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VOTING ON THE BUDGET DEFICIT

Abstract

This paper analyzes a model in which different rational individuals vote over the composition and time profile of public spending. Potential disagreement between current and future majorities generates instability in the social choice function that aggregates individual preferences. In equilibrium a majority of the voters may favor a budget deficit. The size of the deficit under majority rule tends to be larger the greater is the polarization between current and potential future majorities. The paper also shows that a balanced budget is ex-ante efficient. A balanced budget amendment, however, is not durable under majority rule.

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1. INTRODUCTION

Opinion polls show that American voters are well aware of the Federal budget deficit, and disapprove of it. However, there are clear indications that it is politically very difficult to reach an agreement about how to balance the budget. In particular, several polls show that even though voters dislike deficits, they are not in favor of any specific measure which would reduce them.¹

Two explanations for this apparent inconsistency of opinions are commonly proposed. One is that voters do not understand the concept of budget constraint, and suffer from "fiscal illusion". However, it is quite difficult to reconcile this notion with the standard assumption of individual rationality.² The other is that disagreement amongst voters generates cycling and prevents the existence of a stable majority in favor of balancing the budget. As a result, individual preferences about intertemporal fiscal policy cannot be aggregated in a consistent way, and no action can be taken to balance the budget. However, this argument fails to explain why the absence of a stable political equilibrium should result in a budget deficit, rather than in a surplus or a balanced budget: in principle, it seems that any outcome could be observed if the political equilibrium is indeterminate.

This paper provides an alternative explanation of budget deficits which is not based on either individual irrationality or non-existence of equilibrium. The central ingredient of our explanation is the inability of current voters to bind the choices of future voters. This inability, coupled with potential disagreement between current and future majorities, introduces a time inconsistency in the dynamic social choice problem that determines the size of budget deficits: the policies desired by the current majority would
not be carried out if future majorities exhibit different preferences. The awareness of this possibility may induce the current majority to run a budget deficit in excess of what would be ex-ante optimal for society as a whole. This explains why it is hard to agree on how to eliminate deficits, even if there is a consensus that they may be socially sub-optimal.

Our results have a simple economic intuition. Consider a rational voter who is presented with a number of options on how much to spend in the current period, and over what items. His vote is not just an abstract vote on the intertemporal profile of spending. It is also a vote on how to dispose of the resources acquired through a debt issue (or lost through a surplus). Suppose that there is uncertainty about the future composition of public spending, because the identity of future majorities is still unknown. Then there is a fundamental asymmetry: whereas the majority who runs a budget deficit also chooses how to allocate the debt proceeds, the allocation of the burden of repaying the debt is not under its control. Under appropriate conditions this asymmetry prevents the current majority from fully internalizing the costs of budget deficits, the more so the greater is the difference between its preferences and those of the likely future majority.

The paper also shows that if this asymmetry is removed, and the vote on the deficit is taken behind a "veil of ignorance" on how the debt proceeds are spent, then the voters are unanimous in choosing a balanced-budget. That is, in this model a balanced budget is ex-ante efficient. This implies that current voters would like to precommit future governments to a balanced budget rule. However, no current majority wants the rule to be binding on itself. Thus, a balanced-budget rule is enforceable only if a qualified majority is required to abrogate it. This constraint may imply a sub-
optimal lack of flexibility in reacting to unexpected events. Therefore, in this situation as in many others, society has to choose on the tradeoff between rules and discretion.

Our results are related to those of other papers on intertemporal politico-economic models of fiscal policy. In particular, Alesina-Tabellini (1987a,b) and Tabellini (1987) analyze a general equilibrium model in which two ideologically motivated parties randomly alternate in office and disagree on the optimal composition of public spending, or on the level of transfers and taxation of different constituencies. Persson-Svensson (1987) consider a government who is certain to be replaced in the future by a successor which wants to expand the size of public spending. In these papers as in the present one, public debt is a strategic variable used by the current policymaker to influence the actions of future policymakers. In this earlier literature, however, either the political equilibrium is exogenously given (as in Persson-Svensson (1987)), or voters have to chose between two alternative policies presented to them by two ideological candidates with fixed positions (as in Alesina-Tabellini (1987a) and Tabellini (1987)). In the latter case, the equilibrium is such that both candidates choose the same deficit, even though they choose a different composition of government spending. Thus, in effect in these papers voters do not have a choice on the deficit. In particular, the question remains of whether the equilibrium deficit would disappear if a "middle ground" political entrepreneur who promises to balance the budget enters the political arena. The present paper improves the characterization of the political equilibrium by assuming that there are no constraints on the policy options available to the voters. Any proposal can be voted upon in a pairwise comparison, and the voters directly vote on the size of the deficit. This paper also
improves on the existing literature by providing more precise positive implications and by evaluating the normative properties of the political equilibrium (which the previous literature failed to do).

The idea that state variables can be used to influence future voting outcomes can provide important insights in understanding other public choice problems, besides those concerning budget deficits. For example, Glazer (1987) exploits this insight to investigate the choice of durability in public capital projects. He shows that uncertainty about future voting outcomes generates a bias towards overinvesting in long run projects. Several other potentially fruitful applications of this idea come to mind, such as to privatization decisions or to defense policy issues.

Finally, it should be noted that our argument is completely different from the idea that deficits occur because the current generation does not internalize the costs of taxing future generations: in our model everybody has the same time horizon. In an overlapping generations model with no bequests, on the other hand, current voters would be unanimously in favor of the largest possible budget deficit, so as to redistribute the income of future generations towards themselves. In such a model, the equilibrium would always be a corner solution and the size of budget deficits would be determined exclusively by the borrowing capacity of the government.3

The rest of the paper is organized as follows. Section 2 describes the basic model. The political equilibrium is computed in Section 3. Section 4 discusses the normative and positive implications of these results for the issue of balance budget amendments. The last section provides a brief summary and suggests some extensions.
2. THE MODEL

A group of heterogeneous individuals has to decide by majority rule on the consumption of two public goods, \( g \) and \( f \). The group is endowed with one unit of output in each period, and it can borrow or lend to the rest of the world at a given real interest rate, for notational simplicity and no loss of generality taken to be 0. The world lasts two periods, and all debt outstanding has to be repaid in full at the end of the second period. Thus, the group faces the intertemporal constraint:

\[ g_1 + f_1 - b \leq 1 \]  
\[ f_2 + g_2 + b \leq 1 \]

(1a)

(1b)

where subscripts denote time periods and \( b \) denotes debt. In addition, the non-negativity constraints hold: \( g_i, f_i \geq 0, \ i = 1,2 \). Hence, (1b) immediately implies \( b \leq 1 \). Throughout the paper we assume that in equilibrium \( b < 1 \). The extension to the case in which \( b = 1 \) in equilibrium is straightforward, and just involves some changes in notation. At the beginning of each period, the group votes on how much to consume of each public good in that period. Thus, the group cannot precommit in period 1 to choosing a particular quantity of \( g_2 \) and \( f_2 \) for the following period.

The preferences of the \( i^{th} \) member of the group are:

\[ W^i = E\left\{ \sum_{t=1}^{2} [\alpha^t u(g_t) + (1-\alpha^t)u(f_t)] \right\} \]

(2)

where \( u(\cdot) \) is concave, strictly increasing, twice continuously differentiable, and satisfies the Inada condition: \( u'(0) \rightarrow \infty \). \( E(\cdot) \) denotes the expectation operator. With no loss of generality, we assume that voters do not discount the future; thus the rate of time preference is equal to the world real interest rate. This eliminates any incentive to borrow or lend
other than those which are the explicit focus of this paper.

The parameter $\alpha_i$ which identifies voter $i$ is distributed over the [0,1] interval. With only a minor change in notation, all the results can be extended to allow for values of $\alpha_i$ greater than 1 or negative, in which case one of the public goods would create disutility for individual $i$.

This specification of individual preferences allows for disagreement about which proportion of the two public goods to consume. However, it assumes some homogeneity amongst the individuals in the group. Specifically, it implies that individual preferences belong to the class of "intermediate preferences" defined by Grandmont (1978). This class has the following useful property: individual preferences are indexed by the parameter $\alpha_i$ and the distribution of preferences within the group is fully summarized by the distribution of $\alpha_i$. As shown by Grandmont (1978), since $\alpha_i$ is a scalar, it follows that preferences are single peaked and the median voter result applies: provided that all policy options can be compared, and that they are voted on pairwise, the group decisions under majority rule coincide with the most preferred policy of the individual corresponding to the median value of $\alpha$, denoted $\alpha^m$. Thus, the political equilibrium can be computed by solving the problem of maximizing (2) subject to (1), with $\alpha_i = \alpha^m$ in (2).

A crucial feature of the model is that even though individual preferences remain stable over time, the identity of the median voter need not be the same in periods 1 and 2. Thus in period 1 there may be uncertainty about the identity (and hence the preferences) of the median voter in period 2. This is the reason for having the expectations operator in equation (2). Changes in the identity of the median voter over time may be due to: (1) random shocks to the costs of voting that affect the
participation rate (see Ledyard (1984) for a formalization of this idea); or (ii) changes in the eligibility of the voting population (for instance, because of minimum age requirements, or because of geographical movements of the population). The extent to which these events change the median voter's preferences, in turn, depends on the underlying distribution of individual preferences. If the latter are tightly concentrated around a unimodal distribution, then the preferences of the median voter would tend to be relatively stable over time. But on the other hand, if the underlying distribution of individual preferences within society is polarized around very different values of $\alpha$, then relatively "small" shocks of the above mentioned type (i) and (ii) can result in large changes in the median voter preferences. This point is discussed more at length in subsection 3.4 below.

This simple setup can be interpreted as a stylized version of several richer models. The most direct interpretation is that of a "club" with a fixed endowment to be allocated to different uses. With only some minor changes, the club can be interpreted as a society in which taxes are fixed and individual economic agents have access to a linear storage technology. In an interior equilibrium, the real rate of interest on public debt has to equal the technologically given rate of return on storage. Alternatively, the model can be interpreted as applying to a small open economy having access to international capital markets, again with given taxes. In this context, government debt can be either domestic or external. The extension to a model with endogenous distortionary taxation significantly increases the complexity of the analysis, without qualitatively changing the basic message of this paper. Alesina-Tabellini (1987a) illustrate this extension in a model with a much simpler political structure, in which the policy is
chosen by the government and the voters essentially do not vote on the budget deficit.

3. POLITICAL EQUILIBRIUM

3.1 The Last Period

Consider the last period, and let $\alpha_2^m$ denote the value of $\alpha^i$ corresponding to the median voter in period 2. Two cases are possible, depending on the value of $\alpha_2^m$.

If $1 > \alpha_2^m > 0$, then the median voter is at an interior optimum. In this case, his choices satisfy the following first order condition:

$$\alpha_2^m u'(g_2^m) - (1-\alpha_2^m) u'(1-b-g_2^m) = 0$$  \hspace{1cm} (3)

Equations (3) and (1b) implicitly define the equilibrium values $g_2^m$ and $f_2^m$ as a function of $\alpha_2^m$ and $b$. Let us indicate these functions as $g_2^m = G(\alpha_2^m, b)$ and $f_2^m = 1 - b - g_2^m = F(\alpha_2^m, b)$. Applying the implicit function theorem to (3) and (1b), it can be shown that, for $1 > \alpha_2^m > 0$, $G_\alpha = -F_\alpha > 0$, $-1 < G_b < 0$ and $-1 < F_b < 0$, where $G_\alpha$, $G_b$, $F_\alpha$ and $F_b$ denote the partial derivative of $G(\cdot)$ and $F(\cdot)$ with respect to $\alpha_2^m$ and $b$ respectively.

For future reference, this interior optimum is illustrated in the diagram of Figure 1. The downward sloping line denotes the opportunity set faced by the median voter in period 2, as a function of the debt inherited from the past, $b$. The equilibrium in period 2 occurs at point $E$, where the median voter's indifference curve is tangent to his budget line. The upwards sloping line $EP_2$ is the median voter's income expansion path. It traces out the equilibrium combinations of $g_2$ and $f_2$ as $b$ varies. The income expansion path is not necessarily linear: Its slope can be shown to equal $R(g_2^m)/R(f_2^m)$, where $R(\cdot) = -u''(\cdot)/u'(\cdot)$ is the coefficient of
FIGURE 1
absolute risk aversion of \( u(\cdot) \). Thus, with decreasing absolute risk aversion \((R' < 0)\), the slope of the income expansion path is greater than 1 if point \( E \) lies above the 45° line (i.e., if \( \alpha^m_2 < \beta \), as in Figure 1); it is smaller than 1 if \( E \) lies below the 45° line (i.e., if \( \alpha^m_2 > \beta \)).

Intuitively, the slope of the income expansion path reflects how the concavity of the indifference curves at the equilibrium point varies with income. If \( u(\cdot) \) has decreasing absolute risk aversion, then the indifference curves become flatter as income increases -- i.e., the two public goods become closer substitutes. This in turn implies that the tangency point \( E \) diverges away from the 45° line as income increases. The opposite holds if \( u(\cdot) \) has decreasing absolute risk aversion. Let us define a more (less) balanced composition of public spending as corresponding to a point more (less) distant from the 45° degree line in Figure 1. Then, except in the limit case of constant absolute risk aversion, the size of debt inherited from the past determines how balanced the composition of public spending is in period 2. In particular, in the more plausible case in which the two goods are closer substitutes at high levels of income (i.e., if \( R' < 0 \)) a larger value of \( b \) implies a more balanced composition of \( g^*_2 \) and \( f^*_2 \), for any given value of the median voter preferences (except in the case \( \alpha^m_2 = \beta \)). And conversely, a smaller value of \( b \) drives the equilibrium away from the 45° line, and hence brings about a more unbalanced composition of \( g^*_2 \) and \( f^*_2 \), for any given \( \alpha^m_2 = \beta \). This effect of public debt on the second period equilibrium composition of expenditures plays a major role in the next section, where the incentives to issue public debt in the first period are analyzed.

Finally, if instead \( \alpha^m_2 = 1 \) or \( \alpha^m_2 = 0 \), then the median voter of period 2 is at a corner. He sets \( g^*_2 = 1 - b \) and \( f^*_2 = 0 \) if \( \alpha^m_2 = 1 \); and
conversely he sets $g^*_2 = 0$ and $f^*_2 = 1 - b$ if $a^m_2 = 0$. Thus, if $a^m_2 = 1$, we also have $G_b = -1$ and $F_b = 0$, and if $a^m_2 = 0$ we have $G_b = 0$ and $F_b = -1$. In this case, the income expansion path is given by the horizontal or vertical axis of Figure 1, if $a^m_2 = 1$ or $a^m_2 = 0$ respectively.

3.2 The First Period: Preliminary Results

In period 1 there is uncertainty about the identity of the median voter of period 2. As a result, from the point of view of the voters in period 1, the parameter $a^m_2$ in (3) is a random variable. The policy most preferred by the median voter of period 1 (whose preferences are denoted by $a^m_1$) can be found by solving the following optimization problem, where $E(\cdot)$ is the expectation operator over the random variable $a^m_2$.

$$\max_{g_1, b} \left( a^m_1 u(g_1) + (1-a^m_1) u(1-g_1 + b) + E[a^m_1 u(G(a^m_2, b)) + E[a^m_2 u(F(a^m_2, b))] \right)$$

The current median voter maximizes an expected utility function, since he is aware that, in the subsequent period, $g_2$ and $f_2$ will be chosen by a median voter with possibly different preferences. The expectation operator is taken with respect to all possible types of future median voters, knowing how each type would behave. Thus, today's median voter chooses the value of the state variable $b$ taking into account how his choice influences the policies chosen by future median voters, based on the distribution of the future preference parameter, $a^m_2$.

The median voter of period 1 makes two choices: he chooses the composition of public goods in period 1 and the amount of government borrowing (lending). If $1 > a^m_1 > 0$, the first order condition relative to $g_1$ is:
\[ a_1^m u'(g_1) - (1-a_1^m)u'(1+b-g_1) = 0 \] (5)

Equation (5) implicitly defines the optimal values \( g_1^* \) and \( f_1^* \) as a function of \( a_1^m \) and \( b \): \( g_1^* = g(a_1^m, b) \), \( f_1^* = f(a_1^m, b) \). Using the same notation as before, it can be shown that, for \( 1 > a_1^m > 0 \), \( 1 > g_b > 0 \) and \( f_b = 1 - g_b \). If instead \( a_1^m = 1 \) (or \( a_1^m = 0 \)), then the median voter in period 1 is at a corner and chooses respectively \( f_1^* = 0 \) and \( g_1^* = 1 + b \) (or \( g_1^* = 0 \) and \( f_1^* = 1 + b \)).

The intertemporal choice is described by the first order condition of problem (4) relative to \( b \), which, for \( b < 1 \), is:

\[ a_1^m u'(g(a_1^m, b)) + E[a_1^m u'(G(a_2^m, b))G_b + (1-a_1^m)u'(F(a_2^m, b))F_b] = 0 \] (6)

where it is understood that \( G_b \) and \( F_b \) are functions of \( a_2^m \) and \( b \). Despite the concavity of \( u(\cdot) \), the second order conditions are not satisfied unless an additional, very mild, condition is imposed. We assume throughout the paper that such a condition is always satisfied for any value of \( a_2^m \) and \( a_1^m \).

The interpretation of (6) is straightforward. The first term on the left hand side is the marginal gain of issuing one more unit of debt; at the optimum, this must coincide with the marginal utility of spending one extra unit on either of the two public goods (good \( g \) in (6)). The second term of (6) is the expected marginal disutility of having to repay the debt, by curtailing public spending tomorrow. This in turn is computed by taking into account that the future composition of public spending depends on the random parameter \( a_2^m \). The solution to equation (6) determines the equilibrium value of debt, \( b^* \), chosen by the median voter in period 1.

In order to assess the sign of \( b^* \), in the next subsection we consider equation (6) at the point \( b = 0 \). If at this point equation (6) is satis-
fied, then \( b^* = 0 \). If instead at \( b = 0 \) the left hand side of (6) is positive, then by the second order condition we know that in equilibrium \( b^* > 0 \). And conversely, if at \( b = 0 \) the left hand side of (6) is negative, then the second order conditions imply \( b^* < 0 \).

3.3 The Equilibrium Level of Debt

Consider first the case in which the median voter at time 1 is certain that he will also be the median voter in period 2 (i.e., if \( \alpha_1^m = \alpha_2^m \) with certainty). The second term in (6) reduces to \( \alpha_1^m u'(\theta(\alpha_2^m, b)) \), so that \( b^* = 0 \) is the only solution to (6) for any value of \( \alpha_1^m \). This should come as no surprise: since its rate of time discount coincides with the real interest rate (they are both zero), in the absence of political uncertainty the median voter chooses to spend an equal aggregate amount in both periods. It is easy to show that \( b^* = 0 \) is also the policy that would be chosen by a benevolent social planner maximizing a weighted sum of individual utilities, for any choice of weights in the planner's objectives. Thus, with no uncertainty and no disagreement between current and future majorities, the political equilibrium lies on the Pareto frontier.\(^7\)

The remainder of this section investigates the case in which \( \alpha_2^m \neq \alpha_1^m \) with positive probability. It is convenient to break down the second term on the left hand side of (6) into the weighted average of two conditional expectations: the expectation conditional on the event that \( 1 > \alpha_2^m > 0 \); and the expectation conditional on the event that \( \alpha_2^m = 1 \) or \( \alpha_2^m = 0 \).

Although special, the second case provides the simplest illustration of why political uncertainty creates incentives to issue public debt. Hence, we consider it first. In this case future median voters are expected to be at a corner, so that they produce only one kind of public good: only \( g_2 \) if \( \alpha_2^m = 1 \), and only \( f_2 \) if \( \alpha_2^m = 0 \). If \( \alpha_2^m \neq \alpha_1^m \) with positive
probability, we have:

Proposition 1: If either \( \alpha_2^m = 0 \) or \( \alpha_2^m = 1 \), then \( b^* > 0 \). Moreover, \( b^* \) tends to be greater the larger is the difference between \( \alpha_1^m \) and the expected value of \( \alpha_2^m \).

Proof: Let \( \alpha_2^m = 1 \) with probability \( \pi \) and \( \alpha_2^m = 0 \) with probability \( 1 - \pi, \ 1 > \pi > 0 \). Then, using (5), equation (6) can be rewritten as:

\[
\alpha_1^m u'(g) - \tilde{\alpha}u'(1-b) = (1-\alpha_1^m) u'(f) - \tilde{\alpha}u'(1-b) = 0
\]

where \( \tilde{\alpha} = \alpha_1^m \pi + (1-\pi)(1-\alpha_1^m) \). Clearly, \( \tilde{\alpha} \leq \text{Max}(\alpha_1^m, (1-\alpha_1^m)) \), with strict inequality if \( \alpha_1^m \neq 1 \). Moreover, at the point \( b = 0 \), \( u'(1-b) \leq u'(g(\alpha_1^m, b)) \) and \( u'(1-b) \leq u'(f(\alpha_1^m, b)) \), with strict inequality if \( 1 > \alpha_1^m > 0 \). Hence, at the point \( b = 0 \) the two left hand sides of (7) are always strictly positive. As argued above, by the second order conditions this implies that \( b^* > 0 \).

In order to prove the second part of the Proposition, note that the expected value of \( \alpha_2^m \) here is just \( \pi \). Fix \( \alpha_1^m \), and consider \( b^* \) as a function of \( \pi \). We have:

\[
\frac{db^*}{d\pi} = \frac{db^*}{d\tilde{\alpha}} \cdot \frac{d\tilde{\alpha}}{d\pi} = \frac{db^*}{d\tilde{\alpha}} (2\alpha_1^m - 1).
\]

Applying the implicit function theorem to (7), we obtain that \( db^*/d\tilde{\alpha} < 0 \).

Hence,

\[
\frac{db^*}{d\pi} > 0 \quad \text{as} \quad \alpha_1^m > 1/2
\]

Thus, if \( \alpha_1^m > 1/2 \), a lower value of \( \pi \) (a higher likelihood that \( \alpha_2^m = 0 \)) increases \( b^* \). And conversely, if \( \alpha_1^m < 1/2 \), a higher value of \( \pi \) (a higher likelihood that \( \alpha_2^m = 1 \)) also increases \( b^* \). Hence, \( b^* \) tends to be larger when the difference between \( \alpha_1^m \) and the expected value of \( \alpha_2^m \) is
greater. Q.E.D.

This result has a simple intuition. An increase in debt today implies a reduction of aggregate spending tomorrow. But since \( \alpha_2^m \) lies outside the (0,1) interval, tomorrow only one kind of public good will be provided. Hence, with positive probability (and with probability 1 if \( 1 > \alpha_1^m > 0 \)), this reduction of spending will affect only the good that will have a low marginal utility from the point of view of today's median voter. Thus, the median voter of period 1 does not fully internalize the cost of issuing debt: he finds it optimal to spend in excess of the current aggregate endowment. Moreover, this incentive to borrow is stronger the lower is the marginal utility of the future public good. This is more likely to happen if the future median voter is more likely to exhibit very different tastes from the current median voter. That is, if \( \alpha_1^m \) is large and the probability of having \( \alpha_2^m = 1 \) is small, or vice versa.

We now show that, under appropriate conditions, this basic intuition extends to the more general case in which \( \alpha_2^m \) lies in the open interval (0,1). With no loss of generality, suppose that over this interval \( \alpha_2^m \) is distributed according to a continuous probability function \( H(\cdot) \), where \( H(\alpha) = \text{prob}(\alpha_2^m \leq \alpha) \). Then (6) can be rewritten as:

\[
\int_0^1 [\alpha_1^m u'(g_1) - v(\alpha_2^m)] dH(\alpha_2^m) = 0
\]

(10)

where \( v(\alpha_2^m) \) is the marginal cost of repaying the debt, given that in period 2 the median voter tastes parameter is \( \alpha_2^m \). After some transformations we obtain:

\[
v(\alpha_2^m) = \frac{u'(g_2) u'(f_2) [\alpha_1^m \lambda(f_2) + (1-\alpha_1^m) \lambda(g_2)]}{u'(g_2) \lambda(g_2) + u'(f_2) \lambda(f_2)}
\]

(11)
where \( g_2^m = G(\alpha_2^m, b) \), \( f_2^m = F(\alpha_2^m, b) \), and where \( \lambda(\cdot) = -u''(\cdot)/[u'(\cdot)]^2 \) is the "concavity index" of \( u(\cdot) \) as in Debreu-Koopmans (1982).

We now assume that \( u(\cdot) \) has the following property. 8

The concavity index of \( u(x), \lambda(x) \), is decreasing in \( x \), for \( 1 > x > 0 \). (c)

That is, we assume that \( u(\cdot) \) becomes less concave in the sense of the index of Debreu-Koopmans (1982) as consumption increases. This hypothesis is more restrictive than decreasing absolute risk aversion: it implies that the coefficient of absolute risk aversion falls more rapidly than marginal utility as consumption increases. Nonetheless, this hypothesis is satisfied for several commonly used utility functions, such as any CES function \( u(x) = x^\gamma/\gamma \) with \( \gamma > 0 \).

The Appendix proves that, at the point \( b = 0 \), \( \alpha_1^m u'(f_1) - v(\alpha_2^m) > 0 \) for any \( \alpha_2^m \neq \alpha_1^m \) if \( u(\cdot) \) satisfies condition (c). Hence, under this condition, at the point \( b = 0 \) the marginal gain of issuing debt exceeds the corresponding expected marginal cost (i.e., the left hand side of (10) is strictly positive at the point \( b = 0 \)). Thus:

\[
\text{Proposition 2: Given that } \alpha_2^m \in (0,1), \ b^* > 0 \text{ if (c) holds}
\]

Next, let us define the probability distribution \( H(\alpha_2^m) \) as "more polarized relative to \( \alpha_1^m \)" then the distribution \( K(\alpha_2^m) \) if, for any continuous increasing function \( f(\cdot) \), the following condition is satisfied:

\[
\int_0^1 f(|\alpha_2^m - \alpha_1^m|) dH(\alpha_2^m) > \int_0^1 f(|\alpha_2^m - \alpha_1^m|) dK(\alpha_2^m)
\]

That is, a more polarized probability distribution assigns more weight to values of \( \alpha_2^m \) that are further apart from \( \alpha_1^m \). The Appendix also proves that, if condition (c) holds, then for any \( b > 0 \) the expression \( [\alpha_1^m u'(g_1^x) - v(\alpha_2^m)] \) is an increasing function of \( |\alpha_2^m - \alpha_1^m| \) (strictly increasing if
|α₂^m - α₁^m| > 0). Then, using (10) and appealing to the second order
conditions, we also have:

Proposition 3: Under the same condition of Proposition 2, b* is larger
the more polarized is the probability distribution of α₂^m relative to
α₁^m over the interval (0,1).

If the concavity index λ(x) is everywhere increasing (constant) for
1 > x > 0, then Propositions 2 and 3 hold in reverse: b* < 0 (b*=0), and
b* is more negative if H(α₂^m) is more polarized. If λ(x) is not mono-
tonc over 1 > x > 0, then the sign of b* is ambiguous.

The role played by condition (c) is highlighted in Figure 2. The
downward sloping line denotes the opportunity set faced by the median voters
in both periods if b = 0. A positive value of b shifts this line to the
right in period 1, and to the left in period 2. A and B denote the
points chosen in periods 1 and 2 by the median voters of type α₁^m and α₂^m
respectively, again for b = 0. For concreteness, it has been assumed that
α₁^m > h > α₂^m. The indifference curves for the median voter of type α₁^m in
periods 1 and 2 are labelled I and II respectively. Finally, the upwards
sloping lines EP₁ and EP₂ denote the income expansion paths of types α₁^m
and α₂^m. Their divergent slopes reflect the assumption of a decreasing
concavity index for u(·), which implies that the coefficient of absolute
risk aversion of u(·) is also decreasing. Hence, as noted in Section 3.1,
the two public goods become closer substitutes at higher levels of
consumption. As a result, the divergence between the choices of the two
types of median voter (points A and B) increases with income. To put it
differently, with a decreasing concavity index for u(·), disagreement
concerning the optimal composition of g and f is a luxury good: it
grows with the overall size of public spending. 10

The ambiguity of the sign of $b^*$ for $1 > \alpha^m_2 > 0$ is due to the opposite influence of two countervailing forces. By running a surplus (by setting $b < 0$), the median voter in period 1 moves $A$ to the left along $E_{P_1}$ and $B$ to the right along $E_{P_2}$; this has the effect of reducing the distance between the indifference curves labeled I and II. Hence, a surplus, $b < 0$, "buys insurance" for the median voter of period 1, since it tends to equalize the median voter's utility in the two periods. This is the force that works in the direction of making $b < 0$ more desirable.

On the other hand, by running a deficit (by setting $b > 0$), the median voter of period 1 moves $B$ to the left along $E_{P_2}$. Since the slopes of $E_{P_1}$ and $E_{P_1}$ are divergent, this has the effect of moving point $B$ closer to point $A$; that is, it moves the future composition of public spending towards the point that is preferred by today's median voter. This is the force that provides the incentive to issue public debt today.

Condition (c) guarantees that this second effect prevails over the first one. This condition is more likely to be satisfied if the slopes of $E_{P_1}$ and $E_{P_2}$ are very divergent from each other (that is, if the substitutability of $g$ and $f$ increases very rapidly with income); or if the indifference curves are very flat (that is if the utility function is not very concave), because in this case the indifference curves labeled I and II are already close to one another.

Combining the results of Propositions 1-3, we can conclude that a positive equilibrium level of debt occurs if: (i) the future median voter has extreme preferences and is at a corner (i.e., $\alpha^m_2 \notin (0,1)$); or (ii) condition (c) on $u(\cdot)$ is satisfied. Moreover, in both cases, the size of debt is larger the greeter is the probability mass assigned to
values of $\alpha_2^m$ that are very different from $\alpha_1^m$; that is, using the previous terminology, the more polarized is the distribution of the future median voter's preferences.

It is worth noting that, in a sense, (i) is the limit case of case (ii); if the future median voter is at a corner, then its income expansion path coincides with either the vertical or the horizontal axis. With reference to Figure 2, then, leaving some debt to the future has always the effect of moving the future composition of public spending in the desired direction, as in the less extreme case (ii).

In a more general model, the future median voter could find himself at a corner even for values of $\alpha_2^m$ belonging to the interior of the $[0,1]$ interval. For instance, this would happen if the utility function $u(\cdot)$ did not satisfy the Inada conditions, so that the indifference curve of Figure 1 could intersect either the horizontal or the vertical axis. Alternatively, it could happen if the public goods $g$ and $f$ had to be provided in some minimum positive amounts (for instance, because of survival reasons): in this case too, the future decisionmaker could find itself at a corner with respect to either $g_2$ or $f_2$, despite the fact that $1 > \alpha_2^m > 0$. The expectation of this event, in turn, would induce the current median voter to run a budget deficit, just like the case of Proposition 1. Obviously, this event would be more likely to happen the closer is $\alpha_2^m$ to either 0 or 1. 11

3.4 Positive Implications

Propositions 1-3 relate the size of budget deficits to the instability of the median voter's preferences over time. This type of instability depends upon the distribution of individual preferences within society. In the remainder of this section we show that the more "homogeneous" are the
preferences of different individuals, *coeteris paribus* the more stable are
the median voter preferences over time.

Consider a family of density functions indexed by $\epsilon$, $\gamma(\alpha, \epsilon)$. For a
given $\epsilon$, let $\gamma(\alpha, \epsilon)$ be the frequency distribution of $\alpha$ over the [0,1]
interval, where $\alpha$ is the parameter that summarizes the preferences of
specific individuals in equation (2). Different density functions are
associated with different values of $\epsilon$. Thus, $\epsilon$ can be thought of as a
perturbation of the distribution of the voters' preferences, associated with
random shocks to the voting participation or to the eligibility of the
voting population.

To any realization of $\epsilon$ is associated a value of the median voter's
preferences, $\alpha^m(\epsilon)$, defined implicitly by:

$$\int_0^\alpha \gamma(\alpha, \epsilon)d\alpha - \frac{1}{2} = 0 \quad (13)$$

The extent to which $\alpha^m$ varies as $\epsilon$ takes different values depends on the
properties of the density function $\gamma(\alpha, \epsilon)$. Specifically, applying the
implicit function theorem to (13) one obtains:

$$\frac{d\alpha^m}{d\epsilon} = -\frac{\int_0^\alpha \gamma(\alpha, \epsilon)d\alpha}{\gamma(\alpha^m, \epsilon)} \quad (14)$$

where $\gamma(\alpha, \epsilon) = \delta(\gamma)/\delta\epsilon$. The numerator of (14) is the area underneath the
density function that is shifted from one side to the other of $\alpha^m$ as $\epsilon$
varyes. According to (14), for a given value of the numerator, the term
$\frac{d\alpha^m}{d\epsilon}$ is larger in absolute value the smaller is $\gamma(\alpha^m, \epsilon)$. That is, if
there are relatively few individuals in the population that share the median
voter's preferences (i.e., if $\gamma(\alpha^m, \epsilon)$ is small for all $\epsilon$), then $\alpha^m$
tends to vary a lot as the distribution is perturbed by random shocks. Conversely, if the median voter preferences are representative of a large part of the population (i.e., if $\gamma(\alpha^m, \epsilon)$ is large), then $\alpha^m$ tends to be stable even in the face of large perturbations to the underlying distribution of voters preferences.

This result is illustrated in Figure 3. Consider the top distribution first. When $\epsilon$ goes from $\epsilon_0$ to $\epsilon_1$, a fraction of individuals corresponding to the area A is moved from the right to the left of $\alpha^m_0 = \alpha^m(\epsilon_0)$, to the area $A^1 = A$. This area is the numerator of (14). The new median voter, $\alpha^m_1 = \alpha^m(\epsilon_1)$, is found by equating the area between $\alpha^m_0$ and $\alpha^m_1$. B, to the area A. Consider now repeating the same perturbation to the distribution in the bottom of Figure 3. Clearly, the same area B now corresponds to a much larger horizontal distance between $\alpha^m_0$ and $\alpha^m_1$. since the frequency of the population around $\alpha^m$ is relatively small, the median voter's preferences here have to shift by much more than in the case of the top distribution of Figure 3. This is the sense in which a more polarized distribution of preferences (such as in the bottom of Figure 3) tends to be associated with more instability and more polarization in the induced probability distribution of the median voter's preferences.

These considerations are suggestive of an empirically testable interpretation for the results of Propositions 1-3. Namely, that more polarized and unstable societies tend to have larger deviations from budget balance than more homogeneous societies. In a more polarized and unstable political system there is a higher probability that future majorities will choose policies that are very different from those chosen by today's majority. According to Propositions 1-3, it is precisely in this case that the current majority is in favor of deviations from balanced budgets, and if
condition (c) holds these deviations are in the direction of budget
deficits. Moreover, according to Proposition 1, debt is more likely to be
issued when the future majorities are likely to have extreme preferences
(for then the likelihood of being at a corner is greater).

Finally, note that even condition (c) itself (which implies that debt
will always be issued in the presence of political uncertainty) can be
interpreted as an instance of polarization: if the utility function $u(\cdot)$
satisfies condition (c), then different types of median voters have diverg-
ent income expansion paths. In this case, current and future majorities
tend to choose different policies in the following sense: they allocate
changes in their budget (cuts or increments) to different items.
According to Proposition 2, this kind of "incremental" polarization always
creates incentives in favor of issuing public debt.

4. CONSTITUTIONAL CONSTRAINTS ON THE BUDGET DEFICIT

The previous results state that in equilibrium a majority of the voters
may be in favor of a budget deficit. This section investigates the
efficiency properties of this equilibrium.

Section 3.4 shows that a social planner certain of being reappointed
would always choose $b^* = 0$, for any weighting of individual preferences.
That is, a balanced budget is always a component of the first best policy.
On this ground, it is tempting to conclude that a budget deficit is
inefficient in this model. However, this argument would be misleading. By
assumption, a social planner can precommit to choosing the composition of
both periods 1 and 2 public goods according to a stable social welfare
function. This assumption is violated in the political equilibrium of the
model and in any real world political regime: the current majority cannot
precommit the spending choices of future majorities. In other words, the solution to the social planner's optimum is not necessarily the optimal social contract for a group of individuals who cannot also precommit the expenditure choices of future governments.

In order to characterize such an optimal social contract, we need to ask what is the optimal level of debt when there is uncertainty about the median voter preferences in both period 2 and period 1. That is, suppose that the decision concerning \( b \) is taken before choosing the composition of public spending in period 1, and under a "veil of ignorance" about the outcome of this choice. Following Rawls (1971) and Buchanan-Tullock (1962), we can think of a constitutional amendment on budget deficits as being chosen in this way. The optimal level of \( b \) for agent \( i \) is then determined as the solution to the following problem:

\[
\max_{b} \mathbb{E}[\alpha^i \{u(g(\alpha^m_1, b) + u(G(\alpha^m_2, b))) + (1-\alpha^i)[u(f(\alpha^m_1, b)) + u(F(\alpha^m_2, b))])
\]

(15)

where \( \mathbb{E} \) is the expectations operator with respect to the random variables \( \alpha^m_1 \) and \( \alpha^m_2 \), and where \( g(\cdot), f(\cdot), G(\cdot) \) and \( F(\cdot) \) are defined implicitly by (5) and (3) of the previous section. If \( \alpha^m_1 \) and \( \alpha^m_2 \) are drawn from the same prior distribution, then it is easy to show that the only solution to (15) is \( b = 0 \), for any value of \( \alpha^i \). Thus, using the terminology of Holmstrom-Myerson (1983), we can conclude that a balanced budget rule is "ex-ante efficient": before knowing the identity of the current majority, the group is unanimous in favoring a balanced budget amendment.\(^{12}\)

If, however, the value of \( \alpha^m_1 \) is known when choosing \( b \), then we are back in the equilibrium examined in the previous section, where a majority might support a deficit and oppose the balanced budget amendment. In other words, each current majority generally does not want to be bound by the
amendment, even though it wants such an amendment for all future majorities. Thus, such an amendment can be approved only if it does not bind the current majority. However, a budget amendment taking effect at some prespecified future date would be irrelevant: if one needs only a simple majority to abrogate the rule, then any future majority would follow the policy described in Section 3 and would abrogate the amendment. Using again the terminology of Holmstrom-Myerson (1983), we can conclude that a balanced budget amendment, though ex-ante efficient, is not "durable" under majority rule.

This problem could be overcome by requiring a qualified majority to abrogate the amendment. In our model with no exogenous uncertainty, the optimal qualified majority would be unanimity. But this requirement would eliminate the flexibility that may be needed to respond to unexpected and exceptional events. Obviously, a budget rule could be contingent on prespecified events, such as cyclical fluctuations of tax revenues or wars. However, since it is very difficult, or even impossible, to list all the relevant contingencies, it might be desirable to retain some degree of flexibility. Thus, requiring a very large majority to abandon (even temporarily) the budget balance constraint may be counterproductive. More generally, in a model with uncertainty and constraints on the degree of "complexity" of the rule, there would be an "optimal qualified majority" corresponding to the optimal point on the tradeoff between commitment and flexibility.

These normative results may contribute to explain why the majority of voters seem to generally favor an abstract notion of balanced budgets, even though when choosing specific policies the same majority may vote in favor of budget deficits (see the literature quoted in footnote 1). Balanced
budgets are ex-ante efficient. Therefore, the majority of voters, asked in a poll if they would like a balanced budget constitutional amendment, would answer "yes". However, the same majority of voters may choose to run a budget deficit in the current period, if uncertain about the preferences of future majorities.

More generally, these results suggest the desirability of institutions that would enable society to separate its intertemporal choices from decisions concerning the allocation of resources within any given period. In evaluating such institutions, there seems to be an inescapable conflict between the goal of preserving sufficient flexibility to meet unexpected contingencies, and the constraints imposed by the requirement of enforcing this separation. Thus, as in many other problems of macroeconomic policy design (such as monetary policy), society has to choose between simple rules and discretion. Interestingly, in the case of budget deficits in the U.S. this conflict has been resolved in different ways at the federal and state government levels. Whereas the federal government and legislature have retained essentially full discretion in their borrowing policies, the constitution of most states in the U.S. forbids the issue of state or local government debt to finance current expenditures. Some states (such as Louisiana) have repeatedly tampered with their constitution so as to sidestep this restriction (see for instance Ratchford (1941)). But in many other cases the constitutional requirement against state deficits has been enforced. These restrictions on public borrowing at the local and state level are likely to reflect the history of frequent defaults of local and state debts, particularly during the XIX century (see Ratchford (1941) and Scott (1893)). But the asymmetry between the federal and state restrictions on public borrowing may also be due to the factors discussed in the previous
paragraphs. The expenditures and revenues of state governments are probably easier to predict than those of the federal government. The sales tax and the property tax that constitute the primary source of states revenue exhibit much smaller cyclical fluctuations than the income tax revenue; and states expenditures are mostly on transportation, education, and law enforcement, which are also relatively easy to predict. Hence, the value of discretion is much higher at the federal than at the state level.

5. SUMMARY AND EXTENSIONS

This paper shows that disagreement between current and future voters about the composition of public expenditure generates a suboptimal path of public debt. Public debt becomes the legacy left by today's voters to the future, so as to influence the choices of future voters, and it tends to increase with the likelihood of disagreement between current and future voters. Thus, this paper establishes a precise link between political polarization and budget deficits. Political polarization can be interpreted as a situation where the preferences of future majorities can be very different from the preferences of today's majority. This can occur if a government with extreme preferences (relative to the historical average) wins the temporary support of a majority of the voters. Alternatively, it can occur in political systems where parties with very different preferences are equally likely to obtain a majority. Thus, the implications of this paper are in principle testable against either time series or cross sectional data. The most natural empirical work along these lines would be a cross sectional comparison of the deficit policies of countries governed by different political institutions and with different degrees of political conflict. Roubini-Sachs (1988) present encouraging evidence along these lines.
Some possible generalizations of the basic framework of this paper are suggested in Section 2. Another feasible extension would be to incorporate an infinite horizon, by applying the dynamic programming solution procedure presented in Alesina-Tabellini (1987a). In an infinite horizon model one could also study the possibility of cooperation between current and future majorities, for instance based upon trigger strategy equilibria. In these equilibria the path of public debt could be brought arbitrarily close to the socially efficient value. However, in order to obtain the socially efficient solution one needs cooperation between successive majorities: cooperation amongst different voters within the same time period would not solve the intertemporal distortions that are the focus of this paper. As such, the reputation mechanism that would be needed to enforce cooperation might require substantial amounts of coordination. In addition, with discounting of the future, the qualitative implications of reputational equilibria may be similar to those of the equilibrium studied in the present paper, as argued in a different context by Alesina (1987, 1988a).

Finally, a natural and yet difficult extension of the basic model would be to allow the voters to choose whether or not to repudiate the debt. Indeed, the results of this paper are driven by a fundamental asymmetry in the commitment technologies available to the voters: even though current voters cannot bind the spending choices of future majorities, nonetheless they are assumed to be able to force future majorities to honor their debt obligations. This asymmetry seems to faithfully reflect a feature of the real world, at least in industrialized economies during the recent decades. But still, the puzzle remains of what is the source of this asymmetry. Some recent interesting literature has investigated the idea that reputational mechanisms create incentives in favor of honoring the internal debt
obligations of previous governments (see in particular Grossman-Van Huyck (1987a)). The political economy approach of this paper suggests a second line of attack: domestic debt repudiation may not be politically viable, because it would be strenuously opposed by the private sector holders of the debt. Recent accounts of historical episodes of debt repayments in Europe during the interwar period lend support to this view (see for instance Alesina (1988b)). Further investigation of this line of thought sets an exciting task for future research.
FOOTNOTES

1 Both recent polls (New York Times, November 1987) and polls taken in the early 1980s (Blinder-Holtz Eakin (1983)) show that a large majority of American voters is in favor of budget balance amendments. A much lower fraction of voters is in favor of any specific measure to reduce budget deficits and there is disagreement on which expenditures (taxes) to reduce (increase), if any.

2 For recent arguments explaining the deficit as the result of "fiscal illusion", see Buchanan et al. (1987) and the references quoted therein. Rogoff-Sibert (1988) has shown that suboptimal budget deficits may be observed before elections if voters are rational but imperfectly informed. This mechanism can explain short budget cycles around elections, but it cannot explain long-lasting and large budget deficits which go well beyond the electoral cycle.

3 Cukierman-Meltzer (1987) analyze an overlapping generations model in which individuals have a bequest motive. Different voters have a bequest motive of different strength, and for some households the bequest motive is so small that they leave zero bequests to their offspring. The equilibrium budget deficit in their model reflects the size of the fraction of the population with an operative bequest motive (i.e., it reflects the preferences for intergenerational redistribution of current voters). Our approach and that of Cukierman-Meltzer are by no means contradictory, even though they emphasize different political and economic determinants of budget deficits.

4 Any expected utility function that is linear in a vector of parameters belongs to this class. Even though linearity was not essential in Grandmont
(1978), it is essential here, since we deal with an expected utility function. There are families of preferences which do not admit linear representation and yet are intermediate preferences. However, in the absence of linearity in the vector of parameters, the property of intermediate preferences would not necessarily be preserved by the expectations operator. The essential property of intermediate preferences is that supporters of distinct proposals are divided by a hyperplane in the space of most preferred points. See also Caplin-Nalebuff (1988).

This setting is reminiscent of that analyzed in Strotz (1956), where a consumer with time inconsistent preferences solves a dynamic optimization problem. See also Peleg and Yaari (1973) and the references quoted therein. In those papers, like here, the time consistent solution is described as the noncooperative equilibrium of a game played by successive decisionmakers.

This second order sufficient condition can be stated as follows:

\[ R(\gamma_2)^3R(\gamma_2) + R(\gamma_2)^2R(\gamma_2)^2 + (1 - \gamma)R'(\gamma_2)R(\gamma_2)^2 + \]
\[ + \gamma R(\gamma_2)^3R(\gamma_2) + \gamma R(\gamma_2)^2R(\gamma_2)^2 + (\gamma - 1)R'(\gamma_2)R(\gamma_2)^2 > 0 \]  

(F.1)

where \( \gamma = \frac{1 - \alpha_1^m}{\alpha_1^m} \frac{1 - \alpha_2^m}{\alpha_2^m} \) and where \( R(\cdot) = \frac{-u''(\cdot)}{u'(\cdot)} \) is the coefficient of absolute risk aversion of \( u(\cdot) \). In turn, a sufficient (but not necessary) condition for (F.1) to hold is that:

\[ R(\gamma_2)R(\gamma_2) + R(\gamma_2)^2 + R'(\gamma_2) > 0 \]

and

\[ R(\gamma_2)R(\gamma_2) + R(\gamma_2)^2 + R'(\gamma_2) > 0. \]

In a more general framework, the socially optimal policy might imply running a deficit or a surplus (for instance, to smooth tax distortions over
time, as in Barro (1979) and Lucas-Stokey (1983)). Here, for simplicity, we abstract from these complications.

8 This condition can also be stated as:

\[ u^\prime(x) > 2[u^\prime(x)]^2/u^\prime(x), \quad 1 > x > 0 \]

or

\[ R^\prime(x) + R^2(x) < 0, \quad 1 > x > 0. \]

9 The same results would go through if the definition of "more polarized" in (12) was stated with respect to other measures of distance between \( \alpha_2^m \) and \( \alpha_1^m \), such as euclidean norm or \( (\alpha_1^m - \alpha_2^m)^2 \).

10 This implication of decreasing absolute risk aversion would remain true even if points A and B lay on the same half of the budget line (that is, if either \( \alpha_1^m, \alpha_2^m > \lambda \), or \( \alpha_1^m, \alpha_2^m < \lambda \), as long as \( \alpha_1^m \neq \alpha_2^m \).

11 These generalizations however would introduce an additional complication. Namely, the probability that the future decisionmaker will be at a corner could now be endogenous, and in particular depend on the borrowing policies of previous governments. This would add another dimension to the problem of choosing the optimal debt policy.

12 If \( \alpha_1^m \) and \( \alpha_2^m \) have the same probability distribution, say \( H(\cdot) \), then the first order condition of (15) with respect to \( b \) can be written as:

\[ \alpha^l \int_0^1 [u^\prime(G(\alpha,b))g_b(\alpha,b) + u^\prime(G(\alpha,b))G_b(\alpha,b)]dH(\alpha) + \]

\[ + (1-\alpha^l) \int_0^1 [u^\prime(F(\alpha,b))f_b(\alpha,b) + u^\prime(F(\alpha,b))F_b(\alpha,b)]dH(\alpha) = 0 \]

It can be shown that if \( b = 0 \), then the terms inside each integral sum to zero. Hence, by the second order conditions, \( b = 0 \) is the solution to (15). Unanimity would be lost if the distributions of \( \alpha_1^m \) and \( \alpha_2^m \) in (15) were different from each other.
A much larger literature has investigated the problem of external debt repudiation, for instance Sachs (1985), Bulow-Rogoff (1987), Grossman-Van Huyck (1987b).
APPENDIX

Consider the function $v(\alpha_2^m)$ for a given value of $b$. This function is continuous in $1 > \alpha_2^m > 0$ (since $u(\cdot)$ was assumed to be twice continuously differentiable). After some algebra, $v'(\alpha_2^m)$ simplifies to:

$$v'(\alpha_2^m) = \frac{u'(\frac{g_1^m}{g_2^m}) \Delta \left( \frac{1 - \alpha_1^m}{\alpha_1^m} - \frac{1 - \alpha_2^m}{\alpha_2^m} \right)}{[R(g_2^m) + R(f_2^m)]^2} \frac{dg_2^m}{d\alpha_2^m}$$

where

$$\frac{dg_2^m}{d\alpha_2^m} > 0 \text{ and } (A.1)$$

$$\Delta = R(g_2^m)[R(f_2^m)^2 + R'(f_2^m)] + [R(g_2^m)^2 + R'(g_2^m)]$$

If $\lambda(x) = -u''(x)/[u'(x)]^2 = R(x)/u'(x)$ is decreasing in $x$ for $1 > x > 0$ (see also footnote 8), then $\Delta < 0$. Hence for any $b$:

$$v'(\alpha_2^m) \leq 0 \text{ as } \alpha_2^m \leq \alpha_1^m \quad (A.2)$$

if (c) holds. These properties imply that, under (c), $v(\alpha_2^m)$ reaches a maximum at the point $\alpha_2^m = \alpha_1^m$, and is strictly decreasing in $|\alpha_2^m - \alpha_1^m|$ if $\alpha_2^m > \alpha_1^m$. Hence, for given $\alpha_1^m$ and given $b$, the expression $[\alpha_1^m u'(g_1^m) - v(\alpha_2^m)]$ reaches a minimum at $\alpha_2^m = \alpha_1^m$ and is strictly increasing in $|\alpha_2^m - \alpha_1^m|$ if $\alpha_2^m < \alpha_1^m$.

Consider now this expression at the point $b = 0$. The discussion on p. 12 of the text implies that, at $b = 0$, $\alpha_1^m u'(g_1^m) - v(\alpha_1^m) = 0$. Since, as shown above, under (c) $\alpha_1^m = \text{argmax } v(\alpha_2^m)$, we have that, if $b = 0$:

$$\alpha_1^m u'(g_1^m) - v(\alpha_2^m) \geq 0$$

with strict inequality if $\alpha_2^m \neq \alpha_1^m$. Thus, under condition (c), $v(\alpha_2^m)$ can be drawn as in the diagram of Figure (4).
Figure 4
REFERENCES


