ALTERNATIVE TESTS OF INTERNATIONAL ASSET SUBSTITUTABILITY

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Abstract

In this paper we utilize recent advances in principal components analysis to examine the question of international asset substitutability for a small open economy. Specifically, we are interested in whether a small open economy can have independent interest rates all along the term structure. Based on Canadian and U.S. government bond yield data, we find strong evidence of high substitutability between the two countries' bonds of all maturities.

1. Introduction

In this paper we utilize recent advances in principal components analysis (PCA) to examine the question of international asset substitutability for a small open economy. Specifically, we are interested in whether a small open economy can have independent interest rates all along the term structure. We test our approach using government bond yields from the U.S. and Canada for the period 1972-1984.

The degree of international asset substitutability is important for small open economies because it determines the extent to which the country can have an independent interest rate policy. The simple Keynesian model of the small open economy with fixed exchange rates predicts that the potential for an independent interest rate diminishes as domestic and foreign bonds become more substitutable. Edwards (1985) noted that the absence of good measures of intermediate degrees of openness means that many empirical studies simply assume an economy is completely open or completely closed. Our analysis produces a simple overall measure of the degree of asset substitutability.

There are various approaches to testing international asset substitutability, and they usually focus on a single, 'representative' asset from each country. One approach is to extend the portfolio model developed by Brainard and Tobin (1968) to examine questions of substitutability between domestic and foreign (or any alternative) assets (for example, Boothe, 1987). Another is to test interest rate parity (for example, Frenkel and Levich, 1975, 1981), and a third is to measure the effect of foreign variables on the determination of the domestic interest rate (Edwards, 1985, and Edwards and Khan, 1985).

However, most countries have assets of numerous maturities, and a single yield will not be representative of other points on the yield curve unless all assets along the yield curve are perfectly substitutable. Studies supporting the preferred habitat or market segmentation theories of the term structure (for example Modigliani and Sutch, 1967, 1968) indicate imperfect substitutability between assets of different maturities. In the past, authorities have sometimes acted on this assumption, for example in the so-called 'Operation Twist' of the 1960s. We think a useful measure of the degree of international asset substitutability must incorporate information from a number of different points along the yield curve.[1] Rather than simply repeat tests on assets of every maturity, we seek to compactly summarize each country's yield curve by reducing the information in a set of bond yields of different maturities to a smaller set of latent factors using PCA. We then apply a new technique that allows us to comparing sets of principal components across countries. Although we are focussing on the question of international substitutability, our approach also provides some evidence on the degree of domestic substitutability and hence of the term structure.[2]

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[1] Beenstock and Longbottom (1981) showed that the world term premium (ten year bond yield minus three month yield) was a significant determinant of the U.K. yield term premium for the period 1969-1978.

[2] A related issue arises in regression tests of Brainard-Tobin asset demand functions. The higher the degree of substitutability between assets, the more collinear are their yields, and hence the less precise the estimates of cross-price elasticities. In the case of assets of different maturity, clearly the collinearity problem would be severe if we were to include yields from every available maturity. An ideal solution would be to find groupings of assets such that within-group substitutability is high and between-group substitutability is low. Alternatively, the latent factors in a set of assets of different maturities contain all the information from the original data, but are uncorrelated.

PCA is one of a family of latent-variable techniques which also includes factor analysis. Several previous studies have applied latent-variable techniques to the domestic term structure and international asset substitutability questions. A well-known problem with the analysis of latent variables is that it is difficult to give economic interpretations to the factors found because they generally do not correspond to specific economic variables. Thus, researchers' interpretations of the latent factors may differ.

Hester (1969) applied PCA to a group of eleven yields on government and corporate assets with maturities from three month to over ten years and found that almost all of the variation could be explained by just two factors, which he identified as the market rate and term structure. Logue et al.(1976) using factor analysis, and found that most of the variation in seven countries' medium- to long-term government bond yields over the 1958-73 period was due to a single "international" factor, which was evidence of a high degree of integration across the countries.[3] Logue and Sweeney (1984) reported that there were two principal components in a set of Eurocurrencies certificate of deposit rates for four maturities from one to twelve months. They analyzed deposits in six currencies (U.S. dollar, French franc, German mark, British pound, Swiss franc, and Japanese yen) from 1977 to 1982 and interpreted the first component as the 'general market factor' and the second as reflecting term premia. Further analysis provided evidence of imperfect international market integration. Simple time series regressions of each country's first

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[3] They interpreted this factor as representing "an optimally diversified international portfolio."

component on the first U.S. component indicated only a weak relationship. Similar regressions for the second components indicated no relation. Furthermore, when separate PCA were done on the set of six countries' rates at each maturity, four or five components were required to explain most of the total variation.[4]

Geweke and Singleton (1981) developed a time series version of factor analysis, called dynamic factor analysis. Singleton (1980a, 1980b) applied the technique to the U.S. term structure and concluded that two factors accounted for most of the variation in a group of five interest rates of different maturities. His results indicated that the factors were common to all maturities, which was evidence against the hypothesis of market segmentation. Geweke and Singleton suggested that possible interpretations for the two factors were real and monetary influences or anticipated and unanticipated disturbances.[5]

The plan for the remainder of our paper is as follows. We begin with a discussion of the interpretation of PCA applied to term structure data. In Section 3 we present some stylized facts describing U.S. and Canadian bond yields from 1972 to 1984. Our PCA results based on U.S. and Canadian data are presented in Section 4. The paper concludes with a discussion of the results and a brief summary.

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[4] This result is consistent with, but does not necessarily imply, the existence of country-specific influences, ie. international market segmentation. If the components were common to all currencies, then there would be a high degree of integration among national capital markets.

[5] Using a different method of analysis, multiple cointegration, Stock and Watson (1987) also found two common factors in a group of four U.S. interest rates of different maturity.

2. Interpreting PCA Results for the Term Structure

In this section we establish some guidelines for interpreting the results from applying PCA to term structure data. PCA is a data-reduction technique and is discussed further in Appendix A.[6] The basic idea is to reduce a set of p input variables to $k < p$ linear combinations of the variables that account for most of the variation in the input group. The linear combinations correspond to a rotation of the original axes in p-space. The choice of a $k < p$ corresponds to a reduction in the number of dimensions.

An easy way to visualize this process is to think of points scattered within a room, a three-dimensional space with axes running along the floor and up a wall. PCA tips and rotates this room until the axes are as close as possible to the scatter of points. The room's shape is maintained (i.e. the axes remain orthogonal); only its orientation in three-space changes. To illustrate the problem of deciding the degree of dimension reduction (i.e. the value of k), suppose the points in the room lie on a diagonal line from one upper corner to the opposite lower corner. The rotation of the room will result in one axis coinciding with the diagonal. The original three-dimensional scatter can then be seen to have one underlying dimension.

In practice we would find exactly one component (i.e. all but one eigenvalue equal to zero) only in the case of perfect collinearity. More commonly, all eigenvalues are non-zero but some are very small, indicating a subset of components explain most of the variation in input

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[6] PCA is also described in most texts on multivariate statistical analysis. See, for example, Johnson and Wichern (1982) and Kendall et al. (1983).

variables. There is no universally agreed-upon criterion for determining how many components to keep (i.e. the value of k) when no eigenvalues are exactly zero. An ad hoc approach is to choose an arbitrary amount of the total variation to be accounted for and to keep as many components as are necessary to reach that total. In practice, criteria such as 80 or 90 percent are used. An alternative is to find a break in the sizes of the eigenvalues (and hence in the contributions of each component to cumulative variance) after which they become uniformly small. Only components up to the break are retained. This is known as a scree test.

A less-arbitrary alternative, proposed by Eastment and Krzanowski (1982), is based on cross-validatory analysis. The goal of cross-validation is to find the smallest number of components that provide a good prediction of the original data. Therefore, components are added successively until there is an insignificant gain in predictive power, as measured by what Eastment and Krzanowksi call the W statistic. They advise that the kth component should be added only if the W statistic is greater than 1. The details of this procedure are given in Appendix B.

We now consider the application of PCA to term structure data. Suppose that the underlying model which generates bond yields of different maturities nests the expectations and market-segmentation theories within it. Let R(1,T) be the yield on a T-period bond at time 1. Denote the one-period yield on a bond from time t to t+1 by R(t). R(1) is known at time 1, but future yields are represented by their expected values, E(R(t)). The yield on a T-period bond is given by the geometric mean of the current and expected future yields and a maturity-specific (market-segmentation) factor (S):[7]

$$R(1,T) = \{1+R(1)(1+E(R(2))\ldots(1+E(R(T))\}^{(1/T)}-1.0 + S(T) \quad (1)$$

If all yields change proportionally, as in an expectations model of the term structure, then one principal component would account for all the variation in the group of yields.[8] At the other extreme, if markets for all maturities are segmented, then PCA would find as many components as there are maturities. In intermediate cases, there may be components common to subsets of maturities, so that there are groups of markets segmented from each other. PCA does not distinguish among components that are common to all input variables, those that are common to subsets of two or more variables, and those that are specific to single variables, although the correlations between variables and components provide descriptive evidence. Factor analysis distinguishes variable-specific factors from common ones, but does not discriminate between the common-to-all and common-to-a-subset cases (again, factor loadings provide descriptive evidence). Singleton (1980a, 1980b) found two common factors in the U.S. term structure using dynamic factor analysis. Although he found no evidence of maturity-specific market segmentation, his results

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[7] For simplicity, we can think of the maturity-specific disturbance as a random shock. If markets for different maturities are segmented, then each maturity class has its own independent shock. If there are subgroups of maturities with high within-group substitutability, then we can think of each group as sharing a common shock.

[8] This is not the same as saying that one economic variable drives all the yields. There may well be many economic variables that go into the determination of expectations.

do not rule out segmentation between subgroups of maturities.[9][10]

The idea of maturity-specific shocks can be easily expanded to deal with domestic and foreign bonds. If the bonds of the small open economy are perfectly substitutable for foreign bonds, then the open economy's bond yields will be explained by the same factors that explain yields in the other country.[11] If we were to run PCA on the set of bond yields from both countries together, then there would be collinearity between the two halves of the data and k would be the same as in the single country analysis. If the two countries had completely independent term structures (ie. zero asset substitutability) then PCA on the both countries' bonds would yield a number of components equal to the sum of both countries' components when analyzed separately. If a subset of components are common across countries (ie. less-than-perfect substitutability) then running PCA on both sets of bond yields jointly should result in fewer components than the sum of the components found in the individual country analyses.

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[9] Note that this assumes that if the bonds along the term structure are correctly aggregated with respect to maturity.

[10] Stock and Watson (1987) also found two common factors, although their technique did not capture specific ones. Hester (1969) and Logue and Sweeney (1984) each found two principal components, but did not identify them as common or specific.

[11] Perfect substitutability does not imply that interest rates are necessarily equal across countries. Rates may differ due to expected exchange rate depreciation or taxes or transaction costs. If the term structure of the forward premium were not flat, then expected exchange rate changes could constitute an additional component in the two-country analysis.

3. Data and Some Stylized Facts

Since the degree of substitutability between bonds along the yield curve is an important issue in this paper, it is not appropriate for us to analyze the yield data usually examined in empirical bond market studies because it is pre-aggregated into an arbitrary set of maturity classes. These arbitrary maturity classes are unlikely to satisfy the condition that there be a high degree of substitutability between bonds within the class and a lower degree of substitutability across classes. Therefore, we employ alternative data sets for the U.S. and Canada, on which we impose the least possible pre-aggregation. For the U.S. we started with monthly data on outstanding Treasury bonds from the Centre for Research on Securities Prices (CRSP).[12] Bonds with special features are excluded.

The data were aggregated in the following way. First we sorted all bonds into 31 one-year maturity classes, taking the unweighted average yield for each class. Since there was not a bond of every (yearly) maturity outstanding in every month, we grouped together the longest bonds with the next longest bonds and continued adding shorter maturities until we had a grouping which had at least one observation in every month. We then began aggregating the next lower class and so on. The result was eight maturity classes: 1, 2 ,3, 4 ,5, 6-9, 10-13 and 14-31 years. The raw Canadian data were exactly comparable to the CRSP data[13] and we used the same maturity groupings (again excluding bonds with

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[12] The U.S. data are described in the 1984 CRSP documentation.

[13] See Boothe and Reid (1986) for a description of the Canadian data.

special features). We confine our analysis to the sample period 1972:1-1984:12. The starting date avoids both the move to flexible exchange rates between the U.S. and Canada and large-scale changes to capital gains taxation in Canada. The end is dictated by the availability of Canadian data.

PCA is based on the assumptions that the sample observations are independent drawings from a distribution with constant mean and variance. We therefore first determine whether these conditions are satisfied in the bond yield data. It is well known that the variance of U.S. interest rates rose substantially as a result of the October 1979 introduction of monetary targeting by the Federal Reserve (see Spindt and Tarhan, 1987). The variance decreased when the Fed reduced emphasis on monetary targets in October 1982. This suggests three subperiods in the data. The top panel of Table 1 reports the variances for the first differences of U.S. yields in these three periods and the results of Bartlett tests for equality of variances.[14] The 1979-82 subperiod had significantly larger variances than either the first or the third period. The third period variances were not consistently equal to those in the first period indicating three separate subperiods. The bottom panel of Table 1 give the variances and Bartlett tests for the Canadian data. The patterns of variances and test statistics are the same as for the U.S. data. Therefore we will analyze the three subperiods separately in what follows.

The time series behavior of monthly bond yields is very close to a random walk. In Table 2 we report the results of Dickey-Fuller (1981)

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[14] Extensive testing did not reveal any other structural breaks in the sample. The qualitative conclusions from Table 1 apply to the levels data as well.

tests for non-stationarity (unit root tests). Based on these tests, all series in the first period were non-stationary in level and stationary in first differences. Although there are substantially fewer than the 100 observations used to compute the tables of Fuller (1976), the same conclusion appears to hold for the second period. In the third period, even first differences seem to be non-stationary, but this is based on only 26 observations and may simply reflect the movement to a new, lower equilibrium level for interest rates over this short period. We deal with the high degree of autocorrelation in the monthly yields by differencing the data.[15]

The yield curves for the two countries varied substantially over the three subperiods as indicated by the descriptive statistics for the two countries presented in Table 3 and the yield curves plotted in Figures 1 to 3. In the first subperiod (1972:1-1979:9), both yield curves displayed an upward slope on average, with long yields being substantially less volatile than short yields. Both curves shifted upward and became flatter over the period. The Canadian yield curve was higher and steeper than its U.S. counterpart (50 basis points at the short end and 80 points at the long end).

The second subperiod (1979:10-1982:9) was a period of substantially increased interest rate volatility for both countries, especially at the

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[15] As Logue et al. (1976) noted, it is intuitively appealing to examine differenced data rather than levels because international asset substitutability implies that interest rates move together rather than that they are equalized. When PCA (or factor analysis) is run on the levels of a highly-autocorrelated series, the first component (factor) tends to account for a larger percentage of total variation than when the data are differenced. This result is illustrated in Logue et al. and reflects the dominant trend component in the levels data.

short end. On average, both yield curves became 'inverted' in this period. The Canadian curve was again higher and steeper than its U.S. counterpart; this time by an even greater margin (120 points at the short end and 90 basis points at the long end).

In the final subperiod (1982:10-1984:12) yield curves for the two countries became less volatile, shifting downward and reverting to their normal upward slope. Both curves were substantially steeper than in the first subperiod, with the U.S. curve steeper than the Canadian one. While the Canadian curve was higher at the short end (by approximately 70 points), the U.S. curve was higher at the long end (by approximately 20 points).

4. Testing and Results

In this section we present the results of PCA applied to term structure data for the U.S. and Canada. We begin by considering the bonds of each country separately, and then apply PCA to the bonds of both countries jointly. Next, we compare the Canadian and U.S. components directly using the technique of Krzanowski (1979). As described in the previous section, the data were differenced and the three subperiods were examined separately.

Table 4 presents the eigenvalues of the covariance matrix for U.S. yields to maturity in the eight maturity classes, the cumulative amount of variation explained by each component, the correlation coefficients between each variable and each component and the Eastment-Krzanowski W statistics.[16] Recall that the signs of eigenvectors are arbitrary; any row can be multiplied by -1 without changing the results.

The 90 percent cumulative variance criterion would suggest two components in the first and third periods and one in the second. The W statistics tell a somewhat different story, identifying two components in the first period and three in each of the other periods. The reason the W statistic tends to indicate more components than the 90 percent criterion is that any component which is relatively highly correlated with an original variable can improve overall predictive power. Some of the components beyond the 90 percent cumulative variance level have such correlations. For instance, in the second period the correlations between the third component and maturities 7 and 8 account for the W statistic indicating three components.

The following patterns of correlation coefficients appear in all three subperiods. The first component has by far the largest correlations with all maturities. The shortest bonds (one year and less) load less heavily on the first component and more heavily on the second. This difference between the shortest bonds and other maturities is not surprising if we assume that short bonds are the primary tool of monetary policy, so that the short yield is exogenously determined. The large influence of the first component on all maturities is consistent with the expectations theory prediction that a change in the short-term interest rate (which alters expected future interest rates) will affect all yields to some

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[16] For reasons of computational accuracy, the cross-products matrix was used rather than the covariance matrix. The eigenvectors of the two matrices are the same, so the principal components derived from them are too. To get the eigenvalues of the covariance matrix from those reported in the tables, simply divide by n-1.

extent (as in equation (1)).[17]

The results from PCA on Canadian bonds in the same eight maturity classes are presented in Table 5. The W statistics suggest slightly different numbers of components than for the U.S.: three in the first period, two or four in the second[18] and two in the third. The Canadian results show the same general pattern of correlations as the U.S.: (1) high correlations between the first component and all yields, although not quite as high for the shortest maturity and (2) the dominant correlation for the second component is with this shortest maturity.

We next applied PCA to both countries together, i.e. sixteen bonds in all.[19] The results for the first six components are presented in Table 6. (The eigenvalues and correlation coefficients for the remaining ten are negligible.) In the first period, three components are required to explain at least 90 percent of the variance, while the W statistic indicates four components. In the second period, only two components are required to reach 90 percent, although the eigenvalues do not become

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[17] The patterns in Table 2 suggest an alternative to the standard aggregation of bonds into 1-5, 5-10 and over 10 year classes. We pursue the question of optimal aggregation of bonds of different maturities in a separate paper.

[18] The W statistic for the fourth component in the second subperiod rises above 1.0 because of the component's correlation with yields in the 10-13 year maturity class. This aberration can be traced to the period of high interest rates in 1981 when the low-coupon bonds in this infrequently-traded maturity class were priced at a substantial discount. New, near-par issues at 10 and 20 years behaved differently because of the differences in tax treatment for capital gains and interest income. Since these special circumstances were short-lived, we treat the second subperiod as having just two components.

[19] The correlation between the fourth Canadian component and the 10-13 year maturity class again contributes to a W statistic above 1.0, this time at the seventh joint component. We continue to assume that it can be ignored.

really small until after the fourth component and the W statistic suggests four components. The cumulative variance reaches 90 percent with two components in the third period, but the eigenvalues drop off after four and the W statistic also identifies four.[20]

The fact that the total number of components retained in the joint analyses is less than the sum of those retained in the individual country analysis indicates that some of the components are common across the countries. The correlation coefficient patterns support this idea. The first component's pattern of correlation coefficients is the same as that for the first component for each country alone, with the shortest maturity tending to have the lowest correlation. The second component is, with one exception, positively correlated with Canadian yields and negatively correlated with U.S. yields. This seems to suggest a country-specific factor. The majority of the remaining retained components concentrate on the shortest maturities, which suggests that they reflect monetary policy, as in the individual country analyses.

From the number of retained components and their correlation patterns we conclude that U.S. and Canadian bonds are highly substitutable. The heavy loadings on the first component from all bonds in both countries show that, to a large extent, yields in both countries move together. However, the finding that the number of joint components is greater than for one country alone and the second joint component's country-specific correlation pattern are indications that substitutability is less than perfect. We further explore the nature of the substitutability by

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[20] The consistency between the scree and W statistic criteria lead us to have more confidence in them than in the arbitrary 90 percent variance benchmark.

directly comparing the two countries' components.

From inspection of Tables 4 and 5, we see that the principal components for the U.S. and Canada are similar, although not identical. Krzanowski (1979) noted that, "visual inspection is not very trustworthy, as two sets of components that are quite different in appearance may in fact define the same subspace of the original multivariate space." (p. 709) Krzanowski showed how principal components from two groups (e.g. two countries) can be compared by calculating the angles between the subspaces generated by each set of principal components and constructing the bisecting vectors. A pair of identical vectors would have an angle of zero degrees, while a pair of unrelated components would have an angle of 90 degrees. This technique is described in Appendix C.

The sum of the eigenvalues of the cross-product matrix between pairs of principal components is a measure of the overall similarity of U.S. and Canadian components. The individual angles and the weights of the input vectors in the bisecting vectors are indications of the nature of the similarity. We applied Krzanowski's technique for comparing principal components across groups to the first three U.S. and Canadian components from Tables 4 and 5. (The number of components to be compared was determined largely from the W statistics.) The comparison results are presented in Table 7.

Looking first at the eigenvalues and corresponding angles, we find that in each period the first two components are extremely close to each other. Based on an unreported Monte Carlo study, Krzanowski interpreted angles of less than 34 degrees as being small. In spite of the lack of better criteria for determining smallness, it is clear that vectors differing by less than 6 degrees can be considered extremely similar. The first component, which is virtually identical in the two countries, becomes even more similar over time (the angle decreases from 3.05 to 0.67 degrees). The third component is clearly less similar across countries, although it is far from orthogonal. The measures of overall similarity are all over 2 (where 3 is the value for identical components). The highest value is in the second period (when all three angles between component pairs are relatively small). We interpret this measure as telling us that the three pairs of components are similar in at least two dimensions.

In order to determine the nature of the similarity between the U.S. and Canadian components, we examine the sets of vectors closest to the pairs of subspaces (the bisectors). Note that each entry in Table 7 gives the contribution of the given maturity class to the bisector and is analogous to an eigenvector element (i.e. it is not scaled, as the correlation coefficients in the previous tables are). The vector closest to the first components from the two countries appears to put equal weight on yields from all maturities, with slightly more emphasis on shorter maturities. The vector closest to the second components has most of its weight on the bonds with maturity up to one year, suggesting that monetary policy accounts for this dimension of similarity. In the second subperiod the patterns of the first and second vectors are more extreme, with the first maturity class having substantially lower weight in the first vector and substantially higher weight in the second vector. In all three periods the longest bonds (10 years and up) have relatively heavy weight in the vector closest to the third components. Looking back at the joint PCA results in Table 6, we see that the long maturities have some relatively large loadings on the third and fourth components, suggesting that the differences in the countries' issuance of long bonds may account for some of the imperfect substitutability the results imply.

5. Conclusions

In this paper we have demonstrated in two ways that there is a very high degree of substitutability between U.S. and Canadian government bonds across the entire term structures for the two countries. First, while there are up to three principal components underlying each country's domestic term structure, there are no more than four components when U.S. and Canadian bonds are analyzed together. This shows that some of the components are common across the countries. Second, when we compared the first three U.S. components to the first three Canadian components, we found an extremely strong similarity in two of the dimensions - the principal component vectors were less than six degrees apart. Differences in the third dimension can be attributed to differences in the issuance of long bonds. We conclude that Canada, as a small open economy, had very little scope for an independent term structure in the 1972-1984 period.

Finally, we note the applicability of the techniques in this paper to other research in finance. The method of comparing components across groups has potential use in issues of domestic as well as international asset substitutability. The cross-validation approach to determining the number of components will be helpful in other latent-variable applications, such as the first step in testing APT models.

Table 1

Sample Variances of Yield First Differences

U.S. data	Maturity Class: 1	2	3	4	5	6	7	8
1972:1 - 1979:9 (n=92)	0.38	0.18	0.12	0.09	0.07	0.06	0.04	0.04
1979:10 - 1982:9 (n=35)	3.26	1.99	1.53	1.27	0.99	0.85	0.59	0.63
1982:10 - 1984:12 (n=26)	0.34	0.31	0.28	0.26	0.27	0.26	0.24	0.21
Bartlett tests for equal variance								
All periods equal	78.7	88.4	95.7	100.1	96.4	97.0	107.1	105.8
First-Second	66.8	83.4	92.6	98.3	96.3	97.1	106.6	105.9
Second-Third	28.2	20.0	17.3	14.9	10.6	9.2	5.3	7.5
First-Third	0.1	3.4	8.0	13.1	20.2	24.1	42.5	33.9

Canadian data	Maturity Class: 1	2	3	4	5	6	7	8
1972:1 - 1979:9 (n=92)	0.15	0.14	0.13	0.11	0.10	0.09	0.07	0.04
1979:10 - 1982:9 (n=35)	1.97	1.53	1.40	1.38	1.10	0.93	0.76	0.78
1982:10 - 1984:12 (n=26)	0.39	0.26	0.24	0.21	0.21	0.18	0.16	0.14
Bartlett tests for equal variance								
All periods equal	96.2	86.4	83.5	95.8	86.0	78.1	86.0	120.0
First-Second	93.9	82.6	79.5	90.9	83.2	75.3	83.9	117.7
Second-Third	15.9	18.1	18.2	20.3	16.3	15.9	15.0	17.0
First-Third	10.1	4.7	3.9	4.7	6.5	4.9	8.1	17.1

Bartlett test statistics for all periods equal are distributed $\chi^2(2)$.
Statistics for two period comparisons are $\chi^2(1)$.

Critical values:

	5%	1%
$\chi^2(2)$	5.99	9.21
$\chi^2(1)$	3.84	6.63

Table 2

Dickey-Fuller Unit Root Tests

Maturity	U.S. levels	U.S. differences	Canada levels	Canada differences
1972:1-1979:9				
1	-1.12	-6.67	-2.32	-5.69
2	-1.16	-6.68	-2.81	-5.43
3	-1.68	-6.93	-3.05	-5.35
4	-1.80	-7.14	-3.17	-5.26
5	-1.91	-6.77	-3.14	-5.14
6	-2.01	-6.56	-3.31	-5.89
7	-2.32	-6.70	-2.83	-5.54
8	-1.80	-6.96	-2.46	-5.32
1979.10-1982.9				
1	-2.76	-4.87	-1.74	-4.04
2	-3.07	-5.00	-2.10	-4.51
3	-3.04	-5.23	-2.12	-4.87
4	-2.74	-5.04	-1.99	-5.03
5	-2.22	-5.16	-2.04	-4.74
6	-2.38	-5.08	-1.80	-4.84
7	-2.12	-5.12	-1.96	-4.85
8	-2.23	-5.25	-1.86	-5.15
1982.10-1984.12				
1	-1.54	-2.68	-2.97	-2.24
2	-2.08	-2.12	-2.77	-1.93
3	-2.33	-2.16	-2.60	-2.01
4	-2.23	-2.38	-2.82	-1.97
5	-2.12	-2.68	-2.73	-2.09
6	-2.14	-2.72	-2.76	-1.97
7	-2.11	-2.68	-2.59	-2.26
8	-2.01	-2.91	-2.48	-2.30

Critical value for n=100 at 95 percent, from Fuller (1976): -3.17

Table 3

Descriptive Statistics: First Differences

1972:1-1979:9

	maturity	mean	std. dev.	skewness	kurtosis
U.S.	1	.086	.612	-1.3768	.2513
	2	.062	.420	-.4099	.6888
	3	.053	.343	-.3372	.8387
	4	.048	.300	-.4522	1.3231
	5	.043	.270	-.3055	1.0389
	6	.039	.248	-.1144	.5263
	7	.037	.194	-.0452	-.0401
	8	.034	.200	-.0757	-.1390
				(.2513)	(.4977)
Canada	1	.081	.387	.3576	2.4570
	2	.073	.370	.1977	2.4720
	3	.066	.362	-.0329	2.3130
	4	.058	.330	-.0463	1.1615
	5	.058	.312	-.1695	1.7621
	6	.046	.305	.2760	2.9276
	7	.042	.258	.2861	1.4442
	8	.036	.205	.0313	1.3534
				(.2513)	(.4977)

1979:10-1982:9

	maturity	mean	std. dev.	skewness	kurtosis
U.S.	1	.074	1.796	-.8189	.6736
	2	.015	1.432	-.6188	1.1825
	3	.045	1.256	-.6577	2.1839
	4	.054	1.149	-.3362	1.1813
	5	.058	1.018	-.4437	0.6797
	6	.065	.941	-.3338	0.2724
	7	.064	.787	-.4794	0.1256
	8	.068	.805	-.4093	0.1863
				(.3295)	(.7681)
Canada	1	.033	1.41	-.5209	1.9935
	2	.040	1.26	-.7745	1.6704
	3	.047	1.19	-.8718	1.7021
	4	.054	1.16	-1.1298	2.1854
	5	.060	1.04	-.7645	1.0287
	6	.072	.95	-.7898	1.1599
	7	.086	.86	-.1485	.7773
	8	.051	.88	-.5012	1.0351
				(.3925)	(.7681)

Table 3 continued

1982:10-1984:12

	maturity	mean	std. dev.	skewness	kurtosis
U.S.	1	-.022	.579	.3210	.0012
	2	-.040	.600	.0327	-.3292
	3	-.042	.581	-.1441	-.0029
	4	-.032	.559	-.0578	.0259
	5	-.031	.546	.1087	-.0849
	6	-.008	.536	-.0679	-.3211
	7	-.004	.527	-.1817	-.1611
	8	-.001	.481	.0231	-.4228
				(.4479)	(.8721)
Canada	1	-.103	.663	-.2439	-.4257
	2	-.094	.555	.0871	-.0042
	3	-.080	.522	.2164	-.2026
	4	-.087	.507	-.3479	-.4467
	5	-.077	.485	.6165	1.0251
	6	-.068	.466	.0847	-.1087
	7	-.072	.409	.2942	-.9112
	8	-.063	.402	.2202	-.8223
				(.4479)	(.8721)

Notes:

1. Skewness and kurtosis are measure relative to the normal distribution (ie. 0.0=normal). Standard errors are in parentheses.

2. The Jarque-Bera Lagrange multiplier test is used to test for normality. The test statistic is distributed as $\chi^2(2)$.

Table 4

Principal Components Analysis
United States - First Differences

	Eigenvalue	%variance explained	Correlation with maturity #: 1	2	3	4	5	6	7	8	W
			1972:2 - 1979:9								
1	69.7	78.6	-.88	-.95	-.94	-.88	-.83	-.88	-.79	-.74	
2	13.4	93.8	.48	-.15	-.29	-.41	-.44	-.35	-.29	-.43	3.86
3	1.9	96.0	.04	-.23	-.10	.04	.09	.17	.36	.26	0.03
4	1.8	98.0	.02	-.14	.07	.18	.27	-.07	-.25	-.34	0.85
5	0.6	98.7	.01	-.00	-.05	.01	.05	.01	-.30	.27	0.13
6	0.5	99.2	-.00	.05	-.14	-.04	.11	.13	.00	-.12	0.00
7	0.5	99.8	.00	.03	-.03	-.04	.14	-.21	.11	.07	0.27
8	0.2	100.0	-.00	-.02	.07	-.12	.05	.04	-.02	.00	0.00
			1979:10 - 1982:9								
1	360.0	90.4	-.90	-.99	-.99	-.97	-.96	-.95	-.93	-.92	
2	32.6	98.6	.42	.03	-.10	-.21	-.26	-.28	-.32	-.34	7.94
3	2.8	99.3	-.05	.10	.07	.08	-.03	-.06	-.16	-.17	1.07
4	1.2	99.6	.01	-.07	.04	.03	.00	.11	-.08	-.07	0.22
5	0.7	99.8	-.00	.04	-.07	.00	-.03	.09	.01	-.03	0.21
6	0.4	99.9	.00	-.00	-.04	.02	.08	-.02	-.06	.00	0.18
7	0.4	99.9	.01	-.02	-.02	.07	-.03	-.03	.04	-.01	0.32
8	0.1	100.0	-.00	-.00	.00	-.01	.02	-.00	.05	-.06	0.00
			1982:10 - 1984:12								
1	56.9	89.6	-.71	-.98	-.99	-.99	-.99	-.98	-.97	-.95	
2	5.5	98.2	.70	-.13	.01	-.07	-.10	-.18	-.21	-.25	5.57
3	0.8	99.5	.12	-.16	-.14	-.06	-.02	.07	.10	.18	2.05
4	0.2	99.8	.00	-.04	-.01	.02	.10	-.02	.02	-.08	0.42
5	0.1	99.8	-.01	.03	-.03	-.01	.04	.01	-.06	.02	0.07
6	0.1	99.9	-.00	.02	-.01	-.05	.00	.04	.03	-.03	0.08
7	0.0	99.9	-.00	.02	-.04	-.03	-.01	-.01	.02	-.01	0.08
8	0.0	100.0	.00	-.01	-.05	.02	-.01	.04	-.02	-.02	0.00

Table 5

Principal Components Analysis
Canada - First Differences

	Eigenvalue	%variance explained	Correlation with maturity #: 1	2	3	4	5	6	7	8	W
1972:2 - 1979:9											
1	67.6	89.7	-.90	-.98	-.98	-.96	-.97	-.94	-.93	-.89	
2	4.1	95.1	.42	.12	.01	-.12	-.16	-.24	-.23	-.29	2.74
3	1.4	97.0	-.14	.11	.16	.14	-.02	-.11	-.20	-.19	1.28
4	0.7	97.9	-.05	.10	.06	-.21	.01	.06	-.01	.02	0.41
5	0.6	98.7	.01	.01	-.08	.02	.06	.17	-.17	-.14	0.68
6	0.4	99.2	-.00	-.03	.01	-.03	.16	-.06	.06	-.17	0.15
7	0.4	99.7	-.01	.01	.00	.01	-.09	.06	.13	-.19	0.78
8	0.2	100.0	-.02	.09	-.09	.02	.01	-.04	.04	.01	0.00
1979:10 - 1982:9											
1	311.0	89.5	-.83	-.98	-.99	-.98	-.96	-.98	-.95	-.97	
2	28.3	97.6	.56	.05	-.10	-.18	-.22	-.18	-.16	-.18	4.86
3	4.0	98.8	-.06	.18	.08	.06	-.11	-.07	-.12	-.14	0.50
4	2.4	99.5	.02	-.07	.03	.09	.04	.03	-.24	.02	1.10
5	0.7	99.7	-.01	.01	.02	-.02	-.05	-.03	-.03	.13	0.00
6	0.6	99.8	.01	-.05	.04	.04	-.08	.01	.05	-.02	0.38
7	0.4	99.9	-.00	-.00	.04	-.05	-.01	.06	-.02	-.02	0.18
8	0.3	100.0	.00	-.02	.04	-.01	.03	-.05	.01	-.01	0.00
1982:10 - 1984:12											
1	46.7	87.1	-.84	-.98	-.95	-.95	-.96	-.96	-.95	-.92	
2	4.2	84.8	.54	-.05	-.15	-.07	-.16	-.21	-.14	-.18	1.39
3	1.1	96.8	.01	-.13	-.23	.15	.00	-.04	.13	.29	0.42
4	0.7	98.2	-.02	.05	-.05	.25	-.04	.04	-.20	-.13	0.73
5	0.4	98.9	.01	-.01	.03	-.01	-.20	.14	.05	.03	0.07
6	0.3	99.4	-.01	-.06	.09	.08	-.04	-.10	.11	-.07	0.35
7	0.2	99.8	-.01	.07	.03	-.00	-.06	-.09	-.05	.11	0.49
8	0.1	100.0	-.01	.08	-.06	-.00	-.02	-.02	.09	-.06	0.00

Table 6

Joint Principal Components Analysis of U.S. and Canada
1972:2 - 1979:9

U.S. - correlation with maturity class:

	Eigenvalue	%variance explained	1	2	3	4	5	6	7	8	W
1	108.0	65.8	-.73	-.88	-.86	-.80	-.73	-.82	-.71	-.75	
2	30.4	84.3	-.54	-.34	-.35	-.33	-.35	-.30	-.30	-.12	7.75
3	12.9	92.2	-.42	.16	.32	.45	.49	.37	.32	.40	4.80
4	4.0	94.6	.01	-.04	-.03	-.06	-.03	.04	.05	.01	1.48
5	2.1	95.9	-.02	.12	.11	.03	.03	-.17	-.42	-.35	0.41
6	1.8	96.9	-.04	.24	.00	-.16	-.23	-.02	.06	.11	0.69

Canada - correlation with maturity class:

	1	2	3	4	5	6	7	8
1	-.77	-.84	-.86	-.87	-.86	-.84	-.83	-.82
2	.46	.50	.46	.40	.45	.42	.40	.37
3	-.10	-.12	-.08	-.05	-.03	.00	-.03	.06
4	-.41	-.09	.00	.12	.17	.23	.23	.27
5	.05	-.07	-.10	-.06	.02	.13	.12	.08
6	-.06	.04	.07	-.07	-.00	-.00	-.05	-.08

1979:10 - 1982:9

U.S. - correlation with maturity class:

	Eigenvalue	%variance explained	1	2	3	4	5	6	7	8	W
1	605.0	81.2	-.84	-.94	-.95	-.93	-.95	-.92	-.93	-.92	
2	68.6	90.4	-.39	-.33	-.27	-.24	-.16	-.18	-.09	-.08	6.13
3	37.8	95.5	.30	-.01	-.12	-.19	-.20	-.26	-.30	-.29	3.55
4	21.0	98.3	-.20	.04	.07	.17	.18	.17	.09	.15	5.91
5	4.0	98.9	.02	-.04	-.00	.01	-.00	.04	.03	.04	0.51
6	2.6	99.2	-.02	.03	.03	.02	-.02	.02	-.10	-.12	0.33

Canada - correlation with maturity class:

	1	2	3	4	5	6	7	8
1	-.76	-.94	-.93	-.92	-.91	-.92	-.90	-.90
2	.32	.26	.32	.33	.31	.34	.30	.36
3	.47	.07	-.03	-.07	-.12	-.10	-.08	-.12
4	.32	.02	-.09	-.17	-.18	-.16	-.13	-.12
5	.05	-.17	-.09	-.07	.10	.06	.13	.13
6	-.02	.06	-.01	-.08	-.03	-.01	.21	.00

Table 6 continued

1982:10 - 1984:12

	Eigenvalue	%variance explained	U.S. - correlation with maturity class: 1	2	3	4	5	6	7	8	W
1	92.8	79.2	-.75	-.96	-.95	-.95	-.93	-.91	-.90	-.87	
2	12.9	90.2	.20	-.16	-.24	-.29	-.34	-.40	-.42	-.45	6.35
3	4.9	94.4	-.52	-.18	-.09	-.04	-.00	-.03	.06	.07	1.31
4	3.4	97.3	-.34	.01	.03	.05	-.01	.04	.03	.08	3.25
5	1.0	98.2	.11	-.13	-.12	-.07	-.02	.04	.07	.16	0.57
6	0.7	98.8	-.02	.02	.03	.00	-.01	-.02	-.01	-.04	0.49

	Canada - correlation with maturity class: 1	2	3	4	5	6	7	8
1	-.74	-.91	-.85	-.91	-.92	-.90	-.91	-.92
2	.49	.36	.42	.24	.23	.28	.20	.08
3	-.21	.08	.20	.12	.20	.24	.28	.24
4	.41	-.09	-.17	-.02	-.08	-.14	.04	.01
5	.02	-.06	-.07	.08	.03	-.08	.04	.22
6	-.02	-.01	-.10	.28	-.07	.07	-.10	-.03

Table 7

Comparison of Principal Components for the U.S. and Canada
1972:2 - 1979:9

Eigenvalues of matrix Z	:	0.9972	0.9914	0.7517
Angle between subspaces (degrees)	:	3.047	5.322	29.890
Overall similarity	:	2.74		

Vectors closest to both subspaces -- weight on maturity class:

		1	2	3	4	5	6	7	8
Component pair	1	-.50	-.44	-.40	-.35	-.32	-.30	-.24	-.20
	2	.79	.08	-.13	-.29	-.28	-.28	-.22	-.25
	3	.31	-.55	-.38	-.16	.10	.29	.46	.34

1979:10 - 1982:9

Eigenvalues of matrix Z	:	0.9998	0.9991	0.9166
Angle between subspaces (degrees)	:	0.745	2.401	16.780
Overall similarity	:	2.92		

Vectors closest to both subspaces -- weight on maturity class:

		1	2	3	4	5	6	7	8
Component pair	1	-.19	-.41	-.42	-.43	-.37	-.35	-.29	-.30
	2	.97	.05	-.08	-.18	-.11	-.09	-.03	-.04
	3	-.05	.59	.27	.23	-.26	-.25	-.42	-.47

1982:10 - 1984:12

Eigenvalues of matrix Z	:	0.9999	0.9948	0.5791
Angle between subspaces (degrees)	:	0.668	4.137	40.447
Overall similarity	:	2.57		

Vectors closest to both subspaces -- weight on maturity class:

		1	2	3	4	5	6	7	8
Component pair	1	.45	.41	.37	.34	.34	.31	.30	.27
	2	.86	-.03	-.13	-.17	-.20	-.24	-.21	-.25
	3	-.19	.48	.54	-.10	-.01	-.05	-.33	-.57

Appendix A

Principal Components Analysis

PCA is a data reduction technique. Given a set of p variables, $X_1,\ldots,X_p$, the object is to find the k(< p) components which account for most of the variation in the X's. This is accomplished by first re-expressing the variables as p orthogonal variables, known as principal components, and then determining the minimum number of components necessary to adequately describe the X variation.

First, consider population principal components. Arrange the input variables in a 1xp row vector X', and let Ω be the variance-covariance matrix of X'. The principal components, $Y = Y_1,\ldots,Y_p$, are linear combinations of the original variables:

$$Y_i = a_i'X, \tag{2}$$

with the pxl vector of weights, a_i, chosen such that $a_i'a_i=1$ and $Cov(Y_i,Y_k)=0$ for $i \neq k$.

Geometrically, the components are a new set of axes in p-space, chosen to be the closest axes to the original data points by minimization of the distance between the original observations and the linear combination a_iX. This is equivalent to finding the components that successively maximize the variance, given by

$$Var(Y_i) = a_i'\Omega a_i. \tag{3}$$

The solution to the optimization problem yields a_i equal to the eigenvector of Ω corresponding to the ith largest eigenvalue of Ω. Note that normalization of eigenvectors to have an inner product of unity means that the signs of the eigenvector components are arbitrary. Any eigenvector can be multiplied by -1 without changing the meaning of the results. The new axes, like the original ones, are orthogonal to each other, which is reflected in the zero covariance between the components Y_i and Y_k.

Given that the a's are eigenvectors, $Var(Y_i)=\mu_i$, the largest eigenvalue of Ω. By construction, the sum of the variances of Y_i's is the same as the sum of the variances of X_i's: tr(Y'Y)=tr(X'X). The proportion of total variance explained by the ith components is thus μ_i/tr(X'X).

The elements of each eigenvector are proportional to the correlation coefficients between Y_i and X_j. Specifically, the correlation coefficient is the element of a_i multiplied by $\sqrt{\mu_i}$ and divided by the standard deviation of X_j. The patterns of the correlation coefficients often provides clues to the interpretation of the components. Division by the standard deviation of X_j assures that the same measurement scale is used in comparisons. The square of each correlation coefficient is the proportion of variation in X_j explained by component i.

Now consider calculation of sample principal components. Let there be n independent observations from each of the p variables drawn from a population with a constant mean vector and covariance matrix. Let X be the n x p data matrix and denote the sample covariance by S. The ith principal component is then Xa_i is now the eigenvector of S corresponding to the ith largest eigenvalue.

Appendix B

Cross-Validatory Analysis

Eastment and Krzanowski (1982) proposed a procedure for choosing the number of components based on cross-validatory analysis. When p input variables are reduced to k components, we can think of using the k-component model to predict the original data points:

$$X_{i,j} = \hat{X}_{i,j} + \varepsilon_{i,j} \tag{4}$$

where $\hat{X}_{i,j}$ is the prediction conditioned on the k components and $\varepsilon_{i,j}$ is the prediction error. The predicted values are calculated by exploiting the relationship between the eigenvalue decomposition on which PCA is based and the sigular value decompostition, X = WSV' where W is the matrix of eigenvectors of XX', V is the matrix of eigenvectors of X'X and S is the diagonal matrix containing the square roots of Ω_t, $w_{i,t}$ s_t, and v_{tj}. Eastment and Krzanowski showed how to calculate this value for each i and j by deleting row i and column j and using all other observations to form the prediction.

The basic idea of cross-validation is to keep adding components until there is an insignificant gain in predictive power. The gain in predictive power from adding the kth component is expressed in terms of the sum of squared prediction errors (denoted the SSE). Eastment and Krzanowski call this the W statistic:

$$W = \frac{\text{(SSE from using k-1) - (SSE from using k)/df1}}{\text{(SSE from using k)/df2}} \tag{5}$$

where df1=n+p-2K abd df2=np-p-df1. The numerator represents the marginal predictive power of the kth component and the denominator represents the average predictive power of all k. Eastment and Krzanowski advise that the kth component should be added only if the W statistic is greater than 1. They show that the conclusions from using the W statistic can be different from the conclusions of more traditional criteria such as the cumulative variance benchmark.

Although both the W statistic and the traditional criteria aim to determine "the number of components to keep," they differ in a fundamental way. The W statistic distinguishes only the number of predictable components. It does not recognize purely random shocks as separate components. In constrast, traditional criteria count any factor contributing to total variance, whether predictable or random.

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Figure 1

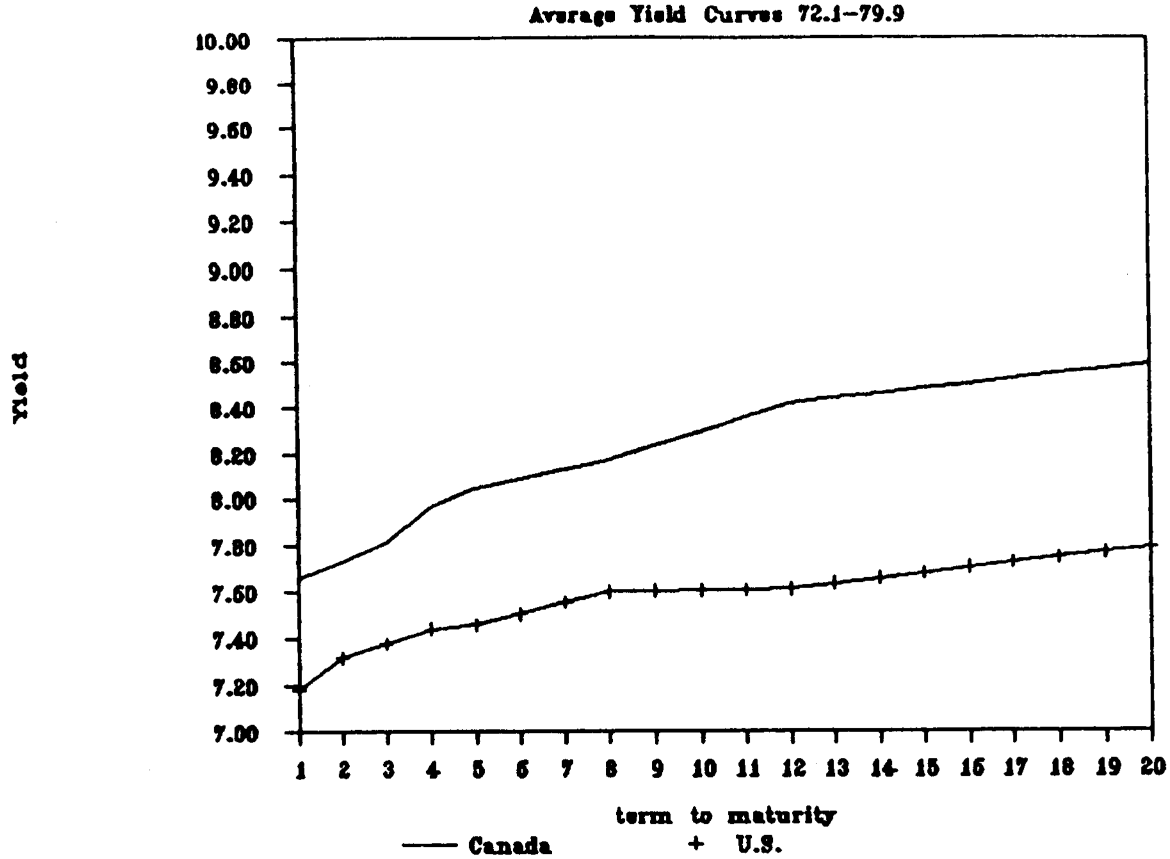


Figure 2

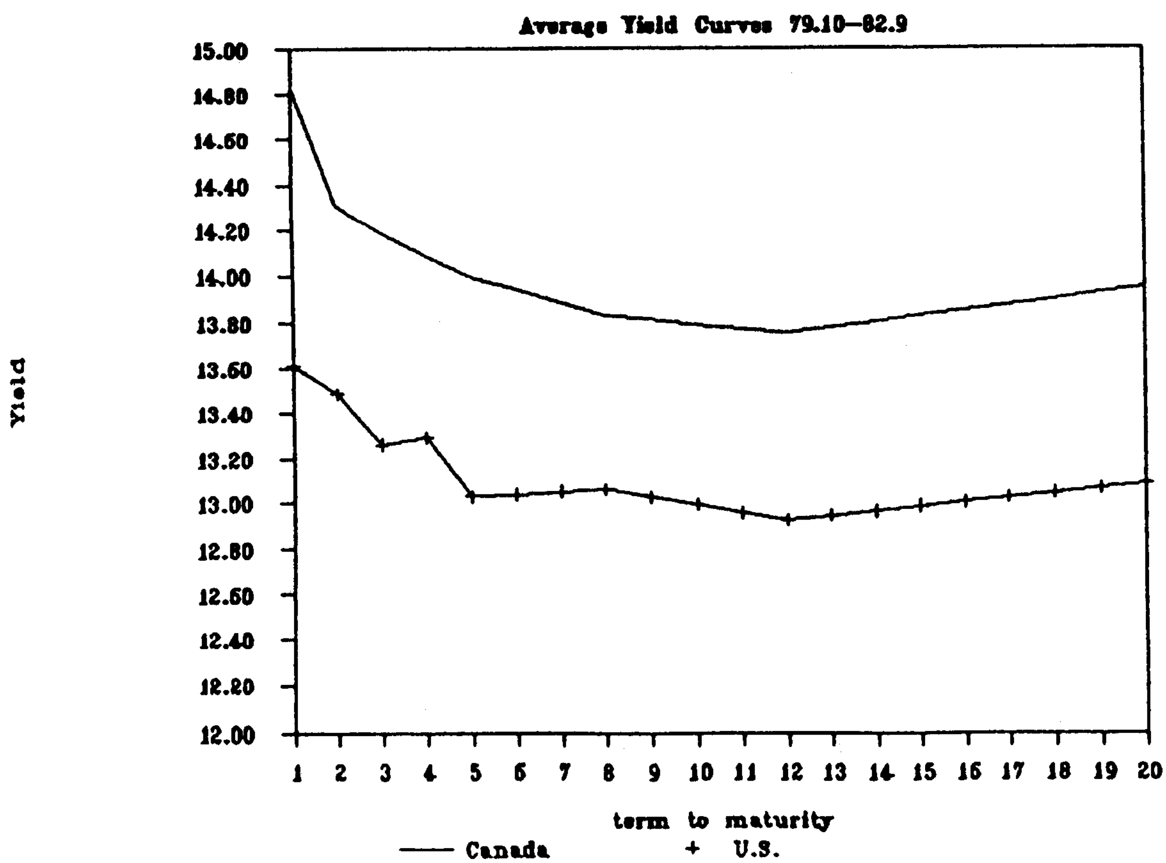
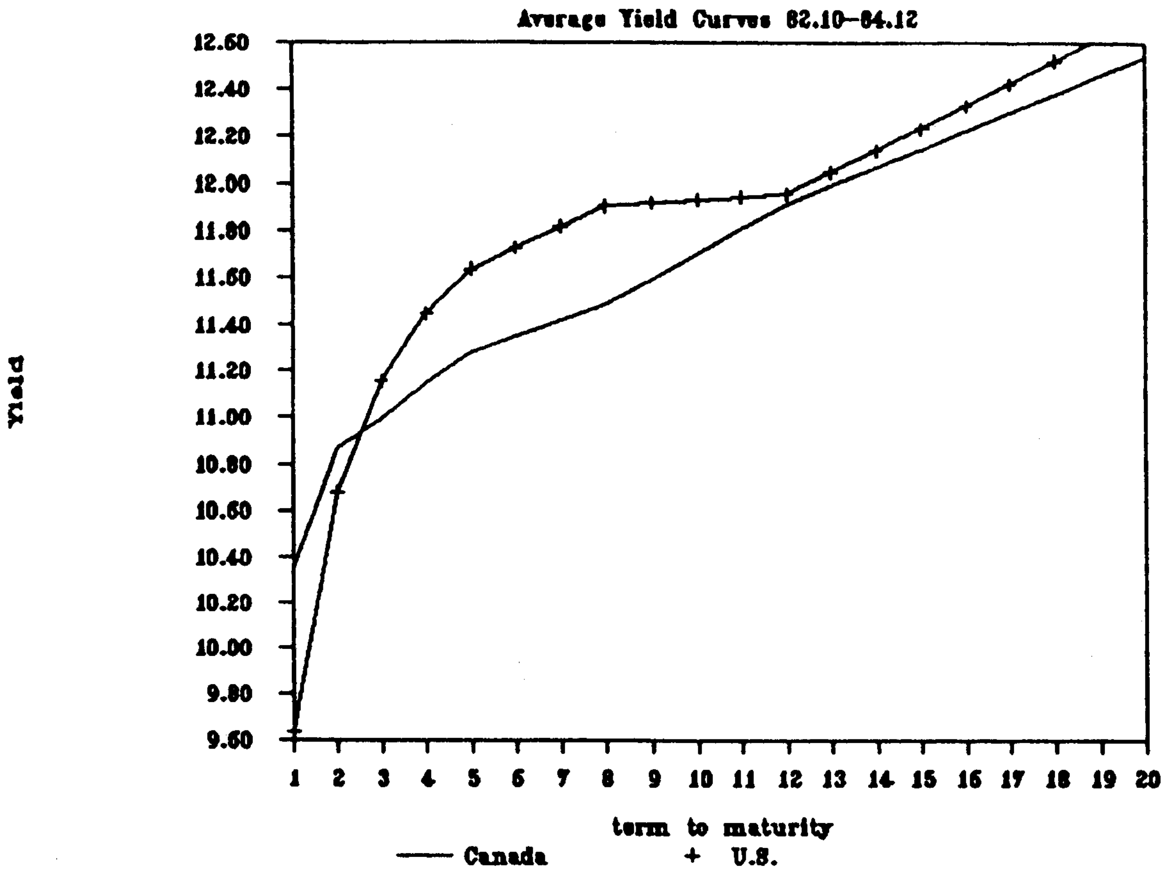


Figure 3


Average Yield Curves 82.10–84.12
Yield
12.60
12.40
12.20
12.00
11.80
11.60
11.40
11.20
11.00
10.80
10.60
10.40
10.20
10.00
9.80
9.60
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20
term to maturity
Canada
+ U.S.