CONFLICT AND RENT-SEEKING: RATIO VS. DIFFERENCE MODELS

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Abstract

The rent-seeking competitions studied by economists fall within a much broader category of conflict interactions that also includes, for example, military combats, election campaigns, industrial disputes, lawsuits, and sibling rivalries. In the rent-seeking literature, each party's success p_i (which can be interpreted either as the probability of victory or as the proportion of the prize won) has been taken to be a function of the ratio of the respective resource commitments. Alternatively, however, p_i may instead be a function of the difference between the parties' commitments to the contest. The Contest Success Function (CSF) for the difference form is a logistic curve in which increasing returns apply up to an inflection point at equal resource commitments, as is consistent with military experience. A crucial flaw of the traditional ratio model is that neither one-sided submission nor two-sided peace between the parties can ever occur as a Cournot equilibrium. In contrast, both of these outcomes are entirely consistent with a model in which success is a function of the difference between the parties' resource commitments.

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Following the seminal contribution of Gordon Tullock [1980], a number of papers[1] have explored various aspects of rent-seeking competitions. In such contests, each of N players invests effort C_i in the hope of gaining a prize of value V. Existing analyses have explored the nature of equilibrium in the short run versus the long run and with identical versus non-identical contestants, the central issue addressed being whether or not under- or over-dissipation of rents will occur.

But the fundamental notion of competitions in which relative success is a function of the parties' respective resource commitments applies far beyond the rent-seeking context. It is applicable, for example, to military combats, election campaigns, industrial struggles (strikes and lockouts), legal conflicts (lawsuits), and even to rivalries between siblings or between spouses within the family. Owing perhaps to failure to perceive these wider implications, the papers in the rent-seeking literature generally do not adopt a <u>general-equilibrium</u> approach, in which the resources available for investment in rent-seeking competitions also have alternative productive or consumptive uses. Thus, rent-seeking efforts may be profitable and yet still fail to generate as much utility as productive or leisure uses of resources. Also, what is very important, a general-equilibrium model would typically make the value of the prize an endogenous variable rather than an exogenously given parameter. I have attempted to provide such a general-equilibrium analysis in Hirshleifer [1988].

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[1] See, e.g., Hillman and Katz [1984], Appelbaum and Katz [1986], Allard [1988].

This note has a much more limited aim, however. My purpose is only to point out that Tullock's basic equation for success in rent-seeking competition represents only one of two major families of possibilities, the other and at least equally interesting family having been totally ignored in the existing literature. Specifically, in Tullock's formula each party's success is a function of the ratios of the respective efforts or inputs C_i. As will be shown, a number of significantly different results are obtained when, alternatively, relative success is determined by the differences among the inputs.

For the purposes of this discussion, it suffices to consider $N=2$ players. In the simplest version of Tullock's basic model, the relative or proportionate outcomes p_i are a function of the ratio of the contest inputs or efforts C_i:

$$(1) \qquad p_1/p_2 = (C_1/C_2)^m$$

Here each p_i may be interpreted either as the party's respective probability of success in a discrete either-or competition or else as the proportionate share of the prize won in a continuous-outcome contest. Since $p_1 + p_2 = 1$, equation (1) is of course equivalent to:

$$(1a) \qquad p_1 = C_1^m/(C_1^m + C_2^m)$$

For given C_2, this may be called the Contest Success Function (CSF) for player #1; the CSF for the other player is defined correspondingly. (I have implicitly been assuming that the two sides' resources have equal effectiveness in the contest. More generally, it would be possible to adjust each side's C_i by an effectiveness coefficient; this straightforward generalization will be omitted here.)

The effect of the "mass effect parameter" m upon the shape of player #1's Contest Success Function is displayed in Figure 1, in which player #2's resource input is arbitrarily fixed at $C_2 = 100$. Regardless of the level of m, we see that $p_1 = p_2 = .5$ when $C_1 = C_2$. If $m \leq 1$, diminishing returns to competitive effort hold throughout. But for $m > 1$, an initial range of increasing returns exists instead. More specifically, taking the second derivative in the usual way, the inflection point along the CSF of player #1 is determined by the condition:

$$C_1/C_2 = [(m - 1)/(m + 1)]^{1/m} \tag{2}$$

Since m cannot meaningfully be zero or negative, we see that, for given C_2, the point of inflection occurs in the positive range of C_1 only if $m > 1$.[2]

While it is often plausible to assume that contest power is a function of the ratio of the forces or efforts committed, this is by no means the only possibly valid functional relation. Nor are all the implications of the ratio form always reasonable. One implication, for example, is that a side investing zero effort must lose everything so long as the opponent commits any finite amount of resources at all, however small, to the struggle. An evident alternative is to make success a function of the numerical difference instead. When the outcome is a function of the difference between the two sides' efforts, a player can have some chance or share of

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[2] In the standard Lanchester equations of military combat (Lanchester [1916 (1956)], Brackney [1959]), the outcome as in Tullock's formulation depends upon the ratio of the forces committed. But for Lanchester the battle result is always fully deterministic, in the sense that the side with larger forces (adjusted for fighting effectiveness) is 100% certain to win. This makes the CSF a step function, which jumps from $p_1 = 0$ to $p_1 = 1$ when $C_1 = C_2$. So Lanchester's formula can be regarded as the limiting case of equation (1a) as the mass effect parameter m goes to infinity.

success even without committing resources to the contest. In struggles between nations, for example, it often happens that one side will surrender rather than resist. While the hope may sometimes be to appease the aggressor, i.e., to make him more friendly, it might even make sense to surrender to a totally unappeasable opponent -- if the submitting nation does not expect to lose absolutely everything after giving up the struggle. And this is reasonable, since in general it will be costly for the victor, even in the absence of resistance, to locate and extract all the possible spoils.

There is one other factor to consider, namely, the location of the inflection point of the CSF. When it comes to military interactions, at least, "God is on the side of the larger battalions". Thus, there is an enormous gain when your side's forces increase from just a little smaller than the enemy's to just a little larger.[3] This implies that the range of increasing returns to player #1's commitment C_1 will extend up to $C_1 = C_2$, or equivalently up to $p_1 = p_2$.[4] But, we have seen, when the ratio form of the CSF is used, increasing returns, if present at all (that is, if $m > 1$), can only hold up to some $C_1 < C_2$.

For the difference form of the CSF, the required conditions are met by the logistic family of curves:

$$p_1 = 1/(1 + \exp\{k(C_2 - C_1)\}) \tag{3}$$

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[3] As seen in the previous footnote, the Lanchester equations of combat take this to the extreme. The larger force is 100% certain of victory; the smaller force has no chance at all.

[4] This is consistent with T.N. Dupuy's study of diminishing returns in combat interactions between Allied and German forces in World War II (see Dupuy [1987], Chapter 11). Dupuy's curves generally show the inflection point displaced slightly from the "equal forces, equal success" point, owing (on his interpretation) to the superior unit effectiveness of the German army.

And of course p_2 is defined correspondingly. (Notice that $p_1 + p_2 = 1$.) In particular, when $C_1 = 0$, player #1 still retains a share of success $p_1 = 1/(1 + \exp(kC_2))$. Figure 2 shows several different CSF curves for varying k, where k is the "mass effect parameter" applicable to the logistic function.

In a military context, we might expect the <u>ratio form</u> of the Contest Success Function to be applicable when clashes take place under close to "idealized" conditions such as: an undifferentiated battlefield, full information, and unflagging weapons effectiveness. In contrast, the <u>difference form</u> tends to apply where there are sanctuaries and refuges, where information is imperfect, and where the victorious player is subject to fatigue and distraction so that the surrendering side can still expect to retain some prizes. And while for the sake of concreteness I have been using military metaphors and examples, analogous statements can evidently be made about non-military struggles like lawsuits or rent-seeking competitions.

One final implication. As has been pointed out, when the ratio form of the CSF applies each side will surely always commit some resources to the contest. If peace is defined by the condition $C_1 = C_2 = 0$, we can say that, as a Cournot equilibrium, under the traditional ratio model peace can never occur!

The demonstration is simple. Side #1 will be seeking to maximize its "profit":

(4) $$\Pi_1 = Vp_1 - C_1$$

where V is the given value of the prize and p_1 is determined as in equation (1a). A similar equation holds of course for player #2. Suppose momentarily it were the case that $C_1 = C_2 = 0$, the parties sharing the prize equally without fighting. Then, assuming only that $V > 0$, under the Cournot assumption either player would be motivated to defect, since even the smallest finite commitment of resources makes the defector's relative success jump from 50% to 100%.

In contrast, two-sided peace may easily hold as a stable Cournot equilibrium under the logistic CSF, even in the sort of partial-equilibrium model traditionally dealt with in the rent-seeking literature.[5] The reason is that the player who defects from $C_1 = C_2 = 0$ does not get the benefit of a discrete jump from 50% to 100% success, but only a marginal increment of gain.

Since some readers have found the proposition above -- that two-sided peace may be a Cournot equilibrium under the logistic model -- hard to credit, a Numerical Example may help drive home the point.

<u>Numerical Example 1</u>

Player #1 seeks to maximize his profit as defined in equation (4), with p_1 given by equation (3) above -- and taking $C_2 = 0$ as given. Finding the derivative in the usual way leads to:

$$\frac{k\,\exp(-kC_2)}{(1+\exp(-kC_1))^2} = \frac{1}{V}$$

For $C_1 = 0$ to be a solution, we must have $V = 4/k$. By symmetry, an analogous equation will hold for player #2. So, for example, if $k = .04$ and $V = 100$, then $C_1 = C_2 = 0$ will indeed be a Cournot equilibrium. In this equilibrium, of course, $p_1 = p_2 = .5$ so that the parties each have profit of 50.

[5] For the analogous result in a general-equilibrium context, see Hirshleifer [1988], Part B.

What about the possibility of <u>one-sided submission</u> rather than two-sided peace? This means that player #1 (say) chooses $C_1 > 0$ while player #2 sets $C_2 = 0$. For such an outcome, some kind of asymmetry must be introduced -- in the parties' valuations of the prize, in the effectiveness of their respective contest efforts, or possibly in the costs of such efforts. But regardless of any such asymmetries, one-sided submission as a Cournot equilibrium can no more occur under the ratio model than could two-sided peace![6] Yet once again, this is entirely possible for the difference form (logistic CSF), as will be illustrated by a second Numerical Example.

Numerical Example 2

Once again player #1 seeks to maximize his profit where p_1 is defined by equation (3) above. Taking $C_2 = 0$ as given, and setting $k = .04$ and $V_1 = 500$, the profit-maximizing solution vector for player #1 is:

$$(C_1, p_1, \Pi_1) = (72.2, .9473, 401.4)$$

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[6]We need look only at asymmetries due to inequalities in valuation of the prize. Specifically, suppose $V_1 > V_2$, suggesting that there might be a Cournot equilibrium with $C_1 > 0$ while $C_2 = 0$. Using the profit equation (4) for player #1, and the generalized version (1a) of the Contest Success Function for the ratio form, the first derivative is:

$$\frac{d\Pi_1}{dC_1} = \frac{V_1 m C_1^{m-1}(C_2^m)}{(C_1^m + C_2^m)^2} - 1$$

Evidently, whenever $C_2 = 0$ this derivative will be negative. So under the ratio form of the CSF, it will never be possible to have an asymmetrical contest outcome with one party having zero and the other having positive commitment of resources.

Reversing the process for player #2, with $C_1 = 72.2$ taken as given, the profit-maximizing[7] solution is:

$$(C_2, p_2, \Pi_2) = (0, .0527, 5.3)$$

The expectations on each side as to the other party's behavior being mutually consistent, this is a Cournot equilibrium.

CONCLUSION: In rent-seeking or other conflict competitions, a closer fit to reality is sometimes achieved by a model in which relative success is a function of the difference rather than the ratio of the parties' resource inputs. And in particular, neither two-sided peace nor one-sided submission can ever occur as a Cournot equilibrium under the traditional ratio model, wheras both are entirely possible under the difference form of the Contest Success Function.

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[7] This is not an interior optimum, but rather a corner solution. Player #2's profit function has a negative first derivative throughout, leading him to cut back effort until the limit of zero is reached.

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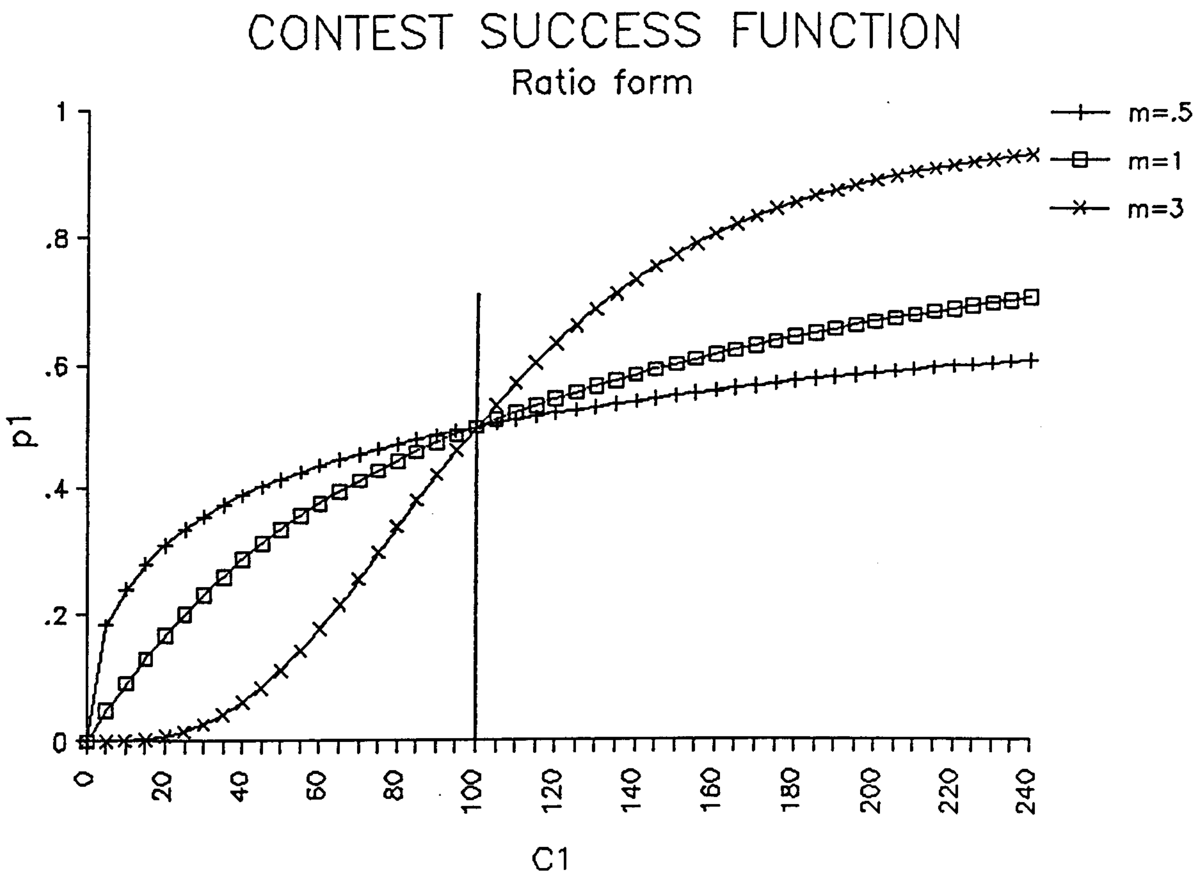

FIGURE 1

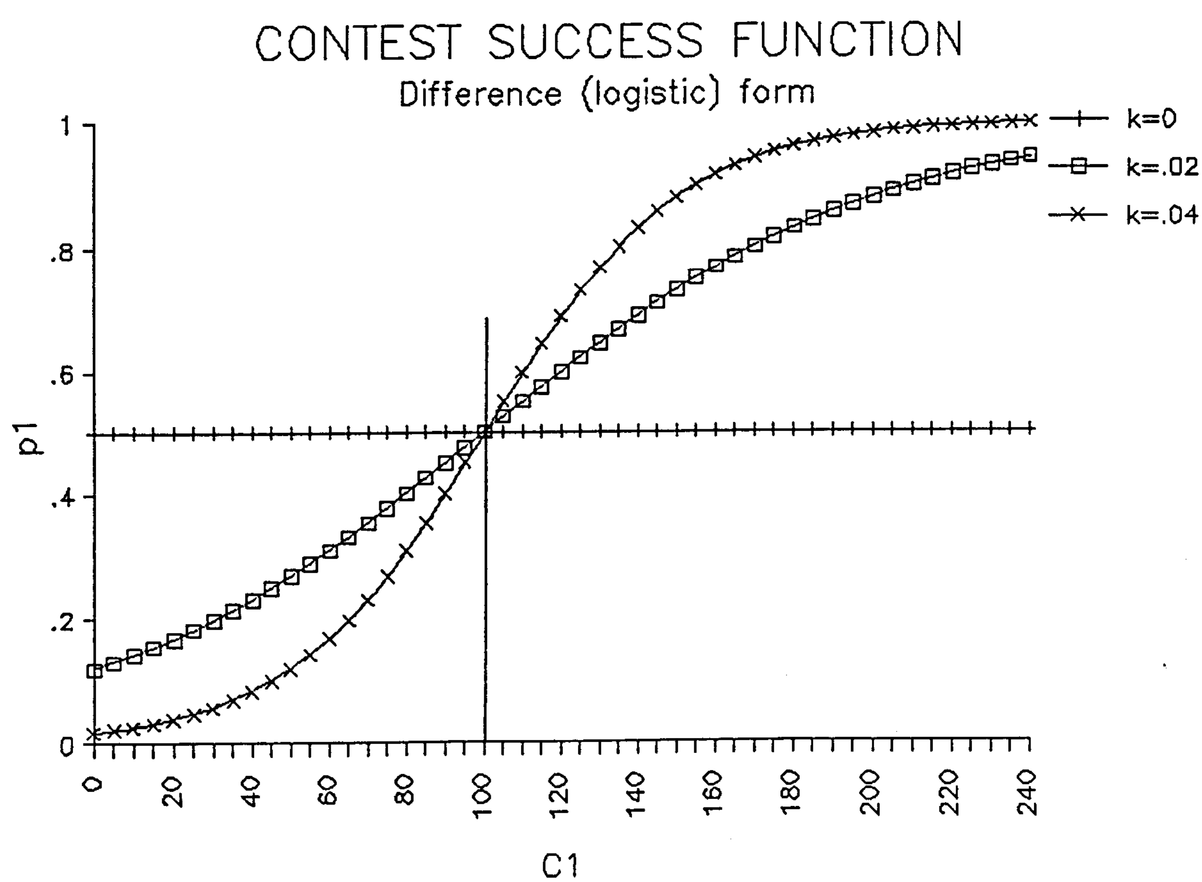

FIGURE 2