MONEY, INFLATION, AND THE EXPECTED REAL INTEREST RATE

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ABSTRACT

This paper tests the exclusion of lagged rates of inflation and money growth from regression equations, with serially correlated disturbances, for the expected real interest rate. Our tests exploit an extension of Mishkin's (1981) observation that under the maintained hypothesis of rational expectations, a regression of the ex post real interest rate on predetermined variables yields valid test statistics for drawing inferences about the correlations of these variables with the expected real interest rate. We discuss some implications of our tests for the empirical literature on the Fisher hypothesis, and for a real business cycle model developed by Litterman and Weiss.

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1. INTRODUCTION

A basic difficulty inherent in testing macroeconomic hypotheses about the determination of expected real interest rates is the lack of a directly observable time series. Mishkin (1981) suggests a simple empirical strategy that circumvents this difficulty by noting that under the maintained hypothesis of rational expectations, the OLS regression of the ex post real interest rate on predetermined variables yields valid test statistics for drawing inferences about the correlations of these variables with the expected real interest rate. Using this empirical strategy, a number of studies [e.g., Mishkin (1981), Barsky (1987)] have found that, in postwar U.S. data, the expected real interest rate is negatively correlated with lagged rates of inflation and/or with lagged growth rates of the money stock.

The present paper implements an extension of Mishkin's empirical strategy by allowing for a serially correlated disturbance in the process generating the expected real interest rate. This extension provides, in particular, a simple way to test the hypothesis that inflation and money growth fail to Granger-cause the expected real interest rate. Our tests have two motivations.

The first motivation is provided by studies [e.g., Garbade and Wachtel (1978), Fama and Gibbons (1982)] that extract a time series for the expected real interest rate by applying a Kalman filter, or related techniques, to observable time series. With the notable exception of Hamilton (1985), most of these studies assume that the expected real interest rate follows a univariate autoregressive process (often restricted to be a random walk) and is not Granger-caused by the rate of inflation.

Our approach provides a simple way to test such assumptions about the process generating the expected real interest rate. More importantly, when the ultimate goal of the empirical exercise is to test some hypothesis about the determinants of the expected real interest rate, our approach has the advantage of yielding valid test statistics.

The second motivation for our tests is provided by an empirical finding of Litterman and Weiss (1985)--henceforth, L&W. Using quarterly postwar U.S. data, L&W fail to reject the hypothesis that the expected real interest rate follows a univariate AR(1) process not Granger-caused by output, the money stock, and the rate of inflation. L&W show that this finding contradicts standard macroeconomic models (including the Lucas-Barro imperfect-information model as well as the conventional IS-LM model) in which monetary shocks have real effects. As an explanation for their empirical finding, L&W formulate a real business cycle (RBC) model in which the expected real interest rate follows an uncaused (in Granger's sense) autoregressive process. In their model, a real shock (not observed by the econometrician) induces spurious Granger-causality running from inflation and the money stock to output.[1]

L&W's empirical strategy begins by estimating a vector autoregression (VAR) for measures of output, the rate of inflation, the money stock, and the nominal interest rate. Assuming that the agents' expected rate of inflation coincides with the VAR forecast, L&W derive the overidentifying restrictions implied by the hypothesis that the variables included in the VAR jointly fail to Granger-cause the expected real interest rate, which is assumed to follow an AR(1) process. L&W conduct a likelihood-ratio test and fail to reject these overidentifying restrictions. Frydman (1986) criticizes L&W's empirical strategy on the grounds that the overidentifying

restrictions it tests are valid only if the information set of market participants consists of the variables included in the VAR.

To allow for the possibility that market participants have access to more information than we do, our tests use a limited-information two-step two-stage least squares procedure, instead of maximum likelihood. Hansen (1982), Hansen and Singleton (1982), and Cumby, Huizinga, and Obstfeld (1983) offer further reasons (e.g., permitting the disturbance terms to be conditionally heteroskedastic) for choosing such a limited-information procedure instead of maximum likelihood. In what follows, Section 2 presents our notation and estimation procedure. Section 3 discusses our data and empirical results. Section 4 contains a brief summary and some concluding remarks.

2. NOTATION AND ESTIMATION PROCEDURE

Let i_t denote the one-period nominal interest rate at date t, q_{t+1} the rate of inflation from date t to date t+1, and r_{t+1} the ex post real interest rate. For continuously compounded rates, we have:

$$i_t = q_{t+1} + r_{t+1}. \tag{1}$$

Let ϕ_t denote the information set of market participants at date t, and define $q_t^e = E(q_{t+1} | \phi_t)$ as the conditional expectation of q_{t+1} given ϕ_t. The market participants' forecast error, v_{t+1}, of the rate of inflation from date t to date t+1 is

$$v_{t+1} = q_{t+1} - q_t^e. \tag{2}$$

Because market participants observe the nominal interest rate at date t, the expected real interest rate, $r_t^e = E(r_{t+1} \mid \phi_t)$, satisfies

$$r_t^e = i_t - q_t^e = r_{t+1} + v_{t+1}. \tag{3}$$

Note that our dating convention uses the time subscript to denote the date at which market participants observe the relevant random variable. Thus, we denote the expected real interest rate from date t to date t+1 (which is known to market participants at date t) by r_t^e, and the corresponding ex post real interest rate (observed at date t+1) by r_{t+1}.

Assuming that the expected real interest rate has a stationary mean, the hypotheses that we would like to test involve estimation of equations of the form:

$$r_t^e = Z_{t-1}\gamma + \sum_{i=1}^{n} r_{t-i}^e \alpha_i + \eta_t, \tag{4}$$

where Z_{t-1} is a stationary vector of lagged variables, γ and $\alpha = (\alpha_1, \alpha_2, \ldots, \alpha_n)$ are coefficient vectors, and η_t is a white noise disturbance.

In our empirical work we will also entertain the possibility that the expected real interest rate is first-difference stationary, in which case we estimate equations of form:

$$\Delta r_t^e = Z_{t-1}\gamma + \sum_{i=1}^{n} \Delta r_{t-i}^e \alpha_i + \eta_t, \tag{4'}$$

where Δ is the difference operator, and Z_{t-1}, γ, α, and η_t are defined as before.

L&W's null hypothesis, that the expected real interest rate follows a univariate autoregression, corresponds to including only a constant series in Z_{t-1}, in equation (4). L&W also set n=1 in their empirical work, but their theoretical model does not restrict the order of the autoregression. The random walk specification adopted by Garbade and Wachtel and by Fama and Gibbons includes only a constant and the error term on the right hand side of equation (4'). To test such univariate autoregressive models, we can include lagged values of other variables among the regressors in Z_{t-1} and test their exclusion from equation (4) or from equation (4').

If lagged values of the expected real interest rate did not appear on the right hand side of equation (4), we could easily test the exclusion of any variable from Z_{t-1}, under the maintained hypothesis of rational expectations, using the procedure of Mishkin (1981). Mishkin's procedure exploits the fact that, because the measurement error that results from replacing the dependent variable r_t^e by r_{t+1} is not correlated with the lagged variables included in Z_{t-1} according to equation (3), the OLS regression of r_{t+1} on Z_{t-1} yields valid test statistics.

However, because r_{t-i}^e in equation (4) is not directly observable and can be correlated with the variables included in Z_{t-1}, we cannot directly apply Mishkin's procedure to this equation. In particular, the finding that lagged rates of inflation or lagged money growth are correlated with r_t^e [e.g., Mishkin (1981), Barsky (1987)] does not contradict L&W's failure to reject the exclusion of these lagged variables from a simple version of equation (4). We modify Mishkin's procedure below to accommodate the inclusion of lagged expected real interest rates on the right hand side of equation (4).

Equations (3) and (4) imply that the ex post real interest rate satisfies

$$(5) \qquad r_{t+1} = Z_{t-1}\gamma + \sum_{j=0}^{n-1} r_{t-j}\alpha_{j+1} + \eta_t - v_{t+1} + \sum_{j=0}^{n-1} v_{t-j}\alpha_{j+1}.$$

The presence of lagged inflation forecast errors, v_{t-j}, on the right hand side of equation (5) reveals that, even if the rate of inflation does not Granger-cause the expected real interest rate, it can Granger-cause the ex post real interest rate. Moreover, since these lagged inflation forecast errors can be correlated with lagged monetary aggregates, evidence that monetary aggregates Granger-cause the ex post real interest rate [e.g., Shiller (1980)] does not necessarily contradict L&W's failure to reject the exclusion of lagged money from their version of equation (4).

Under the null hypotheses that we will test, the white noise disturbance η_t in equation (4) is independent of the inflation forecast error process. Therefore, the moving average error term of equation (5),

$$\eta_t - v_{t+1} + \sum_{j=0}^{n-1} v_{t-j}\alpha_{j+1},$$

is at most of order n [see, for example, Granger and Newbold (1986), pp. 28-29]. Accordingly, under the maintained hypothesis of rational expectations, variables observed at date t-n or earlier qualify as valid instruments for estimation of equation (5) by two-step two-stage least squares [see Hansen (1982); Cumby, Huizinga, and Obstfeld (1983)].[2]

Our tests of equation (4'), which assumes that the expected real interest rate is stationary in first differences rather than in levels, proceed similarly to the tests of equation (4) described above. The only

differences are that the counterpart of equation (5) in the differenced specification has a moving average error term of order n+1, rather than n, and that the set of instruments is lagged an additional period.

3. DATA AND EMPIRICAL RESULTS

Our data are quarterly from 1954:1 to 1987:1. We report our results for the entire sample period as well as the subperiod 1954:1 to 1979:3, which ends before the October 1979 change in the Fed's operating procedure. Our measure of the nominal interest rate is the annualized continuously compounded yield on 13-week Treasury bills. We use the continuously compounded growth rate of seasonally adjusted CPI less shelter as our measure of the rate of inflation. Following L&W, to match the periods over which we measure inflation and the nominal interest rate, we have used the price index for the last month of the quarter (which is based on a sample taken approximately during the middle week of the month) and the weekly average of interest rates during the second or third week of the month.

The measure of output is the seasonally adjusted Index of Industrial Production for the last month of the quarter. We use the seasonally adjusted M1 and M2 series for the last month of the quarter as measures of the money stock. Working with the M1 series facilitates the comparison of our results with L&W's findings, but the M2 series has the advantage of being less sensitive to changes in the definitions of monetary aggregates over our sample period. In all the regressions that do not include the growth rate of M1 among the regressors, we use the lagged growth rates of M2 (rather than M1) in the set of instruments.

Table 1 reports unit-root tests for the relevant time series. The test statistics reported in the first two rows of the Table yield mixed

results about the stationarity properties of the (ex post) real interest rate and the rate of inflation. Specifically, the tests reject the presence of a unit root in both time series when the lag-length parameter k in the Dickey-Fuller regression is set equal to zero (i.e., when the null hypothesis is a random walk), but not when the parameter k is set equal to four. For the first differences of both the real interest rate and the rate of inflation and for the growth rates of output and the money stock the tests reject the presence of a unit root--one of the test statistics for the first differences of the real interest rate rejects the unit-root null only for a test of size 0.10, the remaining test statistics reject this null hypothesis for tests of size 0.05 or smaller.

As several studies (e.g., Evans and Savin, 1984) have emphasized, unit root tests have low power against borderline stationary alternatives, even for a test of size 0.10. Moreover, standard asset-pricing models imply that if the growth rates of the money stock and output (or consumption) do not have unit roots, nor should the rate of inflation and real asset returns.[3] Accordingly, we provisionally attribute the mixed results of the stationarity tests for the real interest rate and the rate of inflation to the low power of these tests and begin our empirical analysis by assuming that the autoregressive representations of these variables do not have unit roots.

Table 2 reports estimates of univariate autoregressive models for the expected real interest rate. The set of instruments for these regressions, and for the other regressions using the undifferenced specification, contains a constant series and four lags each of the nominal interest rate, the rate of inflation, the growth rate of output, and the growth rate of the money stock. The particular lags used vary from regression to

regression, as discussed in the preceding Section, depending on the order of the moving average error term. For our longer sample, L&W's AR(1) specification does not seem adequate for capturing the dynamics of the expected real interest rate. For the shorter sample ending in 1979, the estimates suggest that either the more parsimonious AR(1) specification or the AR(4) specification may be appropriate.

The analogue, in our set up, to the null hypothesis tested by L&W is the hypothesis that the growth rates of output and of the money stock, the rate of inflation, and the nominal interest rate jointly fail to Granger-cause the expected real interest rate.[4] Note that the presence of such Granger-causality would imply that our instruments are correlated with the error terms of the univariate autoregressive models reported in Table 2. Accordingly, a first test for Granger-causality in our set up would involve a diagnostic instrument-residual orthogonality test. Hansen (1982) justifies a test of the orthogonality conditions implied by a model, using the minimized value of the generalized method of moments (GMM) objective function (multiplied by sample size) as the test statistic. Hansen and Singleton (1982) illustrate the use of this statistic for testing the overidentifying restrictions implied by the orthogonality conditions.

The J-statistics reported in Table 2 are the minimized values of the objective functions (multiplied by sample size) for the GMM estimates of our univariate autoregressive models. The p-values reported below the J-statistic are from the χ^2 distribution with 16-n degrees of freedom, where n is the order of the autoregression. Since these p-values do not contradict the instrument-residual orthogonality conditions, the univariate autoregressive models for the expected real interest rate pass Hansen's test.

However, the estimates of higher (than first) order autoregressive models reported in Table 2 frequently reveal significant coefficients on lagged real interest rates, which are linear combinations of the instruments used for the AR(1) models. These significant coefficients suggest that the J-statistics fail to detect the existing correlations between the residuals and some of the excluded variables. To explore this possibility, we turn to tests of the univariate autoregressive models against more precise (bivariate) alternatives.

Table 3 reports tests of univariate autoregressive models for the expected real interest rate against bivariate alternatives that include lags of money growth among the regressors. The columns headed "Wald Test for $\gamma = 0$" report the p-values (marginal significance levels) of the Wald statistic for testing the exclusion of four lags of money growth, beginning with lag m, when n lags of the expected real interest rate are included among the regressors. For both measures of money growth (M1 and M2), the reported p-values are frequently below the 0.05 level for the short sample ending in 1979--thus, money growth appears to Granger-cause the expected real interest rate in this sample. For the longer sample ending in 1987, rejections of the univariate autoregressive models are less frequent.

The most noteworthy feature of the Granger-causality tests reported in Table 3 is the sensitivity of the p-values to the choice of sample and to minor changes in lag-lengths. This sensitivity suggests, on one hand, that the Granger-causality that our tests detect may be spurious. On the other hand, this sensitivity casts doubt on L&W's claim that the failure of their test to detect Granger-causality, running from a vector of time series including the money stock to the expected real interest rate, constitutes

reliable evidence against conventional (monetary) theories of the business cycle.

The last four columns of Table 3 report the p-values of the Wald statistics for testing the hypothesis that the sum of the coefficients on lagged money growth in the estimated equations equals zero. These p-values are well above 0.10 in all cases and, thus, consistent with the hypothesis that sustained changes in the growth rate of the money stock do not affect the expected real interest rate. Geweke (1986) clarifies the sense in which this hypothesis is the empirical counterpart of the theoretical notion of long-run superneutrality of money with respect to the real interest rate.

McCallum (1984) illustrates that the sum of the coefficients on lags of a monetary aggregate, in a regression equation for a real variable, can differ from zero even if anticipated movements in this monetary aggregate have no structural effect on the real variable. This possibility of spurious rejections of long-run superneutrality is not relevant for interpreting the results of Table 3, which do not reject the null hypothesis. McCallum's point, however, should be kept in mind when interpreting the apparent rejections of long-run superneutrality that we report below, for some of the regressions involving the rate of inflation.

Table 4 reports tests of univariate autoregressive models for the expected real interest rate against bivariate alternatives that include either lagged rates of inflation or lagged growth rates of output among the regressors. The results for the rate of inflation are fairly similar to the results for money growth discussed above. We find frequent rejections of the hypothesis that the rate of inflation does not Granger-cause the expected real interest rate in our shorter sample, and less frequent

rejections in our longer sample. For both samples, we find very little evidence against the hypothesis that the sum of the coefficients on lagged inflation equals zero. When the sum of the estimated coefficients was different from zero (using a test of size 0.05) its sign (not reported in the Table) was negative in the regressions using the short sample, but positive in the regressions using the long sample.

In the case of output growth, the results reported in Table 4 are less sensitive to choice of the sample period, than the results for money growth and inflation. In both samples, the hypothesis that output growth does not Granger-cause the expected real interest rate is frequently rejected when lag-lengths are short, but not when longer lag-lengths are allowed for. Once again, we find only a few rejections of the hypothesis that the sum of the coefficients on lagged output growth equals zero. In all the cases where the sum of the estimated coefficients was different from zero (using a test of size 0.05) its sign (not reported in the Table) was negative.

Since the unit-root tests of Table 1 yielded mixed results about the stationarity properties of the real interest rate and the rate of inflation, we reestimated the regressions reported in Tables 3 and 4, assuming this time that the nominal interest rate and the rate of inflation are first-difference stationary. The regression equations for the first differences of the ex post real interest rate have a moving average error term of order n+1, where n is the order of the autoregressive polynomial specified for the first differences of the expected real interest rate. The instruments are a constant series and four lagged observations each on first differences of nominal interest rates, first differences of rates of inflation, growth rates of output, and growth rates of the money stock.[5]

Table 5 reports tests for excluding lags of money growth from autoregressive models for the first differences of the expected real interest rate. Besides the statistical question of whether the levels or the first differences of the real interest rate share the stationarity properties of money growth, there is a conceptual difference between the tests of Table 5 and those of Table 3. Namely, whereas the regression equations of Table 3 can be motivated by the "superneutrality" hypothesis that a change in the growth rate of the money stock does not affect the expected real interest rate, those of Table 5 can be motivated by the "neutrality" hypothesis that a change in the logarithm of the money stock does not affect the expected real interest rate.[6]

The results reported in Table 5, however, are quite similar to those of Table 3. We reject the absence of Granger-causality (running from money growth to the differenced expected real interest rate) frequently for the shorter sample, but less often for the longer sample. And we find little evidence against the hypothesis that the sum of the coefficients on lagged money growth equals zero.

Table 6 reports the results for first differences of the rate of inflation and for the growth rate of output. Both variables seem to Granger-cause the first differences of the expected real interest rate, confirming the results of the undifferenced specifications reported in Table 4. Rejections of the hypothesis that the sum of the coefficients on excluded variables equals zero are more frequent in Table 6 than they were in Table 4. For both output growth and differenced inflation, in all the cases where the sum of the estimated coefficients was different from zero (using a test of size 0.05) its sign (not reported in the Table) was negative.

Overall, the Granger-causality tests reported in Tables 3 to 6 seem to suggest that univariate autoregressive models, with or without a unit root, do not adequately capture the dynamics of the expected real interest rate in our samples. To consider a more general model, we turn to a bivariate specification that includes four lags of output growth among the regressors Z_{t-1} in equations (4) and (4'), under the null hypothesis. The motivation for these bivariate models is the fact that the real interest rate, the rate of inflation, and the growth rate of the money stock fluctuate considerably over the business cycle; therefore, failure to include a business-cycle proxy (such as lagged output growth) among the regressors of the null model can generate spurious Granger-causality running from money growth or inflation to the expected real interest rate.

Table 7 reports Granger-causality tests of our undifferenced bivariate specifications against trivariate alternatives that add lagged money growth or lagged inflation to the list of regressors. The results indicate much less evidence against the bivariate specification than we found (in Tables 3 and 4) against the corresponding univariate specification. The Wald statistics for testing the hypothesis that the <u>sum</u> of the coefficients on lagged money growth or inflation in the regressions of Table 7 equals zero in no case rejected this hypothesis (using a test of size 0.05) and are, therefore, not reported in the Table.[7]

Table 8 reports Granger-causality tests of our differenced bivariate specifications against trivariate alternatives that add lagged money growth or lagged first differences of inflation to the list of regressors. The results indicate somewhat less evidence against the bivariate specification than we found (in Tables 5 and 6) against the corresponding univariate specification. But rejections of the exclusion restrictions are more

frequent for these differenced bivariate specifications (Table 8) than was the case with the undifferenced bivariate specifications (Table 7).

Table 8 does not report the Wald statistics for testing the hypothesis that the sum of the coefficients on the excluded variable in each regression equals zero. The statistics rejected this hypothesis (using a test of size 0.05) in only three cases.[8]

Although our tests find overall less evidence against the bivariate specifications of Tables 7 and 8 (than we found, in Tables 3 to 6, against the corresponding univariate specifications), the p-values for the Granger-causality tests continue to exhibit sensitivity to the choice of sample, stationarity assumption, and to minor changes in lag-lengths. This sensitivity makes it difficult to base any firm conclusions on the results of Tables 7 and 8, without further empirical work along the lines suggested by recent contributions [e.g., Stock and Watson (1987), Christiano and Ljungqvist (1988)] to the money-income causality literature.

4. SUMMARY AND DISCUSSION

This paper implements a simple empirical strategy for testing the exclusion of lagged rates of inflation and money growth from regression equations for the expected real interest rate, while allowing for a serially correlated disturbance term. The empirical strategy exploits an implication of the rational expectations hypothesis for the order of the moving average error term that results from replacing the expected real interest rate, on both sides of the regression equation, by its ex post realization--thus, providing valid instruments for estimation by two-step two-stage least squares.

Our empirical strategy provides an alternative to L&W's strategy for testing the hypothesis that inflation and money growth fail to Granger-cause the expected real interest rate. Unlike L&W, we reject this hypothesis quite frequently--particularly, for our short sample ending in 1979, which represents a more homogeneous sample in terms of the Fed's operating procedure.

Such causality tests complement studies that, following the seminal contributions of Sims (1972, 1980a, 1980b), have focused on whether money Granger-causes output in either a bivariate or a multivariate framework. Several recent contributions [e.g., Bernanke (1986), Eichenbaum and Singleton (1986), King (1986), McCallum (1986)] discuss the relevance, and the limitations, of such causality tests for evaluating monetary and real models of the business cycles. Without attempting to review all the important points made in the literature, we note one caveat that is particularly relevant for interpreting our results. Namely, the finding that nominal variables Granger-cause real variables can arise spuriously--that is, in a world governed by some RBC model--because some relevant real variables have been left out of the researchers' model.[9]

Accordingly, our finding that inflation and money growth Granger-cause the expected real interest rate constitutes evidence against the particular RBC model developed by L&W, rather than against any broad class of RBC models. Our more favorable evidence for a bivariate null model in which the expected real interest rate depends on its own lagged values and on lagged output suggests that a more serious RBC challenge to standard monetary models of the business cycle might come from models involving joint determination of the expected real interest rate and output [see, for example, Diba and Oh (1988)].

The sensitivity of our Granger-causality tests to the choice of sample, stationarity assumption, and to minor changes in lag-lengths seems to warn against placing too much reliance on such tests in evaluating alternative business-cycle models. The only aspect of our findings that seems robust to changes in specification is our failure to reject the hypothesis that the sum of the coefficients on the lagged money or inflation regressors equals zero.

Our findings are also relevant for empirical tests of the Fisher hypothesis that assume the expected real interest rate follows a univariate autoregressive process not Granger-caused by the rate of inflation. Since our results contradict this assumption, further empirical analysis of the relationship between the expected rate of inflation (which is probably correlated with past rates of inflation) and the expected real interest rate seems warranted. Our empirical strategy provides a convenient framework for studying this relationship.

TABLE 1

UNIT-ROOT TESTS

End of Sample: **Number of Lags (k):**	**1979** **0**	**1979** **4**	**1987** **0**	**1987** **4**
Real Interest Rate	-7.75	-2.63	-6.27	-2.20
Rate of Inflation	-5.70	-3.11	-5.27	-2.53
Output Growth	-7.13	-5.34	-8.02	-6.18
Money Growth (M1)	-7.61	-4.21	-10.27	-4.32
Money Growth (M2)	-4.95	-3.79	-6.65	-3.72
Differenced Real Interest Rate	-17.62	-3.21	-20.21	-6.60
Differenced Rate of Inflation	-17.10	-4.78	-20.69	-4.90

Table reports values of the Dickey-Fuller τ_τ statistic. For each time series (x_t) indicated in the first column, regressions of the form:

$$\Delta x_t = \mu + t\gamma + x_{t-1}\rho + \sum_{j=1}^{k} \Delta x_{t-j}\beta_j + \text{error} ,$$

were estimated for k=0 and k=4. The statistic τ_τ was calculated like the t-ratio for ρ. The rejection region, tabulated for a sample of 100 observations in Fuller (1976, p 373), is the set of vales of τ_τ below -3.15 (-3.45) for a test of size 0.10 (0.05).

TABLE 2

ESTIMATED UNIVARIATE AUTOREGRESSIVE MODELS FOR THE EXPECTED REAL INTEREST RATE

	AR(1)	AR(2)	AR(3)	AR(4)
1954-1979 Sample:				
c	0.204	0.342*	0.195	0.042
	(0.167)	(0.166)	(0.143)	(0.112)
r_t	0.808*	0.340	0.462*	0.667*
	(0.121)	(0.261)	(0.188)	(0.225)
r_{t-1}		0.215	0.052	-0.005
		(0.211)	(0.249)	(0.236)
r_{t-2}			0.229	-0.134
			(0.155)	(0.162)
r_{t-3}				0.368*
				(0.128)
J	14.52	15.20	11.78	12.10
(p-value)	(0.49)	(0.36)	(0.55)	(0.44)
1954-1987 Sample:				
c	0.231	-0.042	0.102	0.054
	(0.153)	(0.184)	(0.180)	(0.118)
r_t	0.815*	0.127	-0.001	0.397*
	(0.062)	(0.117)	(0.123)	(0.196)
r_{t-1}		0.816*	0.519*	0.839*
		(0.130)	(0.130)	(0.176)
r_{t-2}			0.420*	0.142
			(0.123)	(0.172)
r_{t-3}				-0.453*
				(0.219)
J	13.04	16.25	12.53	11.59
(p-value)	(0.60)	(0.30)	(0.49)	(0.48)

Table reports regressions of the form:

$$r_{t+1} = c + \sum_{i=0}^{n-1} r_{t-i}\alpha_{i+1} + \text{error},$$

for n = 1, 2, 3, and 4. Asymptotic standard errors are below coefficients in parentheses; * indicates significant t-ratio at the 0.05 level. The statistic J is the minimized value of the GMM objective function multiplied by sample size [see Hansen (1982), Hansen and Singleton (1982)]. The p-values reported below this statistic are from the χ^2 distribution with 16-n degrees of freedom.

TABLE 3

TESTS FOR EXCLUSION OF LAGGED MONEY GROWTH FROM UNIVARIATE AUTOREGRESSIVE MODELS FOR THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)				Wald Test for $\Sigma\, \gamma_j = 0$ (p-values)			
Aggregate:		**M1**	**M1**	**M2**	**M2**	**M1**	**M1**	**M2**	**M2**
Sample:		**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**
n	**m**								
1	1	0.94	0.85	0.07	0.83	0.86	0.77	0.76	0.64
1	2	0.76	1.00	0.21	0.46	0.56	0.95	0.73	0.71
1	3	0.63	0.09	0.22	0.59	0.46	0.48	0.33	0.36
1	4	0.43	0.05	0.20	0.40	0.93	0.70	0.83	0.55
2	1	0.97	0.00	0.18	0.16	0.62	0.19	0.81	0.83
2	2	0.00	0.12	0.00	0.08	0.61	0.78	0.73	0.58
2	3	0.00	0.09	0.00	0.14	0.39	0.72	0.85	0.83
2	4	0.05	0.00	0.00	0.00	0.42	0.76	0.64	0.25
3	1	0.96	0.29	0.17	0.38	0.61	0.77	0.46	0.30
3	2	0.00	0.26	0.00	0.03	0.71	0.46	0.87	0.35
3	3	0.01	0.32	0.00	0.05	0.71	0.96	0.80	0.75
3	4	0.02	0.30	0.00	0.03	0.62	0.95	0.82	0.61
4	1	0.33	0.54	0.38	0.62	0.38	0.14	0.09	0.36
4	2	0.00	0.10	0.00	0.16	0.63	0.28	0.54	0.48
4	3	0.00	0.11	0.00	0.23	0.25	0.34	0.81	0.34
4	4	0.00	0.11	0.00	0.22	0.24	0.26	0.75	0.54

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of money growth from autoregressive models for the expected real interest rate, and for testing the hypothesis that the sum of the coefficients on the four lags equals zero. The regressions are of the form:

$$r_{t+1} = c + \sum_{i=0}^{n-1} r_{t-i}\alpha_{i+1} + \sum_{j=m}^{m+3} \Delta m_{t-j}\gamma_j + \text{error} ,$$

where Δm denotes the growth rate of M1 or of M2.

TABLE 4

TESTS FOR EXCLUSION OF LAGGED INFLATION AND LAGGED OUTPUT GROWTH FROM UNIVARIATE AUTOREGRESSIVE MODELS FOR THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)				Wald Test for $\Sigma \gamma_j = 0$ (p-values)			
Excluded Variable:		q	q	Δy	Δy	q	q	Δy	Δy
Sample:		54-79	54-87	54-79	54-87	54-79	54-87	54-79	54-87
n	m								
1	1	0.28	0.77	0.01	0.04	0.14	0.97	0.98	0.77
1	2	0.17	0.59	0.01	0.02	0.62	0.62	0.41	0.61
1	3	0.17	0.47	0.29	0.06	0.60	0.57	0.16	0.03
1	4	0.06	0.35	0.61	0.10	0.64	0.29	0.55	0.75
2	1	0.01	0.28	0.00	0.04	0.09	0.54	0.30	0.34
2	2	0.00	0.10	0.01	0.00	0.03	0.49	0.71	0.76
2	3	0.01	0.10	0.10	0.04	0.14	0.25	0.01	0.01
2	4	0.00	0.09	0.33	0.05	0.33	0.19	0.06	0.28
3	1	0.00	0.09	0.08	0.08	0.26	0.22	0.65	0.15
3	2	0.01	0.06	0.11	0.09	0.27	0.08	0.18	0.17
3	3	0.01	0.06	0.12	0.06	0.04	0.09	0.02	0.04
3	4	0.01	0.01	0.10	0.12	0.21	0.02	0.99	0.37
4	1	0.06	0.06	0.60	0.23	0.58	0.60	0.91	0.05
4	2	0.01	0.14	0.57	0.22	0.67	0.10	0.27	0.26
4	3	0.04	0.14	0.70	0.35	0.74	0.21	0.20	0.79
4	4	0.00	0.00	0.79	0.22	1.00	0.04	0.24	0.28

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of inflation or of output growth from autoregressive models for the expected real interest rate, and for testing the hypothesis that the sum of the coefficients on the four lags equals zero. The regressions are of the form:

$$r_{t+1} = c + \sum_{i=0}^{n-1} r_{t-i}\alpha_{i+1} + \sum_{j=m}^{m+3} z_{t-j}\gamma_j + \text{error} ,$$

where z is set equal either to the rate of inflation (q) or to the growth rate of output (Δy).

TABLE 5

TESTS FOR EXCLUSION OF LAGGED MONEY GROWTH FROM UNIVARIATE AUTOREGRESSIVE MODELS FOR THE FIRST DIFFERENCES OF THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)				Wald Test for $\Sigma\, \gamma_j = 0$ (p-values)			
Aggregate:		**M1**	**M1**	**M2**	**M2**	**M1**	**M1**	**M2**	**M2**
Sample:		**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**
n	**m**								
1	1	0.64	0.00	0.05	0.16	0.23	0.06	0.60	0.59
1	2	0.00	0.06	0.00	0.13	0.95	0.34	0.95	0.84
1	3	0.00	0.00	0.00	0.05	0.94	0.12	0.94	0.55
1	4	0.01	0.00	0.00	0.00	0.76	0.24	0.84	0.72
2	1	0.30	0.62	0.08	0.68	0.07	0.58	0.25	0.21
2	2	0.00	0.29	0.00	0.05	0.04	0.89	0.34	0.32
2	3	0.00	0.19	0.00	0.04	0.24	0.72	0.42	0.95
2	4	0.00	0.18	0.00	0.09	0.66	0.76	0.84	0.82
3	1	0.08	0.81	0.11	0.36	0.73	0.42	0.10	0.33
3	2	0.00	0.19	0.00	0.05	0.66	0.15	0.32	0.84
3	3	0.00	0.24	0.00	0.10	0.81	0.13	0.54	0.28
3	4	0.00	0.14	0.00	0.10	0.65	0.12	0.75	0.16
4	1	0.89	0.42	0.38	0.01	0.78	0.16	0.65	0.98
4	2	0.36	0.44	0.02	0.01	0.70	0.17	0.37	0.87
4	3	0.33	0.43	0.20	0.36	0.93	0.06	0.82	0.08
4	4	0.00	0.48	0.16	0.41	0.71	0.10	0.99	0.12

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of money growth from autoregressive models for the first differences of the expected real interest rate, and for testing the hypothesis that the sum of the coefficients on the four lags equals zero. The regressions are of the form:

$$\Delta r_{t+1} = c + \sum_{i=0}^{n-1} \Delta r_{t-i}\alpha_{i+1} + \sum_{j=m}^{m+3} \Delta m_{t-j}\gamma_j + \text{error} ,$$

where Δm denotes the growth rate of M1 or of M2.

TABLE 6

TESTS FOR EXCLUSION OF LAGGED FIRST DIFFERENCES OF INFLATION AND LAGGED OUTPUT GROWTH FROM UNIVARIATE AUTOREGRESSIVE MODELS FOR THE FIRST DIFFERENCES OF THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)				Wald Test for $\Sigma\, \gamma_j = 0$ (p-values)			
Excluded Variable:		**Δq**	**Δq**	**Δy**	**Δy**	**Δq**	**Δq**	**Δy**	**Δy**
Sample:		**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**
n	**m**								
1	1	0.01	0.17	0.02	0.03	0.34	0.70	0.98	0.28
1	2	0.03	0.13	0.02	0.01	0.85	0.65	0.72	0.96
1	3	0.01	0.08	0.41	0.12	0.06	0.52	0.18	0.03
1	4	0.02	0.14	0.60	0.13	0.11	0.50	0.83	0.36
2	1	0.00	0.19	0.01	0.05	0.14	0.35	0.61	0.40
2	2	0.01	0.24	0.01	0.04	0.35	0.71	0.08	0.09
2	3	0.01	0.01	0.04	0.06	0.02	0.15	0.01	0.04
1	4	0.02	0.00	0.25	0.06	0.71	0.65	0.39	0.23
3	1	0.00	0.03	0.00	0.12	0.19	0.05	0.16	0.23
3	2	0.00	0.25	0.00	0.12	0.91	0.39	0.83	0.14
3	3	0.00	0.01	0.08	0.11	0.06	0.03	0.02	0.20
3	4	0.00	0.00	0.09	0.08	0.28	0.78	0.01	0.04
4	1	0.06	0.42	0.05	0.00	0.58	0.40	0.29	0.01
4	2	0.19	0.19	0.02	0.00	0.71	0.11	0.23	0.02
4	3	0.14	0.10	0.08	0.36	0.22	0.19	0.04	0.08
4	4	0.02	0.00	0.04	0.01	0.01	0.17	0.00	0.00

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of output growth or of the first differences of inflation from autoregressive models for the first differences of the expected real interest rate, and for testing the hypothesis that the sum of the coefficients on the four lags equals zero. The regressions are of the form:

$$\Delta r_{t+1} = c + \sum_{i=0}^{n-1} \Delta r_{t-i}\alpha_{i+1} + \sum_{j=m}^{m+3} z_{t-j}\gamma_j + \text{error} ,$$

where z is set equal either to the differenced rate of inflation (Δq) or to the growth rate of output (Δy).

TABLE 7

TESTS FOR EXCLUSION OF LAGGED MONEY GROWTH AND LAGGED INFLATION FROM BIVARIATE AUTOREGRESSIVE MODELS FOR THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)					
Excluded Variable:		**ΔM1**	**ΔM1**	**ΔM2**	**ΔM2**	**q**	**q**
Sample:		**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**
n	**m**						
1	1	0.97	0.79	0.18	0.88	0.28	0.81
1	2	0.71	0.91	0.16	0.44	0.14	0.18
1	3	0.75	0.12	0.40	0.81	0.10	0.13
1	4	0.87	0.11	0.80	0.55	0.05	0.21
2	1	0.37	0.36	0.07	0.26	1.00	0.35
2	2	0.01	0.34	0.00	0.08	0.04	0.11
2	3	0.01	0.41	0.00	0.24	0.02	0.16
2	4	0.03	0.56	0.00	0.03	0.01	0.03
3	1	0.94	0.43	0.71	0.38	0.07	0.38
3	2	0.30	0.39	0.00	0.44	0.06	0.21
3	3	0.48	0.25	0.01	0.21	0.39	0.20
3	4	0.77	0.26	0.00	0.34	0.65	0.37
4	1	0.14	0.20	0.26	0.81	0.34	0.43
4	2	0.01	0.09	0.00	0.29	0.09	0.45
4	3	0.01	0.19	0.00	0.25	0.18	0.47
4	4	0.02	0.21	0.00	0.54	0.04	0.06

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of money growth or of inflation from regressions of the form:

$$r_{t+1} = c + \sum_{i=0}^{n-1} r_{t-i}\alpha_{i+1} + \sum_{j=1}^{4} \Delta y_{t-j}\beta_j + \sum_{j=m}^{m+3} z_{t-j}\gamma_j + \text{error} ,$$

where Δy denotes the growth rate of output, and z is set equal to the growth rate of M1 (ΔM1), the growth rate of M2 (ΔM2), or the rate of inflation (q).

TABLE 8

TESTS FOR EXCLUSION OF LAGGED FIRST DIFFERENCES OF INFLATION AND LAGGED MONEY GROWTH FROM BIVARIATE AUTOREGRESSIVE MODELS FOR THE FIRST DIFFERENCES OF THE EXPECTED REAL INTEREST RATE

		Wald Test for $\gamma = 0$ (p-values)					
Excluded Variable:		**ΔM1**	**ΔM1**	**ΔM2**	**ΔM2**	**Δq**	**Δq**
Sample:		**54-79**	**54-87**	**54-79**	**54-87**	**54-79**	**54-87**
n	**m**						
1	1	0.86	0.24	0.08	0.23	0.05	0.16
1	2	0.01	0.32	0.00	0.06	0.13	0.13
1	3	0.00	0.38	0.00	0.22	0.04	0.05
1	4	0.07	0.33	0.02	0.07	0.02	0.04
2	1	0.73	0.54	0.19	0.73	0.04	0.21
2	2	0.00	0.50	0.00	0.65	0.00	0.02
2	3	0.00	0.56	0.00	0.32	0.17	0.05
2	4	0.02	0.44	0.00	0.41	0.14	0.02
3	1	0.02	0.42	0.36	0.61	0.09	0.15
3	2	0.02	0.33	0.00	0.47	0.09	0.12
3	3	0.02	0.36	0.02	0.33	0.10	0.11
3	4	0.06	0.42	0.02	0.30	0.12	0.08
4	1	0.21	0.12	0.92	0.04	0.83	0.62
4	2	0.40	0.09	0.79	0.04	0.57	0.26
4	3	0.52	0.05	0.90	0.57	0.43	0.03
4	4	0.01	0.23	0.83	0.41	0.02	0.43

Table reports the p-values of the Wald statistics for testing the exclusion of four lags of money growth or of the first differences of inflation from regressions of the form:

$$\Delta r_{t+1} = c + \sum_{i=0}^{n-1} \Delta r_{t-i}\alpha_{i+1} + \sum_{j=1}^{4} \Delta y_{t-j}\beta_j + \sum_{j=m}^{m+3} z_{t-j}\gamma_j + \text{error} ,$$

where Δy denotes the growth rate of output, and z is set equal to the growth rate of M1 (ΔM1), the growth rate of M2 (ΔM2), or to the differenced rate of inflation (Δq).

NOTES

1. L&W argue that their RBC model is also supported by the finding of Sims (1980b) that the inclusion of nominal interest rates in a vector autoregression estimated on postwar U.S. data eliminates much of the explanatory power of money-stock innovations for output fluctuations. Sims' finding, however, appears to be sensitive to assumptions about the stationarity properties of the relevant time series--see Stock and Watson (1987) and the studies they cite.
2. For our estimation, we used a GMM program written by Greg Leonard and Dave Runkle. We used the Newey and West (1987) covariance matrix.
3. Rose (1988) argues that the failure of his tests to reject the presence of a unit root in time series of the real interest rate constitutes evidence against the consumption capital asset pricing model.
4. This hypothesis differs from L&W's in that they use the logarithms of output and of the money stock in their test while we use the growth rates.
5. We used M2 as our measure of the money stock in all the regressions that did not include the growth rates of M1 among the regressors.
6. That is, the regressions of Table 5 test for a relationship between the logarithm of the money stock and the level of the expected real interest rate, using a first-differenced specification.
7. For tests of size 0.10, we found only two rejections, both in the short sample--one for inflation with $n=2$, $m=3$; the other for the growth rate of M2 with $n=4$, $m=1$.
8. All three rejections occurred for the differenced rate of inflation: with $n=1$ and $m=3$ in the short sample, and with $n=m=2$ in both samples.
9. See King (1986) for an explicit example of how failure to include a relevant real variable among the regressors of the output equation can lead to spurious Granger-causality running from money to output. See Boschen and Mills (1988) for tests of an empirical RBC model that incorporates a richer specification of the real variables influencing output.

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