

**COSTLY ADJUSTMENT UNDER RATIONAL EXPECTATIONS:
A GENERALIZATION**

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May 1988

UCLA Working Paper No. 480

^{*}I am grateful to Tony Hall for helping me with the computation of the results reported in this paper.

ABSTRACT

This paper provides a generalization of the standard adjustment cost-rational expectations model due to Sargent (1978), which, in addition to the cost of changing the level of the decision variable, also allows for the cost of altering the speed with which decisions are changed. It establishes the existence of a unique stable solution for this more general model, derives an explicit solution for the underlying decision problem, and provides a necessary order condition for identification of the structural parameters. The paper also contains an application of the model to the determination of the employment in the U.K. coal industry over the 1956-83 period.

1. Introduction

In this paper I provide a generalization of the standard adjustment cost-rational expectations model discussed, for example, by Sargent (1978), Kennan (1979), and Pesaran (1987, Example 7.2). In addition to the familiar costs involved in changing the level of the decision variable, I also allow for the cost of altering the speed with which the decision variable is changed. This generalization is of some interest as it furnishes a theoretical justification for the inclusion of a second-order lag of the dependent variable in rational expectations models with future expectations.¹

The plan of the paper is as follows. Section 2 presents the generalized adjustment cost model. Section 3 shows that the solution of the model (say y_t) is unique and has the interesting property that it depends positively on y_{t-1} , and negatively on y_{t-2} . This section also derives the necessary order condition for identification under a fairly general specification of the process generating the exogenous variables, and a set of cross-equation restrictions that can be readily used to test the model's empirical adequacy. The paper ends with an application of the model to the determination of employment in the U.K. coal industry over the 1956-83 period.

2. The Generalized Adjustment Cost Model

Suppose an economic agent is faced with the problem of deciding on y_t in order to achieve stochastic targets y_t^* , determined by

¹For a different generalization of the standard partial adjustment model see Smyth (1984) and Ouliaris and Corbae (1985), where the adjustment parameter is allowed to vary with other variables, such as the rate of interest or the unemployment rate. As shown, for example, in Nickell (1984), theoretical justification for the inclusion of second or higher order lags of the dependent variable in the RE models can also be given by appeal to aggregation over different types of labor or "goods".

$$y_t^* = f(\underline{x}_t, \underline{\gamma}) + u_t, \quad (1)$$

where \underline{x}_t is a $k \times 1$ vector of exogenous or forcing variables, $\underline{\gamma}$ an $m \times 1$ vector of fixed constants, and u_t a serially uncorrelated disturbance term with zero mean. In the context of economic problems, y_t could represent the number of workers to be employed or the level of stock to be held by the economic agent. Similarly, \underline{x}_t could include variables such as real product wage, the real rate of interest, or an index of technological change.

Assume now that in the process of moving towards the target y_t^* , the economic agent incurs two types of adjustment costs. First, the cost of changing y_t , and second the cost of adjusting the speed with which changes in y_t are put into effect. Denoting these adjustment costs, respectively, by $\phi_1(\Delta y_t)$, and $\phi_2(\Delta^2 y_t)$, and the cost of being out of equilibrium by $\phi(y_t - y_t^*)$, the optimization problem facing the economic agent can be written as:²

$$\text{Min}_{y_t, y_{t+1}, \dots} E \left\{ \sum_{r=0}^{\infty} \beta^r [\phi(y_{t+r} - y_{t+r}^*) + \phi_1(\Delta y_{t+r}) + \phi_2(\Delta^2 y_{t+r})] \mid \Omega_t \right\}, \quad (2)$$

where $\Omega_t = (y_t, y_{t-1}, \dots; \underline{x}_t, \underline{x}_{t-1}, \dots; u_t, u_{t-1}, \dots)$ is the information set of the economic agent (but not necessarily that of the observing econometrician) at time t , and $0 \leq \beta < 1$ is the discount factor. In the special case where the functions $\phi(\cdot)$, $\phi_1(\cdot)$ and $\phi_2(\cdot)$ are quadratic, the solution of the above optimization problem leads to linear decision rules. Setting

²Here it is assumed that the economic agent knows, or has completely learned the functional forms of $f(\cdot)$, $\phi(\cdot)$, $\phi_1(\cdot)$, and $\phi_2(\cdot)$ and their parameters, as well as the processes generating the exogenous variables, \underline{x}_t .

$$\phi(y_t - y_t^*) = \frac{1}{2} (y_t - y_t^*)^2,$$

$$\phi_1(\Delta y_t) = \frac{1}{2} \phi_1 (y_t - y_{t-1})^2, \quad \phi_1 > 0,$$

$$\phi_2(\Delta^2 y_t) = \frac{1}{2} \phi_2 (y_t - 2y_{t-1} + y_{t-2})^2, \quad \phi_2 > 0.$$

The decision rule for the current period ($\tau=0$) can be written as:³

$$y_t = \lambda_1 y_{t-1} + \lambda_2 y_{t-2} + \alpha_1 E(y_{t+1} | \Omega_t) + \alpha_2 E(y_{t+2} | \Omega_t) + \theta y_t^*, \quad (3)$$

where

$$\left. \begin{aligned} \theta^{-1} - 1 + \phi_1 + \beta\phi_1 + \phi_2(\beta^2 + 4\beta + 1) &> 0, \\ \alpha_1 - \theta(\phi_1 + 2\phi_2 + 2\beta\phi_2) &> 0, \\ \alpha_2 - \theta\phi_2 &< 0, \\ \lambda_1 - \theta\beta(\phi_1 + 2\phi_2 + 2\beta\phi_2) &> 0, \\ \lambda_2 - \theta\phi_2\beta^2 &< 0. \end{aligned} \right\} \quad (4)$$

3. Solution and Identification of the Model

The solution of equation (3) depends on the roots of the characteristic equation⁴

$$1 - \lambda_1 x^{-1} + \lambda_2 x^{-2} + \alpha_1 x + \alpha_2 x^2. \quad (5)$$

The RE equation (3) has a unique non-explosive solution (relative to y_t^*) if and only if (5) has exactly two roots outside the unit circle with the remaining two roots falling strictly inside the unit circle. I now show that, for positive values of ϕ_1 and ϕ_2 , and $0 \leq \beta < 1$, these

³This decision rule is obtained from the first-order Euler condition for the minimization problem (2), at $\tau = 0$.

⁴See, for example, Whiteman (1983) and Broze et al. (1985), and also Pesaran (1987, Ch. 5) for a survey of different methods of solving linear RE models.

conditions on the roots of (5) are in fact satisfied. Using results in (4) it is easily seen that (5) can be written as:

$$\phi_2 z^2 + \phi_1 z + 1 = 0, \quad (6)$$

where

$$z = 1 + \beta - (x + \beta x^{-1}). \quad (7)$$

Let z_1 and z_2 be the roots of (6), and denote the roots of (7) corresponding to each value of $z = z_i$ by μ_i and μ'_i , $i = 1, 2$. From (7) it now follows that⁵

$$\mu_i \mu'_i = \beta < 1, \quad i = 1, 2,$$

$$(\mu_i - 1)(\mu'_i - 1) = z_i, \quad i = 1, 2.$$

But since ϕ_1 and ϕ_2 are, by assumption, positive we also have (from (6)) that $z_i < 0$, $i = 1, 2$. This together with the above results establishes that equation (5) has two roots, say μ_1 and μ_2 , outside the unit circle, and two roots, μ'_1 and μ'_2 , inside the unit circle. Furthermore, with $\beta \geq 0$, it also follows that $\mu_i, \mu'_i \geq 0$, $i = 1, 2$.

The unique solution of (3) can now be obtained by a variety of methods [see Pesaran (1987, §5.3)]. In general, assuming that the present value expressions

$$\sum_{i=0}^{\infty} \mu_j^{-i} E(y_{t+i}^* | \Omega_t), \quad \text{for } j = 1, 2,$$

exist, we have

⁵Here we are assuming that the roots of (6) are real. This requires that $\phi_1^2 \geq 4\phi_2$.

$$y_t = \psi_1 y_{t-1} + \psi_2 y_{t-2} + \sum_{j=1}^2 \pi_j \sum_{i=0}^{\infty} \mu_j^{-i} E(y_{t+i}^* | \Omega_t), \quad (8)$$

where

$$\begin{aligned} \psi_1 &= \mu'_1 + \mu'_2 > 0, & \psi_2 &= -\mu'_1 \mu'_2 < 0, \\ \pi_1^{-1} &= \phi_2 \mu_1 (\mu_2 - \mu_1), & \pi_2^{-1} &= -\phi_2 \mu_2 (\mu_2 - \mu_1). \end{aligned}$$

The positive and the negative values predicted by the theory for the coefficients of y_{t-1} and y_{t-2} , respectively, provides a quick check on the adquacy of the model in empirical analysis.

In the case where y_t^* can be approximated by the linear function, $y_t^* = \gamma' \underline{x}_t + u_t$, the present value expressions in the above solution will be given by

$$\sum_{i=0}^{\infty} \mu_j^{-i} E(y_{t+i}^* | \Omega_t) = \gamma' \left\{ \sum_{i=0}^{\infty} \mu_j^{-i} E(\underline{x}_{t+i} | \Omega_t) \right\} + u_t,$$

which can be obtained explicitly in terms of the parameters of the process generating \underline{x}_t . For example, when (\underline{x}_t) can be approximated by the r^{th} order vector autoregressive process⁶

$$\sum_{i=0}^r R_i \underline{x}_{t-i} = \underline{\epsilon}_t, \quad (R_0 = I_k), \quad (9)$$

using results in Pesaran (1987, pp. 294-95) we have

$$\sum_{i=0}^{\infty} \mu_j^{-i} E(\underline{x}_{t+i} | \Omega_t) = (I_k + \Phi_{0j})^{-1} (I_k - \sum_{i=1}^{r-1} \Phi_{ij} L^i) \underline{x}_t$$

⁶Notice that the presence of unit roots in the process generating \underline{x}_t are not ruled out here.

where I_k is an identity matrix of order k , and

$$\Phi_{ij} = \sum_{s=i+1}^r \mu_j^{-(s-i)} R_s, \quad i = 0, 1, 2, \dots, r-1, \quad j = 1, 2.$$

Substituting these results in (8), the unique solution of the RE model becomes

$$y_t = \psi_1 y_{t-1} + \psi_2 y_{t-2} + \underline{a}'_{r-1}(L) \underline{x}_t + \sigma u_t, \quad (10)$$

where $\sigma = (1 - \psi_1 - \psi_2) [1 - (\psi_1/\beta) - (\psi_2/\beta^2)]$, and $\underline{a}'_{r-1}(L) = \sum_{i=1}^r \underline{a}'_i L^{i-1}$

represents a $k \times 1$ vector of lag polynomial of order $r-1$ in the lag operator L , defined by

$$\underline{a}'_{r-1}(L) = \gamma' \left\{ \sum_{j=1}^2 \pi_j (I_k + \Phi_{0j})^{-1} \left[I_k - \sum_{i=1}^{r-1} \Phi_{ij} L^i \right] \right\}. \quad (11)$$

From (10) it now follows that the necessary condition for identification of the $k+3$ structural parameters, ϕ_1 , ϕ_2 , β and γ is given by $rk + 2 \geq k + 3$, or $k(r-1) \geq 1$.⁷ When the model is identified the relations in (11) give $k(r-1) - 1$ over-identifying cross-equation restrictions. These cross-equation restrictions are, however, difficult to work with in practice. They involve the roots μ_1 and μ_2 in a highly non-linear form and in general do not readily lend themselves to empirical evaluation. An alternative approach would be to derive the cross-equation restrictions in terms of the reduced form parameters, ψ_1 , ψ_2 , and \underline{a}'_i , $i = 1, 2, \dots, r-1$, and the parameters of the VAR system (9). This can be easily achieved by

⁷The order condition for identification when the i^{th} element of \underline{x}_t (say, x_{it}) follows a univariate autoregressive process of order r_i , is given by $\sum_{i=1}^k r_i \geq k + 1$.

using the factorization method suggested in Pesaran (1987, §7.2.2). Suppose that $r = 2$, then $\underline{a}_1(L) = \underline{a}_1 + \underline{a}_2 L$ should satisfy the polynomial identity

$$\begin{aligned} \underline{a}'_1(L) (L^2 - \delta_1 L - \delta_2) &= \underline{d}'_2 + (\underline{d}'_1 - \underline{d}'_2 R_1) L \\ &+ (\sigma \underline{\gamma}' - \underline{d}'_1 R_1 - \underline{d}'_2 R_2) L^2 - \underline{d}'_1 R_2 L^3, \end{aligned}$$

where $\delta_1 = \psi_1/\beta$, $\delta_2 = \psi_2/\beta^2$, $\sigma = (1 - \psi_1 - \psi_2)(1 - \delta_1 - \delta_2)$, \underline{d}_1 and \underline{d}_2 are $k \times 1$ vectors of the auxiliary parameters and \underline{a}_1 and \underline{a}_2 are the vectors of the reduced form parameters in (10). Equating the coefficients of the powers of L from both sides of this identity yields

$$\left. \begin{aligned} \underline{d}'_1 R_2 &= -\underline{a}'_2, \\ \underline{d}'_2 &= -\delta_2 \underline{a}'_1, \\ \underline{d}'_1 - \underline{d}'_2 R_1 &= -(\delta_1 \underline{a}'_1 + \delta_2 \underline{a}'_2), \\ \sigma \underline{\gamma}' - \underline{d}'_1 R_1 - \underline{d}'_2 R_2 &= \underline{a}'_1 - \underline{a}'_2 \delta_1, \end{aligned} \right\} \quad (12)$$

which can be solved for the following cross-equation restrictions:⁸

$$\beta^2 (\underline{a}'_2 I_k) - \beta \psi_1 (\underline{a}'_1 R_2) - \psi_2 (\underline{a}'_1 R_1 R_2 + \underline{a}'_2 R_2) = 0. \quad (13)$$

Unlike (11), these restrictions do not depend on the roots μ_1 and μ_2 , and can be readily tested by standard classical methods such as the likelihood ratio method, or the Wald procedure.⁹ One only needs to compute consistent estimates of the parameters of the reduced form equations (9) and (10), and their (asymptotic) covariance matrices. Under the assumption that u_t and ϵ_t are serially uncorrelated, the application of the least squares method to relations in (9) and (10) yields consistent estimates of the

⁸Recall that $\delta_1 = \psi_1/\beta$ and $\delta_2 = \psi_2/\beta^2$.

⁹When the discount factor, β , is not known, any one of the k relations in (13) can be used to estimate it, thus leaving the total of $k-1$ over-identifying restrictions for testing purposes.

parameters, and any possible heterogeneity in the variance of u_t can be taken care of via the White (1980) heteroskedasticity-consistent method of computing the covariance matrices of the least squares estimators. Relations in (13) can also be used to obtain the following expression for γ , the parameters of the target equation:

$$\gamma' = \sigma^{-1} (a_1' (I_k - \delta_1 R_1 - \delta_2 R_2 - \delta_2 R_1^2) - a_2' (\delta_1 I_k + \delta_2 R_1)), \quad (14)$$

Now, using the cross-equation restrictions (13), the expression for γ can also be written as:

$$\gamma' = \sigma^{-1} a_1' (S_1 - S_2), \quad (14')$$

where

$$S_1 = (I_k - \delta_1 R_1 - \delta_2 R_2 - \delta_2 R_1^2),$$

and

$$S_2 = (\delta_2 R_1 R_2 + \delta_1 R_2) (I_k - \delta_2 R_2)^{-1} (\delta_1 I_k + \delta_2 R_1).$$

The extension of the results (13) and (14) to higher order VAR systems is relatively straightforward. For example, for $r = 3$ we have the following $2k$ cross-equation restrictions:

$$a_1' (\delta_2 R_1 R_2 + \delta_1 R_2 + \delta_2 R_3) - a_2' (I_k - \delta_2 R_2) - \delta_1 a_3', \quad (15a)$$

and

$$a_3' = (\delta_2 a_2' + a_1' (\delta_1 I_k + \delta_2 R_1)) R_3. \quad (15b)$$

Again, when β is not known, any one of the relations in (15) can be used to estimate it; thus leaving $2k-1$ restrictions to be tested. Similarly, for the structural parameters we have

$$\sigma \gamma' = a_1' (I_k - \delta_1 R_1 - \delta_2 R_2 - \delta_1 R_1^2) - a_2' (\delta_1 I_k + \delta_2 R_1) - \delta_2 a_3',$$

where a_2 and a_3 can be solved out in terms of a_1 using the cross-equation restrictions in (15).

In deriving the above cross-equation restrictions it is assumed that x_t does not contain perfectly predictable variables, such as the intercept term, time trends or seasonal dummies. When these variables are present they could be included in (10), without any loss of generality.

4. An Application

In this section I apply the generalized adjustment-cost model to the determination of employment demand in the U.K. coal industry over the period 1956-1983. I assume that the logarithm of desired employment, y_t^* , measured in man-hours, is determined by the log-linear specification

$$\begin{aligned} y_t^* &= \beta_1 + \beta_2 T_t + \gamma_1 q_t + \gamma_2 w_t + u_t \\ &= \beta_1 + \beta_2 T_t + \gamma' x_t + u_t, \end{aligned} \quad (16)$$

where q_t , w_t , and T_t stand respectively, for the logs of output, real product wage, and a linear time trend ($T_{1980} = 0$) used as a proxy for technical change.¹⁰ The disturbances u_t represent mean zero, serially uncorrelated productivity shocks. Applying the model of Section 3 to (16) yields:

$$y_t = b_0 + b_1 T_t + \psi_1 y_{t-1} + \psi_2 y_{t-2} + a_{-r-1}(L)x_t + \sigma u_t, \quad (17)$$

where $x_t = (q_t, w_t)'$, and ψ_1 , ψ_2 and σ are as defined in the previous section. Taking $r = 2$, we obtained the following results for the unrestricted version of (17):

¹⁰For the data sources and other details, see Pesaran et al. (1987). The log-linear specification for y_t^* can be justified as a cost minimization solution to a firm's employment decision problem under a Cobb-Douglas technology with neutral technical progress.

$$\begin{aligned}
y_t = & 1.0529 - 0.0088 T_t + 1.1661 y_{t-1} - 0.3397 y_{t-2} + 0.4287 q_t \\
& (3.39) \quad (-5.71) \quad (11.80) \quad (-3.68) \quad (6.90) \\
& - 0.4787 q_{t-1} - 0.0895 w_t - 0.0849 w_{t-1} + \hat{\epsilon}_t, \quad (18) \\
& (-8.69) \quad (-2.01) \quad (-1.68)
\end{aligned}$$

$$\bar{R}^2 = 0.9989, \quad \hat{\sigma}_\epsilon = 0.0136, \quad \text{LLF} = 85.23, \quad n = 28 \text{ (1956-1983)},$$

$$\chi_{\text{SC}}^2(1) = 0.29, \quad \chi_{\text{FF}}^2(1) = 0.10, \quad \chi_{\text{N}}^2(2) = 0.22, \quad \chi_{\text{H}}^2(1) = 0.01.$$

The figures in parentheses are t-ratios, $\hat{\sigma}_\epsilon$ is the standard of the regression, \bar{R}^2 is the adjusted R^2 , n is the number of observations. $\chi_{\text{SC}}^2(1)$, $\chi_{\text{FF}}^2(1)$, $\chi_{\text{N}}^2(2)$, $\chi_{\text{H}}^2(1)$ are diagnostic statistics distributed approximately as chi-squared variates (with degrees of freedom in parentheses), for tests of residual serial correlation, functional form misspecification, non-normal errors, and heteroscedasticity, respectively.¹¹

The equation has a good fit, passes all the diagnostic tests and except for the coefficient of w_{t-1} , all the other coefficients are statistically significant at the 5 percent level and have the expected signs. The highly significant positive and negative estimates obtained for ψ_1 and ψ_2 is in accordance with the prediction of the theory and in particular suggests that changes in the speed of adjustment of employment are an important consideration in the U.K. coal industry.¹²

The parameter estimates for the VAR system (9) in the case of the present application are given in Table 1. The estimates of the reduced form parameters of the full model can now be summarized as

¹¹For more details about these test statistics and their computations see Pesaran and Pesaran (1987).

¹²It is important to note that this is not an isolated result. The disaggregate estimates reported in Pesaran et al. (1987) also support the generalized adjustment cost model in the case of 13 of the 40 employment functions estimated for the U.K. economy.

$$\hat{R}_1 = \begin{bmatrix} 0.3597 & -0.1288 \\ 0.0882 & 0.6861 \end{bmatrix}, \quad \hat{R}_2 = \begin{bmatrix} 0.5586 & -0.1074 \\ -0.0810 & 0.0400 \end{bmatrix}$$

$$\hat{a}'_1 = (0.4287, -0.0895), \quad \hat{a}'_2 = (-0.4787, -0.0849),$$

$$\hat{\psi}_1 = 1.1661, \quad \hat{\psi}_2 = -0.3397.$$

Using these estimates in (13) yields the following quadratic equations in $\hat{\beta}$:

$$0.4787\hat{\beta}^2 + 0.2877\hat{\beta} + 0.0575 = 0, \quad (19)$$

$$0.0849\hat{\beta}^2 - 0.05786\hat{\beta} - 0.0094 = 0. \quad (20)$$

Under the REH either of these equations can be used to estimate β . But equation (19) has no real roots, and the only positive root of (20) is equal to 0.82, which yields an annual real rate of discount of 22 percent which is far too high. These results cast serious doubt on the validity of the RE restrictions in the case of the present example.¹³ However, if we ignore the RE restrictions and suppose that β is known to be equal to 0.95, we obtain $\hat{\mu}'_1 = 0.5988$, $\hat{\mu}'_2 = 0.5673$, $\hat{\mu}_1 = 1.5865$ and $\hat{\mu}_2 = 1.6746$, which have the sort of magnitudes predicted by theory (i.e., all are positive with two of the roots falling inside the unit circle, and the other two falling outside the unit circle). Furthermore, from equations (6) and (7) $\hat{z}_1 = -0.2353$, $\hat{z}_3 = -0.2919$, which imply the estimates $\hat{\phi}_2 = 1/(\hat{z}_1\hat{z}_2) = 14.56$, and $\hat{\phi}_1 = -\hat{\phi}_2(\hat{z}_1 + \hat{z}_2) = 7.68$ for the coefficients of the cost function.¹⁴

¹³A formal test of the RE restrictions in (13) can be carried out by means of the Wald procedure, but will not be attempted here.

¹⁴The results are not much affected if instead of the VAR system in Table 1, univariate AR processes are estimated for q_t and w_t .

TABLE 1

Estimates of the Coefficients of the Vector Autoregressive
Equations for Real Output and Real Product Wages
In the Coal Industry, 1956-1983[†]

<u>Regressors</u>	<u>q_t</u>	<u>w_t</u>
Intercept	-0.6705 (-1.15)	-1.5877 (-2.22)
q _{t-1}	0.3597 (1.79)	0.0882 (0.36)
q _{t-2}	0.5586 (2.85)	-0.0810 (-0.34)
w _{t-1}	-0.1288 (-0.63)	0.6861 (2.73)
w _{t-2}	-0.1074 (-0.60)	0.0400 (0.19)
$\hat{\sigma}$	0.0616	0.0757
\bar{R}^2	0.9614	0.5250
$\chi_{SC}^2(1)$	0.27	0.001

[†]q_t = logarithm of real output, w_t = logarithm of real product wage,
 $\hat{\sigma}$ = estimated standard of the equation, \bar{R}^2 = adjusted R², $\chi_{SC}^2(1)$ =

Lagrange multiplier statistic for the test of first order serial correlation. The figures in parentheses are t-ratios.

Overall, there seems to be some support for the hypothesis that apart from the familiar costs of changing employment, there are also additional costs involved in altering the speed with which the employment in the coal industry is changed. There, however, seems to be little evidence in favor of the rational expectations hypothesis that output and real wage expectations are formed rationally (in the sense of Muth) on the basis of a VAR model.

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