The Scope of Bargaining Failures With Complete Information

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ABSTRACT

This paper analyzes bargaining under the split the difference mechanism when there is complete information. There exists a multiplicity of equilibria, they all suffer from unsatisfactory properties. Since anything seems possible, we focus on the completely mixed strategy equilibria but find that such equilibria require that the negotiator with the higher bargaining cost receive higher profits. Allowing the bargaining process to be dynamic does not entirely solve the problem because the offers in the dynamic game can demonstrate chaotic behavior. Moreover when failure costs are low there exist many infinite horizon C.M.S. equilibria. We find that there is significant probability of delay which is consistent with some empirical reality. Finally, if there is asymmetric information over bargaining costs, the negotiator with the higher bargaining costs obtains lower profits. Thus, asymmetric information leads to more plausible properties for most bargaining equilibria.

I. Introduction

"This division of labor....is the necessary, though very slow and gradual, consequence of a certain propensity in human nature which has in view no such extensive utility; the propensity to truck, barter, and exchange one thing for another."[1] Despite Adam Smith's warning of the slow and gradual nature of improvements in efficiency through the bargaining process, most economists seem convinced that if a surplus exists trade will take place, agreements will occur. Furthermore, since all traders are better off when they come to an agreement to share the surplus quickly, economists have focused on the conditions that will lead to immediate trade. Yet, if we believe that profits drive entrepreneurs to enter certain industries, we should also believe that surplus will drive traders to bargain extensively in order to appropriate part of the surplus, even when there are costs to delay and the size of the surplus is well known. If traders compete for larger shares of the surplus, delays may occur in reaching agreements, resources will be consumed, and bargains may fail to occur with significant probability. The models analyzed in this paper attempt to uncover the impact of the rules of bargaining on the expenditure of resources, and the probability that the buyer and the seller can strike a bargain to divide the spoils of trade and commerce.

The Stahl-Rubinstein model of bargaining posits a game in which offers and counter-offers are are tendered until an agreement is reached. The surplus over which the players are bargaining shrinks with successive offers--based on individual discount rates. In the perfect equilibrium of this game the player making the first offer receives a potentially significant advantage and there are no bargaining failures--agreement occurs in the first period. The first period agreement result is frequently criticized from an empirical point of view due to the numerous observations of bargaining delays and failures

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[1] Adam Smith (1976) page 17.

(see, for example, Tracy [1986] and Card [1988]). In addition, it is hard to describe institutional arrangements in the field that allow for first mover advantages. Extensions to the Stahl-Rubinstein model of bargaining where there is asymmetric information concerning individual values or discount rates show that delays in agreements can occur (see Cho [1988]). Nonetheless, these bargaining "failures" are efficient because delay serves the purpose of information transmission. Delay allows for the separation of types provided that the discount rate is not arbitrarily close to one.

To remedy the first mover advantage, analysts have investigated simultaneous move bargaining games. In particular, a "split-the-difference" mechanism for bilateral trading has been developed and analyzed (see Myerson and Satterthwaite [1983], Chatterjee and Samuelson [1983] and Leininger, Linhart, and Radner [1987]). This mechanism is usually described as a game of incomplete information in which the buyer and seller of an object simultaneously submit offers.[2] Trade occurs when the buyer's offer exceeds the seller's offer, and the price paid by the buyer is the average of the two offers. This mechanism has a multiplicity of Bayes-Nash equilibria, some with low efficiency properties (see Leininger, Linhart and Radner [1987]). Thus, it seems that uncertainty concerning the players' valuations is enough to generate delay, failures, or other inefficiencies in bargaining. More importantly, as the uncertainty in values is decreased (the distributions are tightened) bargaining inefficiencies and delays are reduced and in some equilibria become efficient before the uncertainty is completely eliminated (see Broman [1987]).

In addition to asymmetries in information, most of the bargaining models have focused on shrinking surpluses or bargaining costs. However, if players can submit offers arbitrarily quickly and there is no scope for commitment, then bargaining costs are irrelevant. Thus, unless there are significant time intervals required between offers, delay cost can play no role in the equilibrium of the game (this result has been found for durable goods monopoly problems and has been coined the "Coase

[2]There is asymmetric information about buyer's reservation price and the seller's cost of production; the cost of failure is assumed to be zero.

conjecture"--see Coase [1972]; Gul, Sonnenschein and Wilson [1986]; Fudenberg, Levine, and Tirole [1985]). We see delay between offers as being a crucial part of the bargaining process. Negotiators rarely have the leeway to make <u>any</u> offer they see fit at any time the see fit. In fact, they are most often only the representatives of larger bodies like the firm's board of directors or the union membership that constrain their actions. In addition to the rigidities induced by the institutions of bargaining, there are other costs of negotiation failures. For example, strikes are disadvantageous to both parties: the firm relinquishes profits and the workers loose income. In pretrial negotiations the plaintiff and defendant often pay incremental lawyer fees until a settlement is reached. A further empirical example of such institutions is analyzed in Rosenthal [1988] where owners of marshland contemplate draining the marsh and enjoying the returns to better pasture. For the marsh to be drained, however, the owners must agree on a rule to divide the surplus. Every time a proposal fails the owners collectively forego the rent to the surplus.

Clearly such bargaining failures can be the result of asymmetric information (see Mailath and Postlewaite [1989a, 1989b]). Yet it seems that asymmetric information cannot always explain delay or failure to come to an agreement. The open research question we wish to address is: what drives bargaining failures? We shall model the bargaining process using the split-the-difference mechanism with complete information. In addition, if the parties do not come to an agreement they must pay a cost of $c > 0$.[3] The cost c can be interpreted as a direct loss out of current earnings from not agreeing. The cost becomes sunk if there is disagreement but can be avoided if the parties reach an accord. In

[3] We think the ability to commit is crucial in understanding empirical reality. Although in the model we analyze a bargain occurs with probability one if the game is played for sufficiently many periods, a significant proportion of the surplus may be consumed through bargaining, which is what we think occurs in bargaining failures. Hart [1989] has developed a model of strikes when the time between submission of offers is fixed (part of the bargaining institution), so that there is some level of commitment in the offers made by the players.

the dynamic version of this game, disagreement does not affect the size of the surplus because all the cost of failure are borne incrementaly and out-of-pocket. We look at this bargaining game with complete information and symmetric payoffs so that we can focus entirely on failure and delay due to institutional arrangements. Put differently, can there exist significant bargaining failure or delay even when there is complete information and symmetric payoffs?

The model has a multiplicity of Nash equilibria and thus to make any headway we must confront the problem of equilibrium selection. We focus our attention on completely mixed strategy (C.M.S.) equilibria since alternative selection criteria such as focal equilibria and trembling hand perfect equilibria are not robust in both payoffs and conjectures. Of course, when mixing is considered failure and delay become nontrivial and the expected value of the game is very sensitive to the underlying parameters of the bargaining environment. The next section will formally describe the bargaining model for the case of discrete values. Later sections will extend this model to both continuous and dynamic environments.

II. A Description of the Game

The bargaining process can be reduced to the strategic interaction between a buyer and a seller negotiating the terms of trade for an object. We assume the buyer has a reservation price of 1, and the seller's reservation price is 0. If the buyer and seller do not come to an agreement each will incur a "waiting" cost of $c>0$.[4] This model conforms to an agency model of bargaining where a union and management board negotiate a wage contract via intermediaries. Because the representatives do not have full authority, they are allowed only to negotiate within a prespecified range during each bargaining period. The union instructions to their negotiators are to bargain for no less than the wage level w_u, while the negotiators for the firm are instructed to obtain a wage package less than w_b. If

[4]Notice that the complete information symmetric game has a simple definition of efficient outcomes: trade occurs in the first period and joint profits are equal to 1.

the union demands (w_u) are higher than what the management has told its representative it is allowed to offer (w_b), a strike occurs until either the union or the management changes the mandate of their representatives. Thus, we have a bilateral monopoly situation where the players must solve a coordination problem with short commitment.

The split-the-difference mechanism requires each player to simultaneously submit an offer price. Let s denote the offer price for the seller and b the offer price of the buyer. An agreement is reached if $s \leq b$. If an agreement is reached the settlement price occurs half way between the offers, i.e., the seller receives $\frac{(b+s)}{2}$ and the buyer receives $1-\frac{(b+s)}{2}$. If $s>b$, then the parties have not reached an agreement and each player pays c. We shall restrict the offers so that $s \varepsilon [0,1]$ and $b \varepsilon [0,1]$, i.e., if $s>1$ or $b<0$ a failure will occur with probability one (only rational offers can be submitted).

III. The Discrete Game

III.1 *The Model*

Suppose that buyer and seller offers are limited to a finite set $\mathbf{Z}$. The elements of $\mathbf{Z}$, denoted by z_i, are indexed from 0 to n such that $z_i = \frac{i}{n}$, i.e., the offer grid is defined by $\{0, \frac{1}{n}, \frac{2}{n}, \dots, \frac{n-1}{n}, 1\}$. Thus, the actions are given by the seller selecting a $z_i \varepsilon \mathbf{Z}$ and the buyer selecting $z_j \varepsilon \mathbf{Z}$. The payoffs for the seller can thus be written as:

$$\begin{array}{ll} \frac{(z_i+z_j)}{2} & \text{if } i \leq j \\ -c & \text{if } i > j \end{array}$$

In a similar fashion we can describe the payoffs of the buyer as:

$$\begin{array}{ll} 1-\frac{(z_i+z_j)}{2} & \text{if } i \leq j \\ -c & \text{if } i > j \end{array}$$

This payoff structure can easily be represented in matrix form where the column index is the catalog of buyer offers and the row index is the catalog of seller offers. We shall denote the seller payoff matrix as $\mathcal{A}$. In particular, $\mathcal{A}$ is a (n+1) x (n+1) matrix with entries:

$$a_{ij} = \frac{(i+j)}{2n} \qquad \text{for } i \leq j$$

$$a_{ij} = -c \qquad \text{for } i>j$$

Similarly, the buyer payoff matrix ($\mathcal{B}$) is a (n+1) x (n+1) matrix with entries

$$b_{ij} = 1-\frac{(i+j)}{2n} \qquad \text{for } i\leq j$$

$$b_{ij} = -c \qquad \text{for } i>j$$

A strategy for the seller is a probability distribution over $\mathbf{Z}$, $P_s : \mathbf{Z} \mapsto [0,1]^{n+1}$ where $P_s(z_0, \ldots, z_n) = [p_{s0}, \ldots, p_{sn}]$ with $\sum p_{si}=1$ and $p_{si}\geq 0$. Similarly, a strategy for the buyer is a function P_b: $\mathbf{Z} \mapsto [0,1]^{n+1}$ where the range is the set of probabilities $\{p_{b0}, \ldots, p_{bn}\}$ for the buyer offers $z_0, \ldots, z_n$.

III.2 *Equilibrium*

An equilibrium in this game is simply a pair of functions (P_s^*, P_b^*) such that P_s^* maximizes the seller's expected profits π_s;

$$\pi_s (P_s|P_b^*) = P_s A (P_b^*)^T$$

subject to $p_{si}\geq 0$ and $\sum p_{si} = 1$

and P_b^* maximizes

$$\pi_b (P_b|P_s^*) = P_s^* B (P_b)^T$$

subject to $p_{bi}\geq 0$ and $\sum p_{bi} = 1$

where ^{T} denotes the matrix transpose.

We now state an obvious result for mixed strategy equilibria (indifference in payoff for all equilibrium messages).

Lemma 1:
$$\mathcal{A} (P_b^*)^T = [\pi_{si}] = \begin{cases} k & \text{iff } p_{si}^* > 0 \\ < k & \text{if } p_{si}^* = 0 \end{cases}$$

$$(P_s^*) \ \mathcal{B} = [\pi_{si}] = \begin{cases} l & \text{iff } p_{bi}^* > 0 \\ < l & \text{if } p_{bi}^* = 0 \end{cases}$$

Let S denote the set of offers z_i such that $p^*_{si} > 0$ and B the set of offers z_j such that $p^*_{bj} > 0$. We now state the following result (the proof is straightforward and thus will be omitted).

Lemma 2: S = B.

Lemma 2 shows that there must be consistency in the offers of each "rival" or else one side can "take advantage" of the other in its mixed strategy. Lemma 2 also tells us that there are 2^{n+1}-1 non-empty subsets of $\mathbb{Z}$ that are potential equilibria to this game. Notice that Lemma 1 and 2 together tells us that we need only consider matrices $\mathcal{A}$ and $\mathcal{B}$ in which the rows (and corresponding columns) of non-offers are eliminated. For each subset Ω of $\mathbb{Z}$, let $\mathcal{A}_\Omega$ and $\mathcal{B}_\Omega$ denote the reduced matrices to the subset Ω. From Lemma 1 we need only consider the equations:

$$\mathcal{A}_\Omega \, (P^*_b)^T = K \qquad (3.1)$$

$$(P^*_s) \, \mathcal{B}_\Omega = L \qquad (3.2)$$

Where K and L are the appropriate dimensioned matrix of constants k and l respectively. Suppose $\mathcal{A}_\Omega^{-1}$ and $\mathcal{B}_\Omega^{-1}$ exist so that

$$(P^*_b)^T = \mathcal{A}_\Omega^{-1} K \qquad (3.3)$$

$$(P^*_s) = L \, \mathcal{B}_\Omega^{-1} \qquad (3.4)$$

Now, for each k and l there is a unique pair $(P^*_b(k)\ ,\ P^*_s(l))$ that solve (3.3) and (3.4), because $P^*_b(k)$ is a linear function of k and $\sum p^*_{bi}(k) = 1$ (there is only one k that will solve this problem).

Theorem 1: There exists 2^{n+1}-1 equilibria to the split-the-difference mechanism under certainty with offer grid of finesse n and waiting cost c>0.

Proof: (see appendix).

Corrolary 1: If the failure cost of the buyer is larger than that of the seller then all symmetric mixed strategy equilibria are such that the expected profits of the buyer are greater than those of the seller.

Proof: (see appendix).

The number of pure strategy equilibria is n (each element in **Z** is an equilibrium). It is also the case that there is one equilibrium which spans the entire offers of **Z**. Thus, if we are to make any headway to this problem we must go deeper into the decision calculus of the players and attempt to "refine" the set of possible offers to a more restricted and tractable set. The Appendix provides an algorithm to compute the equilibrium strategies, that algorithm will prove useful in the dynamic section.

III.3 *Equilibrium Selection*

An equilibrium to this game is a pair of strategies that are best responses to each other. Therefore the problem for each player is to "divine", or guess what strategy the other player has decided to play. During the introspective process that leads to the realization that *in equilibrium* best response strategies are uniquely defined by the opponents offer set decision, the player has made no conjectures about what his opponent will do. To arrive at an optimal strategy (what to do in the one-shot game) the player must conjecture about the likelihood that his opponent will restrict his offer set to some subset of **Z**. Thus, an equilibrium in conjectures is a set of of restrictions (conjectures) of **Z** where both players "agree" on their opponents choice of Ω.

Definition : A *conjecture* is a mapping C: $\mathbf{Z} \mapsto \Re^{n+1}$ where $C(\mathbf{Z})_i$ is the conjecture that your opponent includes z_i in his offer set. A pair of conjectures $[C(\mathbf{Z})_b, C(\mathbf{Z})_s]$ are said to be *symmetric conjectures* when $C(\mathbf{Z})_{bj} = C(\mathbf{Z})_{sn-j}$.

Thus, a pair of conjectures for the buyer and seller in our game is given by the functions $[C(\mathbf{Z})_b, C(\mathbf{Z})_s]$. Suppose the seller comes to a conclusion about $C(\mathbf{Z})_{si}$, then the seller should realize that due to the symmetry of the payoffs the buyer would set $C(\mathbf{Z})_{bn-i}=C(\mathbf{Z})_{si}$. A player should realize that the reasoning that lead him to the conjecture $C(\mathbf{Z})_i$ should lead to symmetric conjectures by the other player. The game we have described is strictly symmetric (both players have identical costs); thus, when announcing a price the players are actually sending demands for a share of the surplus. symmetric conjectures only support offer sets that are symmetric around $\frac{1}{2}$. Thus, the set of potential equilibria with symmetric conjectures is $2^{\frac{n}{2}}-1$ if n is even, $2^{\frac{n+1}{2}}-1$ if n is odd.

Proposition 1: The only symmetric equilibrium in conjectures that is trembling hand perfect is where both buyer and seller evenly split the surplus, i.e., the offers are $\frac{1}{2}$ and $\frac{1}{2}$.

Proof: (see appendix)

Although this refinement seems very appealing, it is not robust to changes in the bargaining grid and is insensitive to asymmetries in costs. For example, when $\frac{1}{2}$ is not part of the offer set then this refinement clearly does not apply. Indeed there no trembling hand perfect symmetric equilibria in conjectures for this case. Furthermore, any of the pure strategies would seem hard to support without some sort of preplay communication. On the other hand, any mixed strategy based on a subset of offers will not be trembling hand perfect. If we restrict our attention to equilibria that result in symmetric payoffs, then symmetric pairs of offers will generate such payoffs (there are $\frac{n}{2}$ of these equilibria including the C.M.S. equilibrium). However, given that players plan to mix they have acknowledged that they have some uncertainty as to the strategy used by their counterpart. If players cannot rule out the possibility that an offer will be made they should utilize C.M.S.. The equilibrium yielded by C.M.S. is unique and worth a study.

Example : the two offer case

The payoff matrix for the seller is:

$$\text{Seller}\quad \begin{array}{c} \\ 0 \\ 1 \end{array} \overset{\text{Buyer}}{\begin{array}{cc} 0 & 1 \end{array}} \begin{bmatrix} 0 & 0.5 \\ -c & 1 \end{bmatrix}$$

In equilibrium, the seller offers 1 with probability $\frac{1}{1+2c}$ and 0 with probability $\frac{2c}{1+2c}$. The buyer offers 0 with $\frac{1}{1+2c}$ and 1 with probability $\frac{2c}{1+2c}$. Hence the probability that a bargain is struck is

$1-(\frac{1}{1+2c})^2$; expected profits are $\frac{c}{1+2c}$ for each player. The comparative statics are illuminating: as c falls the probability of failure rises and in the limit when there no bargaining costs there are no bargains. Expected profits also fall as c falls and in the limit (costs are zero) profits are zero. Another case of interest is c=1/2 that is the cost of failure are equal to the surplus. Then the probability of failure remains high (3/4) and expect profits are only 1/4. Hence, even when cost are equal to total surplus, fully half the surplus is consumed--in an expected sense--by bargaining.

IV. The Continuous Game

IV.1 *The Model*

Suppose now that the buyer selects his offer b from the interval [0,1] and the seller selects his offer s from the interval [0,1]. Again if s>b then the buyer and seller must pay c. If $s \leq b$ then the buyer receives $1-\frac{(b+s)}{2}$; the seller receives $\frac{(b+s)}{2}$. Thus, a strategy is the selection of a density function over [0,1]. That is, the seller strategy is a pair of functions $f:[0,1] \mapsto \Re^+$, $\alpha:[0,1] \mapsto \Re^+$ and the buyer strategy is the pair $g:[0,1] \mapsto \Re^+$, $\beta:[0,1] \mapsto \Re^+$, where $\int_0^1 f(x)\, d\alpha(x) = 1$, $\int_0^1 g(y)\, d\beta(y) = 1$. Hence, the probability that the seller selects an offer less than x is given by: $\int_0^x f(z)\, d\alpha(z) \equiv \text{prob}[s \leq x]$.

IV.2 *Equilibrium*

An equilibrium then is a set of functions $\{f^*,\alpha^*,g^*,\beta^*)$ such that f^*,α^* maximizes

$$\pi_s(f,\alpha)= \int_0^1 f(x) \left\{ \int_x^1 g^*(y)\frac{(x+y)}{2}\, d\beta^*(y) + \int_0^x g^*(y)(-c)\, d\beta^*(y) \right\} d\alpha(x) \tag{4.1}$$

and g^*,β^* maximizes

$$\pi_b(g,\beta)= \int_0^1 g(y) \left\{ \int_0^y f^*(x)[1-\frac{(x+y)}{2}]\, d\alpha^*(x) + \int_y^1 f^*(x)(-c)\, d\alpha^*(x) \right\} d\beta(y) \tag{4.2}$$

Consider any partition Z of [0,1]. If the functions α and β are step functions with steps at $z \varepsilon Z$ then

$$\int_0^1 f(x)\, d\alpha(x) = \sum_{z \varepsilon Z} f(z_k)\alpha_k \quad \text{and} \quad \int_0^1 g(x)\, d\beta(x) = \sum_{z \varepsilon Z} g(z_k)\beta_k .$$

If the seller restricts his offers to the partition Z_s and the buyer restricts his offers to the partition Z_b then as before (see Lemma 1), in equilibrium, $Z_s = Z_b$.[5] Thus, for any partition $Z = \{z_1, \ldots, z_n\}$ of $[0,1]$ the equations (1') and (2') can be rewritten as:

$$\pi_s(f,\beta) = [f(z_0)\alpha_0, \ldots\ldots, f(z_n)\alpha_n]\ \mathcal{A}_Z\ [g^*(z_0)\beta^*_0, \ldots\ldots, g^*(z_n)\beta^*_n]^T$$

$$\pi_b(g,\alpha) = [f^*(z_0)\alpha^*_0, \ldots\ldots, f^*(z_n)\alpha^*_n]\ \mathcal{B}_Z\ [g(z_0)\beta_0, \ldots\ldots, g(z_n)\beta_n]^T$$

where $\mathcal{A}_Z$ and $\mathcal{B}_Z$ are the appropriately defined matrices as in section I.

<u>Corollary 2</u>: Offers restricted to any partition of [0,1] will be a Nash equilibrium in the continuous case.

Let us now consider the terms

$$\left\{ \int_x^1 g^*(y)\frac{(x+y)}{2}\, d\beta^*(y) \ + \int_0^x g^*(y)(-c)\, d\beta^*(y) \right\} \quad \text{and}$$

$$\left\{ \int_0^y f^*(x)[1-\frac{(x+y)}{2}]\ d\alpha^*(x) \ + \int_y^1 f^*(x)(-c)\, d\alpha^*(x) \right\}.$$

Then, as in Lemma 1 we have:

<u>Lemma 1'</u>: For each $x \varepsilon$ [0,1] and some $\epsilon > 0$ such that $\int_{x-\epsilon}^{x+\epsilon} f(y)\, d\alpha(y) > 0$ we have

$$\left\{ \int_x^1 g^*(y)\frac{(x+y)}{2}\, d\beta^*(y) \ + \int_0^x g^*(y)(-c)\, d\beta^*(y) \right\} = k \tag{4.3}$$

over $[x-\epsilon, x+\epsilon]$. Also, for each $y \varepsilon$ [0,1] and some $\epsilon > 0$ such that $\int_{y-\epsilon}^{y+\epsilon} g(y)\, d\beta(y) > 0$ we have

$$\left\{ \int_0^y f^*(x)[1-\frac{(x+y)}{2}]\ d\alpha^*(x) \ + \int_y^1 f^*(x)(-c)\, d\alpha^*(x) \right\} = l \tag{4.4}$$

over $[y-\epsilon, y+\epsilon]$.

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[5]Notice we are using Riemann Stieltjes integrals to allow for discrete densities. Alternatively, we could consider Lebesgue integration but there is no need at this point to generalize our setting.

Proof: (see appendix).

Since we have found that any subset of [0,1] is a Nash equilibrium we cannot say much about the properties of the equilibria. A focal argument could lead us to select $\frac{1}{2}$, $\frac{1}{2}$ as the equilibrium to this game but we are uncomfortable with this notion because we could always consider a case in which the middle portion of the offer set is missing leaving at least two focal points. If we consider trembling hand perfection we notice that any single point in [0,1] can be supported. Aware of the equilibrium selection problem, we look at the behavior under C.M.S., i.e., f, g, α, β such that f, g>0 and

$\alpha(x) = \beta(x) = y$ (proper Reimann integrals) with $\int_0^1 f(x)\,dx = 1$, $\int_0^1 g(y)\,dy = 1$.

Theorem 2: There exists a unique Nash equilibrium distributions F^* and G^* over [0,1] for the C.M.S. case. In particular, for the buyer we have,

$$F^*(y\,|c) = \frac{(c+l)}{(1+c-y)} - \frac{1}{2(1+c-y)}\cdot\left\{e^{-\int_0^y \frac{1}{(1+c-s)}ds}\right\}\cdot\left\{\int_0^y \frac{(c+l)}{(1+c-s)}\cdot\left\{e^{\int_0^s \frac{1}{2(1+c-x)}dx}\right\}ds\right\}$$

and $F^*(1) = 1$. Since the problem is symmetric the seller distribution is easily determined from the above distribution.

Proof:

Integrating (4.4) and using distribution functions we find:

$$l = -\tfrac{1}{2}\left\{\int_0^y f^*(x)(x)\,d\alpha^*(x)\right\} + F^*(y)\frac{(1+c-y)}{2} - c$$

integration by parts yields:

$$F^*(y) = \frac{(c+l)}{(1+c-y)} - \frac{1}{2(1+c-y)}\int_0^y F^*(s)\,ds \qquad (4.5)$$

Equation (4.5) is a simple Volterra equation of the second kind and can be solved using a differential equation. To solve (4.5) all we need to notice is that $F^* = \frac{d}{dy}\left\{\int_0^y F^*(s)\,ds\right\}$, so that (4.5) is a linear differential equation with the boundary condition $F^*(0) = 0$. Hence the solution will be unique. If we substitute this solution into (4.5) we will obtain the unique C.M.S. equilibrium. The solution to (4.5) is given by:

$$F^{*}=\left\{e^{-\int_0^y \frac{1}{2(1+c-s)}ds}\right\}\cdot\left\{\int_0^y \frac{(c+l)}{(1+c-s)}\cdot\left\{e^{\int_0^s \frac{1}{2(1+c-x)}dx}\right\}ds\right\}$$

Equation (4.5) will be a function of the unknown parameter l which we can solve for since we have the boundary conditions $F^*(0) = 0$ and $F^*(1) = 1$. Now, $F^*(1) = 1 \rightarrow l = -c$ which is not possible since l is the expected value of the game for the seller. However, $F^*(0) = 0$ will result in a feasible value for for l.□

Note: The constraint $F^*(1) = 1$ results in a jump discontinuity at the 1 offer. This result tells us that the players' C.M.S. will place a significant probability (depending on c) at the most "stubborn" offers.

We now turn our attention to some comparative static results and investigate the case of known but asymmetric waiting costs.

IV.3 *Comparative Statics*

We begin by noticing that for any pure strategy equilibrium there will be no bargaining failures and individual waiting costs will not affect the offers made. That is, there is no mapping from bargaining costs to offers for the pure strategy Nash equilibria. It seems totally implausible that players will not change their strategies in the face of known but asymmetric bargaining costs. This supplies even more reason for us to focus on the C.M.S. equilibrium. Figures 1 and 2 supplies the C.M.S. equilibrium for the buyer and seller for different values of c. Notice that the spike at the most stubborn offers increases as c increases. In addition, the probability of negotiation failure increases as c increases. Figure 3 provides a graph of the probability of negotiation failure, i.e., the probability of failure$=\int_0^1 f^*(y)\cdot[1-G^*(y)]\,dy$. Notice that the probability of failure falls rapidly as waiting costs increase but then levels off. At $c = 1$ (cost of failure equal to surplus) the probability of negotiation failure is nearly .25.

In Figure 4 we have graphed the expected value of the game for each party and the cost of negotiation failure. The comparative statics are strait forward - the smaller the negotiation cost the lower the expected value of the game. Thus, lowering the cost of failure can only reduce the expected value of the game to the players even when there is complete information. Extending this idea further let us look at a case in which the the waiting cost of the seller is c_s and the buyer has a cost of c_b. Suppose that $c_s \neq c_b$. It is clear that the we will obtain the same form of the equilibrium distributions, $F^*(y|c_b)$ and $G^*(x|c_s)$ but these will not be symmetric and therefore will differ in expected profit. In particular, from Theorem 2 we can derive the equilibrium expected profits.

Corollary 1′: If $c_b > c_s$ $[c_s > c_b]$ then $\pi_b(f^*|g^*) > \pi_s(g^*|f^*)$ $[\pi_b(f^*|g^*) > \pi_s(g^*|f^*]$.

The above result is rather counterintuitive since the negotiator with the higher waiting cost will obtain a higher expected profit than the negotiator with a lower waiting cost. However, to see the logic of the result in Corollary 2 notice that each player's equilibrium mixed strategy depends on the waiting cost of his counterpart; if your rival has lower waiting cost, then in order to make him indifferent you must place a larger probability on your most stubborn offer. For example, in Figure 1 the cost values in the graph correspond to the buyer's waiting cost, thus as the buyer's waiting cost increases the seller will "give in" more often. This result could be a feature of the one-shot nature of the game, or the fact that we have an environment with complete information. We shall now examine these issues in turn.

V. The Dynamic Game

V.1 *Introduction*

Several problems have emerged from the analysis of the symmetric one-shot game. First, the expected profit of playing the game increases with costs. Second, the player with a higher waiting cost obtains larger profits than the player with lower waiting cost. This is in contrast to the stylized fact of

Figure 1
Seller Offers - Cumulative Distribution

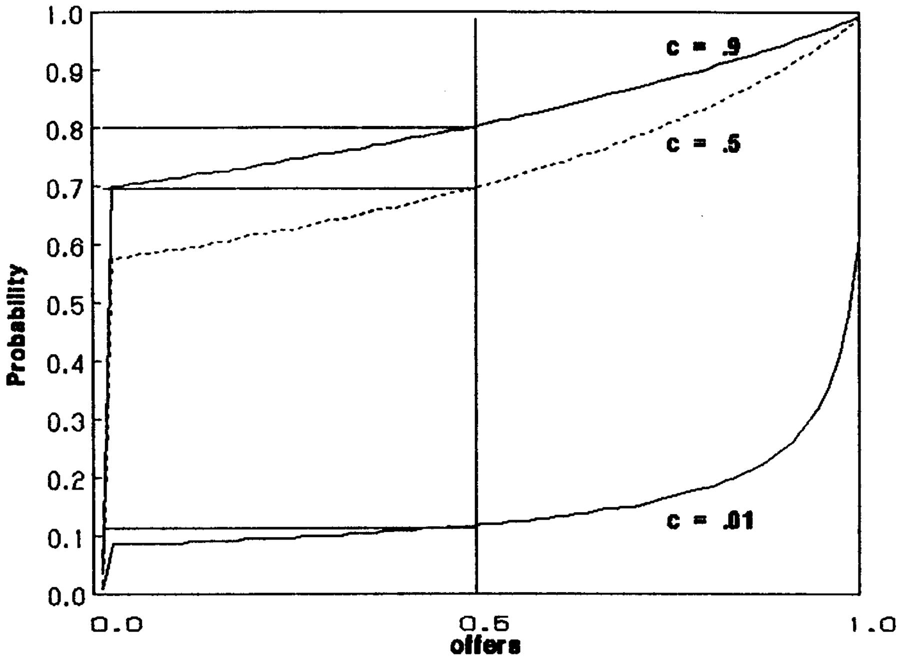


Figure 2
Buyer Bids-Cumulative Distribution

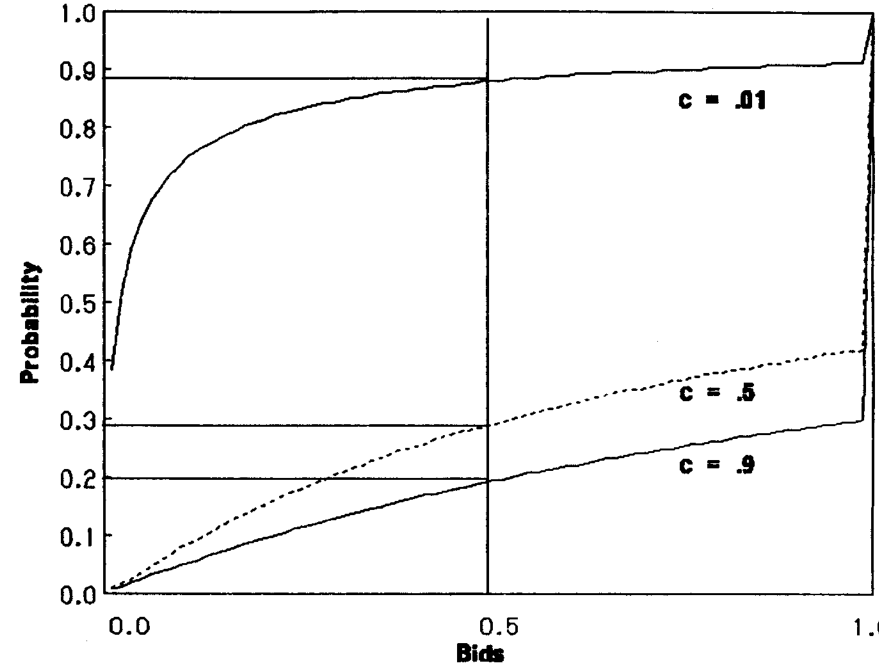


Figure 3
Probability of Bargaining Failure

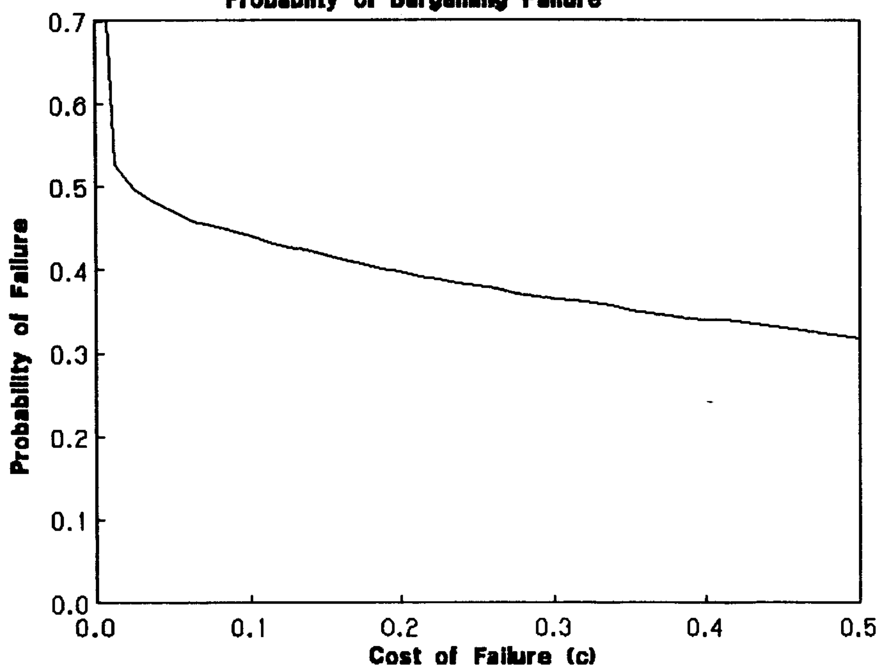
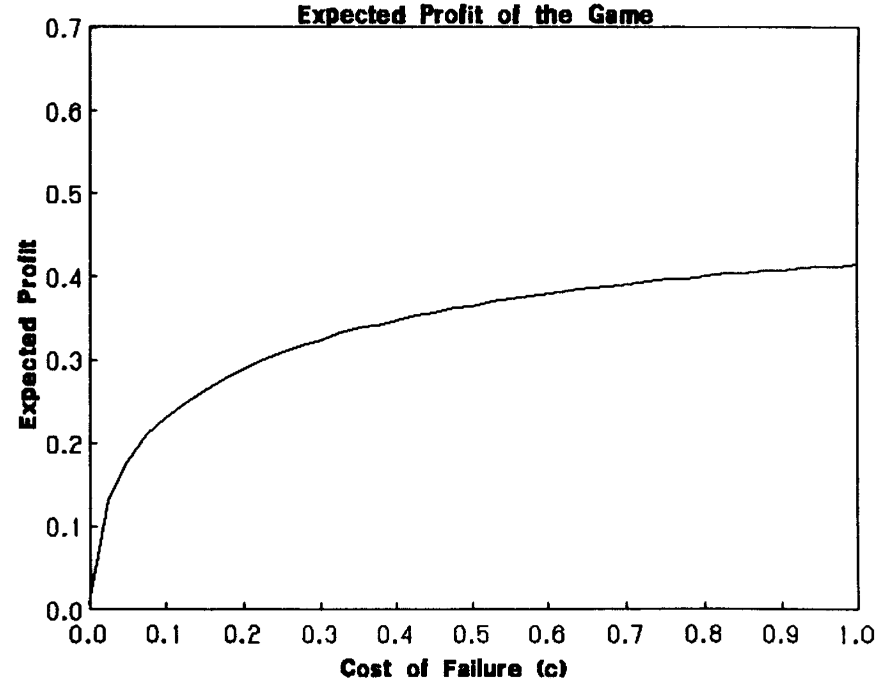


Figure 4
Expected Profit of the Game



bargaining that costs do not increase one's expected share of the pie but rather reduce it. One possible source of our counterintuitive result may have to do with the restriction that the game end after one period.

This section builds upon the previous ones and investigates the question of bargaining failures in a dynamic context. We will maintain that C.M.S. are chosen when anything is possible (recall that any collection of points or subset of [0,1] will yield an equilibrium in the one-shot game). One may wish to focus on pure strategies, but since in the discrete case the number of bargaining points is arbitrary, it is not clear how the coordination problem would be solved. Given the significant probability of failure in the one-shot game equilibria, it is important to investigate how extending the length of play affects the efficiency of trading. Going from one period to a repeated game often allows simplification of the problem because delay may be used to communicate information, however this is not likely to be the case here because there is nothing to communicate. That is, in the complete information context, there is no information concerning individual payoffs to be discovered from failure or delay in reaching an agreement.

To the extent that players cannot solve the coordination problem in the static game, there are strong reasons to believe that in the dynamic game the same situation will prevail, and therefore it is plausible to select out the C.M.S. equilibria. Since everything is known to the players, even after a larger number of periods where bargaining has failed the buyer and the seller have exactly the same beliefs about the each other and therefore should have the same expectations about the equilibrium of the game. The fact that the expected returns do not change is heavily dependent on the structure of the model. In our framework the problem faced by the buyer in period t is exactly the same as in period t-1 because no surplus is consumed by failure and bargaining costs are sunk.

V.2 *The Model*

The dynamic version of the game is similar to the one described in Section II except that if the buyer price is lower than the seller price, the bargain fails and each player must pay c before the next round of offers are made. We will consider both the finite horizon game with a known ending point and the infinite horizon game with discounting, which is akin to a finite horizon game with an uncertain end point. The case where bargaining ends in a fixed number of periods is from an analytical point of view messy but it allows us to examine the issue of convergence towards the infinite horizon equilibrium.

The technical questions are in a sense trivial, since we focus on sequential equilibria. If players arrive at the last period they will play the one-shot game. Working backwards, in any period (t) the buyer and seller will play a game that is similar in payoff to the matricies in Section III, where the entries above the diagonal are the same but the entries below the diagonal are replaced by: $V=d\cdot\Pi^{t-1}-c$, where Π^{t-1} is the expected value of the game in period t and d is the discount rate. Theorem 1 tells us that for each subset of $\mathbf{Z}$ there exists a unique equilibrium to this game. Players will find their strategies by backward induction. Thus the existence of equilibria is not a problem. If we restrict attention to C.M.S. then the equilibria will be unique if the backward induction process is a function--as we will see this poses yet another equilibrium selection problem.

Infinite horizon games are more appealing than games with known-ending points because of the the nature of the process we seek to describe. In this game no surplus is consumed by failure, the mirage of profits remains the same, independent of the number of times the players have failed to come to a bargain. In equilibrium, expected profits must be strictly positive because each player can accept a zero share of the surplus. Hence, no one who sits down at the bargaining table should ever want to get up and leave. Any player who agrees to play this game will not want to stop until a bargain occurs. Thus, the only rational reasons for the game to end is a bargain, death, or some other unexpected event. Therefore, the assumption of an uncertain end to the game, or that the game will go on until the players strike a bargain is the most intuitive.

A strategy for the seller will be a function $P_s^t : \mathbf{Z} \times \mathbf{N} \mapsto [0,1]^{n+1}$ where $P_s^t(z_0, \ldots, z_n, t) = [p_{s0}^t, \ldots, p_{sn}^t]$ and p_{si}^t is the probability that the seller offers z_i in period t. Thus, $\forall$ t we must have $\sum p_{si}^t = 1$ and $p_{si}^t \geq 0$. Similarly, a strategy for the buyer will be a function $P_b^t : \mathbf{Z} \times \mathbf{N} \mapsto [0,1]^{n+1}$ where the range is the sequence of probabilities $[p_{b0}^t, \ldots, p_{bn}^t]$ representing the probability that the buyer offers z_j in period t.

V.3 *Equilibrium*

A sequential equilibrium in this game is a pair of sequences of functions $(\rho_s^{(t)*}, \rho_b^{(t)*})$, where $\rho_s^{(t)*} = \{ P_s^{t*}, P_s^{t-1*} \ldots\}$ and $\rho_b^{(t)*} = \{ P_b^{t*}, P_b^{t-1*} \ldots\}$ such for each t, P_s^{t*} maximizes,

$$\pi_s (P_s^t) = P_s^t \mathcal{A}(\rho_s^{(t-1)*}, \rho_b^{(t-1)*}) (P_b^{t*})^T$$

subject to $\sum p_{si}^t = 1$ and $p_{si}^t \geq 0$

and P_b^{t*} maximizes,

$$\pi_b (P_b^t) = P_s^{t*} \mathcal{B} (\rho_s^{(t-1)*}, \rho_b^{(t-1)*}) (P_b^t)^T$$

subject to $\sum p_{bi}^t = 1$ and $p_{bi}^t \geq 0$

Note: $\mathcal{A}(\rho_s^{(t-1)*}, \rho_b^{(t-1)*})$ is defined to be the matrix of payoffs for the seller that is identical to the matrix $\mathcal{A}$ defined in Section III, except that below the diagonal the entries are the discounted expected returns minus c, if the players use $\rho_s^{(t-1)*}$ and $\rho_b^{(t-1)*}$. $\mathcal{B} (\rho_s^{(t-1)*}, \rho_b^{(t-1)*})$ is defined similarly.

The sequences ρ_s^t and ρ_b^t are the mixed strategies of the players that are best responses for each subgame in the dynamic game. The strategies at time t are dependent on future actions because future actions drive the expected future profits and thus the continuation value of the game. Unlike in the one-shot game where failure to agree lead to a loss there is no reason for the continuation value of the game V to be negative if discounted expected profits are higher than c. Thus, some individual rationality constraints may become binding because individuals have no incentive to demand less than the continuation value of the game and therefore some offers will never be made.

V.4 *Further Problems of Equilibrium Selection*

How many offers are individually rational depends on the continuation value of the game. Offers are individually rational if and only if their minimum return is no lower than the continuation value of the game. There is no a priori reason for the set of acceptable offers to behave consistently over time. For a given set of offers Ω, higher continuation values in period t decrease the expected returns of period t+1. Lower expected profits in period t+1 decrease the continuation value of period t+2 which will increase the expected return in period t+3 and so on. Thus the set of individually rational offers may change abruptly over time, a fact that poses further equilibrium selection problems.

Given that there maybe different sets of acceptable offers at different times, we must select from the set of C.M.S. sequential equilibria the one which seems most plausible for each period. On the one hand, if the set of individually rational offers shrinks for each period of backward induction, it is natural to eliminate the offers that would never be accepted. If, on the other hand, the set of individually rational offer expands, the question arises as to which of the C.M.S. equilibria will be selected. Specifically, should the previous subset of Ω_t be retained for period t+1? Or should we select the largest subset at time t such that all offers are individually rational? The questions are of relevance because it can easily be shown that for the same continuation value of the game (V) the expected profits from the equilibrium where the subset is Ω_t are greater than the expected profits of an equilibrium based on a larger subset of $\mathbf{Z}$.

For consistency we choose the C.M.S. sequential equilibria with the largest cardinality. Thus, any offer which is individually rational to accept in period t will be offered in period t with some (however low) positive probability. Moreover, the fact that in the future an offer will be unacceptable, or that it has been unacceptable in the past, should not influence today's decisions. As a result the C.M.S. are the intuitive extension of the equilibria examined in the one-shot game. Selecting the C.M.S. implies that if $\frac{m+1}{n} > V > \frac{m}{n}$, then $\Omega_t=\{ \frac{m+1}{n}, \frac{m+2}{n}, \ldots, \frac{n-m-1}{n}, \frac{n-m-2}{n}\}$.

V.5 *Expected Profits*

Focusing on sequential equilibria allows a great deal of simplification because instead of focusing on the mixed strategies we can focus on the continuation values. As before, it is easy to show that there is a function which maps continuation values into equilibrium mixed strategies. Thus, for the infinite horizon game we can rewrite our equilibrium definition in terms of continuation values instead of probabilities. Let $\Pi(n,m,V)$ be the expected profit from the C.M.S. equilibrium of the one-shot game with a grid of finesse $\frac{1}{n}$, smallest individually rational offer $\frac{m}{n}$, continuation value V.

Definition: An *infinite horizon C.M.S. sequential equilibrium* to the bargaining game is a pair V_s, V_b such that :

$$V_s= d\cdot\Pi_s(n.m,V_s) -c \text{ ; where } \frac{m}{n}>V_s>\frac{m-1}{n},$$

and

$$V_b= d\cdot\Pi_b z(n,m,V_b) -c \text{ where } \frac{m}{n}>V_b>\frac{m-1}{n}.$$

The equilibria are simply the steady states of V.[6]

We can characterized as follows: $\Pi(n,m,V) = (1-p_m)V + p_m\frac{n-m}{n}$, rearranging terms leads to $\Pi(n,m,V)=V + p_m(\frac{n-m}{n}-V)$, but , $p_m=\prod_{j=m}^{n-m} \frac{2n-2j-2nV}{2n-(2j-1)-2nV}$. Thus we have,

$$\Pi(n,m,V) = \prod_{j=m}^{n-m} \frac{2n-2j-2nV}{2n-(2j-1)-2nV} \left(\frac{n-m}{n}-V\right) +V \tag{5.1}$$

---

[6]We should note that this definition is not exhaustive of C.M.S. sequential equilibria. In fact any pair of sequences $V^*_s=V^0_s,\ldots V^T_s$; $V^*_b=V^0_b,\ldots V^T_b$ is an equilibrium if and only if $V_i^{t+1}=d\cdot\Pi(n,m,V_i^t)-c$ for t in [0,T-1] and for i=s,b. and $V_i^0=d\cdot\Pi(n,m,V_i^T)-c$.

Proposition 2: Over the interval $[-\infty, \frac{m}{n}]$, $\Pi(n,m,V)$:

i) is continuous in V, ii) is decreasing in V, iii) is concave in V, and iv) has a limit of $\frac{m}{n}$ when $V \rightarrow \frac{m}{n}$ and equals $\frac{1}{2}$ when $V \rightarrow -\infty$.

Proof: (see appendix).

In other words the profit function is very well behaved. Proposition 2 will be enough to guarantee that the infinite horizon equilibria exist. However, before discussing that issue we turn to the question of the finite horizon problem.

V.6. *Equilibrium and Efficiency in the Finite Horizon Game*

The significant probability of failure found in the one-shot game suggest that it may be individually rational for individuals to consume a greater part of the surplus attempting to appropriate the rest. Even though we focus on C.M.S., we show that it is possible for the game to achieve full efficiency provided that the continuation value of the game is high enough ($V > \frac{1}{2} - \frac{1}{n}$) however this requires that the interval between offers must be greater than the failure cost. This section makes clear that the incentives for stubbornness remain strong in most dynamic situations (proofs of the following can be found in the appendix).

Corollary 3: For every n, d there exits a c^*such that $c^*$is the argmax of $d \cdot \Pi(n,0,-c) - c$.

Corollary 4: For $c > \frac{1}{n}$ there exist no C.M.S. equilibrium such that the game achieves full efficiency, regardless of the number of periods played.

The interpretation of Corollary 4 is rather simple. Mixed strategies imply a positive probability of failure, however, inefficiency in a repeated game may not be significant provided that the costs of failure are small enough because future profits constrain the subset of individually rational offers Ω_t^*. If future profits are high enough a unique individually rational offer may subsist, however the range of costs where an efficient solution may occur is arbitrarily small even if player are perfectly patient.

Proposition 3: For every n, d>0 there exists c such that $d \cdot \Pi(n,0,-c)-c>0$.

Thus in any two period game the players will randomize on a different set in the fist period than in the second. If c and d are small enough it is not rational for the seller to accept an offer of 0 because his continuation value is positive. For let c' be a solution to $d \cdot \Pi(n,0,-c') = c'$. Because $\Pi(n,0,-c)$ is bounded above and d is less than 1 there must exist at least one c'. Because $\Pi(n,0,-c)$ is strictly concave $c'<\frac{1}{2}$ because the profits are zero when c is zero, c' is unique.

Theorem 5: For n=2, the game achieves full efficiency in two periods if $c<c'$. For n>2, the game does not achieve full efficiency in the first period if the horizon is two periods if C.M.S. are used.

Taken together Corollary 3 and Theorem 5 suggest that except for n=2 (where the dicontinuities are most severe) there is no quick solution to the lack of efficiency of this bargaining game. If c is large enough, a long horizon is worse than the end game in terms of ex-ante expected profits. For small enough values of c a large horizon may lead to full efficiency but the range of costs where the game ends in the first period shrinks as the bargaining grid gets finer. Thus bargaining failures are quite robust unless players can divine a pure strategy equilibrium.

V.7 *Convergence*

Beyond the question of efficiency, the model forces us to confront the problem of convergence. Built into the discrete problem are a set of discontinuities that may lead to chaotic behavior. As the next section will show for $c>c'$ the finite game converges to the steady state infinite horizon equilibrium. The steady state, however, has a lower expected profit lower than that of the one-shot game. Within the interval $[0,c']$ there exist costs such that the discrete finite game forces each player to dramatically alter his strategy from one period to the next, even if more than several hundred periods of play remain and players are not perfectly patient. As Figure 5 shows, for n=10 c=0.18 and t=1 to t=600, each period of play has a specific continuation value and these values follow a chaotic pattern. This kind of chaotic behavior results from the fact that for any given Ω_m high values of V lead to low continuation values and low values of V lead to high continuation values. A finite game will converge to the infinite horizon steady state based on Ω_m if and only if $\frac{m}{n}>V^t>\frac{m-1}{n}$ and $\frac{m}{n}>V^{t+1}>\frac{m-1}{n}$. This condition does not hold in general; thus it is not possible to give either conditions for chaotic behavior or convergence simply based on n, c and d.

The chaotic nature of the strategies displayed in Figure 5 could in fact be related to the convergence of the finite game to an infinite horizon equilibrium which is not a one period steady state but rather a pair of sequences. This possibility remains unexplored, because of the complexity of the strategies involved. Moreover as the next section will show many infinite-horizon-steady-state equilibria exist making the exploration of even more complex equilibria uninteresting. If the finite game had always converged to one of these, it would have resolved an equilibrium selection problem. That is not the case so we must look elsewhere for an appropriate equilibrium.

V.8 *The Infinite Horizon Problem*

This section will address the issue of existence of infinite horizon steady states. First we examine the question when costs are large ($c<c'$) and then when costs are small enough that many equilibria exist (proofs of the Theorem and of the Propositions of this section can be found in the appendix).

V.8.1 *Equilibrium*

Proposition 4: For every n, d and c greater than c' there exist a unique V^* and thus a unique infinite horizon equilibrium to the bargaining game.

Figure 5
C.M.S Contin. Values t periods to end

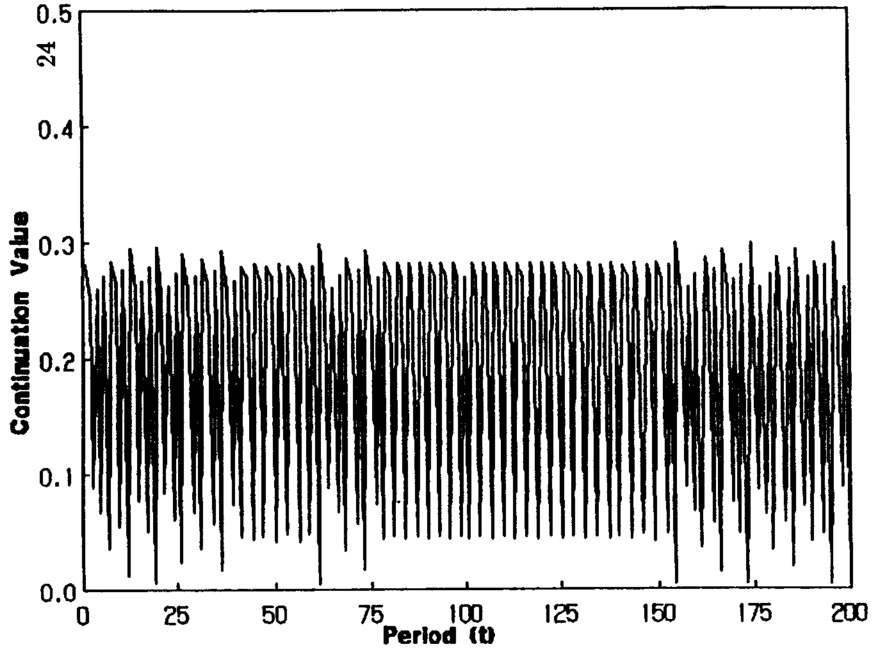


Figure 6
Contin. Value vs. Cost 100 periods to end

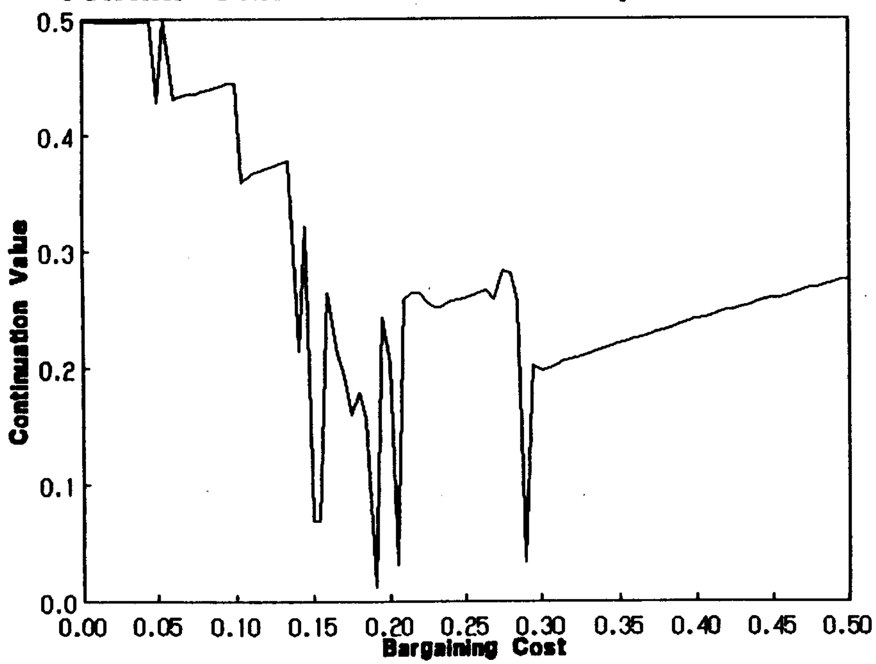
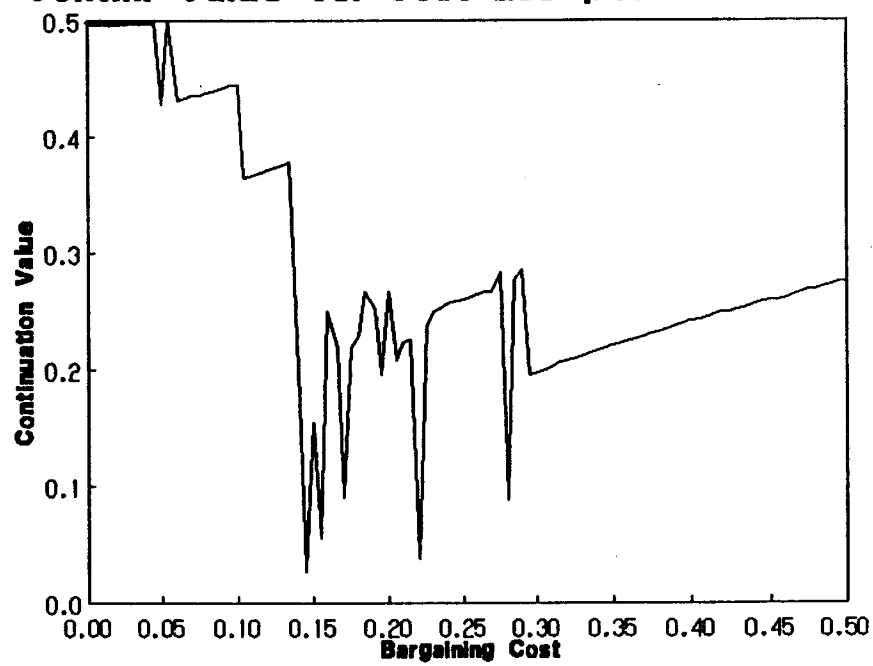


Figure 7
Contin. Value vs. Cost 200 periods to end



The bargaining game does not achieve full efficiency in an infinite horizon for $c>c'$. The disheartening news is that for $c>c'$ the dynamic game is less efficient than the one-shot game. The result is not surprising since the one-shot game is the equivalent of committing not to return to the bargaining table in case of failure. As a result, equilibrium strategies must be less stubborn, and hence the probability of failure falls.

Theorem 6: If $c< c'$, at least one infinite horizon equilibrium exists. Furthermore, the number of infinite horizon equilibria is at most $\frac{n}{2}+1$.

Now, let $\Pi^*(n, m)=\Pi(n, m, \frac{m-n}{n})$; the following Propositions establish the efficiency properties of the C.M.S. infinite horizon equilibrium.

Proposition 5: $\Pi^*(n, m)-\frac{m-1}{n}<\Pi^*(n, m')-\frac{m-1}{n}$ iff $m>m'$. Thus, if C.M.S. infinite horizon equilibrium for c based on Ω_m (because $\Pi^*(n, m)-c <\frac{m-1}{n}$). Then there is no C.M.S. infinite horizon equilibrium for $\Omega_{m'}$, where $m'>m$.

Proposition 6: The infinite horizon game and the finite horizon game have has an efficient C.M.S. equilibrium if and only if $c< \frac{3}{5n}$.

Example (Two offer case):

The seller would like to maximize (at $t = 0$):

$$V_s^0 (p) = pq(0.5) + (1-p)q + (1-p)(1-q)(d\cdot V_s^1-c) \tag{5.2}$$

where p is the probability that the seller offers 0 and q is the probability that the buyer offers 1, and V^1 is the continuation value of the game if there is a bargaining failure. Differentiating (5.2) yields

$$\frac{\partial V_s^0}{\partial p} = q(0.5) - q - (1 - q)(dV_s^1 - c) = 0$$

$$-\frac{q}{2(1-q)} = d\cdot V_s^1 - c \tag{5.3}$$

Equation (5.3) leads to the conclusion that the discounted continuation value of the game ($d\cdot V_s^1$) must always be less than c. Now, in equilibrium p=q and $V_s^1 = V_s^0$ so that substituting (5.3) into (5.2) we find:

$$-dq^2 + q(1 + d + 2c) - 2c = 0 \tag{5.4}$$

Equation (5.4) is a quadratic equation (in q) that is negative at q=0 and positive at q=1; the maximum of the function occurs at $q = \frac{1+d+2c}{2d} > 1$. Thus, (5.4) is strictly increasing on the interval [0,1] and therefore there is a unique solution to this problem. We now state some results we can derive directly from (5.4):

(1) As waiting costs (c) increase the probability of failure $(1 - q)^2$ falls so the game ends sooner.

Reason: $\partial q/\partial c = \dfrac{(2 - 2q^*)}{(1 + d + 2c - d2q^*)} > 0 \rightarrow$ q increases and thus $(1 - q)\downarrow$.

(2) As the discount rate rises the probability of failure increases.

Reason: $\partial q/\partial d = \dfrac{(q^{*2} - q^*)}{(1 + d + 2c - d2q^*)} < 0 \rightarrow$ q decreases and thus $(1 - q)\uparrow$.

(3) When d=0 we return to the one-shot game and $q = \frac{2c}{1 + 2c}$.

Thus, in the complete information game (with an infinite horizon and two offers) the unique symmetric equilibrium has delay (q < 1) and this delay rises as the cost of bargaining falls and the impatience of the players rises. Unless there is a more "suitable" negotiating mechanism or institution there may be no way for the players to resolve their differences instantaneously. Most of the surplus is in fact consumed in the bargaining process.

V.9 *Conclusion*

If the finite game converges to a steady state, it converges to the C.M.S. steady state with the highest expected profit (high profit equilibrium). Furthermore, for all c such that the finite game converges to the same steady state (that is one based on a specific Ω_m) then expected profits rise with costs but at a rate less than 1. The infinite horizon equilibrium for the continuous game is unique for every c and is based on the entire [0,1] interval. Thus there are two (at least) strong candidates for infinite horizon equilibrium based on full randomization. First, the high profit equilibrium, which appealing because it is relatively efficient. Second the C.M.S. steady state based on $\mathbf{Z}$ if we are concerned with the convergence of the discrete case to the continuous case.

Thus, if people are patient enough, low negotiation costs may not be a hindrance to negotiation success, in which case and the dynamic game reverses the results of the static game. Moreover the expectation that individuals with identical costs will arrive at an even split is realized, when players arrive at C.M.S. equilibria $\mathbf{Z}$.

VI. Uncertainty

VI.1 *The Model*

The symmetric cost case, although informative, cannot provide the whole answer to the issue of negotiation failure. Clearly cost are not strictly symmetric. If we abandon the--safe and reassuring--assumption of symmetry we come to a fork in the road. Either we can preserve the complete information setting or we can venture into a world were knowledge about cost is private information. The reader can easily satisfy himself that, in the full information case, asymmetric costs lead to equilibria of the same nature. One disturbing phenomenon in this setting is that profits increase with costs. Thus, the higher cost player has a higher expected profit than the lower cost player, whenever the probability of failure is positive and players mix over subsets of $\mathbf{Z}$ that are symmetric about $\frac{1}{2}$. The idea that higher cost players expect more seems counter-intuitive and reason alone to explore the case of asymmetric information. The model is the same as before except that now each player has his waiting cost drawn independently from a common distribution.

Suppose c is drawn from a common density function f(c), with cumulative density F(c) on the support $[\underline{c},\bar{c}]$. Let the cost drawn by the buyer be denoted by $\hat{c}$. Define $p_{si}(c)$ to be the probability a seller of type c offers $\frac{i}{n}$. A <u>strategy</u> for the seller is a function $P_s:[\underline{c},\bar{c}]\times\mathbf{Z}\mapsto [0,1]^{n+1}$ where $P_s(c,z_0, \ldots ,z_n) = [p_{s0}(c), \ldots ,p_{1n}(c)]$ and $p_{si}(c)$ is the probability that a seller of type c offers z_i. Similarly, a <u>strategy</u> for the buyer is a function $P_b : [\underline{c},\bar{c}] \times \mathbf{Z} \mapsto [0,1]^{n+1}$ where the range is the set of probabilities $[p_{b0}(c), \ldots ,p_{bn}(c)]$ and $p_{bi}(c)$ is the probability that a buyer of type c offers z_j.

VI.2 *Equilibrium*

The profits for a buyer of type $\hat{c}$ of offering $\frac{i}{n}$ given the strategy of the seller are:

$\pi_{ib}(c|\hat{c}) = \sum_{j=0}^{i} p_{sj}(c)\,\frac{2n-2i-i+j}{2n} - \sum_{j=i+1}^{n} p_{sj}(c)\,\hat{c}$, and thus the expected profits from offering $\frac{i}{n}$ are:

$$\pi_{ib}(P_s|\,\hat{c}) = \sum_{j=0}^{i} \frac{2i+\,n-j-i}{2n}\left\{\int_{\underline{c}}^{\bar{c}} p_{sj}\,(c)\,f(c)\,\,dc\right\} - \hat{c}\cdot \sum_{j=i+1}^{n} \int_{\underline{c}}^{\bar{c}} p_{sj}\,(c)\,f(c)\,dc \qquad (6.1)$$

Note that $\pi_{ib}(P_s|\,\hat{c})$ is a linear function of $\hat{c}$.

For a seller of type c' the expected profits of an offer of $\frac{n-j}{n}$ can be described by:

$$\pi_{js}(P_b|\,c') = \sum_{i=j}^{n} \frac{2j+\,n-j-i}{2n}\left\{\int_{\underline{c}}^{\bar{c}} p_{bi}\,(c)\,f(c)\,\,dc\right\} - c'\cdot \sum_{i=0}^{j-1} \int_{\underline{c}}^{\bar{c}} p_{bi}\,(c)\,f(c)\,\,dc \qquad (6.2)$$

Thus, an <u>equilibrium</u> is a pair of functions $P_b^*(c)$, $P_s^*(c)$ such that $P_b^*(c)$ maximizes $P_b(c)\,\Pi(P_s^*(c))$, where $\Pi(P_s^*(c))$ is a 1xn vector whose i^{th} entry is $\pi_{ib}(P_s|c)$; $P_s^*(c)$ maximizes $P_s(c)\,\Pi(P_b^*(c))$, where $\Pi(P^*{}_b(c))$ is a 1xn vector whose j^{th} entry is $\pi_{sj}(P_b|\,c)$.

Because $\pi_{ib}(P_s|\,\hat{c})$ is a linear function of $\hat{c}$, equations (6.1) and (6.2) can be rewritten as:

$$\pi_{bi}(\hat{c}) = \hat{A}_i - \hat{B}_i\cdot\hat{c} \qquad (6.3)$$

$$\pi_{si}(c') = A_i' - B_i'\cdot c' \qquad (6.4)$$

Notice that neither $\hat{A}_i$ nor $\hat{B}_i$ depend on $\hat{c}$. Furthermore, $\hat{A}_i$ is decreasing in i and $\hat{B}_i$ is increasing in i. Thus, each π_{si} is a linearly decreasing function of c'. If $\pi_{si}(c) < \pi_{sj}(c)$ and i>j then a seller of type c will have a dominant strategy of playing i with zero probability, and all sellers with higher cost will have same dominant strategy. For almost every c one $\pi_{sj}(c)$ will be larger than all the other π_{si} so almost all players will have pure strategies.

If $\pi_i(c) > \pi_k(c)$ for all k>i, then $\forall\ \hat{c} > c$, $\pi_i(\hat{c}) > \pi_k(\hat{c})$ for k>j. Thus, if a player of type c makes a demand of $\frac{i}{n}$, all players of type $\hat{c} > c$ will make a demand of no more than $\frac{i}{n}$. Hence, we can restrict our attention strategies where the demanded share of the surplus decreases with cost. Such strategies are completely defined by a vector of "cut-off points", $\Gamma = [\gamma_1, \gamma_2, \ldots, \gamma_n]$ where $\underline{c} \leq \gamma_i \leq \bar{c}$ for the buyer and a vector of "cut-off points", $\Lambda = [\lambda_1, \lambda_2, \ldots, \lambda_n]$ where $\underline{c} \leq \lambda_i \leq \bar{c}$ for the seller. A seller (buyer) with a cost of c between λ_i and λ_{i+1} will demand $\frac{i}{n}$. We consider only the seller's problem because the buyer's problem is strictly symmetric. Let us first rewrite $\pi_i(c)$ in terms of the cumulative density $F(\cdot)$ as follows:

$$\pi_i(c) = \sum_{j=0}^{n-i} \left\{ [F(\gamma_{j+1}) - F(\gamma_j)] \frac{i+n-j}{2n} \right\} - [1 - F(\gamma_{n-i+1})]\ c. \tag{6.5}$$

Then we can look at the difference between offering $\frac{i}{n}$ and $\frac{j}{n}$ as:

$$\pi_i(c) - \pi_j(c) = F(\gamma_{n-j})\frac{j-i}{2n} - \sum_{l=n-i}^{n-j} [F(\gamma_{l+1}) - F(\gamma_l)]\left(\frac{i+n-l}{2n} + c\right)$$

indifference between offering $\frac{i}{n}$ and $\frac{j}{n}$ is equivalent to $\pi_i(c)-\pi_j(c)=0$; because $\pi_j(c)$ is a linear function of c we know that a unique cost satisfies $\pi_i(c)-\pi_j(c)=0$. That cost is defined as :

$$D_{ij}(\Gamma) = \frac{F(\gamma_{n-j})\frac{j-i}{2n}}{F(\gamma_{n-j}) - F(\gamma_{n-i})} - \sum_{l=n-j}^{n-i} [F(\gamma_{l-1}) - F(\gamma_l)] \frac{i+n-l}{2n}$$

let $\mu_i \min_j$ of $D_{ij}(\Gamma)$ then $\lambda_{i+1}^*(\Gamma) = \max\{\mu_i, \lambda_i\}$ and $\lambda_0 = \underline{c}$.

The definition of $\lambda_i^*(\Gamma)$ is somewhat complicated because there exist many distributions of buyer bids such that the best response of seller is to make offers on a subset of **Z**. If offers are skipped that implies either that a seller is indifferent between three offers, or that an offer is strictly dominated by another over the whole range of c. Thus we start at with the offer where the seller demands everything and compare to all other offers because the higher offers are less attractive to higher cost sellers. We want to find the first time demanding everything is equivalent (in terms of profits) to making a less stringent offer ($\min_j$ of $D_{0j}(\Gamma)$). However the cost where demanding 1 is as profitable as demanding less offer may be less than $\underline{c}$ so $\lambda_1^*(\Gamma)= \max\{\mu_0, \underline{c}\}$. Then we repeat the procedure for $\lambda_2^*(\Gamma)$ and so on.

Claim: $\lambda_i^*(\Gamma)$ is a continuous function of Γ.

Proof: each $D_{ij}(c)$ is continuous because $\gamma_j \geq \gamma_i$ if j>i. The minimum of continuous functions is continuous. So μ_i is continuous in Γ. Again the maximum of two continuous function is also continuous so $\lambda_i^*(\Gamma)$ will be continuous in Γ.

Definition: An equilibrium to the asymmetric information game is defined as a pair $\Lambda^*(\Gamma')=\Lambda'$ and $\Gamma^*(\Lambda')=(\Gamma')$.

Definition: A *symmetric equilibrium* to this game occurs iff $\Lambda^*(\Lambda')= \Lambda'$.

Proposition 7:

A) symmetric equilibria exist.

B) In any symmetric equilibrium $\pi_b(c) > \pi_s(c')$ if $c< c'$.

Proof: (see appendix).

Thus, we find the reassuring result that within any symmetric equilibrium players with higher costs will always demand less, and receive lower expected profits than a player with low cost. However, it is also possible to show that shifting the distribution up (for example increasing the mean without changing the variance) will lead to a higher expected return for all players.

VII. Conclusion

This paper has attempted to thoroughly analyze bargaining under the split the difference mechanism when there is complete information concerning individual payoffs and bargaining cost. Unfortunately, in this environment there exists a multiplicity of equilibria. All of the equilibria to this game suffer from unsatisfactory properties; the pure strategy equilibria require coordination and are not sensitive to bargaining costs (including focal equilibria--equal split); mixed strategy equilibria are not trembling hand perfect. Because the traders are arguing over surplus, it is rational to accept any offer, thus we focused on completely mixed strategy equilibria but find that such equilibria require that the negotiator with the higher bargaining cost receive higher profits.

Allowing the bargaining process to be dynamic does not entirely solve all the problems of the one-shot game. The continuation value of the game and thus the offers in the dynamic game can demonstrate chaotic behavior because of the discontinuities of the bargaining grid. Moreover when the failure costs are less than half the surplus, there may exist multiple infinite horizon C.M.S. equilibria. Because of the chaotic nature of the finite horizon game, onecan not 'refine' the infinite horizon equilibrium problem. When the failure costs are more than half the surplus, however, the dynamic game is well behaved (the finite game converges to the unique steady state equilibrium) but the repeated game equilibria are less efficient than one-shot game equilibrium.

These results seem rather negative and not very reassuring to game theorists. Yet, unlike in the Stahl-Rubinstein model we find that there is significant probability of delay, a property of the equilibrium which is consistent with some empirical reality. As bargaining costs fall, the mixed strategy equilibrium approaches a pure strategy in which failure occurs with probability one. This suggests that failure costs play an important role in bringing the parties closer to agreement, and that one should focus on the determinants of bargaining costs to understand the success or failure of negotiations. In the dynamic version of this game efficiency can be increased (provided $c<c'$) and

inefficiency conditions are parameter sensitive. Thus, the comparative static results of the model have some appeal.

Two further results have implications for future research. First, if there is asymmetric information concerning bargaining costs, the negotiator with the higher bargaining costs will obtain lower profits. Thus, the introduction of asymmetric information leads to more plausible properties of the bargaining equilibrium. Second, the assumptions of sequential bargaining models are not innocuous, because of the first mover advantage that is imbeded in these models. We believe that the first mover advantage is decided endogenously, a process that may consume resources and lead to delay. In fact, there seems to be no obvious way to resolve the first mover problem short of adding further institutional arrangements to the bargaining process. The work we have presented leaves some unanswered questions which should be the focus of future work on this topic. There is a need to better understand this model through some empirical investigation. Specifically, an experimental analysis of this bargaining institution should be considered to help establish a link between bargaining costs and bargaining behavior. It seems clear that the bargaining institution matters and thus empirical institutional arrangements should also be modeled in more detail.

Appendix

Proofs

Proof of Theorem 1:

All that is required is to show that $\mathcal{A}_\Omega$ is invertible because that will lead to a unique solution for P_b. Note that $\mathcal{A}_\Omega$ is a matrix with an upper triangle of positive constants and a lower triangle with -c. The determinant of $\mathcal{A}_\Omega$ will be a polynomial of degree $|\Omega|$-1. In particular, the determinant of $\mathcal{A}_\Omega$ (det[$\mathcal{A}_\Omega$]) is given by:

$$\det[\mathcal{A}_\Omega] = \sum_\pi (\text{sgn } \pi)\, a_{1\pi(1)} \cdots a_{m\pi(m)} \quad , \qquad \text{(A)}$$

where $m=|\Omega|$ and π are a permutations on the elements of $\mathcal{A}_\Omega$. Equation (A) is a standard definition for the determinant of a matrix. We can now use the structure of our matrix to determine its value. We shall now show that det[$\mathcal{A}_\Omega$] > 0. The permutation $\pi(i)=i$ results in the product of the diagonal elements, $D \geq 0$. Next, the permutation where two of the elements of $\mathcal{A}_\Omega$ are reversed, i.e. $\pi(i)=j$ and $\pi(j)=i$, results in an odd number of inversions $\rightarrow$(sgn π) = -1. From Equation A, this term in the sum is give by $(-1)\cdot{-c}\cdot b(\pi)$ where $b(\pi)$ is a nonnegative number. At least one of the two elements permutations will be strictly positive. Continuing with permutations where three element positions are reversed the sgn π for these will 1. Thus, these permutations will be given by $(1)\cdot{-(c)^2}\, b^2(\pi)$ where $b^2(\pi)$ is a nonnegative number. At least one of these will be strictly positive. We can continue this argument to obtain elements of the form $(-1)^{\alpha(\pi)}\cdot(-c)^{\beta(\pi)}\cdot b^m(\pi)$ where $b^m(\pi)>0$, and $\alpha(\pi)$ and $\beta(\pi)$ are either both odd or both even. Thus, det[$\mathcal{A}_\Omega$] will be the sum of nonegative terms (at least one of which is strictly positive) so that det[$\mathcal{A}_\Omega$] >0. $\mathcal{B}_\Omega$ has the same properties as $\mathcal{A}_\Omega$.

Since the above arguement holds for any subset Ω of $\mathbf{Z}$ we have 2^n -1 Nash equilibria to this game.□

Algorithm for Computing P_s^*:

We will be examining symmetric C.M.S. equilibria, thus an algorithm to compute the equilibrium probabilities will prove handy. This algorithm can be extended to find the equilibrium probability for any Ω, however such an extention has little relevance to this paper.The difference in expected profits to the seller from offering z_i versus z_{i+1} is: $\pi_i - \pi_{i+1} = p_{n-i}\left(\frac{i}{n}+c\right) - \frac{1}{2n}\sum_{j=0}^{n-i-1} p_j$. The incentive compatibility constraints require that the expected profits of all offers must be equal when players mix. Thus $\pi_i - \pi_{i+1}=0$, which is equivalent to: $p_{n-i} = \frac{1}{(2i+2nc)}\sum_{j=0}^{n-i-1} p_j$. Note that if $c=-\frac{i}{n}$ the equation is undefined but we assumed that c was positive. We can write down what the equilibrium probabilities are solely as a function of p_n as follows:

$$p_0 = p_0$$

$$p_1 = \frac{1}{(2n-2+2nc)}\, p_0$$

$$p_2 = \frac{1}{(2n-4+2nc)}\, p_0 + p_1 \text{ or } p_2 = \frac{1}{(2n-4+2nc)}\,\frac{2n-1+2nc}{2n-2+2nc}\, p_0$$

similarly

$$p_3 = \frac{1}{2n-6+2nc}\,\frac{2n-3+2nc}{2n-4+2nc}\,\frac{2n-1+2nc}{2n-2+2nc}\, p_0$$

by induction

$$p_i = \frac{1}{2n-2i+2nc}\prod_{j=1}^{i-1}\frac{2n-(2j-1)+2nc}{2n-2j+2nc}\, p_0.$$

Now we can solve for p_0 because all the probabilities must add up to one or $\prod_{j=1}^{n}\frac{2n-(2j-1)+2nc}{2n-2j+2nc}\, p_0=1$. Hence $p_0=\prod_{j=1}^{n}\frac{2n-2j+2nc}{2n-(2j-1)+2nc}$ which is always defined for c positive. When c is negative (one get's paid to fail). Then some of the incentive compatibility constraints will bind. For example when one receives more than 0.5 if failure occurs, it always pays to fail (because one is get paid 0.5, which is more than what one can get in any symmetric equilibrium) so $p_0(-0,5) = 1$ and $p_n(-0.5) = 0$.

For $c \in [\frac{-i}{n}, \frac{-i+1}{n}]$ it does not pay to demand less than $\frac{i}{n}$, so $p_j \in [0,....,i]$ are zero and $p_j \in [n-i, ..., n]$ are zero as well. Then the highest offer made with positive probability will be $\frac{n-i+1}{n}$ and

$p_{i+1}=\prod_{j=i}^{n-i-1}\frac{2n-2j+2nc}{2n-(2j-1)+2nc}$. Note that this leads to a discontinuous profit function and failure function for $c\leq 0$.

Proof of Corrolary 1:

Given the algorithm defined above we find that expected profits increase as costs increase because the probability of making the most stuborn offer declines. Thus the profits of the higher cost player will be higher than those of the lower cost player.

Proof of Proposition 1:

We know that $\Omega = \{\frac{1}{2}\}$ is a symmetric equilibrium in conjecture. When a buyer contemplates his equilibrium strategy given that the other player plays a given equilibrium strategy based on Ω he is indifferent between making all offers in Ω. If we assume that his opponent can tremble between a mixed strategy over Ω and making any offer $\omega \varepsilon \Omega$ the buyer strictly prefers offering ω with probability one. In fact ω is a best response to any equilibrium strategy as long as $\omega \in \Omega$. Hence, given Ω, only those equilibria where both player adopt pure strategies are trembling hand perfect. Among the pure strategy equilibria only $\Omega = \{\frac{1}{2}\}$ is symmetric in conjectures.□

Proof of Lemma 1′:

Let $h(x) = \left\{ \int_x^1 g^*(y)(x+y)/2 \; d\beta^*(y) \; + \int_0^x g^*(y)(-c) \; d\beta^*(y) \right\}$. Suppose $\exists \; x_1, x_2 \varepsilon \; [0,1]$ and $\epsilon > 0$ such that $\int_{x_i-\epsilon}^{x_i+\epsilon} f(y) \; d\alpha(y) > 0$, for i=1,2 and h(x) over the interval $[x_1-\epsilon, x_1+\epsilon]$ does not equal h(x) over the interval $[x_2-\epsilon, x_2+\epsilon]$. Now define $h_1(z)$: $[-\epsilon,\epsilon] \rightarrow \Re$ and $h_2(z)$: $[-\epsilon,\epsilon] \rightarrow \Re$ as $h_i(z) = h(x_i + z)$. Since $h(\cdot)$ is continuous, $h_1(z) < h_2(z)$ so that:

$$\pi_s(f,\alpha) = \int_0^1 f(x) \left\{ \int_x^1 g^*(y)(x+y)/2 \; d\beta^*(y) \; + \int_0^x g^*(y)(-c) \; d\beta^*(y) \right\}$$

$$d\alpha(x) = \int_0^{x_1-\epsilon} f(x)\cdot h^*(x)\, d\alpha(x) + \int_{x_1-\epsilon}^{x_1+\epsilon} f(x)\cdot h^*(x)\, d\alpha(x) + \int_{x_1+\epsilon}^{x_2-\epsilon} f(x)\cdot h^*(x)\, d\alpha(x) + \int_{x_2-\epsilon}^{x_2+\epsilon} f(x)\cdot h^*(x)\, d\alpha(x) +$$

$$\int_{x_2+\epsilon}^{1} f(x)\cdot h^*(x) d\alpha(x) < \int_0^{x_1-\epsilon} f(x)\cdot h^*(x)\ d\alpha(x) + \int_{x_1-\epsilon}^{x_1+\epsilon} f(x)\cdot h_2(x)\ d\alpha(x)\ + \int_{x_1+\epsilon}^{x_2-\epsilon} f(x)\cdot h^*(x)\ d\alpha(x)$$

$$+ \int_{x_2-\epsilon}^{x_2+\epsilon} f(x)\cdot h_2(x)\ d\alpha(x)\ .$$ But this contradicts the definition of h^*, β^*.□

Proof of Proposition 2:

i) The profit function will be continuous if p_m is continuous. Since p_m a product of ratios whose numerators and denominators are all continuous in V, with denominators that are strictly positive for all V less than m/n, p_m is continuous in V.

ii) Let us first evaluate the first partial of p_m with respect to c

$$\frac{\partial p_m}{\partial V} = \frac{(\prod_{j=1}^{n} (2n-(2j-1)-2nV)(\sum_{j=1 i\neq j}^{n} \prod (2n-2i-2nV)}{(\prod_{j=1}^{n} 2n-(2j-1)-2nV)^2} - \frac{(\prod_{j=1}^{n} 2n-2j-2nV)(\sum_{j=1 i\neq j}^{n} \prod 2n-(2i-1)-2nV)}{(\prod_{j=1}^{n} 2n-(2j-1)-2nV)^2}$$

or

$$\frac{\partial p_m}{\partial V} = -\ 2n\ \frac{\sum_{i=1}^{n} (\prod_{j\neq i} 2n-(2j-1)-2nV)(\prod_{j\neq i} 2n-2j-2nV)}{(\prod_{j=1}^{n} 2n-(2j-1)-2nV)^2}$$

$$\frac{\partial p_m}{\partial V} = -\ 2n\ p_m \sum_{j=1}^{n-1} \frac{1}{(2j+1)-2nV)(2j-2nV)} < 0.$$

The same method can be used to show that for all other offers in $[\frac{m}{n}, \frac{n-m}{n}]$

$$\frac{\partial p_i}{\partial V} > 0 \text{ for } i>1$$

Now recall that by incentive compatibility $\pi_n=\pi_0$. Note that the coefficients of the first row of the payoff matrix are all positive independent of the value of v. Moreover p_m is the coefficient of the last entry in each row. In the first row the last entry is always $\frac{1}{2}$--corresponding to buyer sending 1 and seller sending 0. In the first row all entries are non negative and no greater than $\frac{1}{2}$; putting less weight on $\frac{1}{2}$ while increasing the weight of all other offers must decrease profits. Thus p_m decreases when V goes up, and so do profits.

iii) The second derivative of profits with respect to costs is:

$$\frac{\partial^2 \Pi(n,m,V)}{\partial V^2} = \frac{\partial^2 p_m}{\partial V^2}\left(\frac{n-m}{n} - V\right) - 2\frac{\partial p_m}{\partial V}$$

The first step involves verifying that the second partial of p_0 is positive, it is then easy to show that the second partial of profits must be positive. For brevity, the paper omits that proof.

All that is required is to differentiate the first partial, then appropriately rearrange terms so that the denominator is the same for all parts of the equation. Comparing the terms we notice that there are $(n-1)^2$ terms in each part of the derivative of the numerator of the first partial and n^2 terms in the derivative of the denominator all of which are negative. The terms from the derivative of the denominator are multiplied by 2. We can select $2(n-1)^2$ in the terms from the denominator which strictly dominate the numerator terms. Thus the second partial of p_0 is positive.

The n terms left over multiplied by $(\frac{n-m}{n}-V)$ are enough to deal with the other part of the second partial of the profit function.

iv) When V=m/n we have $\Pi(n,m,V) = \prod_{j=m}^{n-m} \frac{n-j-m}{n-j-1-m}\left(\frac{n-2m}{n}\right) + \frac{n}{m}$. The last term of this product (j=n-m) is zero so that $\Pi(n,m,V) = \frac{m}{n}$.

Recall that $p_m = \prod_{j=m}^{n-m} \frac{2n-2j-2nV}{2n-(2j-1)-2nV}$. Now as V goes to minus infinity $\frac{2n-2j-2nV}{2n-(2j-1)-2nV}$ goes to 1 for each j. So p_{m+1} goes to 1 this implies that $\pi_m(V)$ goes to $\frac{1}{2}$ when v goes to $-\infty$. Because we look at mixed strategy equilibria the profits of offering $\frac{m}{n}$ are equal to expected profits.□

Proof of Corollary 3:

Follows directly from Proposition 2. The fact that the limit of $\Pi(n,m,V)$ as V goes to infinity is $\frac{1}{2}$ leads to $c^* < \frac{1}{2}$□

Proof of Corollary 4:

The game achieves full efficiency if and only if the players each send $\frac{1}{2}$. If they fully randomize this can only occur if all the other offers are dominated or if V= $d\Pi$-c$>\frac{1}{2}-\frac{1}{n}$. Now, $\Pi(n,m,V)< \frac{1}{2}$ because of individual rationality (since the seller offers $\frac{m}{n}$ with positive probability and the expected profits from such an offer are at most $\frac{1}{2}$. In a mixed strategy equilibrium, expected profits are equal for all offers, thus profits are at most $\frac{1}{2}$). So for the game to achieve full efficiency c cannot be greater than $\frac{1}{n}$.□

Proof of Proposition 3:

First evaluate $\Pi(n,0,-c)$ -c at c= $\frac{1}{n}$. It is easy to show that $p_0(\frac{1}{n})>\frac{1}{2}$ so $\Pi(n,0,\frac{1}{n})$ -c $>\frac{n-3}{2n}$. In fact, $d\cdot\Pi(n,0,\frac{1}{n})$ -c>0 for d $>\frac{2}{n-1}$. Because profits are linear in d, $[\frac{2}{n-1}, 1]$ belongs to $\Delta(n)$.□

Proof of Theorem 5:

The proof involves four steps. We must eliminate the possibility that $V>\frac{n-1}{2n}$ in the second period. So we must investigate whether it is possible for $\Pi(n,0\text{-}c)$ than $\frac{n-1}{2n}$ -c .

1) if $c>\frac{1}{n}$ use Corollary 3.

2) for n>3 if $c<\frac{1}{n}$ then $p_0<\frac{1}{2}$ and $p_0(n,\frac{1}{n}) < p_0(4,\frac{1}{4})$ for n>4. Now, $p_0(4,\frac{1}{4})\simeq 0.2251 <0.5$ so $\Pi(n,0,-c)$ -c=(1- $p_0(n,\frac{1}{n})$)-c+ $p_0(n,\frac{1}{n})$ -c $<\frac{1}{2}-\frac{3}{2}c$. Thus, $\frac{1}{2}-\frac{3}{2}c > \frac{1}{2}-\frac{1}{n}$ iff $c<\frac{2}{3n}$. So the two period game will not achieve full efficiency if c is less than $\frac{2}{3n}$.

3) if $c<\frac{1}{2n}$ note that $p_0(n,\frac{1}{2n}) < p_0(4,\frac{1}{8}) <\frac{3}{10}$ for all n>4.

Thus $\Pi(n,0,-c)$ -c$<\frac{3}{10}-\frac{17}{10}c$

but $\frac{3}{10}-\frac{17}{10}c<\frac{1}{2}-\frac{1}{n}$ for all c>0 if n>4

If n=4 $\frac{1}{2}-\frac{1}{n}=\frac{1}{4}$ and $\frac{3}{10}-\frac{17}{10}c>\frac{1}{4}$ iff $c<\frac{1}{34}<\frac{1}{32}=\frac{1}{8n}$

$p_0(4,\frac{1}{32})<0.09$ so for $c<\frac{1}{32}$

$\Pi(n,0,c)$ -c$<\frac{1}{10}-\frac{19}{10}c$ which is always less that $\frac{1}{4}$. Again if $c<\frac{1}{2n}$ the two period game does not acheive full efficiency.

4) All that remains unaccounted for are the $c \in [\frac{1}{2n}, \frac{2}{3n}]$

Note again that $p_0(n, \frac{2}{3n}) < p_0(4, \frac{2}{12}) < \frac{1}{3}$ for all n>4.

$c > \frac{1}{2n}$ thus $\Pi(n,c) - c < \frac{1}{3} - \frac{5}{6n}$

$\frac{1}{3} - \frac{5}{6n} - \frac{1}{2} + \frac{1}{n} = -\frac{1}{6} + \frac{1}{6n} < 0$ for all n>1.

Thus for all n>3 the game never achieves full efficiency in two stages. For n =3, $\frac{1}{2}$ is not in the offer set so the game cannot achieve full efficiency over symmetric mixed strategies. □

Proof of Proposition 4:

First it is necessary to bound Π^t.

Claim: $\Pi^1 > \Pi^t > \Pi^2$.

Note that $\Pi^t(n,c) = \Pi^1(n,V) = \Pi^1(n, d \cdot \Pi^{t+1}(n.c) - c)$. Now note that $d \cdot \Pi^{t+1}(n.c) > 0$ so V>-c which implies that $\Pi^t(n,c) < \Pi^1(n,c)$ because profits are increasing in costs and therefore decreasing in V. Now note that for t>2 $\Pi^{t-1}(n,c) < \Pi^1(n,c)$. This leads to $V(c, \Pi^{t-1}) < V(\Pi^1)$ and thus $\Pi^t(n,c) > \Pi^2(n,c)$. So Π^∞ must belong to $[\Pi^2, \Pi^1]$ if it exists. A solution exists because $\Pi(n,0,V)$ is continuous for $\Re^+$. Uniqueness is easy because $\Pi(n,V)$ is strictly increasing in c and thus decreasing in V. Thus there must exist a V such that $V = \Pi(n,0,V) - c$.□

Proof of Theorem 6:

The proof of the Theorem follows from our equilibrium selection. Note first that given a cost c players will use the mixed strategy based on Ω_m if and only if V is greater than $\frac{m-1}{n}$ and less than $\frac{m}{n}$; in this range $\Pi(n,m,V)$ is a continuous decreasing function of V thus $d\Pi(n,m,V) - c$ will also be a decreasing continuous function of V. So there is at most one solution to the problem $d\Pi(n,m,V) - c - V = 0$ which is the fixed point condition for an infinite horizon equilibrium. A solution to the problem will exist if $d \cdot \Pi(n,m, \frac{m-1}{n}) - c > \frac{m-1}{n}$ but $\Pi(n,m, \frac{m-1}{n}) < 1/2$ so a solution may not exist except for m=0.

There may also exist solutions for m=k, (k∈ $\{\frac{1}{n};...;\frac{1}{2}\}$ provided that $c< \frac{n-k+1}{2n}$.□

Proof of Proposition 5:

We shall prove this result for m′=m-1 (the extension to m′=m-k is straightforward). We have:

$$\Pi^*(n\ m) = (1-p_m(\tfrac{m-1}{n}))\ \tfrac{m-1}{n}+ p_m(\tfrac{m-1}{n})(\tfrac{n-m}{n})$$

or $$= \tfrac{m-1}{n}+ p_m(\tfrac{m-1}{n})(\tfrac{n+1-2m}{n}).$$

Thus $\Pi^*(n, m)-\frac{m-1}{n}=p_m(\frac{m-1}{n})(\frac{n+1-2m}{n})$

and $\Pi^*(n, m')-\frac{m'-1}{n}=p_{m'}(\frac{m'-1}{n})(\frac{n+1-2m'}{n})$.

Since $m>m'$ $p_m >p_{m'}$

$$\Pi^*(n, m)-\tfrac{m-1}{n}- \Pi^*(n, m')+\tfrac{m'-1}{n}<- p_m(\tfrac{m-1}{n})\tfrac{2}{n}\ +(p_m-p_{m'})(\tfrac{n+1-2m}{n}).$$

Since $p_{m'} = \frac{2n+2-4m}{2n+3-4m}\ \frac{2n-4m}{2n+1-4m}\ p_m$ we have

$$(p_m-p_{m'}) =p_m(1-\frac{2n+2-4m}{2n+3-4m}\ \frac{2n-4m}{2n+1-4m})=p_m\frac{4n+4-8m}{(2n+3-4m)\ (2n+1-4m)}$$

so that $p_m\frac{2}{n}\ +(p_m-p_{m'})(\frac{n+1-2m}{n})= \frac{p_m}{n}\ (\frac{(4n+4-8m)\ (n+1-2m)}{(2n+3-4m)\ (2n+1-4m)}\ -2)$

$\frac{(4n+4-8m)\ (n+1-2m)}{(2n+3-4m)\ (2n+1-4m)}<1$ so that $\Pi^*(n, m)-\frac{m-1}{n}- \Pi^*(n, m')+\frac{m'-1}{n}<0$.□

Proof of Proposition 6:

Let us look at the case where only three offers are still rational ($\Omega=\{\frac{n-2}{2n};\ \frac{1}{2};\ \frac{n+2}{2n}\}$, $V> \frac{n-4}{2n}$ and d=1). Then $\Pi(V,c) = p_0(V)\cdot(\frac{n+2}{2n}-V) + V$. For an efficient C.M.S. Equilibrium it must be rational for the seller to refuse any offer less than $\frac{1}{2}$. So $\Pi(V,c)-c$ must be greater than $\frac{n-2}{2n}$, or $p_0(V)\cdot(\frac{n+2}{2n}-V) + V -c >\frac{n-2}{2n}$. Because Proposition 2 showed that $\Pi(V,c)$ is decreasing in V, we only need to evaluate $\Pi(V)$ at $(\frac{n-4}{2n})$, that yields the condition: $p_0(\frac{n-4}{2n})\cdot(\frac{n+2}{2n}-\frac{n-4}{2n}) + \frac{n-4}{2n} -c > \frac{1}{2}-\frac{1}{n}$. That condition is equivalent to $(3\cdot p_0(\frac{n-4}{2n})-1)\cdot(\frac{1}{n}) > c$. Now $p_0(\frac{n-4}{2n}) = \frac{8}{15}$, that implies that c must be less than $\frac{3}{5n}$ for the game to have a C.M.S.E. that is efficient. It is easy to check that if Ω is of cardinality greater than 3 then c must be even smaller. □

Proof of Proposition 7:

A) let C be the set defined by $\gamma_i \leq \gamma_j$ iff i>j and $\underline{c} \leq \gamma_j \leq \bar{c}$. Clearly C is bounded and C is closed because it contains its boundary. Moreover it is easy to show that C is convex (simply examine convex combinations of elements of C, they are also in C). $\Gamma^*(\cdot)$ maps C into C continuously. Therefore a fixed point exists. Note that this does not prove that C.M.S. equilibria exist for every Ω and every distribution function.

B) Follows directly from (6.3) and (6.4).□

REFERENCES

Broman, E., "The Bilateral Monopoly Model: Approaching Certainty Under the Split-the-Difference Mechanism", mimeo, John Hopkins University, (1987).

Card, D., "Strikes and Wages: A Test of a Signaling Model," N.B.E.R. Working Paper # 2550. (1988)

Chatterjee, K., and W. Samuelson, "Bargaining Under Incomplete Information", *Oper. Res.* 31 (1983), 835-851.

Cho, I., "Uncertainty and Delay in Bargaining: Old Question New Approach", mimeo University of Chicago, (1988).

Coase, R. H., "Durability and Monopoly", *J. Law Economics* 15 (1972).

Fudenberg, D., D. Levine, and J. Tirole, "Infinite horizon Models of Bargaining with One-Sided Information", in Alvin Roth ed., *Bargaining with Incomplete Information* Cambridge: Cambridge University Press, 1985.

Gul, F., H. Sonnenschien, and R. Wilson, "Foundations of Dynamic Monopoly and the Coase Conjecture", *J. of Econ Theory* 39 (1986), 155-190.

Hart, O., "Bargaining and Strikes", *Quarterly Journal of Economics* CIV (1989), 25-45.

Leininger, W., P. Linhart, and R. Radner, "The Sealed-Bid Mechanism for Bargaining with Incomplete Information", AT&T Bell Laboratories, Janurary 1987.

Myerson, R.B., and M. A. Satterthwaite, "Efficient Mechanisms for Bilateral Trading, *J. Econ. Theory* 29 (1983), 265-281.

Nash, J.F., "The Bargaining Problem", *Econometrica*, 18, 155-162.

Mailath, G., and A. Postlewaite, "Workers Versus Firms: Bargaining over a Firm's Value," mimeo, University of Pennsylvania, 1989.

Maitlath, G., and A, Postlewaite, "Asymmetric Information Bargaining Problems with Many Agents", mimeo, University of Pennsylvania, 1989.

Rosenthal, J.-L., *The Fruits of Revolution, Property Rights, Litigation and French Agriculture*; 1700-1860. Caltech PhD dissertation June 1988.

Rubinstein, A.," Perfect Equilibrium in a Bargaining Model", *Econometrica* 50 (1982), 97-110.

Smith, A., *The Wealth of Nations.* Chicago: University of Chicago Press, 1976.

Stahl, I., *Bargaining Theory*, Stockholm: Economic Research Institute, 1972.

Tracy, J., "An Investigation into the Determinants of U.S. Strike Activity," *American Economic Review* 76 (1986) 149-173.