RINCE PREFERENCES *

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ABSTRACT

This paper presents a class of preferences that yield closed form solutions to dynamic stochastic choice problems. These preferences are based on a set of axioms that were proposed by Kreps and Porteus. The Kreps Porteus axioms allow one to separate an agent's attitudes to risk from his or her intertemporal elasticity of substitution. RINCE preferences have the properties of RIsk Neutrality and Constant Elasticity of substitution.

I INTRODUCTION

There are many instances of stochastic intertemporal choice problems that one would like to be able to solve in closed form. But it is generally recognized that, if one maintains the axioms of Von Neumann and Morgenstern (VNM), such problems quickly become intractable. In this paper, I show that a slight weakening of the VNM axioms that was originally explored by Kreps and Porteus [1978, 1979:1, 1979:2], henceforth (KP), allows one to find a convenient parameterization of utility that may be explicitly solved to yield closed form decision rules. These decision rules determine optimal actions as functions of current state variables and of the expected values of certain functions of future state variables.

The parametric structure that I propose exploits the fact that KP preferences are able to separate an agent's attitudes to risk from his or her intertemporal elasticity of substitution. This separation allows one to make the simplifying assumption that agents are indifferent to income risk whilst maintaining a non-trivial preference for the time at which consumption occurs. A decision maker with the preferences that I describe is risk neutral, in the above sense, but he or she displays a constant elasticity of intertemporal substitution in environments where there is no uncertainty. For this reason I refer to these preferences as Risk Neutral Constant Elasticity, abbreviated as RINCE.

The KP axioms take the basic space over which preferences are defined to be a space of temporal lotteries. This space is more complex than the space of lotteries over consumption sequences since elements of the space are distinguished not only by probability distributions over possible payoff sequences but also by the time at which uncertainty resolves. This more complicated structure implies that individuals may express a preference or aversion for the resolution of uncertainty even if knowledge of the future does not yield a planning advantage. This structure may be contrasted to the VNM approach under which the time at which uncertainty resolves does not directly influence one's choices.

II A GUIDE TO THE LITERATURE

This section is intended to place my own work in the context of some recent literature that uses a non Von Neumann-Morgenstern approach to address problems of intertemporal choice under uncertainty. This literature is related to the non VNM approaches discussed by Machina [1982] but it is more directly concerned with choice through time.

Until recently almost all work that deals with intertemporal choice under uncertainty has maintained the complete set of VNM axioms defined on the space of lotteries over intertemporal consumption sequences. The seminal papers that drop this approach are by Kreps and Porteus [1978, 1979:1, 1979:2]. Kreps and Porteus maintain all but one of the VNM axioms, including the independence of irrelevant alternatives, although the space over which these axioms is formulated is more complicated than the space of lotteries over consumption sequences. The axiom that KP relax is a temporal analogue of the reduction of compound lotteries. They do not permit compound gambles to be reduced to simple gambles if such a reduction involves changing the time at which uncertainty is resolved. In the absence of the reduction axiom, one cannot reduce gambles over sequences of lotteries to gambles over sequences of consumption bundles.

An independent series of papers that is related to the KP approach is due to Selden [1978,1979]. Selden's approach deals only with two period decision problems and it is essentially equivalent to the two period version of Kreps and Porteus. The Selden approach, however, cannot easily be generalized to a multi-period context since the most straightforward generalization does not impose the requirement that an agent's plans should be temporally consistent.

A number of extensions of Kreps and Porteus have appeared in work by Chew and Epstein [1987:1, 1987:2], Epstein [1986] and Epstein and Zin [1987:1]. Chew and Epstein have explored the implications of dropping the independence axiom for intertemporal choice problems and Epstein [1986] has formulated an alternative axiomatization of the KP approach and an extension to an important class of infinite horizon problems. The closest work to my own paper is contained in the two pieces by Epstein and Zin [1987:1, 1987:2] and a related, but independent, paper by Weil [1987]. In the first of the Epstein-Zin papers the authors present a parametric class of intertemporal preferences that is based on a generalization of the Kreps-Porteus approach. RINCE preferences are members of this class and, to my knowledge, they are the only members that permit one to obtain closed form solutions to the intertemporal stochastic choice problem that I describe below.

A limited number of papers have applied the KP approach to specific problems. These papers fall into two categories and they all exploit the idea that Kreps-Porteus preferences allow one to disentangle an agent's degree of risk aversion from the intertemporal elasticity of substitution. If one imposes the VNM axioms, and in addition one restricts attention to time separable preferences, these two concepts are inseparable. For example, the preferences given by $U = \sum_{t=0}^{T} \beta^t c_t^{\rho}/\rho$ are often referred to as constant relative risk aversion although they could equally well be called constant elasticity of substitution since one concept is the reciprocal of the other.

Papers by Weil [1987] and Kotcherlakota [1987:1] have examined the possibility that one may explain the equity premium puzzle of Mehra and Prescott [1985] by making use of the flexibility provided by the ability of KP preferences to separate risk from intertemporal substitution. The applied work of Epstein and Zin [1987:2] and the papers by Hall [1985], Attanasio and Weber [1987] and Kotcherlakota [1987:2] address the related issue of whether representative agent models that make use of this separation are able to more accurately model time series data.

III THE STANDARD APPROACH TO INTERTEMPORAL STOCHASTIC CHOICE

Consider the problem faced by a mortal consumer who must make a finite sequence of savings decisions when the future is uncertain. In the standard representation of this problem one assumes that rational choice is characterized by the solution to a dynamic programming problem of the following type:

$$(1)\qquad \max_{\{\hat{c}_t\}_{t=0}^{T}} u(c_0) + E\sum_{t=1}^{T}\beta^t u(c_t) \qquad \text{such that;}$$

$$(2)\qquad a_1 = R_0 a_0 + \omega_0 - c_0;$$

$$(3)\qquad a_{t+1} = \tilde{R}_t a_t + \tilde{\omega}_t - c_t; \qquad t = 1, 2, \ldots, T;$$

$$(4)\qquad R_0 a_0 = \bar{R}_0 \bar{a}_0;$$

$$(5)\qquad a_{T+1} \geq 0.$$

The function $U \equiv \sum_{t=0}^{T}\beta^t u(c_t)$ may be interpreted as a von Neumann-Morgenstern utility function defined over the space of consumption sequences $\{c_t\}_{t=0}^{T}$ where the consumption set is taken to be R_+^{T+1}. The tildes over the variables $\tilde{R}_t$ and $\tilde{\omega}_t$ are used to denote the assumption that they are random variables and the interpretation of the sequence of constraints (3) is that the individual receives endowments $\{\tilde{\omega}\}_{t=1}^{T}$ which may be invested in a single risky asset. The asset a_t is assumed to pay a gross return $\tilde{R}_t$ and in general I shall allow for the possibility that the sequences $\{\tilde{\omega}\}_{t=1}^{T}$, and $\{\tilde{R}_t\}_{t=1}^{T}$ are jointly distributed random variables that may take values in R_+^{2T}. The expectation operator that appears in equation (1) has the interpretation of an expectation taken over the joint probability distribution of $\{\tilde{\omega}_s, \tilde{R}_s\}_{s=t+1}^{T}$ conditional on the realizations of $(\tilde{\omega}_s, \tilde{R}_s)$ for all $s \leq t$.

A solution to equation (1) is represented by a number, $\hat{c}_0$, and a sequence of functions $\hat{c}_t : R_+^{2t} \mapsto R_+$, $t = 1, \ldots, T$, where $\hat{c}_t$ is interpreted as a contingent plan. It represents a list of actions, one for every possible realization of past values of ω and R, that the consumer proposes to undertake in period t.

Stated in this way, this problem is a direct application of expected utility theory which has a distinguished history dating back at least to Bernouilli. But the application of expected utility theory to the choice of intertemporal consumption sequences makes no reference to the temporal nature of the consumer's problem. The axioms of atemporal expected utility theory are typically justified by an appeal to simple thought experiments in which it is suggested that a violation of one or other of the von Neumann-Morgenstern axioms would be irrational; the discussion of the Allais paradox in Raiffa [1970; page 80 ff] is a good example of this approach. But temporal versions of such arguments are not as compelling as their atemporal counterparts. The Kreps-Porteus framework provides a rationalization of a violation of the VNM axioms that can be directly traced to the sequential nature of decisions.

IV THE RELATIONSHIP OF THE KREPS-PORTEUS AXIOMS TO THOSE OF VON NEUMANN AND MORGENSTERN

Kreps and Porteus provide two alternative axiomatizations of their approach. One set of axioms views choice as a sequence of decisions. At each stage in the sequence, the agent ranks alternative pairs of payoffs; each such pair consists of a current consumption bundle and a ticket to a lottery which will take place in the following stage. The prizes in the lottery represent the maximum possible utilities that the agent could hope to achieve in different states of nature. In this formulation of the problem, preferences for one-step-ahead lotteries obey the complete set of VNM axioms. The sequence of one-step decision problems is knitted together with a time consistency axiom. KP also provide a second formulation of the agent's preference ordering in which axioms are formulated directly over a space of temporal lotteries. For the sake of completeness, a description of this second approach is provided below.

To describe the KP axioms it is necessary to introduce some notation. Let d_T be a probability distribution over c_T and let D_T be the set of all such distributions. One may think of the individual, at the beginning of period T, expressing preferences over lotteries for period T consumption; these lotteries are the elements of D_T. Now imagine the individual who stands at the beginning of period T-1. This person must express preferences over uncertain gambles which may resolve partly in T-1 and partly in period T. In the VNM approach these preferences are defined by formulating axioms over the set of lotteries that yield a compound prize of consumption commodities part of which is paid in period T-1 and part of which is paid in period T. In the Kreps-Porteus approach preferences are defined over a more complicated object; that is, the set of lotteries that yield an uncertain consumption payoff in period T-1 and a ticket to another lottery that takes place in period T. In the absence of the reduction of compound lotteries axiom these approaches are not identical.

To formalize this idea one defines recursively the sets, $\{D_t\}_{t=T-1}^{0}$, of probability distributions over $R_+ \times D_{t+1}$. For example, an element of D_{T-1} is a probability distribution, d_{T-1}, which represents the probability of receiving consumption c_{T-1} in conjunction with the lottery ticket d_T. The payoff to the lottery d_{T-2} is the pair (c_{T-2}, d_{T-1}) which is an element of $R_+ \times D_{T-1}$. Carrying this recursion backwards one arrives at the set of *temporal lotteries*, D_0, which is the basic space over which the KP axioms are defined.

An additional piece of notation is required in order to characterize those subsets of D_0 which describe the possible positions at which a decision maker may find him or herself at a given point in time. Let $h_t \equiv \{c_0, c_1, \ldots, c_t\}$ be a consumption *history*. Now define the set $P_t(h_t)$ to consist of those lotteries in D_0 for which the decision maker will recieve the history h_t with probability one. An element of $P_t(h_t)$, denoted $p_t(h_t)$, will give the decision maker a non-stochastic consumption sequence, h_t, and a ticket to a lottery $d_{t+1} \in D_{t+1}$. Notice that if one denotes the first k elements of h_t by $h_k(h_t)$ then $P_k(h_k(h_t)) \supseteq P_t(h_t)$. This follows since one of the possible sequences of lotteries that leads from k to t is the sequence in which the decision maker receives the realizations $\{c_{k+1}, \ldots, c_t\}$ with probability one. It follows that $P_{t-1} \supseteq P_t$ and, by induction, that the sets $\{P_t\}_{t=1}^{T}$ are all contained in D_0.

The key difference between the KP and VNM representations of choice hinges on an agent's attitude towards the timing of the resolution of uncertainty. Imagine standing at the beginning of period zero and choosing between two elements of $P_t(h_t)$ for some $t > 0$. Each of these lotteries contains the same non-stochastic consumption sequence up to date t but possibly different distributions over uncertain events that resolve beyond date t. Now think of mixing two of these lotteries by flipping a coin that comes up heads with probability α and tails with probability $1-\alpha$ *but flip the coin at date $k < t$.* This new mixture is an element of $P_k(h_k)$, where $h_k \equiv h_k(h_t)$. Let the mixed distribution be represented by the quadruple $(k, \alpha; p, p')$ where p and p' are elements of $P_t(h_t)$. A decision maker whose preferences admit an expected utility representation over consumption sequences must be indifferent to the timing of the coin flip in the experiment described above. A KP individual may, on the other hand, prefer either early or late resolution of uncertainty. The following three axioms characterize KP choice.

A: 1 *There exists a complete transitive ordering, $\succeq$, over the elements of D_0.*

A: 2 *The relation, $\succeq$, is continuous on D_0.*

A: 3 *If $p, p' \in P_t(h_t)$ satisfy $p \succ p'$ then $(t, \alpha; p, p'') \succ (t, \alpha; p', p'')$ for all $\alpha \in [0,1)$ and $p'' \in P_t(h_t)$.*

The key axiom, A:3, is a temporal version of the independence of irrelevant alternatives. Kreps and Porteus [1978, page 195] present a representation theorem based on axioms A1, A2 and A3. This theorem asserts that one may represent choice by a sequence of utility functions, $\{w_t\}_{t=T-1}^{0}$, each of which maps $R^+ \times R \mapsto R$. The first argument of each function is consumption in period t and the second argument, denoted v_{t+1}, represents the solution to a programming problem that takes place in period t+1; that is:

$$v_{t+1} = \max_{c_{t+1}} w(c_{t+1}, E_t v_{t+2}).$$

The value of consumption that may be chosen in each of these programming problems is constrained by the sequence of budget sets:

$$a_{t+1} = R_t a_t + \omega_t - c_t; \qquad t = T-1, \ldots, 0.$$

In period T the consumer maximizes a function w_T which is defined over terminal consumption alone. Given the maximal value of utility in period T, v_T, one can construct the sequence of value functions, $\{v_t\}_{t=T-1}^{0}$, which represents the sequence of maximal utilities attainable in each period. In contrast to the VNM approach these value functions will not generally be linear in probabilities. The relationship with von Neumann-Morgenstern choice is given by the following axiom which, in conjunction with the other three axioms, implies the existence of a single von Neumann-Morgenstern utility function over the space of intertemporal consumption sequences.

A: 4 *For all t,* $h_t; \alpha \in [0,1]$ *and* $p, p' \in P_t(h_t), (\mathrm{t}, \alpha; p, p') \sim (\mathrm{t}-1, \alpha; p, p')$.

If axiom A:4 holds then agents are indifferent to the timing of the resolution of uncertainty. In this case their preferences may be reduced to lotteries over intertemporal consumption sequences and KP preferences are identical to VNM. On the other hand — if axiom A:4 does not hold — KP preferences define a much broader class of intertemporal stochastic decision rules. In this sense, axiom A:4 implies that the difference between KP and VNM choice hinges solely on the issue of preference for, or aversion to, the timing of the resolution of uncertainty.

V THE VALUE FUNCTION

Stochastic intertemporal choice problems are usually solved recursively by constructing a sequence of value functions. Beginning with the last period of the problem, one finds the optimal decision rule of a planner who enters period T with a given level of wealth. Given this decision rule, one can proceed to find the optimal allocative decision in period T-1 and, working backwards, one constructs a sequence of decision rules and an associated sequence of value functions. In the case of the expected utility example, equation (1), the sequence of value functions $\{v_t(a_t)\}_{t=0}^{T}$ is defined by the formulae:

$$v_T \equiv w(R_T a_T + \omega_T); \tag{6}$$

$$v_t(a_t) \equiv \max_{c_t}\{w(c_t) + \beta E_t[\tilde{v}_{t+1}(a_{t+1})]\}; \tag{7}$$

such that,

$$(8) \qquad a_{t+1} = R_t a_t + \omega_t - c_t; \qquad t = 0, 1 \ldots, T-1.$$

A great deal is known about the properties of the functions $\{v_t\}$ and for special cases one may obtain closed form solutions for the optimal decision rules. By restricting attention to the case of multiplicative uncertainty (random interest but deterministic endowments) one may obtain closed form solutions to the class of preferences $w(c_t) = c_t^{\rho}/\rho$. On the other hand, with only additive uncertainty (random endowment but deterministic interest rates) one can solve the quadratic case. But the general case of random interest *and* random endowments does not admit a closed form solution except in the trivial situation when w is an affine function. In this case the agent's preferences are linear, not only across states of nature, but also through time.

If one is willing to drop the assumption of timing indifference then the weaker axiom set A:1 – A:3 implies that the choice of intertemporal consumption sequences admits a value function representation where the value functions are defined as follows: [1]

$$(9) \qquad v_T(a_T) \equiv w_T(R_T a_T + \omega_T);$$

$$(10) \qquad v_t(a_t) \equiv \max_{c_t} w(c_t, E_t[\tilde{v}_{t+1}(a_{t+1})]);$$

such that,

$$(11) \qquad a_{t+1} = R_t a_t + \omega_t - c_t; \qquad t = 0, 1, \ldots, T-1.$$

Equation (10) differs from the VNM approach (equation (7)) in that v_t is nonlinear in the expectation operator E_t. This generalization would appear to complicate the problem and make things more, rather than less, difficult. However, by choosing the function w correctly one can find a class of decision problems that yield closed form solutions in a wide variety of situations.

I will return to the value function approach in section X in which I define a class of preferences that admit closed form representations for the sequence of functions $\{v_t\}$. Before taking up this issue, however, I will explore an alternative representation of choice that permits a more direct comparison of the Kreps-Porteus approach with the expected utility framework. This representation is the KP analogue of the expected utility index.

VI THE UTILITY INDEX

In this section I introduce the appropriate notion of the utility index for KP choice. In the case of VNM preferences the utility index is a function which takes, as its domain, the cartesian product of the real line with the space of probability distributions over R_+^T. Current consumption is an element of R_+ and lotteries over future consumption sequences are elements of the set of probability distributions over R_+^T. Decision making under uncertainty is frequently represented as the choice of a set of contingent plans that maximizes such an index subject to a sequence of constraints; that is, in the form of equation (1).

From the perspective of a decision maker at date zero, the utility index for this problem is given by the function;

$$U_0 = u(c_0) + E_0 \sum_{t=1}^{T} \beta^t u(c_t). \tag{12}$$

Because this function is both separable through time and linear in probabilities, one can ignore past choices if plans are reformulated at a later date. That is to say; a decision maker at date τ who uses the index;

$$U_\tau = u(c_\tau) + E_\tau \sum_{t=\tau+1}^{T} \beta^t u(c_t); \tag{13}$$

will make decisions that are consistent with the plans that were formed at date zero to maximize the index U_0. Linearity in probabilities and separability through time are sufficient but not necessary conditions to guarentee consistent planning. Kreps-Porteus preferences are also time consistent but the KP utility index is not linear in probabilities; it is constructed recursively.

Recall the definition of the sequence of sets $\{D_t\}_{t=0}^{T}$ that was introduced in section (IV). One may define a utility index $U_T : R_+ \mapsto R$ and a sequence of indices

$\{U_t\}_{t=T-1}^{0}$, using the following recursion:

$$U_T = w_T(c_T); \tag{14}$$

$$U_t = w(c_t, E_t\tilde{U}_{t+1}); \quad t = 0, 1, \ldots, T-1. \tag{15}$$

The index U_t maps from the space $R_+ \times D_{t+1}$ to the real line and it is the KP analogue of the VNM index defined in equation (12). The structure of this index is closely related to a class of preferences over non-stochastic sequences that Lucas and Stokey [1984] refer to as ***recursive***. Koopmans [1960] was the first to study preferences in this class and in view of the similarity of equation (15) to the Koopmans class I shall refer to $w : R_+ \times R \mapsto R$ as an ***aggregator function.*** Recursive preferences are easy to study because they allow one to construct a solution to a programming problem in steps using Bellman's principle of optimality.

The utility index is a convenient tool for analysing the properties of KP preferences. In the following section I will make use of this concept to discuss some of the implications of the KP axioms and to focus in on those areas in which a KP decision maker will behave in unfamiliar ways. The discussion is centered around two concepts — recursivity and risk aversion.

VII SOME PROPERTIES OF KP PREFERENCES

1. Recursivity

One is entitled to ask why we should complicate the theory of choice under uncertainty by introducing the concept of temporal lotteries. Why not stick to the more basic choice objects; that is, to lotteries over consumption sequences? The answer is that the concept of temporal lotteries allows us to separate risk from intertemporal preferences whilst retaining the very useful property of recursivity.

What would happen if we decided to give up on recursivity? A natural way of generating a change in an agent's attitude to risk without affecting his or her ordinal ranking of non-stochastic sequences would be to apply the multi-commodity analysis discussed in Kihlstrom and Mirman [1974] to the space of distributions over intertemporal consumption sequences. The Kihlstrom-Mirman approach is to define a family of utility functions U_F by taking increasing, concave transformations H_F of a basic utility function U. In the intertemporal context the decision maker would solve the problem:

$$\max_{\{c_t\}_{t=0}^{T}} E_0 H_F[U(c_0, c_1, \ldots, c_T)]. \tag{16}$$

By varying the curvature of H_F one could make the individual more or less risk averse without changing his or her preferences over non-random sequences. But the cost of this approach is that an agent's relative ranking of choices at date t necessarily depends on the entire history of past consumptions *and* on all of the possible choices that might be made in the future. Kreps-Porteus preferences allow one to break the link between risk and intertemporal substitution without giving up on recursivity.

Recursive preferences are defined in the non-stochastic environment by the assumption that the decision maker's ranking over future consumption sequences is independent of his or her ranking over current consumption bundles. The natural extension of this property to choice over temporal lotteries leads to the sequence of recursive indices defined by equations (14) and (15). It is the property of independence of future decisions from events that have occurred in the past that allows one to apply the maximum principle of dynamic programming to choice problems with a recursive structure.

2. A Definition of Risk Aversion

The natural extension — to intertemporal stochastic problems — of the static concept of risk aversion is not so obvious. Pratt's [1964] concept of the risk premium, p, for an expected utility function over wealth, is defined implicitly by:

$$U(\bar{x} - p) = EU(\tilde{x}); \tag{17}$$

where $\bar{x}$ is the mean of the uncertain wealth gamble $\tilde{x}$. For an intertemporal stochastic choice problem, income risk is not the same as consumption risk. For any given preference ordering over temporal lotteries, the way in which the agent assesses income risk will depend not only on the properties of the preference ordering but also on the market structure. This dependence follows from the way in which the functions $\{v_t\}$ are constructed; the market structure enters through the definitions of the budget sets that constrain the sequence of one period decision problems. It follows that any extension of the concept of risk aversion to temporal lotteries must describe either aversion to consumption risk or aversion to income risk; the two concepts will not generally co-incide.

It also follows from the properties of KP preferences that any useful measure of risk aversion should take into account the date at which uncertainty resolves. The following definition extends the Pratt measure of risk aversion to intertemporal consumption sequences. It is a measure of consumption risk that takes into account the issue of temporal resolution.

Let $\tilde{c}_{\tau t}$ represent the stochastic consumption sequence $\{\bar{c}_0, \ldots, \tilde{c}_t, \ldots, \bar{c}_T\}$ where the element $\tilde{c}_t$ is random with mean $\bar{c}_t$ and variance σ^2 and all other members of the sequence $\tilde{c}_{\tau t}$ are known with probability one. Let all uncertainty be resolved at date τ, and let $c^{\mathrm{P}}_{\tau t}$ represent the non-stochastic consumption sequence $\{\bar{c}_0, \ldots, \bar{c}_t - p_{\tau t}, \ldots, \bar{c}_T\}$.

Definition: The ***risk premium*** for the temporal lottery $\tilde{c}_{\tau t}$ is that value of $p_{\tau t}$ for which $U_s(\tilde{c}_{\tau t}) = U_s(c^{\mathrm{P}}_{\tau t})$, where $s < \tau$. That is, the decision maker is indifferent between the lotteries $\tilde{c}_{\tau t}$ and $c^{\mathrm{P}}_{\tau t}$, when these lotteries are evaluated using the utility index U_s.

Notice that the definition of $p_{\tau t}$ is independent of the date at which utility is evaluated providing that this evaluation takes place before the date at which uncertainty resolves. This property follows directly from recursivity. It does *not* follow, however, that $p_{\tau t}$ will be independent of τ, the date at which uncertainty is resolved. The risk premia p_{st} and p_{kt} will not generally be equal when $s \neq k$ since the Kreps-Porteus decision maker will be willing to pay an additional premium to resolve uncertainty either at an earlier or later date. One may capture this idea with the following definition:

Definition: The *timing premium* for early resolution of uncertainty, μ_k, is equal to the difference between the risk premia p_{k+1t} and p_{kt}: that is, $\mu_k = p_{k+1t} - p_{kt}$, where $k < t$.

In the discussion that follows, bear in mind that t refers to the date at which uncertainty occurs — all other elements of the consumption sequence $\{c_s\}_{s=0}^{T}$ are non-stochastic. The idea behind the definition of the timing premium is to compare two lotteries which differ only in the date at which uncertainty resolves. One of these lotteries resolves at date k and the other resolves at date k+1. The timing premium μ_k is the difference in the risk premia associated with these two lotteries where the risk premium of a lottery is the amount of certain period t consumption that the individual would forego in order to avoid the period t consumption risk. If μ_k is positive then the individual would be willing to pay a higher premium to avoid a risk that resolves late than to avoid a risk that resolves early. An individual with preferences of this kind would prefer to avoid the suspense associated with not knowing what will happen. On the other hand, if the timing premium is negative, then the individual would prefer not to know the result of the uncertainty until the very last minute. It is not difficult to come up with anecdotes which suggest that either type of behaviour is reasonable.

The case of preference for early resolution fits more easily with the intuition that we have developed from working with VNM preferences since it usual to think of situations in which there is some planning advantage to be gained from finding out early about the future. This is not what is being described by the above concept. An individual could prefer late resolution of consumption uncertainty but early resolution of income uncertainty in situations where the planning advantage from knowing early outweighs a basic preference for suspense. On the other hand — situations in which individuals express a desire to find out late — are inconsistent with von Neumann-Morgenstern choice. As examples of such situations — think of an individual who plans to watch a football game that will be televised after the match has taken place or a couple who are having a baby but profess a desire not to know the sex of the child until the moment of birth.

VIII COMPUTING RISK AND RESOLUTION PREMIA

In this section of the paper I provide a sketch of a method that may be used to calculate risk premia and timing premia for the class of utility indices given in equations (14) and (15). For this class, the risk premia and the temporal premia are related to each other by the following approximation which is valid for small values of σ^2, the variance of the underlying disturbance.[2]

$$(18) \qquad p_{tt} = (-w_{11}^{t}/w_{1}^{t})\sigma^2/2;$$

$$(19) \qquad \mu_{t-1} = (w_{22}^{t-1}/w_{2}^{t-1})w_{1}^{t}\sigma^2/2;$$

$$(20) \qquad p_{s-1t} = p_{st} - \mu_{s-1}, \qquad s = t, t-1, \ldots, 0;$$

$$(21) \qquad \mu_{s-1} = \mu_{s}(w_{22}^{s-1}/w_{2}^{s-1})(w_{2}^{s})^2/w_{22}^{s};$$

where the terms w_i^s and w_{ij}^s, $i,j \in \{1,2\}$ represent the first and second partial derivatives of the aggregator function, w, evaluated at the certain consumption sequence $\{\bar{c}_v\}_{v=s}^{T}$. The superscript, s, in these expressions refers to the date at which marginal utilities are evaluated; for example:

$$w_1^s \equiv \frac{\partial U_s}{\partial c_s} \equiv \frac{\partial w}{\partial c_s}(c_s, U_{s+1}).$$

Equation (21) is a difference equation which gives the value of the timing premium at date s-1 in terms of the timing premium at date s. For the finite horizon economy the marginal utility terms w_i^s and w_{ij}^s are time dependent constants, that is they do not depend on the value of the timing premium, at least for small values of σ^2. The boundary condition for the difference equation is equation (19) which gives the value of the initial timing premium; that is, the difference in the risk premia — for consumption risk that occurs in period t — between a risk that resolves at date t and a risk that resolves at date t-1.

A simplification of these relationships is available for the case in which the time horizon of the decision maker becomes very long; that is, as $T \to \infty$. An axiomatization of the infinite horizon problem is available in Epstein and Zin [1987:1]. In this case things become notationally less cumbersome since the partial derivatives of the function w become time independent, at least for constant sequences of the form $\{\bar{c}_s = \bar{c}\}_{s=0}^{\infty}$. The timing premia are given by the formulae:

$$\mu_{t-1} = (w_{22}/w_2)w_1\sigma^2/2. \tag{22}$$

$$\mu_{s-1} = w_2\mu_s; \qquad s = t-1, t-2, \ldots, 1. \tag{23}$$

Kreps and Porteus point out that it is the concavity or convexity of w in its second argument that determines whether the individual prefers late or early resolution of uncertainty. Preference for resolution is reflected in the sign of the timing premium. Equation (22) implies that if the aggregator function w is concave in its second argument — if $w_{22} < 0$ — then the decision maker would prefer to find out about period t uncertainty in period t rather than in period t-1. The reverse implication holds if w is convex. From equation (23) it follows that the timing premium will be negative (positive) at all dates s < t if $w_{22} < 0, (w_{22} > 0)$.

In the infinite horizon problem, it is interesting to ask how the timing premium behaves as the date, t, at which uncertainty occurs becomes further and further removed from the date, s, at which the uncertainty is resolved; that is, as $(\mathrm{s}-\mathrm{t}) \to \infty$. It follows from equation (23) that, in this situation, provided $w_2 < 1$, the timing premia will converge to zero; $\mu_s \to 0$, as $(\mathrm{s}-\mathrm{t}) \to \infty$. But the term w_2 is just the marginal rate of intertemporal substitution since in the infinite horizon problem:

$$U_2/U_1 = w_2 U_1/U_1 = w_2. \tag{24}$$

In the case of von Neumann-Morgenstern preferences the aggregator function is linear in its second argument; that is w_{22} is identically zero and w_2 is equal to the rate of time preference, β. It follows that, for VNM preferences the timing premium is zero at all time horizons. In the Kreps-Porteus case, the timing premia may be positive or negative and the rate at which the sequence decays, as (s-t) increases, depends on the endogenous rate of time preference w_2. A positive rate of time preference, $w_2 < 1$, is a necessary and sufficient condition for the sequence of timing premia defined by equations (22) and (23) to converge to zero.

The relationship between the risk premia for different resolution dates is given by equation (20) and the risk premium for a risk that resolves at the same date as the underlying uncertainty is given by equation (18) which is equivalent to the familiar Arrow-Pratt measure of absolute risk aversion in the case of atemporal choice.

IX A HOMOGENEOUS CLASS OF PREFERENCES

In this section I introduce a class of preferences for which the utility index U_s is homogeneous of degree γ in current consumption and in the value of future state dependent consumption. This class, which has been proposed by Epstein and Zin [1987:1] and independently by Weil [1987], has the convenient property of allowing the intertemporal elasticity of substitution and the co-efficient of relative risk aversion to be represented by two separate parameters. It is capable of capturing the behavior either of an individual who prefers early resolution of uncertainty or of one who prefers late resolution. These preferences are defined by choosing the functions w_T and w in equations (14) and (15) to be given by:

$$w_T = z^\gamma \tag{25}$$

$$w(x,y) = (x^\rho + \beta y^{\rho/\gamma})^{\gamma/\rho}. \tag{26}$$

The case in which $\gamma = 1$ is the case that defines RINCE preferences and it is the only member of this class[3] for which one can obtain closed form solutions to intertemporal stochastic choice problems when there is both rate of return and endowment uncertainty. For the special case in which there is no uncertainty, the KP utility index that is induced by equations (25) and (26) takes the degenerate form;

$$U_0 = [\sum_{t=0}^{T} \beta^t c_t^\rho]^{\gamma/\rho}. \tag{27}$$

The parameter γ defines a family of utility functions each of which have the same ordinal properties. The parameter ρ, on the other hand, captures the intertemporal curvature of these functions; ρ is related to the intertemporal elasticity of substitution η by the relationship;

$$\eta \equiv \frac{\partial \log(c_t/c_{t+1})}{\partial \log R_t} = \frac{1}{(\rho - 1)}. \tag{28}$$

In situations for which uncertainty is non-trivial, the parameter γ captures the decision maker's attitude towards risk. It's effect follows from the presence of terms of the form;

$$(E_t x^{\gamma/\rho})^{\rho/\gamma}$$

in the recursive equations that are used to construct the utility index U_0. If there is no uncertainty then these terms collapse. The special case of $\gamma = 1$ corresponds to a type of risk neutrality and it is this property that enables one to generate closed form solutions.

One may also show that, for this class of preferences, the decision maker will prefer early resolution of uncertainty if and only if $\rho > \gamma$. Since ρ is bounded above by 1 it follows that the risk neutral decision maker must prefer late resolution. This property is discussed in more depth in section (XI).

X THE PARAMETERIZATION OF RINCE PREFERENCES

In this section I describe the class of preferences that I call RINCE and I derive an exact solution for the sequence of consumption decisions that would be taken by a decision maker whose preferences were of this type. RINCE preferences are members of the homogeneous class described in section IX for which the homogeneity parameter γ is equal to one. The decision rule that describes optimal behavior in any period is constructed by solving the sequence of value functions described in equations (9) (10) and (11) when the functions w_T and w are given by:

$$w_T(z) \equiv z \tag{29}$$

$$w(x,y) \equiv \begin{cases} (x^\rho + \beta y^\rho)^{\frac{1}{\rho}}; & \text{if } \rho \neq 0; \\ x^{1-\beta} y^\beta; & \text{if } \rho = 0; \end{cases} \tag{30}$$

where $0 \leq \beta < 1$ and $\rho \leq 1$. Before providing explicit functional forms for the decision rules that determine $\{c_t\}_{t=0}^T$, it helps to introduce some additional notation. Define the functions F and $G : R_+ \mapsto R_+$ as follows:

$$F(x) = \begin{cases} (1 + \beta^{\frac{1}{1-\rho}} x^{\frac{\rho}{1-\rho}})^{\frac{1-\rho}{\rho}}; & \text{if } \rho \neq 0; \\ (1-\beta)^{1-\beta} \beta^\beta x^\beta; & \text{if } \rho = 0; \end{cases} \tag{31}$$

$$G(x) = \begin{cases} (1 + \beta^{\frac{1}{1-\rho}} x^{\frac{\rho}{1-\rho}})^{-1}; & \text{if } \rho \neq 0; \\ 1-\beta; & \text{if } \rho = 0. \end{cases} \tag{32}$$

The decision rule for consumption is most conveniently expressed in terms of two variables that resemble a compounded interest rate and a human wealth term. However, this analogy is not exact since these variables involve the parameters of the utility index. More precisely, the sequences of interest terms $\{Q_t\}_{t=0}^T$ and human wealth terms $\{h_t\}_{t=0}^T$ are defined recursively as follows:

$$F(Q_T) = 1; \tag{33}$$

$$Q_t = E_t[\tilde{R}_{t+1} F(\tilde{Q}_{t+1})]; \quad t = 0,\ldots,T\text{-}1; \tag{34}$$

$$h_T = \omega_T; \tag{35}$$

$$h_t = \omega_t + E_t[\tilde{h}_{t+1} F(\tilde{Q}_{t+1})/Q_t]; \quad t=0,\ldots,T\text{-}1. \tag{36}$$

The variable Q_t depends on the moments of the distribution of all future interest rates and on the preference parameters β and ρ. Notice from equation (36) that the terms $F(\tilde{Q}_t)/Q_t$ act as stochastic discount rates on the future endowment sequence $\{\omega_t\}$. The term h_t may be thought of as "perceived human wealth" in view of the analogous role that it plays to human wealth in the non-stochastic case. One may then define total wealth W_t as:

$$(37) \qquad W_t \equiv R_t a_t + h_t; \qquad t = 0, 1, \ldots, T.$$

W_t consists of the market value of physical assets, plus the subjectively discounted value of future endowments. Given these definitions, the sequences of decision rules $\{c_t\}_{t=0}^{T}$ and value functions $\{v_t\}_{t=0}^{T}$ are given by the following equations;

$$(38) \qquad c_t = G(Q_t)W_t; \qquad t = 0, 1, \ldots, T-1;$$

$$(39) \qquad c_T = W_T;$$

$$(40) \qquad v_t = F(Q_t)W_t; \quad t=0,1,\ldots,T.$$

The system of equations (33) - (37) gives explicit rules for determining the values of the variables Q_t and W_t in terms of the conditional moments of the joint endowment-return process $\{\tilde{\omega}_s, \tilde{R}_s\}_{s=t+1}^{T}$. One may therefore summarize the behavior of an individual with preferences of this type by keeping track of two rather simple functional equations.

Some special cases of this model may prove helpful in establishing the relationship of these preferences to more familiar examples of non-stochastic utility functions. Over non-stochastic choice problems the RINCE decision maker will behave very much like a von Neumann-Morgenstern individual whose preferences are described by the additively separable function:

$$(41) \qquad U = \sum_{t=0}^{T} \frac{\beta^t c_t^\rho}{\rho}.$$

For problems of this kind the utility index of RINCE preferences reduces to the function given in equation (27) when the parameter γ is set equal to one. In situations of risky choice, however, it follows from equations (38), (37) and (36) that the decision rule of the RINCE agent is linear in probabilities and that it is only the first moment of the one-step-ahead endowment that affects his or her consumption choice. It is in this sense that RINCE preferences display risk neutrality.

A second case that is of interest is that in which the sequences $\{\tilde{R}_t\}_{t=1}^{T}$ and $\{\tilde{\omega}_t\}_{t=1}^{T}$ are independent of each other and in which each of these sequences is independently distributed through time. In this case the variable h_t is given by the expression:

$$h_t = \omega_t + \frac{E_t[\tilde{\omega}_{t+1}]}{E_t[\tilde{R}_{t+1}]} + \frac{E_t[\tilde{\omega}_{t+2}]}{E_t[\tilde{R}_{t+1}]E_t[\tilde{R}_{t+2}]} + \cdots \frac{E_t[\tilde{\omega}_T]}{\prod_{s=1}^{T-t} E_t[\tilde{R}_{t+s}]}. \tag{42}$$

If ω_t and R_t are non-stochastic then this expression reduces to the familiar definition of human wealth. The expression for consumption given by equation (38) is, in this case, identical to the expression that is given by the 'constant relative risk aversion' preferences described in equation (41). In the case of $\rho = 0$ and stochastic but independent endowment-return processes the consumption function takes the form:

$$c_t = (1 - \beta)W_t; \tag{43}$$

where W_t is the sum of human and non-human wealth terms and human wealth is obtained by discounting the first moments of the endowment process by the first moments of the return process — equation (42). In the case where interest rates are serially correlated, the discount factor will no longer be equal to the mean of the return process because interest rates contain information about the future. Serial correlation will carry a 'resolution premium' which reflects the agent's basic preferences over the timing of the revelation of information. The limiting case of these preferences, as $T \to \infty$, is of special interest since it provides an exact representation of Friedman's permanent income hypothesis. By exploiting the separation between risk aversion and intertemporal elasticity of substitution that is provided by the Kreps-Porteus structure, RINCE preferences are able to incorporate the simplifying assumption that agents are risk neutral without trivializing intertemporal choice.

XI SOME PROPERTIES OF RINCE PREFERENCES

In section VII I introduced the concepts of the risk premium and the timing premium and I showed that an agent will prefer early (late) resolution of uncertainty if the aggregator function w is convex (concave) in its second argument. Since the aggregator function that defines RINCE preferences — equation (30) — is concave it follows that a RINCE decision maker will necessarily prefer late over early resolution. Some insight into this issue can be gained from thinking about the sequence of risk premia defined by equations (18) and (20). Suppose that we want to model an individual who is risk neutral in the sense that he or she is indifferent between a period t consumption risk which resolves at date t, and its mean, for all periods t=1,...,T. This means that the risk premium p_{TT} must be zero. How will this individual respond to period T risks that resolve earlier than date T? It follows from equation (20) that, if he or she is to be risk averse, or at least risk neutral, in the sense that $p_{st} \geq 0$ for all s $\leq$ t then the timing premium must be non-positive; that is, the individual must weakly prefer late resolution.

What is going on here? A priori — it seems plausible that a risk neutral agent could also prefer early resolution of uncertainty. But our concept of risk neutrality, in which the value function is linear in the appropriate measure of expected human wealth, excludes this possibility. What seems to be happening is that the curvature of the period utility function, when preferences are von Neumann-Morgenstern, provides a natural planning advantage to early resolution. That is; von Neumann-Morgenstern preferences generate value functions in which an agent has a natural preference for early resolution of income lotteries. In order to counteract this natural tendency for prefering early resolution; the agent's preferences over consumption lotteries must incorporate a basic desire for late resolution.

Although a preference for late resolution of uncertainty is not necessarily unreasonable, it does have some counter-intuitive implications in the present context. For example, in the case in which T=1 (2 periods), all wealth is non-human and $\rho > 1/2$, it can be shown that an agent would strictly prefer not to be told the realization of the rate of interest before making his or her consumption decision for the first period, even though that information would be used to alter his or her decision if it were available. Suppose, first, that the agent will not find out the realization of the interest rate until period 1. In this case the value of expected utility at date 0 would be given by:

$$v_0 = F(E_0[\tilde{R}_1])R_0 a_0$$

Now suppose that the agent is able to find out the realization of $\tilde{R}_1$ before deciding how much to consume in period 0. In this situation the realized value of utility from the standpoint of period 0 would be given by:

$$v_{0'} = F(R_1)R_0 a_0;$$

and so expected utility at date 0 would be:

$$E_0[v_{0'}] = E_0[F(\tilde{R}_1)]R_0 a_0 < F(E_0[\tilde{R}_1])R_0 a_0 = v_0.$$

The inequality follows from the concavity of F, which in turn follows from $\rho > 1/2$.[4]

This example illustrates that one should be careful when applying RINCE preferences to concrete problems. There will be some situations in which the assumption of risk neutrality and the associated assumption of a preference for late resolution is a relatively harmless simplifying assumption. There will be other situations when it makes no sense. Let the buyer beware!

APPENDIX 1

This appendix provides a sketch of the proof that the closed form solution to the value function described in the text is valid. The proposed solution for v_t is given by:

$$v_t = F(Q_t)W_t. \tag{44}$$

Taking expectations of v_t at t-1 using the identity (37) and the asset accumulation rule one obtains:

$$E_{t-1}(\tilde{v}_t) = E_{t-1}[\tilde{R}_t F(\tilde{Q}_t)][R_{t-1}a_{t-1} + \omega_{t-1} - c_{t-1}] + E_{t-1}[\tilde{h}_t F(\tilde{Q}_t)]; \tag{45}$$

which simplifies, using definitions (34) and (36), to

$$E_{t-1}(\tilde{v}_t) = Q_{t-1}(W_{t-1} - c_{t-1}). \tag{46}$$

By substituting (46) into equation (10) and using the functional form (30) for w one obtains the first order conditions:

$$c_{t-1}^{\rho-1} - Q_{t-1}\rho\beta[Q_{t-1}(W_{t-1} - c_{t-1})]^{\rho-1} = 0; \tag{47}$$

which may be rearranged to give the functional form (38) using the definition of G given in equation (32). By substituting the solution for c_{t-1} at a maximum (equation(38)), into the function w one obtains the expression:

$$v_{t-1} = F(Q_{t-1})W_{t-1}. \tag{48}$$

This establishes that if (44) is a correct representation of the value function at t, then it is also correct at t-1. One completes the proof by establishing that v_{T-1} is described by (44) given the definition of v_T in equation (40).

APPENDIX 2

This appendix describes a method for computing approximate risk premia when preferences are in the KP class. These approximations are computed in the following way. Consider a temporal lottery $\tilde{c}_{\tau t} = \{\bar{c}_0, \ldots, \tilde{c}_t, \ldots, \bar{c}_T\}$ which resolves at date $\tau < t$ and let $\bar{c}_t$ be the mean of $\tilde{c}_t$. For $s < \tau < t$, let $\bar{U}_s$ represent the value of the index U_s evaluated at the reference path $\bar{c}_{\tau t} = \{\bar{c}_s, \ldots, \bar{c}_t, \ldots, \bar{c}_T\}$. Similarly let U_s^p be the value of the risk compensated path $c_{\tau t}^p = \{\bar{c}_s, \ldots, \bar{c}_t - p_{\tau t}, \ldots, \bar{c}_T\}$ and let $\tilde{U}_s$ be the value of $\{\bar{c}_s, \ldots, \tilde{c}_t, \ldots, \bar{c}_T\}$. For all of these definitions, if a path $\{c_v\}_{v=0}^T$ is evaluated at date $s > 0$, the first s-1 elements of the path are dropped. Since preferences are recursive and history independent there is no loss of generality from this convention. Now define:

$$u_s^p = U_s^p - \bar{U}_s; \tag{49}$$

$$\tilde{u}_s = \tilde{U}_s - \bar{U}_s. \tag{50}$$

The terms u_s^p and $\tilde{u}_s$ represent the difference in utilities yielded by the risk compensated path $c_{\tau t}^p$ and the uncertain path $\tilde{c}_{\tau t}$ from the reference path $\bar{c}_{\tau t}$. The value of u_s^p for all $s \leq t$ can be found by solving the following difference equation:

$$u_s^p = w_2^s u_{s+1}^p + o(u_{s+1}^p) \qquad s = t-1, \ldots 0; \tag{51}$$

with boundary condition;

$$u_t^p = -w_1^t p_{\tau t} + o(p_{\tau t}). \tag{52}$$

It is a little more complicated to compute the value of $\tilde{u}_s$ since one must retain the first two terms in the Taylor series expansion of $\tilde{u}_s$ for dates $\tau < s < t$. Making use of the definition, $\epsilon = \tilde{c}_t - \bar{c}_t$, one may keep track of $\tilde{u}_s$ and $(\tilde{u}_s)^2$ with the following pair of difference equations, which hold for $\tau < s < t$:

$$\begin{bmatrix} \tilde{u}_s \\ (\tilde{u}_s)^2 \end{bmatrix} = \begin{bmatrix} w_2^s & w_{22}^s/2 \\ 0 & (w_2^s)^2 \end{bmatrix} \begin{bmatrix} \tilde{u}_{s+1} \\ (\tilde{u}_{s+1})^2 \end{bmatrix} + o((\tilde{u}_{s+1})^2). \tag{53}$$

The boundary conditions for these difference equations are given by:

(54)
$$\begin{bmatrix} \tilde{u}_t \\ (\tilde{u}_t)^2 \end{bmatrix} = \begin{bmatrix} w_1^t & w_{11}^t/2 \\ 0 & (w_1^t)^2 \end{bmatrix} \begin{bmatrix} \epsilon \\ \epsilon^2 \end{bmatrix} + o(\epsilon^2).$$

Now define:

$$W_2^s = \begin{bmatrix} w_2^s & w_{22}^s/2 \\ 0 & (w_2^s)^2 \end{bmatrix};$$

and let;

$$W_1^t = \begin{bmatrix} w_1^t & w_{11}^t/2 \\ 0 & (w_1^t)^2 \end{bmatrix}.$$

Using this more compact notation one may solve the difference equation (53) to obtain the value of $\tilde{u}_s$ and $(\tilde{u}_s)^2$ as functions of ϵ and ϵ^2 :

(55)
$$\begin{bmatrix} \tilde{u}_s \\ (\tilde{u}_s)^2 \end{bmatrix} = \prod_{v=s}^{t-1} W_2^v W_1^t \begin{bmatrix} \epsilon \\ \epsilon^2 \end{bmatrix} + o(\epsilon^2); \qquad \tau < s \leq t.$$

Similarly one may solve (51) to write u_s^p as a function of $p_{\tau t}$:

(56)
$$u_s^p = -\prod_{v=s}^{t-1} w_2^v w_1^t p_{\tau t} + o(p_{\tau t}).$$

At date τ, uncertainty is revealed. Hence the equation that describes the values of $\tilde{u}_\tau$ and $(\tilde{u}_\tau)^2$ takes the form:

(57)
$$\begin{bmatrix} \tilde{u}_\tau \\ (\tilde{u}_\tau)^2 \end{bmatrix} = \prod_{v=\tau}^{t-1} W_2^v W_1^t \begin{bmatrix} 0 \\ \sigma^2 \end{bmatrix} + o(\epsilon^2).$$

Using the definition of $p_{\tau t}$, that is, $u_{\tau t}^p = \tilde{u}_{\tau t}$ it follows that:

(58)
$$p_{\tau t} = A^{12}\sigma^2;$$

where,

$$A = \left[\frac{\prod_{v=\tau}^{t-1} W_2^v W_1^t}{-\prod_{v=\tau}^{t-1} w_2^v w_1^t}\right]$$

and A^{ij} for i, j $\in \{1,2\}$ is the ijth element of A. Using the same notation, one may write the risk premium for a risk that resolves at date $\tau - 1$ as:

$$p_{\tau-1t} = \left[\frac{W_2^{\tau-1}}{w_2^{\tau-1}} A\right]^{12} \sigma^2; \tag{59}$$

which simplifies to;

$$p_{\tau-1t} = \left[A^{12} + \frac{w_{22}^{\tau-1}}{2} A^{22}\right] \sigma^2 = p_{\tau t} + \left[\frac{w_{22}^{\tau-1}}{2w_2^{\tau-1}} A^{22}\right] \sigma^2. \tag{60}$$

A similar calculation implies that:

$$p_{\tau-2t} = p_{\tau-1t} + \left[w_2^{\tau-1} \frac{w_{22}^{\tau-2}}{2w_2^{\tau-2}} A^{22}\right] \sigma^2. \tag{61}$$

Equation (21) follows directly from (60) and (61) together with the definition of μ_{s-1} in equation (20). The boundary conditions (19) and (18) follow directly from evaluating (58) and (60) at τ = t.

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NOTES

1. Throughout this paper I maintain the assumption of payoff history independence. In general, KP preferences at a point in time may depend on the entire consumption history of the consumer. I also assume that the aggregator function, w, is time independent.

2. Appendix 2 provides a description of the way in which these approximations are constructed.

3. This is a conjecture. I do not have a proof of the non-existence of some other class that can be easily solved — but I have been unable to find a counter example.

4. This example was provide by an referee.

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