THE ANALYTICS OF CONTINUING CONFLICT

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Abstract

Conflict and settlement are usually taken to be mutually exclusive conditions. Peace being regarded as normal, traditional scholarship has tried to explain why damaging struggles -- wars among nations or tribes, strikes and lockouts in industry, lawsuits and corporate takeovers, contests for dominance among animals -- ever occur. But individuals, groups, or nations are rarely if ever totally at war or totally at peace. Instead, they typically divide their energies between peaceful productive activities on the one hand, and on the other hand warlike or "appropriative" efforts designed to seize resources previously controlled by others (or to defend against such invasions). The premise here is that both production and appropriation are entirely normal lines of economic activity that all parties will be engaging in to the extent that doing so seems profitable.

The analytic model of this process leads to a steady-state equilibrium with two main features. First, the resources devoted to peaceful production mainly determine the available <u>social total of income</u>. And second, the relative commitments of resources to appropriative combat power mainly determine the <u>fractional distribution</u> of that income. The major features of the model are: (i) a resource partition function, (ii) a social production function, (iii) a combat power function, and (iv) an income distribution equation. The combat power function, essential for modelling economic interactions involving actual fighting or coercive threats, has strikingly different consequences depending upon whether what governs the outcome is the <u>ratio</u> of the resources committed to the struggle by the two sides, or else the numerical <u>difference</u> between them (the latter leading to

an equation in logistic form).

The paper explores three classes of equilibria: the symmetrical Cournot solution, the asymmetrical Stackelberg solutions, and a less familiar contingent-commitment or "hierarchical" solution called here Threat-and-Promise (TAP). In the symmetrical Cournot cases, a remarkable tendency was discovered for the parties to end up with equal final incomes despite initially unequal resource endowments (apart from corner solutions that come about for very large resource disparities). With the ratio form of the combat power function, both parties always devote some resources to coercive effort -- neither "total peace" nor one-sided submission can ever occur. But with the logistic form, both sides may find it more profitable to engage solely in peaceful productive activity.

The asymmetrical Stackelberg protocol led to surprising and somewhat unsatisfactory results. Quite commonly the Stackelberg "leader" ends up no better off than the follower, and in a number of cases the solution is indeterminate. In contrast, in the hierarchical TAP protocol it is always highly advantageous to be the "director" rather than the subordinate. The factors determining which party becomes the director, and how threats and promises can be made credible, were also analyzed. Broadly speaking, though not always, the hierarchical TAP arrangement tends to minimize the resource wastage involved in amassing combat power.

The traditional "harmonistic bias" of orthodox economics has led to a one-sided emphasis upon peaceful production and amicable exchange. Apart from the specific results obtained, methodologically speaking the analysis demonstrates that a general-equilibrium model can be constructed to incorporate the hostile and destructive interactions that characterize real-world social relations among individuals, groups, and nations.

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Contents

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The efforts of men are utilized in two different ways: they are directed to the production or transformation of economic goods, or else to the appropriation of goods produced by others.
-- Vilfredo Pareto

War and peace, we usually think, are mutually exclusive. Peace being taken to be the normal condition, sages in all periods and of all schools and ideologies have tried to understand why war occurs.[1] And similarly for the ordinarily nonviolent struggles of modern life that can be likened to war -- strikes and lockouts, lawsuits, corporate takeover contests, etc. In each case analysts have mainly been concerned to explain why, in view of the potential mutual gains from settlement, costly struggles nevertheless take place.[2]

Thinking in terms of a radical disjunction between war and peace, between conflict and settlement, is certainly valid in many contexts. In addition it has led to deeper study of important phenomena like deterrence, commitment, and signalling of intentions.[3] But in the entirely different approach adopted here, contending parties will typically be simultaneously "at war" and "at peace." Put less paradoxically, an ongoing interaction will be postulated in which intensity of conflict among the parties can vary anywhere along a spectrum. Absolute peace and total war are extremes of the spectrum that will rarely if ever be attained.[4]

The approach adopted here is an economic one. Individuals and groups can choose between two main ways of acquiring income: (1) producing economic goods, versus (2) seizing what other parties have produced (or

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defending against such invasions). Since a continuing interaction between the parties is postulated, the decision variable on each side is not the discrete transition between war and peace but rather the steady-state proportions in which efforts are divided between the two ways of seeking advantage.[5] Following Pareto, the parties may be said to choose between <u>production</u> and <u>appropriation</u>. (Appropriation[6] is a generalization of the more familiar concept of "rent-seeking."[7] When a government agency is in a position to grant special privileges like franchises or patents, private parties are motivated to compete in the political arena for the rents these licenses generate. But the rent-seeking competition for political privilege is only one of many different ways in which control over resources might be achieved or maintained. Appropriation as a broader category covers not only rent-seeking but also attempts to profit from robbery, confiscatory redistribution, or coercive encroachment.)

Production and appropriation both being entirely normal lines of activity, neither requires any more explanation than the other. It is the fact that intrusive efforts (and defending against them) can be as remunerative as production or exchange that makes conflict a permanent feature of life. Think of the decision problem facing a territorial animal. Only the productive time spent exploiting the resources of its territory -- searching for seeds or insects -- yields a direct energy profit. But unless efforts are also devoted to acquiring territory in the first place, and to patrolling and fighting for it whenever challenged, the animal will have a smaller resource base (or none at all) available for productive exploitation. In human affairs as well, individuals and groups must strike a balance between using resources for peaceful production and exchange or else for intrusion and for defense against others' intrusions.

The resources devoted to "peaceful" productive activity mainly determine the aggregate of income available to the society as a whole, while the relative magnitudes of the resource commitments to "warlike" appropriative activity mainly determine the distribution of that income. The outcome of the struggle depends upon the technology of conflict. Like the familiar technology of production, the technology of conflict is a relation between inputs (armies or guns or lawyers) and outputs (incomes or resources gained or lost). For the technology of conflict as for the technology of production, it will be important to specify the regions of decreasing versus increasing returns, the degree of complementarity or substitutability among inputs, etc. (The technologies employed in actual warfare will of course differ in important ways from those that apply to merely metaphorical combats like strikes and lockouts, lawsuits, or rent-seeking competitions.)

Finally, I will be assuming here that all parties are merely self-interested, i.e., neither benevolent nor malevolent with regard to others.[8]

Analytical models are necessarily oversimplified versions of selected aspects of the real world. And in particular, the rather ruthless simplifying assumptions to be made at various points in the logical development that follows cannot fully capture the infinitely varied complexities of the problem addressed. As only one example, the analysis here entirely rules out geography. Obviously, I am not asserting that geography is unimportant (though of course it is less crucial in some contexts than in others), but only that attempting to model it would be unduly complex at this stage of theoretical understanding. The other side of the coin is that there is always much to be said for a mode of analysis that forces us to make our assumptions explicit and to follow logically necessary paths of reasoning. With due sensitivity to the need for caution, I believe that the models of conflict developed here will help illuminate questions like the following:

1. It might seem a plausible generalization that, in conflict situations, "the rich get richer and the poor poorer." But in several of the model variants analyzed, the side less well endowed with resources may end up just as well off as its initially richer opponent! So, the issue is, when does conflict tend to enlarge and when does it tend to reduce initial wealth disparities?
2. As a related question, are wealthier contenders (individuals, groups, or nations) likely to devote <u>absolutely</u> or <u>relatively</u> more effort to fighting in comparison with poorer ones?
3. Can it ever pay one side simply to submit? Under what circumstances if any will <u>both</u> sides choose not to fight at all, so that the extreme of "total peace" is attained? And, in either case, how will the social product then be divided?
4. Will increased productive interdependence between the parties, due for example to greater productive specialization and expanding trade, necessarily reduce the incentive to engage in conflict? Correspondingly, if warfare becomes more destructive does that necessarily tend to promote increased concentration of resources upon productive rather than appropriative activity?
5. Is it more advantageous to have the first or the last move?

A. THE ELEMENTS OF ONGOING CONFLICT

The two-party interaction is the standard and simplest pattern of conflict. In going on to introduce third, fourth, fifth players, etc., the strategic-behavior problem is enormously compounded at each step. At the other extreme, the large-numbers war of each against all, the analysis simplifies once again since all parties can be assumed to engage in the equivalent of "price-taking" behavior. The present study is limited to the straightforward two-party case.

An allegory: On Lake Contention there are two villages. Each village can launch fishing boats or gunboats. The fishing boats catch the fish; the gunboats protect the village's catch and may seize the other village's catch. Yet the hostilities between the villages are limited in several ways. First, each side's home base is invulnerable to attack. Second, in any hostile encounter the outmatched side always surrenders its catch rather than fight. (Thus, there is coercive taking but no actual battle.)

The analytical model to be presented formalizes this parable. But first, it is natural to ask, does the allegory picture any real-world instances of conflict? Like jokes, allegories are normally best left unexplained, but I will suggest some parallels. In the 18th-century imperialist struggles between France and England, the two European powers were the villages and the colonial territories in Africa, America, and Asia the fish in the lake. When the Capone and the Moran mobs strove for supremacy in the 1920's, the two gangs were the villages and the populace of Chicago the fish.[9] On a cynical view, all contests for political power -- whether between Czarists and Bolsheviks in revolutionary Russia, or Democrats and Republicans in the United States -- are ultimately gang struggles to exploit a passive population of citizen-victims as fish.

The analytical model has four main elements: (i) a Resource Partition Function; (ii) a Contestable-income Production Function; (iii) a Combat Power Function; and (iv) an Income Distribution equation. These will be discussed in turn.

1. RESOURCE PARTITION FUNCTION: Each side may divide its endowed resources R_i among three claimant uses: <u>contestable productive effort</u> F_i (fishing boats in the allegory of the two villages), <u>appropriative effort</u> G_i (gunboats in the allegory), and <u>home productive effort</u> H_i. Efforts directed to home production generate income that is safe from appropriation by the other side. (Each village might be able to grow crops that are too low-valued and bulky for raiders to capture.)

(1a) Abstract equation: $\Theta(H_i, F_i, G_i) = R_i$

(1b) Simple example: $F_i + G_i = R_i$

The simpler version (1b) not only takes on an especially elementary linear form, but omits home production H_i. This simplification will be adopted in the main analysis to follow. The absence of home production makes contestable income (fish) all the more vital, thus enlarging the stakes of the contest. Equation (1b) could be generalized in a number of ways. The first of these would be to restore the possibility of devoting resources to invulnerable home production. A second line of generalization would allow for possible comparative advantages: e.g., village #1 might find it easier to produce gunboats and village #2 fishing boats. As a third consideration, in this initial version the parties' respective resource totals R_1 and R_2 are taken as exogenous constants. More generally, the R_i could be made endogenous, for example by allowing for <u>battle damage</u>. (This point will be discussed further below.)

In Fig. 1 we see, in the first quadrant, player 1's opportunities for balancing F_1 against G_1 and, in the third quadrant, player 2's similar opportunities with regard to F_2 and G_2. The illustrated 135° lines correspond of course to the simple linear form of equation (1b).

2. CONTESTABLE-INCOME PRODUCTION FUNCTION: The overall social aggregate of contestable income C is a function of the parties' commitments to contestable production F_i.

(2a) Abstract equation: $C = \Omega(F_1, F_2)$

(2b) Simple example: $C = F_1 + F_2$

(2c) Generalization #1: $C = (a_1F_1 + a_2F_2)^g$

[where the a_i are productivity coefficients and g is an index of returns to scale.]

(2d) Generalization #2: $C = (a_1F_1{}^{1/s} + a_2F_2{}^{1/s})^s$

[where s is an index of productive complementarity.]

Generalizing the simplest linear form (2b), equations (2c) and (2d) both incorporate differential productivity factors a_1 and a_2 for the two sides. Equation (2c) allows also for increasing $(g > 1)$ or decreasing $(g < 1)$ returns to scale, while still retaining the linear form of the isoquant map. Equation (2d) allows for positive or negative productive complementarity. The fourth (lower-right) quadrant of Fig. 1 shows an output isoquant map whose curvature implies positive complementarity $(s > 1)$ between the parties' productive efforts. However, in the analytical development and numerical illustrations that follow, the simple form (2b) will be employed.[10]

3. COMBAT POWER FUNCTION: This function, which summarizes the relevant aspects of the technology of conflict, is the crucial and most novel element of the analysis. It indicates how the resources devoted to combat readiness determine appropriative success as measured by the proportions (p_1 and $p_2 = 1-p_1$) in which the social total of income is divided.[11] As in the allegorical example, combat power may achieve its purpose through coercion even in the absence of actual battle. And that indeed is the assumption made throughout the paper.[12]

(3a) Abstract equation: $p_i = \Phi_i(G_1,G_2)$

(3b) Simple example #1 (ratio form): $p_1 = G_1/(G_1+G_2)$

(3c) Simple example #2 (logistic form): $p_1 = 1/(1+\exp\{G_2-G_1\})$

(3d) Generalized ratio form: $p_1 = b_1G_1{}^m/(b_1G_1{}^m + b_2G_2{}^m)$

[where b_1 and b_2 are constant coefficients of fighting effectiveness and m is an index of "mass effect" in battle.][13]

(3e) Generalized logistic form: $p_1 = 1/(1+\exp\{k(b_2G_2-b_1G_1)\})$

[where k is the index of mass effect in the logistic equation.]

The second (upper-left) quadrant of Fig. 1 shows a number of isoquants of appropriative success p_1 for side #1, using the simple ratio form of equation (3b). This equation leads to straight-line isoquants on G_1,G_2 axes. (Thus, $p_1 = .75$ when $G_1 = 3G_2$.)

As a new feature that does not seem to have been noticed in the literature, two quite different principles may govern relative combat power. The first principle is illustrated by equations (3b) and (3d), where appropriative success depends upon the ratio of the resources committed to coercion -- which is the more familiar condition.[14] The second principle is exemplified by the logistic form of equations (3c) and (3e), where success depends upon the <u>difference</u> between the magnitudes of the resources committed. The divergence between the two formulations is most clearly seen when one party, say side 1, puts in zero appropriative effort. Using the ratio form, it necessarily follows that $p_1 = 0$ -- its appropriative success will be zero. But using the logistic form, where the magnitude of the <u>difference</u> governs, a party committing zero combat effort can nevertheless achieve some degree of appropriative success.

In the allegory of the two villages, the ratio form would tend to apply if fishing takes place by day; under conditions of full visibility, a side totally lacking combat power can expect to lose all its catch to the enemy. But if fishing takes place at night the logistic form would tend to apply; under cover of darkness some of the village's fishing boats may bring home their catch undetected, even in the absence of friendly gunboats to protect them.

Fig.2a illustrates the Combat Power Function using the <u>ratio</u> form of equation (3d), and assuming equal coefficients of fighting effectiveness ($b_1 = b_2 = 1$). Along each curve, G_1 varies while G_2 is held constant; the different curves show what happens when the mass effect parameter m changes. For $m \leq 1$, diminishing returns hold over the entire range. For $m > 1$, however, there is an initial range of increasing returns. The latter condition is more consistent with the technology of warfare, since the chances for victory in battle are known to be very sensitive to the change from having slightly smaller to slightly larger force sizes in comparison with the enemy. The point of inflection in the generalized ratio form is always at a p_1 value less than .5 -- or equivalently, at a force ratio less than unity.[15] In effect, diminishing returns set in too early in any ratio version of the Combat Power Function.

For the logistic form of the Combat Power Function, as illustrated in Fig. 2b, it is again true that $p_1 = .5$ when $G_1 = G_2$, and in addition the point of inflection occurs there as well.[16] In the logistic form the parameter k illustrates the mass effect, and also serves as a scaling coefficient.

4. INCOME DISTRIBUTION EQUATION: As a definitional matter, the income I_i realized by either side will be the sum of its retained fraction of total contestable income C plus any home income J_i, the latter being of course a function of H_i.

(4a) General equation: $I_i = p_i \cdot C + J_i$, where $J_i = \phi(H_i)$

(4b) Simple example: $I_i = p_i \cdot C$

The simple equation (4b) employed here allows only for contestable income.

If the model of conflict were generalized to allow for battle damage and captures, a steady-state solution would still be possible since the parties could simultaneously be investing (consuming less than income in each period) so as to rebuild resources lost, net of war booty gained. These effects would all be incorporated in a Resource Replenishment Function. Postulating a limited form of conflict, without battle damage and resource capture, permits us to forego attempting to model resource replenishment -- which, like geography, raises issues too large to be conveniently addressed here.

B. SYMMETRICAL (COURNOT) EQUILIBRIUM

Following a long-established though still controversial tradition, the equilibrium of the two-party interaction will first be explored under Cournot behavioral assumptions -- where each party's choice is a best response to the other's decision taken as given.[17] Both the ratio and the logistic forms for the Combat Power Function will be examined. However, in the interests of manageability, the simplest versions of the other functions will be employed wherever possible. For convenience these conditions will be repeated here:

(1b) $F_i + G_i = R_i$ Resource Partition Function

(2b) $C = F_1 + F_2$ Contestable-Income Production Function

(4b) $I_i = p_i C$ Income Distribution Function

Home production H_i being excluded by assumption, there is only contestable income C to be distributed.

Interior solutions

Player #1's optimizing problem can be expressed:

(5) $$\text{Max } I_1 = p_1(G_1|G_2) \cdot I(F_1|F_2) \quad \text{subject to} \quad F_1 + G_1 = R_1$$

In this section, we will assume that an interior solution exists (allowing us to defer complications associated with non-negativity conditions). The ratio form of the Combat Power Function will be examined first, using the simplest version (3b):

(3b) $p_1 = F_1/(F_1 + F_2)$ Combat Power Function, Ratio Form

Proceeding by standard constrained-maximization techniques, player #1's Reaction Curve (RC_1), for given F_2 and G_2 choices of player #2, can be written:[18]

(6a) $G_1/G_2 = (F_1 + F_2)/(G_1 + G_2)$ RC_1, ratio form

A corresponding equation applies of course for the other player:

(6b) $G_2/G_1 = (F_1 + F_2)/(G_1 + G_2)$ RC_2, ratio form

In this simple case it is also possible to solve explicitly for player #1's appropriative (coercive) effort G_1 and player #2's corresponding G_2:

(7a) $G_1 = \text{sqrt}\{G_2(R_1 + R_2)\} - G_2$ RC_1, ratio form

(7b) $G_2 = \text{sqrt}\{G_1(R_1 + R_2)\} - G_1$ RC_2, ratio form

Solving (7a) and (7b), the condition for an interior Cournot solution is given by:

(8) $G_1 = G_2 = (R_1 + R_2)/4$ Cournot solution, ratio form

This solution is illustrated by the interior intersection of the two Reaction Curves in Fig. 3a.[19]

Thus, as a first and rather surprising substantive result, despite possible differences in resource endowments the parties devote exactly the same absolute amounts to coercive effort, the quantity depending solely upon the total of the resources available to the two of them together. Answering one of the questions posed in the introductory discussion, therefore, since (in this model) wealthier and poorer players devote the same absolute amounts of resources to appropriative activity, the former commits a smaller and the latter a larger relative amount.

What is even more striking is that, since $G_1 = G_2$ implies $p_1 = p_2$, the initially richer and poorer sides end up with exactly equal achieved levels of income:

(9) $I_1 = I_2 = (R_1 + R_2)/4$ Incomes at interior Cournot solution

So, answering another of the questions posed in the introductory discussion, if the parties differ only in resource endowments and are otherwise symmetrically situated, the opportunity to engage in coercive appropriative activities on equal terms has the effect of equalizing final income between the two! The numerical example shown in Table 1, line 1 (where the richer and poorer sides have resources $R_1 = 200$ and $R_2 = 100$ respectively) corresponds to the interior solution illustrated in Fig. 3a.

Shifting now to the logistic form, equation (3e) will be used with a simplification ruling out differences in fighting effectiveness (i.e., setting $b_1 = b_2 = 1$):

(3e′) $p_1 = 1/(1 + \exp\{k[G_2 - G_1]\})$ Combat Power Function, logistic form

Assuming an interior solution once again, the standard constrained-maximization technique leads to the Reaction Curve equations:[20]

(10a) $G_1 = G_2 + (1/k)\ln\{k(F_1 + F_2) - 1\}$ RC_1, logistic form

(10b) $G_2 = G_1 + (1/k)\ln\{k(F_1 + F_2) - 1\}$ RC_2, logistic form

The intersection of the Reaction Curves occurs at:

(11) $G_1 = G_2 = (R_1 + R_2)/2 - 1/k$ Cournot solution, logistic form

This solution is illustrated in Fig. 4a, and a numerical example is tabulated on line 3 of Table 1 (where $R_1 = 200$ and $R_2 = 100$ as before, and using $k = 0.01333$ as the logistic scaling factor). Notice that the surprising conclusion obtained using the ratio form of the Combat Power Function applies for the logistic form as well: regardless of differences in endowed resource levels, at any interior solution both sides devote equal amounts of resources to coercive appropriative activity and as a result achieve equal levels of final income $I_1 = I_2$.

The logistic form of the Combat Power Function does however lead to a number of distinctively different results. The most evident of these is that, comparing Fig. 4a with Fig. 3a, we see that RC_1 intersects the G_1-axis and RC_2 similarly intersects the G_2-axis at positive quantities. This is of course due to the point already emphasized, that appropriative success (p_1 or p_2, as the case may be) in the logistic model depends upon the numerical difference between the parties' respective coercive efforts. Another point of interest is that the income level achieved by each player under the logistic model is constant all along the other side's Reaction Curve (in the interior range where the non-negativity conditions are not binding). More specifically:[21]

(12) Along RC_2: $I_1 = 1/k$ — Income levels, logistic form
Along RC_1: $I_2 = 1/k$

It follows immediately, of course, that at a logistic interior solution the two players once again achieve the same final income level.

Corner solutions

So far only interior solutions have been considered. A corner solution comes about when the interior solution would require either side's productive effort F_i or appropriative effort G_i to exceed its resource endowment R_i, which of course is impossible. Two inter-related circumstances tend to cause a failure of the interior solution:

(1) For a sufficiently great resource disparity between the players, the interior solution may call for a combat effort on the part of the poorer side that exceeds its resource endowment ($G_i > R_i$). Such a situation, using the simplest ratio form of the Combat Power Function with the mass effect parameter held at $m=1$, is pictured in Fig. 3b; a

numerical instance (where R_1=200 and R_2=40) is shown on line 2 of Table 1.

(2) The second element involved is the magnitude of the mass effect parameter (m for the ratio form, k for the logistic form). The larger is this coefficient, the greater is the marginal product of coercive relative to productive activity. So, as the mass effect parameter rises, for any given resource availabilities both parties would like to devote more effort to combat power, making the poorer side more likely to run into the constraint forbidding $G_i > R_i$. Such a situation is pictured in Fig. 4b for the logistic form, corresponding to the numbers on line 4 of Table 1. Notice that even with resource endowments held at their original levels R_1=200 and R_2=100, increasing the mass effect parameter k alone (from .01333 to .025) dictates a corner solution.

The detailed equations and inequalities allowing for corner solutions under either the ratio or the logistic form are relatively obvious in principle and tedious to set forth in detail, and so will be omitted here. There is one other important issue to mention, however. So far only the upper-bound constraint on appropriative activity, $G_i \leq R_i$, has been considered. What of the lower-bound constraint $G_i \geq 0$, which is of course equivalent to $F_i \leq R_i$? That is, can it ever pay one or both parties to remain totally at the "peace" end of the spectrum of ongoing conflict?

A moment's thought reveals that this cannot occur if the ratio form of the Combat Power Function applies. For, using equation (3d) or any simplified version thereof, if (say) side #1 sets its $G_1 = 0$ then the other party can capture all the contestable income by choosing any positive amount, however small, for its coercive effort G_2. So no lower-bound corner equilibrium is possible. But with the logistic form it is entirely possible to have an equilibrium in which both sides devote zero effort to coercive appropriative activity. Such a situation is illustrated in Fig. 4c, where what might be called "notional" Reaction Curves for the parties are defined only within the respective impermissible negative regions. The numerical illustration on line 5 of Table 1, employing as it does the same resource magnitudes as lines 3 and 4, reveals that the non-coercive equilibrium comes about when the mass effect parameter k is sufficiently small. (In the situation illustrated the "notional" Reaction Curves intersect exactly at the origin, which occurs for $k = .00667$.) As a final point worth noting, while under the logistic Combat Power Function a "total peace" equilibrium is possible in which both sides engage in zero coercive effort, it cannot come about that only one side withdraws from fighting.[22] The underlying reason is that when the Combat Power Function is symmetrical ($b_1 = b_2$) the marginal products of coercive effort are always equal. So, despite possible resource inequalities it will never pay one party to put in less combat effort than its opponent -- unless, indeed, an upper-bound constraint is hit.

C. ASYMMETRICAL EQUILIBRIUM: THE STACKELBERG SOLUTIONS

Suppose, in contrast with the Cournot assumption, that one side moves first, the other last. Under Cournot conditions each party was making its choice in awareness of the other's decision, but here only the second-mover is in that position. The Stackelberg solution to this asymmetrical interaction is based upon the premise that the second-mover or "follower" responds rationally, and therefore predictably (in accordance with the Reaction Curve), to the choice selected by the first-mover or "leader". So, in effect, the leader can optimize along the follower's Reaction Curve. Among the questions of possible interest are the following: (i) Can it be said generally whether it is the leader or the follower who tends to put in the larger coercive effort? (ii) Which way does the advantage lie in terms of income achieved? (iii) Does it make a difference if it is the richer side, or alternatively the poorer one, that becomes the leader? (iv) In comparison with the Cournot solution, do the Stackelberg outcomes tend to come closer to the Pareto-efficient social optimum (which occurs, of course, when $G_1 = G_2 = 0$)? Finally, the Stackelberg equilibrium is not the only solution concept possible for the asymmetrical two-party interaction. A very important alternative -- the Threat-and-Promise solution -- will be taken up in the next section.

It might at first seem obvious that, where the conditions for the Stackelberg equilibrium apply, the advantage must lie with the leader. But this is not necessarily the case. In the traditional analysis of duopoly, for example, in some important situations it is the leader who does better, in other cases the follower. If each duopolist chooses a quantity of output as its decision variable, the advantage lies with the Stackelberg leader; whichever firm is able to fix its output level first can profitably corner most of the market. But if the decision variable is price the advantage tends to lie with the follower instead. Whatever price the first-mover quotes, the opponent can just barely undercut it and take away all or most of the market.[23] Thus it is not intuitively obvious, a priori, whether in ongoing conflict equilibrium the advantage lies with the first-mover or second-mover.

Ratio Model

Starting as before with the Combat Power Function in simple ratio form, and assuming to begin with an interior solution, whichever side is the leader (player #1, let us say) will be attempting to maximize its income I_1 subject to the Reaction Curve condition RC_2 of the follower:

(13) $$\text{Max } I_1 = p_1 \cdot I = G_1/(G_1+G_2) \cdot (R_1+R_2-G_1-G_2)$$

where $$G_2 = \text{sqrt}\{G_1(R_1+R_2)\} - G_1$$

But it is unnecessary to carry out any tedious maximization, since a very simple condition can be shown to hold for either party's income along the other side's Reaction Curve, to wit:[24]

(14) $$I_1 = G_2 \text{ along } RC_2 \quad \text{and} \quad I_2 = G_1 \text{ along } RC_1$$

So player #1 as the leader can simply maximize G_2 in the RC_2 equation, and correspondingly if #2 is the leader. Either way, the result obtained is:[25]

(15) $$G_1 = (R_1 + R_2)/4 = G_2 \qquad \text{Stackelberg equilibrium, ratio form}$$

Thus, for an interior Stackelberg solution the leader and the follower both commit the same amounts of resources to fighting and thus achieve the same level of income. Furthermore, this result is not only qualitatively but even quantitatively exactly the same as the interior Cournot equilibrium!

(Compare line 6 with line 1 of Table 1.)

What about the Stackelberg solutions when a corner equilibrium would be obtained under Cournot conditions? Without going through the details, it may easily be verified once again that -- whether the richer or the poorer side is the leader -- the Stackelberg equilibrium is identical with the Cournot solution. (Compare line 7 with line 2 of Table 1.)

Thus, if the ratio model for the Combat Power Function applies there is no advantage whatsoever to being a Stackelberg leader. In fact, no matter which side is the leader the Stackelberg equilibrium is identical with the Cournot solution.

Logistic Model

Matters are quite different, though perhaps equally unexpected, using the logistic form of the Combat Power Function. Recall that the Stackelberg solution is found by the leader maximizing his income along the follower's Reaction Curve. But we know from equations (12) above that, along either party's Reaction Curve (in the unconstrained region), the income achieved by the other side is a constant (equal to $1/k$). Since the leader attains the same level of income regardless of its coercive effort, there is no determinate Stackelberg equilibrium in pure strategies. More precisely, for the leader (player #1, say) income I_1 is determinate but the optimal coercive effort G_1 is not. It follows that, for the follower, both income I_2 and coercive effort G_2 are also indeterminate. There is no need to linger over the details, but illustrative examples appear in lines 8 and 9 of Table 1.

D. ASYMMETRICAL EQUILIBRIUM: THREAT-AND-PROMISE SOLUTIONS

The Stackelberg solutions to the ongoing conflict problem are, we have seen, seriously unsatisfactory. Even where a determinate equilibrium exists, no advantage seems to attach to being the "leader". Evidently, there is no solid basis for assuming that having the first move in the Stackelberg sense is equivalent to possessing a real leadership role. On the contrary, I will maintain, for a leadership role what is essential is not moving first but rather "having the last word".[26]

More specifically, the Threat-and-Promise (TAP) leadership role to be analyzed here involves contingent precommitment to respond in specified ways to the other party's behavior.[27] It is almost the reverse of the Stackelberg protocol. The Stackelberg leader first selected a level of appropriative (coercive) effort and then allowed the follower to make an optimizing response. In contrast, the TAP leader (to avoid confusion, let us call him or her the "director") allows the other player (the "subordinate") first choice of the quantitative decision variable, i.e., of coercive effort. But before doing so the director will have prespecified a reaction function, a set of threats and promises, that will govern his last-word responses to the subordinate's choice.

Of course, the director wants to select the best reaction function for maximizing his own income. In the present context the director (player D) obviously wants to induce the subordinate (player S) to choose a minimal coercive effort $G_s = M$. (Ideally, the director would like to induce $M = 0$, but this may not always be achievable.) To this end the director must promise to respond in a rewarding way if the subordinate chooses $G_s = M$, and must threaten something unpleasant if any $G_s > M$ is chosen.

It is worthwhile digressing for a moment on threats and promises. These are both pledges to respond in a contingent way to another party's actions. A promise or a threat must be an undertaking to do something you would not otherwise be motivated to do. Put another way, issuing a threat or a promise is a statement that you will not respond to another's action simply in accordance with your own later self-interest, since you can't usefully threaten or promise to do something that you would be wanting to do anyway. (This raises the question of what it is that makes a threat or promise credible, an issue that will be taken up later on.) In the present context the director undertakes not to respond along his standard Reaction Curve RC_d to the subordinate's choice of G_s, but instead to carry out his contingent threats and promises. To avoid confusion with the ordinary Reaction Curve of the Cournot and Stackelberg protocols, let us call the director's prespecified reaction function his Threat-and-Promise (TAP) function.

The threat part of the TAP function is most easily analyzed if the director undertakes to respond with "massive retaliation": in the event that the subordinate chooses any $G_s > M$, the director engages himself to respond with a maximum possible level of coercive effort, $G_d = R_d$.[28] The promise part of the TAP function is a pledge to do something nice, at least relatively speaking, if the subordinate chooses $G_s = M$. But of course the director does not want to be any nicer than he has to be. In fact, the amount that must be promised to the subordinate need only be infinitesimally greater than the maximum the latter could achieve by defiant behavior. We can formalize this as follows:

$$(16) \qquad \begin{cases} G_d = R_d & \text{if } G_s > M \\ G_d = G_d^* & \text{if } G_s = M \end{cases}$$

where G_d^* is determined by (writing I_s as a function of G_s given G_d):

(17) $$I_s(M|G_d^*) = I_s(RC_s|R_d)$$

That is, the subordinate will be permitted to earn by compliance an income level I_s equal to (or, rather, just infinitesimally greater than) what he can achieve by making his best response along his Reaction Curve RC_s to the director's "massive retaliation" choice of $G_d = R_d$.

Ratio model

When the Combat Power Function takes the ratio form, the subordinate cannot actually be induced to set his minimal $G_s = M$ equal to zero. For, were he to do so, his p_s and I_s would also be zero for any positive coercive effort G_d chosen by the director. (The possibility of side-payments is ruled out in this analysis.) On the other hand, any positive M, no matter how small, is in principle achievable. Suppose, for concreteness, that there is some threshold level of coercive effort, the minimum nonzero amount possible, and that this threshold is $M = 1$. In Table 1, lines 10 and 11 represent illustrative compliance solutions, the richer side (#1) being the director in the first case and the poorer side (#2) in the second. (Only interior-equilibrium cases are shown, since the corner equilibria do not raise any new issues in this context.)

Logistic model

Using the logistic form, the subordinate still obtains positive coercive success p_s even if he chooses zero coercive effort. Hence, we might expect a TAP director to be able to induce $G_s = M = 0$. The numerical illustrations on lines 12 and 13 of Table 1 indicate that this is indeed the case. (Once again, only interior-equilibrium cases are shown.)

The numerical examples reveal that results under the Threat-and-Promise protocol differ significantly, in a number of respects, from those achieved under the Cournot and Stackelberg solution concepts. First, all four TAP cases show a tremendous advantage to being the director rather than the subordinate. Second, the question arises whether it is the wealthier or the poorer party who has more to gain from being the TAP director. It turns out there are two countervailing forces: on the one hand the wealthier player can issue a more powerful threat in the event of non-compliance, but on the other hand the less well endowed side has more to gain by forcing the opponent to minimize his fighting effort. It turns out that the first of these forces dominates in the ratio case. Starting with the equal-income (interior) Cournot outcome on line 1 of Table 1, by comparing lines 10 and 11 we see that the richer player #1 gains more from being director than does the poorer player. But the other force dominates in the logistic model; comparing lines 12 and 13 with the logistic equal-income (interior) Cournot outcome of line 3, now it is the less well endowed player #2 who reaps a far larger gain from being director. Third, when the ratio form of the Combat Power Function applies, much less coercive effort in aggregate takes place under the TAP protocol than under Cournot or Stackelberg conditions (compare line 10 or 11 of the Table with lines 1 and 6 for the corresponding Cournot and Stackelberg outcomes). Thus the Threat-and-Promise protocol brings the parties much closer to Pareto-efficient outcomes. On the other hand, when the logistic form is used the opposite may hold! And in fact line 12 of the Table (where the richer party is director) shows greater waste of resources in appropriative effort than under the corresponding Cournot outcome in line 3 of the Table, while line 13 (where the poorer side is director) resembles the ratio cases in showing an improvement over the Cournot outcome.

In view of the advantage always gained by the director, why do we not _always_ observe Threat-and-Promise or, we may say, "hierarchical" equilibria? One major reason is the problem alluded to above, that in order to carry out his commitments the director is required to act in a way inconsistent with his ex-post self-interest. So critical is this difficulty that one may wonder how a director's pledges -- threats or promises -- to respond contingently to the subordinate's behavior can ever be made credible. But recall that we are concerned here with patterns of _ongoing_ conflict, in which all sides are supposed to be taking a long-term point of view. Though there may indeed be a short-run advantage at any single moment of time for the director to default upon his commitments, doing so would destroy what is for him a permanent advantageous arrangement.

Nevertheless, absent the idealized stationarity of the models dealt with here, the TAP solution must inevitably be somewhat fragile. It rests, after all, upon the subordinate's faith that the director will always resist the temptation to realize an immediate short-run gain. Such faith may be hard to sustain in a real world where payoffs and attitudes and personnel are subject to continuous change, often unpredictable in advance and even difficult to substantiate after the event.

How the TAP solution can be sustained over time is related to the question of _which_ player becomes the director. I have argued elsewhere[29] that the emotions, which economists usually disregard as mere awkward obstacles to fully rational behavior, may serve the function of guaranteeing the execution of threats and promises. The "charismatic" quality we look for in our leaders may be an extraordinary capacity to transcend short-run self-interested motivations. If a director is passionately driven by sentiments of magnanimity in response to submission, or outraged anger in the event of defiance, subordinates can be assured that his threats and promises will indeed be carried out. Thus an effective ruler ought not after all follow Machiavelli's advice "not to keep faith when by so doing it would be against his interest, and when the reasons which made him bind himself no longer exist" (The Prince, XVIII). A prince whose pledges cannot be relied on will be less able to induce mutually advantageous cooperation from subordinates and followers.

E. CONCLUSIONS AND INTERPRETATIONS

In view of the evident importance of the subject, it is astonishing how little has been done toward modelling the political economy of conflict.[30] Political scientists, while they have of course produced an enormous literature on conflict, have only recently been developing a tradition of formal theorizing. And the economists, whose selection and training equip them for such analysis, have suffered from a "harmonistic bias" that puts the study of conflict outside their explanatory jurisdiction. The one area where economists and political scientists have succeeded in generating an exciting body of theory on the borderline between the two disciplines, the new field of public choice, provides an illuminating instance. As critics have noted, the central tradition of public choice represents a harmonious quasi-market approach to politics. All agents are assumed to follow the constitutional rules of the game, whether it be majority voting or whatever, as if no enforcement were required. In contrast, the postulate adopted here -- perhaps equally extreme, but in the opposite direction -- is that the only rule that governs in social relations, the only ultimate deciding principle, is force or the credible threat thereof. I have attempted in this paper to show how this aspect of political economy, although scarcely investigated heretofore, is also entirely amenable to analysis in the spirit of economic reasoning.

The main contributions I would claim for this paper lie more in terms of method and approach than specific results. The underlying premise is that individuals, groups, or nations are rarely if ever totally "at war" or totally "at peace", but instead will find it advisable to divide their efforts between productive activity (which generates the income available to them jointly) and appropriative activity (through which each side aims at capturing a larger share.) First and foremost, the analysis outlines the broad features of the steady-state general equilibrium that ensues when the parties interact both productively and conflictually. A second novel point is the crucial role played by the Combat Power Function, which relates the parties' respective coercive efforts to the partition of the overall income. And in particular, the paper explored the strikingly differing consequences when coercive success is a function of the ratio of the respective resources committed to coercive, or alternatively depends upon the difference between the resources respectively committed (leading to an equation in logistic form). A third important feature of the paper is the exploration of different solution concepts, to wit, comparison of the symmetrical Cournot solution versus the asymmetrical Stackelberg and Threat-and-Promise (TAP) outcomes.

While it would be premature to place very great weight upon the specific results obtained, a number of them are surely intriguing, if only to set up a challenging target that other analysts can aim at refuting:

1. In the symmetrical Cournot cases, with both sides identical except for initial disparities in resource availabilities, at any interior solution the poorer-endowed and better-endowed parties engage in exactly the same amount of coercive effort and so end up with the same final income. The advantage of having larger initial resources makes its appearance only when the poorer side is forced to a corner solution (in which it devotes all of its endowed resources to coercive power), the wealthier party then being able profitably to commit a still larger amount. Typically, such a corner solution occurs only when the initial wealth disparity is rather extreme or the "mass effect parameter" makes the marginal product of combat power very large.

2. In the Cournot solutions using the ratio form of the Combat Power Function, both parties always find it advantageous to devote a positive amount of resources to coercion, whether at an interior or a corner solution. When the logistic equation governs instead, both sides -- but never only one side -- may find it optimal to commit no resources at all to combat. So in this latter case "total peace" is indeed a possibility, becoming more likely as the mass effect parameter declines.
3. The asymmetrical Stackelberg protocol leads to surprising and somewhat unsatisfactory results. Surprising in that, quite commonly, the Stackelberg leader (the party having the first move) ends up no better off than the follower. Unsatisfactory, since in a number of important cases the solution is indeterminate. The analysis here casts doubt upon the nature of the "leadership" defined by the Stackelberg model.
4. An alternative leadership concept, termed Threat-and-Promise (TAP), generates more interesting and useful results. The TAP "director" commits himself in advance to a reaction function, and then permits the subordinate to make the first move. It is always advantageous to be the director rather than the subordinate. This then raises the question of who becomes the director, and what is required in order to make his threats and promises credible, topics that could only briefly be treated. A further unexpected point is that the hierarchical TAP arrangement greatly reduces the resource wastage due to fighting when the ratio form of the Combat Power Function applies, but may have the opposite effect when the logistic form holds.

As in all attempts to model complex phenomena, the necessity of making a host of special assumptions limits the applicability of the results attained. And, in particular: (i) The paper analyzes only two-party interactions (and thus does not address issues like alliances or the balance of power);[31] (ii) Full information is assumed throughout, so that factors like deception have been set aside;[32] (iii) The simplified mathematical forms employed for the Combat Power Function do not allow for differences between offensive and defensive weapons, between ground and naval forces, between battle-seeking and Fabian tactics, etc. Furthermore, no provision has been made for battle damage to the resource base, for the possible existence of refuges and sanctuaries or, more generally, for differing kinds of resources with differing vulnerabilities to damage or appropriation by the enemy. (iv) The assumed steady-state equilibrium rules out strategic issues involving timing, e.g., escalation. (v) The effects of distance or of other geographical factors have not been considered.

Most of these limitations (and others could be added to the list) are not really fundamental and, indeed, point the way toward natural generalizations of the basic model offered here. Certain other of the limitations may represent more fundamental flaws, indicating that something more radical in the way of modification is needed. Even so, I submit, the analysis demonstrates the utility of the economic approach to conflict -- whose main themes are (a) <u>the private search for advantage</u>, in which each side is always prepared to employ either warlike or peaceful techniques, depending upon profitability and (b) <u>the overall social balance or equilibrium</u> that results from the associated levels of productive and coercive efforts generated by the contending parties.

A final word on the interpretation of these results. As has been indicated, the model here is more a <u>framework of analysis</u> than a specific "theory." Take the result emphasized above, that in all interior solutions under Cournot interactions the two parties end up with the same final income despite any initial resource disparities. It would be going too far to describe this proposition as a "prediction" emerging out of the theory outlined. For, the special assumptions made -- e.g., that the respective resource bases on each side are invulnerable to attack -- rule out a number of factors that might cut in the opposite direction. Nevertheless, it is surely useful to see that there is an important force working against the seemingly obvious implication that conflict necessarily tends to "make the rich richer and the poor poorer." And indeed, the idea having been suggested, it is not difficult to think of real-world analogs. Consider the common situation in early human history, of a city with relatively advanced industry surrounded by nomadic warlike tribes. The nomads typically found it advantageous to regularly raid or otherwise prey upon the city-dwellers. Thus, the initial wealth disparity between wealthy city and poor nomads was systematically moderated through an ongoing process involving actual combat and/or coerced tribute.

One other real-world implication may be illuminating. Among the modes of social organization observed in animal societies are <u>territoriality</u> and <u>dominance</u>. In the terminology here, the former represents a "symmetrical" and the latter an "asymmetrical" situation. It has been observed that when ecological pressures increase -- owing, perhaps, to overcrowding -- a shift often takes place from a territorial toward a hierarchical dominance pattern.[33] This is readily explicable in terms of the present analysis. The resource wastage due to fighting tends to be, at least when the ratio form of the Combat Power Function applies, a bigger social burden in the symmetrical (Cournot) solutions than in the hierarchical (Threat-and-Promise) solutions. So when ecological pressures become sufficiently great, even accepting a <u>subordinate</u> role in a hierarchical social arrangement may be more profitable than equal status in a world where too many of society's scarce resources are consumed in fighting.

FOOTNOTES

[1]For some modern analytical approaches see Tullock [1974], Wittman [1979], Bueno de Mesquita [1981], and Hirshleifer [1987a] or, in the more traditional literary style Blainey [1973]. Closest to the analysis here, in explicitly allowing for alternative "peaceable" resource uses, is Brito and Intriligator [1985], and there are certain parallels with Becker [1983].

[2]On legal conflicts see, for example, Gould [1973] and Priest and Klein [1984]. On industrial conflicts see Ashenfelter and Johnson [1969].

[3]Classic references include Schelling [1960], Kahn [1961], and McGuire [1967].

[4]Compare Clausewitz [1984 (1832)], Book 1, Ch. 1: "War is merely the continuation of policy by other means."

[5]To some extent the difference between the "disjoint" and the "ongoing" approaches is just a matter of perspective. At any moment of time a primitive tribe may either be peacefully exploiting its territory or else warring upon its neighbors. But over a longer period, it can be perceived, the tribe is choosing to devote a certain proportion of its efforts to each of the two types of activity.

[6]See also Danielsen [1975].

[7]See, for example, Buchanan, Tollison, and Tullock [1980].

[8]The consequences of benevolence and malevolence for the likelihood and scope of conflict are examined in Hirshleifer [1987b].

[9]These contests diverge from the allegory in being somewhat destructive rather than merely coercive. A more accurate instance of the allegory might be a potential lawsuit settled out of court, in accordance with the parties' anticipated though untested "combat power" in the judicial arena. Even in warfare proper, however, nearly bloodless contests are not unknown, e.g., the hostilities among the 15th-century Italian states that attracted the scorn of Machiavelli (see Gilbert [1943], pp. 12-14).

[10]In the allegory of the fishing villages, negative complementarity ($0 < s < 1$) is the most likely assumption. If the fish are not in infinite supply, each village's fishing boats, by thinning out the quarry, reduce the other side's marginal return from fishing.

[11]In Tullock [1980], the resources devoted to appropriative activity (to rent-seeking, in his particular application) determine a probability of winning the prize. For a somewhat analogous analysis of the probability of victory in military combat, see Helmbold [1969]. Given a large number of distinct individual encounters, probability of victory translates into the proportionate measure of appropriative success employed here. In the main-line military operations-research literature, battle interactions are more usually analyzed via the famous Lanchester equations (Lanchester [1956 (1916)]) in which the respective force sizes determine attrition rates over time. The Lanchester equations are deterministic, so that one side or the other wins totally. While probabilistic versions can be and have been developed, the Lanchester formulation does not fit very well into a steady-state model of continuing conflict.

[12]Among rational contenders, actual destructive fighting is only needed as a demonstration of coercive power. Compare Clausewitz [1984 (1832)], p. 97: "[E]ven if no actual fighting occurs, ... the outcome rests on the assumption that if it came to fighting, the enemy would be destroyed .... The decision by arms is for all major and minor operations in war what cash payment is in commerce."

[13]For the generalized logistic as for the generalized ratio form, it is easy to verify that the proportions p_i sum to unity.

[14]For examples in very different contexts see Helmbold [1969] and Tullock [1980]. Tullock also explored certain implications of differing values for the "mass effect" parameter m. For the deterministic Lanchester equations as well, the outcome can be shown to be solely a function of the force ratios (see Brackney [1959]).

[15]Taking the second derivative of $p_1 = G_1^m/(G_1^m + G_2^m)$ w.r.t. G_1 (G_2 being held constant), and setting equal to zero, after standard manipulations we can obtain:

$$G_1/G_2 = [(m-1)/(m+1)]^{1/m}$$

Since m cannot meaningfully be zero or negative, we see that, for given G_2, the point of inflection occurs in the positive range of G_1 only if $m > 1$. A corresponding analysis of course applies for p_2.

[16]Here $p_1 = 1/[1 + \exp\{k(G_2 - G_1)\}]$. Setting the second derivative w.r.t. G_1 equal to zero as before, standard manipulations lead to the simple result: $G_1 = G_2$. Thus the point of inflection occurs where $p_1 = 1/2$. The same analysis of course also holds with regard to p_2.

[17]The Cournot equilibrium remains controversial mainly because it postulates an unreasonable "conjecture" on each side about the other party's response. Specifically, each side is supposed to make its decision in the belief that the other will not react. But then, it is assumed, the other player does indeed respond, making the same mistaken assumption about the first player's reaction, and so on until equilibrium is reached. It is true that the zero-reaction conjectures are consistent at the equilibrium point, but not elsewhere. So the dynamic process leading the parties to the equilibrium is logically flawed. For an analysis of "consistent conjectures" equilibrium see Bresnahan [1981].

[18]Player #1 maximizes $I_1 = p_1(G_1|G_2) \cdot I(F_1|F_2)$ subject to $F_1 + G_1 = R_1$, where $p_1 = G_1/(G_1+G_2)$ and $I = F_1 + F_2$. Following the usual Lagrangian procedure, the first-order conditions are:

$$\partial I_1/\partial F_1 = p_1 \cdot \partial I/\partial F_1 - \lambda = 0$$

$$\partial I_1/\partial G_1 = I \cdot \partial p_1/\partial G_1 - \lambda = 0$$

from which we obtain: $I \cdot \partial p_1/\partial G_1 = p_1$. Taking the derivative and substituting leads to the equation in the text:

$$G_1/G_2 = (F_1+F_2)/(G_1+G_2)$$

A similar result also holds of course for player #2.

[19]The Reaction Curves also intersect at $G_1 = G_2 = 0$, but it is easy to verify that the zero-zero intersection in Fig. 3a is not a Cournot equilibrium. Owing to the ratio form of the Combat Power Function, the probabilities of success p_1 and p_2 are indeterminate when $G_1 = G_2 = 0$. It might at first seem natural to assume that at the origin $p_1 = p_2 = 1/2$, since that is the value approached as G_1 and G_2 go to zero together. But if (say) player #1 chooses $G_1 = 0$, then player #2 would rationally respond

by setting G_2 equal to any small positive magnitude -- since doing so discontinuously improves his fighting success from 50% to 100%. It follows, therefore, that the respective Reaction Curves are defined only over the open interval that does not include the singular point at the origin.

[20]The maximization procedure for player #1 is just the same as before, leading once again to the condition: $I\cdot(\partial p_1/\partial G_1) = p_1$. But now we use the logistic form for p_1:

$$p_1 = 1/[1 + \exp\{k(G_2 - G_1)\}]$$

Taking the derivative and substituting as before, we obtain the text equation:

$$G_1 = G_2 + (1/k)\ln\{k(F_1 + F_2) - 1\}$$

And of course an analogous result holds for player #2.

[21]From the equation defining RC_2, we know that:

$$G_2 - G_1 = (1/k)\ln\{k(F_1 + F_2) - 1\}$$

But $p_1 = 1/[1 + \exp\{k(G_2 - G_1)\}]$, so that:

$$\begin{aligned} I_1 = p_1\cdot I &= (F_1 + F_2)/[1 + \exp\{k(F_1 + F_2) - 1\}] \\ &= (F_1 + F_2)/[1 + k(F_1 + F_2) - 1] \\ &= 1/k \end{aligned}$$

A similar result of course holds for I_2 along RC_1.

[22]The intersection of the Reaction Curve equations, without the boundary conditions, must of course occur along the 45^o line where $G_1 = G_2$. If the intersection occurs in the interior, the G_i are both positive; if at the origin, they are both zero. The only other possibility is a "notional" intersection in the third quadrant. But of course neither player can ever choose a negative fighting effort. So under the conditions leading

to such a "notional" intersection (i.e., for k sufficiently low), the best that each player can do is to set $G_1 = G_2 = 0$.

[23]Actually, in this situation there is no Stackelberg equilibrium in pure strategies, a problem that will recur in our analysis below.

[24]We can start with player #2's Reaction Curve in the form:

$$G_2/G_1 = (F_1 + F_2)/(G_1 + G_2)$$

But $F_1+F_2=I$ and $p_1=G_1/(G_1+G_2)$, so that player #1's income is:

$$G_2 = p_1 \cdot I = I_1$$

An analogous result as usual holds for player #2's income along RC_1.

[25]Since $I_1=G_2$ along RC_2 (from the previous footnote), we know that $dI_1/dG_1 = dG_2/dG_1$. Thus:

$$dI_1/dG_1 = .5 \text{ sqrt}\{(R_1+R_2)/G_1\} - 1$$

Setting the derivative equal to zero and solving, we have for player #1 as leader:

$$G_1 = (R_1+R_2)/4$$

Player #2 as follower reacts along RC_2 so that:

$$G_2 = \text{sqrt}\{(R_1+R_2)G_1\} - G_1$$

Making the substitution we see that $G_2 = G_1$, and of course this is true regardless of which player is the leader.

[26]See also Hirshleifer [1977].

[27]The development here in part follows Thompson and Faith [1981].

[28]This is the maximal threat. Alternatively, there is a minimal threat that would just suffice to induce the desired behavior on the part of the subordinate. In a full-knowledge situation the results are the same either

way, since with proper calculations the threat would always succeed so that no fighting actually occurs. But a minimal threat leaves no margin for error, if the opponent's payoffs or resolve have been miscalculated. On the other hand, if the miscalculation is sufficiently serious even a maximal threat might fail to deter, in which case needlessly great damage will be inflicted when the threatened punishment has to be executed. These considerations have real-world relevance for the design of deterrent ("Mutual Assured Destruction") systems.

[29] Hirshleifer [1987b].

[30] Following the seminal work of Lanchester [1956(1916)], a considerable literature now exists on the mathematical modelling of battles (see, for example, Taylor [1981] and Shubik [1983]). But such operations-research studies do not get into the larger issues examined here -- in particular, how to characterize an equilibrium in which contending parties allocate their endowed resources between combat power and productive activity.

[31] On the balance of power see especially Bernholz [1985].

[32] See, for example, Brams [1977].

[33] See, e.g., Davis [1958] and Krebs [1971].

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Table 1

	R_1	R_2	G_1	G_2	P_1	P_2	I_1	I_2	I	Nature of Solution
COURNOT EQUILIBRIUM										
Ratio Model										
1. $R_1 \geq R_2 \geq R_1/3$	200	100	75	75	.5	.5	75	75	150	Interior (Fig. 3a)
2. $R_2 < R_1/3$	200	40	58.0	40	.592	.408	84.0	58.0	142.0	Corner (Fig. 3b)
Logistic Model										
3. k = .01333	200	100	75	75	.5	.5	75	75	150	Interior (Fig. 4a)
4. k = .025	200	100	109.4	100	.558	.442	50.6	40	90.6	Upper-constrained (Fig. 4b)
5. k = .00667	200	100	0	0	.5	.5	150	150	300	Lower-constrained (Fig. 4c)
STACKELBERG EQUILIBRIUM										
Ratio Model										
6. $R_1 \geq R_2 \geq R_1/3$	200	100	75	75	.5	.5	75	75	150	Interior (whichever player leads)
7. $R_2 < R_1/3$	200	40	58.0	40	.592	.408	84.0	58.0	142.0	Corner (whichever player leads)
Logistic Model (k = .01333)										
8. #1 leads	200	100		Indeterminate			75	Indet.	Indet.	Indeterminate along RC_2
9. #2 leads	200	100		Indeterminate			Indet.	75	Indet.	Indeterminate along RC_1
THREAT-AND-PROMISE EQUILIBRIUM										
Ratio Model										
10. #1 is controller	200	100	26.0	1	.963	.037	262.9	10.1	273.0	Subordinate sets $G_2 = M = 1$
11. #2 is controller	200	100	1	4.5	.182	.818	53.6	240.9	294.5	Subordinate sets $G_1 = M = 1$
Logistic Model (k = .01333)										
12. #1 is controller	200	100	198.9	0	.934	.066	94.4	6.7	101.1	Subordinate sets $G_2 = M = 0$
13. #2 is controller	200	100	0	85.2	.243	.757	52.2	162.6	214.8	Subordinate sets $G_1 = M = 0$

Figure I: F-technology determines income I and G-technology determines fractional divisions p_1, p_2

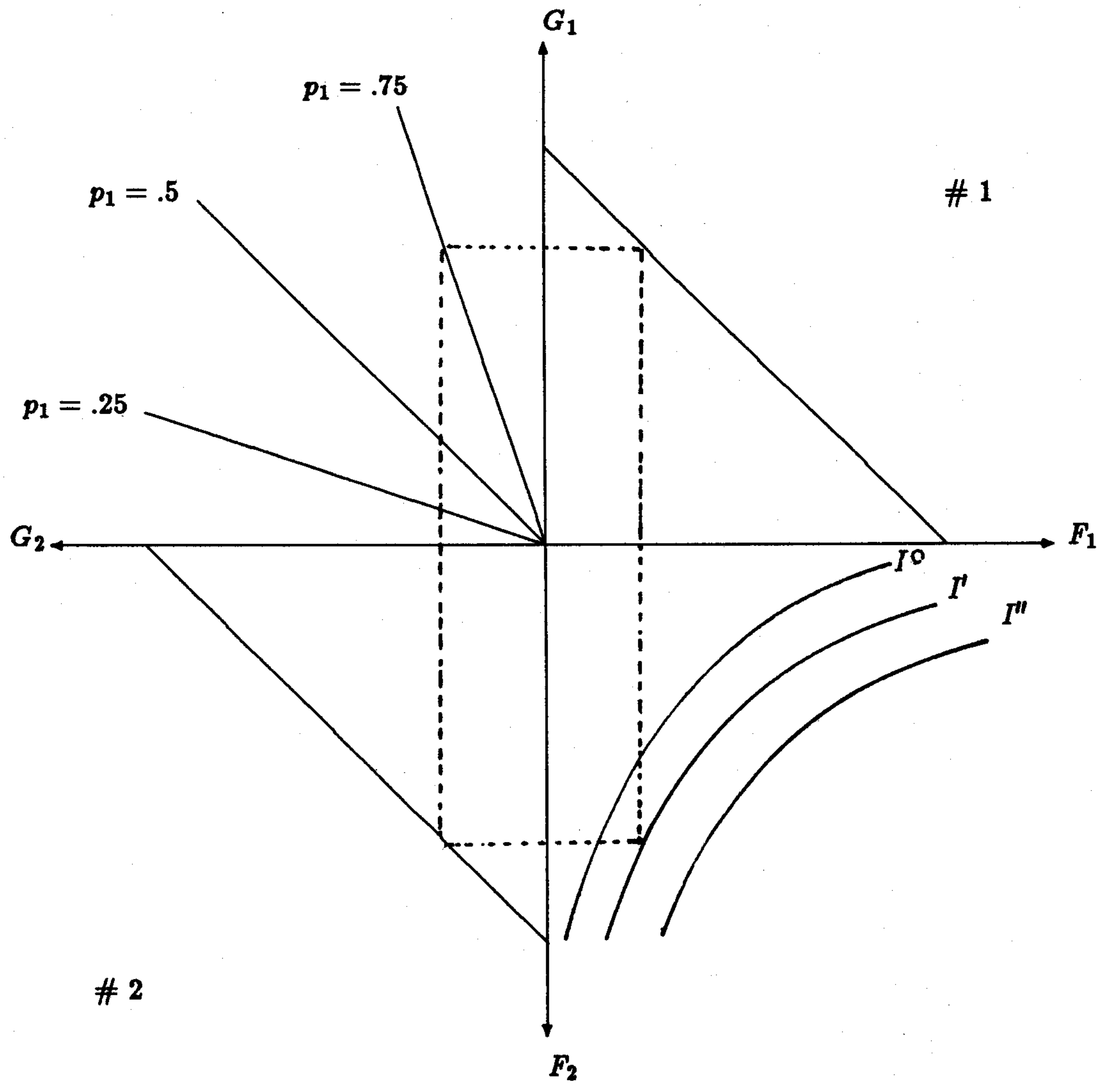
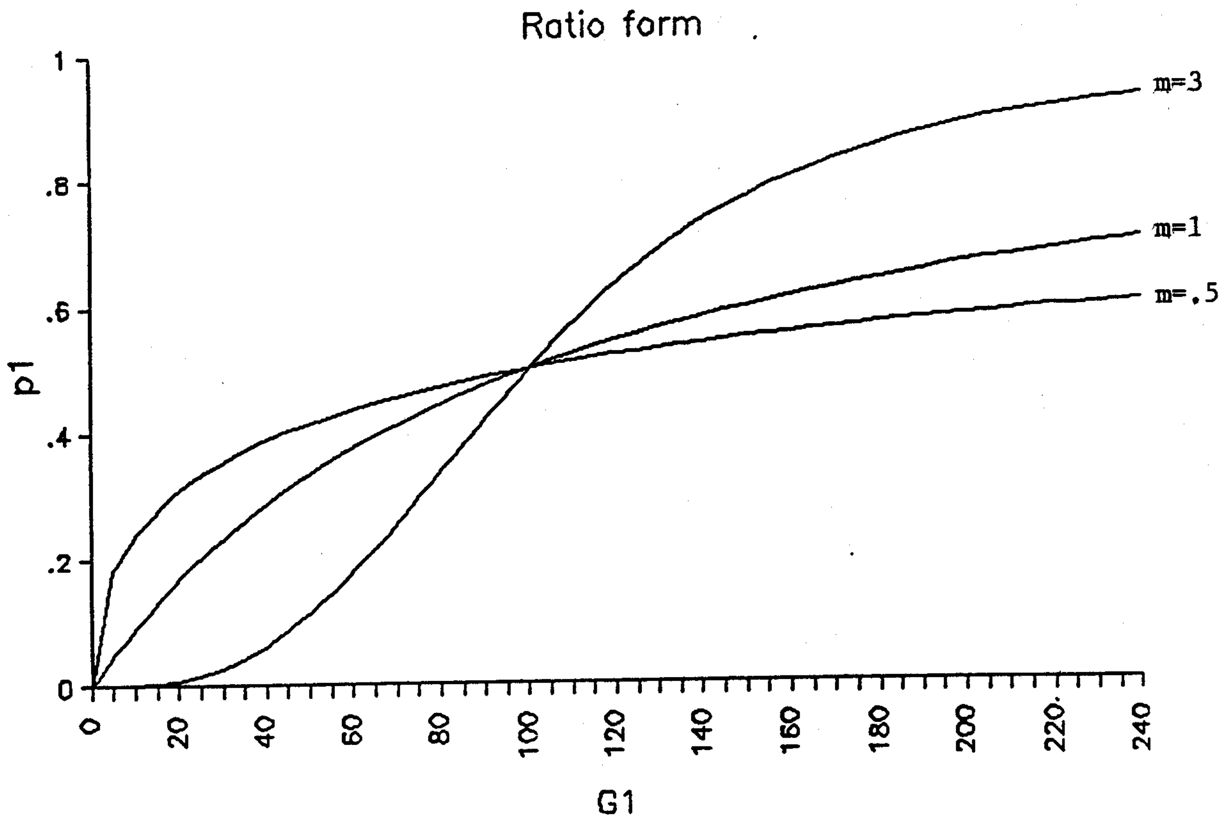


Figure 2

COMBAT POWER FUNCTION



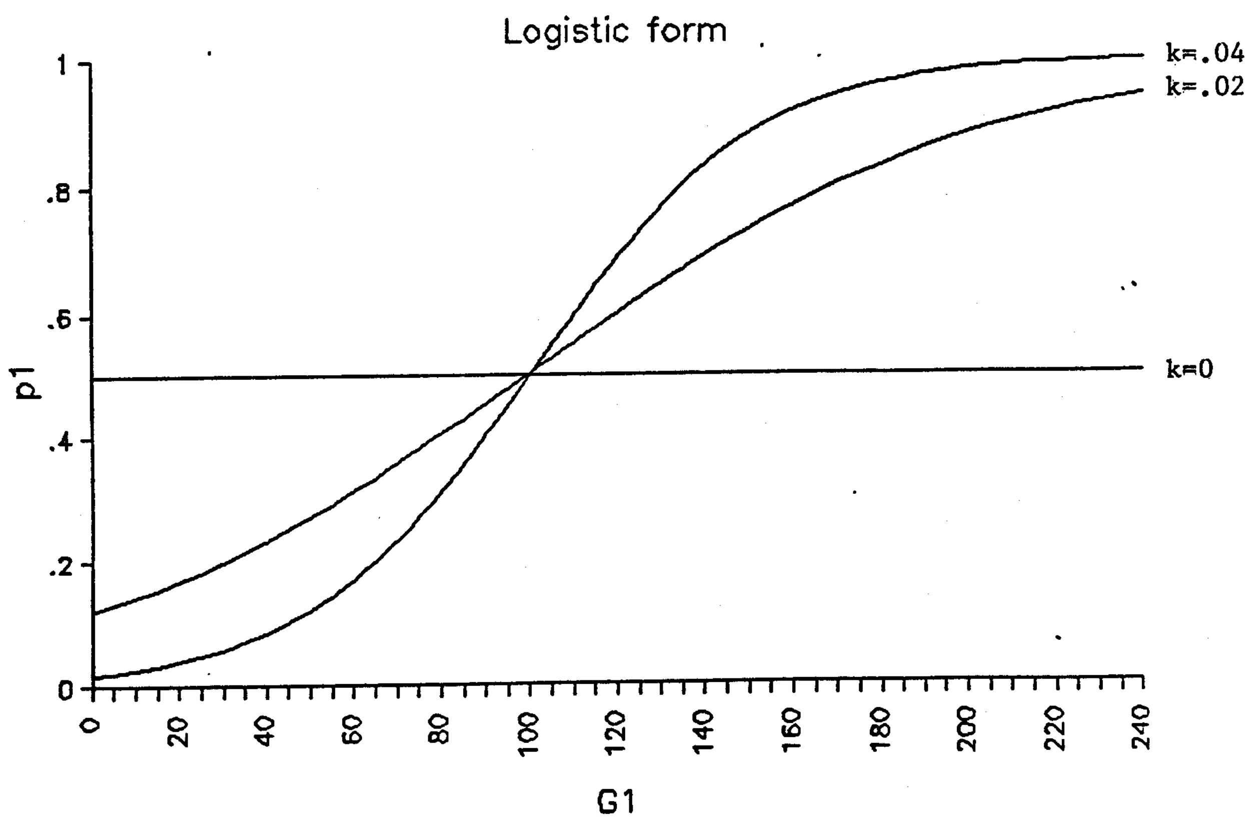
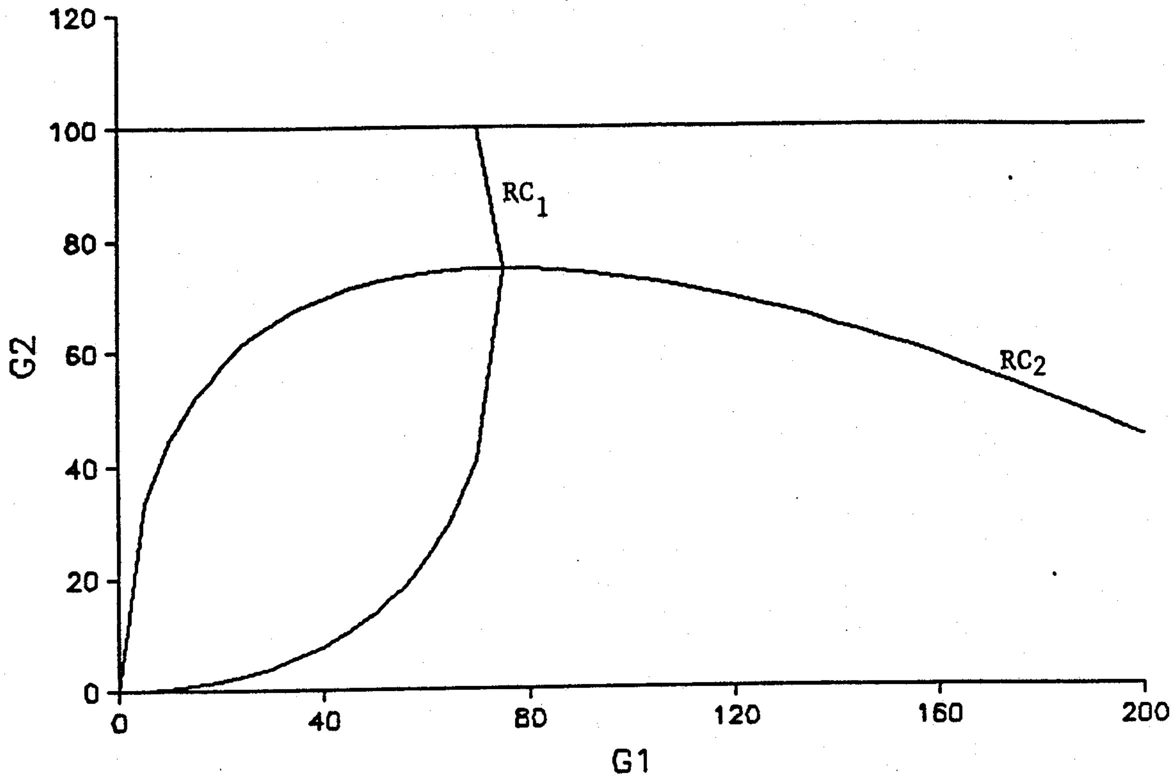


Figure 3

REACTION CURVES AND COURNOT EQUILIBRIUM

Ratio form, interior solution



Ratio form, corner solution

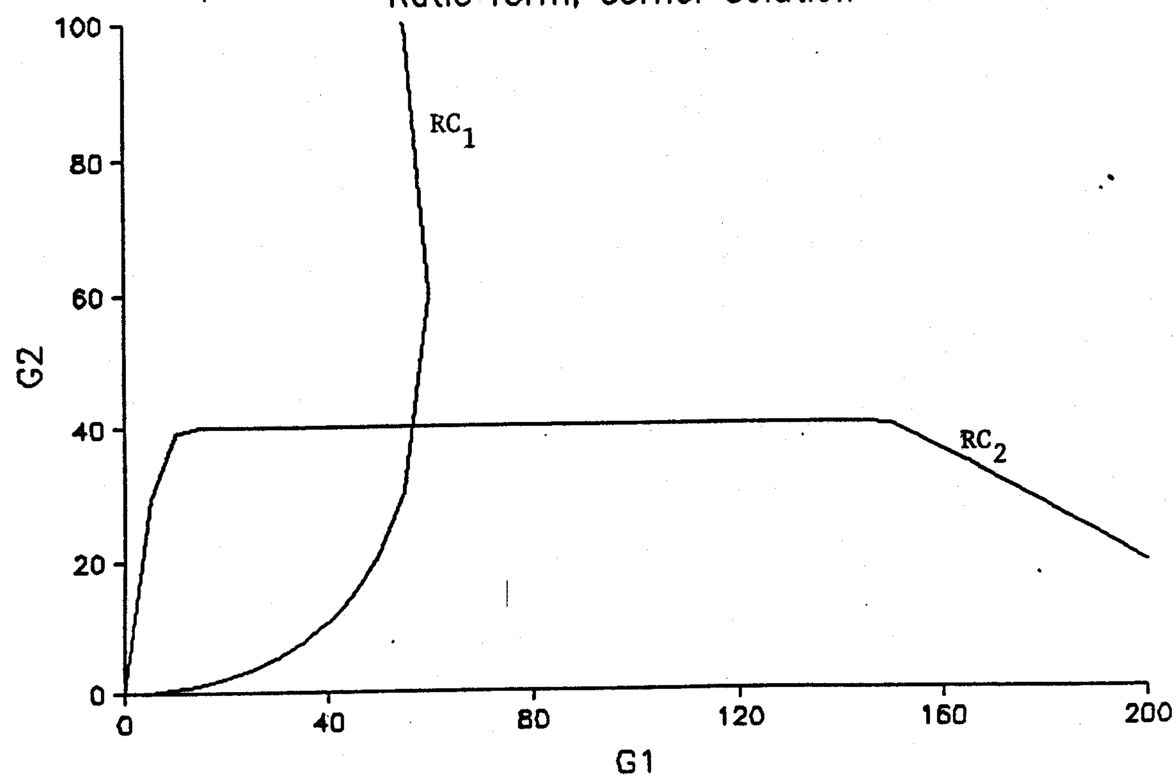


Figure 4

REACTION CURVES AND COURNOT EQUILIBRIUM