

THE INFLATION TAX IN A REAL BUSINESS CYCLE MODEL

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ABSTRACT

This paper develops a model of a competitive economy that incorporates the major features of real business cycle models but includes money. The model is a one sector stochastic optimal growth model with indivisible labor and employment lotteries. Money is introduced via a cash-in-advance constraint for the consumption good. The model is calibrated and simulated to measure the welfare cost of the inflation tax and to determine how the cyclical and steady state properties of the model are affected by inflation. We estimate the welfare cost of a sustained moderate (10%) inflation to be between .11% and .4% of GNP. We also find that the features of the business cycle, as summarized by a set of standard deviations and correlations, are the same in high and low inflation economies. However, when money is supplied erratically, the cyclical properties are affected somewhat and more closely match historical experience. The model also predicts that employment rates will be lower in high inflation economies. We present empirical evidence to support this.

1. Introduction

Current controversies in business cycle theory have much in common with the macroeconomic debates of the 1960's. Twenty years ago Milton Friedman and Walter Heller debated the issue of whether "money matters". In the ensuing years the methods of business cycle research have changed dramatically but the questions have remained much the same. In particular, the issue of how much money matters is as timely now as it was when Friedman and Heller discussed it. In this paper we take the question of whether money matters to mean three things: does money and the form of the money supply rule affect the nature and amplitude of the business cycle? how does anticipated inflation affect the long run values of macroeconomic variables? and, what are the welfare costs associated with alternative money supply rules? These are quite different questions and each implies a distinct sense in which money can affect the economy. Herein we describe a model economy that can be used to address these sorts of questions. The setting is similar to one suggested by Lucas (1987) where money is held due to a cash-in-advance constraint. We use it to provide estimates of the welfare cost of the inflation tax and to study the effect of anticipated inflation on the characteristics of aggregate time series.

Early equilibrium business cycle models were influenced greatly by the monetarist tradition and the empirical findings of Friedman and Schwartz. They were models where unanticipated changes in the money supply played an important role in generating fluctuations in aggregate real variables and explaining the correlation between real and nominal variables (eg. Lucas (1972)). More recently, business cycle research has been focused on a class

of models in which fluctuations associated with the business cycle are the equilibrium outcome of competitive economies that are subject to exogenous technology shocks. In these real business cycle models, as developed by Kydland and Prescott (1982), Long and Plosser (1983), Kydland (1984), Hansen (1985) and Prescott (1986) there is a complete set of contingent claims markets and money does not enter. Considering the importance attributed to money in earlier neoclassical and monetarist business cycle theories, it is perhaps surprising that these real models have been quite successful in replicating the characteristics of aggregate data while abstracting from a role for money. This does not imply that money is unimportant for the evolution of real economic variables, but it is true that the exact role for money in these models is an open and somewhat controversial question.

Not surprisingly, given that the correlation between money and output is a time honored statistical regularity, the absence of money in real business cycle models has been a source of discomfort for many macro-economists. One reaction to this, eg. by Bernanke (1985) and Eichenbaum and Singleton(1986) among others, has been to reexamine the evidence that money "causes" changes in output. Another approach has been to construct models where money plays an essentially passive role but in which the money output correlation can be explained by distinguishing different roles for money (eg. inside and outside money) as in King and Plosser (1984) and Greenwood and Huffman (1987). Yet another reaction has been to argue that there is some role for money over and above technology shocks. This argument is pursued in Lucas (1987).

In this paper we study the quantitative importance of money in a real business cycle model where money is introduced in a way such that shocks to

the money supply are not likely to be important contributors to aggregate fluctuations. The features of our model economy are deliberately chosen to emphasize the influence on real variables of anticipated inflation operating through the inflation tax. Money can have important real effects in this setting: anticipated inflation will cause people to substitute away from activities that require cash, such as consumption, for activities that do not require cash, such as leisure. We analyze the consequence of this distortion for real variables and estimate the magnitude of the welfare losses.

In the following sections we describe, calibrate and simulate a simple one sector stochastic optimal growth model with a real economy identical to that studied by Hansen (1985). The real time series generated by the model fluctuate in response to exogenous technology shocks. The model incorporates indivisible labor and an employment lottery that permits some agents to be unemployed. With the latter features, the model implies a degree of intertemporal substitution that is consistent with observed fluctuations without contradicting microeconomic evidence from panel studies. In addition, the indivisible labor assumption is consistent with the observation that most of the fluctuation in aggregate hours worked is due to fluctuations in employment rather than fluctuations in the average hours worked of an employed worker.

Money is introduced into the model using a cash-in-advance constraint. Economies with this feature have been studied extensively by Stockman (1981), Lucas (1982), Lucas and Stokey (1983,1987) and Svensson (1985). The cash-in-advance constraint applies only to the consumption good. Leisure and investment in our model are credit goods. Thus, if agents in this

economy wish to reduce cash holdings in response to higher inflation, they can only do so by reducing consumption.

In the next section of the paper we lay out the details of our monetary business cycle model and describe the competitive equilibrium of this artificial economy. Solving for an equilibrium in this economy is more difficult than in other real business cycle economies because the inefficiency imposed by the cash-in-advance constraint rules out the use of invisible hand arguments based on the second welfare theorem. In Section 3 we describe how we solve for an equilibrium directly using a suggestion in Kydland (1987).

In Section 4 of the paper we present the results of some simulations of the model under various assumptions about the behavior of the monetary growth rate. Our purpose here is to use our model as an experimental device to study the effect of certain parameter interventions¹. We take a model whose statistical properties have been studied previously and examine how injections of money, operating through a cash-in-advance constraint, alter the conclusions derived from this purely real economy. In this model, when money is supplied optimally, the real economy and its associated steady state paths and cyclical characteristics are identical to those in Hansen (1985). By varying the rate of growth of the money supply we can study how the real allocation and the comovements among variables are altered. In addition we are able to measure the welfare costs of the inflation tax.

The results of the experiments just described are easily summarized. When money is supplied according to a constant growth rate rule that implies

¹See Cooley and LeRoy (1985) for a discussion of parameter and variable interventions.

average inflation rates greater than the rate of discount, individuals substitute leisure for goods, output and investment fall, and the steady state capital stock is lower. The features of the business cycle are unchanged by these constant growth rates. We also report the results of experiments in which money is supplied not at a constant rate but erratically with characteristics that mimic historical experience. In these simulations, the cyclical behavior of real variables are altered slightly: consumption becomes more variable relative to income and the price level becomes quite volatile. In addition, the correlations between these variables and output become smaller in absolute value. It is encouraging that with these changes the cyclical properties of the model more closely match U.S. postwar experience.

Using definitions described in Section 4 we estimate the welfare cost due to the inflation tax of a sustained moderate (10%) inflation to be about 0.4 % of GNP using M1 as the relevant definition of money and a quarter as the period over which it must be held. This is very close to estimates that have been suggested by others. We find the welfare costs to be much lower, about 0.1, when the relevant definition of money is the monetary base and the period over which it is constrained to be held is a month.

Perhaps the most striking implication of our model for the steady state behavior of economic aggregates is that employment rates should be lower in the long run in high inflation economies. This possibility, stated somewhat differently as the proposition that in the long run the Phillips curve slopes upward, has been suggested by others, most notably Friedman(1977). We present evidence that, for a cross section of developed economies during the period 1976-1985, average inflation rates and average employment rates

are negatively correlated.

The conclusions drawn from our simulations reflect only the costs and consequences of money that are due to the inflation tax: there are no informational problems created by the money supply process. We conclude that if money does have a major effect on the cyclical properties of the real economy it must be through channels that we have not explored here.

2. A Cash-In-Advance Model with Production

The economy studied is a version of the indivisible labor model of Hansen (1985) with money introduced via a cash-in-advance constraint applied to consumption. That is, consumption is a "cash good" while leisure and investment are "credit goods," in the terminology of Lucas and Stokey (1983, 1987). In this section we describe the economy and define a competitive equilibrium. In the next section we describe how an equilibrium can be computed using a linear-quadratic approximation of the economy.

We assume a continuum of identical households with preferences given by the utility function,

$$(2.1) \quad E_0 \sum_{t=0}^{\infty} \beta^t (\log c_t + A \log l_t) , \quad 0 < \beta < 1 ,$$

where c_t is consumption and l_t is leisure in time t . Households are assumed to be endowed with one unit of time each period and supply labor to a firm which produces the goods. Households are also engaged in accumulating capital which they rent to the firm.

We assume that households enter period t with nominal money balances equal to m_{t-1} that are carried over from the previous period. In addition, these balances are augmented with a lump sum transfer equal to $(1-g_t)M_{t-1}$,

where M_t is the per capita money supply in period t . The money stock follows a law of motion

$$(2.2) \quad M_t = g_t M_{t-1} .$$

In this paper, we study two economies. In the first, the growth rate of money, g_t , is assumed to be constant. In the other economy, the log of the growth rate of the money supply evolves according to an autoregression of the form:

$$(2.3) \quad \log(g_{t+1}) = \alpha \log(g_t) + \xi_{t+1} .$$

In equation (2.3), ξ_t is an iid random variable with mean $\log(\bar{g})(1-\alpha)$ and variance σ_ξ^2 , where \bar{g} is the unconditional mean of the growth rate g_t . It is assumed that g_t is revealed to all agents at the beginning of period t .

Households are required to use these previously acquired money balances to purchase the nonstorable consumption good. That is, a household's consumption choice must satisfy the constraint,

$$(2.4) \quad p_t c_t \leq m_{t-1} + (g_t - 1)M_{t-1} ,$$

where p_t is the price level at time t . In this paper, attention is focused on examples where this constraint always holds with equality. A sufficient condition for this constraint to be binding is that the growth rate of money, g_t , always exceeds the discount factor, β . Our examples will satisfy this condition.² In our view this assumption is not unreasonable given the

² It can be shown from the first order conditions of the household's problem that the cash-in-advance constraint will be binding (the Lagrange multiplier associated with constraint (2.3) will be positive) if and only if $E_t(g_{t+1}) > \beta$.

observed behavior of the actual money supply.³

As in Hansen (1985), labor is assumed to be indivisible. This means that households can work some given positive number of hours, $h_0 < 1$, or not at all. They are not allowed to work an intermediate number of hours.⁴ Under usual market interpretations, this assumption implies that the consumption set of households is nonconvex. However, we convexify the economy by following Rogerson (1984) and assume that agents trade employment lotteries. That is, households sell contracts which specify a probability of working in a given period, π_t , rather than selling their labor directly. Since all agents are identical, they will all choose the same π_t . Thus, a fraction π_t of the households will work h_0 hours and the remaining $(1-\pi_t)$ households will be unemployed during period t . A lottery determines which of the households work and which do not. Thus, per capita hours worked in period t is given by

$$(2.5) \quad h_t = \pi_t h_0.$$

The market structure described above implies that the period utility function of the representative household as a function of consumption and

³ In addition, to relax this assumption would considerably complicate our solution procedure, forcing us to consider the possibility of both corner and interior solutions.

⁴ The indivisible labor assumption implies that all changes in total hours worked are due to changes in the number of workers. Although over half of the variance in total hours in the U.S. is unambiguously due to fluctuations in employment, there is still a significant percentage that is due to fluctuation in average hours. A model that allows for adjustment along both of these margins is studied in Cho and Cooley (1988).

hours worked is given by⁵

$$\begin{aligned} U(c_t, h_t) &= \log c_t + \pi_t A \log(1-h_t) + (1-\pi_t)A \log(1) \\ &= \log c_t + h_t A (\log(1-h_t)/h_t) . \end{aligned}$$

We rewrite this as,

$$(2.6) \quad U(c_t, h_t) = \log c_t - B h_t , \quad \text{where } B = -A(\log(1-h_t)/h_t)$$

In the remainder of this section, we will discuss the problem faced by a representative agent with preferences given by (2.6) as a stand-in for the individual household with preferences given by (2.1) who is subject to the labor indivisibility restriction.

This representative household must choose consumption, investment (x_t), and nominal money holdings subject to the following budget constraint:⁶

$$(2.7) \quad c_t + x_t + m_t/p_t \leq w_t h_t + r_t k_t + (m_{t-1} + (g_t - 1)M_{t-1})/p_t$$

In this equation, w_t and r_t are the wage rate and rental rate of capital, respectively. Investment is undertaken to augment the capital stock (k_t) owned by the household. The capital stock obeys the following law of motion:

⁵ This derivation assumes that consumption is the same whether or not the household is employed. This result, which holds in equilibrium, follows from the separability of (2.1) in consumption and leisure and is shown formally in Hansen (1985).

⁶ This budget constraint incorporates the fact that consumption and investment sell at the same price even though one is a cash good and the other a credit good. This is because, from the point of view of the seller, sales of both credit goods and cash goods result in cash that will be available for spending at the same time in the following period. Although cash good sales in a given period result in cash receipts in the same period, this cash can not be spent until the next period.

$$(2.8) \quad k_{t+1} = (1-\delta) k_t + x_t, \quad 0 \leq \delta \leq 1.$$

The firm in our economy produces output (Y_t) using the constant returns to scale technology:

$$(2.9) \quad Y_t = \exp(z_t) K_t^\theta H_t^{1-\theta}, \quad 0 \leq \theta \leq 1$$

Capital letters are used to distinguish per capita variables that a competitive household takes as parametric from individual specific variables that are chosen by the household.⁷ The variable z_t is an exogenous shock to technology that follows a law of motion given by

$$(2.10) \quad z_{t+1} = \gamma z_t + \epsilon_{t+1}, \quad 0 \leq \gamma \leq 1,$$

where ϵ_t is an iid random variable with mean 0 and variance σ_ϵ^2 . We assume that z_t , like g_t , is revealed to all agents at the beginning of period t .

The firm seeks to maximize profit, which is equal to $Y_t - w_t H_t - r_t K_t$. The first order conditions for the firm's problem yield the following functions for the wage rate and rental rate on capital:

$$(2.11) \quad w(z_t, K_t, H_t) = (1-\theta) \exp(z_t) K_t^\theta H_t^{-\theta}$$

$$(2.12) \quad r(z_t, K_t, H_t) = \theta \exp(z_t) K_t^{\theta-1} H_t^{1-\theta}$$

A change in variables is introduced so that the problem solved by the households will be stationary. That is, let $\hat{m}_t = m_t/M_t$ and $\hat{p}_t = p_t/M_t$. In addition, let $V(z, \hat{g}, \hat{m}, \hat{K}, \hat{k})$ be the equilibrium maximized present value of the utility stream of the representative household who enters the period with

⁷ In equilibrium, of course, these will be the same.

money balances equal to \hat{m} and a capital stock equal to k when the aggregate state is described by z , g , and K . Implicit in the functional form of V are the equilibrium aggregate decision rules (H and X) and the pricing function (\hat{p}) as functions of the aggregate state, which is taken as given by the households. The function V must satisfy Bellman's equation (primes denote next period values)⁸

$$(2.13) \quad V(z, g, \hat{m}, K, k) = \max \left\{ U(c, h) + \beta E[V(z', g', \hat{m}', K', k') | z, g, \hat{m}, K, k] \right\}$$

subject to

$$(2.14) \quad c + x + \hat{m}'/\hat{p} = w(z, K, H)h + r(z, K, H)k + (\hat{m}+g-1)/(\hat{p} g)$$

$$(2.15) \quad c = (\hat{m}+g-1)/(\hat{p} g)$$

$$(2.16) \quad z' = \gamma z + \epsilon$$

$$(2.17) \quad g' = g \quad (\text{economy 1}) \text{ or}$$

$$\log(g') = \alpha \log(g) + \xi \quad (\text{economy 2})$$

$$(2.18) \quad K' = (1-\delta)K + X$$

$$(2.19) \quad k' = (1-\delta)k + x$$

and c , x , \hat{m}' nonnegative and $0 \leq h \leq 1$. In addition, X , H , and \hat{p} are given functions of (z, g, K) .

A stationary competitive equilibrium for this economy consists of a set of decision rules, $c(s)$, $x(s)$, $\hat{m}'(s)$, and $h(s)$ (where $s = (z, g, \hat{m}, K, k)$), a set of aggregate decision rules, $X(S)$ and $H(S)$ (where $S = (z, g, K)$), a pricing function $\hat{p}(S)$, and a value function $V(s)$ such that:

(i) the functions V , X , H , and \hat{p} satisfy (2.13) and c , x , \hat{m}' , and h are the associated set of decision rules;

⁸ Note that the solution to the firm's profit maximization problem has been substituted into this problem through the functions $w(\cdot)$ and $r(\cdot)$.

(ii) $x = X$, $h = H$, and $\hat{m}' = 1$ when $k = K$ and $\hat{m} = 1$; and

(iii) the functions $c(s)$ and $x(s)$ satisfy $c(s) + x(s) = Y(S)$ for all s .

3. Solution Method

In Hansen (1985) it was possible to compute an equilibrium indirectly by solving for the (unique) equal weight Pareto optimal allocation and invoking the second welfare theorem. In order to obtain an analytic solution to the problem, a linear-quadratic approximation to this nonlinear problem was formed, making it possible to compute linear decision rules. Unfortunately, it is not possible to invoke the second welfare theorem to compute an equilibrium for the economy studied in this paper. This is because money introduces a "wedge of inefficiency" (in the words of Lucas (1987)) that forces one to solve for an equilibrium directly. To get around this, we apply the method described in Kydland (1987) to compute an equilibrium for our cash-in-advance economy.

Kydland's method involves computing a linear-quadratic approximation to the household's problem (2.13). This dynamic programming problem is then solved by iterating on Bellman's equation, requiring that the second equilibrium condition (refer to the above definition of equilibrium) to hold at each step of this recursive procedure. In the remainder of this section, we outline in more detail how this procedure is implemented in our particular case.⁹

The first step is to substitute the nonlinear constraints, (2.14) and (2.15), into the household's utility function (2.6). This is done by first

⁹ This method is similar to the method of Kydland and Prescott (1977), which is described in some detail in Sargent (1981) and Whiteman (1983).

eliminating c by substituting (2.15) into (2.14) and (2.6). The resulting budget constraint is

$$(3.1) \quad x + \hat{m}'/\hat{p} = w(z, K, H)h + r(z, K, H)k .$$

Because of the constant returns to scale technology, requiring that the functions w and r be of the form (2.11) and (2.12) guarantees that equilibrium condition (iii) is satisfied.

The constraint (3.1) can be substituted into the utility function (2.6) by eliminating h . However, we must first eliminate H . This is done by aggregating (3.1) and solving for H . Using (2.11) and (2.12), this implies

$$(3.2) \quad H = \left[(X+1/\hat{p}) / (\exp(z)K^\theta) \right]^{1/(1-\theta)}$$

Equation (3.2) can be substituted into (3.1), and the result substituted into (2.6). The return function for the household's dynamic programming problem is now given by the following expression:

$$(3.3) \quad \log \left[\frac{\hat{m} + \hat{g} - 1}{\hat{p} \hat{g}} \right] - B \left[\frac{[x + (\hat{m}'/\hat{p}) - \theta(X+1/\hat{p})(k/K)](X+1/\hat{p})^{\theta/(1-\theta)}}{(1-\theta)(\exp(z)K^\theta)^{1/(1-\theta)}} \right]$$

In order to obtain an analytic solution to this problem, the above nonlinear return function (3.3) is approximated by a quadratic function in the neighborhood of the steady state of the certainty problem. This approximation technique is described in detail in Kydland and Prescott (1982). The state vector of the resulting linear-quadratic dynamic programming problem is $s = (1, z, \hat{g}, \hat{m}, K, k)^T$ and the individuals' decision (or control) vector is $u = (\hat{m}', x)^T$. In addition, the economy wide variables $U = (\hat{p}, X)^T$ also enter the quadratic return function. Thus, after computing the

quadratic approximation of (3.3), Bellman's equation for the household's problem (2.13) becomes¹⁰

$$(3.4) \quad s^T V s - \max \left\{ \begin{bmatrix} s^T & U^T & u^T \end{bmatrix} Q \begin{bmatrix} s \\ U \\ u \end{bmatrix} + \beta s'^T V s' \right\}$$

subject to (2.16)-(2.19) and a linear function that describes the relationship between U and $S = (1, z, g, K)^T$ perceived by the agents in the model.

To solve for an equilibrium, we iterate on this quadratic version of Bellman's equation. This procedure must involve choosing a candidate for the perceived linear function relating U to S. We start with a guess for the matrix V, call it V_0 , and consider the maximization problem on the right side of (3.4). Once the laws of motion, (2.16) through (2.19), have been substituted into the objective, we obtain from the first order condition for u the linear decision rule

$$(3.5) \quad u = D_1 s + D_2 U .$$

By imposing the equilibrium conditions, $x = X$, $\hat{m}' = \hat{m} = 1$, and $k = K$; we can obtain, from (3.5), a linear expression for U in terms of S that we take as our candidate. That is, we obtain

$$(3.6) \quad U = D_3 S$$

To compute the value function for the next iteration, we eliminate u

¹⁰ This form for Bellman's equation incorporates both certainty equivalence and the fact that the value function will be quadratic.

from (3.4) using equation (3.5) and U using (3.6).¹¹ We are then left with the quadratic form $s^T V_1 s$ on the right side of (3.4), which is used as the value function for the next iteration. This procedure is repeated until V_{j+1} is sufficiently close to V_j to claim that the iterations have converged.

Once this process has converged, we obtain the following equilibrium expressions for X and \hat{p} , (\hat{p} is equal to the inverse of consumption in an equilibrium where the cash-in-advance constraint is always binding):

$$(3.7) \quad X = d_{10} + d_{11}z + d_{12} \log g + d_{13}K$$

$$(3.8) \quad \hat{p} = d_{20} + d_{21}z + d_{22} \log g + d_{23}K$$

Examples of these decision rules for particular parameterizations of the money supply rule are given in the appendix. These equations, which determine investment and consumption, along with the laws of motion (2.16) through (2.18), the expression for hours worked (3.2), and the technology (2.9), are used to simulate artificial time series for various parameterizations of the g_t process. These experiments are discussed in the next section.

4. Results

We use the artificial economy just described to study the interaction between money and the real sector of the economy. We first describe the

¹¹ For some of the parameterizations studied in this paper equation (3.5) can not be formed because Q_{99} and $Q_{10, 10}$, the elements of the quadratic form in (3.4) corresponding to the control variables, are very close to zero. That is, any value for u will satisfy the first order conditions for the problem on the right side of (3.4). However, not every value for u is an equilibrium. In these cases, we use the equilibrium conditions $\hat{m}' = 1$ and $x = X$ to eliminate u from (3.4) and continue to use (3.6) to eliminate U.

cyclical behavior of our economy under various money supply rules. We then use the model to measure the welfare costs of anticipated inflation. Finally, we look for confirmation of the implied steady state behavior of high and low inflation economies in cross section data on several developed countries.

4.1 Cyclical Properties

Statistics summarizing the cyclical behavior of our model economy under various money supply rules, as well as statistics summarizing the cyclical behavior of actual U.S. time series, are presented in Table 1. We will begin by describing how these statistics are computed and then proceed to interpret our results.

The first panel of Table 1 shows the (percent) standard deviations of the set of endogenous variables and their correlations with output that characterize recent U.S. quarterly data. These provide some basis for comparison with the results of our experiments although we wish to stress that ours is not a data matching exercise but an experimental simulation of a model economy. We use quarterly data from 1955,3 to 1984,1 on real GNP, consumption, investment, capital stock, hours worked, productivity, and two measures of the price level, the CPI and GNP deflator.¹² Before computing statistics, the data (both actual and simulated) is logged and detrended

¹² The series for real GNP, investment, hours worked and the price level were taken from the Citibase database. The hours series is based on information from the Current Population Survey. Productivity is output divided by hours worked. The data on the capital stock includes government capital stock and private capital stock (housing) as well as producers durables and structures. The consumption series includes nondurables and services plus an imputed flow of services from the stock of durables. The consumption and capital stock series were provided by Larry Christiano.

using the Hodrick-Prescott filter. The use of this detrending procedure enables us to maintain comparability with prior real business cycle studies by Kydland and Prescott (1982) and Hansen (1985).

In order to derive results from the artificial economies, we follow Kydland and Prescott (1982) by choosing parameter values based on growth observations and the results of studies using microeconomic data. In order to make comparisons with Hansen (1985) meaningful, we set the parameters describing preferences and technology to the same values used in that study. Those values, which were chosen under the assumption that the length of a period is one quarter, are $\beta = .99$, $\theta = .36$, $\delta = .025$, $B = 2.86$, and $\gamma = .95$. The standard deviation of ϵ , σ_ϵ , is set equal to .00721 so that the standard deviation of the simulated output series is close to the standard deviation of the actual output series. We experiment with different values for the parameters describing the money supply process.

Given a set of parameter values, simulated time series with 115 observations (the number of observations in the data sample) are computed using the method described in the previous section. These series are then logged and filtered and summary statistics calculated. We simulate the economy fifty times and the averages of the statistics over these simulations are reported. In addition, we report the sample standard deviations of these statistics, which are given in parentheses.

The columns of the second panel of Table 1 show the percent standard deviations and correlations that result from all of the simulations of our model economy where the money supply grows at a constant rate. These results confirm that when money is supplied at a constant growth rate, even one that implies a high average inflation rate, the features of the business

cycle are unaffected. In particular, the statistics summarizing the behavior of the real variables are the same as would be obtained in the same model without money--the "indivisible labor" model of Hansen (1985).

The remaining two panels of Table 1 show the results of simulations with an erratic money supply. That is, we assume a money supply rule of the form (2.3). We calibrate this money supply process (that is, choose values for α and σ_ξ) so that the money supply varies in a way that is broadly consistent with post war experience. We proceed by assuming that the Fed draws money growth rates from an urn with the draws being serially correlated, as in equation (2.3). We determined the characteristics of that urn from data on M1 and the regression (standard errors in parentheses)

$$\Delta \log(M1)_t = -.00798 + 0.481 \Delta \log(M1)_{t-1} \quad \hat{\sigma} = .0086$$

$$(.0014) \quad (.082)$$

where M1 is the average quarterly value. We intentionally averaged to smooth the data somewhat and increase the implied persistence.¹³ The results of this regression lead us to set α equal to .48 and σ_ξ equal to .009. To insure that the money growth rate always exceeds the discount rate, as is required for the cash-in-advance constraint to be always binding, we draw ξ_t from a lognormal distribution. This implies that $\log(g_t)$ will never become negative.

The statistics reported in Table 1 show that volatility of the money supply has a small but significant impact on the cyclical characteristics of

¹³ This equation is open to criticism as a description of the historical sample. Although we cannot reject its adequacy there maybe a left over moving average piece in the residuals. This in turn could imply that some portion of the innovation in the money growth rate is permanent. See eg. Schwert(1987). We chose to ignore this because the estimated autoregression seems to capture the features that are appropriate for our experiment.

the economy. Virtually all the effect of volatility in the money supply is in the standard deviations of consumption and prices and their correlation with output. In particular, consumption and prices become more volatile and their correlation with output becomes lower in absolute value. It is worth noting that the numbers in these panels are more in keeping with historical experience (see first panel) than are the results from constant growth rate economies. In addition, comparing the third and fourth panels we find that, although the price level does become more volatile, increases in the average growth rate of money has little effect on the cyclical properties of the real variables.

4.2 Welfare Costs of the Inflation Tax

In this section estimates of the welfare costs of the inflation tax are presented that are derived by comparing steady states of our growth model assuming different growth rates of the money supply.¹⁴ Measuring the welfare costs of anticipated inflation is an old issue in macroeconomics. Martin Bailey (1956) provided a classic answer to this question by considering the area under the demand curve for money, the welfare triangle, evaluated at an interest rate embodying the steady state rate of inflation as a measure of the net loss to individuals from the inflation tax. Stanley Fischer (1981) and Robert Lucas (1981) updated Bailey's estimates and they supply a thoughtful discussion of some of the awkward assumptions underlying the welfare triangle approach (eg. that government expenditures are financed

¹⁴ A somewhat similar approach to that taken here appears in a recent paper by Danthine, Donaldson, and Smith (1987). They simulate a model where money appears in the utility function and without investment or leisure. They also demonstrate a decline in welfare with inflation.

by non-distorting taxes). They also discuss some of the subsidiary costs of inflation that are ignored by those calculations.

We chose to measure the welfare costs by comparing steady states because, as explained above, the cyclical characteristics of this economy are unaffected by the average growth rate of the money stock. Thus, our discussion of welfare is based on the steady state properties of a version of our economy where the money supply grows at a constant rate and the technology shock in equation (2.9) is replaced by its unconditional mean.

The welfare costs for various annual inflation rates, along with the associated steady state values for output, consumption, investment, the capital stock, and hours worked, are presented in Table 2. We show results based on two different assumptions on the length of time that the cash-in-advance constraint is binding. The numbers displayed in the top panel reflect the assumption that the relevant period over which individuals are constrained to hold money is a quarter. This is consistent with the calibration of the model in the previous section. In addition, if we assume a unitary velocity as is implied by our model and if we assume that the "cash good" corresponds to consumption of nondurables and services then this would be consistent with defining money as M_1 , based on evidence from the 1980's.¹⁵

The results given in the bottom panel of Table 2 are based on the assumption that the relevant period over which individuals are constrained

¹⁵ This conclusion is based on the fact that the ratio of the stock of M_1 to quarterly consumption of nondurables and services has been close to one since the late 1970's. Unfortunately, this result does not hold over a long period of time--the ratio has been as high as 3 early in the postwar period. The same caveat applies to the observation concerning the monetary base made below.

to hold money is a month. It turns out that monthly consumption of non-durables and services corresponds roughly to the monetary base during the 1980's. The steady states in this second panel were computed using different parameter values for the discount factor and depreciation rate of capital in order maintain comparability to the quarterly results. The values assigned were $\beta = .997$ and $\delta = .008$, which are the monthly rates that correspond to the quarterly rates assumed above. We also scale the production function to reflect monthly output levels by multiplying the right hand side of equation (2.9) by $1/3$. The values for the growth rate of the money supply (g) that correspond to the desired annual inflation rates are also different for the monthly model. We indicate these values in the table.

The welfare measure we use is based on the increase in consumption that an individual would require to be as well off as under the Pareto optimal allocation. The Pareto optimal allocation for our economy is equivalent to the equilibrium allocation for the same economy without the cash-in-advance constraint, or, equivalently, for a version of the model where the money supply grows at a rate such that the cash-in-advance constraint is never binding. It turns out that for the model studied in this paper, the cash-in-advance constraint is not binding if the growth rate of money is equal to the discount factor, β .¹⁶ To obtain a measure of the welfare loss associated with growth rates that are larger than β , we solve for ΔC in the equation

¹⁶ We restrict the growth rate, g , to be greater than or equal to β . This insures that nominal interest rates will not be negative (see Lucas and Stokey (1987)). When we set $g=\beta$, the initial price level is no longer uniquely determined. However, the real allocation and rate of inflation are uniquely determined and the allocation is Pareto optimal.

$$(4.1) \quad \bar{U} = [\ln(C^* + \Delta C) - 2.86H^*] ,$$

where \bar{U} is the level of utility attained (in the steady state) under the Pareto optimal allocation ($g=\beta$), and C^* and H^* are the steady state consumption and hours associated with the growth rate in question (some $g>\beta$).

The results of the welfare calculations expressed as a percent of steady state real output ($\Delta C/Y$) and steady state real consumption ($\Delta C/C$) are shown in the bottom rows of both panels of Table 2. The welfare cost of a moderate (10%) inflation is .387 % of GNP when the period over which individuals are constrained is a quarter. This magnitude may be compared to the estimates of .3 % provided by Stanley Fischer or .45% obtained by Robert Lucas based on an approximation of the area under a money demand function.¹⁷ It is interesting that their exercise, which holds output constant but allows velocity to vary, yields the same answer as our exercise which holds velocity constant but allows output to vary. While an estimate of roughly .4% of GNP sounds small, at current levels of GNP it would amount to \$15.2 billion of real GNP. The welfare costs of very high inflation rates which are not uncommon throughout the world seem extremely high.

If the relevant period over which individuals are constrained is a month then the welfare costs are considerably reduced being only 0.11% at a 10% annual inflation rate and slightly more than 1.5% at a 400% annual inflation rate. Evidently the period over which individuals are constrained, and by implication the definition of the money balances on which individuals are taxed, makes a big difference in the welfare costs of inflation.

¹⁷ Fischer and Lucas use different definitions of money (high powered money and M1 respectively) and different estimates of the interest elasticity.

Since there is a big difference in the estimates it is worth considering what some of the biases might be. Our larger estimates come from assuming that individuals are constrained for one quarter, which is roughly consistent with assuming that the appropriate monetary aggregate is M1. However, a large part of M1 consists of checkable deposits. To the extent that these earn competitive interest they will be shielded from the inflation tax. At the other extreme, the monetary base consists of currency and reserves. Since these are clearly subject to the inflation tax, the monthly data provides a lower bound on the magnitude of the welfare loss. It seems reasonable that in economies with sustained high inflations many individuals will be able to shield themselves against the inflation tax. If the institutions did not exist to facilitate this, one would expect them to evolve in very high inflation economies. For this reason, our model may not be very reliable for analyzing hyperinflation. On the other hand these estimates abstract from many of the subsidiary costs of inflation that are believed to be important. Among these are distortions caused by non-neutralities in the tax system and adjustment costs or confusion caused by the variability of inflation.

4.3 Steady State Implications of Inflation

As shown in Table 2, anticipated inflation has a significant influence on the steady state path of the economy. Steady state consumption, output, hours, investment and the capital stock are all lower whenever the growth rate of the money supply exceeds the optimal level ($g-\beta$). The consumption of leisure increases because agents substitute this "credit good" for the consumption good in the face of a positive inflation tax on the latter.

Lower hours worked leads to lower output and therefore lower consumption, investment and capital stock. The share of output allocated to investment does not change with higher inflation. This result is obtained despite the fact that consumption is a cash good and investment is a credit good since, in the steady state, investment will provide consumption in the future that will be subject to exactly the same inflation tax as consumption today.

A striking implication of higher inflation rates in our model economy is that they are accompanied by lower employment rates.¹⁸ The "menu of choices" available to the monetary authority involves giving up low inflation only to obtain higher unemployment. This result, that the operational Phillips curve is upward sloping, is also obtained by Greenwood and Huffman (1987) for their model economy. Milton Friedman (1977) in his Nobel lecture presented some evidence for this phenomenon by plotting data from several countries. Here we present some statistical evidence that supports the negative correlation between employment rates and inflation rates using a cross section of countries.

Figure 1 shows the relation between the average rate of employment and the average rate of inflation from 1976 to 1985 for 23 countries.¹⁹ The solid line depicts the regression of employment rates on inflation rates. There is a statistically significant negative correlation between inflation

¹⁸ The variable HOURS in Table 2, which corresponds to per capita hours worked, is actually the employment rate multiplied by a constant (h_0), given the assumption of indivisible labor.

¹⁹ The countries are Austria, Belgium, Denmark, Finland, France, W. Germany, Greece, Ireland, Italy, Netherlands, Norway, Portugal, Spain, Sweden, Switzerland, UK, Canada, U.S., Australia, New Zealand, Japan, Chile and Venezuela. Population data are taken from Summers and Heston (1988) and the remainder of the data are taken from the International Labor Office (1987).

rates and employment rates. The coefficient of the inflation rate in a regression of the employment rate on the inflation rate and a constant is $-.5$ with a standard error of $.17$. The most extreme observation in the graph pertains to Chile. When that is eliminated the conclusions are essentially unchanged; the coefficient of inflation is $-.44$ with a standard error of $.22$. These results provide striking confirmation of the phenomenon displayed in our model economy.

5. Conclusions

In this paper we incorporate an interesting paradigm for money holding, the cash-in-advance model, in a stochastic optimal growth model with an endogenous labor leisure decision. We have shown that the solution and simulation of such a model is quite tractable. The model and solution procedure provide a basis for studying the influence of inflation on the path of the real economy and its cyclical characteristics. In addition, the solution procedure we have used could be employed to study the affects of other distortions as well.

We have used this model as the basis for estimating the welfare cost of the inflation tax and studying the long run features of economies with different inflation rates. The fact that our estimates are well within the range of estimates obtained by other methods and the empirical implications are confirmed in cross sectional data is very encouraging. This suggests to us that the approximations and simplifications we have made in writing down a tractable model of a competitive economy incorporating money may not be too serious. This is not to argue that econometric estimation of many of the parameters we have simply specified might not yield further insights

into these problems. What we find appealing about this approach is that all the features of the economy, from the decision rules to the specification of technology and preferences are explicit. Nothing is hidden. This makes it a valuable environment for experimental exercises like those considered here, and for positive exercises, for example where one would model the behavior of the monetary authority.

Although we have shown that anticipated inflation can have significant affects on the long run values of real variables, our model economy predicts that the business cycle will be the same in a high inflation economy as in a low inflation economy. When money is supplied erratically, the characteristics of the business cycle are altered somewhat. These changes in the characteristics of the cycle occur solely because of changes in allocations that result from the changing conditional expectation of inflation. Unexpected inflation has no role in this model. However, we speculate that the most important influence of money on short run fluctuations are likely to stem from the influence of the money supply process on expectations of relative prices, as in the natural rate literature. That is, if money does have a significant effect on the characteristics of the cycle it is likely to come about because the behavior of the monetary authority has serious informational consequences for private agents.

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TABLE 1

Standard Deviations in Percent and Correlations with Output for
U.S. and Artificial Economics

| <u>Series</u> | Quarterly U.S. Time Series* (1955.3-1984.1) | | Economy with Constant** Growth Rate (\bar{g} -.99-1.15) | |
|---------------|--|--------------------------------|---|--------------------------------|
| | <u>Standard Deviation</u> | <u>Correlation with Output</u> | <u>Standard Deviation</u> | <u>Correlation with Output</u> |
| Output | 1.74 | 1.00 | 1.76 (0.22) | 1.00 (0.00) |
| Consumption | 0.81 | 0.65 | 0.51 (0.07) | 0.87 (0.02) |
| Investment | 8.45 | 0.91 | 5.71 (0.74) | 0.99 (0.00) |
| Capital Stock | 0.38 | 0.28 | 0.48 (0.09) | 0.07 (0.07) |
| Hours | 1.41 | 0.86 | 1.34 (0.18) | 0.98 (0.00) |
| Productivity | 0.89 | 0.59 | 0.51 (0.07) | 0.87 (0.03) |
| Price Level | (CPI) 1.59 | -0.48 | | |
| | (GNP Deflator) 0.98 | -0.53 | 0.51 (0.07) | -0.87 (0.02) |

TABLE 1 (Cont.)

| Economy with Autoregressive Growth Rate ($\bar{g} = 1.015$)** | | Economy with Autoregressive Growth Rate ($\bar{g} = 1.15$)** | |
|--|------------------------------------|---|------------------------------------|
| <u>Standard Deviation</u> | <u>Correlation with Output</u> | <u>Standard Deviation</u> | <u>Correlation with Output</u> |
| 1.73 (0.22) | 1.00 (0.00) | 1.74 (0.22) | 1.00 (0.00) |
| 0.62 (0.07) | 0.72 (0.07) | 0.65 (0.07) | 0.70 (0.05) |
| 5.69 (0.76) | 0.97 (0.01) | 5.69 (0.77) | 0.97 (0.01) |
| 0.48 (0.10) | 0.06 (0.07) | 0.48 (0.10) | 0.06 (0.06) |
| 1.33 (0.17) | 0.98 (0.01) | 1.33 (0.17) | 0.98 (0.01) |
| 0.50 (0.07) | 0.87 (0.03) | 0.50 (0.07) | 0.87 (0.03) |
| 1.70 (0.34) | -0.27 (0.16) | 1.93 (0.27) | -0.25 (0.16) |

*The U.S. time series reported on are real GNP, consumption of nondurables and services plus the flow of services from durables, gross private domestic investment (all in 1982 dollars). The capital stock series includes nonresidential equipment and structures, residential structures and government capital. The hours series is total hours for persons at work in nonagricultural industries as derived from the Current Population Survey. Productivity is output divided by hours. All series are seasonally adjusted, logged and detrended. The output, investment, hours and price level series were taken from the Citibase database. The consumption and capital stock series were provided by Larry Christiano.

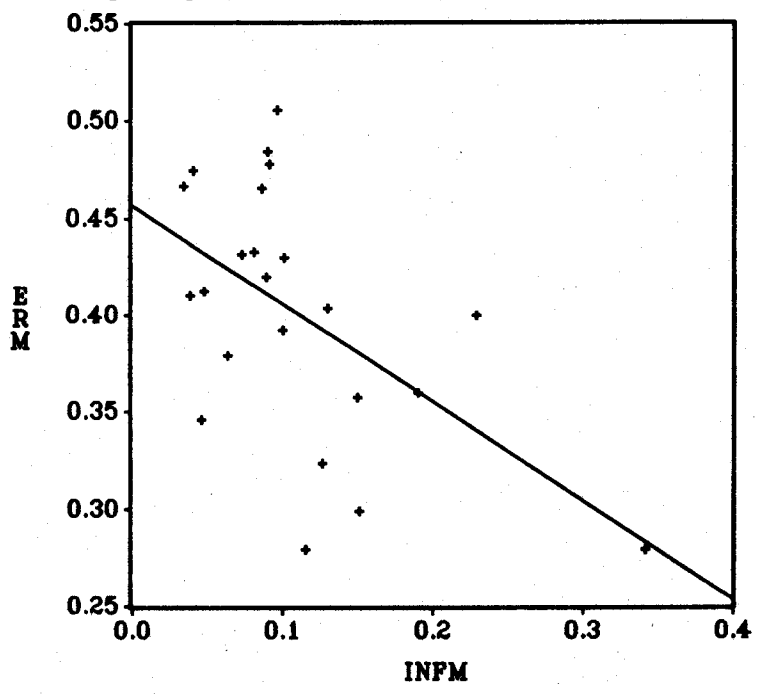
**The percent standard deviations and correlations with output are sample means of statistics computed for each of fifty simulations. Each simulation is 115 periods long, which is the same number of periods as the U.S. sample. The sample standard deviations of these statistics are in parentheses. Each simulated time series was logged and detrended using the same procedure applied to the U.S. sample before the statistics were calculated.

TABLE 2

Steady States and Welfare Costs Associated with Various Annual
Growth Rates of Money

| | | <u>Annual Inflation Rate</u> | | | | |
|-----------------------------|-------------------------|------------------------------|--------|--------|--------|--------|
| | | -4% | 0.0 | 10% | 100% | 400% |
| <u>Quarterly Constraint</u> | | | | | | |
| | $g - \beta$ | 1.0 | 1.024 | 1.19 | 1.41 | |
| Steady State: | Output | 1.115 | 1.104 | 1.077 | 0.927 | 0.783 |
| | Consumption | 0.829 | 0.821 | 0.801 | 0.690 | 0.582 |
| | Investment | 0.286 | 0.283 | 0.276 | 0.238 | 0.201 |
| | Capital Stock | 11.432 | 11.318 | 11.053 | 9.511 | 8.027 |
| | Hours | 0.301 | 0.298 | 0.291 | 0.250 | 0.211 |
| Welfare Costs: | $\Delta C/C \times 100$ | 0.0 | 0.144 | 0.520 | 4.014 | 10.215 |
| | $\Delta C/Y \times 100$ | 0.0 | 0.107 | 0.387 | 2.984 | 7.596 |
| <u>Monthly Constraint</u> | | | | | | |
| | $g - \beta$ | 1.0 | 1.008 | 1.06 | 1.12 | |
| Steady State: | Output | 0.387 | 0.386 | 0.383 | 0.364 | 0.345 |
| | Consumption | 0.286 | 0.285 | 0.283 | 0.269 | 0.255 |
| | Investment | 0.101 | 0.101 | 0.100 | 0.095 | 0.090 |
| | Capital Stock | 12.663 | 12.624 | 12.524 | 11.910 | 11.272 |
| | Hours | 0.303 | 0.302 | 0.300 | 0.285 | 0.270 |
| Welfare Costs: | $\Delta C/C \times 100$ | 0.0 | 0.040 | 0.152 | 0.981 | 2.137 |
| | $\Delta C/Y \times 100$ | 0.0 | 0.030 | 0.112 | 0.724 | 1.578 |

Figure 1
Average Employment and Inflation Rates 1976-1985



Appendix

DECISION RULES FOR SELECTED CASES

Constant Growth Rate

$$\bar{g} = .99$$

$$\hat{P} = 1.84778 + -0.56736 Z + -0.05610 K$$

$$X = 0.66517 + 1.77463 Z + -0.03318 K$$

$$\bar{g} = 1.00$$

$$\hat{P} = 1.86644 + -0.57309 Z + -0.05724 K$$

$$X = 0.65852 + 1.75688 Z + -0.03318 K$$

$$\bar{g} = 1.15$$

$$\hat{P} = 2.14634 + -0.65911 Z + -0.07569 K$$

$$X = 0.57260 + 1.52768 Z + -0.03318 K$$

Autoregressive Growth Rate

$$\bar{g} = 1.015$$

$$\hat{P} = 1.88633 + -0.58175 Z + 0.55474 \log g + -0.05898 K$$

$$X = 0.64419 + 1.73073 Z + 0.30219 \log g + -0.03318 K$$

$$\bar{g} = 1.15$$

$$\hat{P} = 2.07319 + -0.66585 Z + 0.63537 \log g + -0.07726 K$$

$$X = 0.52716 + 1.51216 Z + 0.26423 \log g + -0.03318 K$$