THE LUCAS CRITIQUE POLICY INVARIANCE AND MULTIPLE EQUILIBRIA*

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ABSTRACT

The Lucas Critique of Econometric Policy Evaluation argues that the parameters of econometric models are subject to theoretical cross equation restrictions which follow from the fact that the endogenous variables of the models are chosen optimally by forward looking agents. In this paper I argue that these facts alone are insufficient to generate such restrictions. I present an example of a simple economic model in which there exists a forecast rule for future prices which is independent of the parameters of the process that generate the exogenous variables of the model. This example is one in which there exist multiple stationary rational expectations equilibria but only one of these equilibria is supported by a process invariant forecast rule which is immune to the Lucas Critique. In models in which there exist multiple stationary rational expectations equilibria, immunity to the Lucas Critique is proposed as a natural criterion for selecting an equilibrium.

1 Introduction

The Lucas Critique of Econometric Policy Evaluation has become part of every graduate student's training. According to the Lucas Critique it is inappropriate to estimate econometric models of the economy, in which endogenous variables appear as unrestricted functions of predetermined variables, if one proposes to use such models for the purpose of evaluating alternative economic policy regimes. The reason is that the estimated parameters of such a model would be functions of more fundamental "structural" parameters in combination with other parameters which describe the characteristics of the policy rule itself. Any change in policy regime would change the characteristics of the reduced form response — of endogenous to pre-determined variables — by altering the expectations of agents whose behaviour is summarized by the reduced form model. In short; the parameters of unrestricted econometric models are not invariant to changes in policy regime.

The logic of the Lucas Critique relies on Samuelson's correspondence principle. This principle states that the endogenous variables of econometric models may be described as non-trivial functions of the exogenous variables. The application of the correspondence principle implies that a shift in policy regime (an infinite sequence of exogenous variables) will induce a shift in the equilibrium price sequence (an infinite sequence of perfectly anticipated prices).[1] Since agents' expectations of future prices are implicitly contained in the parameters of the reduced form of an econometric model it follows that any change in policy regime will necessarily involve a change in these parameters.

My reservations about the general applicability of this argument are based on the recent discovery that general equilibrium models may contain infinite numbers of equilibria.[2] In order to apply the correspondence principal to a macroeconomic model it is necessary that the equilibria of the model be locally isolated. If this

[1] I have stated this argument in terms of perfect foresight models but the argument carries over equally well to stochastic economies in which an equilibrium is described as a sequence of distribution functions.

[2] I am referring in particular to the work of Kehoe and Levine [8] who show that for open sets of parameter values the equilibria of overlapping generations economies may be indeterminate.

is not the case, then an exogenous parameter shift may move the set of equilibria but it will leave the economist with no clear prediction about the direction of movement of the endogenous variables. An agent who lives in a world in which there is an infinite number of rational expectations equilibria will clearly face a difficult problem in deciding how to act.

Underlying this paper is the idea that agents may solve the dilemma, of how one should forecast future prices, by fixing on a rule that maps from current and past observables to expectations of future prices; that is, they will use a type of 'adaptive expectations.' I show, in a model in which there exist multiple rational expectations equilibria, that there is a unique rational expectations equilibrium which is supported by a process invariant forecast rule. This rule can be used to correctly forecast the probability distribution of inflation, however, the rule is independent of the parameters of the probability distribution of the fundamentals of the economy. In short, this forecast rule is immune to the Lucas Critique of Econometric Policy Evaluation.

2 A Brief Review of the Lucas Critique

In this section I present the simplest possible example of a dynamic economic model that demonstrates the idea behind the Lucas Critique. This example consists of the following two equations:

$$y_t = \alpha E_t[y_{t+1} \mid I_t] + x_t + u_t \tag{1}$$

$$x_t = \gamma x_{t-1} + \delta + v_t \tag{2}$$

The scalar variable y_t is endogenous and equation (1) is a structural equation that makes y_t a function of its own expected value at date t+1. The rational expectations property is contained in the assumption that the expectation in this equation is evaluated using the true distribution of y_{t+1}, conditional on the information set I_t, which is assumed to contain all contemporaneous and past observations of y_t, x_t, u_t and v_t. The second equation represents a policy rule that governs the evolution of the exogenous variable x_t. The terms u_t and v_t are assumed to represent independently and identically distributed, serially uncorrelated, white noise error terms.

A key assumption that is sometimes obtainable as a restriction from economic theory is the inequality:

$$0 < \alpha < 1. \tag{3}$$

Using this assumption one may iterate equation (1) into the future to find y_t as a weighted sum of all future expected values of x_t. Inequality (3) guarentees that this sum converges and that it may be represented by the expression:

$$y_t = E_t[\sum_{s=0}^{\infty} \alpha^s (x_{t+s} + u_{t+s})]. \tag{4}$$

It is also typical to assume that the policy process represented by equation (2) is a stationary stochastic process. This assumption is guarenteed by the restriction:

$$0 < \gamma < 1. \tag{5}$$

By iterating equation (2), one may describe the value of the policy variable x_{t+s} as a function of past realizations of the policy shock v_t and of the value of x at date t:

$$x_{t+s} = \gamma^s x_t + \sum_{m=0}^{s-1} \gamma^m (\delta + v_{t+s-m}). \tag{6}$$

Substituting (6) into (4) one obtains the unique rational expectations equilibrium, which describes the value of the endogenous variable y_t as a function of the contemporaneous value of the policy variable x_t and of the contemporaneous disturbance term u_t:

$$y_t = \frac{\delta\alpha}{(1-\alpha)(1-\alpha\gamma)} + \frac{x_t}{1-\alpha\gamma} + u_t. \tag{7}$$

Prior to Lucas' article [9] it had been common practice to estimate the parameters of simple econometric models and to assume that these parameters would remain invariant to changes in policy regime. In the context of the model that was introduced above this practice would be equivalent to regressing y_t on x_t during a period over which γ was constant. This in itself is not a problem providing one does not then use this model to predict the effects on y_t of a policy intervention in which γ is altered. It is clear from equation (7) that, in this particular example, the forecasts that one obtains from such an exercise will be inaccurate since they do not take account of the fact that the coefficient of x_t is itself a function of the policy parameter, γ.

A critical step in the above argument is the assumption that the parameter α is less than 1. If this assumption is violated then it is no longer possible to describe y_t as a function of all expected future values of x_t, since the infinite sum in equation (4) does not converge. This does not, however, imply that a rational expectations equilibrium does not exist. Indeed, if the parameter α is greater than 1, then one can find multiple rational expectations equilibria. Consider, for example, any process in the class:

$$y_t = \frac{y_{t-1}}{\alpha} - \frac{x_{t-1}}{\alpha} + (\epsilon_t - \bar{\epsilon}) - \frac{u_t}{\alpha}; \tag{8}$$

where ϵ_t is a disturbance term with a conditional mean of $\bar{\epsilon}$. It is clear that equation (1) is satisfied for any process governing the evolution of ϵ_t provided only that ϵ_{t+1} is orthoganal to the information set I_t. It is also true that equation (8) defines a stationary process for y_t for any value of α that is greater than unity. Notice, however, that the parameters of equation (8) are independent of the parameters of the policy process, equation (2). It follows that agents who lived in an environment that was well described by this model would be able to find stable forecasting rules for future values of endogenous variables that were independent of the parameters of the policy process. An econometrician who studied such an economy could run a regression of y_t on y_{t-1} and x_{t-1}, and he could use this regression to analyze the effect of policy interventions in which γ was altered. In short, the Lucas Critique does not hold in this environment.

3 A Simple Economic Example

The mechanical example that I provided in section (2) is, hopefully, illuminating but the reader is entitled to remain skeptical at this point. Perhaps there are no interesting economic models in which the relevant parameter restrictions are satisfied. To allay these doubts; in this section of the paper I show that one of the most commonly used models of inflation — the Cagan hyperinflation model — contains an equilibrium in which the Lucas critique does not hold. The version of the model that I shall present is non-linear and it also contains an equilibrium that displays the policy dependence properties that one normally associates with rational expectations equilibria. It is, of course, possible to argue that this latter equilibrium is more reasonable in some sense although it is not clear on what grounds such an argument would be based. I shall take the opposite point of view and I shall claim that, in a world of the type that I shall present below, one would expect the economy to be situated at an equilibrium at which the Lucas critique did not hold. Further, I shall argue that, it is precisely because the Lucas critique does not hold that an equilibrium of this kind is compelling. I base this argument on the idea that policy processes are likely to be considerably less stable than many other aspects of an agent's environment. If the policy process changes frequently, in unpredictable ways, then agents must invest effort into forecasting these changes and correcting their behaviours accordingly. If, however, their optimal forecasting rules are independent of policy parameters then shifts in policy will provide no incentive for agents to adapt their actions to changing circumstances. Forecasting rules will be robust, in such an environment, to fluctuating policies. It is precisely this robustness property that leads me to suggest that "Lucas proof" equilibria may be more likely to characterize actual economies than their more familiar policy dependent counterparts.

As an example of a model that displays both types of equilibria consider the following simple structure;

$$\frac{M_t}{P_t} = E_t\left[\frac{P_t}{P_{t+1}} \mid I_t\right] \tag{9}$$

$$M_t = M_{t-1} + g_t P_t. \tag{10}$$

Equation (9) is a demand for money function in which M_t represents the demand for money balances, P_t is the price of commodities and I_t represents information available at date t. Equation (10) represents a money supply process in which the government prints enough money in period t to purchase the real quantity of resources g_t at endogenously determined prices P_t. I shall typically assume that the sequence $\{g_t\}_{t=0}^{\infty}$ is randomly drawn from some known distribution with compact support $[a, b]$ where $0 < a < g_t < b < 1/4$. However, it is convenient for preliminary purposes to investigate the case in which g_t is non-stochastic and is described by the rule, $g_t = \bar{g}$, for all t. In this case one may solve equations (9) and (10) to find a single difference equation in real balances:

$$\frac{M_t}{P_t} = \left(\frac{M_{t-1}}{P_{t-1}}\right)^2 + \bar{g}. \tag{11}$$

This equation is graphed in figure 1 which illustrates the point that equation (11) possesses two stationary states equal to the the roots of the quadratic:

$$x^2 - x + \bar{g} = 0. \tag{12}$$

Both roots are real for values of $\bar{g}$ such that $0 < \bar{g} < 1/4$. Associated with each of these stationary values of money balances is a stationary inflation factor, $\pi_{t+1} \equiv P_{t+1}/P_t = \pi$, given by the expressions:

$$\pi_1 = \frac{1}{x_1(\bar{g})}, \qquad \pi_2 = \frac{1}{x_2(\bar{g})}; \tag{13}$$

where x_1 and x_2 are the roots of equation (12).

In the case in which g_t is stochastic one may obtain a functional equation that characterizes equilibrium sequences of real balances, analogous to equation (11). This functional equation takes the form:

$$\left(\frac{M_t}{P_t}\right)^2 = E_t\left[\frac{M_{t+1}}{P_{t+1}} - g_{t+1}\right]. \tag{14}$$

If one lets $\bar{g}$ represent the mean of g_t then equation (14) may be approximated by the following stochastic difference equation around either of the two stationary states, x_1 or x_2.

$$\frac{M_t}{P_t} = \psi_i + \alpha_i E_t\left[\frac{M_{t+1}}{P_{t+1}} - g_{t+1}\right]; \qquad i = 1, 2. \tag{15}$$

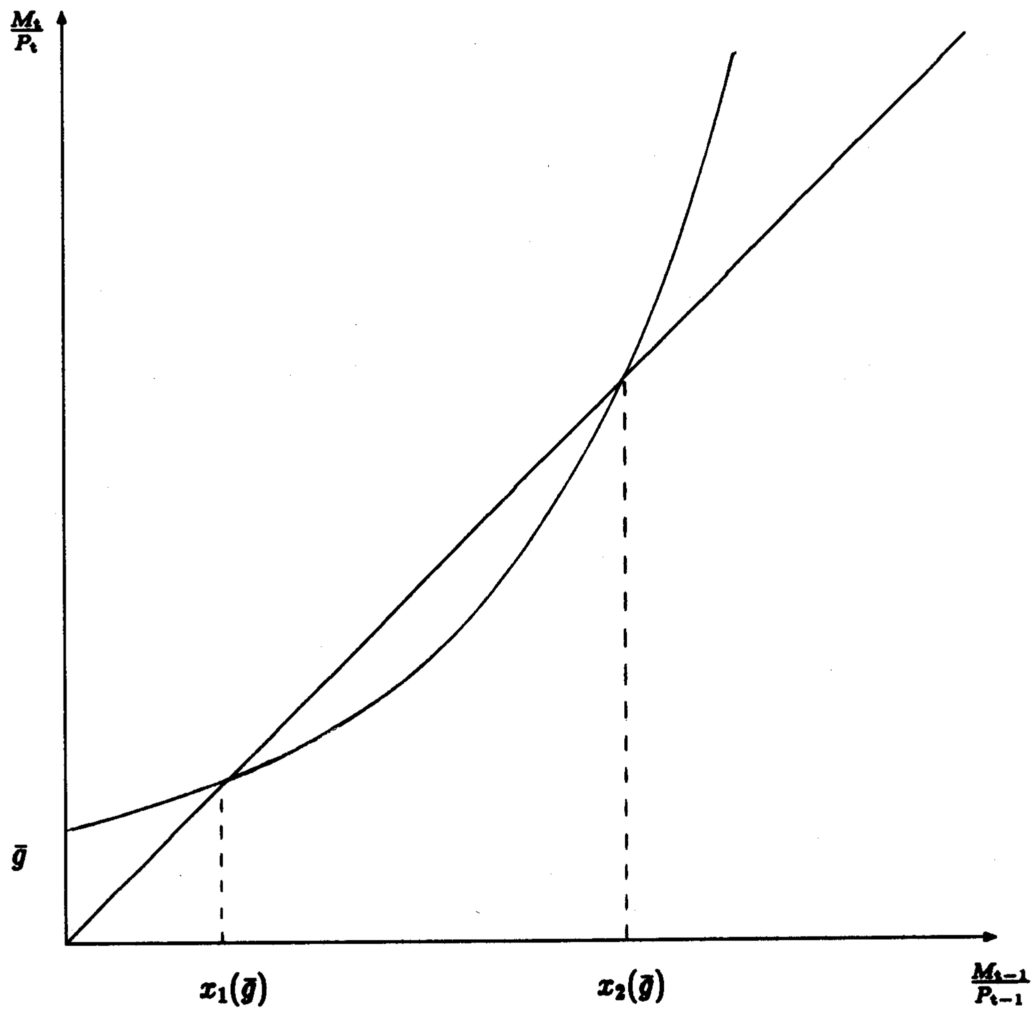

Figure 1: The Set of Perfect Foresight Equilibria

The subscript i on the parameters ψ and α denotes the values of these parameters at the stationary states x_i, for i=1,2. One may show that the value of α_i at the two stationary states is respectively greater than and less than unity; that is:

$$\alpha_2 < 1 < \alpha_1. \tag{16}$$

It follows that this model describes a world in which there are multiple stationary rational expectations equilibria.[3] In light of my discussion in section 2, one of these equilibria — the stationary stochastic rational expectations equilibrium that is associated with the steady state of the non-stochastic model x_2 — is locally unique. But associated with the steady state x_1, one may show that there exists a continuum of stationary rational expectations equilibria.

4 Standard Rational Expectations Models

The usual approach to linear rational expectations modelling is to impose the inequality $0 < \alpha < 1$ as a theoretical restriction. It is typically argued that some restriction of this kind must follow from the transversality conditions of an individual agent's optimizing problem. If one takes this approach then the backwards instability of the perfect foresight dynamics in the neighborhood of a steady state operates as a uniqueness criterion.[4] There are many stochastic difference equations that satisfy equation (15) in the neighborhood of the stationary state at which $\pi_t = \pi_2$, but only one of these stochastic difference equations remains in the neighborhood of π_2 as $t \to \infty$. The stationarity condition picks the value of π_t to be a function of g_t. There is a simple analogy with a nonstochastic difference equation which explains what is happening here.

To begin with I will analyse a non-stochastic model which illustrates the point that the price level must jump in order to maintain equilibrium in a standard

[3] I expand upon this statement below in section 6.

[4] By 'backwards instability' I mean that the difference equation that characterises perfect foresight price sequences, $P_{t+1} = f(P_t)$, diverges for a given initial condition, $P_0 = \bar{P}$, in the neighbourhood of the stationary state.

rational expectations equilibrium. The experiment is the following one. Suppose that the distribution of g_t is concentrated on $\bar{g}$ for $t = 1, 2, \ldots, T-1$. Suppose further that all agents believe that $g_t = \bar{g}$ for all t; but at t=T they are surprised by a previously unannounced shift to the policy $g_t = g^*$ for $t = T, T+1, \ldots, \infty$.[5] What happens in this environment if prices do not jump in response to the policy announcement? In a linear model the answer to this question is clear; if prices do not jump then the economy will not remain at the stationary state.

For example, It would be consistent with the difference equation (14) (and with the demand and supply equations (9) and (10)) if agents were to believe that prices, from dates t=T on, would be given by the equation:

$$\frac{P_t}{P_{t+1}} = \left(\frac{P_{t-1}}{P_t}\right)^2 + g_t \tag{17}$$

where $g_T = \bar{g}$ and $g_t = g^*$ for $t = T+1, \ldots, \infty$.[6] This is an example of a rule for forecasting prices that is inconsistent with the existence of a stationary rational expectations equilibrium in a standard rational expectations model, where I am using the adjective standard to mean that the functional equation that describes equilibrium sequences of distribution functions is solved forwards. Notice in particular, that if agents use equation (17) to forecast future prices, that the inflation factor between dates T-1 and T will not change in response to the announced change in policy regime; that is, with these beliefs, the price level at date T will not jump. In a linear rational expectations economy these beliefs are not consistent with the assumption that the economy is in a stationary rational expectations equilibrium since, if prices are governed by (17), they will not converge back to a new stationary state. The only set of beliefs that *is* consistent with a stationary equilibrium in this linearized version of the model is that which, in equilibrium, causes the price level to jump to a point that implements the long run solution $\pi_t = [1/x_2(g^*)]$.

In a non-trivial *stochastic* rational expectations equilibrium the price level must

[5]In a stochastic environment, the policy regime is described by a perfectly anticipated sequence of distribution functions.

[6]In the linearised model this forecasting rule will be locally approximated by the linear equation: $(P_t/P_{t+1}) = (1/\alpha_2)[(P_{t-1}/P_t) - \psi_2] + g_t$ around the stationary state π_2.

move each period, in step with the exogenous disturbances of the model, in a manner which ensures that the present discounted value of the expectations of future values of the inflation rate is bounded. But, once one recognizes that the complete model is non-linear, there are other possibilities. These possibilities arise from the fact that non-linear models may contain other stationary equilibria around which one may linearize. Further, these alternative equilibria may violate the parameter restriction $0 < \alpha < 1$, that was imposed on the linear model described above.

5 Rational Expectations Economies in which there are Multiple Solutions

As an example of a model in which the restriction $0 < \alpha < 1$ may be violated consider the linearized version of the Cagan hyperinflation model around the non-stochastic steady state π_1. At this steady state, the functional equation that characterizes equilibrium sequences is well approximated by a linear stochastic difference equation in which the coefficient of the future expected value of the state variable is greater than unity. That is; α_1 in equation (15) is greater than 1.

Consider what would happen, close to this stationary state, if agents were to use the forecast rule given by equation (17). Using this rule to predict future prices, it follows from equation (9) that the demand for money in period t will be given by the expression:

$$\frac{M_t}{P_t} = \left(\frac{P_{t-1}}{P_t}\right)^2 + g_t. \tag{18}$$

It follows further, from equation (10), that the supply of money will be given by the equation:

$$\frac{M_t}{P_t} = \left(\frac{M_{t-1}}{P_{t-1}}\right)\left(\frac{P_{t-1}}{P_t}\right) + g_t. \tag{19}$$

Equating demand and supply one obtains the following expression:

$$\frac{P_{t-1}}{P_t} = \frac{M_{t-1}}{P_{t-1}}; \tag{20}$$

which determines the price level in period t. Notice that, from lagging equation (18) and substituting it into the right-hand-side of (20), this equilibrium condition implies that if agents use the forecast rule given by equation (17) to predict future prices, that the rule will be self-fulfilling. That is, the sequence of actual prices will be given by the same equation as the forecast rule.

Recall that, in a standard linear rational expectations model, a forecast rule of this kind is inconsistent with a stationary equilibrium since the difference equation (17) will diverge from the stationary state π_2. But in the non-linear economy this forecast rule may cause prices to converge to a different rational expectations equilibrium around the stationary state π_1. In the perfect foresight example that I described above; the economy will move from the steady state $\pi_2(\bar{g})$ to the steady state $\pi_1(g^*)$ in response to an unanticipated increase in spending from $\bar{g}$ to g^*. Suppose, alternatively, that the government follows a stochastic policy in which g_t is drawn from a probability distribution with mean $\bar{g}$. Under reasonable assumptions about the support of this distribution; one may show that the forecast rule (17) will cause the inflation rate to converge to a stationary distribution around the non-stochastic stationary state $\pi_1(\bar{g})$. Furthermore, this forecast rule will be rational in the sense that if agents use the rule to forecast prices, then actual prices will be described by the same conditional distribution as forecast prices.

6 Sunspot Equilibria

The existence of backward looking equilibria of the kind outlined above has been known for some time. Typically, however, these equilibria are perceived to represent a problem for the rational expectations research agenda that must somehow be resolved. The problem arises from the fact that there exist not one, but many, stochastic rational expectations equilibria around a stationary state of the perfect foresight model for which the perfect foresight dynamics are stable. For example, in the above model, suppose that agents use the following forecast rule:

$$\frac{P_t}{P_{t+1}} = \left(\frac{P_{t-1}}{P_t}\right)\left(\frac{P_{t-1}}{P_t} - (\epsilon_t - \bar{\epsilon})\right) + g_t + (\epsilon_{t+1} - \bar{\epsilon}); \tag{21}$$

where ϵ_{t+1} and ϵ_t represent independently distributed "sunspot" variables with mean $\bar{\epsilon}$, which have no relevance for economic activity other than the fact that agents believe that they affect prices. Using the fact that agents use the forecast rule (21), it follows that the actual price level will be found by equating the following demand and supply equations:

$$\frac{M_t}{P_t} = \left(\frac{P_{t-1}}{P_t}\right)\left(\frac{P_{t-1}}{P_t} - (\epsilon_t - \bar{\epsilon})\right) + g_t; \tag{22}$$

$$\frac{M_t}{P_t} = \left(\frac{M_{t-1}}{P_{t-1}}\right)\left(\frac{P_{t-1}}{P_t}\right) + g_t. \tag{23}$$

Equating the right hand sides of equations (22) and (23) and substituting for M_{t-1}/P_{t-1} by lagging equation (22) one period, one obtains the following expression which determines the price level in period t:

$$\frac{P_{t-1}}{P_t} = \left(\frac{P_{t-2}}{P_{t-1}}\right)\left(\frac{P_{t-2}}{P_{t-1}} - (\epsilon_{t-1} - \bar{\epsilon})\right) + g_{t-1} + (\epsilon_t - \bar{\epsilon}). \tag{24}$$

Equation (24) affirms that the forecast rule given in (21) is self-fulfilling in the sense that if agents use this rule to forecast prices in period t+1, then the same rule will determine prices in period t. But since the sunspot variable ϵ_t is arbitrary, it follows that there are many rational expectations equilibria of this kind. Furthermore, since all of these equilibria are described by Markov processes which converge to a bounded interval, all of them can be described by invariant distributions; that is, as *stationary* rational expectations equilibria.[7] It is for this reason that many economists are reluctant to accept sunspot equilibria as positive models of economic phenomena. Since the sunspot variable is arbitrary, one is left with a model that cannot be refuted by observation.

7 Immunity to the Lucas Critique as a Selection Criterion for Economies with Multiple Equilibria

There have been numerous attempts to narrow down the number of equilibria in models of the kind that I have outlined above. This is not the place to attempt a

[7] For a much more complete analysis of this argument in the context of a fully specified maximising model see Farmer and Woodford [5]. The existence of invariant distributions for the endogenous variables of a model that is very similar to the one discussed above is established, in [5], using the results of Futia [6].

comprehensive survey of this literature although I think that it is fair to say that none of the approaches that is currently available has gained universal acceptance.[8] I make no apology for introducing yet another selection criterion and I am unable to offer any deep argument in terms of, for example, out of equilibrium learning rules.[9] My case rests on, what I hope will be perceived as, its plausibility.

The criterion that I propose is that one should select a rational expectations equilibrium that is invariant to changes in the distribution functions of future values of the exogenous variables of the model. To put it another way, I am looking for a rule for forecasting prices at period t+1 which maps from realizations of exogenous and endogenous variables at dates t+1 or earlier, into realizations of period t+1 prices. This rule may depend on the *realizations* of exogenous and lagged endogenous variables, but it must be independent of the parameters of their distribution functions. I am proposing that one should search for a rational forecasting rule that is immune to the Lucas Critique.

This way of selecting equilibria eliminates sunspots because the sunspot forecast rule, equation (21), depends on the mean of the distribution of ϵ. This dependence implies that if the distribution of the sunspot variable changes in an unpredictable way then the forecast rule described by (21) will turn out, ex-post, to have been inaccurate. In this sense, I am treating a sunspot variable in exactly the same way as any other exogenous variable of the model. Given this criterion for choosing between equilibria, one may show that the above model contains a unique 'Lucas proof' rational expectations equilibrium which is implemented by the forecast rule described in equation (17). Notice that this rule does not imply that changes in policy regime cannot affect prices, rather, it implies that these changes feed into

[8] McCallum [12], Taylor [13] and Evans [3],[4] represent just a few of the solutions that have previously been proposed to the problem; the book by Whiteman [15] is also a good source of material on this issue.

[9] A number of authors have argued that a particular equilibrium should be selected if one can show that some plausible learning mechanism converges to it. For example, the papers by Evans [3] and Marcet and Sargent [11]. But since it is known that there are examples of plausible learning mechanisms that converge to sunspot equilibria (Woodford [17]), it does not seem likely that this approach will allow us to isolate a unique equilibrium.

prices slowly.

My reasons for arguing in favor of a Lucas proof equilibrium, when it exists, are based on the idea that agents forecast future prices using ad hoc rules of thumb. This concept in itself is not, of course, inconsistent with standard rational expectations theory. Typically one argues that, even though agents may use ad hoc schemes to predict the future, there is only one forecasting rule that is consistent with the observed distribution of actual prices. Furthermore, if the fundamentals of the economy change, then agents must recompute their forecast rules in a way that is consistent with the new parameters that govern the evolution of these fundamentals. This view of the economy suggests that there may be certain events that one identifies as regime changes which are followed by a period of instability as agents learn about their new environment. Such a view requires at least a limited degree of confidence in the idea that the fundamentals of the economy remain stable and predictable for reasonable lengths of time.

An alternative description of the world in which we live, and one which seems to me to be a good deal more realistic, is one in which the distribution function of future fundamentals is both unknown and unknowable: a world of Knightian uncertainty rather than of risk. The great advantage of a Lucas proof forecast rule is that it continues to operate perfectly well in such a world. Agents do not need to know the distribution of an infinite sequence of future fundamentals.

8 How Special are Models with Lucas Proof Equilibria?

A number of questions arise when one considers the above selection criterion. In this section I will describe some of the characteristics of the Lucas proof equilibrium that arises in the Cagan hyperinflation model and I will discuss which of these characteristics are likely to generalize to other macroeconomic models. It is clear that if this criterion is to be taken seriously then it must be useful in a reasonably large class of environments.

The first obvious characteristic of the equilibrium forecast rule, equation (17), is the property that t+1 prices are completely predetermined in period t. This characteristic is special to the particular example that I have presented and, in general, there may be examples of models in which the forecasting rule for prices is conditional on the realization of an exogenous variable. For example, the Cagan demand for money example may be derived from a two period overlapping generations economy with quadratic preferences in which agents supply n_t units of labor in period t and consume c_{t+1} units of output in period $t+1$. This example is discussed in depth in Farmer and Woodford [5]. In more complicated examples of the overlapping generations model, in which agents may be endowed with a random amount of the commodity in their second period of life, the amount of the old agent's endowment in period t+1 will directly affect t+1 prices. In examples of this kind prices at date t+1 may depend upon the realization of fundamentals at date t+1 but they will still be independent of the parameters of fundamental distribution functions.

One question which arises in the model with predetermined prices is; how do markets clear? For example, suppose that there is an unanticipated increase in government expenditures. In this situation agents will increase their holding of real money balances in the rational expectation that the rate of inflation between periods t and t+1 will be lower than agents would otherwise have anticipated. The expectation is self-fulfilling in the sense that the increase in the demand for money that it induces raises the tax base of the inflation tax by exactly enough to generate the revenues required to finance a higher level of government expenditure.[10]

What is important in generating models that display Lucas proof equilibria is that the perfect foresight version of the model should contain a locally indeterminate stationary state; that is, there should exist non-stationary equilibria of the model that converge to this stationary state as one perturbs the set of initial conditions

[10]Some readers will undoubtedly be unhappy with this example because of the implication that higher government deficits are associated with lower rates of inflation. In defence of this implication it is worth noting that recent work by Bordo [2] suggests that equilibria of this type are consistent with observations from time series data on Israel. I am unaware of any strongly counter factual implications for low inflation countries and the recent evidence in the United States, of persistently high deficits but consistently low inflation rates, seems to be in accord.

of the model in every possible direction. Since this property holds in overlapping generations models for open sets of parameter values[11] there is a large class of well specified general equilibrium economies that fit the bill.

In order for every structural equation of a macroeconometric model to be invariant to changes in the processes generating the exogenous variables one requires indeterminacy of the highest degree; that is, the perfect foresight dynamics must converge back to the stationary state when the initial conditions of the model are perturbed in every possible direction. This is a much stronger condition than the existence of indeterminacy of some degree, which is all that is required for the existence of sunspot equilibria. My own investigations of the dynamics of *monetary* models in the class discussed by Blanchard [1] and Weil [14] (economies with infinite numbers of infinite horizon agents) suggests that there is a very useful subclass of macroeconomic models with exactly this property of which the familiar representative agent model is a degenerate special case. This subclass corresponds, in perfect foresight economies, to an assumption of logarithmic preferences.

It is also worth pointing out that if complete indeterminacy does not hold there may still exist a process invariant forecast rule for some subset of the variables. For example, generalizations of the class of models that I referred to above, in which agents are assumed to have constant elasticity preferences, contain equilibria which may be described in terms of a nonlinear difference equation involving three variables: human wealth, the rate of interest and aggregate non-human wealth. In examples of these models one can find a process invariant forecast rule for the rate of interest that involves only contemporaneous and lagged values of aggregate non-human wealth and lagged values of the interest rate itself. In response to an unanticipated policy change agents in this economy could continue to use the same forecast rule for interest rates but, using this rule, their own computations of human wealth would not be policy invariant. This observation suggests that behavioural equations such as the consumption function or the demand for money, which will depend on human wealth, are unlikely to remain structurally stable in

[11]See Kehoe Levine [8].

the face of regime changes. However, some subset of the reduced form equations of an econometric model may well display such stability as a direct result of the existence of an invariant forecast rule for the interest rate.

From the perspective of linear stochastic models, the existence of a Lucas proof equilibrium for every endogenous variable requires that the backward dynamics of the model should be stable. More precisely, consider a perfect foresight version of the linear model, by imposing the assumption that all stochastic terms are degenerate. This model will have a representation of the form:

$$Y_t = AY_{t-1} + BX_t; \tag{25}$$

$$X_t = CX_{t-1}; \tag{26}$$

where Y and X are n and m vectors of endogenous and exogenous variables.[12] A sufficient condition for the existence of an equilibrium in which the Lucas Critique does not hold is that the roots of the matrix A should all lie within the unit circle. In models of this class the Lucas proof equilibrium will correspond to the minimal state variable equilibrium discussed by McCallum [12]. In models in which the relevant parameter restrictions are not satisfied, no such equilibrium will exist.

9 Conclusion

What, if anything, should we learn from the economic example that I have discussed in this paper. The response of some readers will undoubtedly be that it is a theoretical curiosum that does not merit serious consideration. This conclusion is suggested by the fact that the model of choice for a good part of the profession, the representative agent paradigm, does not contain equilibria of the kind that I have described. It cannot contain such equilibria since there is a direct equivalence in models with a finite number of infinitely lived agents between the set of competitive equilibria and the solution to an appropriately specified planners problem.

[12] Any linear model in which expectations of future variables appear on the right-hand-side of the equations describing the endogenous variables can be written in this way by a suitable redefinition of variables.

This equivalence allows one to rule out indeterminate steady states by appealing to the transversality conditions of a programming problem.

Whilst it is correct to assert that the representative agent paradigm in its strictest form cannot contain the type of equilibria that I have described, small departures from this structure do display such equilibria. In particular the attempt to include money in the representative agent model using a cash-in-advance constraint leads directly to the type of model that I have analysed in this paper.[13] The two period overlapping generations model is well known to possess a stable equilibrium of the kind required to construct a Lucas proof forecast rule and more general versions of the overlapping generations economy possess similar properties.

Whether or not the arguments that I have presented prove to be persuasive will depend in large part on whether one is able to develop tractable macroeconomic models in which the criterion that I suggest for selecting equilibria is useful. It is my suspicion that many researchers have remained skeptical of studying models with multiple equilibria because it is widely believed that they do not lead to refutable predictions.[14] One obvious way out of this dilemma is to isolate one of the possible equilibria of a model and to confront the implied predictions of this equilibrium with data. The equilibrium that I propose is one possible candidate which has clearly different implications for the behavior of time series data from that of standard rational expectations models.

One implication of a Lucas proof equilibrium follows directly from its definition. The existence of such an equilibrium implies that one should be able to isolate structurally stable econometric models which exhibit parameter constancy over reasonably long periods of time. The feasability of such a task is currently in dispute; indeed it is the assertion that such a research agenda is not likely to

[13] The paper by Wilson [16] was the first work that I know of which analyses the dynamics of a model of this kind in depth. Wilson shows that the cash-in-advance constraint allows one to reduce the dynamics of an infinite horison model to first order difference equation in real balances which behaves a lot like the two period overlapping generations model. Woodford [18] uses a similar technique to construct sunspot equilibria. Lucas [10], on the other hand, is careful to introduce sufficient structure to rule out such equilibria.

[14] See Woodford [19] for a counter argument to this position.

succeed which inspired the Lucas critique in the first place. Recently, however, David Hendry[15] has claimed, in a series of papers, that the Lucas critique is not only refutable, but refuted by the data. Hendry has been strongly criticized in some circles because his position is perceived to be a-theoretical. According to this line of thought, the rational expectations hypothesis is so obviously useful as an organizing principle that any evidence that appears to contradict it must be taken to represent a refutation of some other aspect of the modelling exercise. It is at least possible that the equilibria that I have discussed in this paper may provide a reconciliation of some of Hendry's results on parameter constancy with the theory of rational expectations equilibrium.

I would like to close this discussion by reiterating what I am, and what I am not, claiming about the Lucas Critique. I am not saying that all macroeconomic models contain an equilibrium which is supported by a process invariant forecast rule. But in an important subclass of models that display multiple stationary rational expectations equilibria; one can isolate one equilibrium that does display this property. This much is, I believe, indisputable. Whether or not these facts prove to be interesting to other researchers will depend on whether or not a class of models that displays this property turns out to be useful in explaining macroeconomic data. Recent interest in models of multiple equilibria leads me to be believe that this property will prove to be a useful way of generating refutable predictions in a variety of situations. Whether these predictions will indeed be refuted remains an open question.

[15]See, for example, Hendry [7].

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