TECHNICAL PROGRESS AND AGGREGATE FLUCTUATIONS

Gary D. Hansen

University of California, Los Angeles

UCLA Dept. of Economics
Working Paper #546
November 1988
Revised: February 1989

I have benefited from conversations with Larry Christiano, Ed Prescott, Charles Whiteman and workshop participants at Columbia University and the University of Rochester.

Abstract

We compare the cyclical fluctuations exhibited by a stochastic growth model under various stochastic processes governing technical change, all of which are highly persistent. We find that the results obtained are quite sensitive to the precise form of this stochastic process. In particular, the results depend on whether growth is deterministic or stochastic and, in each case, are quite sensitive to the persistence of an innovation. The model does a relatively poor job of accounting for features of observed business cycles when technical change is difference-stationary and does a better job when technical change is trend-stationary but highly persistent.

1. Introduction

Beginning with the seminal work of Long and Plosser (1983) and Kydland and Prescott (1982), a great deal of research effort has been devoted to modeling the business cycle as an equilibrium outcome of optimizing agents responding to highly persistent shocks to technology.[1] The various contributions to this "real business cycles" research program have argued, using a variety of empirical methodologies, that fluctuations in technical progress can account for a large fraction of observed fluctuations in aggregate economic time series.

Although it is general practice in this literature to assume a highly persistent stochastic process for technical change, individual contributions differ in the precise specification of this process. This paper investigates, using a version of the stochastic growth model originally studied in Hansen (1985), how the results obtained from the model are sensitive to the exact form of this stochastic process. We find that the fluctuations exhibited by the model differ significantly depending on how technical progress is modeled, even though innovations to the process are long lived in all cases considered. In particular, the results depend crucially on whether growth is deterministic or stochastic (whether technical progress is trend-stationary or difference-stationary) and, in each case, are quite sensitive to the persistance of an innovation.

The assumption that the underlying shocks are highly persistent is

---

[1] A source of examples of research along these lines is the special issue of the Journal of Monetary Economics on real business cycles, March/May 1988.

necessary for simple versions of the stochastic growth model to display fluctuations similar to those observed in U.S. data (see King, Plosser and Rebelo (1988)). In addition, the work of Nelson and Plosser (1982) and others, where it is argued that many macroeconomic time series are likely to contain unit roots, provides empirical evidence in favor of persistent underlying shocks. These two facts have motivated much of the recent emphasis on relatively permanent technology shocks over transitory monetary shocks in accounting for the business cycle. Further evidence is provided by Prescott (1986) who shows that when technical change is identified with the Solow residuals computed from U.S. time series, technology shocks are both large and highly persistent.[2]

In this paper, we consider many of the stochastic processes governing technical progress that have been assumed in previous contributions to this literature. One specification, which is perhaps the most common, is to abstract entirely from per capita growth and assume that technical change follows a stationary (but highly persistent) AR(1) process (e.g. Kydland and Prescott (1982, 1988), Hansen (1985), Hansen and Sargent (1988) and Prescott (1986)).[3] Others, such as Altug (1984) and Hansen, Sargent and McGrattan (1988), have added to this specification a deterministic geometric trend. Still others, motivated by the work of Nelson and Plosser (1982) and others, have postulated that technical progress follows a random walk with drift (see, for example, King, Plosser, Stock and Watson (1987)). Christiano

---

[2] Solow residuals are computed by taking first differences of log output and subtracting first differences of log inputs (capital and hours worked), weighted by constant factor shares.

[3] Actually, Kydland and Prescott (1982, 1988) assume that the shock to technology is the sum of a persistent and transitory component. The persistent component follows an AR(1) while the transitory component is iid.

(1987b, 1988) also assumes a stochastic trend but allows for a moving average component as well. We find that our model does a relatively poor job of accounting for features of observed business cycles when unit roots are present in the model. Much better results are obtained when innovations to technical change are assumed to be temporary but highly persistent.

An additional issue raised in this paper is whether the properties of the fluctuations displayed by the model depend crucially on whether growth is explicitly included in the model. That is, is it reasonable to separate the study of short run business cycles from the study of long run growth as so many have done in the past? This issue is an important one given that available evidence, as argued by Lucas (1977) and Prescott (1983), suggests that all market economies exhibit aggregate fluctuations with roughly the same properties, in spite of the fact that the growth rates of these economies differ greatly. We find that our model displays similar fluctuations in both a growing and nongrowing economy for all specifications of technical progress we consider.

The model studied in this paper is the "indivisible labor economy" of Hansen (1985) modified to include the various stochastic processes for technical progress considered. In any given period, individuals either work some positive number of hours or not at all. This feature introduces unemployment into the model; in each period some individuals work and others do not. Another feature of the model, as originally demonstrated by Rogerson (1988), is that, independent of the willingness of individuals to substitute leisure over time, the elasticity of substitution between leisure in any two periods is infinite for the social planner that rationalizes the competitive allocation. For this reason, the model is consistent with large amounts of intertemporal substitution at the aggregate level and relatively little intertemporal substitution at the individual level.[4] As a result, simulations of this model presented in Hansen (1985) show that hours worked are over twice as volatile as productivity. This is an encouraging result given that hours are significantly more volatile than productivity in U.S. time series. Previous real business cycle models without the indivisible labor assumption had been unable to account for this fact. An issue to be considered in this paper is whether this result continues to hold under alternative specifications of technical progress.

The remainder of the paper is organized into three additional sections. In the next section, the economy is described including the stochastic processes governing technical progress that will be considered. The competitive equilibrium for the model is obtained by solving a social planner's problem. Since this problem can not be solved analytically, we obtain a solution by approximating the problem with one that has a quadratic objective and linear constraints, following Kydland and Prescott (1982). This method, and how it is implemented in the presence of nonstationary technical progress, is discussed in the third section. In section 4, results obtained from simulating the model under the various specifications of technical change are presented. In addition, further insights on the sensitivity of the results to these specifications are obtained by studying impulse response functions derived from the model. Some concluding remarks are given in section 5.

---

[4] Therefore, the theory presented here is consistent with the low estimates of this elasticity found when studying panel data (see Altonji (1986) or MaCurdy (1981)).

2. A Real Business Cycle Model with Growth

The economy to be studied is a version of the indivisible labor model of Hansen (1985) assuming alternative stochastic processes for exogenous technological change. The economy is populated by a continuum of identical infinitely lived households and a single competitive firm. Each household wishes to maximize expected discounted lifetime utility,

$$(2.1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log c_t + A \log l_t \right\} \quad 0 < \beta < 1 ,$$

where c_t and l_t are consumption and leisure at time t, respectively.[5] Households have a time endowment equal to one, so $0 \leq l_t \leq 1$.

Following Rogerson (1988) and Hansen (1985), we assume that labor is indivisible. That is, in each period households are allowed to supply h_0 units of labor or none at all, where h_0 is a constant between zero and one.[6] This implies that l_t can only take on the values of $1-h_0$ or 1, so the consumption set for this economy is nonconvex. Since we wish to exploit the connection between the competitive equilibrium for this economy and the solution of a concave dynamic programming problem, we convexify the economy by allowing agents to sell employment lotteries, following Rogerson (1988).

---

[5] Here we have restricted attention to a constant relative risk aversion utility function with a coefficient of relative risk aversion equal to one. In Hansen (1986) it is shown that our results would be the same if we had assumed other reasonable values for this coefficient.

[6] The motivation for assuming indivisible labor is discussed in Hansen (1985). The motivation is partly based on the fact that over half of the variance in total hours worked in the U.S. is unambiguously due to fluctuation in employment as opposed to fluctuation in average hours per worker. The indivisible labor assumption implies that all changes in total hours worked reflect changes in employment. This, however, is inconsistent with the fact that there is still a significant percentage that is due to fluctuation in average hours. Models that allow for adjustment along both margins include Cho and Cooley (1988) and Hansen and Sargent (1988).

Thus, agents choose a probability of working in a given period, π_t, rather than a number of hours. A random drawing determines whether the agent will be employed or unemployed. Since all agents are identical, they all choose the same π_t. It follows that per capita hours worked in period t is given by

$$(2.2) \qquad h_t = \pi_t h_0 \quad .$$

The period utility function of the representative agent as a function of consumption and hours worked can be derived from (2.1) using the assumption that individual households choose lotteries to maximize expected utility. That is,[7]

$$U(c_t,h_t) = \log c_t + \pi_t A \log(1-h_0) + (1-\pi_t)A \log(1)$$
$$= \log c_t + h_t A (\log(1-h_0)/h_0) \ , \quad \text{or}$$

$$(2.3) \qquad U(c_t,h_t) = \log c_t - B h_t \ , \quad \text{where } B = -A(\log(1-h_0)/h_0) \ .$$

The household receives income, y_t, in period t from selling labor and capital services. This income can be used to purchase output that can either be consumed (c_t) or invested (x_t), so that

$$(2.4) \qquad c_t + x_t \le y_t \quad .$$

Investment is undertaken to augment the capital stock (k_t) owned by the household. The capital stock, which depreciates at the rate δ each period, obeys the following law of motion:

---

[7] This derivation uses the fact that, in equilibrium, consumption is the same whether or not the household is employed. This result follows from the separability of (2.1) in consumption and leisure and is shown formally in Hansen (1985).

$$(2.5)\qquad k_{t+1} = (1-\delta)\, k_t + x_t\ , \quad 0 \le \delta \le 1\ .$$

The firm produces output using the constant returns to scale technology given by

$$(2.6)\qquad y_t = \exp\{(1-\theta)(\mu t + z_t)\} k_t^{\theta} h_t^{1-\theta}\ , \qquad 0 \le \theta \le 1\ ,$$

where k_t is the per capita capital stock in period t, h_t is hours of labor employed, and $\exp\{(1-\theta)(\mu t+z_t)\}$ is technical progress. The parameter $\mu \ge 0$ is (approximately) equal to the growth rate of output in a certainty version of the model. We will experiment with various specifications for the stochastic process governing z_t. Agents are assumed to observe z_t at the beginning of period t, so all period t decisions are made knowing the value of the technology shock.

Before describing the various specifications for the stochastic process governing z_t, it is useful to summarize the model by stating the social planning problem for the economy. Since the conditions for the second welfare theorem (see Debreu (1954)) are satisfied, the solution to this problem can be supported as a competitive equilibrium.

$$(2.7)\qquad \text{Maximize}\ \ E_0 \sum_{t=0}^{\infty} \beta^t \left\{\log c_t - B\, h_t\right\}$$

subject to (2.4)-(2.6), a given stochastic process for z_t, and k_0, z_0 given.

We now describe three stochastic laws of motion for z_t that will be considered in this paper. As explained in the introduction, these all correspond to specifications that have been assumed in the real business cycle literature.

Example 1 Stationary AR(1)

This process, which is assumed in Kydland and Prescott (1982, 1988) and Hansen (1985), is given by

$$z_{t+1} = \gamma z_t + \epsilon_{t+1} , \qquad 0 < \gamma < 1 , \tag{2.8}$$

where ϵ_t is an independent and identically distributed random variable drawn from a normal distribution with mean 0 and standard deviation σ_ϵ. This implies that the unconditional mean of z_t is zero and that the unconditional mean of $\exp(z_t)$, which appears in (2.6), is approximately equal to one.

In this example technical change evolves as serially correlated deviations from a geometric steady state growth path.[8] The innovations to the growth process, ϵ_t, are temporary, but in the examples we consider they are assumed to be quite persistent (that is, γ is assumed to be close to one).

Example 2 Random Walk

This process is simply a special case of (2.8) with γ set equal to one. In addition, we assume that the initial shock, z_0, is equal to zero so that the unconditional mean of z_t is also equal to zero as in example 1. In this example, technical change follows a random walk with drift. That is, if we let $g_t = \mu t + z_t$, then

[8] Although this process for z_t is the same as the one assumed in Hansen (1985) and Kydland and Prescott (1982, 1988) (for the persistent component of technical change), those studies assume an economy that does not display long run growth ($\mu = 0$).

$$(2.9) \qquad g_{t+1} = \mu + g_t + \epsilon_{t+1} \quad ,$$

where ϵ_t is drawn from the same distribution as in example 1. We treat this as a separate example because a different solution method is employed in this case.

We study this example since it has been argued in Nelson and Plosser (1982), as well as elsewhere, that econometric tests fail to reject the hypothesis that many U.S. aggregate time series contain unit roots. Assuming that technical change has a unit root will induce a unit root in the artificial time series for output, consumption, investment and capital. In addition, Prescott (1986) argues that, to a first approximation, the Solow residuals computed from U.S. data imply that the empirical counterpart to our technology shock behaves like a random walk with drift.

<u>Example 3</u> Random walk with autocorrelation in ϵ_t

When the technology shock follows a random walk, the Solow residual studied in Prescott (1986) corresponds to ϵ_t. Prescott reports the following serial autocorrelations of the Solow residuals: $\rho_1 = -0.21$, $\rho_2 = -0.06$, $\rho_3 = 0.04$, $\rho_4 = 0.01$ and $\rho_5 = -0.05$.[9] These autocorrelations reveal significant negative first order serial correlation in ϵ_t. To capture this, we consider the following specification of the z_t process (again assuming that $z_0 = 0$):

$$(2.10) \qquad z_{t+1} = z_t + \epsilon_{t+1} \quad \text{where}$$
$$\epsilon_{t+1} = \rho\, \epsilon_t + \xi_{t+1} \, , \qquad -1 < \rho < 0 \; .$$

---

[9] Christiano (1988) reports autocorrelations of the Solow residuals that are similar to these using a different measure of capital.

In this example ξ_t is assumed to be independently and identically distributed from a normal distribution with mean zero and standard deviation σ_ξ. In this specification, which is similar to the specification assumed in Christiano (1987b, 1988), an innovation to the z_t process is not purely temporary (as in example 1) nor is it purely permanent (as in example 2). Instead, a fraction $1/(1-\rho)$ of an innovation to the z_t process is permanent while the remainder is temporary.

For all of our examples, we are able to solve problem (2.7) analytically only if we set δ, the depreciation rate of the capital stock, equal to 1. In this case, it turns out that the equilibrium law of motion for the capital stock depends only on the current value of z and does not depend at all on the serial correlation properties of z (see King, Plosser, and Rebelo (1988)). We choose not to focus on this case, not only because it is unrealistic, but also because it implies that employment remains constant over time (is independent of fluctuations in z). In order to solve problem (2.7) assuming a realistic value for the depreciation rate, we employ the quadratic approximation methods described in the next section.

3. Solution Method

The solution method employed involves rewriting these problems in a form that will make it possible to use the quadratic approximation technique described in Kydland and Prescott (1982).[10] This entails substituting all

---

[10] There are reasons for believing that a quadratic approximation is quite accurate for this model. For example, Danthine, Donaldson and Mehra (1988) compute an exact solution to the "divisible labor economy" in Hansen (1985) by using value function iteration on a discrete state space. They find that this solution is very close to the solution obtained using a quad-

nonlinear constraints into the return function (2.3) and taking a quadratic approximation of this around the steady state of the certainty version of the model formed by setting z_t equal to its unconditional mean. The problem then becomes one of maximizing a quadratic objective subject to linear constraints, one that can be solved using standard methods.

For the model studied in this paper, it is not possible to immediately form a quadratic approximation. Since the economy grows at the rate μ, the certainty version of the model does not possess a constant steady state. A change in variables is introduced to solve this problem. A different change in variables is introduced in the different examples depending on whether the example displays geometric growth (is trend-stationary) or stochastic growth (is difference-stationary).

After substituting the nonlinear technology constraint ((2.4) and (2.6)) into the objective function of problem (2.7), the objective becomes

$$(3.1) \qquad E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log\left[\exp((1-\theta)(\mu t+z_t)) k_t^{\theta} h_t^{1-\theta} - x_t \right] - B\, h_t \right\}$$

The problem is to maximize this objective subject to (2.5) and either (2.8) or (2.10). In the steady state, k_t and x_t grow at the rate μ while h_t is a constant. We now describe the change of variables introduced in the various examples.

Method 1 For Economies with Deterministic Growth

Consider the following change of variables:

---

ratic approximation. Christiano (1986) has performed a similar exercise assuming 100% depreciation of the capital stock, so that the model can be solved analytically, and arrived to a similar conclusion.

$$(3.2) \qquad \hat{k}_t = k_t/\exp(\mu t) \qquad \hat{x}_t = x_t/\exp(\mu t)$$

A stationary version of problem (2.7) is obtained by substituting (3.2) into (3.1), thereby eliminating k_t and x_t. That is, the variables k_t and x_t, which are growing over time in the solution to the certainty version of (2.7), are replaced by variables that are stationary. For example 1, the solution to the uncertainty version of this problem is obtained by solving the following functional equation (where primes denote next period values):[11]

$$(3.3) \qquad v(z,\hat{k}) = \text{Max} \left\{ \log(\exp\{(1-\theta)z\}\hat{k}^{\theta}h^{1-\theta} - \hat{x}) - B\, h_t + \beta\, E\, v(z',\hat{k}') \right\}$$

$$(3.4) \qquad \text{subject to} \quad z' = \gamma\, z + \epsilon' \quad , \quad \epsilon' \sim N(0,\sigma_{\epsilon}^2)$$

$$(3.5) \qquad \text{and} \quad \hat{k}' = [(1-\delta)/\exp(\mu)]\, \hat{k} + [1/\exp(\mu)]\, \hat{x}$$

<u>Method 2</u> For Economies with Stochastic Growth

When the technology shock follows a random walk it is possible to continue using the transformation described above, setting γ equal to one. However, the solution obtained from a quadratic approximation of problem (3.3) under the random walk assumption might be relatively inaccurate given that the quadratic approximation is only accurate when the variables take on values close to the steady state. When z_t follows a random walk the variables may wander far from their steady state values.

An alternative solution procedure that avoids this problem is to use

---

[11] When implementing this change in variables, a constant term will appear in the objective function (3.1). This constant is ignored since it does not affect the solution to the optimization problem.

the following transformation instead of (3.2):[12]

$$(3.6) \qquad \tilde{k}_t = k_t/\exp(\mu t+z_{t-1}) \qquad \tilde{x}_t = x_t/\exp(\mu t+z_t)$$

When this change of variables is implemented, the problem no longer contains the variable z_t. Instead, ϵ_t replace z_t as a state variable. In addition, using (3.6) rather than (3.2) in the random walk case reduces the number of iterations needed to solve the dynamic programming problem by a factor of seven. The following functional equation is solved in this case:

$$(3.7) \qquad v(\epsilon,\tilde{k}) = \text{Max} \left\{ \log(\exp(-\theta\epsilon)\tilde{k}^{\theta}h^{1-\theta} - \tilde{x}) - B\ h_t + \beta\ E\ v(\epsilon',\tilde{k}') \right\}$$

$$(3.8) \qquad \text{subject to} \quad \tilde{k}' = [(1-\delta)/\exp(\mu+\epsilon)]\ \tilde{k} + [1/\exp(\mu)]\ \tilde{x}$$

$$(3.9) \qquad \text{and} \quad \epsilon' \sim N(0,\sigma_{\epsilon}^2)$$

To solve example 3, where ϵ follows a first order Markov process, we use the same transformation (3.6) and obtain the same functional equation except that (3.9) is replaced with

$$(3.10) \qquad \epsilon' = \rho\ \epsilon + \xi' \ , \qquad \xi \sim N(0,\sigma_{\xi}^2)$$

Except for the change of variables, (3.2) or (3.6), the solution procedure is the same for all three examples. Since (3.8) is nonlinear in the state variables, before forming the quadratic approximation we substitute this into the objective by eliminating $\tilde{x}$. So that our procedure is the same for all of the examples, we do the same when solving problem (3.3). Thus $\hat{k}'$ (or $\tilde{k}'$) becomes a decision variable in the dynamic programming problem being solved.

---

[12] Christiano (1988) makes use of a transformation similar to this one in his model of inventory accumulation where technical change also follows a random walk.

Before forming the quadratic approximation, still another transformation is introduced. For the cases where z_t follows a random walk, Christiano (1987a) has shown, using a model that is very similar to the one studied here, that a much better approximation can be obtained by approximating around the logs of capital and hours rather than the levels. This procedure will yield decision rules that are log-linear in the state variables rather than linear.[13] Thus, the following additional change in variables is made (the same transformation is made when Method 2 is used except that the "hats" should be replaced with "tildes").

$$(3.11) \qquad l\hat{k} = \log \hat{k}, \quad l\hat{k}' = \log \hat{k}', \quad lh = \log h$$

After introducing these change of variables, we take the first three terms of a Taylor series expansion of the return function at the steady state to obtain a quadratic return function. The problem can then be solved using standard methods (see Sargent (1987)) to obtain linear decision rules for $l\hat{k}'$ and lh as a function of the state variables. If we are using Method 1, we obtain the following linear functions of z and $l\hat{k}$:[14]

---

[13] Our simulation results are unaffected by approximating around logs rather than levels when $\gamma < 1$. However, this choice makes a significant difference when z follows a random walk, especially when Method 2 is used. In particular, approximating around levels leads to much more variability in consumption than if the log approximation is used. The explanation given by Christiano for this fact is that the decision rules obtained from the level approximation are not monotone in ϵ while the decision rules from the log approximation are monotone. In the interest of using a consistent solution procedure, we approximate around logs in all of the examples we consider.

[14] The rest of this discussion assumes that we are using Method 1. The simulation procedure is the same under Method 2 except that ϵ replaces z as a state variable and we use transformation (3.6) rather than (3.2).

$$(3.12) \qquad l\hat{k}_{t+1} = d_{11} + d_{12}z_t + d_{13}l\hat{k}_t \qquad \text{and}$$

$$(3.13) \qquad lh_t = d_{21} + d_{22}z_t + d_{23}l\hat{k}_t$$

Next, given values for z_t and k_t for $t = 0$, we can compute simulations using a pseudo random number generator to draw realizations of ϵ_t. Equation (3.4) is used to simulate values for z_t, $t > 0$. From these values we can calculate $\hat{k}_t$ for $t > 0$ and h_t for $t \geq 0$ using the decision rules (3.12) and (3.13). Values are obtained for $\hat{x}_t$ from (3.5), and the change of variables (3.2) is used to compute k_t and x_t. Finally, y_t is obtained using the production function (2.6), and c_t is obtained by subtracting investment from output. Productivity is computed by dividing output by hours worked. Results from simulating the economy using this procedure are discussed in the next section.

4. Results

In this section, the above solution procedure is used to compute linear decision rules for capital and hours and simulate artificial time series for output, consumption, investment, capital stock, hours worked and productivity. The decision rules obtained for each example are reported in table 6. For each example considered, fifty simulations of 115 periods are formed.[15] Given that our interest is in the cyclical properties of this time series, the data from the simulations are logged and detrended using

---

[15] We compute simulations for 115 periods since this is the number of quarters in the postwar U.S. sample reported on in Table 1. For each simulation, we use the steady state values of the state variables as initial conditions and actually compute numbers for 215 periods. Before filtering and computing summary statistics, we throw out the first 100 of these. The same set of random numbers are used as innovations to the z_t process in each example considered.

the filter described in Hodrick and Prescott (1980).[16]

Statistics (standard deviations and correlations) summarizing the behavior of the cyclical component are computed for each simulation and the average of these over the fifty simulations are reported in tables 2 through 5. The sample standard deviations of these statistics are also reported. In table 1, statistics describing the cyclical component of quarterly U.S. data from 1955,3 to 1984,1 are given. Although this paper does not attempt to fit the model to the data, these statistics provide a benchmark for comparing the statistics computed from the various simulations.

Before solving the model and computing the simulations, we must choose values for the parameters. Since the basic model is the same as the one studied in Hansen (1985), we begin with the parameterization employed there.[17] We begin with an economy without growth, so μ is equal to zero. The discount factor, β, is set equal to .99 and the depreciation rate of capital, δ, is set equal to .025. These values were chosen so that, in the economy without growth, the real interest rate is 4% and the annual depreci-

---

[16] We follow Kydland and Prescott (1982 and 1988), Hansen (1985), and Prescott (1983 and 1986) (among others) in using this filter as an operational definition of the cyclical component of a time series. Although this definition, like any definition, is inherently arbitrary, we adopt it since it has been used so often in the literature and is easy to implement. The filter involves choosing smoothed values $\{s_t\}_{t=1}^{T}$ for the series $\{x_t\}_{t=1}^{T}$ which solve the following problem:

$$\min \{(1/T)\Sigma_{t=1}^{T}(x_t - s_t)^2 + (\lambda/T)\Sigma_{t=2}^{T-1}[(s_{t+1} - s_t) - (s_t - s_{t-1})]^2\},$$

where $\lambda > 0$ is the penalty on variation (variation is measured by the average squared second difference). A larger value of λ implies that the resulting $\{s_t\}$ series is smoother. As in all of the papers referenced above, we choose λ=1600. The cyclical component is obtained by taking $d_t = x_t - s_t$.

[17] Most of these values were originally taken from Kydland and Prescott (1982). Prescott (1986) defends the choice of these values by appealing to growth observations and studies using micro data.

ation rate is 10%. Capital's share in production, θ, was set equal to .36. The parameter multiplying hours in the utility function (2.3), B, was set equal to 2.86 following Hansen (1985). Finally, we parameterize the law of motion for z_t as in Hansen (1985) by setting γ equal to .95 and setting σ_ϵ so that the standard deviation of output from the simulations is equal to the standard deviation of actual GNP reported in table 1. This implies a value for σ_ϵ equal to .00715/(1-θ).[18] Prescott (1986) measures this parameter using postwar U.S. time series and arrives at a value close to this one.

These parameter values imply that consumption is 74% of GNP in the steady state, which is on average the same percentage found in the data used to construct table 1. In addition, these parameter values imply that hours worked in the steady state is equal to .3, which is consistent with households spending roughly one third of their time engaged in market activities (this provides the criteria for assigning a value to the parameter B).

These steady states are computed assuming that the growth rate (μ) is equal to zero. For most of the results discussed in this paper, an average annual growth rate of output equal to two percent is assumed. The implied value for μ is .005. Assuming this value for μ, the steady state values are slightly changed--consumption becomes 73% of output and hours worked increases to .31. The reason these values are affected is clear from equations (3.5) and (3.8). Once a positive growth rate is assumed, the

[18] The value .00715 is the standard deviation of an innovation to the technical change process. This is the number that should be compared with measurements given in Prescott (1986). Since z is multiplied by (1-θ) in the production function (2.6), it is necessary to divide this number by (1-θ) to get σ_ϵ. This value for the standard deviation of an innovation to this process is maintained throughout all examples studied. Thus, σ_ξ, in example 3, is set equal to this same value.

depreciation rate of the "effective" capital stock is no longer 10% per year. Ten percent of the current period capital stock will disappear, but the remaining capital will be more productive. To maintain the depreciation rate of the "effective" capital stock equal to 10% per year, we must set δ so that $(1-.025) = (1-\delta)/\exp(\mu)$. This implies a value of δ equal to .02. In addition, when these parameter values are used ($\mu = .005$ and $\delta = .02$) the steady states turn out to equal those obtained in the no growth case with δ equal to .025.

We now discuss findings obtained by simulating the model to determine how the various specifications of technical progress described in section 2 affect the cyclical properties of the model. We begin by studying how increasing the growth rate from zero to two percent per year affects this behavior. We then consider other examples where technical change is trend-stationary (example 1 in section 2). Following that we consider examples where technical change is difference-stationary (examples 2 and 3 in section 2). We finish the section with a discussion of impulse responses computed from the model that illustrate the general intuition underlying our simulation results.

<u>Deterministic Growth</u>

In panel A of table 2 summary statistics are given for our initial parameterization without growth. An important feature of this example is that the standard deviation of hours worked is significantly larger than the standard deviation of productivity. This is a feature also observed in actual data (see table 1) that is difficult to account for without the indivisible labor assumption (see Hansen (1985)). We will find that many of the examples we consider are also unable to account for this feature, even though we assume indivisible labor throughout. Other features of the business cycle accounted for by this model are the relatively large fluctuations in investment compared to output, the relatively small fluctuations in consumption and capital compared to output, and the slightly smaller fluctuations in hours relative to output. In addition, the model displays contemporaneous correlations among the various variables that, with a few exceptions, compare favorably to the correlations reported in table 1.

When growth is added to the model by setting μ equal to .005 (leaving δ unchanged), our results are changed slightly (see panel B of table 2). However, all of the features of the business cycle referred to in the previous paragraph continue to be displayed by the model. Therefore, it appears that abstracting from growth does not affect the properties of the fluctuations displayed by real business cycle models with exogenous technical progress.[19] In addition, this model implies that economies with different growth rates will display similar fluctuations, at least when the growth rates are close to the ones assumed here.

If the depreciation rate is adjusted as described above so that the steady states are the same as in the zero growth case (that is, set δ = .02), the cyclical results are essentially identical to the ones given for the zero growth case (see panel C of table 2). This is to be expected given that the decision rules reported in table 6 for these two cases are not very

---

[19] It is an open question whether this conclusion would carry over to models with endogenous growth, such as those studied by Lucas (1988) and Romer (1986).

different.[20] In fact, this conclusion turns out to hold for all of the examples studied in this paper, although we report evidence for only this case. All of our subsequent examples assume the positive growth rate (μ = .005) and a depreciation rate equal to .02.

The next issue to be addressed is how the cyclical component is affected when the amount of serial correlation in the technology shock is increased or decreased by changing the value of γ. In table 3 we present summary statistics obtained by simulating the model using the same parameter values as in panel C of table 2 except that γ is set equal to .90 (in panel A) and .99 (in panel B). We find that as the serial correlation is increased, the cyclical variability of output, investment, capital and hours is reduced, while the variability of consumption and productivity is increased. In table 5, it is shown how the relative volatility of consumption to output is increased when γ is increased while the relative volatility of investment and hours compared to output is reduced. In addition, the volatility of hours relative to productivity is significantly reduced. These results follow from the fact that increasing the persistence of the technology shock reduces the coefficient on z_t in both decision rules reported in table 6 and leaves all other coefficients the same.

In addition to the effect on volatility, the correlations of the various variables with consumption and productivity are significantly affected by changing the value of γ. Consumption and productivity become more highly correlated with output, investment and hours, and less correlated with the capital stock as the amount of serial correlation is in-

[20] Christiano and Eichenbaum (1988) demonstrate in a footnote that these decision rules would be exactly the same if the production function is multiplied by a constant determined by the growth rate.

creased. The other correlations are not affected much.

Stochastic Growth

As explained in section 3, when technical progress follows a random walk it is possible to solve the model using Method 1, by setting γ equal to one, or by using Method 2. We solved the model both ways and found that the same summary statistics were obtained.[21] This we interpret as evidence for the quality of the quadratic approximation given that the two methods involve approximating quite different objective functions.

The summary statistics for this example are presented in panel A of table 4. These results are not surprising given the effect, discussed above, of increasing the serial correlation in the z_t process. The standard deviations and correlations are affected in the same general way as when γ was increased from .90 to .99. The same applies to the relative volatilities reported in table 5.

Clearly, when technical progress is assumed to follow a random walk with drift, this growth model does a less satisfactory job of accounting for the properties of aggregate fluctuations observed in U.S. data. An exception to this is that consumption fluctuates significantly less than output even when innovations are permanent (see also Christiano (1987b)). However, the statistics given in table 5 indicate that the volatility of investment relative to output is much smaller than in the data. In addition, the ratio of the standard deviation of hours to the standard deviation of productivity

---

[21] The decision rules obtained from using both methods are given in the table 5. Note that the decision rules obtained using Method 1 continue to display the same pattern discussed above as γ is increased: The coefficients on z are closer to zero while the other coefficients are unaffected.

is 1.6 in U.S. time series while this ratio is about 2.7 when $\gamma = .95$ as in Hansen (1985). Once technical progress is assumed to follow a random walk, productivity actually fluctuates <u>more</u> than hours worked!

These results lead to the conclusion that an innovation to technical progress must not be completely permanent if the model is to account for important features of aggregate fluctuations observed in U.S. data. A way to introduce a temporary component to the technology shock and still have a stochastic trend is to introduce some negative serial correlation in ϵ_t as in example 3 of section 2. This is consistent with the empirical findings in Prescott (1986) and Christiano (1988). Prescott finds that the first order autocorrelation of the Solow residuals (ϵ_t) computed from U.S. time series is equal to -0.2 and Christiano, using different data, finds it to be -0.1. We experiment with both of these values for this statistic, which corresponds to the parameter ρ in our model (see equation (2.10)). With this specification, a small part of a given innovation is temporary while the rest is permanent. Most of the temporary effect dies off after one period (see Figure 2B). Just as for the examples with a deterministic trend, the decision rules given in table 6 reveal that as ρ is increased (in absolute value) from zero to -.01, and than to -.02, the coefficient on ϵ_t is affected, but the other coefficients remain the same.

Summary statistics for these cases are given in panels B and C of table 4. Comparing panel A with panels B and C we find that adding the temporary component to the technology shock reduces the variability of all of the variables except hours worked. However, the variability of investment and the capital stock is affected very little. This result differs from the results for the deterministic growth examples where reducing the persistence of the technology shock (reducing γ) increased the volatility of all variables except consumption and productivity. The correlations are affected in the same way as in the deterministic growth case: by reducing the amount of persistence in the technology shock, we observe that consumption and productivity are less correlated with output, investment and hours and slightly more correlated with the capital stock. However, the correlations are affected by a smaller amount in this case.

Adding first order autocorrelation in ϵ_t does reverse the counterfactual implication of the original random walk example that productivity fluctuates more than hours worked. However, the ratio of the standard deviation of hours to the standard deviation of productivity is still significantly less than what is found in the data, 1.22 compared to 1.58 with ρ equal to -0.2. Thus, the model with stochastic growth is unable to account for the large observed fluctuations in hours worked relative to productivity, even when a reasonable amount of negative serial correlation in ϵ_t is allowed for. In addition, this negative serial correlation does not increase the volatility of investment relative to output very much (see table 5).

Analysis of Impulse Response Functions

It is perhaps surprising that the results reported above are so sensitive to how technical progress is modeled. This is especially true given that simulations of technical progress from our various examples look quite similar, as illustrated in figure 1.[22] However, figures 2A and 2B

---

[22] Figure 1 shows a plot of simulated "technical progress" for each of the examples considered. Each simulation uses the same initial condition and the same set of realizations of the random variables. The simulations

show that the response of z_t to an innovation is very different in the examples considered. For example, when $\gamma = .90$, it takes about fifty periods for a shock to die out, while it takes twice that long when $\gamma = .95$. When a stochastic trend is assumed, as in figure 2B, the temporary effect of a shock is relatively small and lasts only two periods, leaving only the permanent effect.

In figures 3 and 4 the responses of log $\hat{k}$ and log $\hat{x}$ are plotted.[23] When innovations are temporary, as in figures 3A and 4A, the capital stock increases relatively quickly in response to a positive innovation. This is most pronounced when $\gamma = .90$. In this case, the response of investment in the first period after the shock is larger and dies off more quickly than in the cases where the shock is more persistent. This leads to the higher variability of investment in this case. When an innovation is mostly permanent, as in figures 3B and 4B, there is very little temporary effect on investment or the capital stock. These responses mostly reflect gradual transition to the higher steady state--even when $\rho = -0.2$. This explains the very slight affect on the volatility of these series when ρ is increased in absolute value (see table 4).

Hours, however, show a significant temporary response even in the cases where a stochastic trend is assumed (see Figures 5A and 5B). This response becomes stronger as ρ is increased in absolute value, explaining the

---

were actually run for 215 periods and the last 115 periods are plotted. Technical progress is given by $\exp((1-\theta)(\mu t+z_t))$.

[23] In Figure 3B, where Method 2 was used to solve for the decision rules, we obtained impulse responses for log $\hat{k}$ using the fact that log $\hat{k}_t$ is equal to log $\tilde{k}_t + z_{t-1}$. An analogous transformation enables us to compute impulse responses for log $\hat{c}$, log $\hat{x}$ and log $\hat{y}$. In addition, in order to compute the consumption and investment responses, a linear approximation was used. See Hansen and Sargent (1988) for details.

behavior of the volatility of hours in table 4. This also accounts for the response of output shown in figure 6B. However, it is still the case that the response of hours is significantly stronger when an innovation is completely temporary (see figure 5A). This leads one to expect hours (and output) to be more volatile in these cases (see tables 2 and 3).

The relatively small response of consumption to a temporary shock, as predicted by the permanent income hypothesis, is illustrated in figure 7A. The response of consumption to a permanent shock is shown in figure 7B. This plot confirms Christiano's (1987b) finding that in a model like this one, consumption rises only gradually in response to a permanent shock in contrast to the predictions implied by simple versions of the permanent income hypothesis. In fact, for all values of ρ considered, when z_t is difference-stationary there is no temporary spike in the response of consumption at all. This explains why the volatility of consumption is significantly less than output even when a shock is entirely permanent.

5. Conclusion

The results presented in this paper indicate that temporary shocks to technical progress are crucial for this model to display important features of observed aggregate fluctuations. In an economy with essentially permanent shocks there is not enough incentive to increase the capital stock quickly in response to a positive innovation to produce the relative volatility of time series that is observed in the U.S. economy. These results bode poorly for real business cycle models which assume difference-stationary technical change. However, in an economy where technical progress is a stationary process fluctuating around a geometric trend, fluctuations are exhibited that have more in common with those observed in U.S. time series. In addition, the properties of these fluctuations are the same for any reasonable growth rate.

It is striking how different the implications of the model are under these alternative assumptions concerning technical progress. The conclusion one draws from these results depends on how seriously one believes that macroeconomic time series contain unit roots. If one has very strong priors in favor of unit roots, one must interpret these results as evidence against real business cycle models. Of course, the econometric tests carried out in work by Nelson and Plosser (1982), King, Plosser, Stock and Watson (1987) and others can not reject a highly persistent AR(1) process any more than they can reject difference-stationarity. In addition, recent work by Cochrane (1988) and DeJong and Whiteman (1988), using alternative methods, present evidence in favor of trend-stationarity over difference-stationarity. Thus, another interpretation of these results, if one takes the model studied in this paper seriously, is that the observation of large volatility of investment relative to output and large volatility of hours relative to productivity (or real wages) in the real world is additional evidence that the underlying shocks, and hence aggregate time series, are trend-stationary rather than difference-stationary.

References

Altonji, J.G., 1986, "Intertemporal Substitution in Labor Supply: Evidence From Micro Data," Journal of Political Economy 94, S176-S215.

Altug, S., 1985, "Time to Build and Aggregate Fluctuations: Some New Evidence," Manuscript (Federal Reserve Bank of Minneapolis, MN).

Cho, J.O. and T.F. Cooley, 1988, "Employment and Hours Over the Business Cycle," Manuscript (University of Rochester, Rochester, NY).

Christiano, L.J., 1986, "On the Accuracy of Linear Quadratic Approximations: An Example," Manuscript (Federal Reserve Bank of Minneapolis, MN).

______________, 1987a, "Dynamic Properties of Two Approximate Solutions to a Particular Growth Model," Manuscript (Federal Reserve Bank of Minneapolis, MN).

______________, 1987b, "Is Consumption Insufficiently Sensitive to Innovations in Income?," American Economic Review 77, No. 2, 337-341.

______________, 1988, "Why Does Inventory Investment Fluctuate So Much?," Journal of Monetary Economics 21, 247-280.

Christiano, L.J. and M. Eichenbaum, 1988, "Is Theory Really Ahead of Measurement? Current Real Business Cycle Theories and Aggregate Labor Market Fluctuations," Manuscript (Federal Reserve Bank of Minneapolis, MN).

Cochrane, J.H., 1988, "How Big is the Random Walk in GNP?," Journal of Political Economy 96, 893-920.

Danthine, J.P., J.B. Donaldson and R. Mehra, 1988, "On Some Computational Aspects of Real Business Cycle Theory," Manuscript (Sloan School of Management, MIT, Cambridge, MA).

Debreu, G., 1954, "Valuation Equilibrium and Pareto Optimum," Proceedings of the National Academy of Sciences 40, 588-592.

DeJong, D.N. and C.H. Whiteman, 1988, "Trends and Random Walks in Macroeconomic Time Series: A Reconsideration Based on the Likelihood Principle," Manuscript (University of Iowa, Iowa City, IA).

Hansen, G.D., 1985, "Indivisible Labor and the Business Cycle," Journal of Monetary Economics 16, 309-327.

__________, 1986, "Growth and Fluctuations," Manuscript (University of California, Los Angeles, CA).

Hansen, G.D., E.R. McGrattan and T.J. Sargent, 1988, "An Equilibrium Model of Straight Time and Overtime Employment for the United States," Manuscript (Hoover Institution, Stanford University, Stanford, CA).

Hansen, G.D. and T.J. Sargent, 1988, "Straight Time and Overtime in Equilibrium," Journal of Monetary Economics 21, 281-308.

Hodrick, R.J. and E.C. Prescott, 1980, "Post-War U.S. Business Cycles: An Empirical Investigation," Working Paper (Carnegie-Mellon University, Pittsburgh, PA).

King, R.G., C.I. Plosser and S.T. Rebelo, 1988, "Production, Growth and Business Cycles I. The Basic Neoclassical Model," Journal of Monetary Economics 21, 195-232.

King, R.G., C.I. Plosser, J. Stock and M. Watson, 1987, "Stochastic Trends and Economic Fluctuations," Manuscript (University of Rochester, Rochester, NY).

Kydland, F.E. and E.C. Prescott, 1982, "Time to Build and Aggregate Fluctuations," Econometrica 50, 1345-1370.

__________, 1988, "The Workweek of Capital and its Cyclical Implications," Journal of Monetary Economics 21, 343-360.

Long, J.B., Jr. and C.I. Plosser, 1983, "Real Business Cycles," Journal of Political Economy, 91, 39-69.

Lucas, R.E., Jr., 1977, "Understanding Business Cycles," Carnegie-Rochester Conference Series on Public Policy 5, 7-29.

__________, 1988, "On the Mechanics of Economic Development," Journal of Monetary Economics 22, 3-42.

MaCurdy, T.E., 1981, "An Empirical Model of Labor Supply in a Life-Cycle Setting," Journal of Political Economy 89, 1059-1085.

Nelson, C.R. and C.I. Plosser, 1982, "Trends and Random Walks in Macroeconomic Time Series," Journal of Monetary Economics 10, 139-162.

Prescott, E.C., 1983, "Can the Cycle be Reconciled with a Consistent Theory of Expectations? or a Progress Report on Business Cycle Theory," Manuscript (Federal Reserve Bank of Minneapolis, MN).

__________, 1986, "Theory Ahead of Business Cycle Measurement," Quarterly Review, (Federal Reserve Bank of Minneapolis, MN).

Rogerson, R., 1988, "Indivisible Labor, Lotteries and Equilibrium," Journal of Monetary Economics 21, 3-16.

Romer, P., 1986, "Increasing Returns and Long Run Growth," Journal of Political Economy 94, 1002-1073.

Sargent, T.J., 1987, "Dynamic Macroeconomic Theory," (Harvard University Press, Cambridge, MA).

Table 1 Summary Statistics from Quarterly U.S. Data (55,3-84,1)[24]

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.74	1.00					
Consumption (C)	.81	.65	1.00				
Investment (X)	8.45	.91	.42	1.00			
Capital Stock (K)	.63	.05	.17	-.10	1.00		
Hours (H)	1.41	.86	.50	.79	.15	1.00	
Productivity (Pr)	.89	.59	.47	.54	-.14	.10	1.00

---

[24] The U.S. time series reported on are real GNP, consumption of nondurables and services plus the flow of services from durables, and gross private domestic investment (all in 1982 dollars). The capital stock series includes nonresidential equipment and structures. The hours series is total hours for persons at work as derived from the Current Population Survey. Productivity is output divided by hours. All series are seasonally adjusted, logged and detrended. The standard deviations have been multiplied by 100. The output, investment, capital stock and hours series were taken from the Citibase database. The consumption series was provided by Larry Christiano.

Table 2 Summary Statistics Assuming Deterministic Growth[25]

A. No growth (μ = 0, δ = .025)

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.74 (0.22)	1.00					
Consumption (C)	.50 (0.08)	.86 (0.03)	1.00				
Investment (X)	5.68 (0.73)	.99 (0.00)	.78 (0.04)	1.00			
Capital Stock (K)	.48 (0.09)	.07 (0.06)	.55 (0.07)	- .06 (0.06)	1.00		
Hours (H)	1.32 (0.17)	.98 (0.00)	.76 (0.05)	1.00 (0.00)	- .12 (0.05)	1.00	
Productivity (Pr)	.50 (0.07)	.87 (0.02)	.99 (0.00)	.80 (0.03)	.54 (0.07)	.77 (0.04)	1.00

B. Growth (μ = .005, δ = .025)

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.70 (0.22)	1.00					
Consumption (C)	.54 (0.08)	.87 (0.02)	1.00				
Investment (X)	5.18 (0.67)	.99 (0.00)	.78 (0.04)	1.00			
Capital Stock (K)	.52 (0.10)	.10 (0.07)	.57 (0.07)	- .05 (0.06)	1.00		
Hours (H)	1.26 (0.16)	.98 (0.01)	.75 (0.05)	1.00 (0.00)	- .11 (0.05)	1.00	
Productivity (Pr)	.54 (0.08)	.88 (0.02)	1.00 (0.00)	.80 (0.03)	.56 (0.07)	.75 (0.04)	1.00

---

25 The statistics reported are the sample means of statistics computed for each of fifty simulations of 115 periods. The standard deviations have been multiplied by 100. The sample standard deviations of these statistics are reported in parentheses. Each simulated time series was logged and detrended using the same procedure that was applied to the U.S. sample reported on in Table 1.

C. Growth (μ = .005, δ = .02)

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.74 (0.22)	1.00					
Consumption (C)	.50 (0.08)	.86 (0.03)	1.00				
Investment (X)	5.69 (0.74)	.99 (0.00)	.78 (0.04)	1.00			
Capital Stock (K)	.48 (0.09)	.07 (0.06)	.55 (0.07)	- .06 (0.06)	1.00		
Hours (H)	1.33 (0.17)	.98 (0.00)	.76 (0.05)	1.00 (0.00)	- .12 (0.05)	1.00	
Productivity (Pr)	.50 (0.07)	.87 (0.02)	.99 (0.00)	.80 (0.03)	.54 (0.07)	.77 (0.04)	1.00

Table 3 Effect of Serial Correlation in Technological Change[26]

A. Reduced Serial Correlation (γ = .90)

Series	Standard Deviation	Y	C	X	K	H	Pr
Output (Y)	1.89 (0.24)	1.00					
Consumption (C)	.43 (0.07)	.76 (0.04)	1.00				
Investment (X)	6.58 (0.85)	.99 (0.00)	.67 (0.05)	1.00			
Capital Stock (K)	.52 (0.10)	.05 (0.06)	.67 (0.07)	- .07 (0.05)	1.00		
Hours (H)	1.58 (0.20)	.99 (0.00)	.64 (0.06)	1.00 (0.00)	- .12 (0.05)	1.00	
Productivity (Pr)	.43 (0.07)	.78 (0.03)	.99 (0.00)	.69 (0.04)	.66 (0.06)	.66 (0.05)	1.00

B. Increased Serial Correlation (γ = .99)

Series	Standard Deviation	Y	C	X	K	H	Pr
Output (Y)	1.44 (0.19)	1.00					
Consumption (C)	.62 (0.09)	.96 (0.01)	1.00				
Investment (X)	3.98 (0.52)	.99 (0.00)	.90 (0.02)	1.00			
Capital Stock (K)	.35 (0.07)	.07 (0.07)	.35 (0.08)	- .06 (0.06)	1.00		
Hours (H)	.86 (0.11)	.98 (0.01)	.88 (0.03)	1.00 (0.00)	- .13 (0.06)	1.00	
Productivity (Pr)	.62 (0.08)	.96 (0.01)	1.00 (0.00)	.91 (0.02)	.35 (0.08)	.88 (0.03)	1.00

---

26 See footnote for Table 2.

Table 4 Summary Statistics Assuming Stochastic Growth[27]

A. Random Walk

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.31 (0.17)	1.00					
Consumption (C)	.68 (0.09)	.98 (0.01)	1.00				
Investment (X)	3.25 (0.42)	.99 (0.00)	.94 (0.02)	1.00			
Capital Stock (K)	.28 (0.06)	.07 (0.07)	.28 (0.08)	- .06 (0.06)	1.00		
Hours (H)	.67 (0.09)	.98 (0.01)	.91 (0.02)	.99 (0.00)	- .14 (0.06)	1.00	
Productivity (Pr)	.68 (0.09)	.98 (0.01)	1.00 (0.00)	.94 (0.01)	.28 (0.08)	.91 (0.02)	1.00

B. First Order Autocorrelation in ϵ_t (ρ = -0.1)

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.26 (0.15)	1.00					
Consumption (C)	.62 (0.09)	.96 (0.01)	1.00				
Investment (X)	3.22 (0.37)	.99 (0.00)	.91 (0.01)	1.00			
Capital Stock (K)	.26 (0.05)	.07 (0.07)	.32 (0.07)	- .08 (0.06)	1.00		
Hours (H)	.68 (0.08)	.97 (0.01)	.86 (0.02)	.99 (0.00)	- .17 (0.05)	1.00	
Productivity (Pr)	.62 (0.08)	.96 (0.01)	1.00 (0.00)	.91 (0.01)	.32 (0.07)	.86 (0.02)	1.00

27 See footnote for Table 2.

C. Increased First Order Autocorrelation in ϵ_t (ρ = -0.2)

Series	Standard Deviation	Correlation Matrix					
		Y	C	X	K	H	Pr
Output (Y)	1.22 (0.13)	1.00					
Consumption (C)	.58 (0.08)	.94 (0.01)	1.00				
Investment (X)	3.26 (0.33)	.98 (0.00)	.86 (0.02)	1.00			
Capital Stock (K)	.24 (0.05)	.06 (0.06)	.35 (0.07)	- .10 (0.05)	1.00		
Hours (H)	.71 (0.07)	.96 (0.01)	.80 (0.02)	.99 (0.00)	- .19 (0.05)	1.00	
Productivity (Pr)	.58 (0.08)	.94 (0.01)	1.00 (0.00)	.86 (0.01)	.35 (0.07)	.80 (0.02)	1.00

Table 5 Relative Volatilities[28]

Statistic	Data	$\gamma=.90$	$\gamma=.95$	$\gamma=.99$	$\gamma=1.0$	$\alpha=-.1$	$\alpha=-.2$
σ_c/σ_y	.47	.23	.29	.43	.52	.49	.48
σ_x/σ_y	4.86	3.48	3.27	2.76	2.48	2.56	2.67
σ_k/σ_y	.36	.28	.28	.24	.21	.21	.20
σ_h/σ_y	.81	.84	.76	.60	.51	.54	.58
σ_h/σ_{pr}	1.58	3.67	2.66	1.39	.99	1.10	1.22

---

[28] The relative volatilities reported in this table are the ratios of standard deviations reported in tables 1 through 4. In particular the numbers in the seven columns correspond to statistics reported in tables 1, 3A, 2C, 3B, 4A, 4B and 4C, respectively.

Table 6 Decision Rules

Deterministic Growth

No growth ($\mu = 0$, $\delta = .025$)

$$\log H_t = -0.03978 + 0.94181\ z_t + -0.47654 \log K_t$$

$$\log K_{t+1} = 0.14174 + 0.09935\ z_t + 0.94182 \log K_t$$

Growth ($\mu = .005$, $\delta = .02$)

$$\log H_t = -0.04105 + 0.94297\ z_t + -0.47684 \log \hat{K}_t$$

$$\log \hat{K}_{t+1} = 0.14113 + 0.09913\ z_t + 0.94200 \log \hat{K}_t$$

Reduced Serial Correlation ($\gamma = .90$)

$$\log H_t = -0.04105 + 1.13848\ z_t + -0.47684 \log \hat{K}_t$$

$$\log \hat{K}_{t+1} = 0.14113 + 0.11638\ z_t + 0.94200 \log \hat{K}_t$$

Increased Serial Correlation ($\gamma = .99$)

$$\log H_t = -0.04105 + 0.61551\ z_t + -0.47684 \log \hat{K}_t$$

$$\log \hat{K}_{t+1} = 0.14113 + 0.07024\ z_t + 0.94200 \log \hat{K}_t$$

Stochastic Growth

Random Walk (Using Solution Method 1)

$$\log H_t = -0.04105 + 0.47702\ z_t + -0.47684 \log \hat{K}_t$$

$$\log \hat{K}_{t+1} = 0.14113 + 0.05802\ z_t + 0.94200 \log \hat{K}_t$$

Random Walk (Using Solution Method 2)

$$\log H_t = -0.04103 + 0.47685\ \epsilon_t + -0.47685 \log \tilde{K}_t$$

$$\log \tilde{K}_{t+1} = 0.14113 + -0.94200\ \epsilon_t + 0.94200 \log \tilde{K}_t$$

Serial Correlation in ϵ_t (ρ = -0.1)

$$\log H_t = -0.04103 + 0.57408\ \epsilon_t + -0.47685 \log \tilde{K}_t$$

$$\log \tilde{K}_{t+1} = 0.14113 + -0.93342\ \epsilon_t + 0.94200 \log \tilde{K}_t$$

Increased Serial Correlation in ϵ_t (ρ = -0.2)

$$\log H_t = -0.04103 + 0.65603\ \epsilon_t + -0.47685 \log \tilde{K}_t$$

$$\log \tilde{K}_{t+1} = 0.14113 + -0.92619\ \epsilon_t + 0.94200 \log \tilde{K}_t$$

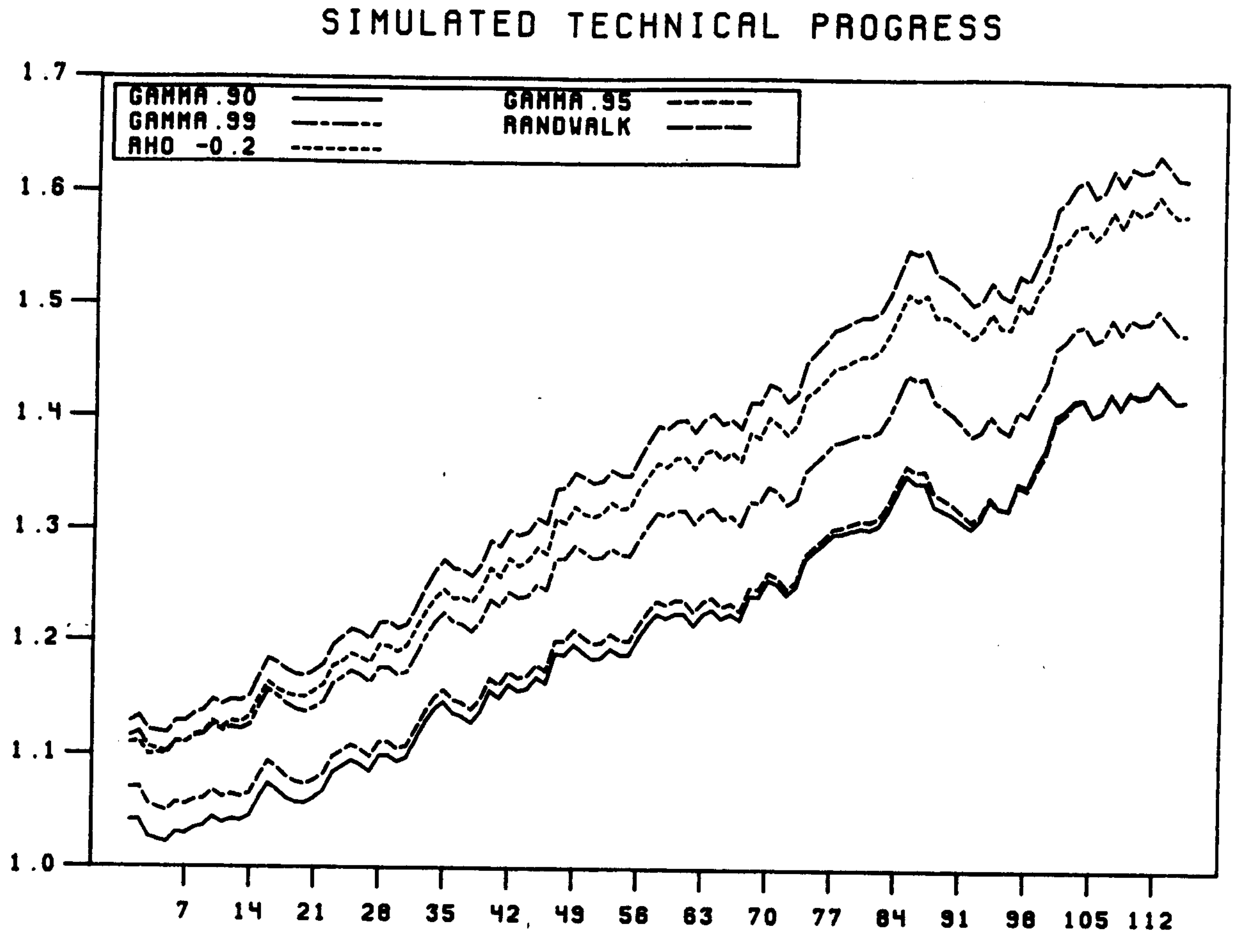
SIMULATED TECHNICAL PROGRESS
GAMMA .90
GAMMA .95
GAMMA .99
RANDWALK
RHO -0.2
1.7
1.6
1.5
1.4
1.3
1.2
1.1
1.0
7
14
21
28
35
42
49
56
63
70
77
84
91
98
105
112

FIGURE 1

RESPONSE OF Z TO INNOVATION IN Z

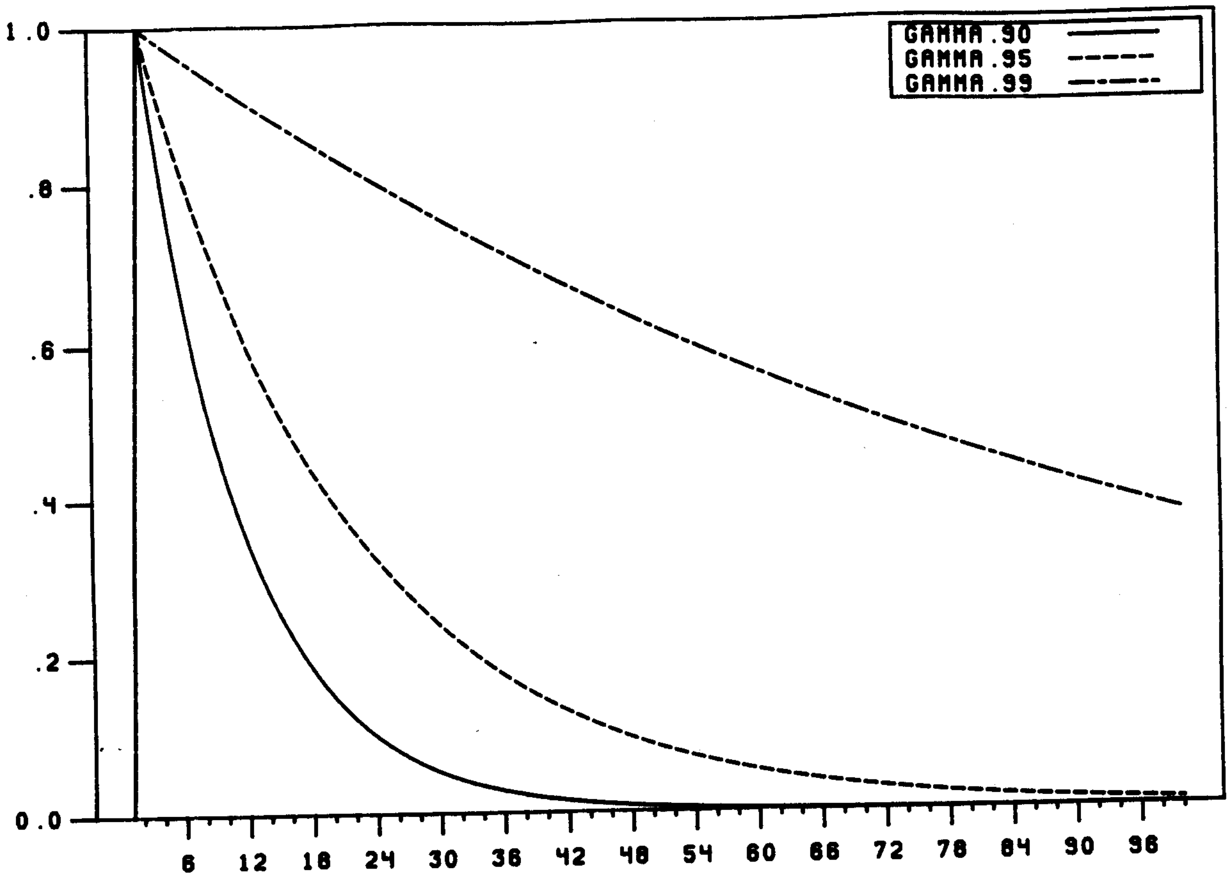

FIGURE 2A

RESPONSE OF Z TO INNOVATION IN Z

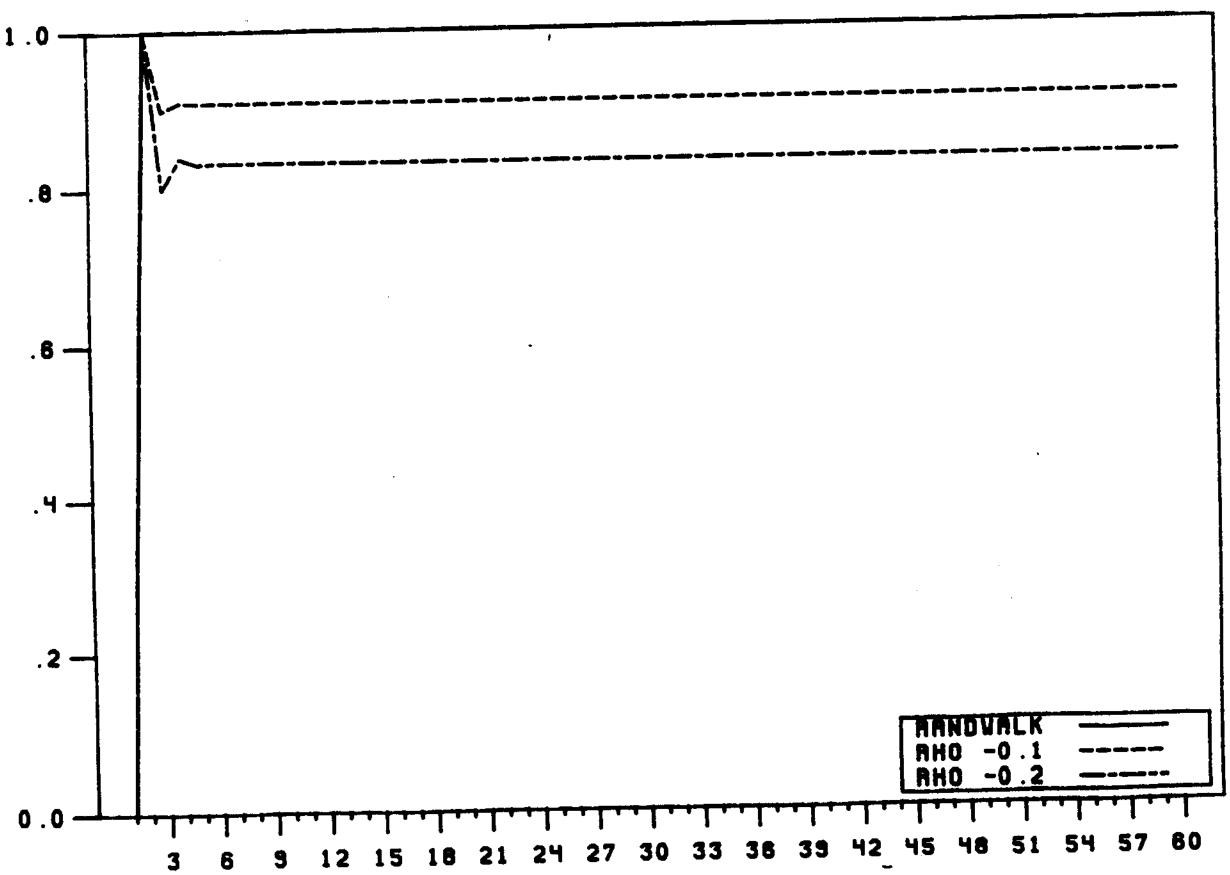

FIGURE 2B

RESPONSE OF LOG $\hat{K}$ TO INNOVATION IN Z

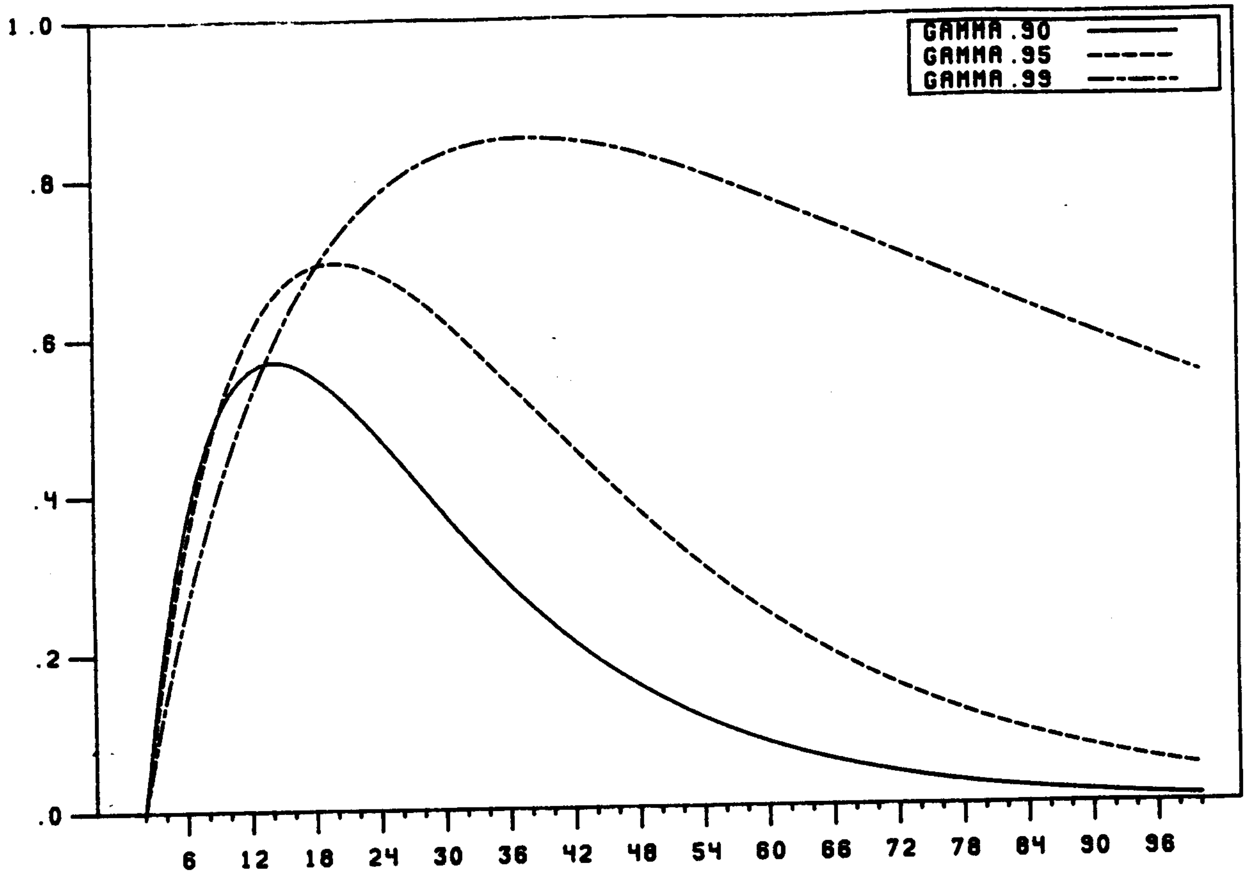

FIGURE 3A

RESPONSE OF LOG $\hat{K}$ TO INNOVATION IN Z

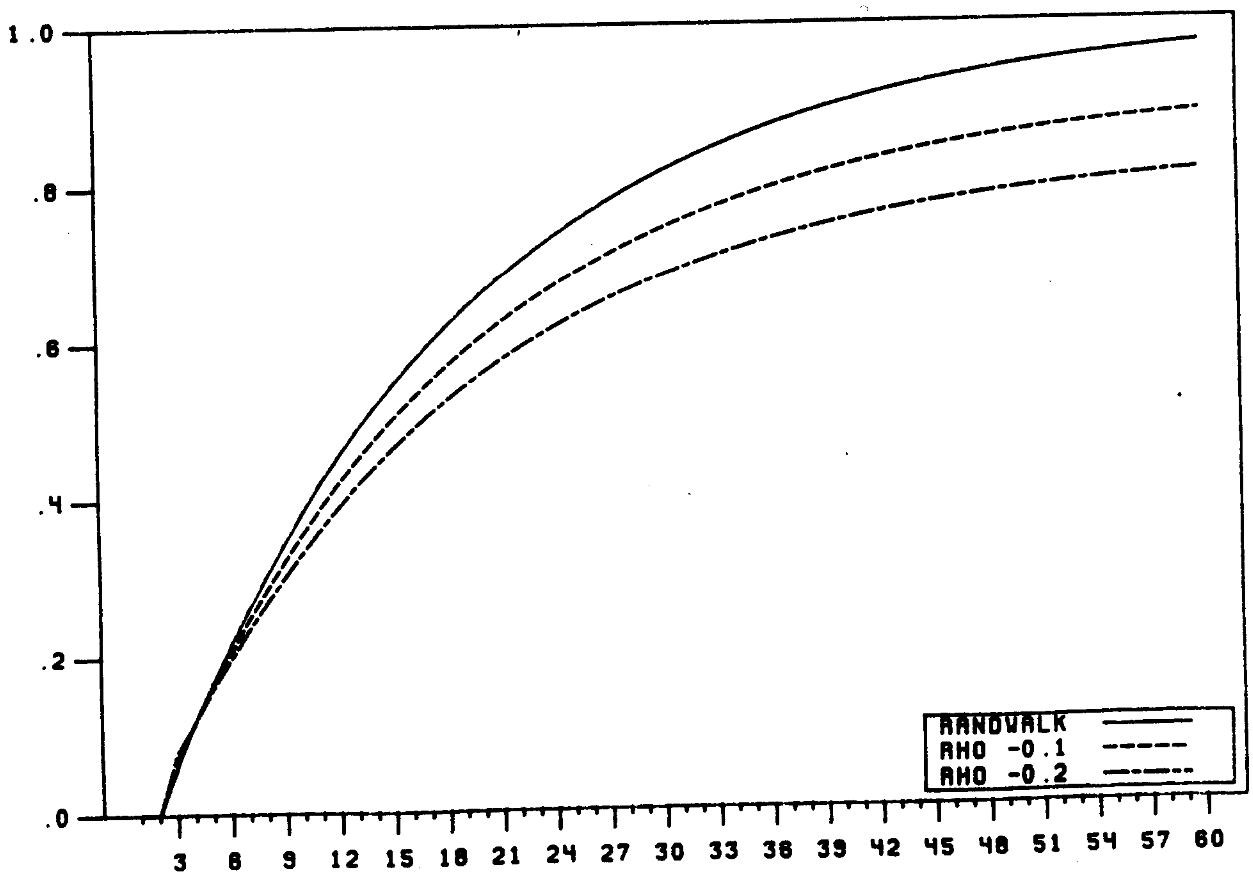

FIGURE 3B

RESPONSE OF LOG $\hat{X}$ TO INNOVATION IN Z

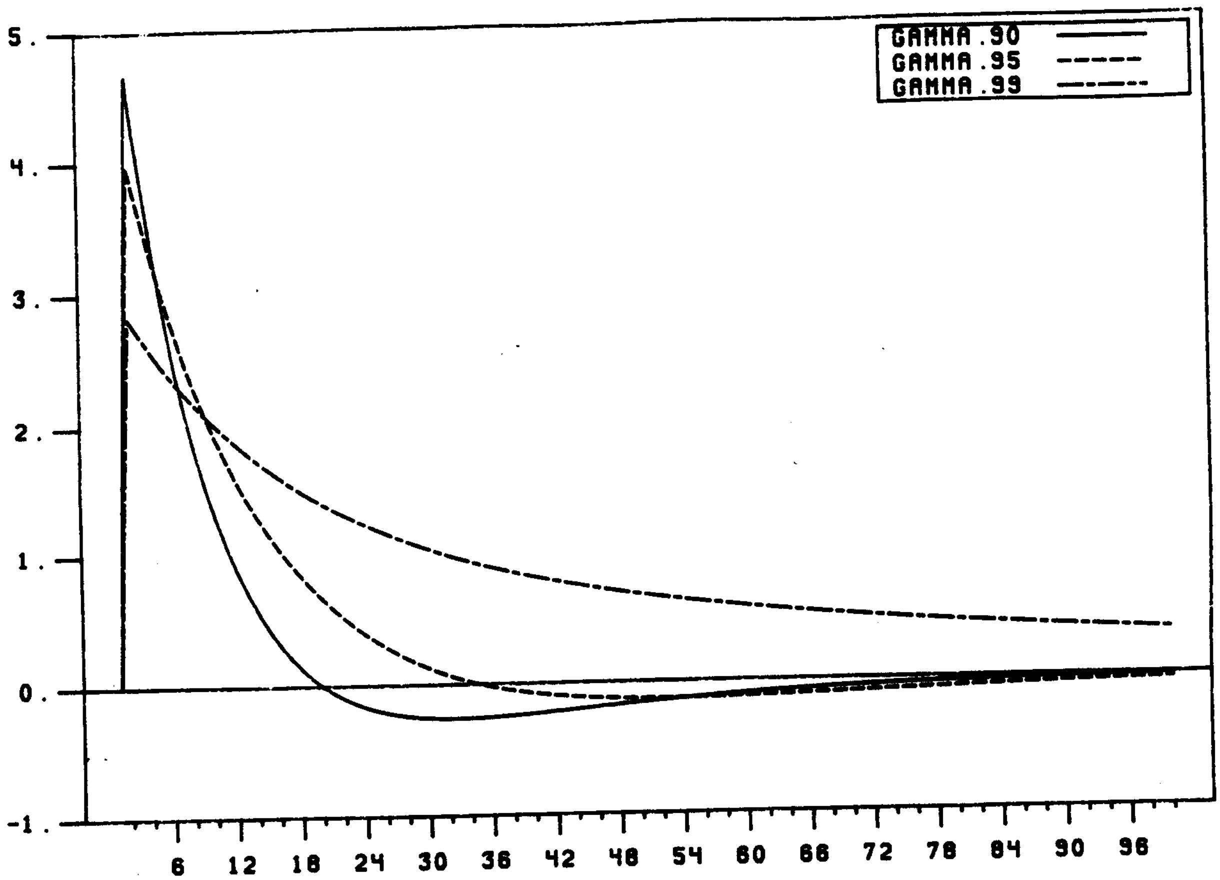

FIGURE 4A

RESPONSE OF LOG $\hat{X}$ TO INNOVATION IN Z

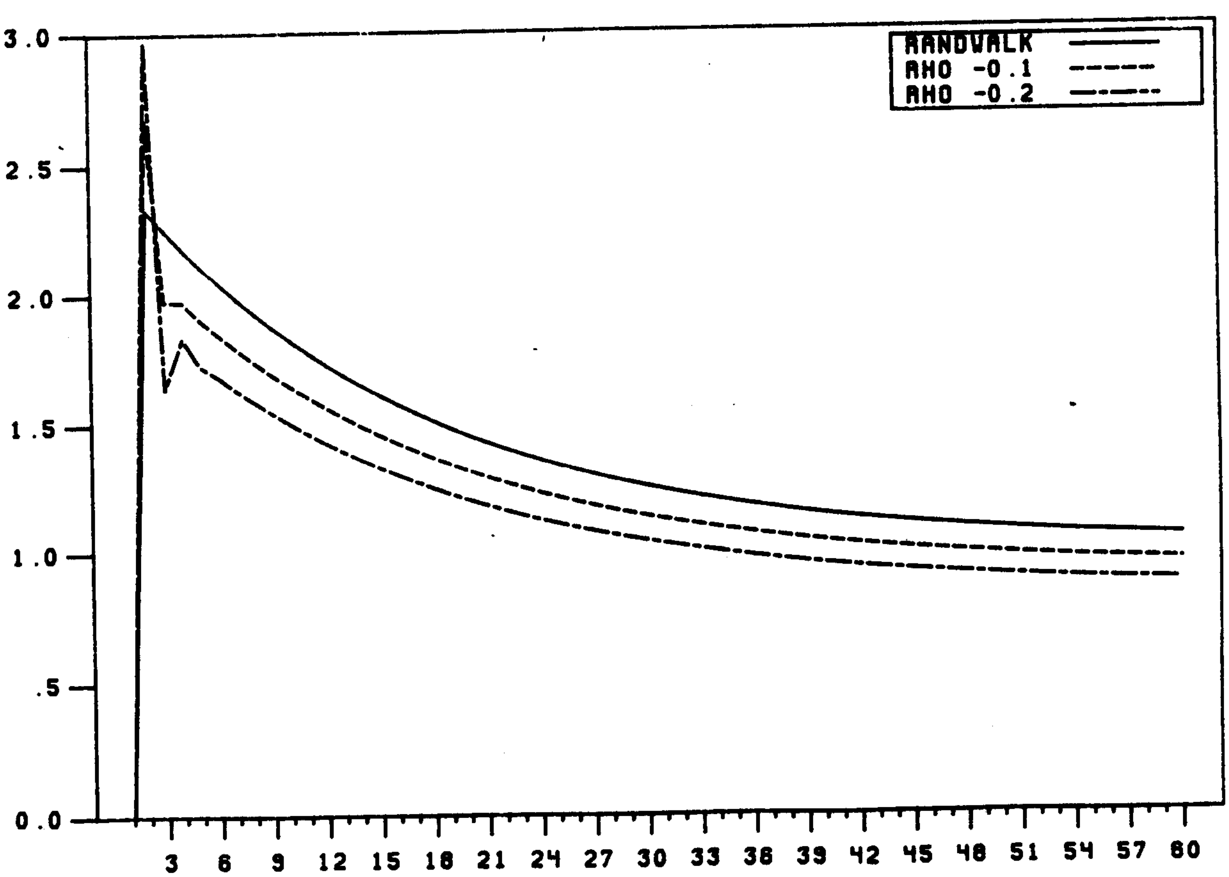

FIGURE 4B

RESPONSE OF LOG H TO INNOVATION IN Z

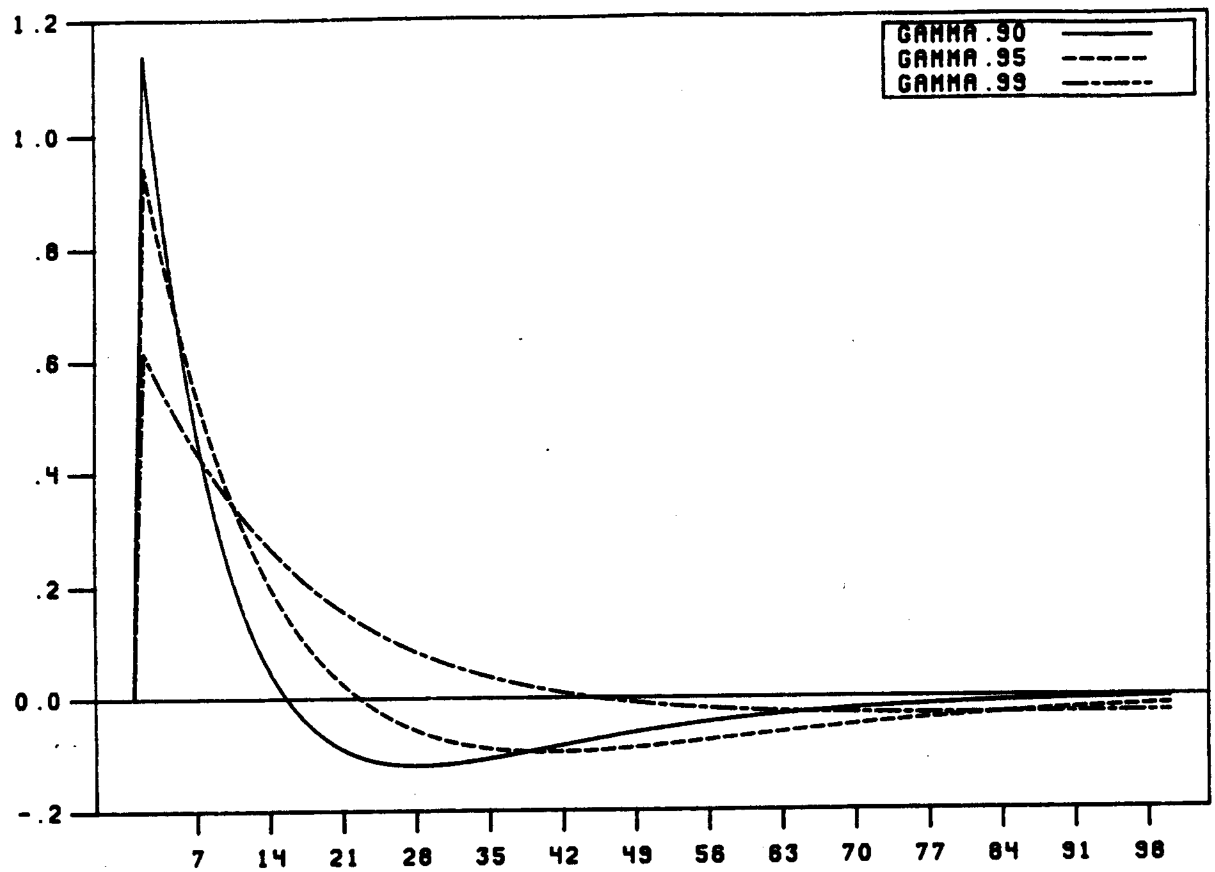

FIGURE 5A

RESPONSE OF LOG H TO INNOVATION IN Z

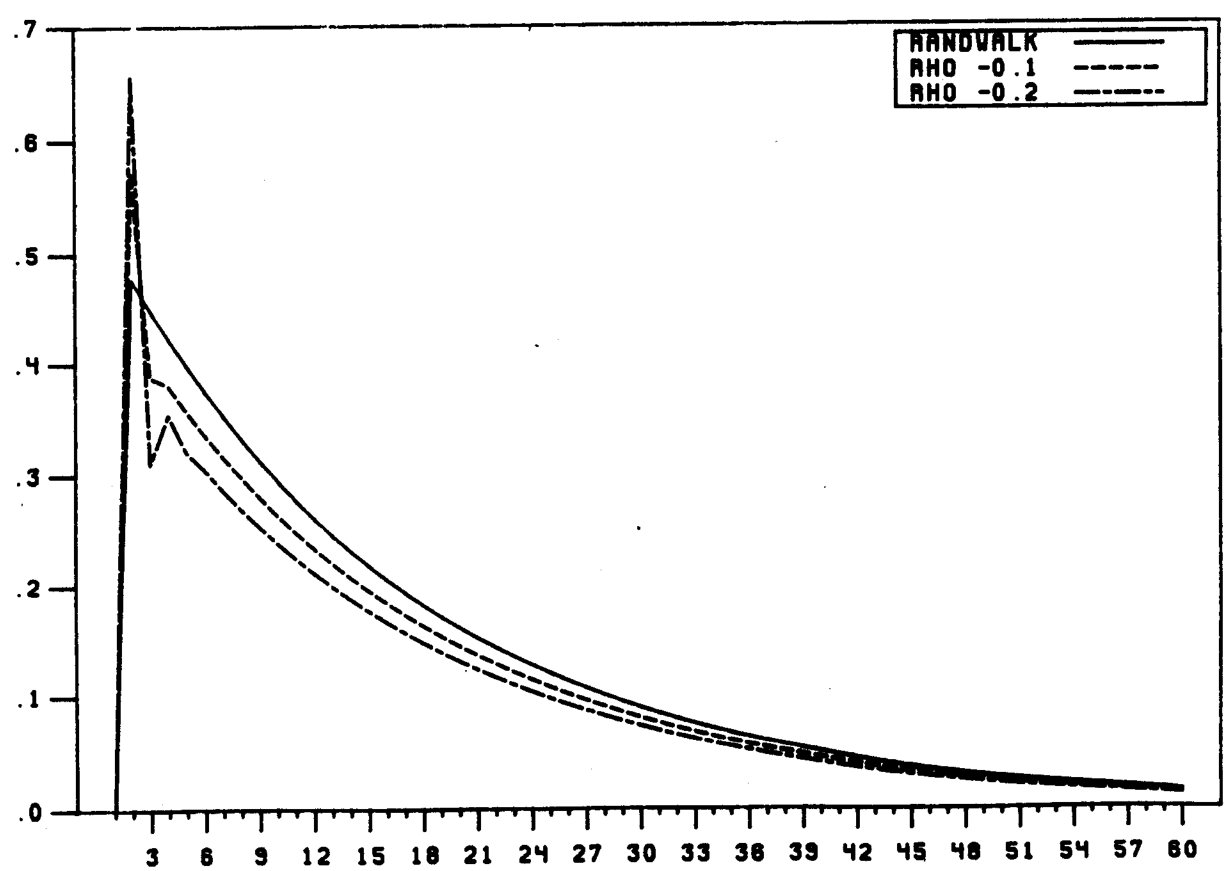

FIGURE 5B

RESPONSE OF LOG $\hat{Y}$ TO INNOVATION IN Z

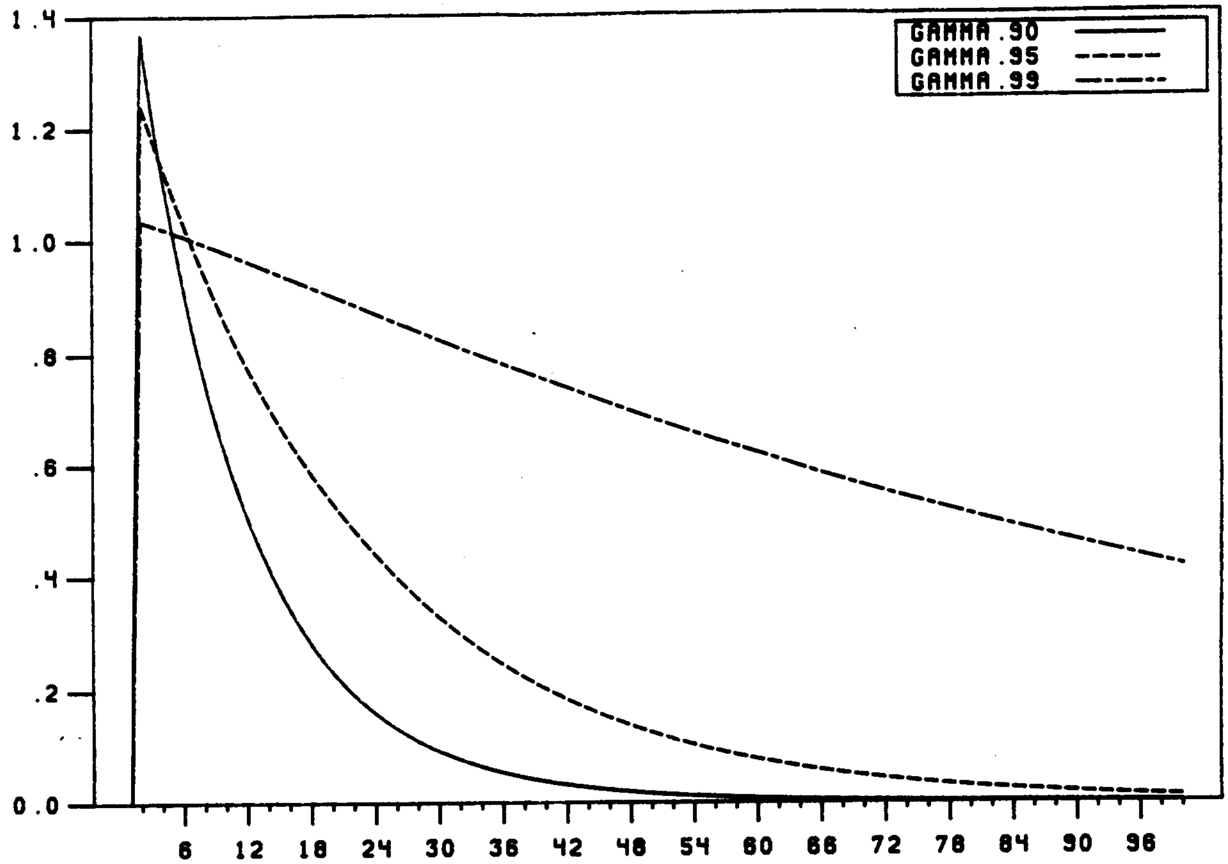

FIGURE 6A

RESPONSE OF LOG $\hat{Y}$ TO INNOVATION IN Z

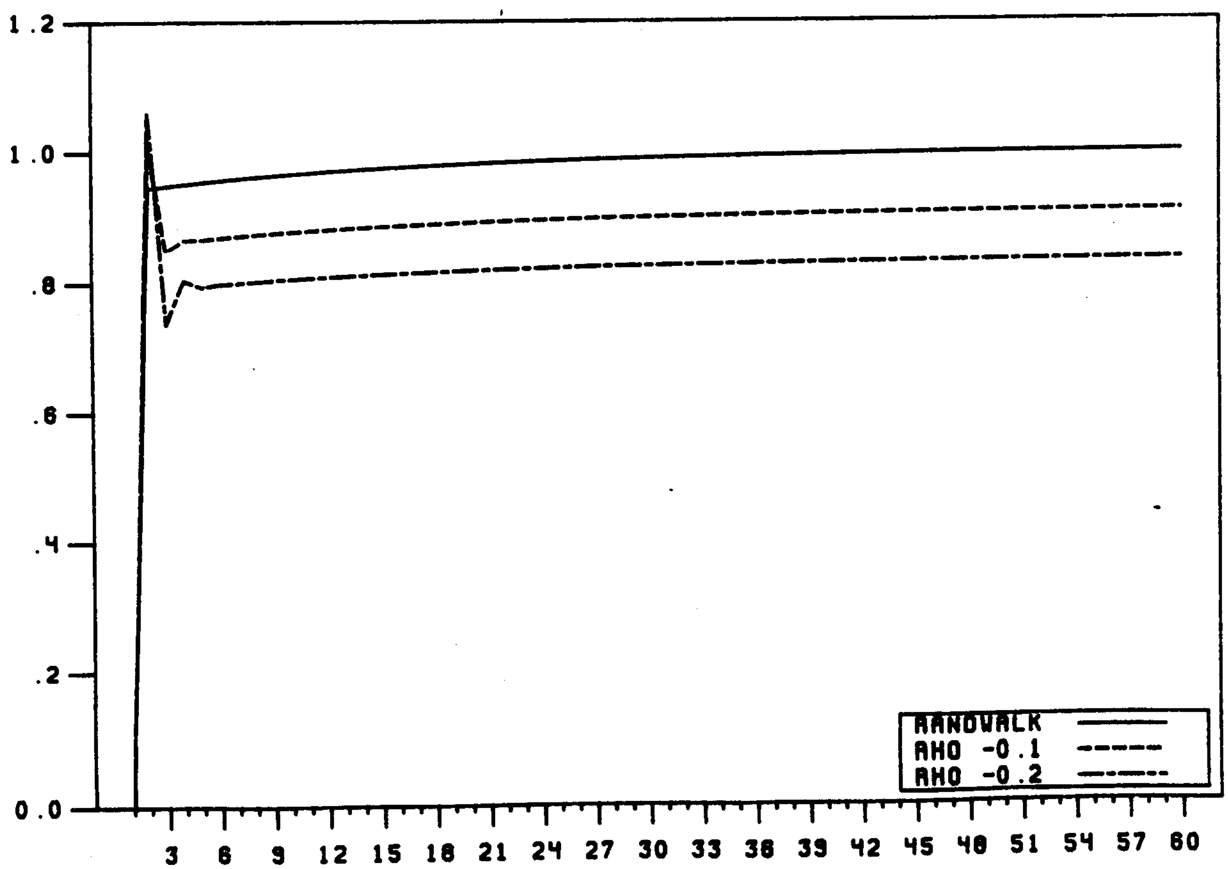

FIGURE 6B

RESPONSE OF LOG $\hat{C}$ TO INNOVATION IN Z

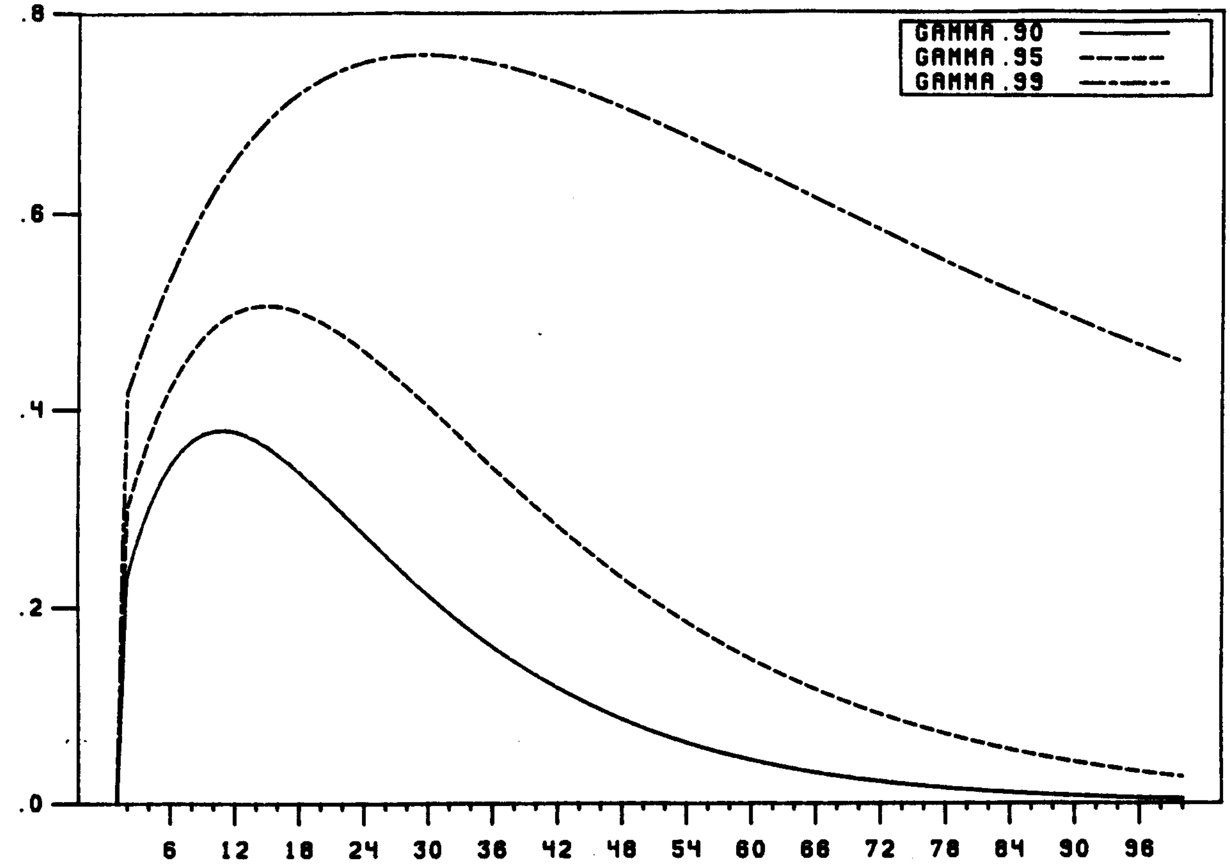

FIGURE 7A

RESPONSE OF LOG $\hat{C}$ TO INNOVATION IN Z

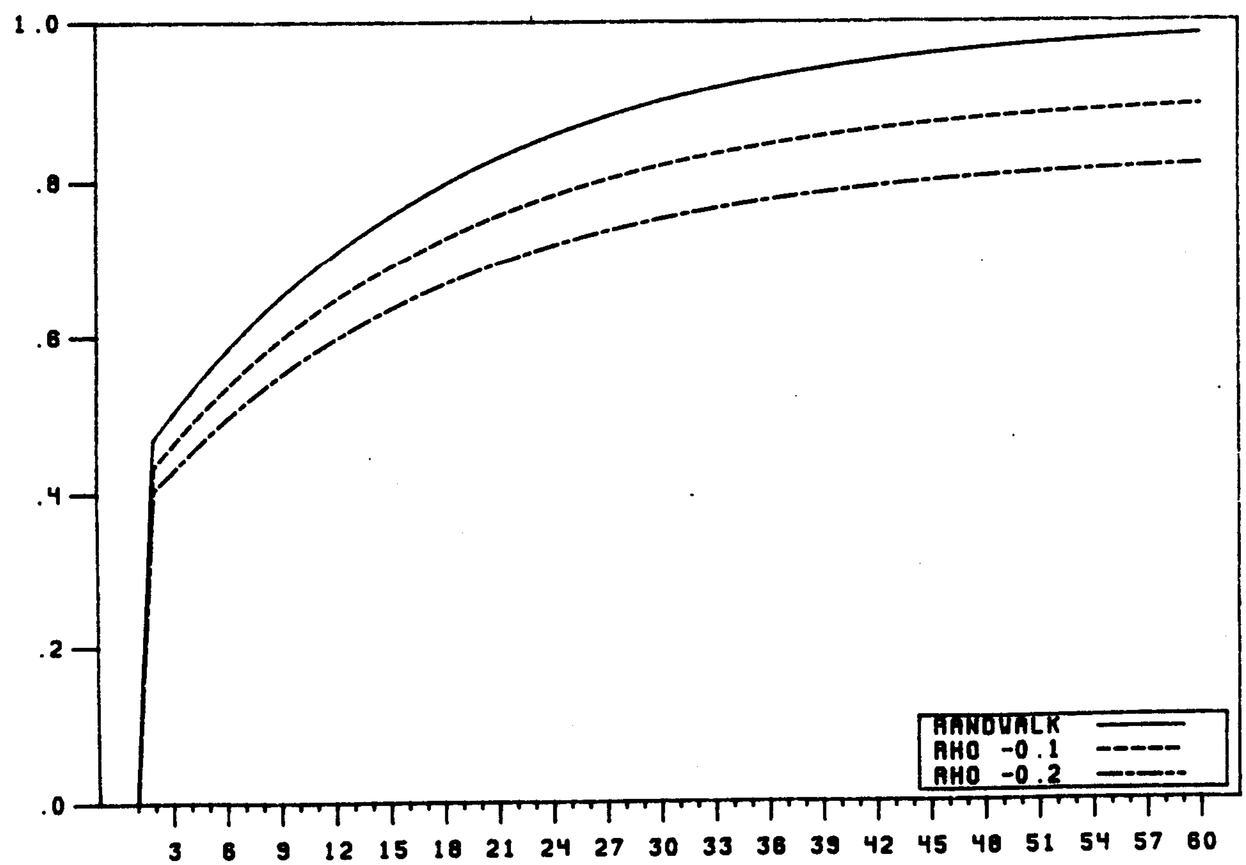

FIGURE 7B