(i,j) = \bar{\alpha}\}$ by {customer j knows the offer of firm i} and {customer j does not know the offer of firm i} we get back the market intransparency model. Thus, all results established are valid in both variants of the model.

9. Welfare Effects of Market Intransparency

In this section we analyze how the distribution of customers' expected utilities is affected by the publicity level of firms, μ.

Let all firms be distributed according to a canonical discrete distribution $H_K^+ \in \bar{D}(h_K^*, \ell)$. Any given customer buys a contract in a sub-interval $[C,G] \subset [C,D]$ if and only if he knows the offer of at least one firm in $[C,G]$. Thus, the probability that any given customer succeeds in buying in the interval $[C,G]$ is given by

$$(37) \qquad \nu([C,G]|H_K^+) = 1 - (1-\pi)^{H_K^+([C,G])}$$

$$= 1 - e^{-\int_C^G \Gamma dw} \; e^{-\rho[H_K^+([C,G]) - \int_C^G h_K^* dw]}.$$

Since $\rho|H_K^+([C,G]) - \int_C^G h_K^* dw| \leq \rho\ell$, which converges to zero if K grows to infinity, we have

$$(38) \qquad \lim_{K\to\infty} \nu([C,G]|H_K^+) = 1 - e^{-\int_C^G \Gamma dw}$$

which is independent of μ, K and π.[13]

This means that the publicity level of firms, μ, has no influence on the probability distribution of customers' sales if the market size K converges to infinity, for finite market size its influence is ambiguous and depends on which discrete offer distribution $H_K^+ \in \bar{D}(h_K^*, \ell)$ has emerged.[14]

This result may seem contra-intuitive. For any fixed offer distribution, an increase of μ increases the probability $\nu([C,G])$ to find an offer in $[C,G]$. However, with (suitably) increased μ, the old offer distribution $H_K^+(\mu)$ does not form any longer a canonical discrete distribution (wrt. the initial approximation error ℓ). The new canonical continuous distribution H_K^* requires a smaller density in $(C,D]$ since $1/\rho = 1/-\ln(1-\mu/K)$ has decreased in (16). Although customers know the offers of a firm now with a higher probability, there are now less firms in

[C,G] to choose from.

10. Real Insurance Markets

Real insurance markets are quite intransparent for customers. However, one has to distinguish between industry insurance and household insurance.

While industrial customers have to cover large amounts of risk and are able to become well-informed about the offers at the market, private households insure much smaller amounts and, correspondingly, apply smaller means to improve their information and understanding. According to an Allensbach-inquiry (Allensbach 1980), in the German life insurance market, only 28% of those insured had compared different offers, and only 33% knew that there were considerable price differentials. This intransparency is aggravated by the multitude of tariffs, deductibles, stipulations, and obliging services. Even in the relatively transparent FRG-automobile insurance market, where contracts are quite uniform due to governmental regulation, there are price differences up to 31% (Capital, 1981).[15]

In industrial insurance markets on the other hand, high amounts of insurance justify high search costs. With expertise, brokers and often in-house insurance departments, customers gain insight into the market. However, in classifying the insurance object of a customer into the rating system of the insurer there always is ample opportunity for subjective judgment. High insurance amounts even justify specific contractual provisions fitted to the customer's special needs. Thus, premium and other terms of the contract are not set uniformly by the insurer, but rather are negotiated individually. Due to short notices, customers have an opportunity to revise their contract or change their insurer practically at any time. Thereby, the relations between firms and their customers are not anonymous, each

insurer knows the mostly few other insurers which compete for the same customer. Hence, an oligopolistic situation is given. Each insurer reacts quickly on the actions of its competitors. This causes permanent, unsystematic variations of the offers. On the one hand this aggravates the customers' search problem. On the other hand it prevents firms from predicting their competitors' offers. Consequently, large price and profit differentials emerge in the industry insurance market, and prices and market shares fluctuate considerably.[16]

The empirical comparison of insurance markets meets the predictions to be drawn from our models. Household insurance markets exhibit a low degree of market transparency and a relatively stable distribution of offers. These properties are reflected in the market intransparency model with positive switching costs and a low degree of market transparency. Industry insurance markets, on the other hand, have a much higher degree of transparency. With a high degree of transparency, the intransparency model leaves for each firm considerable profit possibilities, exploitable by offer variations, and incentives for offer randomization to avoid predictable behavior. The mixed-strategy Nash pooling equilibrium provides an idealized description of industry insurance markets since in real markets, too, each firm reacts quickly on actions of its competitors and the subjective classification of insurance objects can be viewed as a randomization of offers.[17,18] The higher degree of heterogeneity in industry insurance markets can also be interpreted in accordance with our models. With a high degree of market transparency any firm with a constant offer would be exposed to losses since it presents profit possibilities to other firms. Each firm can reduce this risk of being exploited by other firms both by randomizing its offer or by offering a more specialized product as discussed

in Section 8.[19] This incentive for product specialization is much higher in industry insurance markets than it is in household insurance markets.

11. Summary

In this paper we showed that special market features like market intransparency, product and preference heterogeneity, and offer randomization can be essential for an understanding of markets.

Each of these features ensures that each firm holds only a small, but positive market share. Small variations of the terms of any firm's contract then cause only small changes of other firms' market shares and risk compositions. On the contrary, in the original Rothschild-Stiglitz model, a marginal variation of the offer of a single firm can lead to a total change in all other firms' risk compositions. Those discontinuities are reason for the nonexistence of Nash equilibria in the original model.

If there exists no Nash equilibrium in the original R-S model, then market intransparency ensures a market solution according to which many different contracts are offered simultaneously. If the offered contracts are arranged corresponding to a canonical distribution, then each firm makes zero expected profits and, by a variation of its offer, no single firm can gain expected profits above some positive boundary which converges to zero if the population of customers increases to infinity. Thus, these canonical discrete distributions form Nash equilibria if each firm incurs small, but positive switching costs for each variation of its offer and the market size is large enough.

If the population of customers is too small, then there always remain small incentives for firms to vary their offer. However, in case of positive switching costs and imperfect information of firms about their

profit possibilities the analysis suggests that the canonical distribution is stable. If the actual distribution deviates from the canonical distribution then the exploitation of profit possibilities leads to a reduction of the deviation. The greater the deviation, the greater is the expected speed of a correction.

These local instabilities become more severe if the market size decreases or the publicity level of firms increases. In the extreme case, if switching costs are zero, then there exists a mixed-strategy Nash equilibrium, even if customers are perfectly informed about all offers. The probability distributions of the mixed-strategies of the firms have a form related to the canonical distribution.

This corresponds to real insurance markets. In household insurance markets customers only compare very few offers and, thus, there is a low degree of transparency. Contracts are offered uniformly to all customers and are rather constant in time. In industry insurance markets on the other hand, customers invest high amounts in search for good offers and, thus, the degree of transparency is much higher. Here, contracts are negotiated individually. For competing firms the outcomes of these bargains are uncertain. Therefore, whereas the mixed-strategy version of the model depicts the features of industry insurance markets, the market intransparency model with positive switching costs provides a good description of household insurance markets.

In all versions of the presented model, the welfare of customers remains constant if the publicity level of firms is increased. Indeed, customers will be informed about more offer then, but the offer density of the canonical distribution will reduce in response and,thus, there are less firms with good contracts to choose from.

Footnotes

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[1]In Zink (1985) we analyze the case of arbitrary many contracts per firm. There we construct an Epsilon-Nash equilibrium where each firm offers two contracts. Although this leads to a separation of risks by self-selection, there are no fair prices. Each firm will make profits with the offer chosen by good risks, while the offer chosen by bad risks produces a loss of equal amount. No firm, however, can drop its loss-making contract since then the other contract would be demanded by both risk types and turn unprofitable.

[2]The existence and form of Epsilon-Nash equilibria remains valid if we refuse to complete the preference ordering. As one can see later, the only difference is that firms, which would otherwise offer in point B (defined later), then have to offer below the indifference curve $\bar{U}^S(B)$ sufficiently near to B.

[3]For simplicity, we assume that once a customer has bought a contract, he never reenters the market. Otherwise, we would have to consider the

extent to which customers accumulate information about firms and the extent to which firms accumulate information about customers.

[4]Finitely many firms would suffice, too. In that case, the minimal number, however, would be determined endogenously and depend on the market size K and the publicity level of firms μ. To keep the model simple, in that case we would have to assume that there are three firms which are perfectly known, two of which offer in A and one of which offers in B (points A and B are defined later).

[5]If customers search for offers and they determine an optimal number of costly search steps in advance (as done by Stigler, 1961), then again each offer is known with a probability which is the same for all firms. Then, however, for each customer the knowledge about firms is negatively correlated. If customers search sequentially and make the decision of a further search step dependent on the outcome of previous steps, then each firm will be known with a different probability and for each customer, again, the informational relations to firms are negatively correlated.

However, this negative correlation is no property to be desired in any case. Especially, if the quality of the goods is not homogeneous and requires a complex judgment, then the offers are tested simultaneously. Each customer undertakes activities by which he acquires information gradually and about many firms simultaneously. E.g., customers study consumer journals, talks to specialists, etc. In these cases a positive correlation between the informational relations seems to be more likely. For simplicity, in this paper we assume them to be stochastically independent.

Extending our model, we could assume that the probability π increases with the expected utility it offers. The only consequence would be that the canonical distribution (defined later) would be of a more complicated and

more convex form.

[6]Under any canonical discrete distribution there are infinitely many firms in A and in B. Thus, the expected number of customers in each of these firms is zero. We accept this unrealistic feature for the sake of simplicity. See also Zink (1985) where we assume that offers in A and B are known with certainty.

[7]This uniqueness result can be strengthened. Actually, we prove that the reference distribution on $[C,D]$ with the density Γ is uniquely determined by the properties stated in Proposition 1 not only within the set of absolutely continuous distributions on $[C,D]$, but also within the set of all (right-continuous) distributions on $[C,D]$.

[8]This is the mathematical concept of a measure, although it is not a probability measure. See Halmos (1956).

[9]One may conjecture that H_K^* is the only (general) distribution H with

$$q(P|H) \begin{cases} = \beta(P) & \text{for } P \in \text{supp } H \\ < \beta(P) & \text{for } P \in \bar{A}_0 \backslash \text{supp } H \end{cases}$$

where $\text{supp } H := \{P \in \bar{A}: H(\bar{A}') > 0 \text{ for all open subsets } \bar{A}' \text{ containing } P\}$ and $\bar{A}_0 := \{P \in \bar{A}: E[Q^g(P)|H] + E[Q^s(P)|H) > 0\}$. However, a rigorous proof seems to be technically subtle.

[10]Lemma B in Appendix B implies that we can ease condition (17) to allow quality proportions to be equal to the respective fair odds values.

[11]We only claim uniqueness of mixed-strategy Nash equilibria within the set of pooling-strategy combinations.

[12]Let me remark that I proved the main properties of the market intransparency model before I was aware of the paper of Rosenthal and Weiss

(and before that paper was published).

[13]The expected number of customers per (additional) firm at G is givens by

$$K\pi(1-\pi)^{\int_C^G h_K^+ dw} \to \mu e^{-\int_C^G \Gamma dw} \quad (K\to\infty).$$

[14]The same result holds true in the mixed-strategy version of the model developed in Section 7. With the notation introduced there we get:

$$\nu([C,G]) = 1 - \prod_{j\in I_M} [1-\pi\phi_j] = 1 - e^{-\frac{N}{N-1}\int_C^G \Gamma dw}$$

which is independent of K and μ.

[15]Nevertheless, sales react distinctly on prices. For the FRG automobile insurance market, Finsinger (1984, p. 110) finds an elasticity of -12.

[16]See also Müller-Manske (1969), Farny (1983), and Röper (1978).

[17]To describe industry insurance markets by mixed-strategy equilibria, we would drop the assumption that no customer ever reenters the market once he has bought a contract. Then, in equilibrium customers would not accumulate information about firms since offers are in permanent variation. However, one had to consider how firms could learn about the riskiness of individual customers. If losses occur seldomly enough, though, firms could not accumulate information about individual customers.

[18]A mixed-strategy equilibrium can also evolve from deterministic decision rules if the payoff functions of all players are sufficiently disturbed and each player can only observe the disturbances of his own payoff function. This property was first published by Harsanyi (1973).

[19] In the simple version of the heterogeneity model an increased specialization of a firm i would correspond to a reduction of the probability of $\{Z_{ij}=0\}$ in (36).

APPENDIX A

Proposition A: Let the market quality proportion $\bar{q}$ be so small that no intersection point D exists. Then a discrete distribution H forms a separating Nash equilibrium if $H(\{A\}) = H(\{B\}) = \infty$ and $H(\{\bar{A}'\}) = 0$ for any subset $\bar{A}'$ with $\{A,B\} \cap \{\bar{A}'\} = \phi$. ***

Proof of Proposition A: Let H fulfill the condition given in Proposition A. Since each customer knows at least one offer in A and one offer in B, all low risks buy in B and all high risks in A. Thus, all firms make zero profits. It remains to be shown that no firm can gain positive expected profits by varying its offer. Any contract $P \neq B$ above or on the $\bar{U}^g(B)$-indifference curve but not above the $\{\beta=\infty\}$-line would be preferred by both risk groups and, thus, would receive a quality proportion of $\bar{q}$. However, P would make non-positive profits since we assumed that $\bar{U}^g(B)$ lies above the market fair odds line. Contracts $P' \neq A$ below $\bar{U}^g(B)$ are demanded either by no customers at all or only by high risks in which case they produce losses. ***

APPENDIX B

Lemma B: Any distribution $H \in \tilde{D}(0,\infty)$, which fulfills

$$(B1) \qquad q(P(G,S)|H) \begin{cases} = \beta(P(G,S)) & \text{for } G = S \\ \leq \beta(P(G,S)) & \text{for } G \neq S \end{cases}$$

for all points $P(G,S) \in \hat{A}$, is absolutely continuous on $[C,D]$ with a

density given by (16) ***

Proof: Let H be an element of $\tilde{D}(0,\infty)$ fulfilling (B1). First, we show that the absolutely continuous part of H fulfills (16).

For any two points $G,S \in [C,D]$, (B1) and (13) imply

(B2) $$q(P(G,S)|H) = \tilde{q}e^{\rho\{H([C,S])-H([C,G])\}} \leq \beta(P(G,S)) \quad \text{if} \quad w_S > w_G$$

(B3) $$q(P(G,S)|H) = \tilde{q}e^{\rho\{H([C,S))-H([C,G))\}} \leq \beta(P(G,S)) \quad \text{if} \quad w_S < w_G.$$

Since $\tilde{q} = \beta(P(G,G))$ we get from (B2) and (B3)

(B4) $$\lim_{\substack{w_S \to w_G \\ w_S > w_G}} \frac{H([C,S]) - H([C,G])}{w_S - w_G} \leq \lim_{\substack{w_S \to w_G \\ w_S > w_G}} \frac{1}{\rho} \frac{\ln \beta(P(G,S)) - \ln \beta(P(G,G))}{w_S - w_G}$$

(B5) $$\lim_{\substack{w_S \to w_G \\ w_S < w_G}} \frac{H([C,S)) - H([C,G))}{w_S - w_G} \geq \lim_{\substack{w_S \to w_G \\ w_S < w_G}} \frac{1}{\rho} \frac{\ln \beta(P(G,S)) - \ln \beta(P(G,G))}{w_S - w_G}$$

Since V is continuously differentiable, $\beta(P(G,S))$ is continuously differentiable in $\{P \in \hat{A}: \beta(P) < \infty\}$. Thus the limits on the right hand sides of (B4) and (B5) coincide. Since any distribution function is differentiable almost everywhere (wrt. the Lebesgue measure; see, for example, Chung, Theorem 1.3.1), the limits on the lefthand sides of (B4) and (B5) will exist and be equal for almost every w_G. Thus, for almost every w_G there is equality in (B4) and (B5), and the absolutely continuous part of H fulfills (16).

Now, we show that the singular part of H,

$$(B6)\qquad F(w_G) := \begin{cases} H([C,G]) - \int_{w_C}^{w_G} h_K^* dw & \text{if } w_G \in [w_C, w_D] \\ 0 & \text{if } w_G \in [0, w_C) \\ H([C,D]) - \int_{w_C}^{w_D} h_K^* dw & \text{if } w_G \in (w_D, W_1^o) \end{cases}$$

(see, e.g., Chung; F might be singular continuous) vanishes. First, we show that there exists a point $x_0 \in [w_C, w_D]$ with

$$(B7)\qquad \lim_{\substack{y \to 0 \\ y>0}} \frac{F(x_o+y) - F(x_o-y)}{2y} \geq \frac{1}{2} \frac{F(w_D)}{w_D - w_C}.$$

To see this, we partition $[w_C, w_D]$ for each $n \in N$ in 2^n subintervals of equal length,

$$(B8)\qquad J_k^n = [w_C + (k-1)2^{-n}(w_D - w_C),\ w_C + k2^{-n}(w_D - w_C)],\quad k = 1, \ldots, 2^n.$$

For at least one $k_0(n) \in \{1, \ldots, 2^n\}$, F must increase over $J_{k_o(n)}^n$ by not less than $F(w_D)/[2^n(w_D - w_C)]$,

$$(B9)\qquad F(w_C + k_o 2^{-n}(w_D - w_C)) - \lim_{x \downarrow w_C + (k_o - 1)2^{-n}(w_D - w_C)} F(x) \geq \frac{F(w_D)}{2^n (w_D - w_C)},$$

where we can choose these intervals such that for each $n \in N$

$$(B10)\qquad J_{k_o(n)}^n \supset J_{k_o(n+1)}^{n+1}.$$

Let $x_o := \lim_{n \to \infty} J_{k_o(n)}^n$. Then x_o fulfills (B7).

Now we can show that $F(w_D) > 0$ would contradict with (B1). Let $G \in [C,D]$ be such that $w_G = x_0$. For each $n \in N$ let $P_n := P(G^n, S^n) := P(w_G - 1/n, w_G + 1/n)$ be a sequence of contracts converging to G from the

right of [C,D]. Then we get from (B2), (B6), and (B7)

$$(B11)\qquad \lim_{n\to\infty} \frac{\ln q(P_n|H) - \ln q(G|H)}{2/n} = \lim_{n\to\infty} \rho \frac{H([C,S^n]) - H([C,G^n])}{2/n}$$

$$= \lim_{n\to\infty} \frac{\ln q(P_n|H_K^*) - \ln q(G|H_K^*)}{2/n} + \lim_{n\to\infty} \rho \frac{F(w_G+\frac{1}{n}) - F(w_G-\frac{1}{n})}{2/n}$$

$$\geq \lim_{n\to\infty} \frac{\ln \beta(P_n) - \ln \beta(G)}{2/n} + \frac{1}{2}\frac{F(w_D)}{w_D-w_C},$$

where the last inequality follows from (B7) and the fact that $q(P|H_K^*)$ is continuously differentiable in P and tangent with $\beta(P)$ in [C,D] by definition. Hence, $F(w_D) > 0$ would imply that there are some contracts P_n with $q(P_n|H) > \beta(P_n)$ contrary to (B1). Thus, $H = H_K^*$ and Lemma B is proven. ***

APPENDIX C

Lemma C1: Condition (17) holds true in $\hat{A}$ if

$$(C1)\qquad \frac{d}{dw_G}\frac{d}{dw_S} \ln \beta(P(w_G,w_S)) < 0$$

for all points $P = P(G,S) \in \tilde{A}$, where $\tilde{A}$ is defined to be the interior of $\{P \in \hat{A}$: in P the curve $\bar{U}^g(P)$ is flatter than $\{\beta = \beta(P)\}\}$. ***

Remark: For each $G \in [C,D]$ there is a point $T_G \in \hat{A}$ such that in T_G the line $\{\beta = \beta(T_G)\}$ is tangent to $\bar{U}^g(G)$ (see Figure 6). The curve of these points is upward sloping and $\tilde{A}$ lies to the right of this curve (see Figure D in Appendix D).

Proof of Lemma C1: From (19) we have seen that $q(P) = \beta(P)$ for all $P \in [C,D]$. Thus it suffices to show for all $P \in \hat{A}$ that the growth rate

differential

$$(C2) \qquad \phi(P(G,S)|H_K^*) := \frac{d}{dw_S} \ln q(P(w_G,w_S)|H_K^*) - \frac{d}{dw_S} \ln \beta(P(w_G,w_S))$$

is negative if $w_C \le w_G < w_S \le w_D$ (i.e., if $\beta(P) > \bar{q}$) and positive if $w_C < w_S < w_G \le w_D$ (i.e., if $\beta(P) < \bar{q}$).

From (14), (16) and (15) we get

$$(C3) \qquad \frac{d}{dw_S} \ln q(P(w_G,w_S)|H_K^*) = \rho h_K^*(w_S) - \Gamma(w_S) - \frac{d}{dw}\Big|_{w=w_S} \ln \beta(P(w_S,w)).$$

Inserting (C3) into (C2) we get from (C1)

$$(C4) \qquad \phi(P(G,S)|H_K^*) \gtrless 0 \quad \text{if} \quad w_S \lessgtr w_G,$$

provided $P(G,S) \in \tilde{A}$. For $P \in \hat{A} \setminus \tilde{A}$ we have $w_C \le w_S \le w_G$ and $d[\ln \beta(P(G,S))]/dw_S \le 0$ and $d[\ln q(P(G,S)|H_K^*)]/dw_S \ge 0$. Thus, (C4) is true for all $P \in \hat{A}$. ***

Lemma C2: Inequality (C1) holds true for all points $P(G,S) \in \tilde{A}$ if the risk aversion of customers is bounded by

$$(C5) \qquad \frac{V'(W_1)}{V'(W_2)} > 2\left(1 + \frac{1-p^g}{p^g} \frac{p^s}{1-p^s}\right)^{-1} \quad \text{for all} \quad (W_1,W_2) \in \tilde{A}. \qquad ***$$

Remark: Before we prove this lemma we note that there are utility functions V which fulfill (C5) but exclude the existence of Nash equilibria in the original R-S model. Since the right side of (C5) is smaller than unity, the inequality holds if V' decreases suitably slowly with increasing income. Condition (C5) is independent of $\bar{q}$. For given V, p^g, p^s, however, Nash equilibria are excluded in the original R-S model if $\bar{q}$ is large enough.

Proof of Lemma C2: For $P \in \tilde{A}$ the derivative in (C1) is equal to

$$\text{(C6)} \qquad \lim_{\substack{h\to 0\\ k\to 0}} \frac{1}{k}\left[\frac{\ln \beta(P(w_G+k,w_S+h)) - \ln \beta(P(w_G+k,w_S))}{h} - \frac{\ln \beta(P(w_G,w_S+h)) - \ln \beta(P(w_G,w_S))}{h}\right].$$

Since β is continuously differentiable it suffices to show that for all points $P(G,S)$ of $\tilde{A}$ there is an $\epsilon = \epsilon(P) > 0$ with

$$\text{(C7)} \qquad [\ln \beta(P(w_G+k,w_S+h) + \ln \beta(P(w_G,w_S))] - [\ln \beta(P(w_G+k,w_S)) - \ln \beta(P(w_G,w_S+h))] < -\epsilon\, hk$$

respectively

$$\text{(C8)} \qquad \beta(P(w_G+k,w_S+h))\ \beta(P(w_G,w_S)) < e^{-\epsilon hk}\ \beta(P(w_G+k,w_S))\ \beta(P(w_G,w_S+h))$$

for all small positive h and k.

It suffices to prove inequality (C8) for a special sequence $(h,k) \to (0,0)$. For every h we choose k such that $\beta(P(w_G,w_S)) = \beta(P(w_G+k,w_S+h))$ holds true. As can be seen from Figure C1 such a choice is always possible in $\tilde{A}$.

Let Q be any point in $\tilde{A}$. According to Figure C2 we construct the points R, R', P' and P as corner points of a parallelogram with center Q, edges the slopes of which are equal to those of $\tilde{U}^g$ and $\tilde{U}^s$ in Q, $\beta(R) = \beta(R') = \beta(Q)$, and a horizontal distance δ between P' and Q which is chosen arbitrarily but so that all five points lie in $\tilde{A}$.

According to (C7) and (C8) it suffices to prove the inequality

$$\text{(C9)} \qquad \beta(R)\beta(R') < e^{-\epsilon\delta^2}\ \beta(P)\beta(P'),$$

where the assumption of straight indifference curves in the neighborhood of

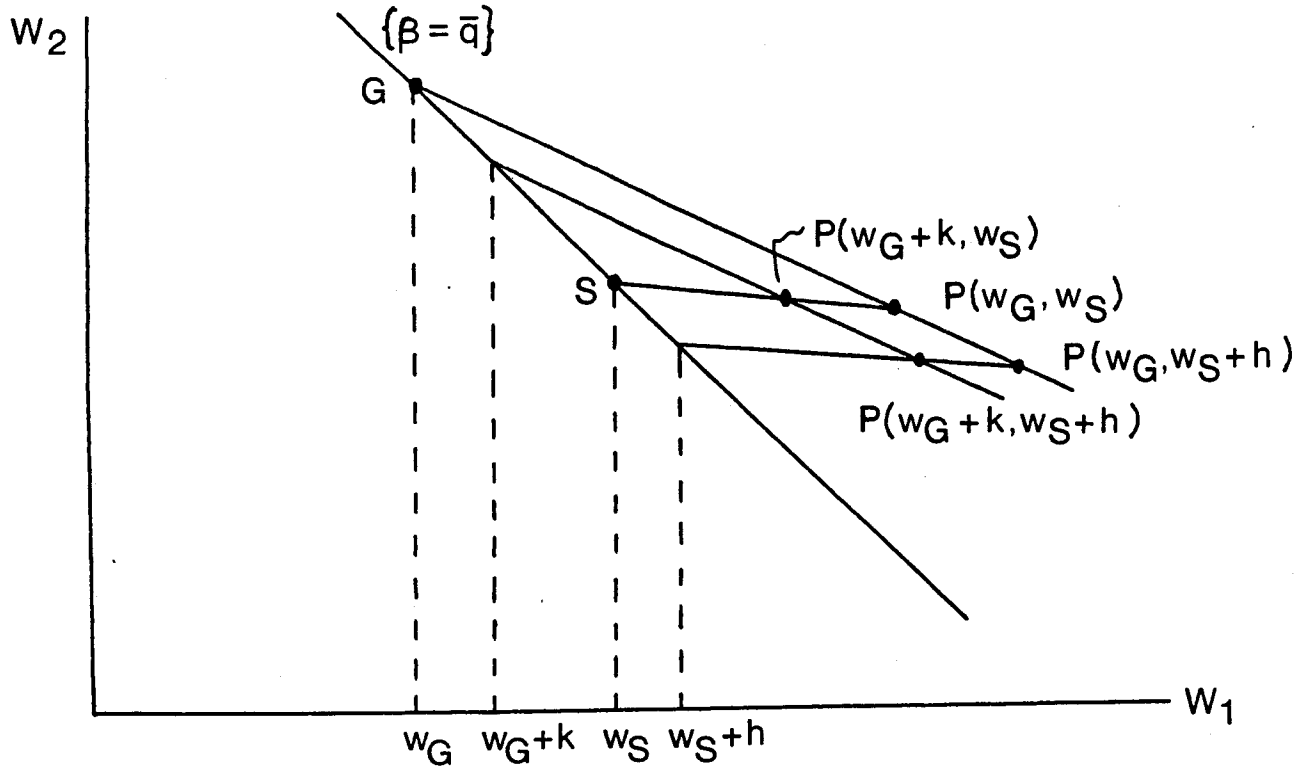

FIGURE C1

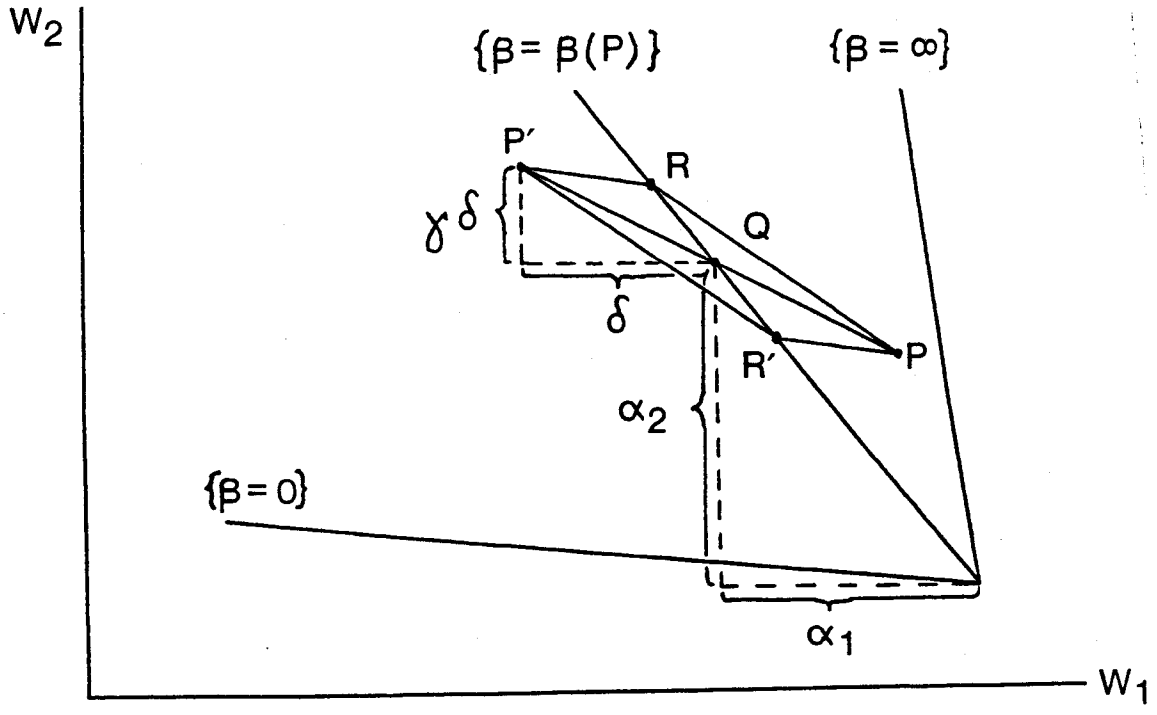

FIGURE C2

Q implies a mistake which is negligible in the limit.

Let the fair odds values be expressed in (α_1,α_2) coordinates according to (9). If Q has coordinates (α_1,α_2) then, by construction, we have $P' = (\alpha_1+\delta,\alpha_2+\gamma\delta)$ and $P = (\alpha_1-\delta,\alpha_2-\gamma\delta)$ where $-\gamma$ is the slope of the line $\overline{P'P}$. Using the function

$$\Lambda(\delta) := -\frac{(1-p^s)(\alpha_1+\delta) - p^s(\alpha_2+\gamma\delta)}{(1-p^g)(\alpha_1+\delta) - p^g(\alpha_2+\gamma\delta)} \tag{C10}$$

we have to show $\Lambda^2(0) < e^{-\epsilon\delta}\,\Lambda(\delta)\,\Lambda(-\delta)$. We transform Λ to

$$\Lambda(\delta) = -\frac{[\alpha_1-p^s(\alpha_1+\alpha_2)] + \delta[1-p^s(1+\gamma)]}{[\alpha_1-p^g(\alpha_1+\alpha_2)] + \delta[1-p^g(1+\gamma)]}. \tag{C11}$$

Using the third binomial formula, we have to show:

$$\frac{[\alpha_1-p^s(\alpha_1+\alpha_2)]^2}{[\alpha_1-p^g(\alpha_1+\alpha_2)]^2} < \frac{[\alpha_1-p^s(\alpha_1+\alpha_2)]^2 - \delta^2[1-p^s(1+\gamma)]^2}{[\alpha_1-p^g(\alpha_1+\alpha_2)]^2 - \delta^2[1-p^g(1+\gamma)]^2}\, e^{-\epsilon\delta^2} \tag{C12}$$

or equivalently

$$\frac{[1-p^s(1+\gamma)]^2}{[1-p^s(1+\frac{\alpha_2}{\alpha_1})]^2} \Big/ \frac{[1-p^g(1+\gamma)]^2}{[1-p^g(1+\frac{\alpha_2}{\alpha_1})]^2} < \phi(\epsilon,d), \tag{C13}$$

where

$$\phi(\epsilon,\delta^2) := \frac{1-e^{\epsilon\delta^2}}{\delta^2}\,\frac{[\alpha_1-p^g(\alpha_1+\alpha_2)]^2}{[1-p^g(1+\gamma)]^2} + e^{\epsilon\delta^2}. \tag{C14}$$

Using the relation $\epsilon\delta^2 < (e^{\epsilon\delta^2}-1)/\delta^2 < 2\epsilon\delta^2$, for a suitable constant k and suitably small values of δ we have $\phi(\epsilon,\delta^2) > 1 - k\epsilon\delta^2$. Therefore, it

suffices to show that the left side of (C13) is smaller than unity. For that, we look at the signs of the differences in this expression.

From the slopes of the lines $\{\beta=0\}$, $\{\beta=\beta(Q)\}$ and $\{\beta=\infty\}$ we get $(1-p^s)/p^s < \alpha_2/\alpha_1 < (1-p^g)/p^g$ and thus

$$1 - p^s(1 + \frac{\alpha_2}{\alpha_1}) < 0 < 1 - p^g(1 + \frac{\alpha_2}{\alpha_1}). \tag{C15}$$

Since in all points of $\tilde{A}$ the indifference curves are flatter than the $\{\beta=\infty\}$-line we have $\gamma < (1-p^g)/p^g$ resp. $1 > p^g(1+\gamma)$. Hence, two cases have to be distinguished (i) $1 < p^s(1+\gamma)$ and (ii) $1 > p^s(1+\gamma)$ where in case (i) the line $\overline{P'P}$ is steeper than the $\{\beta=\infty\}$-line, and in case (ii) it is flatter. In case (i) the left side of (C13) is equal to

$$\left[\frac{p^s(1+\gamma)-1}{p^s(1+\frac{\alpha_2}{\alpha_1})-1} \; \frac{1-p^g(1+\frac{\alpha_2}{\alpha_1})}{1-p^g(1+\gamma)}\right]^2; \tag{C16}$$

hence it suffices to show

$$[p^s(1+\gamma)-1]\;[1-p^g(1+\frac{\alpha_2}{\alpha_1})] < [p^s(1+\frac{\alpha_2}{\alpha_1})-1]\;[1-p^g(1+\gamma)]. \tag{C17}$$

But this follows straight away by using $\gamma < \alpha_2/\alpha_1$ and $p^s > p^g$.

With this, (C1) holds if case (i) is valid, that is if

$$\gamma > \frac{1-p^s}{p^s}. \tag{C18}$$

The slope γ can be estimated from below by the arithmetic mean of the slopes of the indifference curves $\tilde{U}^g$ and $\tilde{U}^s$ in Q,

(C19) $$\gamma \geq \frac{1}{2} \frac{V'(W_1)}{V'(W_2)} \left(\frac{1-p^g}{p^g} + \frac{1-p^s}{p^s}\right).$$

With this, (C18) holds if

(C20) $$\frac{V'(W_1)}{V'(W_2)} > 2 \frac{1}{1+ \frac{1-p^g}{p^g} \frac{p^s}{1-p^s}} .$$

This completes the proof. ***

APPENDIX D

Lemma D: Let $\tilde{A}$ be defined as in Lemma C1. If for some $\epsilon > 0$

(D1) $$E[R(P)|H_K^+] < \epsilon$$

for all $P \in \tilde{A}$, then (D1) holds true for all $P \in \bar{A}$ which have a positive expected number of customers. ***

Proof: Let (D1) hold true in $\tilde{A}$. Corresponding to Figure D, we will compare the expected profit of contracts from several domains $A_k \subset \bar{A}$, $k = 1,2,\ldots,6$, with the expected profit in suitable points of $\tilde{A}$.

Let a contract $P \in A_1 := \{P \in \bar{A}: U^g(D) \leq U^g(P) \leq U^g(C)$, in P $\bar{U}^g(P)$ is steeper than $\{\beta = \beta(P)\}\}$ move along $\bar{U}^g(P)$ to the right until it enters $\tilde{A}$. Then its quality proportion does not decrease, but its fair odds value decreases. Thus (D1) holds true in P.

Let $P \in A_2 := \{P \in \bar{A}: U^g(P) > U^g(C), U^s(P) \geq U^s(C)\}$. Then $q(P) = \bar{q}$ but $\beta(P) > \bar{q}$ which implies losses in P.

Let a contract $P \in A_3 := \{P \in \bar{A}: U^s(C) > U^s(P), U^g(P) > U^g(C)\}$ move along $\bar{U}^s(P)$ to the left until it enters $\tilde{A}$. Then its quality proportion remains constant but its fair odds value decreases. Thus (C1) holds true in P.

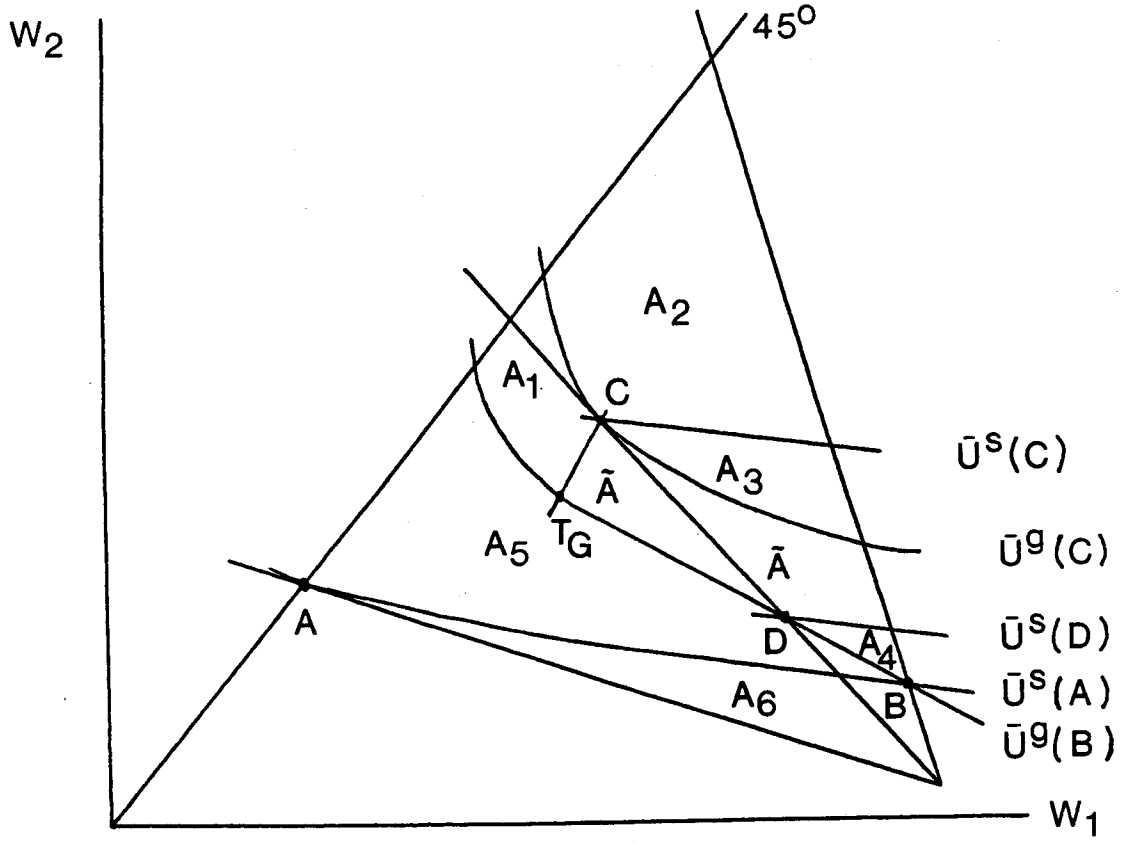

FIGURE D

Let a contract $P \in A_4 := \{P \in \tilde{A}: U^g(P) \geq U^g(D),\ U^s(P) < U^s(D)\}$ move along $\bar{U}^g(P)$ to the left until it enters $\tilde{A}$. Then its quality proportion remains constant but its fair odds value decreases. Thus (D1) holds true in P.

Let $P \in A_5 := \{P \in \tilde{A}: U^g(P) < U^g(D),\ U^s(P) \geq U^s(A),\ P \neq A\}$. Then $q(P) = 0$ but $\beta(P) > 0$ which implies losses in P.

Let $P \in A_6 := \{P \in \tilde{A}: U^s(P) < U^s(A)\}$. Then no customer applies for P.

Since $\tilde{A} \cup \cup_{k=1}^{6} A_k = \tilde{A}$ we have completed the proof. ***

APPENDIX E

Lemma E: Except for sets of measure zero, the function Γ in Proposition 1 is uniquely determined by the properties listed in this proposition. Its shape is described in equation (15). ***

Proof: Let $\hat{\Gamma}$ be any nonnegative integrable function on $[w_C, w_D]$ which differs from Γ on a set of positive measure. Let $\ell \geq 1$ and fixed. We show that then there exists a point $P \in \hat{A}$ with

$$\text{(E1)} \qquad \lim_{K\to\infty} E[R(P)|H_K^+] > 0$$

for any sequence $(H_K^+)_{K=1}^{\infty}$ with $H_K^+ \in \tilde{D}(\hat{\Gamma}/\rho, \ell)$.

We use the uniqueness result in Lemma 1. For any $K \in N$, any $\hat{H}_K \in \tilde{D}(\hat{\Gamma}/\rho, 0)$ and any $G \in [C,D]$ we have $q(G|\hat{H}_K) = \bar{q}$. Thus, according to Lemma 1, for any $K \in N$ there exist a point $P_o = P_o(G,S) \in \hat{A}$ and a real number $\delta > 0$ with

$$\text{(E2)} \qquad q(P_o|\hat{H}_K) = \bar{q}e^{\int_G^S \hat{\Gamma} dw} > (1+\delta)\,\beta(P_o).$$

Since the quality proportion $q(P_o|\hat{H}_K)$ does not depend on K, we can choose P_o and δ independent of K. For any discrete distribution $H_K^+ \in \tilde{D}(\hat{\Gamma}/\rho, \ell)$ we can estimate the expected profit possibilities in P_o with the help of the first two equations in (22) and (E2),

$$\text{(E3)} \qquad E[R(P_o)|H_K^+] = -c^s EQ^s \left[\frac{q(P_o|H_K^+)}{\beta(P_o)} - 1\right]$$

$$> -c^s EQ^s \left[(1+\delta)\,\frac{q(P_o|H_K^+)}{q(P_o|\hat{H}_K)} - 1\right]$$

$$= -c^S EQ^S \left[(1+\delta)e^{\rho[H_K^+((G,S])\ -\ \int_G^S \hat{\Gamma}/\rho dw]} - 1\right]$$

$$\geq -c^S EQ^S \left[(1+\delta)e^{-\rho\ell} - 1\right].$$

This implies (E1) and, thus, proves the lemma. ***

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