UNDERINVESTMENT IN ENTRY DETERRENCE:

WHEN AND WHY

by

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ABSTRACT

This paper considers environments in which incumbent firms cannot collude on an investment in entry deterrence, and attempts to identify the circumstances under which the free rider problem leads to underinvestment. What I show is that underinvestment can arise if some factor is present which smooths the return to investing in entry deterrence. In particular, in the first part of the paper I consider an environment in which underinvestment can arise if either: i) uncertainty is present; ii) entry deterrence has the effect of delaying the date at which entry occurs; or iii) entry deterrence decreases the market share of the entrant upon entry. At the end of the paper I then consider how these results relate to recent papers by McLean and Riordan, and Eaton and Ware.

I. Introduction

The recent literature in Industrial Organization has witnessed a proliferation of papers on the topic of entry deterrence. The first wave of papers in this area concentrated on the different methods by which a single incumbent firm (or group of colluding incumbents) may attempt to deter entry.[1] More recently, attention has moved towards settings characterized by multiple incumbents, where these firms interact in a noncooperative fashion. By moving in this direction the focus of the research has shifted towards such interesting topics as the role of the free rider problem and the significance of firms moving in a sequential fashion.[2] This paper also considers settings in which multiple incumbents act in a noncooperative fashion, and in particular is concerned with the circumstances under which the free rider problem causes incumbent firms to underinvest in entry deterrence.

Consider an oligopoly in which member firms cannot collude on an investment in entry deterrence. In such an environment entry deterrence has the properties of a public good. A public good in this context means that, because of the effect on the probability of entry, increasing the investment in entry deterrence yields a return to the oligopoly as a whole, some of which is not reflected as a return to the actual investor. Given such a situation, the free rider problem suggests there should be an underinvestment. That is, the investment in entry deterrence should be less than that which would maximize the expected joint profits of the oligopoly.

Two early papers which considered the free rider issue were that of Bernheim (1984) and Gilbert and Vives (1986). In each of these papers the formal analysis uncovered little or no evidence of the free rider problem leading to underinvestment. Additionally, at least in the Gilbert and Vives

paper, it was suggested that the free rider problem may not be a significant factor whenever entry deterrence is an issue. Given the intuitive appeal of the above argument, if such a conclusion were correct it would truly be quite surprising.

One property shared by the Bernheim model and that of Gilbert and Vives is that the exact investment needed to deter entry is known with certainty. That is, in each model the incumbent firms face a critical investment in entry deterrence such that, if the actual investment is less than this critical level, then the probability of entry is one. If, on the other hand, the actual investment is greater than this critical level, then the probability of entry is zero. In Waldman (1987) I explored what happens in these models when uncertainty is introduced. The conclusion drawn was that once uncertainty is introduced the free rider problem may lead to underinvestment, but it will not do so in all cases. In particular, introducing uncertainty to the model considered by Bernheim does cause the incumbent firms to underinvest. However, this is not the case for the model considered by Gilbert and Vives. In that model, in the absence of a potential entrant the incumbent firms overinvest in the entry deterring activity.[3] The subsequent result is that in the presence of a potential entrant, the free rider problem does not lead to underinvestment even after uncertainty is introduced.

This paper will further investigate the importance of the free rider problem, where attention will be focused on the case originally considered by Bernheim. That is, the focus will be on environments in which in the absence of a potential entrant there is no overinvestment in the entry deterring activity. What I will demonstrate is that, although the presence of uncertainty is in many cases clearly sufficient for the free rider problem to

lead to underinvestment, it is not a necessary condition. In fact, what will be shown is that in relation to the existence of an underinvestment, there are a variety of different properties of models which can serve the same role as uncertainty.

In order to grasp this point one must first understand how the presence of uncertainty can cause the free rider problem to become an important factor. As indicated above, in the Bernheim model an absence of uncertainty means that there is a critical investment in entry deterrence such that an ϵ increase in the investment causes the probability of entry to change from one to zero. At this margin the oligopoly faces an infinite return to investing in entry deterrence, while everywhere else the oligopoly faces a zero return. Consider now what happens when the oligopoly cannot collude on the investment. In that case the relevant return is the return to the oligopoly divided by the number of incumbent firms. Dividing by the number of incumbents does not change the return, however, since the initial return was either infinite or zero. Hence, one should not be surprised that in the absence of uncertainty the free rider problem is not an important factor. Introducing uncertainty, however, eliminates this critical investment level, and the straightforward intuition is then that the free rider problem should be important.

In other words, uncertainty plays the role of smoothing the return to investing in entry deterrence, i.e., the introduction of uncertainty means the whole return does not occur at a single critical point. Hence, it is not the presence of uncertainty per se which is important, but rather that the return to investing in entry deterrence is a smooth function. Further, as will be demonstrated, there are a number of factors besides the presence of uncertainty which can serve this role.

Before proceeding, one point of interest is the relationship between the argument presented above and the recent free rider results of McLean and Riordan (1987). They consider a model in which firms sequentially enter an industry, where each firm has the option of undertaking an entry deterring investment upon entry. They demonstrate that in such an environment the free rider problem will frequently lead to underinvestment, and attribute this result to the fact that investments are made sequentially.

> "...underinvestment in entry deterrence is a distinct possibility. There exist cases where a group of early entrants would mutually benefit from a more severe entry deterring posture, but fail to achieve this because of the sequential nature of decision-making."
> (McLean and Riordan (1987), p. 3)

I will show this reasoning is incorrect. The reason they find an underinvestment is not primarily that investments are made sequentially, but rather that their model is characterized by multiple potential entrants. That is, the presence of multiple potential entrants is simply a way of smoothing the return to investing in entry deterrence and, as discussed above, underinvestment can arise as long as the return to investing in entry deterrence is a smooth function.[4]

The outline for the paper is as follows. Section II considers a model characterized by a single potential entrant who has a choice as to which date to enter the industry. Here it is shown that, even in the absence of uncertainty, underinvestment can occur if investing in entry deterrence either delays the date at which entry occurs, or decreases the market share of the entrant upon entry. Section III considers a model where an underinvestment arises whose nature is identical to that found by McLean and Riordan. As indicated above, here it is shown that in such a setting underinvestment is

not primarily due to investments being made sequentially, but rather to the fact that the presence of multiple potential entrants smooths the return to investing in entry deterrence. This section also discusses the relationship between the analyis in this paper and the recent results of Eaton and Ware (1987). Section IV presents some concluding remarks.

II. Different Reasons for Underinvestment

This section considers a continuous time model in which the potential entrant has a choice as to which date to enter the industry. The particular model analyzed is similar to ones considered in Bernheim (1984) and Waldman (1987). The structure is as follows. There are N risk neutral incumbent firms who produce a homogeneous product and who face a single potential entrant, where incumbents and entrant all discount the future at a constant rate k. At date zero each incumbent can invest in entry deterrence, but the incumbents are unable to collude on the investment. Let z_i denote the investment in entry deterrence of incumbent firm i, and let $Z = \sum_{i=1}^{N} z_i$. After the incumbents have chosen their investments in entry deterrence, the potential entrant observes the aggregate investment and then decides whether or not to enter. In particular, entry occurs if there exists an entry date for which the present discounted value for profits is strictly positive.

At each date in time each of the N or N+1 firms in the industry chooses an output level, where this choice is made noncooperatively, i.e., at each date in time the firms in the industry play a Cournot game. Further, the industry demand curve at date t is given by $P(X^I) = A(t) - bX^I$, where X^I is industry output and A(.) is twice continuously differentiable. That A(.) is a function of the date allows us to capture the idea that industry demand may

vary over time.[5]

On the cost side of the model each firm (incumbent or entrant) incurs a cost cx when it produces x units of output. There is also an entry cost. For the incumbents the entry cost has been incurred in the past and is therefore not relevant to the analysis. For the potential entrant the entry cost depends on the aggregate investment in entry deterrence. In particular, the entry cost equals $S(Z)$, where $S'>0$, $S(0)=0$ and $S(\infty)=\infty$.

This completes the description of the model, and we can now proceed to the analysis.[6] Our main interest is in whether or not the incumbent firms underinvest in entry deterrence. Hence, the first step is to establish a benchmark with which later results can be compared. Let Z^* denote the total investment in entry deterrence which the oligopoly would make if it could collude on the investment. Note, in defining Z^* we retain the assumption that at each date in time the N or N+1 firms in the market play a Cournot game, i.e., the incumbent firms are not assumed to act cooperatively as regards output decisions. Z^* is defined by (1).

$$(1) \qquad Z^* = \arg \max_Z N[\int_0^{\hat{t}(Z)} \Pi_N(t)e^{-kt}dt + \int_{\hat{t}(Z)}^\infty \Pi_{N+1}(t)e^{-kt}dt] - Z,$$

where $\Pi_N(t)=(A(t)-c)^2/b(N+1)^2$, $\Pi_{N+1}(t)=(A(t)-c)^2/b(N+2)^2$, and $\hat{t}(Z)$ is the date of entry (note: entry not occurring will be denoted as $\hat{t}(Z)=\infty$).

Our interest of course is on what happens when the incumbent firms cannot collude on the investment in entry deterrence. The first proposition deals with the case where the demand curve is shrinking over time.

<u>Proposition 1</u>: Suppose $A' \leq 0$ for all t, $\lim_{t \to \infty} A(t) > c$. Then,

     i) There exists a critical value Z' such that if $Z \geq Z'$ then $\hat{t}(Z) = \infty$, while

     if $Z < Z'$ then $\hat{t}(Z) = 0$.

     ii) There exists an equilibrium for which $Z = Z^*$.

<u>Proof</u>: Let Z' be such that

$$(2) \qquad \int_0^\infty \Pi_{N+1}(t) e^{-kt} dt - S(Z') = 0.$$

$A' \leq 0$ implies $\Pi'_{N+1} \leq 0$. Now suppose entry occurs at date $\hat{t}$. Then profits for

the entrant, denoted Π_E, are given by

$$(3) \qquad \Pi_E = \int_{\hat{t}}^\infty \Pi_{N+1} e^{-kt} dt - S(Z) e^{-k\hat{t}},$$

Taking the derivative with respect to $\hat{t}$ yields

$$(4) \qquad (kS(Z) - \Pi_{N+1}(\hat{t})) e^{-k\hat{t}}.$$

Given (4), if $\hat{t}$ does not equal 0 or ∞, then $kS(Z) - \Pi_{N+1}(\hat{t}) = 0$. Given this, if

$A' < 0$ for some $t < \hat{t}$, then $(kS(Z) - \Pi_{N+1}(t)) e^{-kt} \leq 0$ for all $t < \hat{t}$ and strictly negative

for some $t < \hat{t}$. Hence, for this case $\hat{t}$ must equal 0 or ∞. If $A' = 0$ for all $t < \hat{t}$,

then $(kS(Z) - \Pi_{N+1}(t)) e^{-kt} = 0$ for all $t < \hat{t}$. Hence, the firm is at least as well

off entering at 0 as at $\hat{t}$. However, (2) tells us that if $kS(Z) - \Pi_{N+1}(0) = 0$,

then $Z \geq Z'$. Given (2) and that the firm is at least as well off entering at

date 0, we have $\hat{t}(Z) = \infty$. Or overall, $\hat{t}(Z) = 0$ or ∞. Finally, given (2), we now

have that if $Z \geq Z'$ then $\hat{t}(Z) = \infty$, while if $Z < Z'$ then $\hat{t}(Z) = 0$. This proves i).

     For the proof of ii) there are two cases. The collusive result could be

that the incumbent firms invest zero, or it could be that the incumbent firms

invest just enough to totally deter entry, i.e., Z'. I can prove ii) for this

latter case by demonstrating that if N-1 of the incumbents set $z_i = Z'/N$, then the remaining incumbent will also have an incentive to invest Z'/N. It is obvious that this firm cannot have an incentive to invest more that Z'/N, because Z'/N deters entry and the entry deterring investment serves no other role. On the other hand, if he invests less than Z'/N, then entry occurs. However, because we are dealing with the case where deterring entry is jointly profit maximizing, this investment level is also dominated by the investment level Z'/N.

The proof for the other case follows similarly.

Proposition 1 tells us that if the demand curve shrinks over time, the free rider problem is not a significant factor. That is, there always exists an equilibrium for which the total investment in entry deterrence equals the collusive investment level.[7] That this is the case should not be surprising given the discussion in the Introduction. i) tells us that when the demand curve is shrinking, the return to investing in entry deterrence is not a smooth function. That is, there is a critical investment in entry deterrence, Z', such that investing less means that entry occurs immediately while investing more means that entry never occurs. In turn, as indicated in the Introduction, in such an environment the free rider problem will typically not play a significant role.

In my previous work (Waldman (1987)) I showed that one way to make the free rider problem an important factor is to introduce uncertainty. This could be done for the present model by assuming that the entry cost equals $\theta S(X)$, where θ is a privately observed draw from a random variable which has a cumulative distribution function $G(.)$: $G(0)=0$, $G(\bar{\theta})=1$, $G'(\theta)>0$ and $G''(\theta)$ exists for $\theta \epsilon (0, \bar{\theta})$. That θ is privately observed means that incumbent firms

base their investments in entry deterrence solely on the distribution function $G(.)$, while the potential entrant bases his entry decision on the actual realization of θ.

The following proposition demonstrates that the introduction of such uncertainty does indeed cause the free rider problem to become an important factor.

<u>Proposition 2</u>: Suppose $A' \leq 0$ for all t, $\lim_{t \to \infty} A(t) > c$, and that the entry cost is given by $\theta S(X)$, where θ has the properties described above. Then,

    i) If entry occurs, it occurs at date zero.

    ii) If $Z^* > 0$, any equilibrium must be characterized by $Z < Z^*$.

<u>Proof</u>: i) follows from the same logic as in the proof of Proposition 1. Let $E(Z)$ be the probability of entry given an investment in entry deterrence of an amount Z. i) allows us to rewrite equation (1) as follows.

$$(5) \qquad Z^* = \arg \max_{Z} N[[1-E(Z)]\int_0^\infty (\Pi_N(t) - \Pi_{N+1}(t))e^{-kt}dt] - Z$$

If $Z^* > 0$, then (5) yields the following first order condition

$$(6) \qquad -N \frac{dE(Z^*)}{dZ} \int_0^\infty (\Pi_N(t) - \Pi_{N+1}(t))e^{-kt}dt - 1 = 0.$$

Let $Z_{-i} = \sum_{j \neq i} z_j$. The maximization problem faced by incumbent firm i is given by (7).

$$(7) \qquad \max_{z_i} [1-E(Z_{-i}+z_i)]\int_0^\infty (\Pi_N(t) - \Pi_{N+1}(t))e^{-kt}dt - z_i$$

Suppose $Z = Z^*$, and let incumbent firm i be such that $z_i > 0$. (7) now yields

$$(8) \qquad -\frac{dE(Z^*)}{dZ} \int_0^\infty (\Pi_N(t) - \Pi_{N+1}(t))e^{-kt}dt - 1 = 0.$$

Comparing (6) and (8) immediately yields $Z \neq Z^*$.

I now need only demonstrate that $Z>Z^*$ also yields a contradiction. Suppose $Z>Z^*$, and consider incumbent firm i where $z_i \geq Z/N$. There are two cases to consider. Case 1 is $Z_{-i} \leq Z^*$. We know it is not profitable for the incumbent firms as a group to increase the investment from Z^* to Z. Further, the return to firm i for increasing the investment from Z^* to Z is strictly less than the return faced by the group, while the cost is the same. Hence, we have a contradiction. The other case is $Z_{-i}>Z^*$. Given $z_i \geq Z/N$, the expected profits for firm i are less than it would get if the group were to collude, i.e., given that all the firms are treated symmetrically in the collusive agreement. However, because $Z_{-i}>Z^*$, firm i could invest zero and do better than it would if the group were to collude, i.e., a contradiction.

As indicated in the Introduction, the reason the presence of uncertainty causes the free rider problem to become an important factor is that it smooths the return to investing in entry deterrence. The main point of this paper, however, is that there are other properties of models which can serve this same role. We now turn our attention in this direction. Proposition 3 demonstrates that, even without introducing uncertainty, underinvestment will arise if the demand curve is assumed to grow over time.

<u>Proposition 3</u>: Suppose $A'>0$ for all t, $A(0)>c$. Then,

   i) There will be a range of values for Z such that $0<\hat{t}(Z)<\infty$.

   ii) If $0<\hat{t}(Z^*)<\infty$, any equilibrium must be characterized by $Z<Z^*$.

Proof: Let $\hat{Z}$ be such that

(9) $$\int_0^\infty \Pi_{N+1}(t)e^{-kt}dt - S(\hat{Z}) = 0.$$

$A' > 0$ implies $\Pi'_{N+1} > 0$, which given (9) implies

(10) $$\Pi_{N+1}(0) < kS(\hat{Z})$$

and

(11) $$\lim_{t \to \infty} \Pi_{N+1}(t) > kS(\hat{Z}).$$

~~We know $\Pi_E = 0$ if $t = \infty$. Hence,~~ Taken together (9), (10) and (4) yield $0 < \hat{t}(\hat{Z}) < \infty$. In fact, $\hat{t}(\hat{Z})$ must be such that

(12) $$kS(\hat{Z}) - \Pi_{N+1}(\hat{t}(\hat{Z})) = 0,$$

where we know such a value exists given (11). Further, extending the argument above, it is easy to demonstrate that there is a neighborhood of values around $\hat{Z}$ such that $0 < \hat{t}(Z) < \infty$. This proves i).

Suppose $0 < \hat{t}(Z^*) < \infty$. Then taking the derivative of the right hand side of (1) with respect to Z yields the following first order condition.

(13) $$N[\frac{d\hat{t}(Z^*)}{dZ}(\Pi_N(\hat{t}(Z^*)) - \Pi_{N+1}(\hat{t}(Z^*)))]e^{-k\hat{t}(Z^*)} - 1 = 0$$

The maximization problem faced by incumbent firm i is given by (14).

(14) $$\max_{z_i} \int_0^{\hat{t}(Z_{-i}+z_i)} \Pi_N(t)e^{-kt}dt + \int_{\hat{t}(Z_{-i}+z_i)}^\infty \Pi_{N+1}(t)e^{-kt}dt - z_i$$

Suppose $Z = Z^*$, and let incumbent firm i be such that $z_i > 0$. (14) now yields

(15) $$[\frac{d\hat{t}(Z^*)}{dZ}(\Pi_N(\hat{t}(Z^*)) - \Pi_{N+1}(\hat{t}(Z^*)))]e^{-k\hat{t}(Z^*)} - 1 = 0.$$

Comparing (13) and (15) immediately yields $Z \neq Z^*$.

I now need only demonstrate that $Z > Z^*$ also yields a contradiction, and that can be shown using the same argument as in the proof of Proposition 2.

As suggested by the previous discussion, underinvestment arises in Proposition 3 because the introduction of a growing demand curve in this model is simply another way of smoothing the return to investing in entry deterrence. The logic is as follows. i) tells us that, given a growing demand curve, entry can occur at a date other than date zero. Even more to the point, equation (12) tells us that, given a range of values for Z for which entry occurs after date zero, a small change in the investment will cause a small change in the date of entry. In other words, because of the manner in which the date of entry depends on the investment, in such a range the return to investing in entry deterrence is a smooth or in this case differentiable function. In turn, if the collusive investment level falls in such a range, then the fact that the return to investing in entry deterrence is a smooth function means there will be an underinvestment in entry deterrence.

Another way of smoothing the return to investing in entry deterrence is to have investments in entry deterrence decrease the market share of the entrant if entry occurs. One way to capture this notion in the present model is to let the marginal cost of production for the potential entrant depend on the aggregate investment in entry deterrence. Proposition 4 demonstrates that when the return to investing in entry deterrence is smoothed in this way, we again find that the model exhibits an underinvestment.

<u>Proposition 4</u>: Suppose $A' \leq 0$ for all t, $\lim_{t \to \infty} A(t) > c$, and that the marginal cost of production for the potential entrant is given by $\hat{c}(Z)$, where $\hat{c}(0) = c$ and $\hat{c}' > 0$. Then,

i) There exists a critical value Z' such that if $Z \geq Z'$ then $\hat{t}(Z) = \infty$, while if $Z < Z'$ then $\hat{t}(Z) = 0$.

ii) If Z_1 and Z_2 are such that $Z_1 < Z_2 < Z'$, the market share of the entrant at any date is smaller for Z_2 than for Z_1.

iii) If $0 < Z^* < Z'$, then $Z < Z^*$.

<u>Proof</u>: i) follows from the same arguments as in the proof of Proposition 1. Consider some date t and let x_i denote the output of incumbent firm i, x_j denote the output of incumbent firm j, $j \neq i$, and $\hat{x}$ denote the output of the potential entrant. If entry occurs, incumbent firm i faces the following maximization problem.

$$(16) \qquad \max_{x_i} (A(t) - (N-1)bx_j - b\hat{x} - bx_i - c)x_i$$

(16) yields the following first order condition.

$$(17) \qquad A(t) - (N-1)bx_j - b\hat{x} - bx_i - c - bx_i = 0$$

Given that in equilibrium $x_i = x_j$, (17) implies

$$(18) \qquad x_i = \frac{A(t) - b\hat{x} - c}{b(N+1)} \ .$$

If entry occurs, the entrant faces the following maximization problem.

$$(19) \qquad \max_{\hat{x}} (A(t) - Nbx_i - b\hat{x} - \hat{c})\hat{x}$$

(19) yields

$$(20) \qquad A(t) - Nbx_i - b\hat{x} - \hat{c} - b\hat{x} = 0,$$

or

(21) $$\hat{x} = \frac{A(t) - Nb x_i - \hat{c}}{2b} .$$

(18) and (21) imply

(22) $$x_i = \frac{A(t) - 2c + \hat{c}}{b(N+2)}$$

and

(23) $$\hat{x} = \frac{A(t) + Nc - (N+1)\hat{c}}{b(N+2)} .$$

Let MS denote the market share of the entrant. MS is given by

(24) $$MS = \frac{A(t) + Nc - (N+1)\hat{c}}{2A(t) + (N-2)c - Nc} .$$

Taking the derivative with respect to $\hat{c}$ yields

(25) $$\frac{dMS}{d\hat{c}} = - \frac{1 + N(1 - MS)}{2A(t) + (N-2)c - Nc} < 0.$$

This proves ii).

If $0 < Z^* < Z'$, then Z^* is defined by

(26) $$Z^* = \arg \max_{Z} N \int_0^{\infty} (A(t) - Nb x_i(t) - b\hat{x}(t) - c) x_i(t) e^{-kt} dt - Z,$$

where $x_i(t)$ and $\hat{x}(t)$ are defined by (22) and (23). (26) yields the following first order condition.

(27) $$N \int_0^{\infty} \left[(A(t) - 2Nb x_i(t) - b\hat{x}(t) - c) \left. \frac{dx_i(t)}{dZ} \right|_{Z=Z^*} - b x_i(t) \left. \frac{d\hat{x}(t)}{dZ} \right|_{Z=Z^*} \right] e^{-kt} dt - 1 = 0$$

If $0 < Z < Z'$, the maximization problem faced by incumbent firm i is given by

(28) $$\max_{z_i} \int_0^{\infty} (A(t) - Nb x_i(t) - b\hat{x}(t) - c) x_i(t) e^{-kt} dt - z_i .$$

Suppose $Z = Z^*$, and let incumbent firm i be such that $z_i > 0$. (28) now yields

(29) $\int_0^\infty [(A(t) - 2Nbx_i(t) - b\hat{x}(t) - c) \frac{dx_i(t)}{dZ} \bigg|_{Z=Z^*} - bx_i(t) \frac{d\hat{x}(t)}{dZ} \bigg|_{Z=Z^*}] e^{-kt} dt - 1 = 0.$

Comparing (27) and (29) immediately yields $Z \neq Z^*$.

I now need only demonstrate that $Z > Z^*$ also yields a contradiction, and that can be shown using the same argument as in the proof of Proposition 2.

Just as in Proposition 1, there is a critical investment in entry deterrence, Z', such that investing less means entry occurs immediately while investing more means that entry never occurs. There is a difference here, however, which is captured in ii) of Proposition 4. That is, even if entry is to occur, the incumbent firms may invest in entry deterrence because it now has the effect of decreasing the market share of the entrant. The result is that the return to investing in entry deterrence is smooth over the interval $(0, Z')$, and hence there will be underinvestment if the collusive investment level falls in this range.

III. A Reconsideration of the McLean/Riordan Results

McLean and Riordan (1987) consider a model in which firms sequentially enter an industry, where each firm has the option of undertaking an entry deterring investment upon entry. They demonstrate that in such an environment the free rider problem will frequently lead to underinvestment, and attribute this result to the fact that investments are made sequentially. In this section I will show that their results are not primarily due to the sequential nature of investing, but rather that what is crucial is that the presence of multiple potential entrants smooths the return to investing in entry deterrence. To make this point, I will construct a sequential entry model closely related to the model of the previous section, where underinvestment

arises for reasons identical to those behind the underinvestment results of McLean and Riordan. I will then demonstrate the following two points. First, underinvestment can only arise when there are multiple potential entrants. Second, whenever there is underinvestment in this sequential investment model, there is also underinvestment in a closely related model where incumbent firms are assumed to move simultaneously.[8]

There are three or four firms, denoted firms 1, 2, 3 and 4, which sequentially decide whether or not to enter an industry, where entry occurs if a firm anticipates strictly positive profits. It is assumed that upon entry each firm can purchase either zero or one unit of entry deterrence. This assumption serves both to simplify the analysis, and make this model more closely related to that of McLean and Riordan. Denote by z_i the investment in entry deterrence of firm i. If firm j decides to enter it incurs an entry cost $S(Z^j)$, where $Z^1=0$ and $Z^j=\sum_{i=1}^{j-1} z_i$ for j=2, 3, 4. After the entry decisions the firms which have entered interact in the market. Let Π_n denote the profits of a firm in the market when there are n total firms in the market, and the firm's entry cost was equal to zero. It is assumed that the interaction in the market is such that $\Pi_n > \Pi_{n+1}$ for n=1, 2, and 3.[9]

Before proceeding to the analysis, it is necessary to describe what is meant by an underinvestment in this section. In particular, following McLean and Riordan, there will be underinvestment among firms 1,...j if, as a group, these firms could have invested more in entry deterrence and yielded each individual firm a higher payoff.[10]

The first step is to demonstrate that underinvestment cannot occur unless there are multiple potential entrants. This is captured in Proposition 5.[11]

<u>Proposition 5</u>: If there are only three potential firms, then an equilibrium never involves underinvestment.

<u>Proof</u>: An underinvestment is only a possibility if: i) the second firm cannot be deterred ($\Pi_2 - S(1) > 0$); ii) the third firm is deterred if and only if each of the first two firms purchases a unit of entry deterrence ($\Pi_3 - S(2) \leq 0 < \Pi_3 - S(1)$); and iii) deterring the third firm makes each of the first two firms better off ($\Pi_2 - S(1) - 1 > \Pi_3$). This is because the free rider problem can only arise if it is necessary for two or more firms to coordinate behavior, and this is the case only when the above three conditions hold. Suppose all three conditions hold and consider firm 1. If the third firm is not deterred, the first firm must get a payoff equal to Π_3. However, the first firm could enter and purchase a unit of entry deterrence. Given $\Pi_2 - S(1) - 1 > \Pi_3$, this would result in firm 2 entering and purchasing a unit of entry deterrence. That is, by purchasing a unit of entry deterrence the first firm gets a payoff equal to $\Pi_2 - 1$ which we know is strictly greater than Π_3. Hence, there is no underinvestment since the unique equilibrium is that the third firm is deterred.

The logic behind Proposition 5 is straightforward. When there is only a single potential entrant, the return to investing in entry deterrence is not a smooth function. That is, in the case where there is a potential for underinvestment (see the proof), there is a critical investment in entry deterrence, $z^3 - 2$, such that investing this amount means the probability of entry is zero, while investing less means the probability of entry is one. Further, as was demonstrated in the last section, when the return to investing in entry deterrence is not a smooth function, then no underinvestment arises.[12]

One question which might be asked at this point is why do I refer to the three firm case as having only a single potential entrant. The reason is suggested by the discussion contained in the proof of Proposition 1. A free rider problem can only occur if more than a single firm needs to invest, and thus that the second firm is a potential entrant is not really relevant. In other words, in terms of finding the free rider problem it is really only the third firm which is a potential entrant.

We can now consider what happens when there are multiple potential entrants. Note, below $\tilde{Z}_n$ is such that $\Pi_n - S(\tilde{Z}_n) = 0$ for $n = 2, 3, 4$.

Proposition 6: If there are four potential firms, then the unique equilibrium involves underinvestment if and only if: i) $\tilde{Z}_4 \leq 1 < \tilde{Z}_3 \leq 2$; ii) $\Pi_4 < \Pi_2 - S(1) - 1 < \Pi_3 - S(1)$; and iii) $\Pi_3 - 1 < \Pi_4$.

Proof: Here it is shown that, given conditions i), ii) and iii), the unique equilibrium involves underinvestment. In the Appendix I show that a unique equilibrium never involves underinvestment if these three conditions are not all satisfied.

Suppose the first firm enters and purchases a unit of entry deterrence. Given $\Pi_2 - S(1) - 1 < \Pi_3 - S(1)$, the second firm will choose to enter but not purchase a unit of entry deterrence — thus allowing the third firm to enter. Hence, the payoff to the first firm would be $\Pi_3 - 1$.

Suppose the first firm enters and does not purchase a unit of entry deterrence. Given $\Pi_3 - 1 < \Pi_4$, the result would be that all four firms would enter since neither the second or third firm would have an incentive to invest in entry deterrence. This has a payoff to the first firm equal to Π_4. Since $\Pi_3 - 1 < \Pi_4$, the unique equilibrium is that all four firms enter. Finally,

given $\Pi_2 - S(1) - 1 > \Pi_4$, this equilibrium involves underinvestment since both the first and second firms would be better off if they each purchased a unit of entry deterrence and deterred the third firm.

Towards the end of their paper McLean and Riordan work through an example which illustrates how underinvestment can arise in their model, and the nature of the underinvestment in Proposition 6 is identical to what occurs in that example (see McLean and Riordan (1987), pp. 27-28). That is, the first two firms would like to deter the third firm, but if the first firm were to invest, the second firm would free ride on that investment and allow the third firm to enter. Realizing this, the first firm decides not to invest and the resultant equilibrium is that all four firms enter.

McLean and Riordan attribute this result to the fact that investments are made sequentially. I will now show this is incorrect. Consider the following variant of the model analyzed in this section. Rather than all firms deciding whether or not to enter in a sequential fashion, let there be two incumbent firms and two potential entrants. The incumbents simultaneously choose investments in entry deterrence, and the potential entrants then sequentially decide whether or not to enter the market (note: it is assumed that upon entry a potential entrant cannot invest in entry deterrence). Proposition 7 demonstrates that whenever there is underinvestment in the sequential game analyzed previously, there is also underinvestment in this game where investments are chosen simultaneously.[13]

Proposition 7: In the simultaneous game described above, the unique equilibrium involves underinvestment if conditions i), ii) and iii) of Proposition 6 hold.

Proof: Given $\tilde{Z}_3 \leq 2$ and $\Pi_2 - 1 > \Pi_4$, an incumbent would prefer an equilibrium where both firms invest and deter both potential entrants over an equilibrium where both potential entrants enter. Suppose incumbent firm i purchases a unit of entry deterrence. Given $\Pi_2 - 1 < \Pi_3$, if the other firm anticipates firm i's investment, it will decide not to invest. That is, it is not an equilibrium for both firms to invest. Suppose incumbent i does not purchase a unit of entry deterrence. Given $\Pi_3 - 1 < \Pi_4$, if the other firm anticipates this behavior it will again decide not to invest. Hence, there is an underinvestment since the unique equilibrium is that both potential entrants enter.[14]

Now that we see the underinvestment is not due to investments being made sequentially, the question which arises is what does lie behind the result.[15] The answer is that the underinvestment is driven by the same factor which caused underinvestment in the previous section. That is, the presence of multiple potential entrants is simply another way of smoothing the return to investing in entry deterrence. The logic is simple. With a single potential entrant there is a critical investment in entry deterrence such that investing this amount or more means that no entry occurs, while investing less means that "full" entry occurs. Now consider what happens when, for example, there are two potential entrants. This opens the possibility of two critical investment levels. The lower level of investment would deter one potential entrant, while the higher level would deter both. That is, the return to investing in entry deterrence is now smoother than in the single potential entrant case and, as demonstrated in Propositions 6 and 7, the result is that underinvestment may occur.

As a final point, one might ask what is the relationship between the results of this section and the recent results of Eaton and Ware (1987). They consider a sequential entry model where underinvestment never arises. Without going into the details of their model, it is still possible to understand the relationship between the papers. In the model considered by Eaton and Ware, the same investment in entry deterrence will deter one potential entrant, two potential entrants, three potential entrants, etc. That is, in contrast to the model considered by McLean and Riordan and the model analyzed in this section, in the model of Eaton and Ware the introduction of multiple potential entrants does not smooth the return to investing in entry deterrence. Further, as is not surprising given the analysis of the present paper, since the return to investing in entry deterrence is not a smooth function in their model, no underinvestment ever arises.[16]

IV. Conclusion

This paper considers environments in which incumbent firms cannot collude on an investment in entry deterrence, and attempts to identify the circumstances under which the free rider problem leads to underinvestment. Consistent with my earlier work (Waldman (1987)), I show that underinvestment can arise if some factor is present which smooths the return to investing in entry deterrence. What is new here is that the return to investing in entry deterrence can be a smooth function for reasons other than that uncertainty is present. In particular, in the first model considered I show that, in addition to the presence of uncertainty, underinvestment can arise if entry deterrence either delays the date at which entry occurs, or decreases the market share of the entrant upon entry.

At the end of the paper I then consider the relationship between the results described above and the recent free rider results of McLean and Riordan (1987). They find that underinvestment can occur in a sequential entry model, where each firm has the option of undertaking an entry deterring investment upon entry. Their explanation is that the result is due to investments being made sequentially. I demonstrate that this is incorrect. Rather than being due to sequential decision making, an underinvestment arises in their model because of the presence of multiple potential entrants. That is, the presence of multiple potential entrants is simply another way of smoothing the return to investing in entry deterrence and, as we already know, underinvestment can arise as long as the return to investing in entry deterrence is a smooth function.

Finally, although it might be somewhat obvious, I would like to end by pointing out that the basic message of this paper is really a message about public goods rather than one about entry deterrence. That is, nothing in the analysis depends crucially on the fact that the public good took the form of an investment in entry deterrence. Hence, the basic message is really that for any type of public good, if agents do not cooperate on voluntary contributions that are being made for the provision of that public good, there will be underprovision as long as the return to contributing is a smooth function.

Appendix

<u>Proof of Proposition 6 continued</u>: Here it is shown that a unique equilibrium never involves underinvestment if conditions i), ii) and iii) are not all satisfied. To show this I will consider each case separately. Also, due to space considerations, proofs are somewhat abbreviated.

Case 1: i) and ii) hold, but $\Pi_3 - 1 \geq \Pi_4$.

As in the case in the text, the first two firms are better off deterring the third than having all four firms enter. However, if the first two firms invest zero, it is now an equilibrium for the third firm to deter the fourth.

Case 2: i) holds, but $\Pi_2 - S(1) - 1 \geq \Pi_3 - S(1)$.

The first two firms are better off deterring the third firm than having the third firm enter. There is no underinvestment, however, because if firm 1 purchases a unit of entry deterrence, firm 2 now has an incentive to also purchase a unit and deter the third firm.

Case 3: i) and iii) hold, $\Pi_2 - S(1) - 1 < \Pi_3 - S(1)$, but $\Pi_2 - S(1) - 1 \leq \Pi_4$.

As in the case in the text, the equilibrium here is that all four firms enter. This is not an underinvestment, however, because firm 2 is now at least as well off in the four firm equilibrium as it would be if the third firm were deterred.

Case 4: i) holds, $\Pi_2 - S(1) - 1 < \Pi_3 - S(1)$, but $\Pi_2 - S(1) - 1 \leq \Pi_4$ and $\Pi_3 - 1 \geq \Pi_4$.

An equilibrium here is that the third firm deters the fourth. This is not an underinvestment, however, since the second firm is strictly better off with this than it would be if the third firm were deterred.

Case 5: $\bar{Z}_3 > 2$.

An underinvestment here would mean that there is an equilibrium where the fourth firm enters, but by investing more the first three firms could deter entry and make each of the first three firms better off, i.e., $\Pi_3 - S(2) - 1 > \Pi_4$ and $\bar{Z}_3 \leq 3$. Suppose both these conditions hold. If the first two firms each purchase a unit of entry deterrence, the third firm will have an incentive to purchase a unit also. If the first firm purchases a unit, the second firm will realize the previous statement is true and will also purchase a unit. Realizing this, the first firm will decide to purchase a unit. That is, there is no underinvestment since the unique equilibrium is that each of the first three firms purchases a unit of entry deterrence and in this way deter entry.

Case 6: $1 < \bar{Z}_4 < \bar{Z}_3 \leq 2$.

There are three possible ways this could lead to an underinvestment.

Sub-case 1: The equilibrium is that the second and third firms invest and deter the fourth, and firms 1 and 2 could be made better off by deterring the third, i.e., $\Pi_3 - S(1) - 1 \geq \Pi_4 - S(1)$ and $\Pi_2 - 1 > \Pi_3$. Suppose the first firm were to invest. Given $\Pi_2 - 1 > \Pi_3$, the second firm would have an incentive to invest and deter the third firm. Realizing this, the first firm will decide to invest. That is, there is no underinvestment since the unique equilibrium is that each of the first two firms purchases a unit of entry deterrence and deter the third firm.

Sub-case 2: The equilibrium is that all four firms enter, and firms 2 and 3 could be made better off by investing and deterring the fourth firm, i.e., $\Pi_3 - S(1) - 1 > \Pi_4$. Suppose firm 1 decided not to invest, and firm 2 decides to invest. Given $\Pi_3 - S(1) - 1 > \Pi_4$, the third firm would have an incentive to invest and deter the fourth firm. Realizing this, if the first firm does not invest

then the second firm will. Hence, it is not an equilibrium for all four firms to enter.

Sub-case 3: The equilibrium is that all four firms enter, firms 2 and 3 could not be made better off by investing and deterring the fourth firm, but firms 1 and 2 could be made better off by investing and deterring the third firm, i.e., $\Pi_2-S(1)-1>\Pi_4$ and $\Pi_3-S(1)-1\leq\Pi_4$. If exactly one of the first two firms invests, the third firm might not invest. Suppose each of the first two firms anticipates the third firm will behave in this way. Also, suppose the first firm invests. The second firm will then invest and deter the third firm. Realizing this, the first firm will decide to invest. That is, it is an equilibrium for the first two firms to invest and deter the third.

Case 7: $\tilde{Z}_4<\tilde{Z}_3\leq1$.

This can never lead to underinvestment because the free rider problem can only arise if it is necessary for two or more firms to coordinate behavior, and this is never the case when the above condition holds.

Footnotes

[1]See, for example, Spence (1977), Salop (1979), Dixit (1980), Grossman (1981), Spulber (1981), Kreps and Wilson (1982), Milgrom and Roberts (1982), Fudenberg and Tirole (1983), Ware (1984), and Bulow et al (1985). See also Gilbert (1986) for a survey of the literature.

[2]See, for example, Prescott and Visscher (1977), Bernheim (1984), Gilbert and Vives (1986), Eaton and Ware (1987), Harrington (1987), McLean and Riordan (1987), and Waldman (1987).

[3]In the Gilbert and Vives model the entry deterring investment is simply the aggregate output chosen by the noncooperating incumbent firms. Hence, overinvestment in the absence of a potential entrant simply means that the Cournot output exceeds the collusive output.

[4]Harrington (1987) also considers a model in which an underinvestment result arises. However, it is of a very weak form in that whenever there is an underinvestment equilibrium, there is also an equilibrium where the incumbent firms invest optimally. This is similar to the original Bernheim analysis where it was suggested that the underinvestment equilibrium is not very plausible (see also Bernheim and Whinston (1987) and Proposition 1 of the current paper).

[5]The assumption of a linear demand curve is not crucial, but serves to simplify the analysis.

[6]One real world interpretation of the above model is that it is a model of lobbying activity. That is, at date zero the legislature is considering a bill which will affect the cost of entering this industry, where such a bill will not be considered at any later date. Further, the N incumbents at date zero have the opportunity to lobby the legislators, where by doing so they can

raise the cost of entry and in turn possibly deter it from occurring.

[7]See Bernheim and Whinston (1987) for a discussion of why this equilibrium is more plausible than any underinvestment equilibrium that might also exist.

[8]I do not use the exact model employed by McLean and Riordan because in their model the importance of having multiple potential entrants is somewhat obscured (see footnote 12). Note, the model I do look at is quite closely related to the one considered by Bernheim (1984). There are two main differences. First, in my model investments in entry deterrence are of a discrete nature. Second, in Bernheim only the entry decision is of a sequential nature, while here both the entry decision and the investment decision is sequential.

[9]A specification which ensures this condition is that the demand curve is linear or concave, cost conditions are as in the previous section, and, after the entry decisions have all been made, the firms in the market play a Cournot game.

[10]A less restrictive definition would be that there is underinvestment among firms $1,...j$ if, as a group, these firms could have invested more in entry deterrence and increased the total profits of the group. With this definition, underinvestment could actually be the unique equilibrium even with only a single potential entrant. Note, however, if we were to move away from the assumption that investments are discrete, then, even with this less restrictive definition, for underinvestment to be the unique equilibrium there would have to be multiple potential entrants.

[11]Following McLean and Riordan, attention is restricted to subgame perfect equilibria.

[12] In their model, McLean and Riordan find that underinvestment can arise even with only three firms. As they state, however, the type of underinvestment which arises in the three firm case has unappealing properties, and "more plausible kinds of underinvestment in entry deterrence emerge with more than three potential entrants." (McLean and Riordan (1987), p.12)

[13] Assuming that potential entrants have the opportunity to invest in entry deterrence upon entry would have no effect on the statement of Proposition 7. This was not assumed because my goal is to show that investments being made sequentially is not an important element of the McLean/Riordan analysis. Additionally, it is also not important that potential entrants sequentially decide whether or not to enter the market. In particular, the statement of Proposition 7 would remain unchanged if it was instead assumed that potential entrants simultaneously decide whether or not to enter, but are restricted to pure strategies.

[14] The proof in the text assumes that firms are restricted to pure strategies, however, it is easy to extend the proof to the case where firms have the option of employing mixed strategies.

[15] With one additional assumption it would be possible to guarantee that for the sequential game analyzed there is always a unique equilibrium (the assumption required is one analogous to Assumption 2 of McLean and Riordan's analysis). In turn, it would then be possible to show that for the sequential game an underinvestment equilibrium could only arise if conditions i), ii), and iii) all hold, while for the simultaneous game analyzed an underinvestment equilibrium could arise even if i), ii), and iii) did not all hold (although it would not be unique). Hence, one might argue that allowing investments to

be made sequentially actually reduces the probability of observing the free rider problem. Note, finally, additional evidence in this direction is that, in contrast to Proposition 5, for the simultaneous game underinvestment equilibria can exist even when there are only three potential firms (although such an equilibrium would not be unique).

[16]In the Eaton and Ware analysis, each firm upon entry has the opportunity to invest in sunk capacity. Further, the model is such that, as long as all the sunk capacity will be used upon entry, there is a constant critical investment in sunk capacity which will always deter the "next" firm. This is the sense in which the same investment in entry deterrence will deter one potential entrant, two potential entrants, three potential entrants, etc. What happens in their model is that entry is deterred as soon as there are enough firms in the market such that the incumbent firms can credibly commit to using this critical amount of capacity if further entry were to occur.

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