The Stock-Flow Analysis of Investment

by

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ABSTRACT

The paper considers the microfoundations of the investment function at the level of the firm and industry. The present value of a unit of capital (a "machine") will decrease with the rate of accumulation of machines. A firm's demand price for machines will decrease, therefore, with what it believes the industry accumulation rate to be. This demand-price relationship, however, cannot simply be inverted to give us investment demand as a function of price.

In the model presented here, the individual firms must form their estimates of the present value of machines without being able to observe the aggregate rate of investment directly. The elasticity of the number of machines demanded by a firm with respect to the difference between its own best estimate of the present value and the market price of a machine is assumed to be finite. Aggregation over firms of these relationships gives the market demand schedule at a point in time.

Firms revise the expectations underlying their estimates of the value of machines by comparing expected and realized (imputed) rental values. Under certain "nice" assumptions, the result is a fairly well-coordinated aggregate expansion process even though the individual firm lacks information on how fast its competitors are expanding.

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1. After two decades of concentration on issues raised by the monetarists, macrotheoretical debate seems once more to be turning back to the problem of the intertemporal coordination of economic activities. The older Keynesian literature gave a central role to investment fluctuations but left the theory of investment in an unsettled state. This is perhaps nowhere more obvious than in the textbook literature where one is used to finding standard models of consumption or money demand, for instance, which recur from text to text with only minor modifications but where no particular investment model has achieved canonical status. There are several reasons for this. There are genuine difficulties that remain to be resolved; in addition, the earlier literature generated some conundrums that caused unnecessary confusion; finally, the modelling options are simply very numerous, and often without compelling criteria for choosing among competing alternatives, so that a plethora of reasonable investment models is possible. In this chapter we hope to dispose of a couple of conundrums, clarify the difficulties, and discuss the modelling choices. In the main, we will follow an analytical line of development that starts with Clower (1954) and includes Eisner and Strotz (1963), Witte (1963), and more recently Mussa (1977), Hansson (1981), and LeRoy (1983).

2. Although our ultimate concern is with the macroeconomic investment function, this paper will deal with capital accumulation at the level of the firm and industry while treating aggregate demand and the interest rate as parametric. We want to keep a bridge to macro clearly in view, however. Keynes' *marginal efficiency of capital* construct (Fig. 1) will serve this purpose. The MEC-schedule is a short period construct that takes as given the capital stock inherited from the past as well as the current "state of long-term expectation". The MEC-function must be updated, therefore, between periods, as these conditions change. Changes in the capital stock relative to the expected future demand for output imply changes in the prospective earnings accruing to capital goods and thus shifts of the MEC locus. But also the state of long-term expectation might also change, and MEC shift, if current market outcomes cause last period's estimates of future return streams to be revised.

FIGURE 1

This will not suffice to clarify exactly what is and what is not held constant when a "given" state of long-term expectation is invoked as a *ceteris paribus* clause. How one is to conceive the long-term expectations as being structured is indeed one of the difficulties of the subject to which we will have to return below.

As is well known (Hirschleifer [1958], and also [1970], pp. 66-69, 76ff), the MEC-schedule cannot be interpreted as an array of investment projects ordered by their internal rates of return. It must be conceived of, instead, as a reduced form showing, for different rates of interest, the corresponding equilibria in the markets for newly produced capital goods. We will focus on one such market; for concreteness we may consider a market for shoe machines, for instance, so that we have the shoe manufacturing industry on the demand side and the shoe machine industry on the supply side. This imaginary "shoe machine" should be thought of as a homogeneous unit of physical capital in which the shoe industry's capital stock can be measured.

3. The next question is how the market equilibria behind the MEC-schedule are best to be modelled. The immediate choice is between a Walrasian "price-into-quantity" and a Marshallian "quantity-into-price" approach.[1]

The theory of investment, however, is a bit of a lacuna in both traditions. Marshall dodged the topic, letting it fall between the two stools of short period and long period analysis. In the Marshallian long period, the capital stock is assumed to have adjusted to full stationary equilibrium. In the short period, it is treated as if parametric. A succession of such short periods, therefore, does not a long run make. A theory of how the capital stock is adjusted -- a theory of investment -- is lacking in this analytical structure.

The Walrasian inheritance is hardly more promising. With prices parametric at the individual experiment level, the optimal capital stock of a firm may or may not be determinate. Even if it is determinate,[2] however, the rate at which the individual firm would choose to close the gap between the desired and its actual stock is indeterminate in the usual setup. The Walrasian model may give us a demand function for capital, it is said, but it will not give us an investment demand function (e.g., Witte [1963] or Treadway [1969]). It has become conventional to "solve" this problem by invoking adjustment costs, usually at the firm level, but a number of other modelling options exist.

A Marshallian construction of the capital goods market promises at least to dispose of this particular Walrasian predicament. Instead of trying to find a relation between the desired rate of growth of the capital stock and the price of machines, we focus on the demand price for machines $P(t)$ as a function of the rate of growth of the stock, g. The demand price is defined as the maximum price at which the demanders would absorb a given quantity. In this particular application, the demand price would simply be the present value of the stream of future rentals imputed to a machine.

That the present value of a newly acquired machine will be inversely related to the rate of interest r at which future rentals are discounted is obvious enough. It is only slightly less obvious that it must be inversely related to the current rate of industry investment: the larger the stock of machines in relation to product demand at future dates, the lower the imputed rental per machine. Consequently, we can draw the demand-price as a downward-sloping function of the flow-rate of new investments, as in Fig. 2.

FIGURE 2

Here, the supply-price schedule has been drawn horizontal, portraying the constant cost case, simply in order to illustrate the point that neither diminishing returns on the supply side nor increasing adjustment costs on the demand side of the market are needed to ensure a determinate solution for the rate of industry investment.

The intersection of the two schedules represents, of course, a short-run equilibrium in the market for machines. It is worth noting that, at the equilibrium price, $P*$, the "gap" between desired and actual stock is always zero. Correspondingly, at the equilibrium rate of output of new machines, $\dot{K}*$, the ratio of the market value of physical capital to its supply price -- the counterpart in the present model to Tobin's q -- is never different from unity. In the present model, therefore, we cannot conceptualize investment as the "speed" with which a stock-gap is closed; neither can we explain investment as a function of q.

The demand price locus will shift down with a rise in the rate of interest. The equilibrium rate of accumulation of new machines will thus be inversely related to the rate of interest. This construction of Keynes' MEC is presupposed in Leijonhufvud [1968, pp. 157-85] and is done formally in LeRoy [1983]. The same model of business investment is carefully developed in Hansson [1981].

4. In the literature, stock-flow analysis is frequently carried out with the aid of a two-panel diagram (Fig. 3) stemming from Clower's original paper. Here stock-demand and -supply are said to determine price (left panel) to which the rate of flow-supply then adjusts (right panel). Time-integrals of flow-supply show up as rightward shifts of the stock-supply schedule. Price, as a consequence, declines and the rate of accumulation tapers off. With stationary stock-demand and flow-supply functions, a stationary state with zero net accumulation, i.e., a full stock-flow equilibrium, will eventually be reached.

FIGURE 3

In the context of this kind of stock-flow analysis it is sometimes asserted that the flow magnitudes do not influence price in the short run but only over the longer run. If the flow-supply schedule in Fig. 3 were to shift rightwards, for instance, price in the present period (being determined by stock-demand and -supply) would supposedly be unaffected; the accumulation process would be speeded up, however, and so therefore would the rate of price decline (cf., e.g., Witte [1963]). Of course, this lack of any dependence of the current price on the flow-conditions contradicts the analysis in this paper. The relationship between the two approaches therefore deserves some comment.

The proposition that flow supply does not affect price is often advanced as if it were a simple dimensional truism. And it is true, of course, that a purely random "blip" in the instantaneous flow density of supply (or of consumption) should be of no consequence whatsoever. The question is how a "permanent" shift[3] in flow-supply conditions should be treated. This question, in turn, dissolves into two: (i) whether we are dealing with capital goods or final consumer goods; and (ii) whether changes in the aggregate rate of accumulation become known to the individual investing firms right away.

In his early stock-flow work, Clower tended to favor consumption illustrations: agents had to decide, for instance, how much beer to hold in their refrigerators, at which flow-rate to drink, and at which rate to produce. In this kind of example, it seems a reasonable approximation to treat stock-demands as stationary as long as there is no interdependence between individual demands (such as when one beer-drinker envies the other's well-stocked refrigerator, for instance). Individual stock-demand functions are derived in the usual Walrasian manner, assuming that the agent optimizes facing parametric prices: these are aggregated to obtain a market stock-demand schedule which will be independent of the aggregate stock in existence and of the aggregate rate of accumulation.[4] When we are dealing with capital goods, however, it is necessary to recognize that the value of a machine to one producer depends *(ceteris paribus)* on how many machines his competitors have and will have in operation. Witte took over the Clowerian apparatus without carrying out the modifications necessary to take into account this interaction between the individual firms' capital goods demands. In this respect, his well-known "Social Investment Function" is seriously flawed.

We can obtain a Marshallian counterpart to Clower's Walrasian model by considering the special case of a perfect information accumulation process, i.e., a process producing no surprises such as would compel agents to reappraise the values they had put on their capital assets. In this particular case, we get a demand-price surface [eq. (1)] which is stationary through the process:

(1) $$P^d = d(K, \dot{K})$$

(2) $$P^s = s(\dot{K})$$

(3) $$P^d = P^s = P^*$$

This version of the model can be represented by a three-dimensional diagram (Fig. 4). The locus of intersection between the supply-price and demand-price surfaces (for a net accumulation process) shows the paths of the stock, K, of the market price, P^*, and of the net rate of accumulation, $\dot{K}$. The larger the already accumulated stock, the lower the price and the slower the rate of accumulation. Under appropriate conditions, the processes will terminate in a stationary stock-flow equilibrium (P^*, K^*) with $\dot{K} = 0$.[5] This is all very similar to the Clower-Witte process, yet the construction is not the same.

FIGURE 4

Consider, on the one hand, the Clower stock-demand schedule, $D(P)$, and, on the other, the $\dot{K} = 0$ intersection of the demand-price surface in Fig. 4, $P^s(K \mid \dot{K} = 0)$. Although we can draw them both in the (P, K) plane, they need *not* be congruent. Indeed, they normally will not be. Even if by chance we had a case in which they happen to be congruent, the one should *not* be taken to be the inverse of the other.[6]

The stock-demand schedule, $D(P)$ aggregates the quantities demanded by n optimizing, price-taking agents. The demand-price schedule, $P^d(K)$, represents the "market valuation" of a machine when the aggregate stock is K and K is staying constant (e.g., at $\underline{K}$ in Fig. 5). In our perfect foresight example, $P^d(\underline{K})$ is, of course, the "true" value of a unit of K. It is now obvious why the demand and the demand-price schedules are not congruent. Under perfect foresight, the Clower-Witte stock-demand construction (applied in this instance to a capital good) should be infinitely elastic around the "true" value, $P(\underline{K})$: anyone able to buy the rental stream accruing to a machine for less will have a capital gain and anyone paying more will incur a capital loss. So the two schedules would look as in Fig. 5.

FIGURE 5

We note in passing the implied conclusion that, in the case of a capital good (at least), the stock-demand function cannot be drawn independently of the stock-supply. In Fig. 5, if the stock were to grow to $K' > K$, we should redraw the $D(P)$-schedule so as to show it as being infinitely elastic around the corresponding lower $P^d(K')$-level.

5. With the distinction between stock-demand and demand-price functions thus clarified, it is also that much clearer that we have a problem. The stock-demand schedule is presumed to be built up in the usual way from individual maximizing decisions. But it is not the appropriate construction for the determination of the market price and the aggregate rate of investment. The demand-price function does provide the appropriate construction for our purpose. But whose decisions is it supposed to represent? And does it have any "foundation" in maximizing behavior?

The demand-price function shows the "market" valuation of machines depending on the existing stock and the current rate of accumulation. This "market" valuation we may think of as performed by a "representative" transactor. The analytical sequence whereby the market conceptual experiment *precedes* the derivation of propositions about "representative" behavior is characteristically Marshallian and contrasts, of course, to the usual Walrasian procedure of building up market excess demand function by aggregating the results of prior individual conceptual experiments.[7] In the case of the usual Marshallian flow-schedules, however, the demand-price (supply-price) is the price which just suffices to induce the representative agent to maintain his rate of consumption (rate of output) constant. In the present instance, we cannot associate the demand-price with a corresponding quantity-decision. We do not know how many machines the representative agent will buy (or sell). His valuation is a price-setting decision and, therefore, does not emerge from the kind of budget-constrained optimization problem that is standard in the modern competitive analysis literature and the solution to which is stated in terms of desired quantities. We can only say that the representative transactor would stand ready to buy if the market price were to settle below his valuation and, of course, to sell if it rose above. Although these behavior statements are not of the usual form, they are obviously nonetheless expressing behavior that is in substance profit-maximizing.

Now, in rational expectations models of the "first generation" a representative agent construct has generally been regarded as a perfectly adequate microfoundation for the macro behavior functions.[8] From that standpoint, our construction of the market schedule would presumably be acceptable. In such a model, one would meet our perfect foresight case once more, albeit in the frills of stochastic dress. All agents would know the structure of the system and would have the same information (and know that this is so)[9] and so would come to the same, thus truly "representative", valuation; beliefs about the structure stay constant through the process and need not be updated between periods; -- and the macro-economist would not need to bother himself with questions about who bought how much once the market aggregates are determined.

Another aspect of this particular Marshallian construction might be judged more deeply troubling from a Walrasian point of view, however. In the canonical version of general competitive analysis, the economy is decentralized by means of the price-system. Agents reach their decisions by combining the *private* knowledge of their own tastes, endowments and production possibilities with the *public* information of market prices. The information content of prices suffices to coordinate their decisions. In particular, no transactor needs to know aggregate quantities. But our representative agent cannot rely simply on prices. He needs to know the aggregate net rate of accumulation[10] in order to form his demand-price for machines. In certain simple settings, we might perhaps defend the assumption that he infers the aggregate rate of accumulation from the market price -- but it is not obvious that we are justified in assuming, in general, that he *can* know it.

A rational expectations model with a representative firm that has the requisite information on the aggregate net capital accumulation rate will be the most convenient vehicle for equilibrium growth theory. But for macrotheories (not all of them Keynesian) that see fluctuations in investment as central to the subject, such a model will not do. For such purposes we need an investment model that will at least allow for the possibility that aggregate investment is not perfectly coordinated but may proceed at the wrong rate. This could happen, for instance, if the representative agent does not know the aggregate investment rate and misconstrues the behavior of competitors. To describe such a process, to see where coordination errors might come in, and to study the corrective market feedback mechanisms, we need a model of individual firm behavior and of the interaction of firms in the market. This means that we must consider the "determinateness" of individual quantity decisions and how to model their aggregation.

6. Before going further with the flow of the argument, we had better take stock a bit. The stock-flow modelling alternatives are almost endless and we do not want to get lost among all the things one can do.

Investment problems, we have noted, are missing in the Marshallian layout of price theory that, despite the dominance of a different neoclassical tradition, still holds its own in standard price theory textbooks. The teaching of the Marshallian part of the subject proceeds through a taxonomy of cases: Competition, Imperfect Competition and Monopoly; Diminishing, Constant and Increasing Returns;[11] the Short Run and the Long. What the stock-flow approach aims at, of course, is in part to replace the organization of the subject into long run stationary cases and short run cases analyzed *as if* stationary with an analysis that concentrates on determining the levels *and* rates of change of the capital stock in the short run -- and from which one will get long run results when the solution-values of the time-derivatives are zero.[12]

In the present context, we may use the terms "scale" and "growth" in place of "capital" and "investment" (both understood, as before, to be measured in "number of machines"). The terms may be applied at the plant, the firm, and/or the industry levels.

The highly prized talent of "thinking like an economist" has by and large been acquired when one's second nature steps in to dictate all the convexity-assumptions needed to determine the quantities for which one requires determinate solutions in the particular case. The investment literature suggests that for many economists this instinct is only weakly developed with regard to growth-rates. Everyone knows that the scale of the constant cost competitive firm is indeterminate, that the supply of such an industry cannot be built up by aggregation (even if one knew the number of firms in it), but that the scale of the industry will still be taken care of by the Law of Demand. In the macroinvestment literature, on the other hand, considerable consternation was at one time expressed over the apparent non-existence of a "demand-for-investment function" at either the firm or industry level.

The *scale* of a plant, a firm, or an industry may be made determinate by assuming either declining demand price or increasing cost for the body in question. Increasing cost cases are usually divided into technological and pecuniary diseconomies. One can obviously do the same for *growth*- variables -- no great ingenuity is required. What this gets us, unfortunately, is a taxonomy with a truly lamentable number of Empty Boxes. We should consider the growth-rate of the plant, of the firm, and of the industry and, at each level, ask whether we have reason to assume that declining (marginal) revenue or rising cost (or both) determines the respective growth-rate. Such a taxonomy would give us a quick survey of the characteristics of any particular model and thus tell us pretty much at a glance what it can and cannot be expected to do for us.

If this stock-flow scale/growth approach were to replace the short run/long run approach in teaching, however, ambitions to "cover the subject" would have to be curbed. If the distinction between technological and pecuniary diseconomies were to be maintained also on the growth-side, the result would be 18 binary modelling choices. Satisfaction with such a powerful generalization of Empty Box Theory is tempered by the realization that this confronts us with $2^{18} = 262{,}144$ possible cases. While one would like to believe that a quarter-million or so of these can be shown to be meaningless or uninteresting, there threatens to be enough left over to tax the patience of students, not all of whom start off with a passion for partial equilibrium analysis in the first place. [13]

Among all these possibilities, which one has the first call on our attention? We believe that all the central problems with this kind of investment theory will be addressed by focusing on the "perfectly competitive" case where there are no diseconomies of growth, whether technical or pecuniary, to the individual firms. The supply curve of the capital goods producing industry is assumed to show diminishing returns (although this is not essential). If we can set out a convincing model for this case, other complications can be added "to taste" without significant difficulty.

7. It may help to have a concrete example in view at this point. The housing market in a large metropolitan area like Los Angeles may serve. Instead of our previous shoe machine, the homogeneous "unit of capital" is now a standard two-bedroom apartment (with a palm tree lit by purple and orange floodlights in front). A very large number of large and small investors make up the demand-side of the market and the housing construction industry is also competitive. All investors may be assumed equally efficient in managing apartments, i.e., in producing housing services from a given stock. The individual investor has to forecast future demand in the apartment rental market. What an apartment unit is worth today, given future rental demand, depends upon the size of the present stock and the net rate of accumulation today and at future dates.

At this point, the immediate analytical problem is two-fold. First, assuming that all apartment managing firms make the same forecasts of the future time-profile of rental values, how do we make the number of new apartments demanded add up to the quantity supplied by the construction industry? Second, allowing some distribution of subjective forecasts, how do we "prevent" the most optimistic firm, which puts the highest demand price of anyone on apartments, from buying the entire output (and, indeed, from going on a takeover rampage through the industry)? Highest bidder takes all, of course, does not give us a representative firm model. Either problem can be "solved" in a number of more or less obvious ways. The question is what assumptions will do this in the most palatable way.

If the most optimistic firm were to exercise a perfectly elastic demand for housing at a price equal to its estimate of the present value of an apartment, the "winner's curse"[14] would be a recurrent feature observed in all capital goods markets. Obviously, the market must have evolved institutionalized restraints to prevent financial institutions from being sucked under by some plunging optimist. It is equally obvious, however, that transactors will normally want to guard themselves against bankruptcy without being forcibly held back from the brink by lenders. We do not, therefore, want to rely on investors facing interest rates that rise with leverage or credit rationing (although one or the other may be potentially ever-present) in order to explain why the individual firm demands a finite amount of capital at and below a price corresponding to its estimate of the present value of a unit of capital stock.

Let $P_i(B_i;\, \gamma_i)$ be the i^{th} firm's best estimate of the present value of a unit of capital. Here, B_i is *what it knows or infers* about the present aggregate capital stock, K, and γ_i, similarly, what it knows about its rate of growth. Firm i's beliefs about future demand are taken as given. Both B_i and γ_i may change with time t. The arguments of $P(.)$ will often be omitted and the firm's estimate denoted as P_i(t) for simpler notation. Let the quantity of additional capital demanded by the firm be an increasing function of the difference between P_i, thus defined, and the market price P. For example,

$$\Delta K_{it}/K_{it} = s_i\ \gamma_i + f\ [P_i(t) - P(t)] \tag{4}$$

where $s_i > 0$ the firm i's share of the industry. This is a quantity adjustment rule analogous to the one underlying the usual Marshallian short run supply-function When $P_i(t)$ differs from the market price, the firm does not plunge but bets a limited amount on its own estimate of present value being correct.

In this example, the second term would vanish if and when the P_i's of individual firms converged to the market price. Such convergence would then produce "Gibrat's Law", i.e., that all firms tend to grow at the same proportional rate.[15] This rate, g, of which γ_i is the i-th firm's estimate, will be an increasing function of the market price.

Given (4), the model of the market for newly produced capital goods can be closed by aggregating these capital-stock adjustments over all firms and requiring that the price be such as to equate the net aggregate incremental demand for capital with the output of the capital goods producing industry.

Our example, however, does not make clear what information firms rely on in forming their beliefs about how fast their competitiors are expanding. The simplest case, of course, would be to assume that the aggregate rate of (net) investment, $\dot{K}_t$, is more or less directly observable--that it is accurately reported by some trade publication, perhaps. But we will not want always to make this assumption. When $\dot{K}_t$ cannot be known directly, firms would have to try to infer how fast their competitors are expanding by *ex post* observations of the rentals earned per unit of capital. We concentrate on this case. It opens up the possibility of considering further complications, such as those that would arise, for instance, in the case of lengthy gestation periods for new capital to get on line.

This, then, is the direction in which our exploration of the stock-flow analysis of investment points. But it remains to actually put the model together more explicitly.

8. We first describe the well-known present value maximization problem by a representative firm, and derive the differential equation and the solution for the demand price for capital stock (new machines). We abstract from taxes and installation costs and omit the distinction between gross and net investment, since the information problem facing the firms can be brought out without these complications. See Hayashi (1982) for the effects of these on Tobin's q and on the investment decision by the representative firm.

The standard way of posing the optimal investment problem for the representative firm is to formulate the intertemporal maximization problem where the firm maximizes

$$\int_{t_o}^{\infty} [P_x X - P\ I - P_L L] e^{-R(t_o,t)} dt$$

with respect to X, I, and L, where the time argument has been dropped for shorter notation, and where P_x, P and P_L are the prices of the output, the investment good and labor, respectively. This optimization is carried out subject to constraints:

$$X = F(K,L), \quad \text{and} \quad \dot{K} = I - \delta K.$$

Here

$$R(t_o,t) = r(t - t_o),$$

where r is the instantaneous rate of discount. Since this interest rate may be taken to be constant without loss of generality, we assume that $R = r$.

In this formulation, the firm takes the future time paths for the output price, the price of the investment good and the wage rate as known. They are actually expected or anticipated time paths by a representative firm, so that the resulting present value should be interpreted accordingly.

The maximization, sketched in the Appendix, produces the differential equation for the demand price for the investment good by the representative firm:

$$\dot{P} = -P_x F_K + \rho P \tag{5}$$

where ρ is defined to be the sum $r+\delta$. An intuitive interpretation of (5) may be obtained by considering an income source yielding a finite stream of earnings with certainty. The change in the value of this asset from one period to the next will have two components. First, the value will diminish by the earnings encashed during the period and, second, it will increase because the remainder of the discounted income stream is now one period closer in time. These two components correspond to the two terms of (5). The expression $P_x F_K\ (I, L^*)$ is denoted by V for brevity, called the rental price of a new capital good. The symbol L^* in the marginal product of capital denotes the optimal level of employment obtained by the marginal product condition. Thus $V = V(K, w)$, where w is the real wage rate assumed to be known to all firms. V depends negatively on K. Since the coefficient ρ is positive, (5) is an unstable differential equation. The solution can be obtained only by imposing the boundedness condition on P that it remains bounded for large t, which allows us to write the solution as

$$P(t) = \int_t^{\infty} V[\,K(\tau),w\,]e^{-\rho(\tau-t)}d\tau \tag{6}$$

The future time path of the capital stock in the argument of V is an expected path. (This expression shows that any expected disturbance in the marginal value product of capital expected in the future will induce an immediate change in the demand price for the investment good).

To proceed without too much algebraic complications, we assume a parametric functional form for the rental price

$$V(K(\tau),w) = \bar{V}(w)K(\tau)^{-\lambda}\ ,\ \tau>t\ , \tag{7}$$

for some positive number λ. We drop w from now on from $\bar{V}(w)$.

When the capital stock is expected by the representative firm to grow at the rate γ, the integral in (6) can be carried out by substituting $K(\tau) = K(t)e^{\gamma(\tau-t)}$ into (6) to yield the demand price for the investment good by the representative firm

$$P[K(t),\gamma] = \bar{V}\, K(t)^{-\lambda}/(\lambda\gamma + \rho), \tag{8}$$

where we now explicitly carry the expected growth rate γ as an argument in the demand price for new machines. This formula shows that it is inversely related to the expected growth rate and the interest rate, among other things.

9. The next task is to drop the assumptions of a representative agent and of perfect foresight used in the preceding section. These assumptions need then to be replaced by an explicit learning model for individual firms. We require a learning model that can be shown to be stable and to converge to rational expectations of the growth path of the industry.

We proceed by positing such a limiting path, called the reference path, and examining the dynamics of the capital stocks of firms in the neighborhood of it. This method is used in order to avoid the technical details of convergence proofs and to focus, instead, on demonstrating the existence of a stable adjustment rule in situations reasonably close to rational expectations paths.

Suppose that firm i's capital stock is growing at the rate denoted by g_i, so that the level of gross investment by firm i is

$$I_i(t) = (g_i + \delta)K_i(t). \tag{9}$$

Denote its share of the industry capital stock by

$$s_i(t) = K_i(t)\,/\,K(t).$$

Here $K(t)$ denotes the industry-wide capital stock. Differentiating the definitional relation for the aggregate stock $K = \sum_i K_i$ with respect to time, the industry growth rate g is related to the growth rates of the individual firms by $g = \dot{K}/K = \sum_i s_i g_i$.

All firms observe the values of K at periodic time intervals by observing V of (7). The industry-wide capital stock cannot be measured exactly due to the presence of other industries supplying close substitutes to the output of "our" industry. This fact is modelled by introducing a stochastic "fudge"-factor, N, to be specified below, and replacing $K(t)$ by $K(t)N(t)$ in (7). Firm j calculates its demand price for new capital stock by (8). Since it does not in general know $K(t)$ exactly, it uses its estimate, $B_j(t)$, and the estimate of the industry growth rate $\gamma_j(t)$. These will be defined precisely later. The supply price schedule is taken to be an increasing function of the actual growth rate g.

The next task is to see if the model is well posed by applying perturbation analysis to it, i.e., we examine the model responses to small perturbations and see if the responses are stable. To derive adjustment rules and examine the implied stability properties, it is convenient to examine the behavior of this model for "small" deviations from a reference path, however chosen. Accordingly, define lower-case variables k_j by

$$K_j(t) = K_j^0(t)[\,1 + k_j(t)\,]$$

and k in the same way. The variables with superscript 0 define standard or reference time trajectories. Both $K^0(t)$ and $K_j^0(t)$ grow at a common rate g^0. These standard growth paths may not be known to firms.[16] For the time being, we postpone examination of this point and concentrate on convergence, i.e., the stability property of these variational variables.

Denote the deviations of the actual rates of growth of the industry and of firm j by $\delta g(t)$ and $\delta g_j(t)$ respectively. The dynamics of the capital stock growth is given by

$$k_j(t+1) = k_j(t) + \delta g_j(t)$$

and

$$k(t+1) = k(t) + \delta g(t). \tag{10}$$

All firms observe the logarithm of the rental price with a period lag. The log of (7), $1n\bar{V} - \lambda\ 1nK(t) - \lambda n(t)$ is observed, where $n(t)$ is $lnN(t)$ and is serially uncorrelated (and log-normally distributed with mean zero) by assumption. It is convenient to measure the observation from the reference level, $1n\bar{V} - \lambda lnK^0(t)$. Therefore, let

$$v_{t+1} = -\lambda[\,k(t) + n(t)\,] \tag{11}$$

be the signal observed by all firms at time $t+1$. As will become evident later, the only part of the observed signal that matters is the prediction error which is independent of the reference level, so long as all firms choose the same reference level. The offset by 1 of the time index is due to the fact that the rental price prevailing during the period $(t, t+1)$ becomes known only at time $t+1$. There are other possible specifications for what the firms may be able to observe. For example, suppose firm j observes (7) directly after K is replaced by KN. Denote its logarithm by y_t. Then changes in this signal is related to the industry growth rate by

$$y_t - y_{t-1} = -\lambda[\,g(t-1) + n_t - n_{t-1}\,].$$

Actually, the rental price is continuously changing with time. One may thus suppose that the firms observe the "average" rent prevailing during the period $[\,t, t+1\,]$ at time $t+1$. Then the deviational signal observed is a linear combination of $k(t)$ and $\delta g(t)$. This paper uses (11). Implications of using alternative specifications can be similarly analyzed.

Firm j's information set is thus postulated to be $D_{j,t} = (D_{j,t-1}, v_t)$. Define firm j's estimates of the industry-wide capital stock and its deviational growth rate by

$$\beta_j(t) = E(k(t) \mid D_{j,t}) \tag{12}$$

and

$$x_j(t) = E\ (\delta g(t) \mid D_{j,t}). \tag{13}$$

Advancing the time index by 1 in (12), we note from (10) that

$$\beta_j(t+1) = E[\, k(t) + \delta g(t) \mid D_{j,t}, \nu_{t+1}\,].$$

New information contained in the new observation ν_{t+1} is the innovation $e_{t+1} = \nu_{t+1} - E\ (\nu_{t+1} \mid D_{j,t})$. Using this innovation we can rewrite the above as

$$\beta_j(t+1) = E(k(t) + \delta g(t) \mid D_{j,t}) + E\ [\, k(t) + \delta g(t) \mid e_{t+1}\,].$$

Firm j therefore updates its estimate by[17]

$$\beta_j(t+1) = \beta_j(t) + x_j(t) + h_{j,t}\ e_{t+1}\ . \tag{14}$$

In the absence of noise, a positive value of e_{t+1} indicates that firm j's estimate of the industry-wide capital stock is too high. Therefore the constant $h_{j,t}$ is expected to be negative. We later show that negative h's are needed for stability. Note from (11) and (12) that

$$e_{t+1} = -\lambda[\, k(t) + n(t)\,] + \lambda\beta_j(t).$$

Firm j also observes the gap between its demand price at its intended investment rate and the supply price of new capital stock $P_j(t) - P^s$. Firms are assumed to observe the instantaneous market clearing supply price that depends on the current growth rate $g(t)$, i.e., $P^s(g)$. (Alternatives such as delayed signals may be more destabilizing to the adjustment process). By expressing this gap as $P_j(t) - P_j^0(t) + P^s(g^0) - P^s[\, g(t)\,]$, where $K^0(t)$ and g^0 are so chosen that the equality $P_j^0(t) = P^s(g^0)$ holds in the absence of noise, and where firm j invests to keep its market share on the reference path, i.e., $I_j^0 = s_j I^0$, this gap can be expressed as

$$P_j(t) - P^s[\, g(t)\,] = P^s(g^0)[\, p_j(t) - \alpha\delta g(t)\,] \tag{15}$$

where $\alpha = P^{s\prime}(g^0) / P^s(g^0) > 0$ is assumed.[18] Here the variable $p_j(t)$ is defined as the deviational logarithm of the demand price for new capital stock by firm j, i.e.,

$$P_j(t) = P_j^0(t)[\ 1+p_j(t)\ ].$$

It is equal to

$$p_j(t) = \lambda\zeta_j(t) \tag{16}$$

where $\zeta_j(t) = \beta_j(t) + \xi x_j(t)$, and $\xi = (\lambda g^0 + \rho)^{-1}$. This equation implies that firm j lowers its demand price for new capital stock if $\zeta_j(t)$ is positive, i.e., when firm j believes that the industry as a whole has too much stock (indicated by a positive $\beta_j(t)$) or is growing faster than sustainable (a positive $x_j(t)$).

Assume that the firm j invests by the rule which is a linearized version of the earlier (4)

$$I_j = I_j^0 + \mu_j\ [\ P_j - P^s\ ] \tag{17}$$

where the first term on the right, defined to be $s_j^0 I^0$, is the investment rule to keep $K_j^0(t)$ on the reference path. The constant μ_j is expected to be smaller the more risk-averse and less well informed is the firm (cf., footnote 18). The deviational investment, denoted by i_j, is defined to be $(I_j - I_j^0)/I_j^0$. From (15) it equals

$$i_j = \omega_j\ [\ p_j - \alpha\delta g\ ], \tag{18}$$

where $\omega_j = \mu_j p^s(g^0)\ /\ I_j^0$. This constant is larger for firms with smaller market share because $I_j^0 = s_j I^0$. By assumption α is positive. If P^s is independent of the growth rate, then α is zero, and no information on growth rate deviation is available this way. (See footnote 18) Since in this deviational system

$$\delta g_j(t) \approx i_j(t)\ ,$$

use of (16) and (18) leads to

$$\delta g_j(t) = -\lambda\omega_j\zeta_j(t) - \alpha\omega_j\ \delta g\,(t). \tag{19}$$

Since the weighted sum of δg_i, with weights s_j is equal to δ g, we obtain from (19)

$$\delta g(t) = \frac{\lambda}{1+\alpha\sum\theta_j} \sum j\,\theta_j \zeta_j \tag{20}$$

where $\theta_j = \omega_j s_j^0$. When (20) is substituted back into (19), it shows that the actual deviational growth rate of firm j depends on the difference between its demand price for new capital stock and the industry-wide average demand price, or the difference between ζ_j and its weighted average $[\, \alpha / (1 + \lambda\alpha\sum\theta_i\zeta_i\,)]$. Error in estimating the growth rate depends on both the error in estimating the industry capital stock and the error of the firm's estimate of the growth rate.

From (10), (14), and (19) the dynamics for the entire industry are described by

$$\begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ \beta_M \\ k \end{bmatrix}_{t+1} = \begin{bmatrix} \phi_1 & 0 & 0 & 0 & 0 \\ 0 & \phi_2 & 0 & \vdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \psi_1 & \psi_2 & \vdots & \phi_M & 1 \end{bmatrix} \begin{bmatrix} \beta_1 \\ \beta_2 \\ \vdots \\ k \end{bmatrix}_t + \begin{bmatrix} x_1 \\ \vdots \\ x_M \\ \sum\limits_i v_L x_i \end{bmatrix} + \begin{bmatrix} h_1 \\ \vdots \\ h_M \\ 0 \end{bmatrix} v_{t+1}$$

where

$$\phi_i = 1 + \lambda h_{j,t}\,, \quad \psi_i = \lambda\theta_i \,/\, (1 + \alpha\sum_i \theta_i)\,, \quad \text{and} \quad v_t = \xi\psi_i\,,\ i=1,\ldots,M\,. \tag{21}$$

So far, there is no dynamically systematic way to update the estimate of δg, unless $k(t)$ is assumed to be known because no independent observation is available on δg. Either we need an assumption on the distribution of δg or distribution of δg_j or firms need another signal to update their estimates of the growth rate deviation. We consider the case where firms have no other observation or estimates of the distribution. Then, firms have to deduce the growth rate from past observations on rental values as shown below. From (10) and (11)

$$\delta g(t) = k(t+1) - k(t) \approx -\frac{1}{\lambda}\,[\,\nu(t+2) - \nu(t+1)\,] - [\,n(t+1) - n(t)\,].$$

Firm j estimates $\delta g(t)$ then by taking the conditional expectation of the above, where $D_j(t)$ is the conditioning information set of firm j. Firm j needs to estimate the changes in the rental price two periods into the future.

Since the conditioning information set is spanned by $\nu(t)$, $\nu(t-1)$, . . . , we approximate this conditional expectation expression by $\pi_{1j}\nu(t) + \pi_{2j}\nu(t-1)$, where π's are some constants. (In reality, the conditional expectation also depends on earlier ν's. They are neglected in this approximation). We thus adopt an expression

$$x_{j,t} = -\lambda\,[\,\pi_{1j}k(t-1) + \pi_{2j}k(t-2)\,] - \lambda\,[\,\pi_{ij}n(t-1) + \pi_{2j}n(t-2)\,]. \tag{22}$$

Substituting (21) into (20), the total dynamics can be put in a state space representation for the state vector $z_t = (\beta_t\; k'_{t-1}\; k'_{t-2})'$ as

$$z_{t+1} = \Phi z_t + G e_t, \tag{23}$$

where

$$\Phi = \begin{bmatrix} \phi & -\lambda\hat{h} & -\lambda\,\hat{\pi}_1 & -\lambda\,\hat{\pi}_2 \\ \psi' & 1 & -\lambda\hat{\nu}'\,\hat{\pi}_1 & -\lambda\hat{\nu}'\,\hat{\pi}_2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix},$$

and

$$G = \begin{bmatrix} \hat{h} & \hat{\nu}'\,\hat{\pi}_1 & \hat{\pi}_2 \\ 0 & \hat{\nu}'\,\hat{\pi}_1 & \hat{\nu}'_2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad \textit{and} \quad e_t = -\lambda \begin{bmatrix} n_t \\ n_{t-1} \\ n_{t-2} \end{bmatrix}.$$

The vector $\hat{h}$, $\hat{\nu}$ and π_i, $i = 1, 2$ are made up of h_j, ν_j and $\pi_{i,j}$, $i = 1, 2$, $j = 1, \ldots, M$. The matrix ϕ is a diagonal matrix with elements ϕ_j.

We next derive conditions under which the dynamics are stable. For this purpose, we calculate the characteristic polynomical of the matrix Φ. To consider a simple case suppose that the same filter gain is used by all firms. If π_1 and π_2 are both zero, there is one root greater than one, making the dynamics unstable. In other words, if firms use the same filter gains and if they do set their estimates of the growth rate at zero, then the dynamics are unstable. If π_1 is not zero, the dynamics are stable even if π_2 is zero, if

$$\hat{\psi}'(\xi\hat{\pi}_1+\hat{h}) > 0, \quad \textit{and} \quad \hat{\psi}'\hat{\pi}_1 \, [\, \xi+\lambda(\hat{h}-1) \,] < 1.$$

By continuity, then, there are stable regions for suitable choices of ϕ's and h's even if firms do not choose the same values for these parameters.

10. This is as far as we take the story. But, obviously, it is not the end of it. The coordination of investment decisions when firms have incomplete information is a large subject that deserves further exploration in several directions.

The paper has focused on one particular information problem, to wit, that the firm typically lacks direct, reliable information on the rate of accumulation by its competitors. Without such information, the future rentals cannot be imputed to capital goods, even if the future growth of industry demand were known with confidence. Without, in turn, an estimate of the future time-profile of rental values, the firm cannot calculate its current demand price for additions to its capital stock.

One does not pose an information problem of this sort, of course, in order to proceed to assume it away. Thus, we have not been willing to assume that all firms share a rational expectations estimate of future rentals (which is then foreordained to be validated by their aggregate behavior). Instead, the model assumes that, while firms have a rational appreciation of the qualitative features of the situation, they do not all have the same expectations, nor are their expectations necessarily distributed around some "representative" or average expectation which conforms to the solution of the model.

Our explicit model shows how, in spite of this information problem, a fairly well-coordinated expansion process can result without any centralization of information (or, of course, of decision-making). Two assumptions combine to produce this result. The first is that the individual firm will not "plunge" in an attempt dramatically to increase its market share when it finds itself calculating a demand-price higher than the current market price for new capital goods. The second is that it can check its own expectations about the time-profile of capital rentals against current realizations more or less continuously and that these *ex post* rentals accurately reveal errors in the firm's forecasts of the aggregate industry accumulation rate with a minimum lag.

These are also the assumptions that should come first under scrutiny when less well-behaved industry growth-processes are to be analyzed. Thus, for example, if some firms think they have a definite cost advantage (or product advantage) over the competition, they may well plunge for sharply increased market shares, causing industry growth to overshoot. If, in our model, firms do not have the same short run cost, so that $V(w)$ differs across firms, the $P_i - P^s$ discrepancy observed by the individual firm is no longer a reliable signal on which to base its own investment decision. Under these circumstances, $P_i - P^s$ is no longer proportional to $b_i - \beta_i$ but depends also on the distribution of costs by firms in the industry. Without reliable information on this cost-distribution, firms will not be able to relate the observed signal to the estimation error. When innovation in an industry proceeds at a high rate, the result may thus be sharp fluctuations in growth. Several branches of the computer industry in the early eighties come to mind as illustrations.

Similarly, a lengthy gestation period of investments may delay the information feedback and introduce lags in (8) and (9). These lags are known to be destabilizing and may cause sizeable fluctuations of the actual around the equilibrium time-path of capital accumulation. In cases of this sort, more complicated schemes for updating firms' expectations may be analyzed by setting up state equations for the estimation errors and formulate appropriate Kalman filters for them, i.e., state space models for the observed data series. Whether or not firms actually use such sophisticated decision strategies, recent booms-and-busts in office building construction, for instance, show that individual investment decisions are not always coordinated in such a way as to produce smooth aggregate accumulation paths.

Information lags due to lengthy gestation periods may be the most plausible reason for fluctuations in the growth of individual industries. But for macroeconomic pruposes this source of potential instability seems to us to be of somewhat limited interest. In that context, it is rather the feedback loop from aggregate investment to real income and demand and thus to observed rentals that is likely to cause trouble. The study of this problem, however, would require imbedding the present construction in a larger macroeconomic model -- a task beyond the purview of this paper.

Footnotes

[1]The terminology is Hicks'. Cf., his (1956), Chapter IX.

[2]These matters of determinateness are discussed more fully in Section 6 below.

[3]"Permanent" in this context should not be taken too literally. A government subsidy to the capital goods producing industry, for instance, may not last forever but must obviously be analyzed differently from such random and evanescent disturbances to the rate of capital goods output as may perhaps occur but provide no useful information about the industry supply function.

[4]Actually, this is *not quite* true. Sophisticated beer-drinkers might well speculate on the rate of change of beer-prices in deciding how big a stock to hold. But for present purposes at least we will allow ourselves to abstract from sophisticated beer-drinkers. These yuppie stock-broker types, who cannot ever stop themselves from maximizing, are probably important only in the market for "lite" beers anyhow.

[5]At this point we are ignoring any difference between gross and net investment. This omission is, of course, easily repaired if we allow ourselves the convenient assumption of radioactive depreciation. Until later, however, this will be left as an exercise for those used to handling such materials.

[6]Cf., Leijonhufvud (1974) for a detailed explanation of why Walrasian "price into quantity" schedules may not be treated as inverses of Marshallian "quantity into price" schedules.

[7]Cf., again Leijonhufvud, *op. cit.*. A fuller treatment of the Marshallian case is found in Aoki (1976, pp. 193-202).

[8]The contributions of Hansson (1981) and of LeRoy (1983) also use representative firm constructions.

[9]Cf., E.S. Phelps "The Trouble with 'Rational Expectations' and the Problem of Inflation Stabilization", in Frydman and Phelps (1983).

[10]This distinction may seem overdrawn at first. The Walrasian trick in this context, of course, would be to assume that the representative agent had rational expectations about future prices (so as to be able to compute future machine rentals directly). In the absence of future markets in which these prices would actually be determined, the rational formation of such expectations will require knowledge of the rate of capital accumulation -- and we are back at the point of the text.

[11]The Laws of Return were the target of Clapham's (1922) "empty boxes" attack on the Marshallians. His article, Pigou's reply, and Sraffa's subsequent, more fundamental, attack were reprinted in the AEA *Readings in Price Theory*. Cf. Boulding and Stigler (1952).

[12]This is analogous to the way in which Marshall constructed his short run schedules. These are loci of points at which specified adaptive (quantity adjustment) processes have time-derivatives of zero.

[13] Note that the taxonomic "riches" could easily be made even more abundant by allowing also *concavities* (those Black Holes of the economist's Cosmology). Indeed, increasing returns to scale in both the shoe industry and the shoe machinery industry are assumptions with much to recommend them. This paper, however, does not seem the place to take up this particular cudgel.

[14] The term comes from the literature on mineral rights auctions. Cf., Capen, Clapp, and Campbell (1971). Milgrom and Weber (1982, p. 1094) summarize: "... even if all bidders make unbiased estimates, the winner will find that he has overestimated (on average) the value of the rights he has won at auction". We are grateful to Gerald Garvey for bringing the "winner's curse" to our attention.

[15] Some earlier contributions on firm size distributions are Simon and Bonini (1958), Ijiri and Simon (1964), and Lucas (1978). They consider factors that cause the size distribution to deviate from Gibrat's Law which posits that the proportional growth rates of firms are independent of firm sizes. Taking capital stock as a proxy for firm size, Gibrat's Law states that the rate of change of capital stock is independent of capital stock. Its weaker version states that the expected percentage change in size of the totality of firms in each size stratum is independent of stratum. Simon and Bonini show that empirical deviations of firm size from Gibrat's Law can be reconciled by allowing entry and exit. In our models deviations from Gibrat's Law may be attributable to inhomogeneity of firm specifications. The serial correlation of growth rates in Ijiri and Simon could be explained by slow adjustments. Lucas shows that the earlier works, which did not build on optimization models, can be reconciled with firm optimization, and considers the distribution of managerial talents as a factor in determining the firm size distribution.

[16]However, it may sometimes be reasonable to assume that the reference time path is known to all firms of the industry as the output of a national planning agency or trade association long-range forecast.

[17]Eq. (14) has the same form as a Kalman filter. The "optimal" choice of the filter gain, $h_{j,t}$ requires knowledge of noise covariance which firms may not possess. Here we assume that firms choose them on some *a priori* basis.

[18]A demand for assets of similar structure has been obtained by D. Friedman (1981) who used a quadratic approximation of the utility function involved to provide a rationale for the form of the demand-equation we use in the text. Briefly, consider a stochastic stream of net receipts of the form $m(s) = \mu(s) + [P^s(s) - P(s)]g(s)$. Here μ represents the stream of receipts due to the inherited position, and the second term represent the net addition of assets by the amount g(s) where $P^s - P$ represents the gap between the supply price and the demand price. The utility maximization $E \int_0^\infty U(m(s))\ e^{-\nu t} ds$ with respect to g results in $g(t) = const.$ $[P^s(t) - P(t)]$, where the constant is proportional to $-\dot{U}/U$ and inversely proportional to the variance of the price gap. Although this result does not directly apply to the problem of this paper, one could think of the new investment as generating a stream of net receipts in addition to that from the inherited capital stock. Thus it is not too unreasonable to expect this type of investment rule in our case as well. Furthermore, the previous discussion suggests that the proportional constant μ_j used by firm j may indeed vary from firm to firm since it depends on the degree of risk aversion by the firm as well as on the precision of information it has on the possible price gap distribution.

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Appendix

We drop time argument when no confusion is likely to arise. Form the Hamiltonian of the firm's intertemporal optimization problem:

$$H = e^{-R(t_0,t)}(P_x X - P_I I - P_L L) + \lambda[X - F(K,L)] + \mu(I - \delta K)\,,$$

by incorporation the two constraints with the two Lagrange multipliers, λ and μ. They are related by

$$\dot{\mu} = -H_K = \lambda F_K + \delta\mu\,, \tag{A1}$$

$$0 = H_X = e^{-R} P_x + \lambda\,, \tag{A2}$$

$$0 = H_L = -e^{-R} P_L - \lambda F_L\,, \tag{A3}$$

and

$$0 = H_I = -e^{-R} P_I + \mu\,. \tag{A4}$$

The optimal level of employment is determined by the usual marginal product condition obtained from (A.2) and (A.3)

$$F_L = P_L / P_X = w\,, \tag{A5}$$

where w is the real wage rate. When (A.5) is solved out for L , it is a function of K and w. Denote the optimal L by L*.

Combination of (A.2) and (A.4) produces the differential equation for P_I. The transversality condition $\lim_{t\to\infty} P_I(t)K(t)e^{-R(t_0,t)} = 0$ can be shown to be satisfied. The differential equation for P_I is given by

$$\dot{P}_I = -V + \rho P_I\,, \tag{A.7}$$

where

$$V = P_x F_K(K, L^*)$$

is the marginal value product of capital, and ρ is the sum of r and δ. By imposing boundedness condition (A.8) can be integrated.

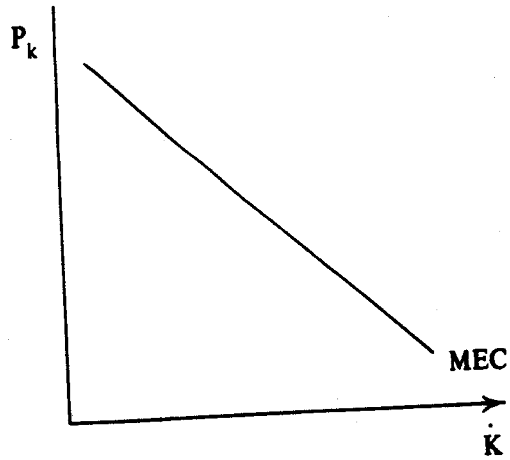

Figure 1

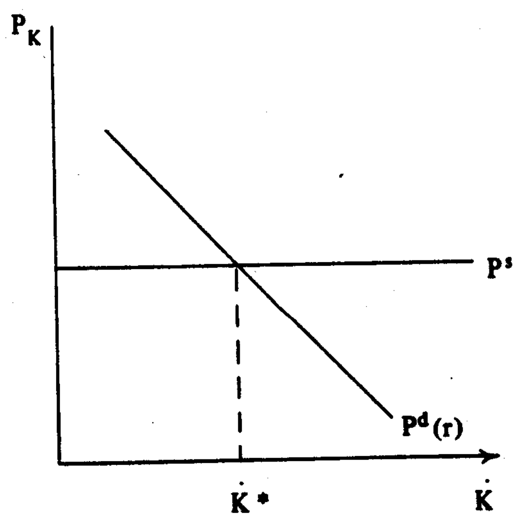

Figure 2

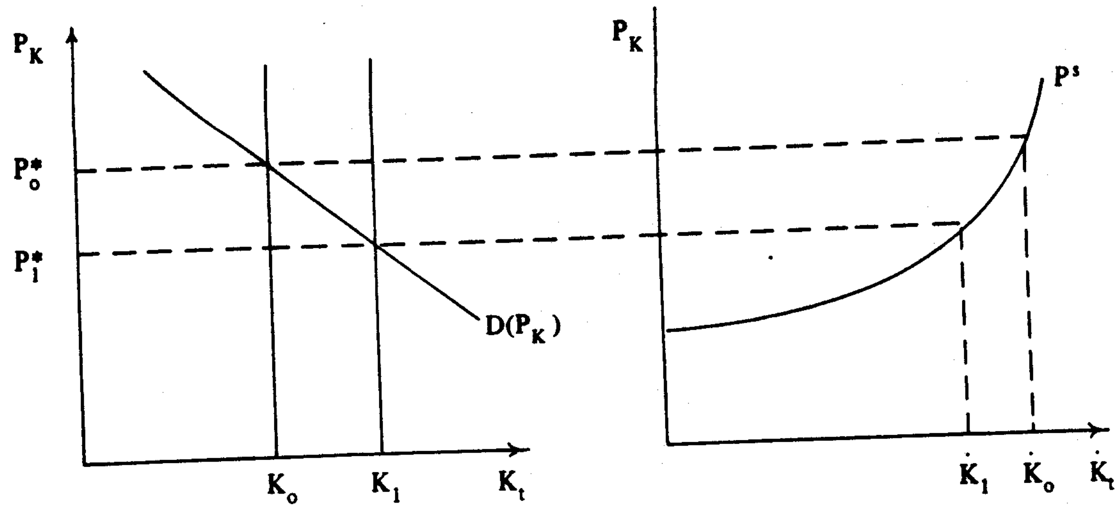

Figure 3

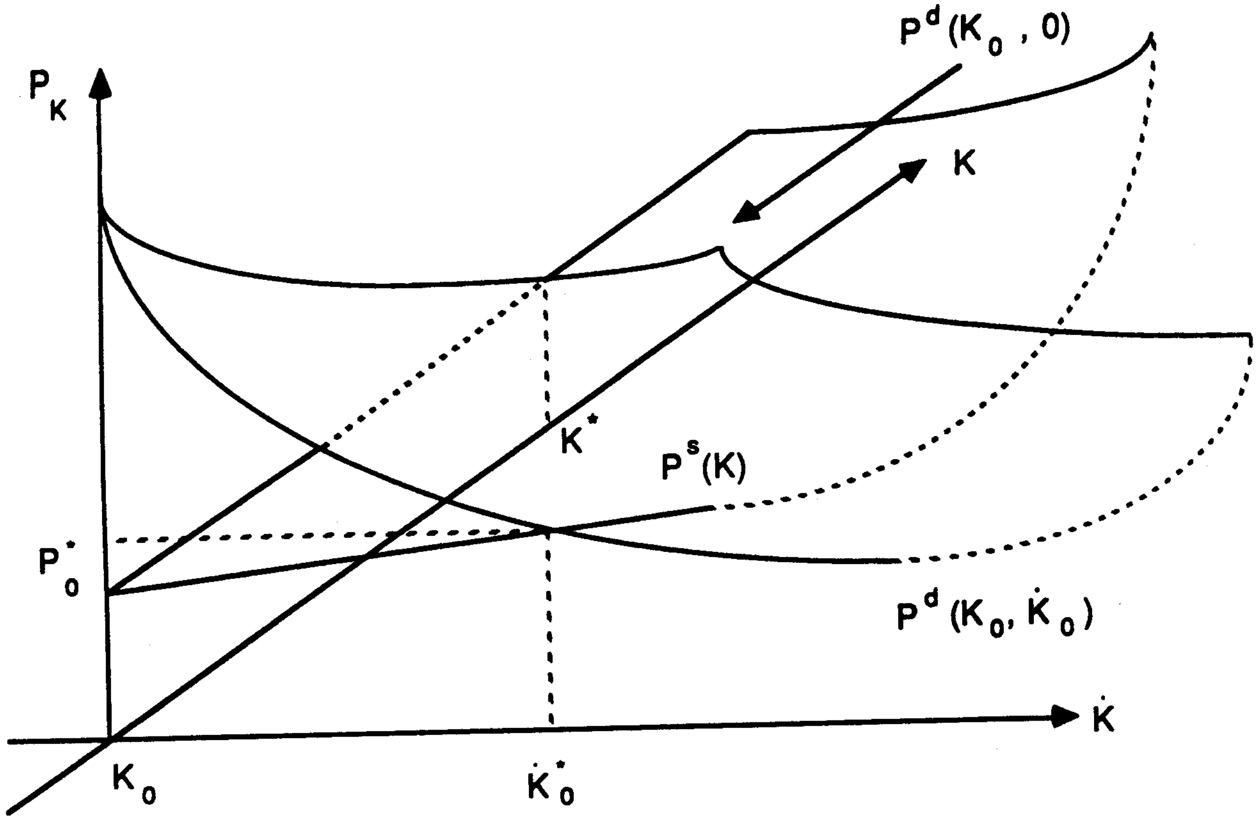

Figure 4

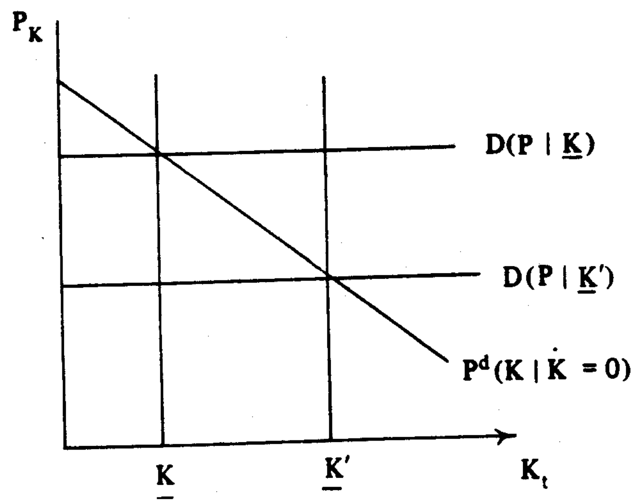
P_K
$D(P \mid \underline{K})$
$D(P \mid \underline{K}')$
$P^d(K \mid \dot{K} = 0)$
$\underline{K}$
$\underline{K}'$
K_t

Figure 5