An Econometric Analysis of Exploration and Extraction of Oil in the U.K. Continental Shelf

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1. Introduction

Modelling of oil exploration and extraction is a formidable undertaking and involves important economic, geological, and political considerations. The modelling task is further complicated by the largely non-quantifiable uncertainties that generally surround the future movements of oil prices and discovery of new oil fields. As a result there are very few serious econometric studies of oil supplies, especially outside OPEC. Those that are available, however, suffer from a number of important shortcomings. For example, the studies by Epple and Hansen (1981) and Farzin (1986) fail to explicitly account for new discoveries and the effect of the exploration process on extraction costs and production decisions. The pioneering study by Fisher (1964), and the subsequent studies by Erickson and Spann (1971), Khazzoom (1971), and MacAvoy and Pindyck (1975), although dealing with the exploration process directly are, as already pointed out by Attanasi et al. (1981), rather *ad hoc* and lack a coherent theoretical and geological basis. By contrast the disaggregated process models put forward by Echbo et al. (1978), and Kaufman et al. (1981) are based on sound geologic-statistical analysis of the exploratory process, but fail to provide explicit estimates of the supply and exploration functions and their responsiveness to oil price movements. The disaggregated process models are highly data-intensive simulation models and are generally difficult to evaluate empirically.[1] In this paper, by building on the theoretical contributions of Pindyck (1978), Uhler (1979), and Devarajan and Fisher (1982), we develop an econometric framework for the analysis of exploration and production policies of "price-taking" suppliers, and derive exploration and output equations for oil which

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[1]On this point also see the comments by Ramsey (1981, pp. 330-32).

explicitly take account of the oil discovery process and the intertemporal nature of exploration and production decisions.[2] We will then apply the framework to an empirical analysis of oil exploration and extraction on the United Kingdom Continental Shelf (UKCS).

Our analysis differs from the other econometric studies in the field in a number of respects:

a) The output and exploration equations estimated in the paper are theory-consistent in the sense that they are <u>both</u> derived as solutions to a <u>single</u> optimization problem. This, for example, means that in contrast to the recent empirical works by Epple (1985) and Hendricks and Novales (1987), we do not take the (shadow) price of oil in the ground as given, but treat it as an endogenous variable and estimate it along with other parameters of the model.

b) We pay careful attention to the problem of expectations formation and consider alternative models for formation of price and cost expectations. In the case of the supply equation we present formal tests of the hypothesis that oil price expectations are formed rationally against the alternatives that they are formed adaptively, or recursively.

c) Our empirical analysis explicitly takes account of the engineering information concerning the pressure dynamics of the petroleum reserves and the geological knowledge pertinent to the discovery process, and presents formal statistical tests of the significance of these factors

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[2]This paper is concerned with non-OPEC oil supply behavior where the available recoverable oil reserves are low relative to the rate of production, and where oil price movements can be taken to be invariant with respect to the oil supply decision. The study of oil supply by the OPEC member countries presents an altogether different set of considerations, some of which have been already reviewed extensively, for example, in Fischer et al. (1975), Griffin and Teece (1982), Gately (1984), Griffin (1985), and Salehi-Isfahani (1986).

for the explanation of output and investment policies of oil companies operating on the UKCS.

The plan of the paper is as follows: Section 2 sets out the optimization framework, derives the necessary Euler conditions, and obtains output and exploration equations suitable for empirical analysis under alternative models of oil price expectations. Section 3 gives a brief account of the history of the exploration and development of oil resources in the UKCS, and presents the empirical results on the output equation in Section 3.1, and on the exploration equation in Section 3.2. The main findings of the paper are summarized in Section 4. There is also a Data Appendix that describes data sources and gives the definition of the variables used in the empirical section of the paper.

2. An Intertemporal Model of Exploration and Extraction

The economic theory of exhaustible resources, beginning with the seminal work of Hotelling (1931), has been primarily concerned with the optimal extraction of a fixed reserve base over time.[3] The question of new reserve discoveries and the interdependence of production and exploration decisions have been largely neglected. Notable exceptions are to be found in the work of Pindyck (1978), Uhler (1979), and Devarajan and Fisher (1982).[4] Pindyck considers the problem of the simultaneous determination of the optimal rates of exploratory activity and production in the context of a continuous-time model under certainty. He shows that the optimal rates of

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[3]The Hotelling work and its various extensions are surveyed, for example, in Peterson and Fisher (1977) and Dasgupta and Heal (1979).

[4]The exploration activity viewed as a method of obtaining better estimates of the size of the reserve base has also been studied, for example, by Loury (1978), Gilbert (1979) and Hoel (1978).

extraction and exploratory effort critically depend on the initial level of reserves.

The key assumption in Pindyck's analysis is the inverse relationship he postulates between extraction costs and the level of available reserves. When initial reserves are small the unit cost of extraction tends to be high and the gains from early exploration can be substantial. But if initial reserves are large the link between extraction costs and reserves will be weak and there will be little or no immediate gain from early exploration. Devarajan and Fisher (1982) allow for the uncertainty that surrounds the discovery process, and in the context of a simple two-period model show that the resource royalty or rent (i.e., the shadow price of the resource in the ground) will not in general be equal to the expected marginal discovery cost when uncertainty is present.[5] Uhler (1979) approaches the exploration and extraction decisions differently and adopts a two-stage process, whereby in the first stage the optimum level of exploratory effort is determined in terms of the price of reserves in the ground, and in the second stage the optimum level of extraction is determined in terms of the well-head price.

The difference between the well-head price and the price of reserves in the ground (or the reserve-price) furnishes the link between the exploration and extraction stages of the production process in Uhler's analysis. Although this approach may be attractive from a pedagogic viewpoint it is not suitable for empirical analysis as it is based on reserve-prices which are in general unobservable and even, assuming exogenously given well-head

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[5]But, as can be seen below, the results obtained by Devarajan and Fisher (1982) do not carry over to the multi-period case where the rate of discovery depends on the cumulative exploratory effort as well as on the current level of exploratory effort.

prices, are endogenously determined.[6]

In this section we build on the contributions of the above studies and develop a multi-period discrete-time econometric model for the analysis of exploration and extraction decisions of a price-taking firm operating under uncertainty. Later on we use this model as the basis of an econometric analysis of oil supplies and exploration activity on the UKCS.

2.1 The optimization framework

We assume that the producers operating on the UKCS are risk neutral and decide on the rates of extraction, $q_t, q_{t+1}, \ldots$; and the rates of exploratory efforts $x_t, x_{t+1}, \ldots$ by maximizing the expected discounted future streams of profits conditional on the information set Ω_{t-1}. That is

$$\underset{\substack{q_t, q_{t+1}, \ldots \\ x_t, x_{t+1}, \ldots}}{\text{Max}} \; E\left\{ \sum_{\tau=0}^{\infty} \beta^{\tau} \Pi_{t+\tau} \mid \Omega_{t-1} \right\}, \tag{1}$$

where $0 \leq \beta < 1$ is the discount factor, and Π_t is the producer's profit defined by

$$\Pi_t = p_t q_t - C(q_t, R_{t-1}) - w_t x_t, \tag{2}$$

where p_t is the well-head price. The cost of development and extraction at time t, $C(q_t, R_{t-1})$, is a convex function which varies positively with the rate of extraction, q_t, and negatively with the level of remaining proven reserves, R_{t-1}. The inclusion of remaining reserves as an argument in the cost function is justified on the basis of engineering information which is available concerning the determinants of the pressure dynamics of

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[6] This criticism also applies to the recent empirical work by Hendricks and Novales (1987).

the petroleum reserves. As is shown, for example, in Uhler (1979), the reservoir pressure, which is one of the important determinants of extraction costs, itself depends on the ratio of remaining reserves to the initial reserves. The inclusion of R_{t-1} in the cost function has important implications for the firm's extraction and exploration policies. By reducing the level of available reserves current extraction raises future extraction costs, while current exploratory efforts by adding to available reserves tend to lower extraction costs in the future. Viewed from this perspective, exploration activity can be seen as a way of keeping down marginal extraction costs in the future.

The other component of a firm's cost function is the cost of exploration which we have denoted by $w_t x_t$, where w_t stands for the unit cost of exploratory effort. The level of exploratory effort, x_t, is usually measured either by the number of exploratory wells drilled or by the square footage of exploratory drilling. In the case of the North Sea only the former measure is available.

In solving the optimization problem (1), the firm faces the following constraint:

$$R_{t+\tau} - R_{t+\tau-1} = d_{t+\tau} + e_{t+\tau} - q_{t+\tau}, \quad \tau = 0,1,2,\ldots \tag{3}$$

where d_t denotes the addition to proven reserves during the period $t-1$ to t from new discoveries, and e_t the revisions/extensions to previously discovered reserves. In general, one would expect e_t to be a function of the size of past discoveries and the extent of the development and appraisal efforts. But to simplify our analysis here we assume the process generating e_t is distributed independently of current or past values of exploratory efforts. The determination of d_t, which is the focus of the literature on the "reserve discovery process", is based on two basic assumptions:[7]

(i) The reservoir size is log normally distributed.

(ii) The exploratory process can be characterized as one of sampling without replacement in proportion to reservoir size.

Under these assumptions it is possible to derive the density function of new discoveries conditional on past discoveries. For our purposes we adopt a simplified version of Kaufman's model and specify that[8]

$$d_t = F(x_t, X_{t-1}) + \nu_t, \tag{4}$$

where X_{t-1} represents the level of cumulative exploratory effort defined by

$$X_t = X_{t-1} + x_t, \tag{5}$$

and ν_t is the unobservable (and unpredictable) component of the discovery function assumed to satisfy the orthogonality condition

$$E(\nu_t | \Omega_{t-1}) = 0. \tag{6}$$

The information set Ω_t is assumed to contain observations on at least the current and past values of R_t, p_t, q_t, x_t, and w_t.

The discovery function (4) represents a generalization of the empirical relationship obtained by Hubbert (1969) and Uhler (1976), and in view of geological characteristics of the oil discovery process is expected to satisfy the following conditions:

(i) $$\partial F_t / \partial x_t > 0, \tag{4a}$$

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[7]See Kaufman (1975), Barouch and Kaufman (1976, 1977), Eckbo (1979) and Uhler (1976).

[8]In his theoretical contribution Pindyck (1978) specifies the discovery function in terms of cumulative discoveries instead of cumulative exploratory efforts. The empirical results by Uhler (1976) and others, however, suggest that (4) may be preferable to Pindyck's specification.

(ii) $$\partial^2 F_t/\partial x_t^2 < 0, \tag{4b}$$

(iii) $$\partial F_t/\partial X_{t-1} < 0, \quad \text{for} \quad X_t \geq X_m, \tag{4c}$$

(iv) $$\lim_{X_{t-1}\to\infty} F(x_t, X_{t-1}) = 0. \tag{4d}$$

The first condition imposes that the marginal product of exploratory effort should be positive for the exploration activity to be worthwhile. Condition (ii) is the familiar diminishing marginal productivity condition. The last two conditions, (iii) and (iv), capture what is known as the "discovery decline phenomenon". Initially under the influence of accumulating geological knowledge the effect of the cumulative exploratory effort on discovery may be positive, but as exploration proceeds the effect of reserves exhaustion begins to dominate and when X_t increases beyond the threshold value of X_m, the discovery rates start to decline, even if the level of exploratory effort, x_t, is maintained. In the limit as $X_{t-1} \to \infty$, $F_t \to 0$, and from (4) it follows that the probability of making new discoveries approaches zero.

2.2 The Euler Equations

Given price and cost expectations (i.e., $p^e_{t+\tau} = E(p_{t+\tau}|\Omega_{t-1})$, and $w^e_{t+\tau} = E(w_{t+\tau}|\Omega_{t-1})$, formed at time t-1), and an initial level of proven reserves, R_{t-1}, relations (1)-(6) completely define the decision environment of the firm. The first order conditions for the firm's optimization problem can now be obtained from the necessary conditions for the unconstrained maximization of the Lagrangian form:

$$\mathcal{L} = E\left[\sum_{\tau=0}^{\infty} \beta^\tau \; G_{t+\tau} \mid \Omega_{t-1}\right], \tag{7}$$

with respect to $q_{t+\tau}$, $x_{t+\tau}$, $R_{t+\tau}$, $X_{t+\tau}$, $\tau = 0,1,2,\ldots$ where[9]

$$G_t = \Pi_t + \lambda_t(d_t + e_t - q_t - R_t + R_{t-1}) + \mu_t(X_t - X_{t-1} - x_t). \tag{8}$$

The auxiliary variables $\lambda_{t+\tau}$ and $\mu_{t+\tau}$, $\tau = 0,1,2,\ldots$ are the Lagrangian multipliers and, as we shall see below, can be interpreted respectively as the (undiscounted) shadow price of reserves in the ground (the reserve-price), and the net value of the marginal product of reserve discovery. The first order conditions, also known as Euler equations, for maximization of (7) can be written as

$$E_{t-1}\left(p_{t+\tau} - \frac{\partial C_{t+\tau}}{\partial q_{t+\tau}} - \lambda_{t+\tau}\right) = 0, \tag{9a}$$

$$E_{t-1}\left(\beta\lambda_{t+\tau+1} - \lambda_{t+\tau} - \beta\frac{\partial C_{t+\tau+1}}{\partial R_{t+\tau}}\right) = 0, \tag{9b}$$

$$E_{t-1}\left(\mu_{t+\tau} + w_{t+\tau} - \lambda_{t+\tau}\frac{\partial d_{t+\tau}}{\partial x_{t+\tau}}\right) = 0, \tag{9c}$$

$$E_{t-1}\left(\mu_{t+\tau} - \beta\mu_{t+\tau+1} + \beta\lambda_{t+\tau+1}\frac{\partial d_{t+\tau+1}}{\partial X_{t+\tau}}\right) = 0, \tag{9d}$$

$$E_{t-1}(R_{t+\tau} - R_{t+\tau-1} - d_{t+\tau} - e_{t+\tau} + q_{t+\tau}) = 0, \tag{9e}$$

$$E_{t-1}(X_{t+\tau} - X_{t+\tau-1} - x_{t+\tau}) = 0, \tag{9f}$$

for $\tau = 0,1,2,\ldots$[10] These Euler equations represent a highly nonlinear set

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[9]The validity of this approach, known as the maximum principle, in the non-stochastic case is demonstrated, for example, by Whittle (1982, Ch. 6).

[10]To simplify the notations we have used $E_{t-1}(\cdot)$ in place of $E(\cdot|\Omega_{t-1})$, and $\partial C_t/\partial q_t$ in place of $\partial C(q_t, R_{t-1})/\partial q_t$, etc. We are also assuming that the transversality conditions, $\lim_{T\to\infty} E_{t-1}(\beta^T\lambda_{t+T}) = \lim_{T\to\infty} E_{t-1}(\beta^T\mu_{t+T}) = 0$, are satisfied.

of stochastic equations and in general do not lend themselves to a closed form solution. However, for the purpose of econometric analysis it is possible to derive extraction and exploration decision rules that, under certain conditions, can be consistently estimated.[11] To see this we focus on current decision variables q_t and x_t and rewrite relations (9) for $\tau = 0$, in the following manner:

$$E_{t-1}(\lambda_t) = E_{t-1}(p_t) - E_{t-1}\left(\frac{\partial C_t}{\partial q_t}\right), \tag{10a}$$

$$E_{t-1}(\lambda_t) = \beta\left\{E_{t-1}(\lambda_{t+1}) - E_{t-1}\left(\frac{\partial C_{t+1}}{\partial R_t}\right)\right\}, \tag{10b}$$

$$E_{t-1}(\mu_t) = E_{t-1}\left(\lambda_t \frac{\partial d_t}{\partial x_t}\right) - E_{t-1}(w_t), \tag{10c}$$

$$E_{t-1}(\mu_t) = \beta\left\{E_{t-1}(\mu_{t+1}) - E_{t-1}\left(\lambda_{t+1} \frac{\partial d_{t+1}}{\partial X_t}\right)\right\}. \tag{10d}$$

Equation (10a) provides the justification for interpreting $E_{t-1}(\lambda_t)$ as the expected shadow price of oil in the ground, since at the optimum it is given by the difference between the expected well-head price and the expected marginal extraction cost. The term $E_{t-1}(\lambda_t)$ also measures the expected resource rent or royalty and, as is argued in Devarajan and Fisher (1982), provides a satisfactory measure of natural resource scarcity. Equation (10b) represents the intertemporal condition for the extraction of oil over time and gives the link between current and future expected resource rents. It states that at the optimum the expected current resource rent should be

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[11] On this see Hansen and Singleton (1982). Here we are also assuming that the solution to the Euler equations in (9) is an interior one, namely one which results in strictly positive values for x_t and q_t.

equal to the discounted value of expected future resource rent minus the discounted expected change in the extraction cost due to changes in the reserve base. In the simple case where $\partial C_{t+1}/R_t = 0$, equation (10b) simplifies to the familiar Hotelling rule and requires that at the optimum the (expected) resource rent should grow at the rate of discount, namely $(1-\beta)/\beta$. Equations (10c) and (10d) give the necessary conditions for the determination of the optimum level of exploratory activity. In view of the interpretation that we have given to λ_t (i.e., the shadow price of oil in the ground) equation (10c) defines $E_{t-1}(\mu_t)$ to be equal to the expected net return to exploration defined as the difference between the expected value of the exploratory effort and the expected unit cost of exploration. Equation (10d) then gives the intertemporal equilibrium condition for $E_{t-1}(\mu_t)$ and states that at the optimum the current expected net return to exploration should be equal to the discounted value of the future expected net return to exploration minus the discounted expected change in the value of marginal product of exploration due to the cumulative effect of exploration, which in practice may turn out to be negative.

2.3 The Output Equation

Since the resource rent (λ_t) and the net benefit to exploration (μ_t) are unobservable, we need to eliminate them from (10) to arrive at an output equation that can be analyzed empirically. For this purpose we first note from (10a) that

$$E_{t-1}(\lambda_{t+1}) = E_{t-1}\left\{E_t(p_{t+1}) - E_t\left(\frac{\partial C_{t+1}}{\partial q_{t+1}}\right)\right\},$$

which, assuming expectations are formed consistently yields

$$E_{t-1}(\lambda_{t+1}) = E_{t-1}(p_{t+1}) - E_{t-1}\left(\frac{\partial C_{t+1}}{\partial q_{t+1}}\right).$$

Substituting this result in (10b) and using (10a) to eliminate $E_{t-1}(\lambda_t)$ we now obtain

$$E_{t-1}\left(\frac{\partial C_t}{\partial q_t}\right) = E_{t-1}(p_t-\beta p_{t+1}) + \beta E_{t-1}\left(\frac{\partial C_{t+1}}{\partial q_{t+1}} + \frac{\partial C_{t+1}}{\partial R_t}\right). \tag{11}$$

This equation does not depend on μ_t or λ_t and can be consistently estimated for a given specification of the extraction cost function, $C(q_t,R_{t-1})$. In our empirical analysis we adopt the following nonlinear form which is quadratic in q_t

$$C(q_t,R_{t-1}) = \delta_0 + \delta_1 q_t + \frac{1}{2}\left(\delta_2 + \frac{\delta_3}{R_{t-1}}\right)q_t^2 + \epsilon_t q_t, \tag{12}$$

where ϵ_t represents unobserved random shocks to marginal extraction cost which are assumed to be orthogonal to the information set Ω_{t-1}, i.e., $E(\epsilon_t|\Omega_{t-1}) = 0$. On <u>a priori</u> grounds we would expect all the parameters of this cost function to have a positive sign except for δ_1 which could be negative so long as $E_{t-1}(\partial C_t/\partial q_t) = \delta_1 + (\delta_2 + \delta_3/R_{t-1})q_t > 0.$[12] Under (12,) equation (11) can now be solved for q_t^*, the optimum or the desired rate of extraction:

$$\begin{aligned} q_t^* = {} & [-(1-\beta)\delta_1/\delta_2]z_{t-1} + \delta_2^{-1}z_{t-1}\,E_{t-1}(p_t-\beta p_{t+1}) \\ & + \beta z_{t-1}E_{t-1}(q_{t+1}) + \beta\gamma z_{t-1}E_{t-1}(h_{t+1}), \end{aligned} \tag{13}$$

where

$$z_t = R_t/(R_t+\gamma), \tag{14}$$

[12]Notice that we are assuming that at the end of the period t-1, the firm knows its own desired output decision for time t.

$$h_t = (q_t/R_{t-1}) - 1/2(q_t/R_{t-1})^2, \tag{15}$$

and

$$\gamma = \delta_3/\delta_2 \geq 0.$$

This result gives the desired level of output as a function of reserves, price expectations, and firm's planned or expected future output and reserves. The relationship between the actual and the desired levels of output is governed by the cost of changing the actual level of output relative to the cost of deviating from the desired output level.[13] In the case of oil production the adjustment costs arise primarily from the need to maintain the reservoir pressure to keep down the costs. The feasible rate of extraction is generally constrained by the reservoir pressure. The maintenance of pressure in the reservoir is often costly and involves injection of fluid and gas into the reservoir, an operation known as the secondary and tertiary recovery process. As a first approximation we assume that the relationship between the actual rate of extraction and the firm's desired rate of extraction can be characterized by the following simple partial adjustment model

$$q_t - q_{t-1} = \phi(q_t^* - q_{t-1}), \quad 0 < \phi \leq 1.$$

Substituting for q_t^* from (13) in the above relation now yields

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1} E_{t-1}(p_t - \beta p_{t+1}) \tag{16}$$
$$+ \alpha_2 z_{t-1} E_{t-1}(q_{t+1}) + \alpha_3 z_{t-1} E_{t-1}(h_{t+1}),$$

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[13] In principle, the adjustment costs can be allowed for explicitly in the optimization problem (1) along the lines demonstrated, for example, in Pesaran (1987, Example 7.2). But this unduly complicates the analysis, and for simplicity of exposition here we have chosen a two-stage optimization process.

where

$$\alpha_0 = -\phi(1-\beta)\delta_1/\delta_2 \le 0; \quad \alpha_1 = \phi/\delta_2 > 0,$$

$$\alpha_2 = \phi\beta \ge 0; \quad \alpha_3 = \phi\beta\gamma \ge 0.$$

From equation (16) it is now clear that the impact of oil price movements on oil supplies depends on the process generating price expectations. For example, in the case where price expectations are formed according to the simple Hotelling rule we have $E_{t-1}(p_{t+1}) = (1+r)E_{t-1}(p_t)$, where r is the real rate of interest, and when $\beta = 1/(1+r)$ then oil prices will have no effect on oil output. This is not, however, a plausible expectations formation model in a world where oil prices are determined under oligopolistic conditions, with OPEC having an important influence on oil prices. As possible models of oil price expectations we consider the rational expectations hypothesis (REH) and the adaptive expectations hypothesis (AEH). Under the former hypothesis we have

$$p_t - E_{t-1}(p_t) = \xi_{t1},$$

$$p_{t+1} - E_{t-1}(p_{t+1}) = \xi_{t2},$$

where $E(\xi_{ti}|\Omega_{t-1}) = 0$, $i = 1,2$. The price expectations term in (16) can now be replaced by:

$$E_{t-1}(p_t-\beta p_{t+1}) = p_t - \beta p_{t+1} + \xi_{tp} \tag{17}$$

where ξ_{tp} satisfies the orthogonality property

$$E(\xi_{tp}|\Omega_{t-1}) = E((\beta\xi_{t2}-\xi_{t1})|\Omega_{t-1}) = 0. \tag{18}$$

Under the adaptive expectations hypothesis, as shown in Pesaran (1987, Ch. 9), we have

$$E_{t-1}(p_{t+1}) = E_{t-1}(p_t) = (1-\theta) \sum_{i=1}^{\infty} \theta^{i-1} p_{t-i}, \quad 0 \leq \theta < 1.$$

and the price expectations term in (16) becomes

$$E_{t-1}(p_t-\beta p_{t+1}) = (1-\beta)(1-\theta) \sum_{i=1}^{\infty} \theta^{i-1} p_{t-i}, \tag{19}$$

which we write compactly as $(1-\beta)\tilde{p}_t(\theta)$. Under the AEH the supply function is upward sloping, but the quantitative effect of oil prices on oil supplies declines with the amount of proven reserves. This is an important feature of the supply equation (16) which distinguishes it from other supply functions that are derived in the literature.

Finally, for the other unobserved expectational variables in (16) we adopt the REH and write

$$E_{t-1}(q_{t+1}) = q_{t+1} - \xi_{tq},$$

$$E_{t-1}(h_{t+1}) = h_{t+1} - \xi_{th},$$

where ξ_{tq} and ξ_{th} denote the expectations errors. Under REH, ξ_{tq} and ξ_{th} are orthogonal to the information set Ω_{t-1} and are also serially uncorrelated, although not necessarily homoscedastic.[14]

Using the above results in (16) we now obtain the following two specifications of the supply equations in terms of the observables, depending on whether price expectations are formed rationally or adaptively. Under the former hypothesis

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[14] More specifically, under the REH the expectations errors ϵ_{ti}, $i = q,h$ are martingale difference processes with respect to the information set Ω_{t-1}. See, for example, Pesaran (1987, Ch. 5).

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 z_{t-1}(p_t - \beta p_{t+1})$$
$$+ \alpha_2 z_{t-1}\, q_{t+1} + \alpha_3 z_{t-1} h_{t+1} + u_t, \tag{20}$$

where

$$u_t = z_{t-1}(\alpha_1 \xi_{tp} - \alpha_2 \xi_{tq} - \alpha_3 \xi_{th}). \tag{21}$$

When price expectations are formed adaptively we have

$$q_t = (1-\phi)q_{t-1} + \alpha_0 z_{t-1} + \alpha_1 (1-\beta)\tilde{p}_t(\theta)$$
$$+ \alpha_2 z_{t-1}\, q_{t+1} + \alpha_3 z_{t-1}\, h_{t+1} + v_t, \tag{22}$$

where

$$v_t = -z_{t-1}(\alpha_2 \xi_{tq} + \alpha_3 \xi_{th}). \tag{23}$$

Notice that under our assumptions the composite disturbances u_t and v_t are also martingale difference processes which are serially uncorrelated and satisfy the orthogonality conditions

$$E(u_t|\Omega_{t-1}) = E(v_t|\Omega_{t-1}) = 0. \tag{24}$$

2.4 The exploration equation

The decision function for the exploratory effort, x_t, is much more complex than the output equation. Eliminating μ_t from (10c) and (10d) we obtain

$$E_t\left(\lambda_t \frac{\partial d_t}{\partial x_t}\right) = E_{t-1}(w_t) - \beta E_{t-1}\left\{w_{t+1} - \lambda_{t+1}\left(\frac{\partial d_{t+1}}{\partial x_{t+1}} - \frac{\partial d_{t+1}}{\partial X_t}\right)\right\}. \tag{25}$$

The unobserved shadow prices, λ_t, can also be eliminated using the martingale difference form of (10a). For empirical purposes we simplify (25) by assuming that the discovery function, $F(x_t, X_{t-1})$ in (4), takes the following exponential form advocated by Uhler (1976, p. 79) in his empirical analysis of the oil and gas discovery in the province of Alberta:

$$F(x_t, X_{t-1}) = Ax_t^{\rho} \exp(b_1 X_{t-1} - b_2 X_{t-1}^2). \tag{26}$$

For positive values of A, b_1 and b_2, and for $0 < \rho < 1$, this function clearly satisfies all the four conditions (4a)-(4d), and captures the discovery decline phenomenon described above. The threshold value for the cumulative exploratory effort in this example is given by $X_m = b_1/2\, b_2$.

Even under (26), the form of the exploration decision rule will still be highly complicated. Here, we consider the relatively simple case where the discount factor, β, is small and the terms involving future expectations of x_t can be ignored. (Later we show that in the case of the U.K. data, this is not an unreasonable first-order approximation.) In this case equation (25) can be solved for the desired level of exploratory effort, x_t^*, in terms of the past cumulative level of exploratory effort, X_{t-1}, and price and cost expectations. Assuming that the stochastic component of the discovery function (4), ν_t, is distributed independently of the exploration decision, x_t, and denoting the expectations of w_t and λ_t formed at time $t-1$ by w_t^e and λ_t^e, respectively, we have[15]

$$\log x_t^* = a_0 + a_1 X_{t-1} + a_2 X_{t-1}^2 + a_3 \log(\lambda_t^e / w_t^e), \tag{27}$$

where

$$a_0 = (1-\rho)^{-1} \log(A\rho); \quad a_1 = (1-\rho)^{-1}\, b_1 \geq 0,$$

$$a_2 = -(1-\rho)^{-1}\, b_2 < 0; \quad a_3 = (1-\rho)^{-1} > 0.$$

To obtain an equation for x_t, the actual rate of exploratory effort, we again employ a simple partial adjustment model with the parameter $0 < \psi$

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[15]To derive (27) substitute $\partial d_t/\partial x_t = \partial F_t/\partial x_t$ from (26) in (25) and solve for $\log(x_t)$. Notice that x_t is a decision variable, and hence will be a part of the information set of the firm at time $t-1$..

≤ 1, and using (27) we write:[16]

$$\log x_t = (1-\psi)\log x_{t-1} + \psi a_0 + \psi a_1 X_{t-1} + \psi a_2 X_{t-1}^2 + \psi a_3 \log(\lambda_t^e/w_t^e). \quad (28)$$

Notice that in this relation the expected shadow price of oil in the ground, λ_t^e, provides the link between the exploration and extraction decisions and is given by (10a), which under (12) simplifies to

$$\lambda_t^e = p_t^e - \delta_1 - \delta_2(q_t^e/z_{t-1}), \quad (29)$$

where $q_t^e = E(q_t|\Omega_{t-1})$ and z_t is already defined by (14).

3. Empirical Results

Using the econometric framework developed in the previous section, we now present an empirical analysis of exploration and extraction for the UKCS oilfields over the period 1978(1)-1986(4) The exploration and development of oil resources in the North Sea began in 1964, but only on a modest scale. But by the mid-1970s, the level of exploration activity (as measured by the number of exploratory wells drilled) started to increase rapidly under two important influences: the discovery of the two major oilfields at Forties and Brent in the early 1970s, and the four-fold increase in oil prices in 1973/74.[17] Since 1975, as can be seen from Figure 1, the level of exploration activity has undergone important cyclical variations and it is our aim here to see whether these variations can be adequately explained by means of the exploration equation (28).

Following the major oil discoveries in the early 1970s, oil production from the offshore fields on the UKCS started in 1975 and rose from 2.33

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[16] The partial adjustment model in this case is taken as $\Delta\log x_t = \psi\log(x_t^*/x_{t-1})$.

[17] For a detailed historical account of the development of oil and gas resources in the U.K., see Hall and Atkinson (1983), and Mabro et al. (1986)

FIGURE 1

Number of exploration wells started on theUKCS

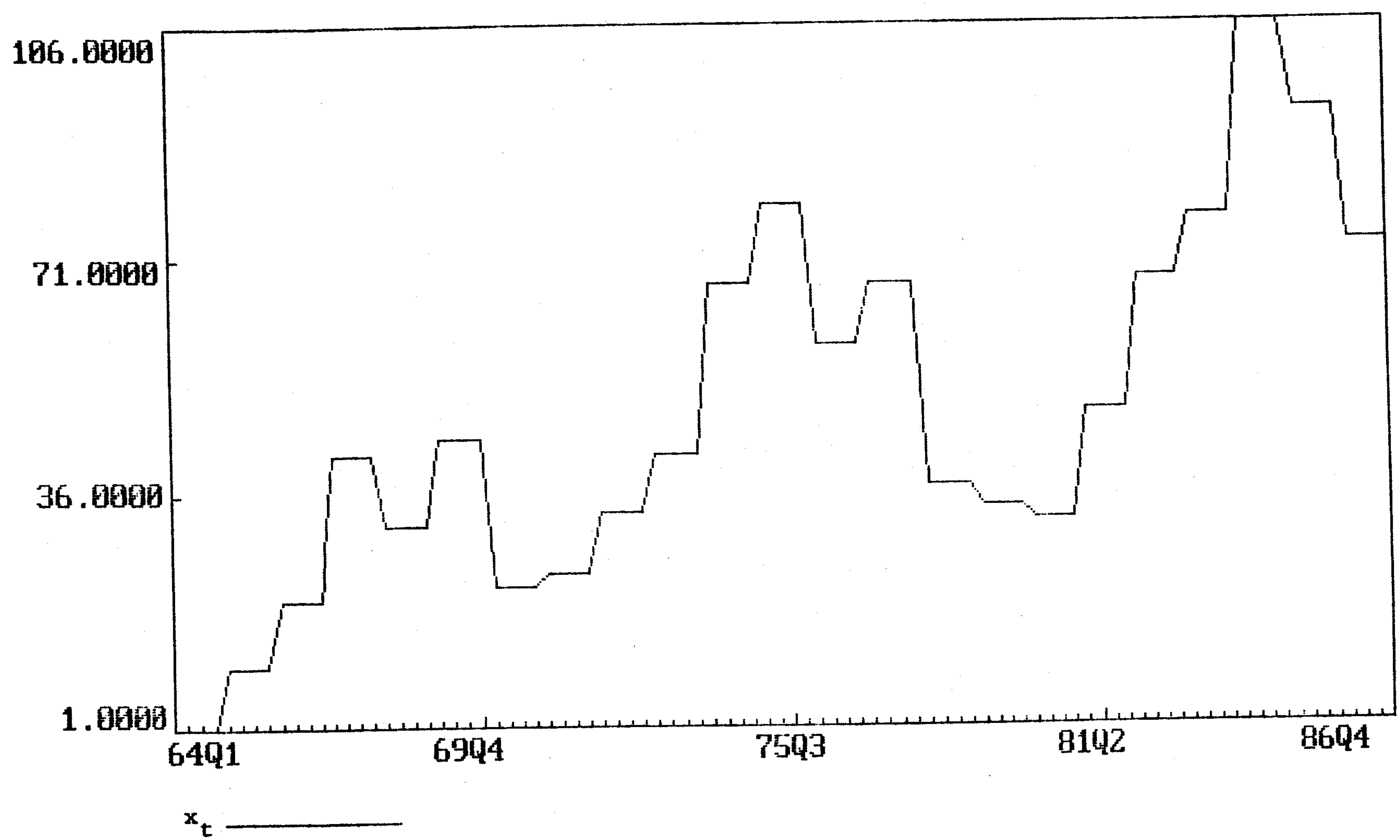
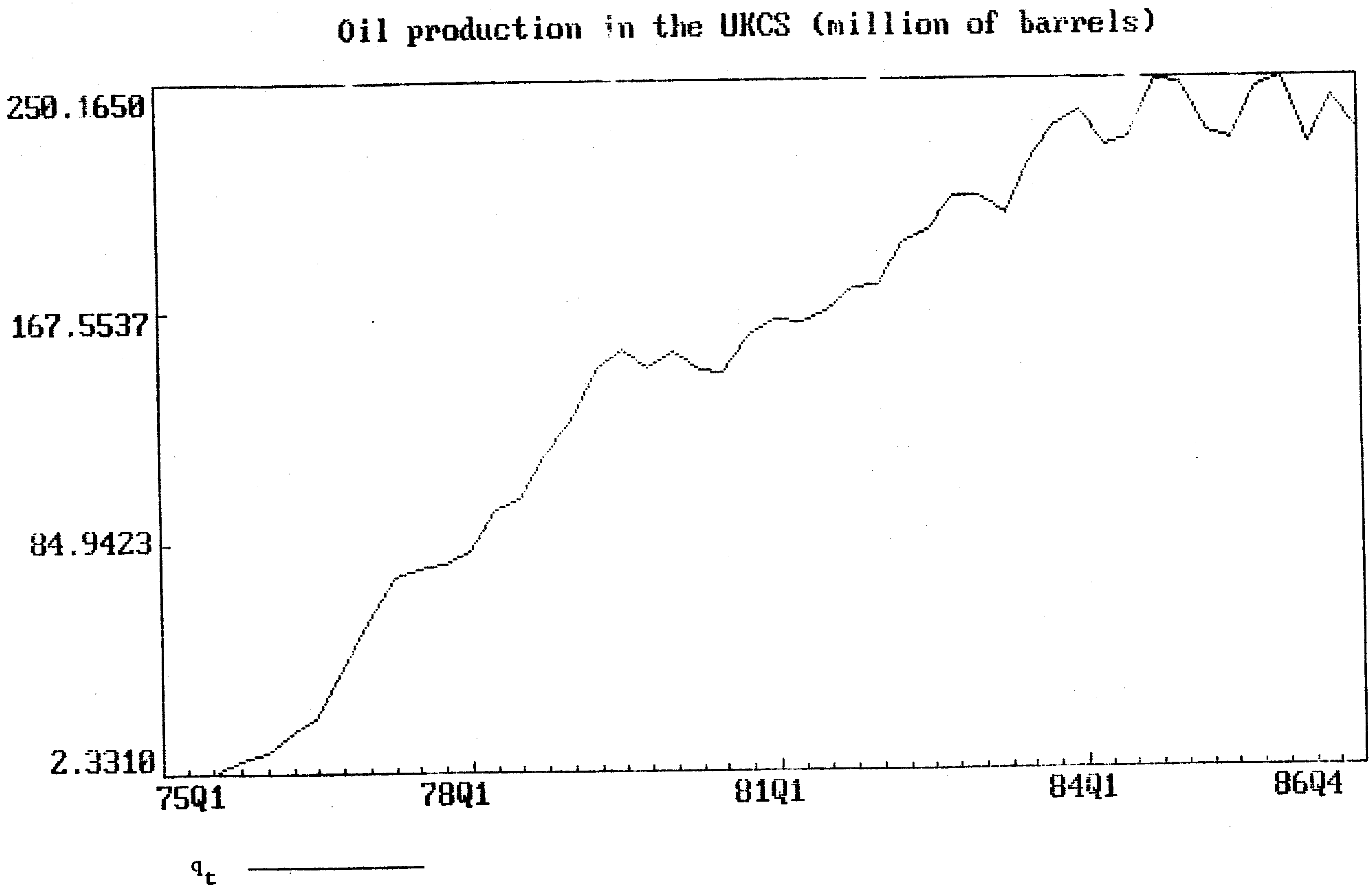


FIGURE 2


Oil production in the UKCS (million of barrels)
250.1650
167.5537
84.9423
2.3310
75Q1
78Q1
81Q1
84Q1
86Q4
q_t

million barrels in 1975(3) to around 230.69 million barrels in 1986(4) (see Figure 2). The quarterly average of growth of oil production over the period 1976(1)-1977(4) amounted to around 36.95 percent as compared with the average rate of 3.3 percent over the 1978(1)-1986(4) period. The initial surge in oil production over the 1976(1)-1977(4) period is directly related to the starting up of production from the major oilfields discovered in the early 1970s, and in view of the discovery decline phenomenon described in the previous section is best regarded as a unique event which is unlikely to repeat itself on the UKCS. So we do not attempt to explain the rapid rise in the oil production in the initial years and base our analysis on the data over the period 1978(1)-1986(4). In their recent work on the temporal pattern of oil production in the UKCS and its likely causes, Mabro et al. (1986, Appendix 1) also concentrated on the post-1978 period and run Ordinary Least Squares (OLS) regressions of oil output on seasonal dummies, a time trend and the nominal price of Brent Crude using monthly data over the period from January 1980 to February 1985. They find evidence of significant seasonal variations in U.K. oil production, but fail to find any evidence of price sensitivity, especially as far as the total oil output is concerned. However, as will be shown below, these results crucially depend on the static linear regression model adopted by these authors and do not hold when appropriate allowances are made for adjustment lags, price expectations and the non-linear effect of proven reserves on oil supplies.

3.1 Estimates of the Supply Function

In estimating the supply equation (20) we need to take account of the correlation that may exist between the regressors $z_{t-1}p_t$, $z_{t-1}p_{t+1}$, $z_{t-1}q_{t+1}$ and $z_{t-1}h_{t+1}$, and the composite disturbance term u_t. The estimation of equation (20) (or (22)) by the OLS method is clearly invalid.

But, since under the REH, u_t is orthogonal to the variables in the information set Ω_{t-1}, a consistent estimate of the parameters of (20) can, in principle, be obtained by the application of Sargan's (1958) generalized instrumental variable (IV) method to (20) using lagged values of q_t, h_t, p_t and R_t as instruments. The possible heteroskedasticity of u_t's can then be taken into account at the hypothesis testing stage by basing inferences on White's (1982) heteroscedasticity-consistent estimator of the covariance matrix of the IV estimators.[18]

However, before applying the IV method to the estimation of rational expectations models such as (20) or (22), it is important to recognize that the orthogonality of the instruments and the disturbances implied by the condition $E(u_t|\Omega_{t-1})$, although necessary is by no means sufficient to ensure the consistency of the resultant estimators. As has been emphasized elsewhere (Pesaran, 1987, Example 7.1), the IV estimation of RE models yields consistent estimates if in addition to the orthogonality condition the population correlation matrix of the instruments and the regressors also has a full rank. This latter condition is extremely difficult to verify, especially in the case of nonlinear RE models such as those that underlie the supply equations (20) and (22).[19] As a result there seems to be no guarantee that

---

[18]An alternative and in many respects a similar method of estimating (20) (or (22)) would be to use the generalized method of moments (GMM) estimators due to Hansen (1982). However, in the present example where u_t's are serially uncorrelated, there is little to choose between the GMM and the IV estimators.

[19]The computation of the population correlation matrix of the instruments and the regressors in the case of equations (20) and (22) requires an explicit solution of the nonlinear RE model (16) which does not seem to be possible.

even under the REH the IV estimators of (20) or (22) will be consistent.[20]

Bearing the above econometric considerations in mind we estimated equation (20) by the IV method using quarterly data over the period 1978(1)-1984(4).[21] We also included seasonal dummies amongst the regressors to take account of possible seasonal variations in output due to weather conditions or other seasonal factors in the North Sea. In order to ensure that the seasonal effects add up to zero over a given year, we used $s_{it} - s_{4t}$, $i = 1,2,3$ as seasonal variables where $s_{it} = 1$ in the i^{th} quarter and zero elsewhere. As instruments we used

$$\underset{\sim}{\phi}_t = \{s_{1t} - s_{4t}, s_{2t} - s_{4t}, s_{3t} - s_{4t}, q_{t-1}, \\ z_{t-1}, z_{t-2}p_{t-1}, z_{t-3}p_{t-2}, z_{t-3}q_{t-1}, \\ z_{t-4}q_{t-2}, z_{t-3}h_{t-1}, z_{t-4}h_{t-2}\},$$

which satisfy the orthogonality condition $E(u_t|\underset{\sim}{\phi}_t) = 0$. Finally, since $z_t = R_t/(R_t+\gamma)$, we estimated the supply equations for a number of different values of γ in the range $0 < \gamma < 10,000$. Overall, the results were only marginally affected by the choice of γ and for simplicity we decided to fix γ at its ML estimate, $\hat{\gamma} = 2000$, obtained under the simple case of $\beta = 0$.

For the supply equation (20) we obtained the following IV estimate:

$$q_t = \underset{(3.514)}{3.416}(s_{1t}-s_{4t}) - \underset{(3.370)}{3.358}(s_{2t}-s_{4t}) \\ - \underset{(2.843)}{2.100}(s_{3t}-s_{4t}) + \underset{(0.370)}{0.443}q_{t-1} - \underset{(17.104)}{4.008}z_{t-1}$$

---

[20]This is a general problem which arises whenever nonlinear RE models are estimated by the IV or the generalized method of moments.

[21]The data sources and definitions are described in the Data Appendix.

$$+ \underset{(1.543)}{0.077}\, z_{t-1}p_t - \underset{(1.763)}{0.103}\, z_{t-1}p_{t+1}$$

$$+ \underset{(0.369)}{0.593}\, z_{t-1}q_{t+1} + \underset{(1941.8)}{933.731}\, z_{t-1}h_{t+1} + \hat{u}_t, \qquad (30)$$

$$\bar{R}^2 = 0.9689, \quad \hat{\sigma} = 8.539, \quad \chi^2_{SM}(2) = 1.33,$$

$$\chi^2_{SC}(4) = 13.59, \quad \chi^2_{FF}(1) = 0.14, \quad \chi^2_{N}(2) = 0.36, \quad \chi^2_{H}(1) = 2.65,$$

where $\bar{R}$ is the adjusted multiple correlation coefficient, $\hat{\sigma}$ is the estimated standard error of u_t, $\chi^2_{SM}(2)$ is Sargan's (1964, pp. 28-29) misspecification statistic distributed asymptotically as a χ^2 variate with 2 degrees of freedom. The figures in { } are White's (1982) heteroscedasticity-consistent estimates of the standard errors of the IV estimates of the regression coefficients adjusted for the degrees of freedom (see MacKinnon and White (1985)). $\chi^2_{SC}(4)$, $\chi^2_{FF}(1)$, $\chi^2_{N}(2)$ and $\chi^2_{H}(1)$ are diagnostic statistics distributed as chi-squared variates (with degrees of freedom in parentheses) for tests of residual serial correlation, functional form misspecification, non-normal errors and heteroscedasticity, respectively.[22]

The above results provide only partial support for the intertemporal optimization hypothesis. On the favorable side all the parameter estimates have the a priori expected signs. From (16) we would expect the signs of the coefficients of the price terms $z_{t-1}p_t$ and $z_{t-1}p_{t+1}$ to be positive and negative, respectively, and those of $z_{t-1}q_{t+1}$ and $z_{t-1}h_{t+1}$ to be positive, and they are. The estimated equation fits relatively well, and explains around 97% of the variations in output over the 1978(1)-1986(4) period. Nevertheless, the parameter estimates are all poorly determined and in fact none are statistically significant at the 5% level! Moreover, the

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[22]The computations reported in this paper are carried out on Data-FIT. For details of the computations and algorithms see Pesaran and Pesaran (1987).

estimate of β (the discount factor) which can be obtained either as the ratio of the estimates of the coefficients of $z_{t+1}p_{t+1}$ and $z_{t-1}p_t$ ($\hat{\beta}$ = 0.103/0.077 = 1.33), or by using the estimates of the coefficients of q_{t-1} and $z_{t-1}q_{t+1}$ (i.e., $\hat{\beta}$ = 0.593/(1-0.443) = 1.06) both exceed their upper bound, and are clearly implausible. The presence of significant residual serial correlation, as indicated by the large value obtained for the statistic $\chi^2_{SC}(4)$, also contradicts the intertemporal model which predicts that under the REH the disturbances of (30) should be serially uncorrelated. Overall, the combination of a good fit and poorly determined estimates suggests that the available data may not contain adequate variations to enable us to arrive at a definite conclusion regarding the empirical validity of the supply equation (20). This seems to be particularly the case as far as the estimation of the discount factor, β, is concerned. In fact it was not possible to reject the hypothesis of $\beta = 0$, at conventional levels of significance. We found the same to be true when we estimated the supply equation (22), where it is assumed that price expectations are formed adaptively. In view of these findings we decided to focus our analysis on the case where $\beta \approx 0$. The results are summarized in the first two columns of Table 1.

This new estimate of equation (20) is still unsatisfactory. The price variable, $z_{t-1}p_t$, is statistically insignificant, and z_{t-1} which is marginally significant has an incorrect sign. On the basis of this result it appears that the U.K. oil production is driven solely by "inertia" and "seasonal" factors. This conclusion is similar to that arrived at by Mabro et al. (1986), which is not surprising considering that both results use current oil prices as a proxy for price expectations, (although we use the IV method and Mabro et al. use the OLS method). The results for equation

TABLE 1*

Estimates of the Oil Supply Equation for $\beta = 0$,

Under Alternative Price Expectations Formation Models

(1978(1)-1986(4))

Regressors	Equation 20[1] (REH)	Equation 22[2] (AEH)	Combined[3] Model
$s_{1t}-s_{4t}$	-0.540 (2.877)	-0.159 (2.623)	-0.162 (2.706)
$s_{2t}-s_{4t}$	-6.081 (2.874)	-5.577 (2.622)	-5.575 (2.706)
$s_{3t}-s_{4t}$	1.305 (2.875)	0.549 (2.632)	0.423 (2.720)
z_{t-1}	19.608 (10.074)	-3.031 (11.057)	1.046 (12.357)
q_{t-1}	0.937 (0.029)	0.694 (0.101)	0.676 (0.106)
$z_{t-1}p_t$	0.0165 (0.459)	-	-0.379 (0.441)
$z_{t-1}\tilde{p}_t(0.96)$	-	6.103 (2.433)	6.643 (2.588)
$\hat{\sigma}$	9.955	9.056	9.347
$\bar{R}^2$	0.958	0.965	0.963
$\chi^2_{SC}(4)$	3.36	6.73	7.13
$\chi^2_{FF}(1)$	0.08	0.000	1.71
$\chi^2_{N}(2)$	0.33	1.79	0.37
$\chi^2_{H}(1)$	0.82	0.002	0.01

(continued)

Table 1 (continued)

*The figures in brackets are the conventional standard errors, $\hat{\sigma}$ is the estimated standard error of the regression, $\bar{R}$ is the adjusted multiple correlation coefficient, and $\chi^2_{SC}(4)$, $\chi^2_{FF}(1)$, $\chi^2_N(2)$ and $\chi^2_H(1)$ are diagnostic statistics distributed as chi-squared variates (with degrees of freedom in parentheses) for tests of residual serial correlation, functional form misspecification, non-normal errors, and heteroscedasticity, respectively.

[1]IV estimates computed using seasonal dummies, z_{t-1}, q_{t-1}, $z_{t-2}p_{t-1}$, and $z_{t-3}p_{t-2}$ as instruments.

[2]OLS estimates.

[3]IV estimates computed using seasonal dummies, z_{t-1}, q_{t-1}, $z_{t-2}p_{t-1}$, $z_{t-3}p_{t-2}$ and $z_{t-1}\tilde{p}_t(0.96)$ as instruments.

Sources and Definitions: See Data Appendix.

(22) which is based on adaptively formed price expectations are, however, much more satisfactory. The price variable, $z_{t-1}\tilde{P}_t(0.96)$, has the correct positive sign and is highly significant.[23] The variable z_{t-1} also has the correct sign, but is statistically insignificant. The lagged oil output exerts an important influence on current production decisions but its effect is considerably smaller under equation (22) than under (20). In order to formally test (20) against (22) (or vice versa) we estimated a combined model, that included both of the supply equations as special cases, by the IV method. The results are given in the last column of Table 1, and conclusively reject the supply equation based on rationally formed price expectations in favor of the supply equation based on adaptively formed price expectations. Nevertheless, it may be argued that the adaptive expectations hypothesis is not necessarily in conflict with the rationality hypothesis and, as shown by Muth (1960), will yield statistically optimal price forecasts if the process generating Δp_t can be represented by a first-order moving average scheme such as $\Delta p_t = \epsilon_t - \theta\epsilon_{t-1}$, with θ being the same as the parameter of the adaptive process. (On this also see Pesaran (1987, Ch. 2)). This argument is not, however, supported by the data. Estimating a first-order moving average process for Δp_t over the periods 1970(1)-1986(4), and 1975(1)-1986(4) we obtained the results

$$\Delta p_t = \underset{(0.2716)}{0.0804} + \underset{(0.11353)}{0.1457}\,\hat{\epsilon}_{t-1} + \hat{\epsilon}_t,$$

$$\Delta p_t = \underset{(0.3268)}{-0.0625} + \underset{(0.1664)}{0.1563}\,\hat{\epsilon}_{t-1} + \hat{\epsilon}_t,$$

respectively. Both estimates reject the presence of a statistically

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[23]The value of 0.96 reported for the adaptive coefficient, θ, is the maximum likelihood estimate of θ computed by the grid search method over the range $0 \leq \theta < 1$.

significant moving-average component in the Δp_t-process, and both give negative estimates for θ.

One reason for the failure of the REH in the present context may be due to the volatility of international oil markets, particularly over the 1978-86 period, and the absence of a satisfactory econometric model which is capable of accounting for these volatile price movements. In such a situation North Sea producers may have no choice but to opt for an alternative, informationally less demanding expectations formation hypothesis, such as the adaptive one.

In view of the above results we adopted the following as our "preferred" output equation:[24]

$$q_t = \underset{(2.575)}{-0.212} (S_{1t}-S_{4t}) - \underset{(2.578)}{5.622} (S_{2t}-S_{4t}) + \underset{(2.582)}{0.614} (S_{3t}-S_{4t})$$
$$+ \underset{(0.075)}{0.712} q_{t-1} + \underset{(1.351)}{5.552} z_{t-1}\tilde{p}_t(0.96) + \hat{v}_t, \quad (31)$$

$$\bar{R}^2 = 0.9661, \quad \hat{\sigma} = 8.920, \quad \text{Durbin's h-statistic} = -0.75,$$

$$\chi^2_{SC}(4) = 4.43, \quad \chi^2_{FF}(1) = 0.07, \quad \chi^2_N(2) = 1.75, \quad \chi^2_H(1) = 0.01.$$

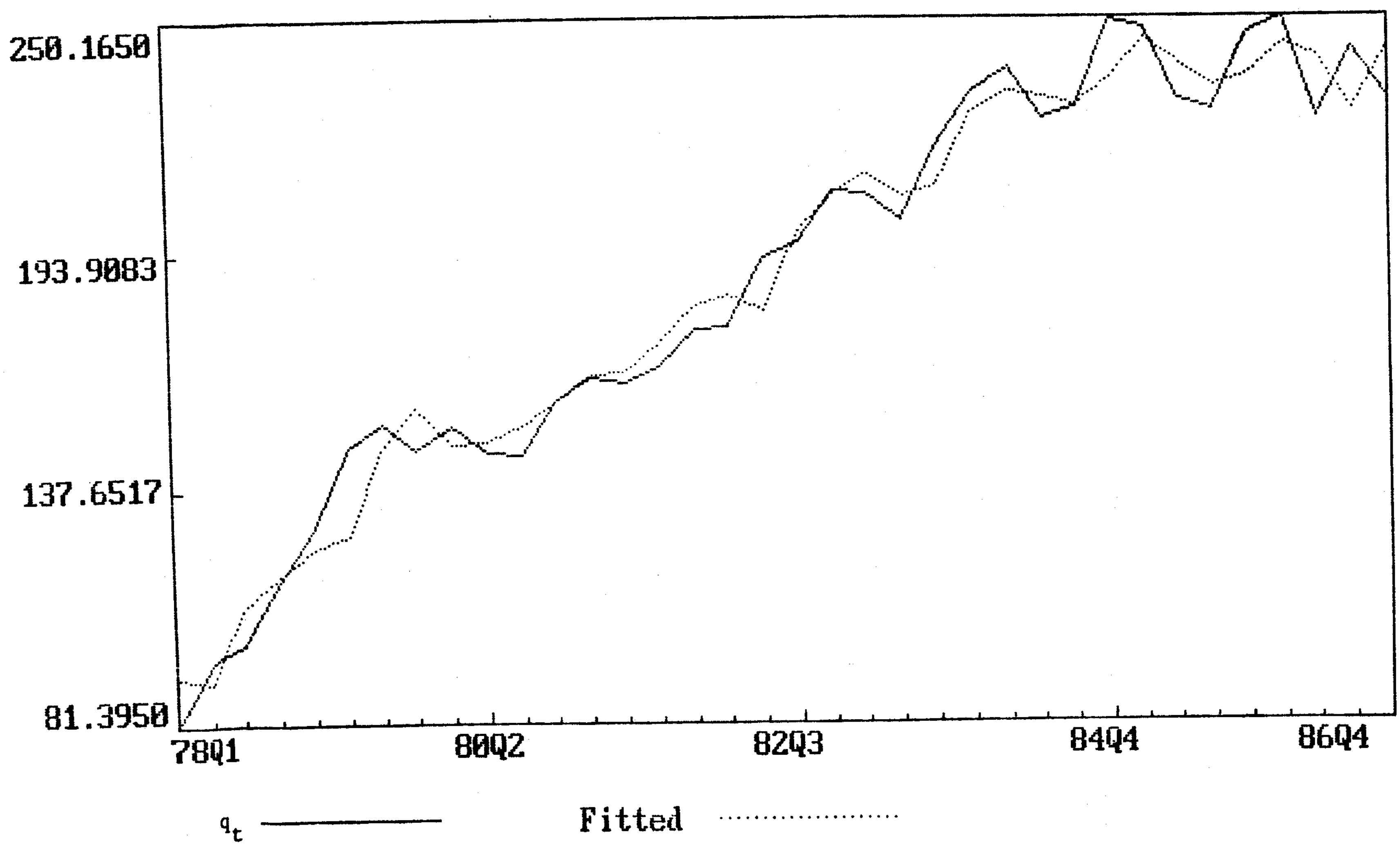
This equation passes the various diagnostic tests, fits well and its coefficients have the correct signs (also see Figure 3). Furthermore, the parameter estimates are not sensitive to the inclusion of a constant term and/or a time trend in (31). Bearing in mind the strong upward trend that exists in $\tilde{p}_t(0.96)$, this last result is particularly important and conclusively rejects the view that the price expectations variable in (31)

---

[24] Notice that we have dropped the statistically insignificant variable z_{t-1}. Also, since we could not reject the homoscedasticity hypothesis for the disturbances, v_t, we give the conventional standard errors in brackets.

FIGURE 3

PLOT OF ACTUAL AND FITTED VALUES FOR THE "PREFERRED" EXTRACTION EQUATION



may be acting as a proxy for a time trend.[25] The statistical significance of the price variable in the supply function is also robust to the choice of the estimation period. Estimating equation (31) over the shorter period 1978(1)-1984(4) yielded the following OLS estimates

$$q_t = \underset{(2.647)}{-1.534}\,(s_{1t}-s_{4t}) - \underset{(2.642)}{2.577}\,(s_{2t}-s_{4t}) - \underset{(2.642)}{0.908}\,(s_{3t}-s_{4t})$$

$$+ \underset{(0.087)}{0.791}\,q_{t-1} + \underset{(1.505)}{4.211}\,z_{t-1}\tilde{p}_t(0.96) + \hat{v}_t, \tag{32}$$

$\bar{R}^2 = 0.968$, $\hat{\sigma} = 8.070$, Durbin's h-statistic $= 0.23$,

$\chi^2_{SC}(4) = 2.16$, $\chi^2_{FF}(1) = 0.01$, $\chi^2_N(2) = 1.34$, $\chi^2_H(1) = 0.01$.

This equation does not show any evidence of seasonal variations in output, but gives estimates for the coefficients of the lagged output and the price variables which are remarkably similar to those reported in (31) for the longer time period.[26].

The supply equation (31) also has a number of other important features that are worth highlighting. The price variable in (31) incorporates the negative effect of declining reserves on the supply price elasticity discussed above, and has the desirable property that with the exhaustion of reserves, oil supplies become less responsive to oil price changes.

---

[25]The t-ratio for the estimated coefficient of the time trend in (31) was equal to 0.25, and the F-statistic for the joint test of zero restrictions on the coefficients of the constant term and the time trend variable in (31) was equal to 0.072.

[26]The F-statistic for the Chow test of the equality of the coefficients of q_{t-1} and $z_{t-1}\tilde{p}_t(0.96)$ in the preferred output equation over the periods 1978(1)-1984(4), and 1985(1)-1986(4) turned out to be equal to 1.897, which is well below the 5 percent critical value of the F distribution with 2 and 26 degrees of freedom. Equation (31) also easily passes Brown et al.'s (1975) cumulative sum (CUSUM) and the CUSUM of squares tests of parameter stability.

Computing price elasticities at sample means of reserves, output and price expectations we obtained the relatively high figures of 0.31 and 1.07 for the short run and the long run elasticities of output with respect to expected prices, respectively. The impact of actual oil price movements on oil supplies is, however, much smaller and builds up only very gradually. At a constant level of reserves the mean lag of output changes behind price changes is over 6 years.

Using (29) we can also obtain the following estimates of the structural parameters[27]

$$\hat{\phi} = 1 - 0.712 = 0.288,$$

$$\hat{\delta}_1 = 0,$$

$$\hat{\delta}_2 = \hat{\phi}/5.552 = 0.0519,$$

$$\hat{\delta}_3 = \hat{\delta}_2\hat{\gamma} = 103.75.$$

Substituting these estimates in (29) we finally obtain the following estimates for the expected shadow price of oil in the ground

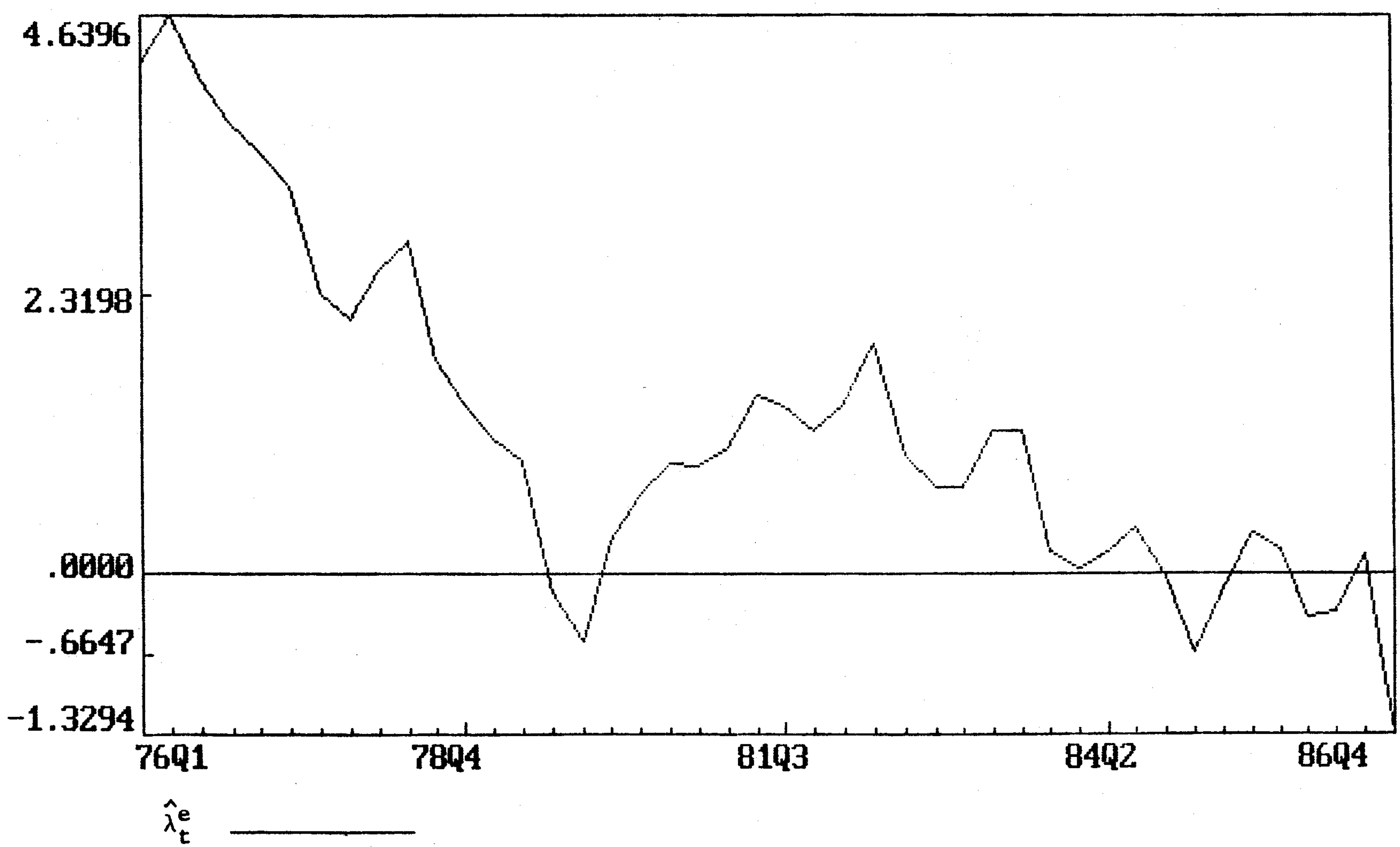
$$\hat{\lambda}^e_t = \tilde{p}_t(0.96) - 0.0519\ \hat{q}^e_t/z_{t-1}, \quad \text{for} \quad t = 1976(1),\ldots,1986(4), \qquad (33)$$

where $\hat{q}^e_t$ is given by the deterministic part of equation (31). The values of $\hat{\lambda}^e_t$ over the period 1976(1)-1986(4) are displayed in Figure 4 and provide an indication of the movement in the expected value of the marginal product of the exploratory effort on the UKCS. The estimates of λ^e_t can also be interpreted as the real rent that North Sea producers expect to earn

[27]Notice that the corresponding estimates based on (32) are $\hat{\phi} = 0.209$, $\hat{\delta}_1 = 0$, $\hat{\delta}_2 = 0.0496$, and $\hat{\delta}_3 = 99.20$, which are not very different from the estimates obtained using (31)..

FIGURE 4

Shadow price of oil in the ground based on the extraction equation ($/b)



on production of a barrel of oil at the margin. The estimates are, however, rather low and decline rapidly under the influences of falling oil prices and rising marginal extraction costs over the period, and in fact become negative during the 1985-86 period, although only slightly.[28] The downward trend in the estimates of λ_t^e also accords with the rising cost of developing new oil fields, and gives further justification to the concern expressed by some commentators in recent years over the decline in investment opportunities in the North Sea. (See, for example, Quinlan (1985)). It can, however, be argued that the rather low, and for some years negative, estimates obtained for the shadow price of oil in the ground are unrealistic and raise doubts about the adequacy of the model and its parameter estimates, in particular the estimate obtained for δ_2 ($= 0.0519$) which plays a key role in the estimation of expected shadow prices, λ_t^e. While we accept that this is a legitimate viewpoint, for the level of aggregation that we are working with, we could not find any evidence of model misspecification.[29] The reason for the relatively high estimate obtained

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[28]The average estimates of $\hat{\lambda}_t^e$ for the years 1985 and 1986 were -0.06 and -0.46.

[29]It is also worth noting that the estimate $\hat{\delta}_2 = 0.0519$ is quite robust to the choice of the discount factor, β. For values of $\beta = 0.1$, 0.2, 0.3, 0.8, 0.95 we obtained the estimates 0.05195, 0.051998, 0.05206, 0.0531, 0.0607 for δ_2, respectively. These estimates are computed by the application of the IV method to

$$\Delta q_t = \text{seasonal dummies} - \phi \tilde{q}_t(\beta) + ((1-\beta)\phi/\delta_2)\ \tilde{p}_t(0.96) + v_t,$$

where

$$\tilde{q}_t(\beta) = q_{t-1} - \beta z_{t-1} q_{t+1} - \beta\gamma z_{t-1} h_{t+1},$$

using $\{\text{seasonal dummies}, q_{t-1}, z_{t-3}q_{t-1}, z_{t-4}q_{t-2}, z_{t-3}h_{t-1}, z_{t-4}h_{t-2}\}$ as instruments. The above equation is a restricted version of (22).

for δ_2 (and hence the low estimates for λ_t^e) may be due to aggregation over oil-fields of different sizes and characteristics, the assumption of risk neutrality, or the measurement errors in the proven reserves estimates. These are important factors and clearly require serious consideration; but accounting for them in a satisfactory manner is clearly beyond the scope of the present paper. Here, we maintain the model specification, but obtain an alternative estimate of δ_2 based on the exploration equation, (28). In this way we also provide some evidence on the validity of the cross-equation restrictions that the optimizing framework imposes on the extraction and exploration equations.

3.2 Estimates of the Exploration Function

In what follows we estimate the exploration equation (28) assuming that w_t^e, the expectations, of the unit cost of exploratory efforts are formed rationally, and that λ_t^e are estimated by $\tilde{\lambda}_t^e = \tilde{p}_t(0.96) - \delta_2\, \hat{q}_t^e/z_{t-1}$. Here we treat δ_2 as a free parameter and estimate it along with other parameters of the exploration equation. To carry out the estimation, we first used the following linear approximation of (28),[30]

$$\log x_t = \psi a_0 + (1-\psi)\log x_{t-1} + \phi a_1 X_{t-1} + \psi a_2 X_{t-1}^2$$
$$+ \psi a_3 \log(\tilde{p}_t(0.96)/w_t) - \psi\delta_2 a_3 V_t + \xi_t, \qquad (34)$$

where $V_t = \hat{q}_t^e/(\tilde{p}_t(0.96)z_{t-1})$, and ξ_t is an unobservable disturbance term. Under the REH, ξ_t is uncorrelated with all the right hand side

[30]This approximation ignores squares and higher order terms in $\delta_2 V_t$, which is not unreasonable considering that for those values of δ_2 where $\tilde{\lambda}_t^e > 0$, we also have $0 < \delta_2 V_t < 1$.

variables in (34), except for w_t.[31] We therefore estimated (34) over the period 1978(1)-1986(1) by the IV method, using $\{1, \log x_{t-1}, X_{t-1}, X_{t-1}^2, \log \tilde{p}_t(0.96), \log \tilde{p}_{t-1}(0.96), \log w_{t-1}, \log w_{t-2}, V_t\}$ as instruments.[32] The results (with the heteroscedasticity-consistent estimates of the standard errors in { }) were as follows:

$$\log x_t = \underset{\{1.79\}}{1.862} + \underset{\{0.0902\}}{0.3718} \log x_{t-1} + \underset{\{0.0016\}}{0.0104} X_{t-1}$$

$$+ \underset{\{10^{-6}\times 0.8711\}}{-10^{-6}\times 5.7630} X_{t-1}^2 + \underset{\{0.1237\}}{0.3048} \log\left(\frac{\tilde{p}_t(0.96)}{w_t}\right) - \underset{\{0.0139\}}{0.0286} V_t + \hat{\xi}_t, \quad (35)$$

$$\bar{R}^2 = 0.9691, \quad \hat{\sigma} = 0.0757, \quad \chi_{SM}^2(3) = 1.03, \quad \chi_{SC}^2(4) = 2.07,$$

$$\chi_{FF}^2(1) = 2.20, \quad \chi_N^2(2) = 0.68, \quad \chi_H^2(1) = 0.87.$$

As can be seen from the above statistics, the estimated equation passes all the diagnostic tests and fits reasonably well. Furthermore, all the parameter estimates have the "correct" signs (in the sense that their signs are as predicted by the theoretical model), and except for the intercept term, they are all statistically significant at the 95 percent level. This equation, however, yields the estimate of 0.0938 (= 0.0286/0.3048) for the parameter δ_2, which is even larger than the estimate obtained using the supply equation (31). But the estimate of δ_2 based on the exploration equation (35) is very poorly determined and is not significantly different

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[31] Since $\tilde{p}_t(0.96)$ and V_t depend only on past observations we also have $E(\tilde{p}_t(0.96)|\xi_t) = 0$, and $E(V_t|\xi_t) = 0$.

[32] The details of the measurement of w_t are given in the Data Appendix. Also, since we did not have quarterly data on the number of exploration wells x_t, we assumed that x_t in each quarter can be approximated by 1/4 of the corresponding annual figure.

from the value of 0.0519, the estimate obtained using the supply equation.[33] This suggests that as far as δ_2 is concerned the estimates obtained from the supply equation and from the linearized version of the exploration equation are not in conflict and it may not be unreasonable to restrict δ_2 to the range $0 < \delta_2 < 0.047$, and then estimate it directly by the IV method using the non-linear form of the exploration equation.[34] With the same set of instruments as before, we obtained the following results,[35]

$$\begin{aligned} \log x_t = \underset{(1.539)}{2.427} + \underset{(0.0798)}{0.3894} \log x_{t-1} + \underset{(0.001238)}{0.009693}\, X_{t-1} \\ - \underset{(10^{-6}\times 0.7014)}{10^{-6}\times 5.407}\, X_{t-1}^2 + \underset{(0.1082)}{0.3450} \log(\tilde{\lambda}_t^e/w_t) + \hat{\xi}_t, \end{aligned} \qquad (36)$$

$\bar{R}^2 = 0.9712$, $\hat{\sigma} = 0.0731$, $\chi^2_{SM}(4) = 1.49$,

$\chi^2_{SC}(4) = 1.54$, $\chi^2_{FF}(1) = 1.67$, $\chi^2_{N}(2) = 0.26$, $\chi^2_{H}(1) = 0.69$.

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[33] Using the approximate formula

$$\hat{V}(\hat{a}/\hat{b}) \approx (1/\hat{b})^2 \{\hat{V}(\hat{a}) + (\hat{a}/\hat{b})^2 \hat{V}(\hat{b}) - 2(\hat{a}/\hat{b})\widehat{COV}(\hat{a},\hat{b})\},$$

the estimate of the standard error of $\hat{\delta}_2$ based on (35) amounted to 0.0576.

[34] The inequality restriction $\delta_2 < 0.047$ is needed in order to ensure that the estimates of the shadow price of oil in the ground, λ_t^e, over the sample period are positive. Otherwise the non-linear form of the exploration equation could not be estimated.

[35] The non-linear IV estimates were computed by minimizing $\underset{\sim}{\xi}' P_f \underset{\sim}{\xi}$ with respect to ψ, δ_2, a_0, a_1, a_2, a_3, where $P_f = \underset{\sim}{F}(\underset{\sim}{F}'\underset{\sim}{F})^{-1}\underset{\sim}{F}'$, and $\underset{\sim}{F}$ represents the matrix of observations on the instruments. The t^{th} element of $\underset{\sim}{\xi}$ in this non-linear case is defined by

$$\xi_t = \log x_t - \psi a_0 - (1-\psi)\log x_{t-1} - \psi a_1 X_{t-1} - \psi a_2 X_{t-1}^2 - \psi a_3 \log(\bar{\lambda}_t^e/w_t),$$

where

$$\bar{\lambda}_t^e = \bar{p}_t(0.96) - \delta_2 \hat{q}_t^e/z_{t-1}.$$

The estimate of δ_2 in this case turned out to be equal to 0.03, which is not very different from the estimate based on the supply equation.[36] The estimates of the shadow prices of oil in the ground for this estimate of δ_2 are displayed in Figure 5, and by construction are all positive but follow the same cyclical pattern as the estimates already shown in Figure 4.

The estimates in (36) are in a number of respects more satisfactory than the ones based on the linearized version, (35). Despite the restriction imposed on δ_2, equation (36) gives a better fit to the data than the linearized version. (The plot of actual and fitted values based on this equation is given in Figure 6.) It also yields parameter estimates that are more precisely determined. Therefore, in what follows we concentrate our attention on the estimates in (36). But first we need to see whether, as predicted by theory, exploration decisions respond to expected price and cost changes symmetrically.[37] For this purpose we computed the following IV estimates,[38]

$$\begin{aligned} \log x_t = {} & \underset{(1.578)}{2.048} + \underset{(0.0871)}{0.3856} \log x_{t-1} + \underset{(0.001587)}{0.009132}\, X_{t-1} \\ & - \underset{(10^{-6}\times 0.8934)}{10^{-6}\times 5.0935}\, X_{t-1}^2 + \underset{(0.1149)}{0.4143} \log \tilde{\lambda}_t^e \\ & - \underset{(0.1741)}{0.3134} \log w_t + \hat{\xi}_t, \end{aligned} \tag{37}$$

$$\bar{R}^2 = 0.9692, \quad \hat{\sigma} = 0.0756, \quad \chi^2_{SM}(3) = 0.95, \quad \chi^2_{SC}(4) = 1.95,$$

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[36] This estimate of δ_2 is accurate up to two decimal points.

[37] We also estimated the non-linear exploration equation using recursive predictions of w_t as a proxy for w_t^e. For an assumed AR(2) specification of the w_t-process we could not find any evidence against the rationality hypothesis maintained here.

[38] These estimates are conditional on $\hat{\delta}_2 = 0.03$, and are computed employing the same set of instruments as those used in the estimation of (35) and (36).

FIGURE 5

Shadow price of oil in the ground based on the exploration equation (\$/b)

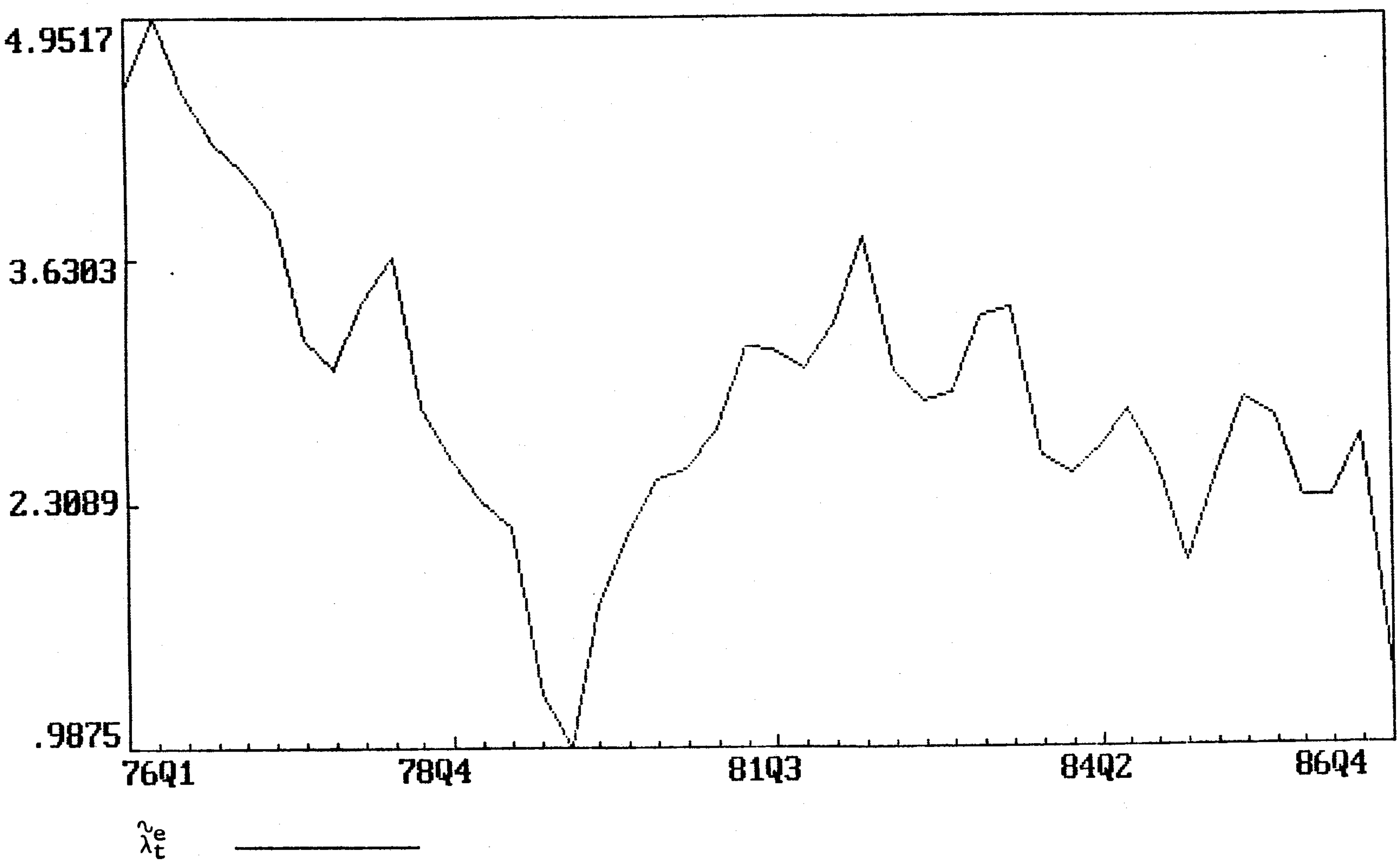
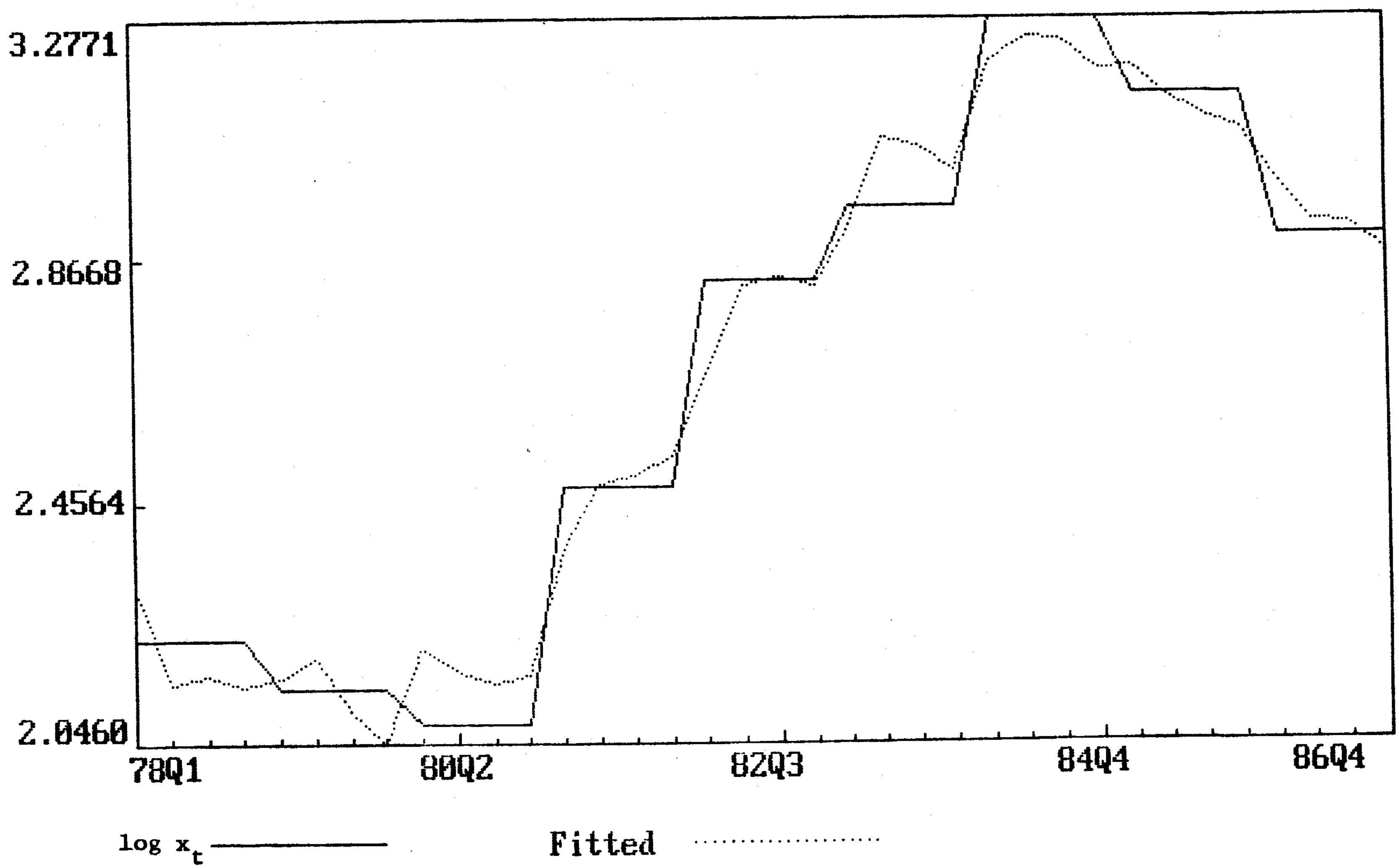


FIGURE 6

PLOT OF ACTUAL AND FITTED VALUES FOR THE "PREFERRED" EXPLORATION EQUATION



$$\chi^2_{FF}(1) = 2.61, \quad \chi^2_N(2) = 0.84, \quad \chi^2_H(1) = 0.91.$$

The estimated coefficients of the price and cost variables are both statistically significant and have the "correct" signs. A casual comparison of the results in (36) and (37) also indicates that the imposition of the *a priori* restriction that the exploration equation should be homogeneous of degree one in nominal variables has had only marginal effects on the parameter estimates. In fact carrying out a formal test of the hypothesis that the coefficients of the price and the wage variables are equal and of opposite signs can not be rejected even at the 10 percent significance level.[39]

In view of the above results we adopted (36), as our "preferred" exploration equation. In this specification the price variable is highly significant and captures the effects of some of the economic and geological factors on the exploration decisions. The elasticity of exploration effort with respect to the relative price variable is estimated to be 0.345 in the short-run, and around 0.57 in the long-run.[40] The price variable in (36) also provides the link between the exploration and the production decisions. A rapid expansion of output, by raising extraction costs, lowers the price of oil in the ground and, for a given level of expected costs, w_t^e, and oil price expectations, p_t^e, reduces the expected return to exploration, and hence the level of exploratory effort.

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[39]The value of the t-statistic for this test (based on White's heteroscedasticity-consistent estimates of the covariance matrix of the IV estimators) was calculated as 0.63.

[40]We also estimated (36) over the shorter sample period, 1978(1)-1984(4), and obtained a statistically significant effect for the price variable with a t-ratio of 2.98. The short-run and the long-run price elasticities estimated on the basis of this shorter period were 0.3164 and 0.49, respectively, which are only marginally different from the corresponding estimates obtained for the whole period.

The results in (36) can also be utilized to obtain an estimate for the threshold value of the cumulative level of exploration effort, X_m. Using the coefficient of X_{t-1} and X_{t-1}^2 in (36), we obtained the estimate $\hat{X}_m =$ 896 wells. This is an interesting result and suggests that the "discovery decline phenomenon" has already begun in the UKCS. (The cumulative number of exploration wells reached the figure of 943 wells in 1984.)

The estimates in (36) are, however, subject to one important shortcoming. The estimate of the less than unit elasticity obtained for the long-run response of the exploration effort to changes in the relative price variable, leads to a negative estimate of ρ in the discovery function (26), which is not plausible. This may be due to the fact that most large discoveries on the UKCS were made before 1978, and our analysis is confined to the post-1978 period. Unfortunately, lack of suitable cost data on exploration activity prevents us from including this early period in our analysis.

4. Summary and Conclusions

In this paper we have put forward an intertemporal econometric model for the joint determination of extraction and exploration decisions of a "representative" profit-maximizing oil producer, facing given price and cost expectations. The econometric model developed here differs from other models of oil supplies that employ an optimizing framework in the important respect that it explicitly takes account of the "discovery decline phenomenon", and the "reserve depletion effects" on extraction and exploration decisions. The analysis also allows for alternative treatments of price and cost expectations and presents formal statistical tests of the REH against the adaptive and the recursive hypotheses.

In the case of the output equation we show that the price responsiveness of oil supplies depends crucially on how price expectations are formed. Oil supplies will be independent of oil prices only in the extreme case where price expectations are formed according to the simple Hotelling rule, namely when oil prices are expected to increase at the real rate of interest. In general, however, we would expect oil prices to have a positive effect on non-OPEC oil supplies, but because of the effect of reserve depletion on extraction costs, the price responsiveness of supplies in our model is not fixed and declines with the level of oil reserves. In general, supplies also depend on expected future values of output-reserve ratios. This dependence arises because of the assumed intertemporal nature of the decision-making process and links current output decisions to the expected effects of current and future exploration decisions.

The exploration equation derived in the paper utilizes geological knowledge concerning the discovery process and provides an explicit link between the extraction and the exploration decisions via the shadow price of reserves. Unlike the studies by Epple (1985), and Hendricks and Novales (1987), the shadow reserve-prices are determined endogenously in the model. The exploration equation explains the desired level of exploratory effort as a function of past cumulative efforts, and the expected shadow reserve-price of oil relative to the expected unit cost of exploration. This equation can in fact be regarded as the dual of the discovery function and provides a simple method of obtaining parameter estimates of the discovery function even in the absence of direct reliable observations on new discoveries.

In the application of the econometric model to U.K. data we found strong positive price effects on oil supplies only in the case of the supply equation with adaptively formed price expectations. The results conclusively rejected the output equation based on rationally formed price expectations against the hypothesis that price expectations are formed adaptively. We also could not find significant intertemporal effects on supply decisions. This is surprising and requires further analysis. Another feature of the estimated output equation which may not be acceptable is the rather low estimates it gives for the expected shadow reserve-price of oil, especially over the 1985-86 period.

The estimation results for the exploration equation were generally satisfactory. The parameter estimates all had the "correct" signs, were statistically highly significant and yielded plausible estimates for the expected shadow reserve-price of oil. The results also provided some support for the view that the post-1985 period marks the beginning of an era of diminished investment opportunities in the U.K. sector of the North Sea.

Overall, the empirical results seem to be satisfactory and give a reasonable degree of support to the theory. A more definitive appraisal of the usefulness of the theoretical framework advanced here for the empirical analysis of output and exploration decisions should, however, await the application of the model to other non-OPEC regions. Meanwhile, there is clearly a great deal of room for further improvements both at the theoretical and empirical levels. Scope for theoretical improvements exists particularly in connection with the explicit modelling of the determinants of the reserve extensions and revisions, the tax system, and the relaxation of the assumption of risk neutrality.[41] At the empirical level, the joint

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[41] A useful account of the U.K. oil tax system can be found in Mabro et al. (1986, Ch. 8). In the case of the U.K. the tax system has remained relatively unchanged over the period covered by this study, and its incorporation into our model is unlikely to have significant effects on our general conclusions.

estimation of the extraction and exploration equations should improve the efficiency of the parameter estimates besides providing additional statistical tests of the adequacy of the model by means of cross-equation restrictions.

Data Appendix

The definitions and sources of the data used in this paper are as follows:[1]

q_t: Total quarterly oil production in the UKCS (in millions of barrels), computed from monthly production data published in various issues of the Petroleum Economist.

PB_t: Average quarterly spot prices of Brent Crude (in dollars). This series is available only over the period 1975(1) - 1986(4), and is taken from various issues of Petroleum Economist.

PAL_t: Average quarterly prices of Arabian Light Crude over the period 1960(1) - 1986(4), taken from various issues of the Petroleum Economist, and OPEC's Secretariat, Vienna.

PX_t: Average quarterly index of export prices of industrial countries (1975 = 1.00), taken from various issues of Financial Statistics, International Monetary Fund. This price index is used as a general deflator in the econometric model.

p_t: Real price of oil computed as $1.0107 \times PAL_t/PX_t$, and PB_t/PX_t over the periods 1960(1) - 1974(1), and 1975(1) - 1986(4), respectively. Notice that the price of Arabian Light Crude has been multiplied by 1.0107 in order to put it on par with the Brent Crude prices. It is assumed that the movements of Arabian Light Crude prices have been appropriate to the exploration decisions on the UKCS before 1975.

[1] I am grateful to Shovan Ray for helping me with the compilation of production and the oil price data used in this study.

YR_t: End of year proven reserves on the UKCS (in millions of barrels),[2] taken from various issues of Development of the Oil and Gas Resources of the UK, Department of Energy, HMSO, London, commonly known as the 'Brown Book'. Proven reserves are referred to as "those reserves which on the available evidence are virtually certain to be technically and economically producible (i.e., those reserves which have a better than 90 percent chance of being produced)."

R_t: End of quarter UK proven reserves computed from YR_t by the method of exponential interpolation in such a way that $R_t = YR_t$ at the end of each year.

x_t: Number of offshore exploration wells started on the UKCS per annum, taken from Table 5.1 in Mabro et al. (1986), and Appendix 2 in the 1987 issue of the Brown Book. Notice that the figures for the years 1972 and 1979 in Mabro et al. (1986), are incorrect and should both be replaced by 34. With these corrections the cumulative number of offshore exploration wells started over the 1964 - 1986 period will be equal to 1109 which is the figure quoted on p. 1 of the 1987 issue of the Brown Book.

$\tilde{x}_t$: Number of exploration and appraisal wells started (onshore and offshore) on the UKCS per annum; various issues of Brown Book.

$\tilde{p}_t(\theta)$: Adaptive expectations of the real oil prices, constructed recursively according to

$$\tilde{p}_t(\theta) = \theta\tilde{p}_{t-1}(\theta) + (1 - \theta)\, p_{t-1},$$

for t = 1960(2) to 1986(4), from the initial value of $\tilde{p}_{1960(1)}(\theta) = p_{1960(1)}$. This method of constructing $\tilde{p}_t(\theta)$ ensures that the price expectations series used in the study will

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[2]Converted from metric tons at a constant factor of 7.49:1.

not be sensitive to the choice of the initial value, even if the value of θ is close to unity.

$EEXP_t$: Total quarterly exploration expenditure including the cost of appraisal wells drilled prior to development approval, (in Pounds Sterling); Brown Books, 1979 (Appendix 11), and 1987 (Appendix 12). Data for exploration expenditure for the period before 1976 are not available.

E_t: Sterling dollar exchange rate (quarterly averages), taken from various issues of Financial Statistics, International Monetary Fund.

w_t: Quarterly real unit cost of exploration wells started on the UKCS (in dollars), computed as $(4\ EEXP_t \times E_t)/(\tilde{x}_t \times PX_t)$. This assumes a uniform distribution for the quarterly number of exploration and appraisal wells started within a year.

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