EXCHANGEABILITY AND THE STRUCTURE OF THE ECONOMY: A PRELIMINARY PROCESS ANALYSIS*

by

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Abstract

This approach to business cycles requires that each agent in the economy be considered. The model begins with these atomic entities -- consumers, firms, and their manifold relations. The basic economic concepts are contracts and the no-arbitrage condition. The entire economy is unified by exchangeability and martingale analysis. From the design of information producing experiments through the spatial distribution of firms and industries, the formation and dissolution of contracts, the hiring of inputs, the production and distribution of output, advertising, the measure and control of fluctuating heterogeneity afflicting all of these processes (this entails the establishment of control limits on the process performance, with the cost of misclassification determining how refined these limits should be) -- all of these activities are studied with an assortment of exchangeable birth and death processes. Stopping times and information are the foundations for this approach.

The methodology is exemplified by a queuing network model of the labor market.

Both firms and individuals are viewed as collections of stochastic contracts where the formation of contracts is the major action of these decisionmaking entities. Equivalently, each agent in the economy is the node of an enormous network composed of relations (implicit and explicit contracts) among the economy's decisionmakers. It is these oscillating networks that comprise the real "business cycle".

1. Introduction

The economy is partitioned into a segmented mosaic of communicating sectors comprising the "industrial" structure. The structure is all-encompassing in that each member of the society belongs to one or more of these partitions. Industries include the typical industries like steel, automobile, etc., but also the less conventional industries: retirement, homekeeping, education from pre-school to post graduate, prisons, and sanitariums.

Each of these industries is composed of firms; a firm is partitioned into teams and each worker belongs to one or more teams.

The firm is a collection of stochastic contracts designed to convert inputs into outputs. For simplicity assume that each firm produces a single output and takes prices as given in both the product and factor markets. The essence of the firm is the rearrangement of inputs into a commodity whose value to buyers (consumers and/or other firms) exceeds its cost. The firm increases production until the "no arbitrage" boundary is hit. Production beyond this critical boundary yields negative returns. The firm is contractually linked with its customers and with its factors of production. We assume that both explicit and implicit contracts are governed by a simple birth and death process. The explicit contracts are legally binding as long as the performance of both parties remains in the "acceptable range" agreed to by both parties. The contract may be terminated whenever the performance of one or both parties lies outside this range.[1]

Similar types of controls are placed on the production process. Whenever a control limit is pierced the process is stopped and searched

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[1]The contractual model by Telser (1980) is similar to the one envisaged here. A complete analysis is presented in Lippman and McCall (1988).

until the faulty mechanism is discovered and fixed. There is an enormous literature on quality control and boundary-hitting processes.[2] The recent literature on the so-called "disorder problem" contains several semimartingale models that are compatible with the structure of our model.

Information flow is the core of the economic process. The success of matchmaking between firms and factors of production[3] hinges on the amount and quality of the continuing information available to each agent. The firm's ability to compete and survive requires knowledge of the best technology for rearranging inputs, information about the consumers' fluctuating utility functions, and the transmission of information about its product to consumers by some mechanism like advertising. Initially the environment is competitive and the information flows in the price mechanism perform many of these on-going coordinating tasks.

The model is spatial as well as temporal. The mobility of firms and individuals is crucial to the efficient operation of the economy. The de Finetti-Savage exchangeability analysis has been generalized to spatial processes by Pitman (1978).[4] Spatial mobility also is based on birth-and-death processes. Integration of the spatial and temporal motion of the economy probably will require simulation. A key feature of this approach is its modularity. One can focus on the formation of contracts, the advertising process, the labor market in a single industry or the flow of resources across particular regions. The same methods can study the hierarchical

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[2] Siegmund (1985) is an excellent survey of these control-limit models.

[3] Mortensen (1986) is a splendid survey of the search literature.

[4] Kinderman and Snell (1980) is a lucid account of this spatial analysis. McCall and McCall (1987) applies multi-armed bandit methods and point processes to the migration decision.

decisions in organizations. Estimation and control of these processes are straightforward.

Three different stochastic partial orderings are important. The first is the mean preserving spread (majorization) usually called a dilation in function space. This is an exchangeable entity easily converted into an operator.[5] The dilation ordering is used to choose among alternative "experiments", while the dilation operator identifies and estimates the model. Rearrangement is the second stochastic ordering. It has a variety of uses, but we only apply it to characterize the production technology. This is a novel application which may require modification. Finally, one of the methods for linking the various production components of the model -- employees, teams, firms and industries -- is the weakness-by-failure ordering described by Norros (1985). This dynamic ordering introduces positive dependence among components and is tailor-made to fit a spatial-temporal-hierarchical analysis based on exchangeability.

Heterogeneity is treated explicitly. Indeed, the dilation is the natural estimator of diversity. There are differences among agents at a moment of time so that one must partition to achieve homogeneity (conditional independence). In addition, agents move among partitions over time <u>and</u> the partitions themselves are susceptible to change. Insurance companies have confronted these problems for centuries and their methods are applied to this problem. The recent research by Speed, Lauritzen, et al. on contingency table analysis also is pertinent for characterizing the heterogeneity process.

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[5]The details are presented in McCall (1988a).

The following section contains a description of birth and death processes and a rather crude analogy of the stochastic model. The latter illustrates a host of analogous processes (usually hydrodynamic like Fisher's (1892)) that are not only equivalent to one another, but also to Brownian motion. They belong to probabilistic potential theory. A rather detailed description of the labor market network is the subject of Section 3, with queuing network theory being applied to labor mobility. Several estimators of economic processes are described. The concluding section summarizes the current status of the model, indicates future research and relates the model to some of the burgeoning research on this topic.

2. Birth and Death Processes and a Mechanical Analogy

Birth and Death Processes

The seminal research on birth and death processes (BDP) was by Karlin and McGregor. In a series of articles they proved that the Stieltjes moment problem has the same structure as BDP. Furthermore, if λ_n and μ_n are the birth and death rates then the process can be represented by a sequence of orthogonal polynomials $\{Q_n(x)\}$ such that

$$Q_0(x) = 1, \; Q_1(x) = (\lambda_0+\mu_0-x)/\lambda_0;$$

$$-xQ_n(x) \;]\; \lambda_n Q_{n+1}(x) + \mu_n Q_{n-1}(x) - (\lambda_n+\mu_n)Q_n(x), \qquad n > 0.$$

The process also produces the random walk polynomials defined by:

$$R_0(x) = 1, \; R_1(x) = x/B_0,$$

$$xR_n(x) = B_n R_{n+1}(x) + D_n R_{n-1}(x),$$

where $B_n = \dfrac{\lambda_n}{\mu_n+\lambda_n}$ and $D_n = 1 - B_n$.

By altering λ_n and μ_n the BDP is equivalent to the Laguerre, Meixner, Charlier, and Gegenbauer polynomials. These polynomials can be represented by urn models.

The Stietjes moment problem is to determine the sequence of numbers representing the sequential moments:

$$m^k = \int y^k d\mu(y), \quad k = 0,1,\ldots,$$

with μ a measure on the half-line. If the measure is supported on an interval, the Hausdorff moment problem emerges.[6] It is well-known that Hausdorff <=> de Finetti (see Feller (1971)).

The moment problems have unique solutions iff the appropriate quadratic form is positive definite. Caratheodory, Schur, et al. obtained integral representations for the convex families of analytical functions corresponding to these moment problems. Krein applied Chebyshev analysis that Karlin and Shapley exploited. The integral representations were generalized by Krein-Milman (Hewitt-Savage used this methodology in their broad generalization of the exchangeability theorem) and then by Choquet. The birth and death process is one of the simplest Markov processes, has been studied intensively, and is probably the most useful stochastic process.

Jackson queuing networks, cooperative and non-cooperative stochastic contracts, Markov decision processes, counting processes, and their martingale representation, population processes and their equilibrium properties, spatial, temporal and hierarchical versions of these processes, their linear programming, and general equilibrium counterparts all have been analyzed using a variety of techniques with operator theory and sophisticated

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[6]Two other famous moment problems are Hamburger's (power) problem and the trigonometric moment problem.

simulations the most recent.

Being semimartingales, birth and death processes preserve their basic properties under aggregation and decomposition; they are easily estimated, optimized, and their ergodic properties are well known as is their behavior in fluctuating environments with heterogeneity. In short, they appear to be appropriate for the economic analysis proposed here.

Historical Background and a Mechanical Analogy

Irving Fisher (1867-1947) was the student of J. Williard Gibbs (1839-1903). Gibbs' study of thermodynamics led to the development of statistical mechanics and provided the foundations for quantum mechanics. Gibbs had a profound influence on Fisher's intellectual development which is manifested clearly in his dissertation. Implicit in this extraordinary thesis is a coherent business cycle theory. It is contained in Fisher's hydrodynamic analogy of consumer behavior.

Fisher's interest in the theory and practice of hydrodynamics was interpreted by most economists as merely another sign of a highly eccentric personality. Actually, it was another dimension of his genius -- a dimension that invented a lucrative card-sorting machine and that eventually led to the co-founding of Remington Rand! If this approach had been applied to stochastic economic fluctuations -- clearly a Gibbsian implication -- it could have generated a stochastic dynamic macroeconomic theory superior in both elegance and usefulness to most modern macroeconomic models.[7]

In modeling business cycles the use of Fourier analysis has been almost irresistible. Statistically, the spectral decomposition of stochastic time

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[7] In a remarkable paper, Samuelson (1952) develops a spatial general equilibrium model exploiting network analogies, linear rogramming and graph theory.

series is quite sensible. Here the procedure is reversed.[8] Rather than concentrating on the aggregate time series, the time series that reflect the stochastic economic decisionmaking of consumers and firms is paramount.

Consider the following mechanical analogy of the economic process.[9] The economy is composed of two giant box spring mattresses, one on top of the other. The lower mattress represents the consumer; the upper represents the industrial sector. Each spring in the lower mattress depicts a consumer, whereas each coil in the industrial sector corresponds to a firm. The consumer coil is composed of a network of smaller springs. Each of these is a stochastic contract or relation that the individual has with other members of the economy. The size of the consumer coil fluctuates as new relationships are formed and old ones terminated. Thus each consumer can be viewed as a vibrating network of relations. There are other birth and death processes determining the number of consumers in the economy and their distribution among various subsectors that are deemed important by the analyst. Each consumer continuously modifies his portfolio of contracts in response to the changing environment. His actions are guided by the no arbitrage condition -- he continues to add and delete contracts until no further net benefits are obtainable. In this general setting his tastes and

[8] A penetrating comparison of dynamic exchangeability/martingale analysis with classical Fourier methods is presented in Kallenberg (1982). Of course, exchangeability martingales, and Fourier analysis are closely related. Nevertheless, the Box-Jenkins procedures adopted by most economists tend to be "static".

[9] An entirely different and ingenious approach has been taken by Grandmont (1985). He models macrobehavior as a <u>deterministic</u> system of nonlinear differential equations. With the passage of time, these equations can evolve in such a way that after some critical point T, it may be impossible to distinguish these deterministic paths from stochastic diffusions. However, Brock, et al. have shown that under certain conditions, it is possible to recover the parameters of the initial system from the seemingly chaotic behavior.

preferences are also random variables and interact with the other environment oscillations so that at any time, the no arbitrage condition isnever satisfied across all nodes.

Each firm is also a vibrating network of contracts. Many of these contractual arrangements are implicit.[10] For example, the firm's contract with an employee is multidimensional. The worker's performance is monitored by the manager and/or by the other member of his production team. His performance may fluctuate according to a semimartingale Markov process composed of both Brownian motion and Poisson jumps. If he can control the drift parameter of the Brownian motion, but has little influence on the jump process, the firm will adjust for this when deciding whether to continue or terminate the contract. The employee reacts in similar fashion to the vicissitudes of the firm, remaining at the firm when an unexpected decline in demand occurs. These implicit arrangements will resolve the moral hazard/adverse selection problem as best they can in this uncertain environment.[11] Of course, mistakes will be made with probability one, just as the eventual bankruptcy of the firm is an almost sure event. Similar arrangements will characterize the contracts with buyers and other suppliers of inputs. The firm is also continuously adjusting its portfolio of contracts in response to the myriad uncertainties that it must face. These include fluctuating demand and technical change, some of which occurs because of its own research and development activities. Once again, it is the no arbitrage condition that drives these responses. The industry is composed of similar

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[10] Rosen (1985) contains an excellent survey of the conventional implicit contract.

[11] For a fine analysis of intertemporal incentives in a stochastic environment, see Holmstrom and Milgrom (1985).

firms (oscillators). The industrial sector is a box spring mattress, with each spring an industry. Some springs tend to vibrate in unison, so that the box spring is a set of industrial groups. Left on its own, the industrial sector would exhibit groups that were growing rapidly, groups of relatively constant growth, and declining group. External shocks may not only alter the interactions among the fluctuating industries, but also influence their overall course.

The joint response of the consumer sector and industrial sector to endogenous and exogenous shocks generates the aggregate semimartingale process, usually called the business cycle and measured by a time series of GNP, employment, or unemployment.

The price and wage distributions depend on the precise form of the contractual relations between firms and factors of production and between firms and buyers. For example, there also may be implicit agreements between firms and groups of buyers such that prices remain within a certain band unless some extraordinary event occurs.[12] The price distributions obviously are influenced by the rate of technological change, the rate of preference change, and a host of other exogenous shocks many of which are induced by government policy.

3. A Network Approach to Labor Market Mobility

It is difficult to underestimate the growth of research and innovations in network theory during the past ten years. The impetus for this extraordinary development resides in the concurrent flourishing of computer and communication technology, artificial intelligence, neurological research,

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[12] The advantages accruing to these customer arrangements are described by Alchian.

and the realization that point processes (infinite particle systems) can be applied to areas as diverse as reliability theory, queuing networks, neurobiology and statistical mechanics.

Network analysis is based on exchangeable Markov processes, martingales, and point processes. The purpose of this section is to present recent innovations in these fields that seem especially pertinent to the production process. Results, extensions and conjectures will all be stated without proof. Our primary purpose is to alert economists to this vast body of seemingly relevant methods. The technical details will be presented in a subsequent paper.

We begin with a brief survey of the network queuing literature.[13] Apparently Erlang (1917) and Engset (1918) were the first to apply network theory to a practical problem (telephone exchanges). There is no doubt though that the key network papers were composed by J.R. Jackson (1957, 1963). They comprise the seminal node from which the subsequent literature proliferated.

As will be evident, there is no single mathematical technique for studying queuing networks. On the contrary, there are few branches that are not included. The major modes of analysis are Markov processes, renewal processes, point processes, semimartingales, diffusion processes, and closely allied deterministic methods developed by Ford and Fulkerson in the 1950s and 1960s and continuing through the recent operator analysis by Massey (1984a,b, 1985).

Typically, a queue is represented by $M|M|S$, where the first M denotes an exponential (Poisson) arrival rate, the second M represents an

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[13] This is based on the excellent survey by Disney and Konig (1985).

exponential service time, and S denotes the number of servers. More generally, the arrival process is composed of i.i.d. interarrival times, a renewal process denoted by a GI-arrival process. If the service times are i.i.d. rv's it is called a GI-service process. The service and arrival processes are assumed to be independent. There are a number of subsidiary stochastic processes associated with this general queuing model.

Let $N(t)$ be the number of customers (reparables) in the queuing network -- those waiting for repair and those being repaired. The stochastic process $\{N(t), t \leq 0\}$ is the queue length process at time t. This is distinguished from the process confronted by the m^{th} arrival at the point, t_m^a. The sequence of values of the queue length process at these distinguished points (embedded points) is called the embedded process. The waiting time process yields a similar duality: the waiting time of the m^{th} arriving part until service is initiated and the time a part would wait if it arrived at an arbitrary point. The latter is called the virtual waiting time. The actual waiting times comprise the embedded process of the continuous time virtual process.

The total time consumed by a part from entry until repair is the sojourn time process $\{S_m, m = 1,2,\ldots\}$.

The output process of the servers is the sequence, $\{t_m^o, m = 1,2,\ldots\}$, of service completion times, t_m^o.

Finally, if some parts arrive at a facility with a "long" waiting time they may be diverted to another repair station. This gives rise to a loss process.

A queuing network is composed of a finite number of these simple service systems. Each system is characterized by nodes connected by arbitrary arcs. The parts move across the arcs. Parts can arrive at a node from outside or inside the system. A switching process is a set of rules that governs the motion of the parts among the nodes from entry to exit. The entire ensemble is called a queuing network. If a queuing network possesses a distribution that is independent of t or m, this is called an equilibrium distribution. The queuing network is a vector-valued stochastic process, $N = (N_1(t),\ldots,N_J(t))$, where each component is the queue length of node j at time t. Disney and König call this the queue length process of the network. If part m moves through the system with stops at nodes $1,\ldots,\ell$ then $W_m = W_{m1} + \ldots + W_{m\ell}$ is its total waiting time. The network sojourn time is defined similarly. The two basic types of network are open and closed. In the open network parts enter the system from several external sources. They all eventually leave the network. In the closed system there are no arrivals nor departures. Each of these systems is a Markov process.

Reversibility

A Markov process $N(t)$ is called time reversible if the process $N(-t)$ has the same finite dimensional distributions as $N(t)$. The time independent probability distribution μ of $N(t)$ is called the equilibrium or invariant measure of the network. Let Q be the generator of the process, that is, $Q(x,y)$ is the rate at which transitions occur between x and y. Then μ is the solution to:

(1) $$\sum_{x \in S} \mu(x)Q(x,y) = 0, \quad y \in S.$$

When the network is time reversible this equilibrium condition becomes:

(2) $$\mu(x)Q(x,y) = \mu(y)Q(y,x), \quad x,y \in S.$$

Here S denotes the state space.[14]

In a repair process the state $N = (N_1, N_2, \ldots, N_J)$ is the number of parts at each repair station. The transition rate from station i to station k has the form $\phi_i(N_i)\lambda_{ik}$, so that the flow may depend on the number of parts at i. The process is time reversible in equilibrium only if $\lambda_{ik} = \lambda_{ki}$. The invariance measure μ is the product measure given that the total number of parts is constant. If ϕ is linear the system has a multinomial equilibrium distribution.

The Jackson Product Theorem

Let Q_t be a Jackson open network. There is a unique equilibrium distribution iff the unique solution of the flow-conservation equations satisfies the light flow condition:

$$\frac{\lambda_i}{\mu_i S_i} < 1, \quad \text{for all } i.$$

The equilibrium distribution $(p(n), n \in S)$ has the product form:

$$p(n) = \prod_{i=1}^{m} p(n_i, \lambda_i, \mu_i, S_i).$$

Thus, the random variables $Q_t(i)$, all i, are mutually independent.

Each repair facility i has an equilibrium distribution identical to that of the $M|M|S_i$ queue with parameters λ_i and μ_i.

Martingales and Queuing Networks

Two important martingales are associated with the birth and death Markov process, Q_t. Suppose the number Q_t of parts in a network at time t is a birth and death random variable with parameters λ_n and μ_n and $E[Q_o] < \infty$.

[14] The first complete discussion of this important concept is contained in Kelly (1979). It is important to note that reversibility is an equilibrium concept.

Also assume that $\Sigma_{n=0}^{\infty} E[\int_0^t \lambda_n P(Q_s=n)ds] < \infty$, $\forall t \geq 0$. Then, both

$$A_t - \int_0^t \lambda_{Q_s} ds \quad \text{and} \quad D_t - \int_0^t \mu_{Q_s} 1(Q_s > 0)$$

are martingales relative to the appropriate probability measure and sigma-algebra. The latter contains all of the pertinent history of the process up to time t. The integral in each of these martingales is called the compensator.

Let L_t be the size of the labor force at time t, where L_0 is the fixed "size" of the firm at $t = 0$. The size of labor force L_t may be decomposed:

$$L_t = A_t - D_t + L_0$$

The sequence of arrival [departure] times is: $(\alpha_1,\ldots,\alpha_a)[\delta_1,\ldots,\delta_d]$, with $a \equiv A_t$ and $d \equiv D_t$. Suppose $\lambda_n = \lambda$ and $\mu_n = \mu$. The likelihood function associated with these $a+d$ arrival and departure times is:

$$\mathcal{L}(\alpha_1,\ldots,\alpha_a;\delta_1,\ldots,\delta_d) = \lambda^a \mu^d \exp[-(\lambda+\mu) \textstyle\int_0^t L_s ds].$$

This likelihood and its various forms are the basis for inference.[15]

An Open Network

The Reiman open network is composed of random arrival processes for identical parts from outside the network to one of K teams or work centers, a random work length within the team, and an assignment mechanism, whereby the part moves among different teams until it eventually leaves the network. Parts are stored while they await assignment to one of the K teams. The parts are moved among teams as their defects are perceived and

[15] An excellent discussion of point processes is contained in Bremaud (1981).

in response to changes in demand. Thus each team is receiving parts that have just entered the firm and also from the K-1 other teams. Reiman (1984) shows that his stochastic network converges weakly to Brownian motion provided the flow is heavy.

Whitt (1984) performed a sensitivity analysis using mixtures of exponential distributions for both the interarrival times and service times. In particular, the mean equilibrium queue length for the $GI|G|1$ queue was approximated, given the first two moments of these mixed exponential distributions. The analysis is based on the Karlin-Studdin methodology.

Whitt presents the following example. Suppose the interarrival distribution has moments $m_1 = 2.0$, $m_2 = 12.0$, and $m_3 = 119.0$. The upper and lower limits (when $\rho = 2/3$) of mean equilibrium queue rate were $\sigma_u = .777$ and $\sigma_\ell = .67$. Including the 3^{rd} moment m_3 changes σ_ℓ to .767, narrowing the bound considerably.

The main point is not the specifics of the analysis, but rather the fact that a sensitivity study like this can be conducted. As the environment changes, so will these extremes and so too must the response of the decisionmaker. Preparations for alternative extremes can be made and the extremes can be altered by changing policies.

Optimizing the Jackson Network

Whittle (1986) also presents a technique for calculating an optimal policy for the Jackson network. The minimization occurs at the design stage and achieves the least cost for the network in equilibrium by manipulating the routing matrix subject to capacity constraints. Whittle also studies an adaptive policy for the Jackson network. These methods are very promising and probably can be linked with the martingale analysis.

An Operator Approach to Queuing Networks

In a series of papers Massey (1984a,b, 1985) has developed a very promising operator methodology for extending the analysis of queuing networks. His methods can accommodate nonstationarity, transience, and alternative queuing disciplines. It appears that dilation theory can streamline these methods and estimate network characteristics. The paper by Lemoine (1986) is also very important for the applications we are contemplating.

The Labor Market as a Queuing Network

The labor market is a hierarchical system composed of industries, firms, teams and the basic employer-employee contract. An individual employee moves among industries, among firms in a given industry, and among teams in a given firm. The motion is both temporal and spatial. Each level of the hierarchy has a search component. Hence, there is a search industry, a search firm and a search team. This network encompasses the entire economy, so it contains a retirement industry, an education industry and a residual industry containing all those individuals who for one reason or another are not members of those industries already cited. The network can be composed so that it is a reversible process (Markov chain) and yet does not violate the institutional and empirical constraints that are critical to the economic analysis. Hence a realistic birth and death process can describe the motion of a labor market and produce an equilibrium distribution.

For reversible Markov chains, Aldous (1982) showed that the following statements are equivalent:

(i) convergence to equilibrium is rapid

(ii) mean hitting times on single states are almost uniform in the initial state

(iii) mean hitting times on a set A of states can be bounded using the stationary measure on A.

Suppose that the general Jackson model suffices for the problem at hand. There are several important economic issues that can be addressed:

Labor Mobility in a Fluctuating Environment

There has been considerable study of random walks in randomly fluctuating environments. These random matrix methods can be incorporated into the birth and death analysis without violating the conditions of the Jackson-type network. The response of industries, firms, and employees to realistic environmental fluctuations could yield significant insights into the economics of oscillations.

The Moment Problem

The moment problem also can be studied in the labor market setting. The moment problem is based on the fundamental research by Karlin, Krein and Shapley. Simply stated: the moment problem is to characterize a probability measure on $\tilde{R} \subset R$ possessing s specified moments such that the expected value of some criterion function on $\tilde{R}$ is minimized.

A simple version of the moment problem is: let Z be a random variable on the closed interval $[\underline{a},\bar{a}]$ and suppose the n moments of Z are $\{m_0,m_1,\ldots,m_{n-1}\}$, where $m_i = E[T_i(X)]$, $i = 0,\ldots,n-1$ and $\{T_0,\ldots,T_{n-1}\}$ comprises a Tchebyshev system.[16] For a function g on $[\underline{a},\bar{a}]$ calculate the upper and lower bounds on $E[g(X)]$. In the network model, we may wish to calculate upper and lower bounds on τ, the time to reach equilibrium or upper and lower bounds on the birth and death processes possessing the same equilibrium distribution. Similar calculations can be made for extreme value

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[16] For a discussion of these, see Karlin and Studdin (1967).

distributions. These calculations are extraordinarily useful for "preparedness decisionmaking". The individual firm and industry must determine how much of this "flexibility insurance" to purchase, but at least they have a guide for calculating its upper and lower values.

The most general moment calculation entails the dilation operator, which was introduced by Halmos and possesses the "symmetric dilation" property. In particular, an operator $\tilde{T}$ in a Hilbert space $\tilde{H}$ is a dilation of the operator T in the Hilbert space H if $\tilde{H} \supset H$ for $x \in$ domT, $\tilde{P}\tilde{T}x = Tx$, where $\tilde{P}$ is the orthogonal projection of $\tilde{H}$ onto H. Thus the dilation operator resembles the (symmetric) U-statistic also discovered by Halmos. An excellent discussion of the dilation operator is in Halmos (1982). The U-statistic will be encountered shortly. I am indebted to W. Brock for introducing me to U. The dilation operator also has been used to solve the basic problem of system sciences: the realization problem.

An Index of Poverty Mobility

In the recent literature on poverty, various measures have been suggested for assessing escape probabilities. The following extremal index appears attractive. Let $s_p, s_{p-1}, \ldots, s_0$ denote the $p+1$ poverty states, $s_p > s_{p-1} > \ldots > s_0 = 0$. For simplicity collapse these $p+1$ states into a single state i. Now let $L_i(T)$ be the maximum time until the individual leaves i in the time interval $(0,T]$. The asymptotic distribution of $L_i(T)$ is double exponential, that is,

$$\lim P[(-q_{ii})L_i(T) - \log(-q_{ii}\pi_i) - \log T \leq z] =$$

$$\exp(e^{-z}), \quad z \in R, \tag{1}$$

where Q is the intensity matrix of the Markov chain and π_i is the steady-state probability of state i.

A stayer-in-poverty would be defined as one who remains in poverty for a period of time that is some simple function of (1).

The Marginal Problem and Simpson's Paradox

The analysis of labor markets frequently is based on information about marginal probability distributions without specifying the joint d.f. This is the source of the notorious Simpson's paradox. The problem is that knowing n marginal distributions is not sufficient information for identifying the joint distribution. The marginal problem is: given specified distributions on $R_1, R_2, \ldots, R_n$ characterize a joint distribution on $R_1 \times R_2 \times \ldots \times R_n$ such that the expected value of a criterion function on R^n is minimized. In principle, this is solved easily by linear programming methods. However, practicing econometricians will not find these LP results of much use when there is (and there always is) fluctuating heterogeneity. In these more complicated circumstances, a dilation operator/realization analysis appears attractive.

Segmented Labor Markets (Phelps Islands)

There is a large literature on the rigidity that pervades a society's labor market. Unless certain entry requirements are satisfied, it is impossible to reach a preferred set of occupations. Formal representation of this segmentation is achieved by postulating a Markov process with a discrete state space S. Let s and $s' \in S$ and denote the transition from s to s' be given by the intensity $\Lambda(s,s')$. The balance equations for the equilibrium distribution $p(s)$ are:

$$(*) \qquad \sum_{s' \in S} [p(s')\Lambda(s',s) - p(s)\Lambda(s,s')] = 0.$$

The states are positive recurrent and s decomposes into n irreducible subsets s(b), where b is the label attached to the ergodic class containing s. Let B denote the set of values of b. Then (*) decomposes into the following partial balance equations:

$$\sum_{s' \in S(b')} [\beta(s')\Lambda(s',s) - p(s)\Lambda(s,s')], \quad b' \in B.$$

Take the n irreducible sets and calculate the equilibrium distribution for each. This is the standard version of the segmented labor market -- no transitions are allowed among the segmented markets. Now suppose that this assumption is relaxed by designating a mobility state within each segmented class, such that motion (in both directions) is permitted between this state and a state outside the segmented market. Whittle refers to this opening of a previously closed network as a relaxation.

4. A Basic Methodology For Appraising The Performance Of The Economic Process

Much of economic theory entails large samples for estimating key parameters and distinguishing among alternative conjectures. When convergence occurs with sufficient speed, asymptotic methods can be used to check the validity of the economic theory. Since the theory is expressed as an economic process, it is important that econometrics be transformed to estimators that are stochastic processes.

When feasible, the process methodology will have the following form.

1. The economic theory is expressed as an economic process. Any sequence of random variables can be regarded as a stochastic process, so this first crucial step should be relatively simple. Since all economic activities occur over time, our instincts impel us to design statistical

procedures that explicitly incorporate time, namely, process estimators.

2. Many economic theories are expressible as exchangeable processes. The second step is to derive the asymptotic estimators associated with the particular process. During the last ten years probabilists have applied invariance principles to obtain the asymptotic properties of random functions of random sequences associated with the underlying economic process. This theory began by demonstrating that a random function of a random walk process converges to Brownian motion. The most desirable results are based on the strong invariance principle and yield almost sure bounds on the approximation error made by some conventional random function. Perhaps of wider applicability in economics is the weak invariance principle. This method yields convergence in distribution. The most significant feature of both methods is that the limiting behavior of functionals of the economic process can be derived from the limiting behavior of the random function. The best reference for weak invariance methods is Billingsley (1968), whereas the more complex strong invariance results due to Strassen and utilizing Skorokhod embedding is described clearly in Shorack and Wellner (1986).

3. Many functionals of the economic process can be calculated from these invariance principles or functional limit theorems. The functional that is most appealing in process economics is the first passage time, that is, the first time that the functional, which presumably would be related to profits if firm behavior was being studied, crosses some critical barrier or boundary $b^*(t)$. This first passage time is expressed by:

$$\inf\{t: B(t) > b^*(t)\}.$$

4. An innovative feature of the analysis is the sequential nature of the econometric process. As long as $B(t)$ remains below $b^*(t)$ or within some critical band, the economic process is deemed "satisfactory". However, just as in quality control when a critical barrier is pierced, the alarm is sounded and the process is modified in an appropriate manner. For example, the production process may be consuming too many resources or producing a product with an unacceptable failure rate or the subpopulation being insured by an insurance company may by crossing a barrier revealing an unacceptable level of heterogeneity in which case additional decompositions are necessary.

In many applications the boundary crossing methodology can be applied directly rather than using functional limit approximations. The techniques are those most suitable for the detection of a disorderly economic process.

The distinguishing feature of this appraisal of economic processes is the smooth transition from economic analysis to stochastic process to statistical decision theory to estimation process to control. Economics, statistics, and operations research are all participants in this process analysis. The behavior of the process is judged with respect to its final output. Hence, to say that a particular statistic is optimal makes little sense in this process environment. Optimality judgments must be reserved for the performance of the entire process.

5. Conclusion

Perhaps the most appealing feature of the exchangeability/martingale business cycles is the variety of functionals that can be calculated and evaluated for alternative policies. For example, altering the employment contract will influence the time until an individual quits or is laid off.

A technical improvement in the production process will affect the average queue length and the expected inventory level. This functional analysis is analogous to comparative statics in deterministic models.[17]

The effect of alternative policies on stationary measures (equilibrium distributions) is also easily measured in this stochastic model. Suppose that unemployment is the process being analyzed and the process is recurrent. Then there is a stationary distribution. The steady-state proportion of individuals in each class, unemployed for m months, $m = 1,2,3,\ldots$ can be calculated. This estimated distribution would correspond to the equilibrium distribution of unemployment. Given the stochastic structure of the system, equilibrium migration distributions and equilibrium layoff distributions, and equilibrium bankruptcy distributions, also can be estimated.

The modularity of the approach implies that the analysis of aggregate economic behavior, industry behavior, firm behavior, or some component of the production process like the fluctuations of equipment failures, the number of stock-outs or the queue length will be similar. Of course, if one wishes to study aggregate responses to stochastic shocks at a fairly detailed level, that is, measure its effect on some component of the firm's production function or its influence on the consumer's purchases of a particular commodity, then a simulation may be the most efficient mode of analysis. Since no model like this has ever been constructed, simulation also would have great pedagogical value for assessing the model and suggesting modifications. Each firm maximizes its own welfare, but there may be institutional or informational problems that prevent it from

[17] An excellent discussion of this functional analysis with many pertinent examples is contained in Karlin and Taylor (1975, 1981).

generating positive and negative externalities. This is also true of our utility maximizing consumers. These externalities may be difficult to isolate in an analytical study, but they should be apparent in a simulation.

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