

AGGREGATION BIAS AND LABOR DEMAND EQUATIONS
FOR THE U.K. ECONOMY*

by

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ABSTRACT

This paper extends the results obtained in Pesaran, Pierse and Kumar (1988), and examines the effect of aggregation on the estimates of long run wage and output elasticities of demand for employment in the U.K. The aggregate and the disaggregate employment functions analyzed in this paper differ from those in Pesaran, Pierse and Kumar (PPK) in two respects: first the functions allow for a longer lagged effect of output on employment, second, in order to deal with some of the econometric difficulties associated with the use of the time trend as a proxy for technical change in estimating the employment functions, the time trend is replaced by a measure of embodied technological change based on the current and past movements of gross investment, a la Kaldor (1961).

The paper also discusses alternative methods of testing for aggregation bias and proposes direct tests of the discrepancy of the macro parameters from the average of the corresponding micro parameters, and derives tests of aggregation bias in the general case where the parameters of interest may possibly be non-linear functions of the micro parameters. The paper also develops a Durbin-Hausman type misspecification test of the disaggregate model. These tests and the goodness-of-fit criteria and the test of perfect aggregation proposed in PPK are then applied to disaggregate and aggregate specifications of the employment functions.

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1. Introduction

The responsiveness of employment to changes in real wages is an issue of considerable importance, particularly for policy analysis, and over the past decade a number of studies have been devoted to this issue in the U.K. Notable examples include the papers by Nickell (1984), Symons (1985), Wren-Lewis (1986), and Burgess (1988) for the manufacturing sector, and by Beenstock and Warburton (1984), Layard and Nickell (1985, 1986) for the private sector and the economy as a whole. In contrast to the earlier work by Godley and Shepherd (1964), Brechling (1965) and Ball and St. Cyr (1966), these recent studies find a significant and quantitatively important effect for real wages on employment. The point estimates of the long run wage elasticity obtained in these studies vary widely depending on the coverage of the data (whether the data set used is economy-wide or just manufacturing), and on the specification of the estimated equations. A recent review of these studies by the Treasury (1985) concludes that the estimate of long run wage elasticity most likely falls in the region $-.5$ to -1 , although for the economy as a whole under the influence of Layard and Nickell's important contributions the "consensus" estimate of this elasticity in the U.K. currently seem to center on the figure of -1 .¹

All the above studies are, however, carried out using highly aggregated data, either at the level of the whole economy or the manufacturing sector, and given the significance of their results for macroeconomic policy it is important that the robustness of their results to the level of aggregation chosen are carefully investigated. In this paper we extend the results obtained in Pesaran, Pierse and Kumar (1988), and examine the effect of

¹This consensus estimate is also the same as the figure obtained by Beenstock and Warburton (1984) for their extended data set.

aggregation on the estimates of long run wage and output elasticities of demand for employment in the U.K. The aggregate and the disaggregate employment functions analyzed in this paper differ from those in Pesaran, Pierse and Kumar (PPK) in two respects: First the functions allow for a longer lagged effect of output on employment. Second, in order to deal with some of the econometric difficulties associated with the use of the time trend as a proxy for technical change in estimating the employment functions,² the time trend will be replaced by a measure of embodied technological change based on the current and past movements of gross investment, a la Kaldor (1957, 1961). This measure of technological change is both statistically less problematic than a simple time trend and more satisfactory from a theoretical standpoint.

In this paper we also discuss alternative methods of testing for aggregation bias and propose direct tests of the discrepancy of the macro parameters from the average of the corresponding micro parameters. We distinguish between the case where the values of the macro parameters are given a priori (say by the consensus estimate), and when they are defined under the disaggregate model. We develop the aggregation bias tests in the general case where the parameters of interest may possibly be non-linear functions of the micro parameters. Since the tests of the aggregation bias, whether of the type discussed here or the one proposed in Zellner (1962), assume the disaggregate model is correctly specified, in this paper we also develop a Durbin-Hausman type misspecification test of the disaggregate model. These tests and the goodness-of-fit criteria and the test of perfect

²The econometric problems involved in the use of time trends in regression equations containing non-stationary variables are discussed, for example, by Mankiw and Shapiro (1985, 1986), and Durlauf and Phillips (1986).

aggregation proposed in PPK will then be applied to the new disaggregate and aggregate specifications of the employment functions.

The plan of the paper is as follows: the next section sets out the disaggregate employment functions and discusses the theoretical rationale that underlies them. Section 3 motivates the use of a distributed lag function in gross investment as a proxy for technological change. Section 4 gives the details of the misspecification test and the test of the aggregation bias. Section 5 presents the empirical results, and the final section provides a summary of the main findings of the paper.

2. Industrial Employment Functions: Theoretical Considerations

In specifying of the employment demand functions we follow the literature on derivation of dynamic factor demand models and suppose that the employment decision is made at the industry level by identical cost minimizing firms operating under uncertainty in an environment where adjustment can be costly. We assume that in the absence of uncertainty and adjustment costs the industry's employment function is given by

$$h_t^* = f(w_t, y_t, a_t) + v_t, \quad (1)$$

where

h_t^* = the desired level of man-hours employment (in logs),

w_t = the real wage rate (in logs),

y_t = the expected level of real demand (in logs),

a_t = an index of technological change,

v_t = mean zero serially uncorrelated productivity shocks.

The actual level of employment, h_t , measured in logarithms of man-hours employed in the industry is then set by solving the following optimization problem

$$\text{Min}_{h_t, h_{t+1}, \dots} E \left\{ \sum_{r=0}^{\infty} \beta^r [(h_{t+r} - h_{t+r}^*)^2 + \frac{1}{2} \phi_1 (\Delta h_{t+r})^2 + \frac{1}{2} \phi_2 (\Delta^2 h_{t+r})^2] \mid \Omega_t \right\}, \quad (2)$$

where $\Omega_t = (h_t, h_{t-1}, \dots; w_t, w_{t-1}, \dots; y_t, y_{t-1}, \dots; a_t, a_{t-1}, \dots; u_t, u_{t-1}, \dots)$ represents the information set of the firm at time t , Δ is the first difference operator, and $0 \leq \beta < 1$ is the real discount factor. The first term in [--] measures the cost of being out of equilibrium, and the second and the third terms stand respectively for the costs of changing the level and the speed which changes in employment are put into effect. The inclusion of the last term in (2) is proposed in Pesaran (1988) and generalizes the familiar adjustment cost-rational expectations models discussed, for example, by Sargent (1978) and Kennan (1979), and is of some interest as it provides a theoretical justification for the inclusion of h_{t-2} in the employment function.³ In practice the speed of adjustment coefficients $(1-\phi_1)$ and $(1-\phi_2)$ could vary with the state of the labor market as argued, for example, by Smyth (1984) and Burgess (1988). Here, however, we shall assume that they are fixed. The unique solution to the above optimization problem is derived in Pesaran (1988) and is given by:

$$h_t = \psi_1 h_{t-1} + \psi_2 h_{t-2} + \sum_{j=0}^{\infty} \theta_j E(h_{t+j}^* \mid \Omega_t). \quad (3)$$

where

$$\psi_1 = \mu_1' + \mu_2' > 0, \quad \psi_2 = -\mu_1' \mu_2' < 0,$$

$$\theta_j = (\mu_1^{-j-1} - \mu_2^{-j-1}) / [\phi_2 (\mu_2 - \mu_1)],$$

³The inclusion of first or higher order lags of h_t in the employment function can also be justified by appeal to aggregation over different types of labor or firms with different adjustment costs (Nickell, 1984).

and μ_1, μ_2, μ'_1 and μ'_2 are the roots of

$$\alpha_2 x^2 + \alpha_1 x + \lambda_1 x^{-1} + \lambda_2 x^{-2} = 1.$$

The reduced form parameters $\alpha_1, \alpha_2, \lambda_1$ and λ_2 are defined in terms of the structural parameters, β, ϕ_1 and ϕ_2 . [See, Pesaran (1988)]. It is important to note that for plausible values of the structural parameters the theory suggests a negative value for the coefficient of h_{t-2} in (3).

Adopting a linear approximation for (1), and assuming that conditional expectations of w_{t+j}, y_{t+j} and a_{t+j} with respect to Ω_t are formed rationally on the basis of an r^{th} order vector autoregressive (VAR) system, the decision rule (3) becomes:

$$h_t = \text{Intercept} + \psi_1 h_{t-1} + \psi_2 h_{t-2} + \underline{c}'_{r-1}(L) \underline{z}_t + u_t, \quad (4)$$

where $u_t = (1 - \psi_1 - \psi_2)(1 - \psi_1/\beta - \psi_2/\beta^2)v_t$, $\underline{z}_t = (a_t, y_t, w_t)'$, and $\underline{c}_{r-1}(L) = \sum_{i=1}^r c_i L^{i-1}$ is a 3×1 vector of lag polynomials of order $r-1$ in the lag operator L . In the case where the variables y_t, w_t and a_t have univariate $AR(r_i)$, $i = y, w, a$ representations, (4) simplifies to

$$h_t = \text{Intercept} + \psi_1 h_{t-1} + \psi_2 h_{t-2} + \left[\sum_{i=1}^{r_y} \gamma_{iy} L^{i-1} \right] y_t + \left[\sum_{i=1}^{r_w} \gamma_{iw} L^{i-1} \right] w_t + \left[\sum_{i=1}^{r_a} \gamma_{ia} L^{i-1} \right] a_t + u_t, \quad (5)$$

which is a generalization of the aggregate employment function (7.2) in PPK.⁴ Under the rational expectations hypothesis (REH), the coefficients

⁴To derive (7.2) in PPK from (5) let $r_y = r_w = 2$, and notice that when a simple linear trend is used as a proxy for a_t , then $a_t = a_{t-1} + b$,

c_i in (4), and γ_{iy} , γ_{iw} , γ_{ia} in (5) will be subject to $3r-4$ and $(r_y+r_w+r_a) - 4$ cross-equation restrictions, respectively. However, given our concern with the problem of aggregation in the present study we do not consider imposing these restrictions, and employ instead the unrestricted version of (5) as our maintained hypothesis.⁵ We then choose the orders of the lag polynomials on h_t , y_t , w_t and a_t empirically. The validity of the RE restrictions at the industry level and the problem of aggregation bias in the context of RE models is beyond the scope of the present paper.

3. Modelling and Measurement of Technological Change

In the empirical analysis of labor demand, technological change, broadly defined to include new scientific, engineering and electronic discoveries and inventions, is generally assumed to occur exogenously; evolving independently of market conditions and government policy interventions. It is inferred either indirectly as a residual using a production function approach, or is represented by linear, piece-wise linear or non-linear functions of time. Neither procedure is satisfactory. The former approach employed, for example, by Layard and Nickell (1985), assumes an a priori knowledge of the production possibilities and involves circular reasoning, while the latter is devoid of a satisfactory theoretical rationale and is adopted by most researchers as a "practical" method of

where b is a fixed constant, and

$$\begin{aligned} (\sum_{i=1}^{r_a-1} \gamma_{ia} L^{i-1}) a_t &= (\sum_{i=1}^{r_a-1} \gamma_{ia}) a_t - b_{i=2}^{r_a-1} (i-1) \gamma_{ia} \\ &= \gamma_a a_t + \text{constant.} \end{aligned}$$

⁵This is similar to the research strategy followed by Nickell (1984) and Burgess (1988).

dealing with a very difficult problem (Arrow, 1962).⁶

Ideally what we need are direct reliable measures of technological change, and there are some data such as expenditure on R&D and the number of patents and product designs granted over a given period that can be used. In the absence of suitable direct measures of technological change, here we adopt an indirect approach and following Kaldor (1957, 1961), postulate a distributed lag relationship between the a_t , the technological change index, and the rate of gross investment, GI_t ,

$$a_t = \text{Intercept} + \sum_{j=0}^{\infty} \lambda_j \log(GI_{t-j}). \quad (6)$$

A static version of this relationship when used in a linear version of (1) yields a log linear approximation to Kaldor's "technical progress function", which relates the rate of change of productivity per worker to the rate of change of gross investment.⁷ According to this model technological progress is "embodied" in the process of capital accumulation and takes place primarily through gross capital formation by the infusion of new equipment and machines embodying the most up-to-date technology into the economy. The formulation (6) can also be justified along the lines suggested by Arrow (1962) in his seminal paper on "learning by doing". Arrow (1962, p. 157) himself uses cumulative gross investment as an index of experience, which is closely related to the distributed lag function in (6).

⁶Notice that the use of time trends in regression equations containing integrated stochastic processes is also subject to important econometric pitfalls and as argued in Mankiw and Shapiro (1985, 1986), and Durlauf and Phillips (1986) can lead to spurious inference.

⁷See in particular Kaldor and Mirrlees (1962, pp. 176-77). Notice, however, that Kaldor's formulation abstracts from the effect of real wages on labor productivity, while ours does not.

The technological progress function (6) is more than a theoretical postulate. It is also based on direct empirical support. Schmookler (1966) in his pioneering work, using patents as a measure of technological change showed there exists strong positive correlations between gross investment and patents in railroads, petroleum refining, and building industries over the period 1873-1940. He also obtained similar results using cross section data. While there is some doubt about the direction of causation in Schmookler's findings, there is little dispute about the existence of a close relationship between gross investment and technological change.⁸ Since our aim here is not to explain the causes of technological change but to estimate its impact on employment demand, we feel that the controversy over the causality of investment-patents relationship has little bearing on our analysis.

The coefficients λ_j , $j = 1, 2, \dots$, measure the impact of past investments on the current state of technological advance, and it is reasonable to assume that they are a decreasing function of the lag length, $j = 1, 2, \dots$. The likely rate of decline of λ_j depends on the importance of the learning by doing component of a_t . Under a pure learning story (λ_j) will be fixed or show a very slow rate of decline. The rate of decline of (λ_j) is likely to be much higher if one adopts Kaldor's idea. Here, for the purpose of empirical analysis we assume the following geometrically declining pattern for λ_j

$$\lambda_j = \alpha(1-\lambda)\lambda^j, \quad j = 0, 1, 2, \dots \quad \alpha, \lambda > 0$$

⁸For a review of more recent evidence see, for example Beggs (1984) and Baily and Chakrabarti (1985). Notice, however, that Beggs uses wage expenditures as a surrogate for investment data and his results may not be directly comparable to those obtained by Schmookler. On this see the comments by Schankerman (1984) on Beggs's paper.

and write (6) as

$$a_t = \text{Intercept} + \alpha d_t(\lambda) \quad (7)$$

where $d_t(\lambda)$ satisfies the following recursive formula:

$$d_t(\lambda) = \lambda d_{t-1}(\lambda) + (1-\lambda) \log(GI_t). \quad (8)$$

Substituting (7) in (5) now yields

$$h_t = \text{Intercept} + \psi_1 h_{t-1} + \psi_2 h_{t-2} + \gamma_y(L) y_t + \gamma_w(L) w_t + \alpha \gamma_a(L) d_t(\lambda) + u_t, \quad (9)$$

where $\gamma_y(L)$, $\gamma_w(L)$, and $\gamma_a(L)$ are lag operator polynomials of orders $r_y - 1$, $r_w - 1$ and $r_a - 1$, respectively. It is clear that in general α is not identifiable, although the decay coefficient, λ , can in principle be estimated from the data. We shall return to the issue of the estimation of (9) in Section 5.

4. Testing For Aggregation Bias: Econometric Considerations

Suppose that for a given value of the decay parameter λ , the variables in (9), namely h_t , y_t , w_t and $d_t(\lambda)$ are observed over the period $t = 1, 2, \dots, n$ for each of the m firms (industries), $i = 1, 2, \dots, m$. Then the disaggregate employment equations can be written in matrix notations as

$$H_d: \underline{h}_i = X_i \underline{\beta}_i + \underline{u}_i, \quad i = 1, 2, \dots, m \quad (10)$$

where \underline{h}_i is the $n \times 1$ vector of observations on the log of man-hours employment in the i^{th} firm (industry), X_i is the $n \times k$, ($k = r_y + r_w + r_a + 3$) matrix of observations on the regressors in (9) for the i^{th} firm (industry). $\underline{\beta}_i$ is the $k \times 1$ vector of the coefficients associated with columns of X_i , and \underline{u}_i is the $n \times 1$ vector of disturbances for the i^{th}

firm (industry). The aggregate equation associated with (9) that satisfies the Klein-Nataf consistency requirement is given by⁹

$$H_a: \underline{h}_a = X_a \underline{b}_a + \underline{v} \quad (11)$$

where

$$\underline{h}_a = \sum_{i=1}^m \underline{h}_i, \quad X_a = \sum_{i=1}^m X_i,$$

and \underline{b}_a is the $k \times 1$ vector of macro parameters. The $n \times 1$ disturbance vector \underline{v} , will be equal to $\underline{u}_a = \sum_{i=1}^m \underline{u}_i$, only if the perfect "aggregation condition"

$$H_\xi: \xi = \sum_{i=1}^m X_i \beta_i - X_a \underline{b}_a = 0, \quad (12)$$

discussed in detail in PPK, are satisfied. Here we focus on the problem of aggregation bias and develop alternative methods of analyzing the extent of this bias in economic applications. In what follows we adopt the following basic assumptions:

Assumption 1: The n elements of the disturbance vector $\underline{u}_i = (u_{it})$, have zero means, finite variances and are serially independently distributed.

They also satisfy the moment condition

$$E|u_{it}|^{2+\delta} < \Delta < \infty, \quad \text{for some } \delta > 0, \quad \text{and all } t.$$

Assumption 2: The disturbance vectors \underline{u}_i are distributed independently of X_j , and $E(\underline{u}_i \underline{u}_j') = \sigma_{ij} I_n$, for all i and j , ($\sigma_{ii} > 0$).

Assumption 3: The matrices X_i have full ranks, and the probability limits

⁹See Lovell (1973), and the discussion in PPK (p. 25).

$$\text{plim}_{n \rightarrow \infty} (n^{-1} X_i' X_j) = \Sigma_{ij}, \quad i, j = a, 1, 2, \dots, m,$$

exist, and the $k \times k$ matrices Σ_{ii} , $i = a, 1, 2, \dots, m$, are non-singular.

We also base our tests on the OLS estimates

$$\hat{\underline{b}} = (X_a' X_a)^{-1} X_a' h_a, \quad \hat{\underline{\beta}}_i = (X_i' X_i)^{-1} X_i' h_i, \quad i = 1, 2, \dots, m,$$

although, in principle, the tests proposed below can also be constructed using the more efficient SURE (Seemingly Unrelated Regression Equations) estimators of $\underline{\beta}_i$, due to Zellner (1962).

4.1 A Direct Test of Aggregation Bias

The problem of "aggregation bias", as originally discussed by Theil (1954) is defined in terms of the deviations of macro parameters from the averages of the corresponding micro parameters.¹⁰ In the context of the linear disaggregate and aggregate models (10) and (11), the vector of aggregation bias is defined by

$$\underline{\eta}_\beta = \underline{b} - \frac{1}{m} \sum_{i=1}^m \underline{\beta}_i. \quad (13)$$

A test of aggregation bias then involves testing the hypothesis $H_0: \underline{\eta}_\beta = 0$. In testing this hypothesis the case where \underline{b} is given a priori (such as the "consensus" estimate of the long-run real wage elasticity discussed in the introduction) should be distinguished from the case where \underline{b} is defined as the pseudo true value of $\hat{\underline{b}}$ assuming that the disaggregate model is correctly specified. In the former case the hypothesis $H_0: \underline{\eta}_\beta = 0$ can be tested using the fact that when \underline{u}_i are normally distributed then under

¹⁰For empirical analysis of aggregation bias see, for example, the papers by Boot and de Wit (1960), Gupta (1971) and Sasaki (1978).

Assumptions 1-3

$$\left(\underline{b} - \frac{1}{m} \sum_{i=1}^m \hat{\underline{\beta}}_i \right) \sim N(\eta_{\beta}, \Omega),$$

where¹¹

$$\Omega = \frac{1}{m^2} \sum_{i,j=1}^m \text{Cov}(\hat{\underline{\beta}}_i, \hat{\underline{\beta}}_j).$$

Using standard results, it is now easily shown that under H_0 and for a fixed m

$$q_1 = \left(\underline{b} - \frac{1}{m} \sum_{i=1}^m \hat{\underline{\beta}}_i \right)' \hat{\Omega}_n^{-1} \left(\underline{b} - \frac{1}{m} \sum_{i=1}^m \hat{\underline{\beta}}_i \right) \stackrel{a}{\sim} \chi_k^2, \quad (14)$$

where $\hat{\Omega}_n$ represents a consistent estimator of Ω . The result (14) holds even if \underline{u}_i are not normally distributed. The statistic q_1 takes \underline{b} as a fixed vector, and tests for the deviation of the average of micro parameters from this fixed vector, on the assumption that H_d holds. In practice, however, it is rare that a "consensus" value for \underline{b} or some of its elements is available, and \underline{b} needs to be chosen in light of the knowledge of the disaggregate model. When H_d holds the pseudo true value of \underline{b} is given by

$$\underline{b} = \text{plim}_{n \rightarrow \infty} (\hat{\underline{b}} | H_d) = \sum_{i=1}^m C_i \underline{\beta}_i, \quad (15)$$

where

$$C_i = \Sigma_{aa}^{-1} \Sigma_{ai}, \quad i = 1, 2, \dots, m, \quad (16)$$

¹¹Note that $\text{Cov}(\hat{\underline{\beta}}_i, \hat{\underline{\beta}}_j) = \sigma_{ij} (\mathbf{X}'_i \mathbf{X}_i)^{-1} (\mathbf{X}'_i \mathbf{X}_j) (\mathbf{X}'_j \mathbf{X}_j)^{-1}$.

satisfy the condition $\sum_{i=1}^m C_i = I_k$. (I_k is an identity matrix of order k). The matrices C_i are the probability limits of the coefficients in the OLS regressions of the columns of X_i on X_a , the "auxiliary" equations in Theil's terminology.

Notice that result (15) holds only when H_d is correctly specified. We will use this result later as the basis of a Durbin-Hausman type test of misspecification of the disaggregate model. For the time being, however, we assume that the disaggregate model H_d is correctly specified and write H_0 as

$$H_0: \sum_{i=1}^m (C_i - \frac{1}{m} I_k) \beta_i = 0. \quad (17)$$

A familiar method of testing (17), originally proposed by Zellner (1962), is to test the micro-homogeneity hypothesis

$$H_\beta: \beta_1 = \beta_2 = \dots = \beta_m.$$

Testing H_β as an indirect method of testing H_0 is, however, rather too restrictive. Although H_β implies H_0 , the reverse is not true. It is possible for $\eta_\beta = 0$ to hold even when the micro-homogeneity hypothesis is rejected. Here we propose a direct test of H_0 based on the OLS estimate of η_β , namely

$$\hat{\eta}_\beta = \hat{b} - \frac{1}{m} \sum_{i=1}^m \hat{\beta}_i. \quad (18)$$

Under H_0 , $\hat{\eta}_\beta$ is given by

$$\hat{\eta}_\beta = \sum_{i=1}^m P_i u_i \quad (19)$$

where

$$P_i = (X'_a X_a)^{-1} X'_a - \frac{1}{m} (X'_i X_i)^{-1} X'_i. \quad (20)$$

This suggests basing a test of H_0 on the statistic

$$q_2 = n^{-1} \hat{\eta}'_{\beta} \hat{\Phi}_n^{-1} \hat{\eta}_{\beta}, \quad (21)$$

where

$$\hat{\Phi}_n = n^{-1} \sum_{i,j=1}^m \hat{\sigma}_{ij} P_i P'_j, \quad (22)$$

and $\hat{\sigma}_{ij}$ is a consistent estimator of σ_{ij} .¹² Notice that except for the extreme case where $X_i = m^{-1} X_a$, matrix $\hat{\Phi}_n$ will in general be non-singular.

Theorem 1: Suppose

- (i) The disaggregate model H_d is correctly specified,
- (ii) Assumptions 1-3 hold,
- (iii) The matrix $\hat{\Phi}_n$ defined by (22) and the matrix $n^{-1}(P_i P'_i)$ both are non-singular and also converge in probability to non-singular matrices.

Then on the hypothesis of no aggregation bias, H_0 , the statistic q_2 defined in (21) is asymptotically distributed as a chi-squared variate with k degrees of freedom.

Proof: See the Mathematical Appendix.

This theorem provides an asymptotic justification for the use of q_2 in testing the null hypothesis of no aggregation bias, and holds for $\sigma_{ij} \neq 0$

¹²In small samples we suggest using the unbiased (and consistent) estimator of σ_{ij} proposed in PPK. [See equation (5.9) in PPK.]

and $m \geq 2$, but requires n , the sample size, to be sufficiently large. This contrasts the asymptotic framework underlying the perfect aggregation test proposed in PPK where n is fixed but m is allowed to increase without bounds.

The test statistics q_1 and q_2 are applicable when the focus of the analysis is on all the elements of $\underline{\beta}_i$. In practice, however, it is often the case that the parameters of interest are subsets, or more generally, non-linear functions of $\underline{\beta}_i$. For example, in the analysis of employment demand, the parameters of special interest are often the long run output and wage elasticities, which assuming $r_y = r_w = 2$, are given (in terms of the elements of $\underline{\beta}_i$) for the i^{th} industry by

$$\epsilon_{iy} = \frac{\beta_{14} - \beta_{15}}{1 - \beta_{12} - \beta_{13}},$$

$$\epsilon_{iw} = \frac{\beta_{16} + \beta_{17}}{1 - \beta_{12} - \beta_{13}},$$

respectively.¹³ To deal with such cases we now consider a generalization of (13) and write the null hypothesis of no aggregation bias as

$$\eta_g = g(\underline{b}) - \frac{1}{m} \sum_{i=1}^m g(\underline{\beta}_i), \quad (23)$$

where $g(\underline{\beta})$ is a $s \times 1$ ($s \geq k$) vector of known functions of $\underline{\beta}$. As before, we distinguish between the case where \underline{b} is given a priori, and when it is determined under H_d by (15).

¹³From (5) note that $\underline{\beta}_i = (\beta_{i1}, \beta_{i2}, \dots, \beta_{ik})' = (\text{Intercept}, \psi_{i1}, \psi_{i2}, \gamma_{1y}, \gamma_{2y}, \dots; \gamma_{1w}, \gamma_{2w}, \dots; \gamma_{1a}, \gamma_{2a}, \dots)$.

Denoting the $s \times k$ derivative matrix $\partial \underline{g}(\underline{\beta}) / \partial \underline{\beta}'$ by $G(\underline{\beta})$ and assuming that $\text{Rank}[G(\underline{\beta})] = s$, the relevant statistic for the test of $\underline{\eta}_g = 0$ when \underline{b} is set a priori is given by

$$q_1^* = \left[\underline{g}(\underline{b}) - \frac{1}{m} \sum_{i=1}^m \underline{g}(\hat{\underline{\beta}}_i) \right]' \hat{\Omega}_n^{-1} \left[\underline{g}(\underline{b}) - \frac{1}{m} \sum_{i=1}^m \underline{g}(\hat{\underline{\beta}}_i) \right], \quad (24)$$

where $\hat{\Omega}_n$ is now defined by

$$\hat{\Omega}_n = \frac{1}{m^2} \sum_{i,j=1}^m \hat{G}_i \text{Cov}(\hat{\underline{\beta}}_i, \hat{\underline{\beta}}_j) \hat{G}_j',$$

and $\hat{G}_i = G(\hat{\underline{\beta}}_i)$. Then on the null hypothesis of $\underline{\eta}_g = 0$ (with \underline{b} set a priori), $q_1^* \stackrel{a}{\sim} \chi_s^2$.

Turning to the case where \underline{b} is defined by (15), Theorem 1 continues to hold with this difference that the appropriate statistic is now given by

$$q_2^* = n^{-1} \hat{\eta}_g' \hat{\Phi}_n^{-1} \hat{\eta}_g \stackrel{a}{\sim} \chi_s^2, \quad (25)$$

where

$$\hat{\eta}_g = \underline{g}(\underline{b}) - \frac{1}{m} \sum_{i=1}^m \underline{g}(\hat{\underline{\beta}}_i), \quad (26)$$

and $\hat{\Phi}_n$ is as defined by (22), although in this more general case P_i is now given by

$$P_i = \hat{G}_a (\underline{X}'_a \underline{X}_a)^{-1} \underline{X}'_a - m^{-1} \hat{G}_i (\underline{X}'_i \underline{X}_i)^{-1} \underline{X}'_i, \quad (27)$$

in which $\hat{G}_a = G(\underline{b})$ and $\hat{G}_i = G(\hat{\underline{\beta}}_i)$. Notice, also that under $\underline{\eta}_g = 0$, the asymptotic distribution of q_2^* will be a chi-squared with $s(\leq k)$ degrees of freedom. The statistics q_1^* and q_2^* are direct generalizations of q_1 and q_2 and will reduce to them in the case where $\underline{g}(\underline{\beta}) = \underline{\beta}$.

4.2 A Misspecification Test of the Disaggregate Model

The tests of aggregation bias advanced above are based on the assumption that the disaggregate model H_d is correctly specified. In particular the tests based on the q_2 and q_2^* statistics assume that estimating the macro-parameters directly from the regression of \underline{h}_a on X_a , or indirectly by utilizing the expression $\sum_{i=1}^m C_i \beta_i$ should not make any difference asymptotically, in the sense that both are consistent estimators of \underline{b} under H_d . This implication of the disaggregate model can be tested by means of a Durbin-Hausman type misspecification test and suggests basing a test of H_d on the statistic

$$\hat{\eta}_s = \hat{\underline{b}} - \sum_{i=1}^m \hat{C}_i \hat{\underline{\beta}}_i, \quad (28)$$

where \hat{C}_i represents a consistent estimator of C_i defined by (16).¹⁴

Using the least squares estimates

$$\hat{C}_i = (X_a' X_a)^{-1} X_a' X_i, \quad i = 1, 2, \dots, m$$

we have

$$\hat{\eta}_s = (X_a' X_a)^{-1} X_a' e_d, \quad (29)$$

where

$$e_d = \sum_{i=1}^m (h_i - X_i \hat{\beta}_i) = \sum_{i=1}^m M_i y_i, \quad (30)$$

and

$$M_i = I_n - X_i (X_i' X_i)^{-1} X_i'$$

¹⁴See Durbin (1954) and Hausman (1978). Also see Ruud (1984), and Pesaran and Smith (1986) for a unified treatment of misspecification tests in the context of simultaneous equation models.

Since $(X'_a X_a)$ is by assumption a non-singular matrix, a test based on $\hat{\eta}_s$ and $X'_a e_d$ will be equivalent and for simplicity we use the latter statistic. Suppose now X_a and X_i have the same p variables in common and write¹⁵

$$X_a = [X_{a1} \mid X_{a2}]; \quad X_i = [X_{a1} \mid X_{i2}], \quad \text{for all } i,$$

where the $n \times p$ matrix X_{a1} contains the observations on the common set of variables. It is now easily seen that

$$X'_a e_d = \begin{bmatrix} 0 & : & X'_{a2} e_d \\ p \times 1 & & (k-p) \times 1 \end{bmatrix},$$

and the appropriate statistics to base the misspecification test on is given by the non-zero components of $X'_a e_d$, namely $X'_{a2} e_d$. Under H_d , we have

$$X'_{a2} e_d = \sum_{i=1}^m X'_{a2} M_i u_i, \quad (31)$$

which suggests the following theorem.

Theorem 2: Suppose

- (i) Assumptions 1-3 hold,
- (ii) The matrices $n^{-1}(X'_{a2} M_i X_{a2})$ are non-singular in finite samples, and also converge in probability to non-singular matrices,
- (iii) The matrix

$$\hat{V}_n = n^{-1} \sum_{i,j=1}^m \hat{\sigma}_{ij} (X'_{a2} M_i M_j X_{a2}), \quad (32)$$

is non-singular for a finite n , and converges in probability to the non-

¹⁵ Examples of such variables include the intercept term, time trends and seasonal dummies.

singular matrix, V . Then on the hypothesis that the disaggregate model is correctly specified the test statistic

$$q_3 = n^{-1} \underline{e}' \underline{X}_{a2} \hat{V}_n^{-1} \underline{X}'_{a2} \underline{e}_d, \quad (33)$$

is asymptotically distributed as a chi-squared variate with $k-p$ degrees of freedom.

Proof: See the Mathematical Appendix.

This theorem complements Theorem 1 and in a sense precedes it. Since Theorem 1 assumes the validity of the disaggregate specification, it is important that the misspecification test of Theorem 2 is carried out before testing for aggregation bias. It is also worth noting that since in general $\sum_{i=1}^m \hat{C}_i \hat{\beta}_i$ is not necessarily a more efficient estimator of $\underline{b} = \sum_{i=1}^m C_i \beta_i$ than $\hat{\underline{b}}$, the familiar Hausman formula for the covariance of $\hat{\eta}_s$, namely $\text{Cov}(\hat{\underline{b}}) - \text{Cov}(\sum_{i=1}^m \hat{C}_i \hat{\beta}_i)$ is not valid. However, when β_i are estimated by the SURE method, the resultant estimators, say $\tilde{\beta}_i$ will be efficient and the covariance difference formula

$$\text{Cov}(\tilde{\eta}_s) - \text{Cov}(\hat{\underline{b}}) - \text{Cov}\left(\sum_{i=1}^m \hat{C}_i \tilde{\beta}_i\right) \geq 0,$$

applies. But even in this case to avoid some of the computational problems that arise because of the possible singularity of $\text{Cov}(\hat{\underline{b}}) - \text{Cov}(\sum_{i=1}^m \hat{C}_i \tilde{\beta}_i)$, a direct deviation of the variance of $\tilde{\eta}_s$ along the above lines seems to be more desirable.

5. Empirical Results

In this section the theoretical considerations on employment functions of Sections 2 and 3, and the statistical methods of Section 4 are brought together in the estimation of disaggregate and aggregate employment

functions for the U.K and the analysis of aggregation bias. The data employed is taken from the Cambridge Growth Project Databank, and full details are provided in the Data Appendix. Figures are available annually for the period 1954-84, and except for some public sector services, the whole of the U.K. economy is covered, with data provided on a 41-industry basis. As in PPK, industry 4 (mineral oil and natural gas) is excluded from the analysis, and both the disaggregate and the aggregate specifications are based on the remaining 40 industry groups (i.e., $m = 40$). Although our data set starts in 1954, all the equations are estimated over the period 1956-84, and the data for the years 1954 and 1955 are used to generate ^{the} lagged values of employment, output and real wages that are included in the employment function [see equation (9)]. For the technical change variable $d_t(\lambda)$, we employed the recursive formula given by (8), for $t = 1955, 1956, \dots, 1984$ and experimented with different methods of initializing the recursive process. We also experimented with different estimates of the decay rate, λ .

5.1 Initialization of the $d_t(\lambda)$ Process

We tried two methods for generating the initial value, $d_{1954}(\lambda)$. In one set of experiments we derived $d_{1954}(\lambda)$ assuming that the process generating $\log(GI_t)$ in the pre-1954 period can be characterized by a random walk and that on average $E[\log(GI_{1954})] = E[\log(GI_{1953})] = \dots = \log(\bar{GI})$, where we estimate \bar{GI} by the average of gross investment over the 1954-58 period. Under these assumptions, the estimate of $d_{1954}(\lambda)$, which

we denote by \hat{d}_{01} , is given by¹⁶

$$\hat{d}_{01} = \log(\bar{GI}). \quad (34)$$

As an alternative procedure we followed the backward forecasting procedure proposed in Pesaran (1973), and derived the following alternative estimate for $d_{1954}(\lambda)$,

$$\hat{d}_{02} = \left\{ \frac{\hat{\rho}\lambda}{\hat{\rho} - (1-\lambda)} \right\} \log(GI_{1954}). \quad (35)$$

This estimate assumes that in the pre-1954 period $\log(GI_t)$ follows the first order autoregressive process

$$\log(GI_t) = \rho \log(GI_{t-1}) + \epsilon_t, \quad t = 1954, 1953, \dots,$$

and that ρ can be estimated consistently by the OLS method using data over the period 1954-84.

5.2 Estimation of the Decay Rate Parameter, λ

In the initial experiments we assumed a decay rate of $\lambda = 0.10$ and estimated the employment equations under both methods of initializing the $d_t(\lambda)$ process described above. We found that the technological variable, $d_t(\lambda)$ showed significantly in about half of the industries, and of these the majority demonstrated the better fit using \hat{d}_{01} (i.e., had the larger log likelihood value, LLF) as opposed to \hat{d}_{02} . The difference between LLF obtained in most industries was well below 1, and in only two cases did the difference exceed 2. In both of these \hat{d}_{01} proved to be the more

¹⁶Notice that

$$d_{1954}(\lambda) = \lambda \sum_{i=0}^{\infty} (1-\lambda)^i \log(GI_{1954-i}),$$

and under $E[\log(GI_{1954})] = E[\log(GE_{1953})] = \dots = \log(\bar{GI})$, we have

$$d_{01} = E[d_{1954}(\lambda)] = \log(\bar{GI}).$$

satisfactory measure. In view of these preliminary results we decided to initialize the $d_t(\lambda)$ process with \hat{d}_{01} . However, we note that, apart from the size of the coefficient on the constant in the estimated equations, there was little qualitative difference between results obtained using either of the two alternative initialization methods.

Using \hat{d}_{01} we also estimated the industrial employment equations by the grid search method, for values of λ in the range (0.0, 0.30). Again restricting attention to those industries with significant technological change effects, we found for about half of these industries the maximum likelihood estimates of λ fell within this interval, with many of the rest located on the $\lambda = 0.0$ bound. In general, however, we found the results to be qualitatively robust to the choice of the decay parameter in the range (0.0, 0.30). In the absence of any strong evidence of a more appropriate estimate of λ , therefore, we decided to maintain our original choice of $\lambda = 0.10$, in the remainder of the empirical work.

5.3 The Estimated Equations

The most general set of equations that we considered are presented in Table 1. This includes among the explanatory variables two lagged dependent variables, h_{t-1} and h_{t-2} , and current and lagged values of industry output, wages, and technological change $(y_t, y_{t-1}, y_{t-2}, w_t, w_{t-1}, d_t, d_{t-1})$.¹⁷ This equation follows from the theoretical discussion of Sections 2 and 3, by setting $r_y = 3$ and $r_w = r_a = 2$ in (9). Also included in the list of explanatory variables are current and lagged aggregate output measures, \bar{y}_{ta} and $\bar{y}_{t-1,a}$ ($\bar{y}_{ta} = 1/m \sum_{i=1}^m y_{ti}$). These variables were shown to be

¹⁷For each industry the technological variable $d_t(\lambda)$, or d_t for short, is computed using (8), with the initial value given by (34) and the decay parameter, $\lambda = 0.10$.

important in the empirical work of PPK, and it is clearly necessary to consider their influence here also. Their inclusion can be justified on the grounds that agents could use this aggregate information in the formation of their conditional expectations of y_{t+j} , w_{t+j} which we have shown to be important in explaining current employment. This unrestricted model differs from that in PPK by excluding the time trend, and by including y_{t-2} , d_t and d_{t-1} . Replacing the time trend by d_t and d_{t-1} alone caused a serious deterioration in the performance of many of the industrial equations, and in particular many became unstable. The inclusion of a second lagged output term remedied this in most of the equations however, and Table 1 represents a satisfactory set of results. The fit of most of the equations is satisfactory, with \bar{R}^2 falling below 0.90 only for industry 5 (Petroleum Products). Short run elasticities of employment with respect to wages, employment and technological change are generally of the expected sign, although as the standard errors of the coefficients (shown in brackets) indicate, the equations are in many cases over-parameterized.

For this reason, a specification search was carried out on these equations to obtain a more parsimonious set of results, and these are presented in Tables 2 and 2(a). Coefficients with t-values less than one (in absolute value) were omitted. Some a priori incorrectly-signed coefficients were also constrained to zero where the constraints were not violated by the data. Specifically, we expect the coefficient on h_{t-2} , and the long run wage and technological change effects to be negative. The χ^2 statistic for testing the validity of linear restrictions imposed on the parameters of the unrestricted equations to obtain the results of Table 2 are given in the second column in Table 2(a). It can be seen that the imposed restrictions are not rejected for any industry, at the conventional

levels of significance.

The overall performance of the equations in Table 2 is good and in line with those of PPK. Real wages show up significantly (and negatively) in most industries, with no long run wage effect found only in industries 22, 33, 37, and 40. The output variable also performed well, showing significantly and positively in all but 3 industries (6, 20, 38), the last one of which shows a strong positive aggregate output influence. Only 15 of the industries failed to demonstrate any technological change effects, although there are problem industries (10, 11, 26, 36, and 39) for which the technological change variables are (in sum) incorrectly signed. Other industries with a priori implausible parameter estimates include 23, 26, and 31, in which an unexpected positive second lagged dependent variable appears, and industry 25 which is unstable. Industry 20 also remains problematic: the differenced form reported in the PPK paper could not be improved upon, and this equation is retained here.

The histograms in Figures 1.1-1.3 illustrate the long run elasticities of employment with respect to industrial output, wages, and technological change as obtained from the results in Table 2. Figure 1.1 shows the long run output elasticities for 38 industries, omitting industries 20 and 25; the two industries with incorrectly signed output effects show to the left of the vertical axis, while industry 21 demonstrates an implausibly high positive output elasticity (the equation for this industry has a coefficient on the lagged dependent variable in excess of 0.9). The output elasticities for the bulk of the industries, however, lie within the interval (0,1.5) and the mean long run output elasticity is 0.97. The histogram in Figure 1.1 provide a clear illustration of the variability in the responsiveness of employment to output changes across industries, and this is confirmed by a standard

deviation around the mean of 0.86. Similar observations can be made on the long run real wage and technological change elasticities, which have means (standard deviations in brackets), $-0.68(0.66)$, and $-0.41(0.81)$ respectively.

The preferred equations set out in Table 2 show the technological change variable d_t to be an adequate replacement for the time trend in some industries, but not all. Of the 24 industrial equations in which a significant time trend was found in PPK, 15 are improved upon, in terms of the equation standard errors, by their equivalent estimate in Table 2, while 9 fit less well in the absence of the time trend. Moreover, there are a further 11 equations which did not previously involve a time trend whose standard error is lower in Table 2 than that in PPK, demonstrating the extra explanatory power of the additional lagged output and technological change variables. Since we prefer to replace a time trend with a variable with a more satisfactory theoretical basis, and given that the fit of this new set of equations is generally higher, these results, taken as a whole, can be seen as an improvement over those obtained previously.

Having made these points, however, closer comparison of the results in Tables 2 and 2(a) with those in PPK reveals that in some cases the diagnostic test statistics on the new set of equations are less reasonable than those previously found, and in all 16 industries have a preferable specification in the PPK paper. The superiority of the original equations in so many industries cannot of course be ignored, and for this reason we present a third set of industrial equations in Tables 3 and 3(a) which are an amalgamation of the results in Table 2 and those in PPK. (The PPK results are labelled "*"). These results represent the most satisfactory set of equations that we have been able to obtain for explaining employment at the industrial level in the U.K. As before, the long run coefficients

are represented diagrammatically in the histograms of Figures 2.1-2.3. Estimated coefficients are once again largely of the expected sign, and of a reasonable magnitude. The mean and standard deviation (in brackets) of the plotted long run elasticities are 0.86(0.88), -0.54(0.58), and -0.27(0.72) for output, wages, and technological change respectively, confirming the considerable variability of long run estimates across the industries and providing a reasonable a priori case for the use of disaggregated analysis.

5.4 Comparison With the Aggregate Relations

The following unrestricted and restricted aggregate employment equations, corresponding to the results discussed above, were also estimated:

Unrestricted aggregate equation:

$$\begin{aligned}
 h_{ta} = & -137.45 + 0.49689 y_{ta} + 0.19565 y_{t-1,a} + 0.11375 y_{t-2,a} \\
 & (3.77) \quad (6.39) \quad (1.32) \quad (1.03) \\
 & + 0.33110 h_{t-1,a} + 0.18986 h_{t-2,a} - 0.34110 w_{ta} \\
 & (1.48) \quad (1.17) \quad (-5.13) \\
 & - 0.087043 w_{t-1,a} - 0.046365 d_{ta} - 0.23405 d_{t-1,a} \quad (36) \\
 & (-1.00) \quad (-0.13) \quad (-0.82)
 \end{aligned}$$

$$\bar{R}^2 = 0.998, \quad \hat{\sigma} = 0.3316, \quad n = 29(1956-1984),$$

$$\chi_{SC}^2(1) = 3.23, \quad \chi_{FF}^2(1) = 1.27, \quad \chi_N^2(2) = 0.67, \quad \chi_H^2(1) = 3.51.$$

The figures in brackets are t-ratios, $\hat{\sigma}$ is the standard error of the regression, \bar{R}^2 is the adjusted R^2 , n is the number of observations. $\chi_{SC}^2(1)$, $\chi_{FF}^2(1)$, $\chi_N^2(2)$, $\chi_H^2(1)$ are diagnostic statistics distributed approximately as chi-squared variates (with degrees of freedom in parentheses), for tests of residual serial correlation, functional form misspecification, non-normal errors, and heteroscedasticity, respectively. [For more details about these test statistics and their computations see Pesaran and Pesaran (1987).]

Restricted aggregate equation:

$$\begin{aligned}
h_{ta} = & -99.28 + 0.49854 y_{ta} + 0.67897 h_{t-1,a} \\
& (-4.84) \quad (11.06) \quad (17.37) \\
& - 0.31216 w_{ta} - 0.12049 d_{t-1,a} \quad (37) \\
& (-7.53) \quad (-2.33)
\end{aligned}$$

$$\bar{R}^2 = 0.997, \quad \hat{\sigma} = 0.3209, \quad n = 29(1956-1984),$$

$$\chi_{SC}^2(1) = 1.83, \quad \chi_{FF}^2(1) = 2.46, \quad \chi_N^2(2) = 1.68, \quad \chi_H^2(1) = 5.29.$$

LM test on exclusion of $(y_{t-1,a}, y_{t-2,a}, h_{t-2,a}, w_{t-1,a}, d_{ta}) = 4.47$, cf $\chi^2(5)$

LM test on exclusion of $(y_{t-1,a}, y_{t-2,a}, h_{t-2,a}, w_{t-1,a}, d_{ta}, T_t) = 5.90$, cf $\chi^2(6)$

where h_{ta} , w_{ta} , and d_{ta} are the aggregate measures of employment, wages, and technological change derived from the industrial figures, and T_t is a linear time trend ($T_{1980}=0$).

To check for the possible effect of the simultaneous determination of output, employment and real wages on the OLS estimates, we also estimated the restricted aggregate equations using the instrumental variable method.

With $z_t = (1, h_{t-1,a}, h_{t-2,a}, y_{t-1,a}, y_{t-2,a}, w_{t-1,a}, w_{y-2,a}, d_{t-1,a})$ as instruments, we obtained:

$$\begin{aligned}
h_{ta} = & -86.86 + 0.4708 y_{ta} + 0.7025 h_{t-1,a} \\
& (-3.36) \quad (7.11) \quad (13.48) \\
& - 0.2783 w_{ta} - 0.1365 d_{t-1,a} \quad (38) \\
& (-4.69) \quad (-2.21)
\end{aligned}$$

$$\bar{R}^2 = 0.997, \quad \hat{\sigma} = 0.3258$$

Sargan's misspecification statistic = 4.40 cf $\chi^2(3)$,

$$\chi_{SC}^2(1) = 1.53, \quad \chi_{FF}^2(1) = 0.48, \quad \chi_N^2(2) = 3.83, \quad \chi_H^2(1) = 5.23,$$

These clearly differ only marginally from the OLS results in (37).

The parameter estimates in (36) and (37) imply long run elasticities with respect to aggregate output, real wages and technological change of (1.68, -.89, -.59) for the unrestricted equation and (1.55, -.97, -.38) for the restricted equation.

5.5 Tests Of Misspecification And Aggregation Bias

Applying the Durbin-Hausman misspecification test developed in Section 4.2 to the set of unrestricted disaggregated results of Table 1 and the unrestricted aggregate equation (36) we obtained a value of 48.56 for q_3 statistic defined by (33). The test statistic in this case has 7 degrees of freedom (there being 3 common regressors between the aggregate and the disaggregate specifications; namely the intercept term, y_{ta} and $y_{t-1,a}$) so the result implies strong rejection of the orthogonality of the disaggregate residuals to the aggregate variables. This might be due to omitted variables, such as industry-specific variables, from the disaggregate equations.

Table 4 presents the prediction criteria developed in PPK for the aggregate equations (36) and (37) and the disaggregate equations of Tables 1-3. In each case the disaggregate model outperforms the aggregate equation. The superiority (in terms of predictive performance) of the specifications in Table 3 over those in Table 2 can also be seen in the estimates presented in Table 4. The computation of the statistic for the test of perfect aggregation also provides evidence in favor of the disaggregate model. In the case of the unrestricted version the value of the test statistic is 89.6 which is approximately distributed as $\chi^2(29)$. This strongly rejects the null hypothesis of perfect aggregation.

Finally, the methods for directly testing for aggregation bias developed in Section 4.1 were applied to the aggregate and disaggregate employment equations, and the results obtained are summarized in Table 5. In the introduction to the paper, it was noted that much policy debate had centered around the extent to which employment is influenced by real wage levels and that a consensus view had emerged, based on a number of empirical

studies carried out at the aggregate level, suggesting a long run wage elasticity of unity. This consensus view is precisely of the type that was discussed in Section 4.1, where it was proposed that the average of the micro-parameters be compared to an *a priori* consensus estimate as a first test of aggregation bias. The first row of Table 5 shows the result of this test carried out on the estimated relations set out in Tables 1-3. The average of the estimated long run wage elasticities obtained on the basis of the results in the three tables is -0.66, -0.68, and -0.54 respectively, and these were each compared to the consensus value of -1. As is clear, the hypothesized unit elasticity is accepted in the case of the unrestricted specifications, but when a more precisely determined set of results are considered, as in Tables 2 and 3, the hypothesis is rejected.

Of course, the proposed consensus view is chosen rather arbitrarily, so that the more interesting test of aggregation bias proposed in Section 4.1 is that based on the q_2^* statistic, in which a pseudo true aggregate elasticity is obtained through estimation of the aggregate relation. This has the advantage that the same data and the same general specification is used in estimating the aggregate and the disaggregate elasticities, so that differences between the two are entirely explained through differences in the level of aggregation employed. Row 2 of Table 5 again considers the important wage elasticity issue for the three sets of results in Tables 1-3. The average wage elasticity across industries in Table 1 is compared to -0.89 (the estimated long run wage elasticity of equation (36)), while the averages from Tables 2 and 3 are compared to -0.97 (obtained from restricted equation (37)). Once again, the poorly determined set of equations in Table 1 provide no evidence of aggregation bias. A similar conclusion is also obtained from the results of Table 2. However, the hypothesis of no

aggregation bias based on the more satisfactory estimates in Table 3 is firmly rejected at the 5% level, providing strong evidence in support of the claim that the aggregate relation overstates the responsiveness of employment to changes in wages. Similar tests on aggregation bias are reported in rows 3 and 4, for the long run output and technological change elasticities in turn. Here, average output elasticities of 1.63, 1.23, and 1.24 are obtained from Tables 1, 2, and 3 respectively, while the corresponding average estimates for the technological change elasticity were -0.46, -0.41, and -0.27. These estimates are compared to long run output and technological change elasticities of 1.68 and -0.59 from the unrestricted equation (36), and of 1.55 and -0.38 from the restricted aggregate equation (37). In none of these tests is there any evidence of aggregation bias in the estimated coefficients.

6. Concluding Remarks

The application of the statistical methods developed in this paper to the study of employment equations in the U.K. provides some important insights for academics and policymakers alike. The estimated industrial employment equations show that there is a wide diversity in the responsiveness of labor demand to different influences across industries, illustrated most clearly by the histograms discussed in the previous section. In itself, this provides strong support for employing disaggregated analysis rather than aggregate analysis, since the latter cannot capture the structural detail that clearly exists.¹⁸ The result of the test for perfect

¹⁸ Indeed, the relatively poor diagnostic statistics obtained in the case of some of the industrial equations indicate that there is likely to be scope for further structural detail in the form of industry specific variables, and the use of different functional forms across industries.

aggregation confirms that this detail is important even if we are interested only in the prediction of aggregate employment levels, discounting the possibility that errors in disaggregate relations might be offsetting ones. Further, the results of the aggregation bias tests show that the emphasis of policymakers on the importance of wage restraint in attempts to reduce unemployment may be misplaced. These tests confirm the view put forward in PPK that labor demand equations estimated at the aggregate level significantly overstate the extra employment that might be achieved through wage reductions, however these are achieved. In fact, a wage elasticity of around -0.6 is suggested by the disaggregate results, considerably less than the unit elasticity that has become the consensus view in the U.K. and which is supported by our own aggregate estimates.¹⁹ The results do not, however, provide any evidence of aggregation bias in the long run estimates of output and technological change elasticities. Taken together, therefore, these results provide a clear illustration of the gains to be made from disaggregate analysis, and of the dangers involved in aggregation.

¹⁹Of course, estimated wage elasticities obtained in unconditional labor demand equations would be somewhat higher as reduced wage inflation helps encourage higher output levels.

TABLE 1.

UNRESTRICTED INDUSTRIAL LABOUR DEMAND EQUATIONS *

	lnpt/40	y_t	y_{t-1}	y_{t-2}	h_{t-1}	h_{t-2}	w_t	w_{t-1}	\bar{y}_{ca}	$\bar{y}_{c-1, a}$
1 Agriculture etc.	-8.3291 (101.8650)	0.3996 (0.1641)	0.3135 (0.1996)	-0.2215 (0.1442)	0.5188 (0.2661)	0.0008 (0.1542)	-0.5179 (0.1006)	0.0399 (0.1531)	-0.1477 (0.1740)	-0.1409 (0.1704)
2 Coal Mining	-99.6098 (66.1970)	0.3380 (0.0499)	-0.5043 (0.0953)	0.1258 (0.1254)	1.3944 (0.2044)	-0.3702 (0.2355)	-0.2331 (0.0369)	-0.0772 (0.0777)	0.0075 (0.1254)	0.1290 (0.1402)
3 Coke	-426.9027 (183.1692)	0.4256 (0.1479)	0.2996 (0.2329)	0.1716 (0.2395)	0.0496 (0.2288)	0.1848 (0.1285)	-0.6430 (0.1066)	0.0465 (0.1225)	0.2449 (0.3889)	0.1604 (0.4312)
4 Mineral Oil & Nat.Gas										
5 Petroleum Products	69.7436 (233.7634)	0.7333 (0.4544)	-0.0163 (0.4349)	0.0576 (0.2395)	0.6440 (0.2651)	-0.0224 (0.2815)	-0.4099 (0.1412)	0.0223 (0.1804)	0.1563 (0.8680)	-1.0622 (0.8383)
6 Electricity etc.	-26.1434 (51.5244)	0.0030 (0.1887)	0.3339 (0.2606)	-0.2057 (0.1605)	0.8984 (0.2033)	-0.3624 (0.1964)	-0.1186 (0.0768)	-0.1291 (0.0845)	0.0347 (0.1893)	0.3182 (0.2049)
7 Public Gas Supply	184.9195 (131.9965)	-0.1753 (0.2299)	0.6582 (0.2748)	-0.6325 (0.2016)	0.5349 (0.1630)	0.0739 (0.2029)	-0.3584 (0.0857)	0.2904 (0.0858)	-0.2904 (0.2853)	0.1382 (0.2625)
8 Water Supply	-187.1010 (93.4028)	1.0830 (0.4370)	0.2506 (0.5354)	-0.4169 (0.4807)	0.5300 (0.1454)	0.0286 (0.1480)	-0.4472 (0.1069)	0.3289 (0.1258)	-0.0699 (0.3251)	0.7324 (0.2887)
9 Minerals & Ores nes	79.1062 (171.5604)	0.2078 (0.1627)	-0.0659 (0.1775)	-0.1183 (0.1319)	0.5591 (0.2462)	0.2587 (0.1974)	-0.1445 (0.0952)	0.0586 (0.1142)	-0.5481 (0.4423)	0.4380 (0.6235)
10 Iron & Steel	-114.7884 (76.2388)	0.3066 (0.1046)	0.2143 (0.1108)	-0.1191 (0.1014)	0.5330 (0.2617)	0.1296 (0.1655)	-0.1959 (0.1471)	-0.0661 (0.1674)	0.2340 (0.3665)	-0.2408 (0.3609)
11 Non-ferrous Metals	-31.6300 (33.2585)	0.2543 (0.1174)	-0.1371 (0.1294)	-0.0952 (0.1154)	1.0529 (0.1748)	-0.2393 (0.1712)	-0.1066 (0.0465)	-0.0500 (0.0576)	0.6953 (0.2225)	-0.6961 (0.1985)
12 Non-metallic Min.Pr.	-412.1530 (163.0320)	0.3213 (0.1825)	0.0723 (0.1916)	-0.1101 (0.1668)	0.4784 (0.2423)	0.4562 (0.2947)	-0.3170 (0.1253)	-0.1929 (0.1451)	0.5393 (0.3488)	0.2921 (0.4147)
13 Chemicals & MM Fibres	-185.8193 (45.5295)	0.0643 (0.1694)	0.0698 (0.1161)	-0.0238 (0.0925)	0.0445 (0.2309)	0.4134 (0.1742)	-0.3640 (0.0978)	-0.0953 (0.1229)	0.3515 (0.3008)	0.2586 (0.2523)
14 Metal Goods nes	54.6916 (125.6380)	0.2179 (0.1241)	0.0515 (0.1540)	-0.1951 (0.1097)	0.5656 (0.2116)	0.1800 (0.1728)	-0.2192 (0.1143)	0.0742 (0.1190)	0.1296 (0.3236)	-0.2402 (0.3611)
15 Mech. Engineering	-61.0479 (69.0331)	0.4429 (0.1756)	-0.2644 (0.1849)	0.0449 (0.1384)	0.0449 (0.2231)	-0.0847 (0.2248)	-0.2644 (0.1317)	-0.3027 (0.1531)	-0.1211 (0.1878)	0.4252 (0.2083)
16 Office Machinery etc	-469.2709 (238.0055)	0.3130 (0.1262)	-0.1861 (0.1643)	0.2028 (0.1074)	1.0064 (0.2490)	0.1025 (0.2770)	-0.6552 (0.2125)	-0.0961 (0.2181)	0.4731 (0.3203)	0.2497 (0.3034)
17 Elect. Engineering	116.3987 (26.1750)	0.3691 (0.0762)	-0.0526 (0.1130)	0.0530 (0.0950)	0.1906 (0.1719)	0.1288 (0.1140)	-0.2311 (0.0838)	0.0905 (0.0842)	-0.0759 (0.1322)	0.5089 (0.1589)
18 Motor Vehicles	-26.1567 (88.6237)	0.5670 (0.0721)	-0.2497 (0.1452)	-0.0747 (0.1808)	0.8065 (0.1991)	0.0150 (0.2676)	0.1033 (0.1286)	-0.2076 (0.1413)	0.2030 (0.2125)	-0.0142 (0.2216)
19 Aerospace Equipment	222.3672 (112.9182)	0.0106 (0.0956)	-0.0980 (0.0983)	0.0719 (0.0900)	0.8612 (0.2301)	-0.3314 (0.2567)	-0.0477 (0.1001)	-0.1448 (0.1029)	-0.2439 (0.2800)	0.2692 (0.2864)
20 Ships & Other Vessels	-234.0763 (75.5925)	0.5828 (0.1078)	-0.1889 (0.1700)	-0.3240 (0.1530)	1.0843 (0.1868)	0.1554 (0.2188)	0.0276 (0.0654)	0.0009 (0.0839)	0.8235 (0.2530)	-0.4509 (0.2490)
21 Other Vehicles	-168.3526 (135.2829)	0.2968 (0.1062)	0.1114 (0.1206)	-0.0248 (0.1278)	1.0604 (0.2377)	-0.1252 (0.2550)	-0.1545 (0.0741)	0.1153 (0.0652)	0.2878 (0.2332)	-0.1187 (0.2443)
22 Instr. Engineering	467.8993 (122.7449)	0.0551 (0.1390)	0.1219 (0.1263)	0.3384 (0.1302)	0.2943 (0.1752)	-0.2474 (0.1310)	-0.1954 (0.0850)	0.2810 (0.0897)	-0.0956 (0.2179)	0.1420 (0.2458)
23 Manufactured Food	-96.1864 (138.7636)	0.7245 (0.3199)	-0.3829 (0.2337)	0.1395 (0.2045)	0.4446 (0.1915)	0.2049 (0.1648)	-0.1728 (0.0757)	-0.0398 (0.1034)	-0.0032 (0.1593)	0.2371 (0.1614)
24 Alcoholic Drinks etc	-106.2884 (190.7833)	0.4993 (0.5409)	0.0207 (0.5076)	-0.3969 (0.5117)	1.0974 (0.2355)	-0.1643 (0.3246)	-0.1113 (0.1279)	0.0734 (0.1178)	0.0755 (0.6244)	0.2879 (0.4363)
25 Tobacco	150.5910 (238.0243)	1.3370 (0.4483)	-0.4851 (0.4499)	-0.5259 (0.4688)	0.6519 (0.2606)	1.0220 (0.3253)	0.0061 (0.0687)	-0.1805 (0.0846)	-2.0127 (0.5811)	0.0303 (0.5378)
26 Textiles	-163.5445 (110.9788)	0.4008 (0.1809)	0.0199 (0.1746)	-0.3391 (0.1335)	0.3040 (0.2162)	0.1939 (0.1579)	-0.4269 (0.0869)	-0.1452 (0.1585)	0.2241 (0.2388)	0.1107 (0.3882)
27 Clothing & Footwear	-50.6793 (62.0796)	0.5050 (0.1060)	0.0437 (0.1899)	-0.0218 (0.1111)	0.5797 (0.2890)	-0.0361 (0.1783)	-0.4216 (0.0871)	0.0527 (0.1505)	-0.1067 (0.1944)	-0.0706 (0.1993)
28 Timber & Furniture	57.3350 (83.7562)	0.2824 (0.1185)	0.0633 (0.1490)	0.0163 (0.1127)	0.3392 (0.2469)	0.0730 (0.1428)	-0.2734 (0.0935)	0.0580 (0.0981)	0.0904 (0.2622)	-0.0689 (0.3858)
29 Paper & Board	92.5460 (49.1811)	0.1240 (0.2041)	0.1082 (0.1692)	0.0645 (0.1207)	0.3571 (0.2419)	0.3316 (0.1362)	-0.1795 (0.0769)	0.1637 (0.1279)	0.4807 (0.3626)	0.1567 (0.3013)
30 Books etc	116.8745 (38.6891)	0.2547 (0.1161)	0.0013 (0.1355)	-0.0353 (0.0777)	0.8079 (0.2747)	-0.2086 (0.2297)	-0.0864 (0.0590)	-0.0626 (0.0617)	-0.1167 (0.1828)	-0.2457 (0.1840)
31 Rubber & Plastic Pr.	-114.0693 (68.7654)	0.3664 (0.2515)	-0.0228 (0.2248)	-0.3178 (0.1387)	0.4959 (0.2956)	0.3564 (0.2238)	-0.2941 (0.2183)	0.0733 (0.1284)	0.0953 (0.4141)	0.0950 (0.4318)
32 Other Manufactures	157.9064 (71.7964)	0.3169 (0.0739)	0.0061 (0.1356)	0.0167 (0.1194)	0.6974 (0.1986)	-0.0339 (0.2011)	-0.1025 (0.0938)	0.0520 (0.0909)	0.1378 (0.2044)	-0.6710 (0.2084)
33 Construction	0.1708 (72.0577)	0.3346 (0.1097)	-0.4336 (0.1464)	0.1650 (0.0958)	1.1032 (0.1785)	-0.2506 (0.1212)	-0.3106 (0.0893)	0.4361 (0.1073)	0.3935 (0.1970)	0.0579 (0.1980)
34 Distribution etc	165.8094 (89.2530)	-0.0730 (0.2295)	0.6536 (0.3158)	-0.1662 (0.1553)	0.7386 (0.2320)	-0.2110 (0.1739)	-0.1118 (0.1285)	-0.0537 (0.1296)	-0.0828 (0.1893)	-0.5416 (0.2348)
35 Hotels & Catering	42.4162 (100.6322)	0.3120 (0.2517)	0.3603 (0.4124)	-0.2291 (0.3271)	0.5370 (0.3275)	0.0360 (0.2834)	-0.3282 (0.1593)	0.1610 (0.1738)	-0.0757 (0.2217)	-0.1791 (0.2135)
36 Rail Transport	120.7537 (103.6046)	0.3027 (0.1210)	0.4311 (0.1312)	-0.0013 (0.1354)	0.4013 (0.1979)	-0.1819 (0.1413)	-0.1381 (0.1143)	-0.0703 (0.1043)	-0.1976 (0.2297)	-0.3762 (0.2246)
37 Other Land Transport	191.4749 (77.5060)	-0.0685 (0.1657)	0.2417 (0.1952)	-0.2581 (0.1712)	0.9714 (0.2405)	-0.3262 (0.2238)	0.0266 (0.0591)	0.0380 (0.0650)	0.0908 (0.1671)	-0.0147 (0.1733)
38 Sea, Air & Other	63.4575 (132.3929)	0.2006 (0.1897)	-0.5264 (0.2355)	-0.1053 (0.1519)	1.1557 (0.2143)	-0.2569 (0.2906)	0.2130 (0.1420)	-0.0463 (0.1521)	0.6873 (0.2573)	0.6733 (0.3391)
39 Communications	259.2154 (82.3347)	0.5147 (0.2695)	-0.4403 (0.4696)	-0.2152 (0.3093)	0.6941 (0.2071)	-0.3138 (0.2094)	-0.1371 (0.1147)	0.1934 (0.1024)	-0.1709 (0.2188)	0.1549 (0.1711)
40 Business Services	354.3580 (95.7801)	0.0441 (0.1496)	0.1689 (0.1514)	-0.1074 (0.1404)	0.3361 (0.2678)	-0.1646 (0.2481)	0.0769 (0.0859)	-0.1400 (0.0939)	-0.0283 (0.1137)	-0.2971 (0.1173)
41 Miscell. Services	-248.5543 (145.5570)	0.4159 (0.1826)	-0.2700 (0.1973)	0.2990 (0.1843)	1.0218 (0.2370)	0.0167 (0.2849)	-0.2817 (0.1605)	0.1537 (0.1468)	0.0955 (0.2302)	0.1960 (0.2065)

* For source of data see the Appendix.
 $\hat{\sigma}$ is equation standard errors.

Standard errors in brackets, R^2 is adjusted
multiple correlation coefficient.

LLF is the maximized value of log-likelihood function.

/Continued

TABLE 1. (Continued)

	d_t	d_{t-1}	\bar{R}^2 (LLF)	$\hat{\sigma}$
1 Agriculture etc.	-0.4305 (0.4127)	0.3869 (0.3583)	0.9982 (90.1369)	0.0141
2 Coal Mining	-0.4265 (0.2022)	0.3679 (0.2374)	0.9986 (86.5871)	0.0160
3 Coke	-1.3583 (0.4749)	1.8793 (0.5582)	0.9710 (53.1491)	0.0506
4 Mineral Oil & Nat.Gas	---	---	---	---
5 Petroleum Products	0.5526 (0.3451)	-0.6812 (0.3579)	0.8841 (44.8866)	0.0672
6 Electricity etc.	0.6020 (0.2726)	-0.7048 (0.2477)	0.9917 (87.3926)	0.0155
7 Public Gas Supply	0.2513 (0.1922)	-0.1382 (0.2368)	0.9779 (69.6787)	0.0286
8 Water Supply	-2.8566 (0.7126)	2.0447 (0.5450)	0.9594 (67.4242)	0.0309
9 Minerals & Ores nes	-0.2124 (0.3888)	0.1637 (0.4321)	0.9706 (63.7009)	0.0351
10 Iron & Steel	0.4557 (0.1928)	-0.2938 (0.2185)	0.9923 (69.8891)	0.0284
11 Non-ferrous Metals	0.2762 (0.1125)	-0.0547 (0.1306)	0.9906 (78.8431)	0.0208
12 Non-metallic Min.Pr.	-0.0472 (0.4862)	-0.2831 (0.4049)	0.9932 (82.9853)	0.0181
13 Chemicals & MM Fibres	0.4584 (0.2036)	-0.3184 (0.2028)	0.9819 (89.0752)	0.0146
14 Metal Goods nes	1.1805 (0.6468)	-1.0864 (0.4716)	0.9893 (83.2485)	0.0179
15 Mech. Engineering	0.6855 (0.4794)	-0.5910 (0.5108)	0.9917 (90.3325)	0.0140
16 Office Machinery etc	-0.7454 (1.1230)	0.3020 (1.1174)	0.9331 (71.1734)	0.0272
17 Elect. Engineering	0.5176 (0.2558)	-1.3098 (0.2954)	0.9894 (99.4433)	0.0102
18 Motor Vehicles	-0.0786 (0.3168)	-0.2601 (0.3650)	0.9868 (81.3391)	0.0191
19 Aerospace Equipment	-0.0917 (0.5831)	-0.6019 (0.4519)	0.9837 (67.3423)	0.0310
20 Ships & Other Vessels	-0.8088 (0.5227)	0.8595 (0.3407)	0.9881 (72.3533)	0.0261
21 Other Vehicles	0.9568 (0.4453)	-0.8510 (0.3900)	0.9968 (72.1643)	0.0262
22 Instr. Engineering	0.9770 (0.4345)	-2.4352 (0.5578)	0.9727 (87.4193)	0.0155
23 Manufactured Food	0.6892 (0.4081)	-1.0381 (0.3510)	0.9862 (88.2687)	0.0151
24 Alcoholic Drinks etc	-0.5152 (0.6288)	0.2831 (0.5310)	0.9033 (68.1601)	0.0301
25 Tobacco	1.6853 (0.9638)	-0.6736 (0.7384)	0.8996 (56.2608)	0.0454
26 Textiles	-0.1934 (0.3831)	0.5008 (0.3380)	0.9984 (86.4858)	0.0160
27 Clothing & Footwear	-0.0193 (0.3359)	0.1237 (0.2419)	0.9980 (94.1785)	0.0123
28 Timber & Furniture	0.3085 (0.3788)	-0.4176 (0.3563)	0.9851 (89.4445)	0.0145
29 Paper & Board	0.1453 (0.4445)	-1.5894 (0.4768)	0.9942 (84.5039)	0.0171
30 Books etc	0.7865 (0.3149)	-0.5611 (0.2934)	0.9427 (96.8764)	0.0112
31 Rubber & Plastic Pr.	0.1452 (0.7177)	0.0405 (0.6053)	0.9795 (82.5569)	0.0183
32 Other Manufactures	0.5953 (0.2537)	-0.4945 (0.1999)	0.9918 (91.1621)	0.0136
33 Construction	-0.4806 (0.3877)	0.1353 (0.3364)	0.9804 (89.9104)	0.0142
34 Distribution etc	0.6452 (0.5798)	-0.4648 (0.4992)	0.9548 (88.7516)	0.0148
35 Hotels & Catering	-0.1462 (0.3011)	0.2173 (0.2850)	0.8996 (77.6065)	0.0218
36 Rail Transport	1.1548 (0.2405)	-0.7263 (0.1962)	0.9979 (84.8173)	0.0170
37 Other Land Transport	0.3518 (0.2886)	-0.4643 (0.2605)	0.9740 (85.5580)	0.0165
38 Sea, Air & Other	-0.3834 (0.4332)	0.3880 (0.3686)	0.9196 (76.7924)	0.0224
39 Communications	0.7280 (0.3443)	-0.5472 (0.2974)	0.9416 (84.0346)	0.0174
40 Business Services	0.9987 (0.3955)	-0.7970 (0.3127)	0.9942 (93.5970)	0.0125
41 Miscell. Services	-0.1665 (0.4956)	-0.1150 (0.4013)	0.9483 (76.1713)	0.0229

TABLE 2.

RESTRICTED INDUSTRIAL LABOUR DEMAND EQUATIONS *

	inpt/40	y_t	y_{t-1}	y_{t-2}	h_{t-1}	h_{t-2}	w_t	w_{t-1}	$\bar{y}_{t,a}$	$\bar{y}_{t-1,a}$
1 Agriculture etc.	5.9025 (57.5851)	0.4080 (0.1365)	0.3215 (0.1440)	-0.1981 (0.1169)	0.4782 (0.0669)		-0.5134 (0.0836)		-0.3886 (0.0480)	
2 Coal Mining	-24.6379 (10.1993)	0.2845 (0.0338)	-0.4268 (0.0716)	0.2247 (0.0957)	1.3624 (0.1359)	-0.4332 (0.1680)	-0.2194 (0.0297)			
3 Coke	-81.9147 (66.9629)	0.3733 (0.1351)	0.5137 (0.2122)		0.1876 (0.1151)		-0.4294 (0.0790)		-0.1354 (0.1158)	
4 Mineral Oil & Nat.Gas										
5 Petroleum Products	-92.6973 (88.2755)	0.3475 (0.1847)			0.7915 (0.1253)		-0.2882 (0.1086)			
6 Electricity etc.	42.7381 (26.8282)		0.5112 (0.1342)	-0.3648 (0.1475)	1.1202 (0.1862)	-0.4619 (0.1803)	-0.1490 (0.0746)			
7 Public Gas Supply	112.7709 (44.4602)		0.4462 (0.1803)	-0.5338 (0.1653)	0.6934 (0.1060)		-0.2975 (0.0681)	0.2300 (0.0774)		
8 Water Supply	-167.9418 (62.5607)	1.4846 (0.3476)			0.4752 (0.1039)		-0.4094 (0.0976)	0.1846 (0.1085)		
9 Minerals & Ores nes	172.9158 (79.1246)	0.2655 (0.1265)			0.6931 (0.0790)		-0.1494 (0.0622)		-0.5337 (0.2560)	
10 Iron & Steel	-155.7089 (20.5070)	0.3796 (0.0562)	0.1997 (0.0711)		0.5533 (0.0854)		-0.3402 (0.0596)			
11 Non-ferrous Metals	-21.3448 (27.8342)	0.2912 (0.1040)	-0.1825 (0.1138)		1.0929 (0.1544)	-0.3288 (0.1229)	-0.1061 (0.0439)	-0.0520 (0.0496)	0.6320 (0.2043)	-0.7019 (0.1920)
12 Non-metallic Min.Pr.	-236.8126 (65.3042)	0.3842 (0.1125)			0.8437 (0.0450)		-0.2505 (0.1029)	-0.1262 (0.1188)	0.3674 (0.2118)	
13 Chemicals & MM Fibres	-99.1032 (28.1020)				0.5831 (0.0710)		-0.2735 (0.0329)		0.5764 (0.0769)	
14 Metal Goods nes	-51.0504 (67.1158)	0.2663 (0.1006)			0.6006 (0.0653)		-0.2355 (0.0763)		0.2716 (0.2663)	
15 Mech. Engineering	-101.2152 (51.8451)	0.4909 (0.0750)	-0.3408 (0.1484)	0.1184 (0.0960)	0.5996 (0.2087)	-0.2925 (0.1792)	-0.1979 (0.1133)	-0.3348 (0.1163)		0.4802 (0.1746)
16 Office Machinery etc	-67.1178 (75.6691)				0.2278 (0.0600)		-0.2389 (0.1049)		0.2548 (0.1399)	
17 Elect. Engineering	106.1219 (25.5818)	0.3463 (0.0462)			0.3886 (0.0646)		-0.2592 (0.0771)	0.1351 (0.0637)		0.3684 (0.1099)
18 Motor Vehicles	-74.0164 (48.8105)	0.5451 (0.0470)	-0.2618 (0.1197)		0.8395 (0.1614)	-0.1165 (0.0909)	-0.2102 (0.0666)		0.2625 (0.1099)	
19 Aerospace Equipment	246.9802 (75.2084)				1.1388 (0.1495)	-0.6421 (0.1727)	-0.1737 (0.0720)			
20 Ships & Other Vessels	-0.7667 (0.3086)	0.4809 (0.1171)	-0.4809 (0.1171)		1.4717 (0.1543)	-0.4717 (0.1543)			0.5103 (0.2000)	-0.5103 (0.2000)
21 Other Vehicles	-165.4705 (58.7346)	0.3896 (0.0706)			0.9154 (0.0419)		-0.1680 (0.0552)	0.1064 (0.0564)	0.2078 (0.1235)	
22 Instr. Engineering	423.2630 (55.7285)		0.2115 (0.0825)	0.3539 (0.0987)	0.3177 (0.1267)	-0.2486 (0.1064)	-0.2377 (0.0581)	0.2377 (0.0581)		
23 Manufactured Food	-172.9101 (63.0103)	0.5982 (0.1660)			0.4844 (0.1266)	0.3037 (0.1327)	-0.2337 (0.0498)			
24 Alcoholic Drinks etc	-96.0446 (69.2247)				1.1312 (0.1750)	-0.3072 (0.2359)	-0.0956 (0.0705)		0.4479 (0.1457)	
25 Tobacco	155.5488 (182.3643)	1.3459 (0.4049)	-0.4827 (0.4058)	-0.5240 (0.3982)	0.6473 (0.2205)	1.0209 (0.2796)		-0.1781 (0.0668)	-2.0136 (0.5337)	
26 Textiles	-69.3333 (39.2933)	0.3637 (0.0675)			0.5652 (0.0657)		-0.4337 (0.0753)		0.0882 (0.1207)	
27 Clothing & Footwear	-68.9489 (11.9600)	0.4514 (0.0372)			0.5364 (0.0411)		-0.3756 (0.0284)			
28 Timber & Furniture	27.5732 (13.7004)	0.3925 (0.0352)			0.5144 (0.0572)		-0.2885 (0.0595)	0.1108 (0.0681)		
29 Paper & Board	39.4291 (31.8083)	0.4375 (0.0640)	0.1880 (0.0929)		0.4661 (0.0659)		-0.2130 (0.0477)			
30 Books etc	96.1742 (32.3071)	0.4094 (0.0946)	-0.2040 (0.0681)		1.3273 (0.1955)	-0.6121 (0.1602)	-0.0578 (0.0477)		-0.2125 (0.1434)	
31 Rubber & Plastic Pr.	-81.7005 (16.8687)	0.4581 (0.0427)			0.5365 (0.1198)	0.3160 (0.1195)	-0.2692 (0.0700)			
32 Other Manufactures	56.9103 (51.0746)	0.2992 (0.0602)			0.7367 (0.0902)		-0.0805 (0.0810)		0.4048 (0.1692)	-0.6459 (0.1125)
33 Construction	3.3516 (27.4087)	0.3475 (0.0858)	-0.3710 (0.1167)	0.1345 (0.0835)	0.9814 (0.0957)	-0.2355 (0.0955)	-0.3435 (0.0704)	0.3435 (0.0704)	0.3916 (0.1500)	
34 Distribution etc	141.2744 (41.2202)		0.7842 (0.1615)	-0.2726 (0.1207)	0.6360 (0.0917)		-0.0409 (0.0356)			-0.5717 (0.1527)
35 Hotels & Catering	-58.7494 (44.4425)	0.3544 (0.1150)			0.7096 (0.1022)		-0.3876 (0.1191)	0.1959 (0.1094)		
36 Rail Transport	-50.9886 (25.3141)	0.1307 (0.0894)	0.3372 (0.1055)		0.5187 (0.0978)		-0.2545 (0.0718)			
37 Other Land Transport	118.4359 (34.6439)		0.2123 (0.1302)	-0.2016 (0.1434)	1.0003 (0.1803)	-0.2638 (0.1884)				
38 Sea, Air & Other	67.2451 (92.1103)	0.1608 (0.1269)	-0.3506 (0.1263)		1.2952 (0.1460)	-0.6002 (0.1863)	-0.2432 (0.1204)	0.1835 (0.1255)		0.3148 (0.1521)
39 Communications	309.7212 (57.1105)				0.5777 (0.1677)	-0.4744 (0.1616)	-0.0937 (0.0631)	0.0860 (0.0664)		
40 Business Services	209.6513 (49.1545)	0.3108 (0.0718)			0.6781 (0.1759)	-0.3104 (0.1680)				-0.1633 (0.0486)
41 Miscell. Services	-39.9043 (33.3057)	0.2123 (0.0790)			0.8264 (0.0970)		-0.1408 (0.0747)			

* See notes to Table 1.

/Continued

Table 2. (continued)

	d _t	d _{t-1}
1 Agriculture etc.		
2 Coal Mining	-0.1226 (0.0326)	
3 Coke	-1.4606 (0.4225)	1.4606 (0.4225)
4 Mineral Oil & Nat.Gas		
5 Petroleum Products	0.3281 (0.2698)	-0.5234 (0.2702)
6 Electricity etc.	0.2793 (0.2408)	-0.3007 (0.1838)
7 Public Gas Supply		
8 Water Supply	-3.0775 (0.6623)	2.4681 (0.5508)
9 Minerals & Ores nes		
10 Iron & Steel		0.1489 (0.0636)
11 Non-ferrous Metals	0.2278 (0.0572)	
12 Non-metallic Min.Pr.		-0.2609 (0.1336)
13 Chemicals & MM Fibres	0.2720 (0.1678)	-0.2720 (0.1678)
14 Metal Goods nes	0.4410 (0.3831)	-0.6038 (0.3240)
15 Mech. Engineering		
16 Office Machinery etc	1.0366 (0.5489)	-1.5206 (0.5314)
17 Elect. Engineering	0.2694 (0.1979)	-0.9762 (0.2051)
18 Motor Vehicles	-0.2471 (0.0892)	
19 Aerospace Equipment	-0.6598 (0.2197)	
20 Ships & Other Vessels		
21 Other Vehicles	0.7687 (0.3109)	-0.7687 (0.3109)
22 Instr. Engineering	1.2396 (0.3043)	-2.6097 (0.3537)
23 Manufactured Food	0.6483 (0.3462)	-0.8005 (0.3177)
24 Alcoholic Drinks etc		-0.1047 (0.0868)
25 Tobacco	1.7244 (0.6965)	-0.6987 (0.6201)
26 Textiles		0.2960 (0.1265)
27 Clothing & Footwear		
28 Timber & Furniture	-0.1174 (0.0282)	
29 Paper & Board		-0.5252 (0.1523)
30 Books etc		
31 Rubber & Plastic Pr.		
32 Other Manufactures		
33 Construction	-0.2830 (0.0705)	
34 Distribution etc		
35 Hotels & Catering		
36 Rail Transport	0.8608 (0.2211)	-0.6958 (0.1953)
37 Other Land Transport	0.4509 (0.2304)	-0.5162 (0.1937)
38 Sea, Air & Other		
39 Communications	1.0539 (0.2010)	-0.9139 (0.1745)
40 Business Services		
41 Miscell. Services		

TABLE 2(A).

**SUMMARY AND DIAGNOSTIC STATISTICS FOR THE RESTRICTED
EMPLOYMENT EQUATIONS OF TABLE 2**

Industry	\bar{R}^2	χ^2_r	$\hat{\sigma}$	$\chi^2_{SC}(1)$	$\chi^2_{FF}(1)$	$\chi^2_N(2)$	$\chi^2_H(1)$
1 Agriculture etc.	0.9983	5.00 (5)	0.0136	0.08	5.96	0.02	0.06
2 Coal Mining	0.9987	4.02 (4)	0.0155	0.00	0.25	1.18	0.32
3 Coke	0.9656	10.10 (5)	0.0551	0.00	13.82	0.50	2.96
4 Mineral Oil & Nat.Gas	---	---	---	---	---	---	---
5 Petroleum Products	0.8874	6.94 (6)	0.0663	0.00	1.89	1.93	0.33
6 Electricity etc.	0.9909	7.59 (4)	0.0163	1.08	1.26	0.22	2.58
7 Public Gas Supply	0.9773	8.14 (6)	0.0290	4.60	1.11	4.12	5.42
8 Water Supply	0.9520	10.06 (5)	0.0336	0.01	0.38	0.71	0.00
9 Minerals & Ores nes	0.9760	3.84 (7)	0.0318	1.36	0.16	32.70	0.00
10 Iron & Steel	0.9919	8.63 (6)	0.0291	0.20	4.59	2.08	0.77
11 Non-ferrous Metals	0.9912	1.31 (2)	0.0202	4.25	2.19	2.75	0.05
12 Non-metallic Min.Pr.	0.9937	4.82 (5)	0.0174	1.82	4.70	2.09	4.48
13 Chemicals & MM Fibres	0.9808	9.67 (7)	0.0151	4.76	3.53	1.65	1.29
14 Metal Goods nes	0.9898	5.39 (5)	0.0174	0.21	5.96	1.69	4.74
15 Mech. Engineering	0.9916	4.60 (3)	0.0141	0.17	0.21	0.02	1.09
16 Office Machinery etc	0.9291	7.85 (5)	0.0280	0.38	1.10	0.76	4.48
17 Elect. Engineering	0.9893	5.82 (4)	0.0103	1.79	0.01	0.41	1.12
18 Motor Vehicles	0.9887	1.43 (4)	0.0176	4.16	0.88	1.55	1.58
19 Aerospace Equipment	0.9847	7.20 (7)	0.0301	3.56	1.04	0.06	2.78
20 Ships & Other Vessels	0.9818	16.13 (8)	0.0323	0.45	0.61	0.40	4.46
21 Other Vehicles	0.9972	2.85 (5)	0.0243	0.90	2.10	0.33	2.59
22 Instr. Engineering	0.9759	2.44 (4)	0.0146	1.57	1.29	2.14	1.02
23 Manufactured Food	0.9856	7.50 (5)	0.0154	0.43	0.48	6.76	2.46
24 Alcoholic Drinks etc	0.9119	5.49 (6)	0.0288	0.10	1.43	0.71	3.57
25 Tobacco	0.9101	0.02 (2)	0.0430	4.95	16.07	0.09	0.04
26 Textiles	0.9985	5.09 (5)	0.0155	4.21	2.46	2.20	2.03
27 Clothing & Footwear	0.9984	4.32 (8)	0.0110	0.36	1.92	0.62	0.03
28 Timber & Furniture	0.9873	3.76 (6)	0.0133	0.08	3.73	0.91	0.85
29 Paper & Board	0.9932	10.81 (6)	0.0186	5.30	0.14	1.73	6.16
30 Books etc	0.9341	9.52 (5)	0.0120	1.76	0.00	0.00	1.13
31 Rubber & Plastic Pr.	0.9834	2.51 (6)	0.0165	0.14	2.45	0.50	0.01
32 Other Manufactures	0.9908	9.82 (6)	0.0144	0.22	0.37	1.11	0.02
33 Construction	0.9819	2.42 (3)	0.0137	0.02	0.10	0.73	0.27
34 Distribution etc	0.9603	4.63 (6)	0.0139	0.32	1.96	0.23	2.47
35 Hotels & Catering	0.9169	4.17 (7)	0.0198	0.58	1.88	0.45	0.63
36 Rail Transport	0.9975	9.84 (5)	0.0183	0.03	0.38	2.66	0.86
37 Other Land Transport	0.9766	4.12 (5)	0.0157	0.13	0.98	1.02	0.02
38 Sea, Air & Other	0.9278	2.87 (4)	0.0212	2.23	7.95	1.88	0.91
39 Communications	0.9388	7.59 (5)	0.0178	0.53	1.51	1.16	4.09
40 Business Services	0.9940	9.26 (7)	0.0128	0.98	2.01	1.98	0.17
41 Miscell. Services	0.9512	8.14 (8)	0.0222	0.06	0.47	0.39	1.91

Notes:

- χ^2_r is the chi-squared statistic for the test of r linear restrictions on the parameters of unrestricted employment equations (see Table 1). The value of r is given in brackets after the statistic.
- $\chi^2_{SC}(1)$ is the first order LM test of residual serial correlation.
- $\chi^2_N(2)$ is a test of normality of the errors.
- $\hat{\sigma}$ is equations' standard errors.
- $\chi^2_{FF}(1)$ is Ramsey's RESET test of order 1.
- $\chi^2_H(1)$ is a heteroscedasticity test of order 1.
- \bar{R}^2 is the adjusted multiple correlation coefficient.

The underlying regressions and the test statistics reported in this table are computed on Data-FIT package. For details of relevant algorithms and references - see Pesaran and Pesaran (1987).

TABLE 3.

COMPOSITE RESTRICTED INDUSTRIAL LABOUR DEMAND EQUATIONS*

	lnpt/40	y_t	y_{t-1}	y_{t-2}	h_{t-1}	h_{t-2}	w_t	w_{t-1}	$\bar{y}_{t,a}$	$\bar{y}_{t-1,a}$
1 Agriculture etc.	5.9025 (57.5851)	0.4080 (0.1365)	0.3215 (0.1440)	-0.1981 (0.1169)	0.4782 (0.0669)		-0.5134 (0.0836)		-0.3886 (0.0480)	
2 Coal Mining	-24.6379 (10.1993)	0.2845 (0.0338)	-0.4268 (0.0716)	0.2247 (0.0957)	1.3624 (0.1359)	-0.4332 (0.1680)	-0.2194 (0.0297)			
3 Coke	-351.5712 (44.6561)		0.6330 (0.1471)				-0.3005 (0.0418)		1.0448 (0.1564)	
4 Mineral Oil & Nat. Gas										
5 Petroleum Products	-70.7959 (71.7711)	0.3640 (0.1324)			0.5185 (0.1348)		-0.3144 (0.0869)			
6 Electricity etc.	42.7381 (26.8282)		0.5112 (0.1342)	-0.3648 (0.1475)	1.1202 (0.1862)	-0.4619 (0.1803)	-0.1490 (0.0746)			
7 Public Gas Supply	-47.1096 (97.2188)		0.0611 (0.0659)		0.4191 (0.1524)		-0.1507 (0.0496)		0.5379 (0.1827)	
8 Water Supply	-167.9418 (62.5607)	1.4846 (0.3476)			0.4752 (0.1039)		-0.4094 (0.0976)	0.1846 (0.1085)		
9 Minerals & Ores nes	172.9158 (79.1246)	0.2655 (0.1265)			0.6931 (0.0790)		-0.1494 (0.0622)		-0.5337 (0.2560)	
10 Iron & Steel	-349.9558 (58.8686)	0.1083 (0.0893)			0.4978 (0.0832)		-0.3873 (0.0777)		1.1803 (0.2928)	
11 Non-ferrous Metals	-84.8257 (30.7245)	0.1817 (0.1286)	-0.3091 (0.1273)		1.2461 (0.1458)	-0.4796 (0.1229)	-0.0756 (0.0481)	0.0756 (0.0481)	0.5854 (0.1789)	
12 Non-metallic Min.Pr.	-280.5702 (60.6439)	0.3101 (0.1511)			0.6919 (0.0877)		-0.2356 (0.1075)	-0.2214 (0.0959)	0.5170 (0.2901)	
13 Chemicals & MM Fibres	-125.0557 (23.8339)				0.6205 (0.0693)		-0.2810 (0.0337)		0.6049 (0.0773)	
14 Metal Goods nes	-32.2448 (25.5280)	0.4365 (0.0444)			0.5798 (0.0542)		-0.1671 (0.0817)			
15 Mech. Engineering	-101.2152 (51.8451)	0.4909 (0.0750)	-0.3408 (0.1484)	0.1184 (0.0960)	0.5996 (0.2087)	-0.2925 (0.1792)	-0.1979 (0.1133)	-0.3348 (0.1163)		0.4802 (0.1746)
16 Office Machinery etc	-67.1178 (75.6691)			0.2278 (0.0600)	0.8571 (0.0721)		-0.2389 (0.1049)		0.2548 (0.1399)	
17 Elect. Engineering	106.1219 (25.5818)	0.3463 (0.0462)			0.3886 (0.0646)		-0.2592 (0.0771)	0.1351 (0.0637)		0.3684 (0.1099)
18 Motor Vehicles	-74.0164 (48.8105)	0.5451 (0.0470)	-0.2618 (0.1197)		0.8395 (0.1614)	-0.1165 (0.0909)	-0.2102 (0.0666)	0.2625 (0.1099)		
19 Aerospace Equipment	200.3920 (53.1219)	0.0732 (0.0654)			0.7560 (0.1659)	-0.4659 (0.1440)	-0.1252 (0.0674)			
20 Ships & Other Vessels	-0.7667 (0.3086)	0.4809 (0.1171)	-0.4809 (0.1171)		1.4717 (0.1543)	-0.4717 (0.1543)			0.5103 (0.2000)	-0.5103 (0.2000)
21 Other Vehicles	-165.4705 (58.7346)	0.3896 (0.0706)			0.9154 (0.0419)		-0.1680 (0.0552)	0.1064 (0.0564)	0.2078 (0.1235)	
22 Instr. Engineering	423.2630 (55.7285)		0.2115 (0.0825)	0.3539 (0.0987)	0.3177 (0.1267)	-0.2486 (0.1064)	-0.2377 (0.0581)	0.2377 (0.0581)		
23 Manufactured Food	-172.1572 (76.0519)	0.6697 (0.1734)			0.3177 (0.1742)	0.2237 (0.1560)	-0.1962 (0.0645)		0.5103 (0.2000)	-0.5103 (0.2000)
24 Alcoholic Drinks etc	-15.1802 (73.4889)	0.2933 (0.1167)			0.7283 (0.1239)		-0.0945 (0.0919)	0.0591 (0.0882)		
25 Tobacco	-213.3698 (80.8449)	0.7424 (0.2840)			0.7367 (0.2225)	0.2633 (0.2225)				
26 Textiles	-69.3333 (39.2933)	0.3637 (0.0675)		-0.2759 (0.0871)	0.5652 (0.0657)		-0.4337 (0.0753)		0.0882 (0.1207)	
27 Clothing & Footwear	-68.9489 (11.9600)	0.4514 (0.0372)			0.5364 (0.0411)		-0.3756 (0.0284)			
28 Timber & Furniture	27.5732 (13.7004)	0.3925 (0.0352)			0.5144 (0.0572)		-0.2885 (0.0595)	0.1108 (0.0681)		
29 Paper & Board	-44.7394 (13.2869)	0.4680 (0.0652)	0.1585 (0.0925)		0.3644 (0.0842)		-0.2503 (0.0433)			
30 Books etc	96.1742 (32.3071)	0.4094 (0.0946)	-0.2040 (0.0681)		1.3273 (0.1955)	-0.6121 (0.1602)	-0.0578 (0.0477)		-0.2125 (0.1434)	
31 Rubber & Plastic Pr.	-64.4432 (14.2846)	0.5398 (0.0588)	-0.1401 (0.0963)		0.6844 (0.0818)		-0.1820 (0.1007)			
32 Other Manufactures	60.3555 (20.0274)	0.2345 (0.0435)			0.6028 (0.0933)				0.4274 (0.1287)	-0.4274 (0.1287)
33 Construction	3.3516 (27.4087)	0.3475 (0.0858)	-0.3710 (0.1167)	0.1345 (0.0835)	0.9814 (0.0957)	-0.2355 (0.0955)	-0.3435 (0.0704)	0.3435 (0.0704)	0.3916 (0.1500)	
34 Distribution etc	141.2744 (41.2202)		0.7842 (0.1615)	-0.2726 (0.1207)	0.6360 (0.0917)		-0.0409 (0.0356)			-0.5717 (0.1527)
35 Hotels & Catering	-58.7494 (44.4425)	0.3544 (0.1150)			0.7096 (0.1022)		-0.3876 (0.1191)	0.1959 (0.1094)		
36 Rail Transport	-50.9886 (25.3141)	0.1307 (0.0894)	0.3372 (0.1055)		0.5187 (0.0978)		-0.2545 (0.0718)			
37 Other Land Transport	118.4359 (34.6439)		0.2123 (0.1302)	-0.2016 (0.1434)	1.0003 (0.1803)	-0.2638 (0.1884)				
38 Sea, Air & Other	67.2451 (92.1103)	0.1608 (0.1269)	-0.3506 (0.1263)		1.2952 (0.1460)	-0.6002 (0.1863)	-0.2432 (0.1204)	0.1835 (0.1255)	0.3148 (0.1521)	
39 Communications	14.3221 (41.3966)	0.9014 (0.1808)	-0.4533 (0.1966)		0.8261 (0.1727)	-0.2785 (0.1579)	-0.1686 (0.0822)	0.1565 (0.0807)		
40 Business Services	209.6513 (49.1545)	0.3108 (0.0718)			0.6781 (0.1759)	-0.3104 (0.1680)				-0.1633 (0.0486)
41 Miscell. Services	-39.9043 (33.3057)	0.2123 (0.0790)			0.8264 (0.0970)		-0.1408 (0.0747)			

* See notes to Table 1.

/Continued

Table 3 (continued)

	d_t	d_{t-1}	T_t
1 Agriculture etc.			
2 Coal Mining	-0.1226 (0.0326)		
3 Coke			-1.3100 (0.1752)
4 Mineral Oil & Nat.Gas	---	---	---
5 Petroleum Products			-0.5087 (0.1297)
6 Electricity etc.	0.2793 (0.2408)	-0.3007 (0.1838)	
7 Public Gas Supply			-0.6014 (0.1995)
8 Water Supply	-3.0775 (0.6623)	2.4681 (0.5508)	
9 Minerals & Ores nes			
10 Iron & Steel			-0.9045 (0.2732)
11 Non-ferrous Metals			-0.5749 (0.1517)
12 Non-metallic Min.Pr.			-0.3729 (0.2148)
13 Chemicals & MM Fibres			
14 Metal Goods nes			-0.1231 (0.0976)
15 Mech. Engineering			
16 Office Machinery etc	1.0366 (0.5489)	-1.5206 (0.5314)	
17 Elect. Engineering	0.2694 (0.1979)	-0.9762 (0.2051)	
18 Motor Vehicles	-0.2471 (0.0892)		
19 Aerospace Equipment			-0.6788 (0.1586)
20 Ships & Other Vessels			
21 Other Vehicles	0.7687 (0.3109)	-0.7687 (0.3109)	
22 Instr. Engineering	1.2396 (0.3043)	-2.6097 (0.3537)	
23 Manufactured Food			-0.4510 (0.1973)
24 Alcoholic Drinks etc			-0.4844 (0.1411)
25 Tobacco			-0.3959 (0.1161)
26 Textiles		0.2960 (0.1265)	
27 Clothing & Footwear			
28 Timber & Furniture	-0.1174 (0.0282)		
29 Paper & Board			-0.3259 (0.1040)
30 Books etc			
31 Rubber & Plastic Pr.			-0.3192 (0.1872)
32 Other Manufactures			-0.3233 (0.0653)
33 Construction	-0.2830 (0.0705)		
34 Distribution etc			
35 Hotels & Catering			
36 Rail Transport	0.8608 (0.2211)	-0.6958 (0.1953)	
37 Other Land Transport	0.4509 (0.2304)	-0.5162 (0.1937)	
38 Sea, Air & Other			
39 Communications			-0.6566 (0.2354)
40 Business Services			
41 Miscell. Services			

TABLE 3(A).

**SUMMARY AND DIAGNOSTIC STATISTICS FOR THE RESTRICTED
EMPLOYMENT EQUATIONS OF TABLE 3**

Industry	\bar{r}^2	x_T^2	$\hat{\sigma}$	$x_{SC}^2(1)$	$x_{FF}^2(1)$	$x_N^2(2)$	$x_H^2(1)$
1 Agriculture etc.	0.9983	5.21 (6)	0.0136	0.08	5.96	0.02	0.06
2 Coal Mining	0.9987	6.84 (5)	0.0155	0.00	0.25	1.18	0.32
3 Coke (*)	0.9771	10.15 (8)	0.0449	0.24	0.67	0.27	1.87
4 Mineral Oil & Nat.Gas	---	---	---	---	---	---	---
5 Petroleum Products (*)	0.9178	13.40 (8)	0.0566	0.48	0.01	1.83	0.85
6 Electricity etc.	0.9909	14.25 (5)	0.0163	1.08	1.26	0.22	2.58
7 Public Gas Supply (*)	0.9719	23.36 (7)	0.0322	1.29	0.00	4.86	1.42
8 Water Supply	0.9520	10.18 (6)	0.0336	0.01	0.38	0.71	0.00
9 Minerals & Ores nes	0.9760	3.90 (8)	0.0318	1.36	0.16	32.70	0.00
10 Iron & Steel (*)	0.9933	12.88 (7)	0.0265	0.08	0.19	1.42	0.43
11 Non-ferrous Metals (*)	0.9864	13.33 (5)	0.0250	0.01	3.47	0.20	1.89
12 Non-metallic Min.Pr. (*)	0.9935	12.21 (6)	0.0177	1.11	0.23	0.76	3.15
13 Chemicals & MM Fibres (*)	0.9795	11.78 (9)	0.0156	3.51	1.80	0.96	1.14
14 Metal Goods nes (*)	0.9877	12.31 (8)	0.0192	0.09	0.27	0.38	1.00
15 Mech. Engineering	0.9916	4.63 (4)	0.0141	0.17	0.21	0.02	1.09
16 Office Machinery etc	0.9291	9.49 (6)	0.0280	0.38	1.10	0.76	4.48
17 Elect. Engineering	0.9893	11.58 (5)	0.0103	1.79	0.01	0.41	1.12
18 Motor Vehicles	0.9887	4.82 (5)	0.0176	4.16	0.88	1.55	1.58
19 Aerospace Equipment (*)	0.9878	6.31 (7)	0.0268	0.90	0.30	1.81	1.30
20 Ships & Other Vessels	0.9818	16.91 (9)	0.0323	0.45	0.61	0.40	4.46
21 Other Vehicles	0.9972	12.07 (6)	0.0243	0.90	2.10	0.33	2.59
22 Instr. Engineering	0.9759	2.46 (5)	0.0146	1.57	1.29	2.14	1.02
23 Manufactured Food (*)	0.9837	13.89 (6)	0.0164	1.69	2.78	1.33	4.38
24 Alcoholic Drinks etc (*)	0.9232	15.42 (7)	0.0269	1.32	0.02	0.94	2.06
25 Tobacco (*)	0.8796	16.63 (9)	0.0497	0.25	8.22	0.65	7.62
26 Textiles	0.9985	5.59 (6)	0.0155	4.21	2.46	2.20	2.03
27 Clothing & Footwear	0.9984	4.37 (9)	0.0110	0.36	1.92	0.62	0.03
28 Timber & Furniture	0.9873	6.05 (7)	0.0133	0.08	3.73	0.91	0.85
29 Paper & Board (*)	0.9927	12.00 (7)	0.0192	1.09	1.33	1.74	4.41
30 Books etc	0.9341	9.55 (6)	0.0120	1.76	0.00	0.00	1.13
31 Rubber & Plastic Pr. (*)	0.9818	8.01 (7)	0.0173	0.21	1.59	0.96	1.03
32 Other Manufactures (*)	0.9917	13.49 (8)	0.0137	0.37	0.21	1.12	0.00
33 Construction	0.9819	2.58 (4)	0.0137	0.02	0.10	0.73	0.27
34 Distribution etc	0.9603	13.35 (7)	0.0139	0.32	1.96	0.23	2.47
35 Hotels & Catering	0.9169	5.59 (8)	0.0198	0.58	1.88	0.45	0.63
36 Rail Transport	0.9975	11.38 (6)	0.0183	0.03	0.38	2.66	0.86
37 Other Land Transport	0.9766	8.48 (6)	0.0157	0.13	0.98	1.02	0.02
38 Sea, Air & Other	0.9278	6.26 (5)	0.0212	2.23	7.95	1.88	0.91
39 Communications (*)	0.9351	8.03 (5)	0.0184	1.56	0.48	0.14	1.81
40 Business Services	0.9940	9.33 (8)	0.0128	0.98	2.01	1.98	0.17
41 Miscell. Services	0.9512	8.20 (9)	0.0222	0.06	0.47	0.39	1.91

See the notes to Table 2(a).

TABLE 4

Relative Predictive Performance of the Aggregate and the
Disaggregate Employment Functions*

	<u>Unrestricted Specifications</u>	<u>Restricted Specifications</u>	
	(Table 1)	(Table 2)	(Table 3)
Disaggregate criterion	0.1007	0.0856	0.0737
Aggregate criterion	0.1100 ^a	0.1030 ^b	0.1030 ^b

*Results exclude industry 4 (Mineral Oil and Natural Gas).

^aCorresponds to the unrestricted aggregate equation (36).

^bCorresponds to the restricted aggregate equation (37).

TABLE 5

Tests of Aggregation Bias[†]

	<u>Unrestricted</u>	<u>Restricted</u>	
	<u>Specifications</u>	<u>Specifications</u>	<u>Specifications</u>
	(Table 1) ^a	(Table 2) ^b	(Table 3) ^b
q ₁ [*] (1) [wages]	0.63	5.13	17.20
q ₂ [*] (1) [wages]	0.32	2.46	5.18
q ₂ [*] (1) [output]	0.00	1.68	1.66
q ₂ [*] (1) [technology]	0.07	0.05	0.34

[†] Square brackets indicate variables over which restrictions are imposed; figures in round brackets show the number of restrictions imposed, s . Test statistics are compared to $\chi^2(s)$. The q_1^* and q_2^* statistics are computed using the results (24) and (25), respectively.

^a Results compared to unrestricted aggregate equation (36)

^b Results compared to restricted aggregate equation (37).

FIGURE 1.1

Long Run Industry Output Elasticities from Table 2

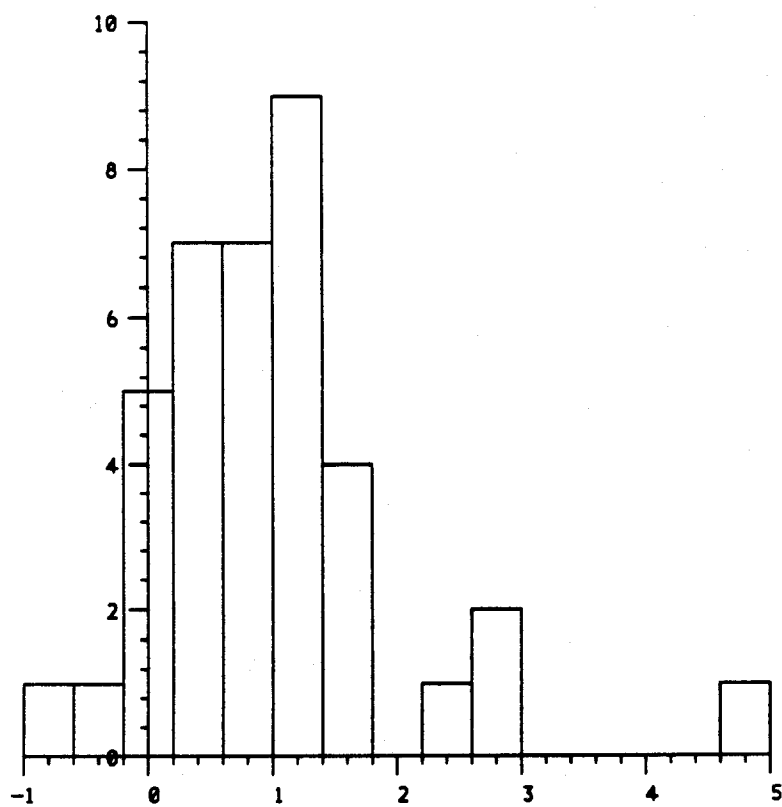


FIGURE 1.2

Long Run Industry Real Wage Elasticities from Table 2

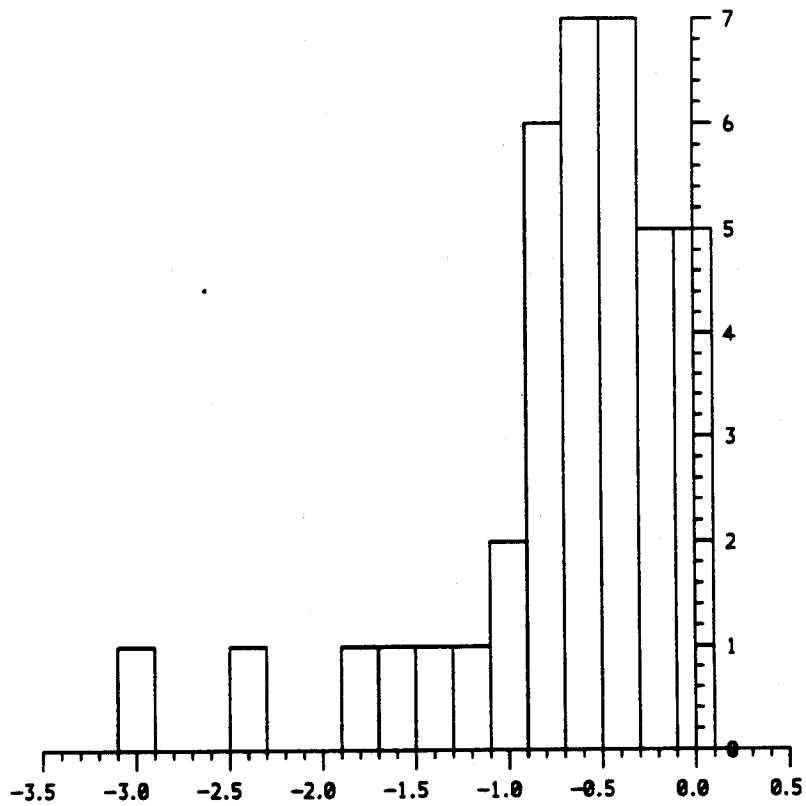


FIGURE 1.3

Long Run Industry Productivity Elasticities from Table 2

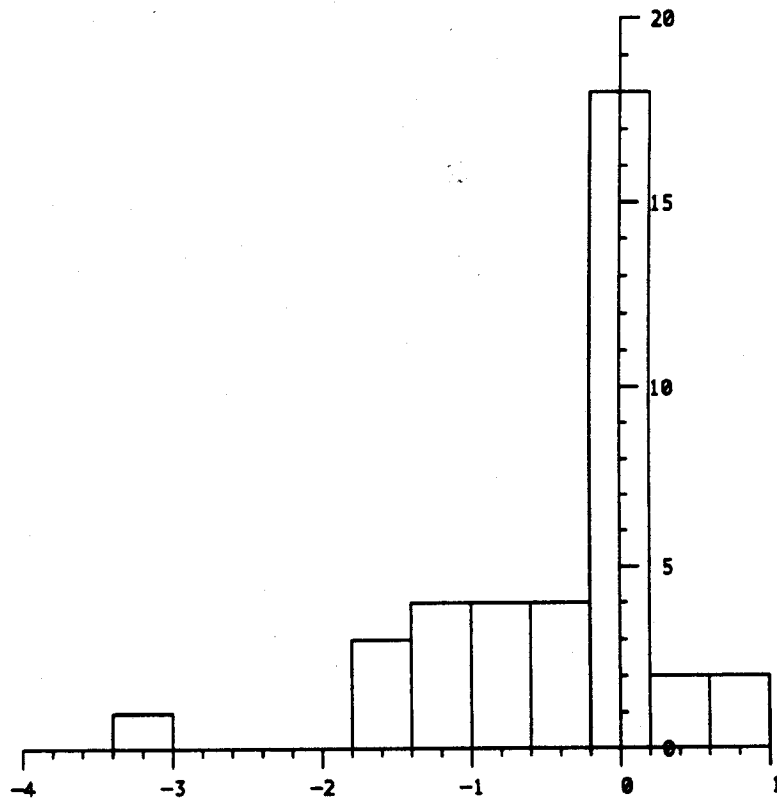


FIGURE 2.1

Long Run Industry Output Elasticities from Table 3

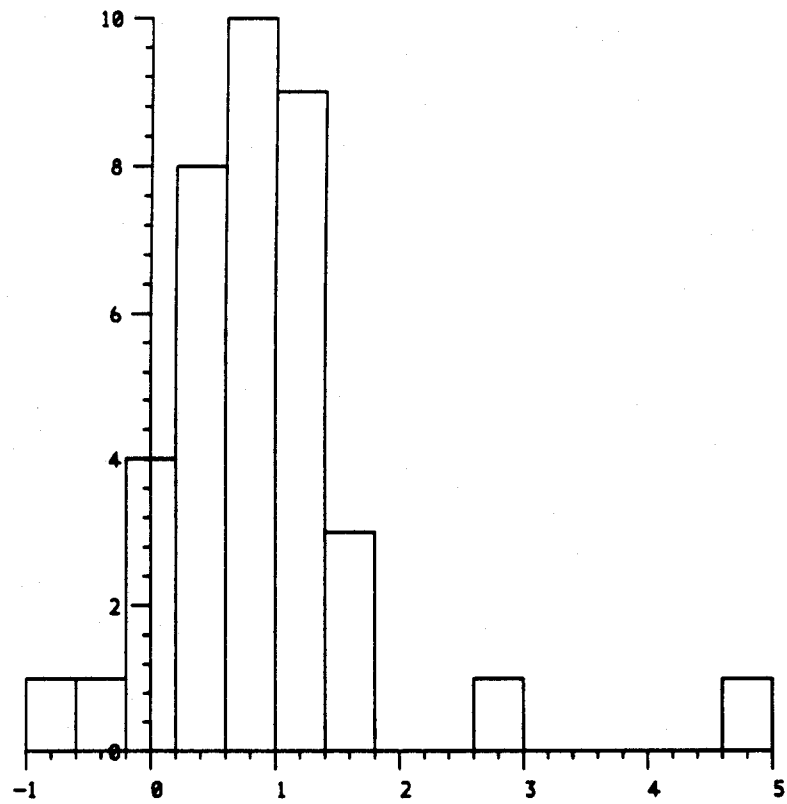


FIGURE 2.2

Long Run Industry Real Wage Elasticities from Table 3

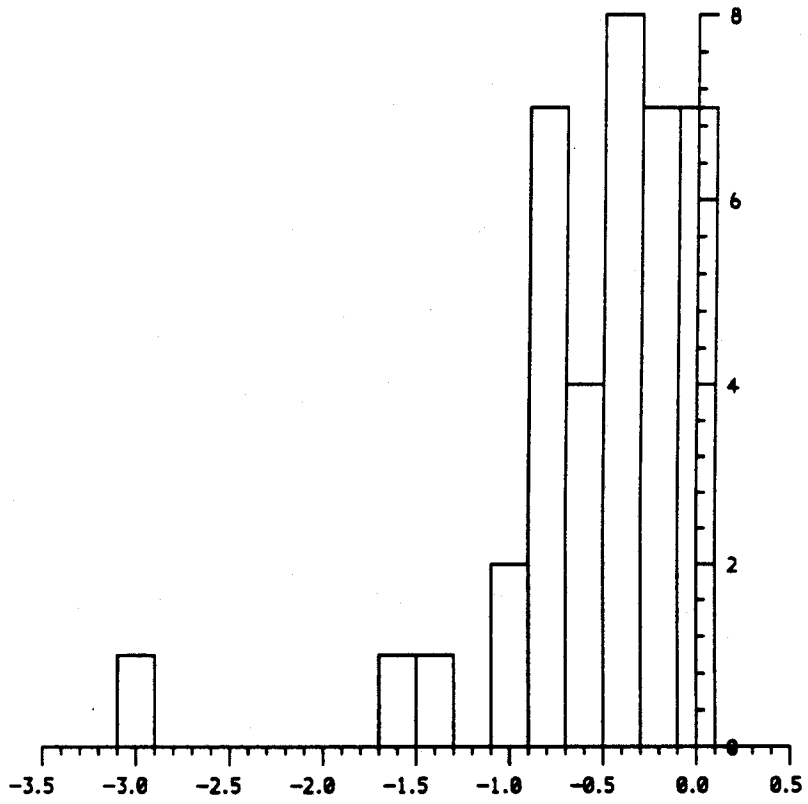
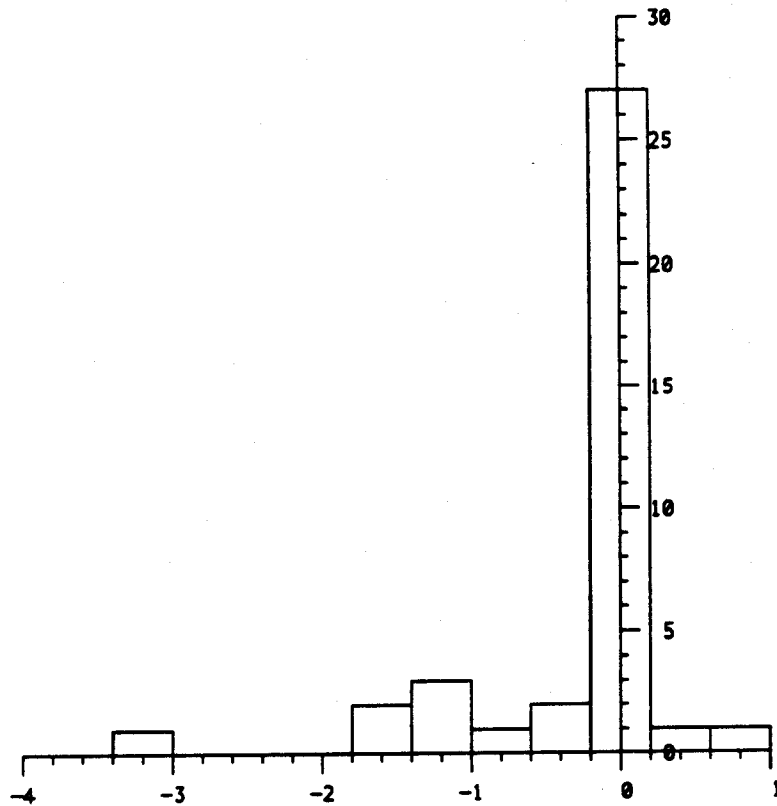


FIGURE 2.3

Long Run Industry Productivity Elasticities from Table 3



MATHEMATICAL APPENDIX

Proof of Theorem 1: Under H_0 defined by (17), the statistic q_2 in (19) can be written as

$$q_2 = \frac{d'd}{-n-n} \quad (A1)$$

where

$$\underline{d}_{-n} = \sum_{i=1}^m \underline{z}_{in}, \quad (A2)$$

and

$$\underline{z}_{in} = n^{-1/2} \hat{\Phi}_n^{-1/2} P_i u_{i-1}, \quad i = 1, 2, \dots, m. \quad (A3)$$

The matrices $\hat{\Phi}_n$ and P_i are defined by (22) and (20) in the text, respectively. The proof we offer here has two stages: we first show that for each i and for any real $k \times 1$ vector $\underline{\lambda}$ such that $\underline{\lambda}'\underline{\lambda} = 1$, $\underline{\lambda}'\underline{z}_{in} \stackrel{a}{\sim} N(0, \phi_{ii})$ where $\phi_{ii} > 0$. Using this result in (A2), we then show that $\underline{\lambda}'\underline{d}_{-n} \stackrel{a}{\sim} \underline{\lambda}'\underline{d}$, where $\underline{d} \sim N(0, I_k)$. From this it follows that $\underline{d}_{-n} \stackrel{a}{\sim} N(0, I_k)$, and $\frac{d'd}{-n-n} \stackrel{a}{\sim} \chi_k^2$. [See, for example, Proposition 5.1 in White (1984).]

Under Assumptions 1-3 it readily follows that

$$\text{plim}_{n \rightarrow \infty} (\hat{\sigma}_{ij}) = \sigma_{ij},$$

$$\text{plim}_{n \rightarrow \infty} (\hat{\Phi}_n) = \Phi,$$

where

$$\Phi = \sum_{i,j=1}^m \sigma_{ij} Q_{ij},$$

and the matrices Q_{ij} defined by

$$Q_{ij} = \text{plim}_{n \rightarrow \infty} (n^{-1} P_i P_j')$$

$$= \Sigma_{aa} - \frac{1}{m} \Sigma_{aa}^{-1} \Sigma_{aj} \Sigma_{jj}^{-1} - \frac{1}{m} \Sigma_{ii}^{-1} \Sigma_{ia} \Sigma_{aa}^{-1} + \frac{1}{m^2} \Sigma_{ii}^{-1} \Sigma_{ij} \Sigma_{jj}^{-1},$$

are finite for all i and j and are non-singular for $i = j$. Now noting that by assumption Φ is also non-singular we have

$$\lambda' z_{in} \stackrel{a}{\sim} n^{-\frac{1}{2}} \mu' P_i u_i = n^{-\frac{1}{2}} \sum_{t=1}^n \delta_{it} u_{it} \quad (A4)$$

where $\underline{\mu} = \Phi^{-\frac{1}{2}} \underline{\lambda}$, and δ_{it} stands for a typical element of vector $P_i' \underline{\mu}$. It is now easily seen that under assumptions of the theorem, the conditions for the application of the version of Liapounov's Theorem cited in White (1984, Theorem 5.10) to the right hand side of (A4), which is a sum of independently, but non-identically distributed random variables, are met and

$$\lambda' z_{in} \stackrel{a}{\sim} N(0, \phi_{ii}),$$

where

$$\phi_{ii} = \sigma_{ii} \mu' Q_{ii} \mu > 0.$$

Therefore, asymptotically $\lambda' d_{-n} = \sum_{i=1}^m \lambda' z_{in}$ is distributed as a linear function of m normal variates and itself will be distributed normally with zero mean and variance²⁰

²⁰Notice that since $\lim_{n \rightarrow \infty} V(\lambda' z_{in} z_{jn}') = \sigma_{ij} \mu' Q_{ij} \mu = \phi_{ij}$, then

$$\begin{aligned} \lim_{n \rightarrow \infty} V(\lambda' d_{-n}) &= \lim_{n \rightarrow \infty} V(\sum_{i=1}^m \lambda' z_{in}) = \sum_{i,j=1}^m \phi_{ij} \\ &= \mu' (\sum_{i,j=1}^m \sigma_{ij} Q_{ij}) \mu = \mu' \Phi^{-1} \mu = \underline{\lambda}' \Phi^{-\frac{1}{2}} \Phi^{-\frac{1}{2}} \underline{\lambda} = 1. \end{aligned}$$

$$\lim_{n \rightarrow \infty} V(\underline{\lambda}' \underline{d}_{-n}) = \underline{\lambda}' \underline{\lambda} = 1.$$

Hence, for a finite m ,

$$\underline{d}_{-n} \stackrel{a}{\sim} N(0, I_k),$$

and

$$\underline{d}'_{-n-n} \stackrel{a}{\sim} \chi_k^2.$$

Q.E.D.

Proof of Theorem 2: The proof is similar to that presented for Theorem 1.

Under H_d the statistic q_3 defined by (33) can be written as

$$q_3 = \underline{d}'_{-n-n} \underline{d}_{-n}$$

where \underline{d}_{-n} is defined by (A2), but \underline{z}_{in} is now given by

$$\underline{z}_{in} = n^{-k/2} \hat{V}_n^{-k/2} \underline{X}'_{a2} M_{i-1} u_i.$$

Since by assumption \hat{V}_n converges in probability to a non-singular matrix, say V , we also have

$$\underline{\lambda}' \underline{z}_{in} \stackrel{a}{\sim} n^{-k/2} \underline{\lambda}' V^{-k/2} \underline{X}'_{a2} M_{i-1} u_i = n^{-k/2} \underline{\mu}' \underline{X}'_{a2} M_{i-1} u_i,$$

where $\underline{\lambda}$ is now a $(k-p) \times 1$ vector of constants such that $\underline{\lambda}' \underline{\lambda} = 1$, and $\underline{\mu} = V^{-k/2} \underline{\lambda}$. Denoting the t^{th} element of $M_{i-1} \underline{X}'_{a2} \underline{\mu}$ by η_{it} we now have

$$\underline{\lambda}' \underline{z}_{in} \stackrel{a}{\sim} n^{-k/2} \sum_{t=1}^n \eta_{it} u_{it}, \quad (\text{A5})$$

which is a sum of independently, but non-identically distributed random variables. As in the proof of Theorem 1, it is easily seen that under assumptions of Theorem 2, the Liapounov's theorem (White, 1984, Theorem 5.10) is applicable to (A5) and

$$\underline{\lambda}' \underline{z}_{in} \stackrel{a}{\sim} N(0, \psi_{ii}).$$

where

$$\psi_{ii} = \underline{\mu}' \left(\text{plim}_{n \rightarrow \infty} (n^{-1} \mathbf{X}'_a \mathbf{M}_a \mathbf{X}_a) \right) \underline{\mu} > 0.$$

Hence, by a similar reasoning as in the proof of Theorem 1, we have

$$\underline{\lambda}' \underline{d}_{-n} \stackrel{a}{\sim} N(0, 1),$$

and

$$\underline{d}_{-n} \stackrel{a}{\sim} N(0, \mathbf{I}_{k-p}),$$

which establishes that

$$q_3 = \underline{d}'_{-n} \underline{d}_{-n} \stackrel{a}{\sim} \chi^2_{k-p}.$$

Q.E.D.

DATA APPENDIX

With the exception of data on industrial investment, the data used in this study are the same as that employed in Pesaran et al. (1988), and are taken from the Cambridge Growth Project (CGP) Databank. For the sources of the data and the classifications of industry groups see the Data Appendix and Table A in PPK. For convenience, Table A is reproduced at the end of this Appendix.

Data on industrial investment in vehicles, in plant and machinery, and in buildings are available separately for the period 1954-84, from which total gross investment is constructed. There is not a one-to-one correspondence between the Blue Book (BB) industrial classifications for which the data is published and our own, however. Where the BB data is more disaggregated, this causes no problem, since we simply amalgamate the appropriate industries. There remains six areas in which the BB data is more aggregated than our own. These are listed in Table B.

In these cases, we have made the simplifying assumption that the investment reported by BB classification can be divided equally over the (more disaggregate) CGP industrial groups. This procedure is satisfactory if the CGP industry groups within the BB classifications show similar investment growths over the 1954-84 period. This is likely to be the case for the Coal and Coke industries, but is less likely to hold in the case of the BB industry classifications 13 and 17.

TABLE A

Classification of Industry Groups

(In Terms of the 1980 Standard Industrial Classification)

<u>Industry Groups (CGP classification)</u>	<u>Division, Class or Group</u>
1. Agriculture, Forestry and Farming	0
2. Coal Mining	1113, 1114
3. Coke	1115, 1200
4. Mineral Oil and Natural Gas	1300
5. Petroleum Products	140
6. Electricity, etc.	1520, 1610, 1630
7. Public Gas Supply	1620
8. Water Supply	1700
9. Minerals and Ores n.e.s.	21, 23
10. Iron and Steel	2210, 2220, 223
11. Non-Ferrous Metals	224
12. Non-Metallic Mineral Products	24
13. Chemicals and Manmade Fibers	25, 26
14. Metal Goods n.e.s.	31
15. Mechanical Engineering	32
16. Office Machinery, etc.	33
17. Electrical Engineering	34
18. Motor Vehicles	35
19. Aerospace Equipment	3640
20. Ships and Other Vessels	3610
21. Other Vehicles	3620, 363, 3650
22. Instrument Engineering	37
23. Manufactured Food	41, 4200, 421, 422, 4239
24. Alcoholic Drinks, etc.	4240, 4261, 4270, 4283
25. Tobacco	4290
26. Textiles	43
27. Clothing and Footwear	45
28. Timber and Furniture	46
29. Paper and Board	4710, 472

Table A (cont.)

<u>Industry Groups (CGP classification)</u>	<u>Division. Class or Group</u>
30. Books, etc.	475
31. Rubber and Plastic Products	48
32. Other Manufactures	44, 49
33. Construction	5
34. Distribution, etc.	61, 62, 63, 64, 65, 67
35. Hotels and Catering	66
36. Rail Transport	71
37. Other Land Transport	72
38. Sea, Air and Other	74, 75, 76, 77
39. Communications	79
40. Business Services	81, 82, 83, 84, 85
41. Miscellaneous Services	94, 98, 923, 95, 96, 97

TABLE B

Blue Book and Cambridge Growth Project Industrial Classifications

<u>CGP Classification</u>	<u>BB Classification</u>
2. Coal	2. Coal and coke
3. Coke	
9. Minerals and ores nes	8. Metals
10. Iron and steel	9. Other minerals
11. Non-ferrous metals	
12. Non-metallic mineral products	
16. Office machinery	13. Electrical and instrument engineering
17. Electrical engineering	
22. Instrument engineering	
19. Aerospace equipment	15. Transport, other than motor vehicles
20. Ships	
21. Other vehicles	
24. Drink	17. Drink and tobacco
25. Tobacco	
29. Paper and board	21. Paper, printing and publishing
30. Books	

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