UCLA Dept. of Economics
Working Paper #566

Efficient and Individually Rational Bayesian Mechanisms Only Exist on Perfectly Competitive Environments

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May 1989

Abstract

Necessary and sufficient conditions are given for a Bayesian Nash equilibrium of a mechanism to be simultaneously *ex-post* efficient and *ex-ante* individually rational. These conditions require (A) that each individual receive his/her expected marginal product and (B) that the sum of the expected marginal products over individuals equal the total expected gains from trade. Some consequences of this characterization are: (1) the above conditions are equivalent to the conditions required for a dominant strategy mechanism to be efficient and individually rational which, in turn, is equivalent to the condition that the environment must be perfectly competitive; (2) such mechanisms are generally impossible to construct in environments with a finite number of individuals; and, (3) for the sealed-bid double auction model of exchange where players types are independently and uniformly distributed, as the number (n) of traders increases the rate at which the environment converges to one that permits the necessary and sufficient conditions (A) and (B) to be satisfied is $O(1/n)$.

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1 Introduction

The dominant strategy solution concept is known for being very demanding. For an unrestricted domain of economic environments, Hurwicz [1972] showed that it is impossible to construct a mechanism that is simultaneously dominant strategy (DS), Pareto-optimal (PO), and individually rational (IR) — DSPOIR, for short. In Makowski and Ostroy [1987a], we took another tack by characterizing the class of environments on which a DSPOIR mechanism does exist, assuming quasi-linear preferences. We showed that such a mechanism had to be ***perfectly competitive***, i.e., a Walrasian mechanisms with the extra property that Walrasian prices are non-manipulable. Such a mechanism exists only on very special environments which we call perfectly competitive environments; hence the impossibilty results with finite numbers of individuals. (In fact, impossibility can occur in economic environments even without the IR condition — see Green and Laffont [1977],Walker [1980],and Hurwicz and Walker [1987].)

*Research supported by a grant from the NSF. We thank John Roemer and Lloyd Shapley for helpful comments.

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The DS solution concept is often criticized as being too demanding. Would weakening the solution concept to Bayesian Nash equilibrium lead to a more robust existence result? Bayesian equilibrium is really the only alternative if we wish to preserve the incomplete information feature implicit in the DS notion of equilibrium. It is logically weaker than DS since a DS mechanism is equivalent to one that is Bayesian under ***any*** probability beliefs. Arrow [1979], d'Aspremont and Gérard-Varet [1975,1979], Holmström [1977,1979] and others have demonstrated that Bayesian mechanisms which are PO can be exhibited over a wide range of non-perfectly competitive environments.

The question we address is: Do Bayesian mechanisms exist that are simultaneously PO and IR — BPOIR, for short — for non-perfectly competitive environments; or, does weakening the solution concept from DS to Bayesian buy nothing in terms of existence when a mechanism must also satisfy PO and IR? As the title of the paper suggests, our main result is that the latter is the case. The conclusion we draw both from this result and our earlier one is a uniqueness property of perfectly competitive environments: with incomplete information about the characteristics of others, the ***only*** environments for establishing efficient and incentive-compatible outcomes, when participation is voluntary, are the perfectly competitive ones.

In our [1987a] we characterized DSPOIR mechanisms as those exhibiting exact full appropriation, i.e., everyone receives a reward exactly equal to his/her ***marginal product*** to society. Analogously, but less demandingly, we will characterize BPOIR mechanisms as exact full appropriation mechanisms in ***expectations***, i.e., everyone receives a reward exactly equal to their expected marginal product. It will also be shown that while there are many BPOIR mechanisms, they form a single equivalence class in the sense that they all reward each individual with the same expected utility. So, weakening the standard of incentive compatibility from DS to Bayesian may buy something in terms of the number of incentive compatible POIR mechanisms, provided a DSPOIR mechanism exists; however, all of these mechanisms are expected utility equivalent to the DSPOIR mechanism. The latter may be regarded as the standard bearer of the equivalence class, not only because it is consistent with the more demanding and attractive DS solution concept, but also because it corresponds to the traditional

Walrasian *and* perfectly competitive market mechanism.

We call attention to three corollaries of our main result.

- A Walrasian mechanism is not BPOIR on a universal domain, i.e., one including non-perfectly competitive environments.

Thus, even under the weaker Bayesian standard of incentive compatibility, Walrasian equilibria are not implementable unless the environment is perfectly competitive, in which case they are implementable in dominant strategies.

- There exists no BPOIR mechanism on a universal domain.

Thus, perfectly competitive environments are uniquely suited for realizing efficiency in a non-coercive way when there is incomplete information; outside of them, there is an impossibility result.

- Since any two-person economy with gains from trade is necessarily non-perfectly competitive, there exists no BPOIR mechanism for two-person exchange.

This result was first proved by Myerson and Satterthwaite [1983] who focus exclusively on two-person trading. The benefit of viewing their result from our framework is that one sees more clearly the economic rationale for non-existence: it arises from the unavoidable absence of sufficient competition between the traders.

Section 2 presents the framework we shall be using, a standard demand-revealing model with quasi-linear preferences. To obtain our results we shall build upon (i) the characterization results of d'Aspremont and Gérard-Varet and Holmström and (ii) some results appearing in our "Vickrey-Clark-Groves Mechanisms and Perfect Competition", henceforth referred to as VCG. Section 3 establishes the marginal productivity in expectations framework used in Section 4 to prove our main results. Section 5 exhibits — within the family of double-auction-type models — (i) a perfectly competitive environment and a BPOIR mechanism on it, (ii) some impossibility results when the environment is extended to include non-perfectly competitive economies, and (iii) an asymptotic result on the predominance of perfect competition (and therefore the existence of BPOIR) as the number of individuals increases. The concluding Section 6 contains some remarks on the work of others.

2 The Model

The set of possible populations is $V = \times_i V_i$, where V_i represents the possible types for individual $i = 1, \ldots, n$. While it is common knowledge among individuals that they are in some population $v \in V$, each individual may only be sure of his/her own type, v_i.

There are $\ell + 1$ commodities. The first ℓ we call "y- commodities" and the last we call "money". Let $Y_i \subset \mathbf{R}^\ell$ represent i's possible trades in the y-commodities. We assume for all i that Y_i is independent of v_i and $0 \in Y_i$.

The set of Y-feasible outcomes for any population is defined by

$$Y = \{y = (y_1, \ldots, y_n) \in \times_i Y_i : \sum y_i = 0\}.$$

Note that this framework may apply to models of net trades in private goods or, through the identification of public goods with personalized joint supply, to models with net trades in public goods.

Adding money transfers to the model, let $x_i = (y_i, m_i) \in Y_i \times \mathbf{R}$ represent a possible trade for i in both the y-commodities and money. The set of *feasible outcomes* for the population is defined by

$$X = \{x = (y, m) \in Y \times \mathbf{R}^n : x_i = (y_i, m_i) \text{ and } \sum x_i = 0\}.$$

Each i has a *quasi-linear* utility function over y-commodities and money of the form,

$$u_i(x_i; v_i) = v_i(y_i) + m_i.$$

Since all individuals evaluate money in the same way, the only distinguishing feature of i's actual type is the function $v_i : Y_i \rightarrow \mathbf{R}$. We also assume that for all i, $v_i(\cdot)$ is continuous and $v_i(0) = 0$.

For any $v \in V$, the set of Y-*efficient* outcomes is

$$PO_Y(v) = \arg\max_Y \sum v_i(y_i).$$

The set of *Pareto optimal* outcomes in v is

$$PO(v) = \arg\max_X \sum u_i(x_i; v_i).$$

Notice that quasi-linearity implies

$$x = (y, m) \in PO(v) \iff x \in X \text{ and } y \in PO_Y(v),$$

i.e., the Pareto optimal outcomes are simply those which are y-efficient and for which the money components of the allocation sum to zero.

Assume that each V_i is a compact metric space. Let (V, S, P) be a probability space where $S = \times_i S_i$ and S_i is the Borel sets of V_i. The probability measure P represents the prior distribution of populations in V. P is assumed to be common knowledge and supp $P = V$.

From the point of view of any individual i, others' possible types are given by $V^i = \times_{j \neq i} V_j$, with typical member v^i. Let $(V^i, S^i, P(\cdot|v_i))$ be another probability space, where $S^i = \times_{j \neq i} S_j$. The measure $P(\cdot|v_i)$ represents i's posterior beliefs about the probability of others' types given that he is a type v_i.

Throughout $f : V \to Y \times \mathbf{R}^n$ will denote a *mechanism* where $f(v) = (y(v), m(v))$. Interpret $f_i(v) = (y_i(v), m_i(v))$ as the outcome to individual i in the population v under the mechanism f. A mechanism is assumed to be P-measurable.

Under the mechanism f an individual i of type v_i will receive the expected payoff

$$E_f U_i(v_i) = \int_{V^i} u_i(f_i(v_i, v^i), v_i) dP(v^i | v_i),$$

where $v = (v_i, v^i)$, provided that i reports his type truthfully. If, however, i reports that his type is v_i' when he is actually v_i, the expected payoff will be

$$E_f U_i(v_i'; v_i) = \int_{V^i} u_i(f(v_i', v^i), v_i) dP(v^i | v_i).$$

By construction, $E_f U(v_i) = E_f U(v_i; v_i)$.

We shall say that the mechanism $f \in BPO_Y[V]$, the set of ***Bayesian, Y-efficient mechanisms*** on V, if

(1) $$\forall v, \quad y(v) \in PO_Y(v), \text{where} f(v) = (y(v), m(v)),$$

(2) $$\forall i \; \forall v_i \; \forall v_i', \quad E_f U_i(v_i) \geq E_f U_i(v_i'; v_i).$$

Notice that condition (1) does not place any restrictions on the feasibility of the money component of the allocation, and therefore the range of f need not lie in the set of feasible outcomes X. Condition (2) is the Bayesian incentive compatibility constraint. It says that truthful revelation maximizes each individual's *expected* utility, regardless of his type.

How should individuals be rewarded so that the mechanism is Y-efficient (condition (1)) subject to the Bayesian incentive compatibility condition (2)? Evidently, this involves restrictions on the money component of the mechanism. To describe these restrictions, it is useful to introduce some definitions that will also be employed below.

The *gains from trade* in v is given by

$$g(v) = \sum v_i(y_i(v)),$$

where $y(v)$ is any element of $PO(v)$. Note that while $y(v)$ may not be unique, $g(v)$ certainly is. We shall assume that g is continuous on V rather than derive it from prior assumptions on each V_i and Y_i. (In the model of Section 5, the derivation is immediate.)

The *expected* gains from trade *conditional on individual i being of type* v_i is

$$Eg(v_i) = \sum_{j \neq i} \int_{V^i} v_j(y_j(v_i, v^i)) dP^i(v^i | v_i) + \int_{V^i} v_i(y_i(v_i, v^i)) dP^i(v^i | v_i),$$

where y satisfies condition (1).[1] Subtract from $Eg(v_i)$ the value of the second term on the RHS to obtain

$$Eg^i(v_i) = Eg(v_i) - \int_{V^i} v_i(y(v_i, v^i)) dP(v^i | v_i),$$

the expected gains from trade to everyone except i when he is of type v_i and the mechanism $f = (y, m)$ is used.

A mechanism f is a *Groves mechanism in expectations*, denoted by $f \in$ EG[V], if it satisfies condition (1) and $\forall i \; \forall v_i$, there exists $h_i : V^i \to \mathbf{R}$, such

[1] $Ek(x)$ denotes the expected value of k conditional on the realization x.

that,

$$(3) \qquad Em_i(v_i) \equiv \int_{V^i} m_i(v_i, v^i)dP(v^i|v_i) = Eg^i(v_i) + Eh_i(v_i),$$

where

$$Eh_i(v_i) = \int_{V^i} h_i(v^i)dP(v^i|v_i),$$

The term $Eh_i(v_i)$ is dependent on v_i only to the extent that $P(\cdot|v_i)$ is. Since there is nothing that v_i can do to effect this quantity, it is a lump sum. Thus, condition (3) says the expected money payment to an individual of type v_i is the expected gains to everyone except i plus a lump sum. In the following section we shall provide an alternative marginal product interpretation of this formula.

Just as it is easy to verify that any Groves scheme is DS, it is easy to verify that any Groves scheme in expectations is Bayesian incentive compatible. Further, provided individuals' types are chosen independently, d'Aspremont and Gérard-Varet [1975,1979] and Holmström [1977,1979] have shown that a Groves mechanism in expectations is also necessary for Bayesian incentive compatibility.

Specifically, say that *individuals' types are independently distributed* if for any i, (V, S, P) can be written as the product of two probability spaces (V_i, S_i, P_i) and (V^i, S^i, P^i), i.e., if $A \times B \subset S_i \times S^i$, then $P(A \times B) = P_i(A) \cdot P^i(B)$. Thus, for any v_i, $P(\cdot|v_i) = P^i(\cdot)$. With independence, the expected lump sum term $Eh_i(v_i)$ does not depend on v_i.

We conclude this section with a summary of the relationship between BPO_Y and Groves mechanisms in expectations under independence, which follows from Holmström [1977,1979].

Theorem 1 *Suppose that for each i V_i is a convex set and individuals' types are independently distributed. Then,*

$$\text{EG[V]} = \text{BPO}_\text{Y}\text{[V]}.$$

3 Groves Mechanisms in Expectations as Marginal Product Mechanisms in Expectations

In VCG we showed that any Groves mechanism is equivalent to a marginal product mechanism, i.e., a mechanism that rewards each individual with his marginal product, plus perhaps a lump sum. We shall demonstrate here that this equivalence can be extended to expected values. This equivalence will be used in the next section to show the limitations on lump sum transfers in BPO_Y mechanisms resulting from the assumption of individual rationality.

Recalling that $g(v)$ is the gains from trade in the population v, let

$$g^i(v) = \max_{Y^i} \sum_{j \neq i} v_j(y_j),$$

where $Y^i = \{y = (y_1, \ldots, y_n) \in Y : y_i = 0\}$. Thus, $g^i(v)$ is the gains from trade when i is excluded (i.e., when i makes the null trade. This is effectively the gains from trade in the population v^i. The continuity assumption for g will also be assumed to apply to g^i.

The contribution of the characteristics v_i (including Y_i) to the the gains from trade in v is measured by the *marginal product* of i in v, where

$$MP_i(v) = g(v) - g^i(v).$$

The *expected marginal product of i* conditional on his being of type v_i is

$$EMP(v_i) = Eg(v_i) - Eg^i.$$

The first term on the RHS was defined in Section 2 as the expected gains to the population conditional on i being of type v_i while the second term, $Eg^i = \int_{V^i} g^i(v_i, v^i)dP(v^i|v_i)$, is the expected gains when i is effectively absent from the population.

Say that the mechanism f is in the set of *marginal product mechanisms in expectations*, denoted EMP[V], if f satisfies (1) and $\forall i\ \forall v_i$ there exists $H_i : V^i \to \mathbf{R}$ such that

$$E_f U_i(v_i) = EMP(v_i) + EH_i(v_i), \tag{4}$$

where $EH_i(v_i) = \int_{V^i} H_i(v^i)dP(v^i|v_i)$. Condition (4) says that if i is of type v_i he expects to be rewarded with his expected marginal product plus an expected lump sum. (Note: the dependence of EH_i on v_i is only via $P(v^i|v_i)$ as in the definition of $Eh_i(v_i)$, above.)

To establish the identity between Groves and MP mechanisms in expectations, let

$$Ev_i = \int_{V^i} v_i(y_i(v_i, v^i))dP(v^i|v_i),$$

where y_i is the i^{th} component of an allocation satisfying (1). From the formula for a Groves mechanism in expectations, condition (3),

$$Em_i(v_i) = Eg^i(v_i) + Eh_i(v_i) = Eg(v_i) - Ev_i + Eh_i(v_i).$$

Since

$$E_f U_i(v_i) = Ev_i + Em_i(v_i),$$

if we add Ev_i to both sides of the formula for $Em_i(v_i)$ and set $H_i(v^i) \equiv h_i(v^i) - g^i(v)$ — recall that $g^i(v)$ depends only on v^i and is therefore a lump sum with respect to i — we obtain

$$\begin{aligned} E_f U_i(v_i) &= Ev_i + Em_i(v_i) = Ev_i + Eg(v_i) - Ev_i + Eh_i(v_i) \\ &= Eg(v_i) + Eh_i(v_i) = Eg(v_i) - Eg^i + EH_i(v_i) = EMP(v_i) + EH_i(v_i). \end{aligned}$$

Thus, any Groves mechanism in expectations can be written as a marginal product mechanism in expectations. For the converse, just run the above argument in reverse.

Summarizing,

Lemma 1 EG[V] = EMP[V].

4 Individual Rationality and Feasibility: Characterizing BPOIR Mechanisms

Our concern is with mechanisms involving voluntary, non-coercive participation; hence, mechanisms that are "individually rational." Say that the mechanism f is ***individually rational in expectations***, written $f \in$ EIR[V], if $\forall i\ \forall v_i$,

$$E_f U_i(v_i) \geq u_i(0; v_i) = 0.$$

Notice that a stronger *ex post* notion of individual rationality is possible: $\forall i\ \forall v_i,\ u_i(f_i(v), v_i) \geq 0$. But we shall restrict ourselves to the weaker concept since voluntary exchange must surely satisfy it, while it need not satisfy the stronger notion, at least when agreements must be made prior to revelation and without recourse.

To characterize BPOIR mechanisms we shall make two restrictions concerning the populations in V. The first is an assumption that there exist types exhibiting no complementarities with others in the expected gains from trade, in the sense that others can expect to do as well with such a type present or absent. More formally,

$$(A.1) \qquad \forall i, \exists v_i^0 \in V_i \text{ such that } \mathrm{EMP}(v_i^0) = 0.$$

In the example of Section 5, such a type is illustrated by a seller with a reservation value at least as high as that of any buyer or a buyer with reservation value at least as low as any seller.

Suppose $f \in$ EMP[V] $\cap$ EIR[V]. Then (4) and the definition of v_i^0 implies

$$\forall i,\ E_f U_i(v_i^0) = EH_i(v_i) = \int_{V^i} H_i(v^i) dP(v^i | v_i^0) \geq 0.$$

Therefore, we have

Lemma 2 *Assuming (A.1), if $f \in$ EMP[V] $\cap$ EIR[V], where $E_f U_i(v_i) = EMP(v_i) + EH_i(v_i)$, then $\forall i$,*

$$EH_i(v_i^0) \geq 0.$$

Let EMP^0[V] be the family of EMP[V] mechanisms in which the lump sum term is identically zero. Since $f \in$ EMP^0 implies that $E_f U_i(v_i) =$ EMP$_i(v_i)$ and since $MP_i(v) = g(v) - g^i(v) \geq 0$ because $0 \in Y_i$, we can conclude that

Lemma 3 EMP^0[V] $\subset$ EIR[V].

Denote by F[V] the set of *feasible* mechanisms f such that $\forall v\ f(v) \in X$. Recall from the definition of X that a mechanism is feasible if its y component always lies in Y *and* the the money components sum to zero.

From Theorem 1 and Lemma 1 we know that if $f \in \text{EMP}^0[\text{V}]$, then $f \in \text{BPO}_Y[\text{V}]$; and from Lemma 3 we can conclude that $f \in \text{BPO}_Y\text{IR}[\text{V}]$ ($\equiv \text{BPO}_Y[\text{V}] \cap \text{EIR}[\text{V}]$). While such a mechanism is required to satisfy condition (1), which necessarily means that it is Y-feasible, there is no guarantee that $f \in \text{F}[\text{V}]$, i.e., the sum of the money payments is not necessarily zero. And, it is the latter restriction that makes the mechanism PO rather than merely PO_Y.

There is one situation in which EMP^0 will be PO. Call V a *full appropriation* environment if

$$\forall v,\ \sum MP_i(v) = g(v),$$

i.e., if the sum of the individual marginal products *always* sums to the gains from trade. In such an environment each individual can fully appropriate the gains from trade that the individual, through his characteristics, confers on the population.

On full appropriation environments we could construct an f such that $u_i(f_i(v); v_i) = MP_i(v)$ *and* $f \in \text{F}[\text{V}]$. This f would clearly exhibit

$$E_f U_i(v_i) = \text{EMP}(v_i).$$

From the previous results, this would guarantee BPOIR. (In fact, we showed in VCG that such a mechanism would guarantee DSPOIR.) The question that remains is whether there are any other situations — mechanisms and environments — in which it is possible to establish BPOIR.

Economic environments, at least those with finite numbers of individuals, do not typically exhibit the full appropriation property. Rather, they exhibit what can be called *overfull appropriation*, i.e.,

$$(A.2) \qquad \forall v,\ \sum MP_i(v) \geq g(v).$$

For example, if v is a population having a non-empty core it will satisfy (A.2). Hence, any private good economy with a Walrasian equilibrium or any collective good economy with a Lindahl equilibrium will exhibit overfull appropriation. (See Proposition 1, below, where (A.2) is verified for the double-auction model and Section 6.1 for a class of models exhibiting "underfull" appropriation.)

Let EMP*[V] = EMP^0[V] ∩ F[V] be the set of ***full appropriation mechanisms in expectations*** on V. A mechanism f belongs to this set precisely when

(a) $f \in \mathrm{PO}_Y[\mathrm{V}]$, i.e., satisfies condition (1).

(b) $\forall i \forall v_i,\ E_f U_i(v_i) = \mathrm{EMP}(v_i)$

(c) $f \in \mathrm{F[V]}$.

The characterization of BPOIR is described in

Theorem 2 *If V satisfies the hypotheses of Theorem 1 and (A.1) and (A.2), then*

$$\mathrm{BPOIR[V]} = \mathrm{EMP^*[V]}.$$

Theorem 2 says that if $f \in$ BPOIR[V], it must give each type its expected marginal product. So, for any $f, f' \in$ BPOIR[V],

$$E_f U_i(v_i) = E_{f'} U_i(v_i) = \mathrm{EMP}(v_i).$$

In terms of *ex ante* expected utility, the equivalence class of all BPOIR mechanisms yields the same expected utility. Thus it is meaningful to speak of "the" full appropriation mechanism in expectations. The following result shows that there is a natural focal point of this equivalence class of BPOIR mechanisms, namely the mechanism that rewards each i with exactly $\mathrm{MP}_i(v)$ for each $v \in V$ rather than $\mathrm{EMP}_(v_i)$. In our earlier VCG paper, it is shown that this focal point is the unique DSPOIR mcehanism.

Theorem 3 *If V satisfies the hypotheses of Theorem 2, then*

$$\mathrm{BPOIR[V]} \neq \emptyset \iff \forall v, \sum MP_i(v) = g(v).$$

To relate these results to perfect competition theory, suppose that V is a class of populations for which Walrasian equilibria exist. (An especially simple class is given in Section 5.) Say that V is a class of *perfectly competitive* populations if each $v \in V$ possesses a Walrasian equilibrium with the extra property that everyone truly faces perfectly elastic supplies and demands, i.e., with the extra property that price-taking is truly rational. In VCG, we show that a perfectly competitive population and one in which there is full appropriation are equivalent. Combining this the results of VCG and Theorems 2 and 3 above, BPOIR *mechanisms only exist on families of perfectly competitive populations.* Alternatively put, if the family of populations is not perfectly competitive then ndividuals will justifiably believe that they possess some non-negligible monopoly power. And in their efforts to exercise this power, the tension between efficiency (PO) and selfish Bayesian bargaining burst into inefficiency; formally, BPOIR becomes impossible.

Proof of Theorem 2: By Lemma 3,

$$f \in EMP^*[V] \Rightarrow f \in EMP[V] \cap F[V] \cap EIR[V].$$

By Theorem 1 and Lemma 1,

$$f \in EMP[V] \cap F[V] \cap EIR[V] \Rightarrow f \in BPO_Y[V] \cap F[V] \cap EIR[V] = BPOIR[V].$$

Conversely, by Theorem 1 and Lemma 1,

$$f \in BPOIR[V] \Rightarrow f \in EG[V] \cap F[V]\ EIR[V] \Rightarrow f \in EMP[V] \cap F[V] \cap EIR[V].$$

Hence, we need only show that for all i, $EH_i = 0$ to show that $f \in EMP^*[V]$.

From Lemma 2, if $f \in EMP[V] \cap EIR[V]$, then $\forall i, EH_i \geq 0$. Note that by independence, EH_i does not depend on v_i. Therefore,

$$E_f U_i(v_i) = EMP(v_i) + EH_i \geq EMP(v_i);$$

from which it follows that

$$E_f U_i \equiv \int_{V_i} E_f U_i(v_i) dP_i(v_i) \geq \int_{V_i} EMP(v_i) dP_i(v_i) \equiv EMP_i.$$

Summing over i,

$$(*) \qquad \sum E_f U_i = E_f \sum U_i = E \sum u_i(f_i(v); v_i) = Eg \geq \sum EMP_i.$$

If $EH_i > 0$ for some i, the inequality would be strict, contradicting (A.2). So, $f \in$ BPOIR $\Rightarrow f \in$ EMP*, completing the argument for Theorem 2.

Proof of Theorem 3: When $\forall v, \sum MP_i(v) = g(v)$, then by choosing f such that $\forall i\ \forall v_i, u_i(f_i(v); v_i) = MP_i(v)$, we can certainly satisfy $f \in$ EMP* which, by Theorem 2, implies that $f \in$ BPOIR.

For the converse, by Theorem 2 we must show $f \in$ EMP$^* \Rightarrow \forall v$, $\sum MP_i(v) = g(v)$. If, on the contrary, this is not the case then by (A.2) there exists a v such that $\sum MP_i(v) > g(v)$. By continuity of g and g^i there is an open set $W \subset V$ such that $\sum MP_i(w) > g(w), w \in W$. From this we can conclude $P(W) > 0$ since otherwise if $P(W) = 0$, then supp P would be contained in the closure of $V \backslash W$, contradicting the hypothesis assumed throughout that supp $P = V$. Therefore, by (A.2),

$$\sum EMP_i > Eg.$$

But this contradicts $(*)$ in the proof of Theorem 2 which was necessary for $f \in EMP^*$.

5 The Double-Auction Model

In this section we shall apply our results to a simple and well-studied class of environments with quasi-linear preferences, the sealed-bid version of the double-auction model. Here we can explicitly make the connection between BPOIR mechanisms and perfect competition stated in the Introduction. Given our characterization result and our claim that BPOIR mechanisms require perfectly competitive environments, one might jump to the conclusion that BPOIR is impossible in models with a finite number of individuals because perfect competition is impossible with small numbers. While it is correct that perfectly competitive configurations of parameter values are rare with small numbers of individuals, we shall see that they do exist in the double-auction model. We shall also show that as the number of individuals increases, the frequency of (almost) perfectly competitive populations increases so that what is a rare event for small numbers becomes common for large.

In terms of the model of Section 2, the double-auction model is defined by

(i) $\ell = 1$;

(ii)

$$Y_i = \begin{cases} [0,1] & \text{if } i \in B \\ [-1,0] & \text{if } i \in S \end{cases}$$

where B and S are a partition of the n individuals into buyers and sellers, respectively;

(iii)

$$\forall i, V_i = \{v_i \in [0,K] : v_i(y_i) = v_i y_i,\ \ y_i \in Y_i\}.$$

Thus, there is just one y-commodity and money. Letting $x_i = (y_i, m_i) \in \mathbf{R}^2$, the utility of x_i to a type v_i is

$$u_i(x_i; v_i) = v_i y_i + m_i.$$

Note, each individual enjoys constant marginal utility from the commodity if $i \in B$ and suffers constant marginal cost if $i \in S$, and no individual can trade more than one unit.

Let $V = V_1 \times \cdots \times V_n = [0,K]^n$. The triple (π, y, m) is a *Walrasian equilibrium* for v if

(WE.1) y is a feasible allocation, i.e., $y_i \in Y_i$ and $\sum y_i = 0$,

(WE.2) $\forall i,\ \ v_i y_i - \pi y_i \geq v_i y_i' - \pi y_i'$,

(WE.3) $m_i = \pi y_i$, so that the budget constraint $\pi y_i + m_i = 0$ is satisfied.

A description of any v along with its Walrasian equilibria can be illustrated in a demand-and-supply diagram. The demand schedule consists of the $\{v_i\}, i \in B$, arranged in descending order while supply consists of

$\{v_i\}, i \in S$, arranged in ascending order. Walrasian equilibrium price(s) and quantity (or quantities) occur at the intersection of the two schedules.

In Theorem 3 we gave necessary and sufficient conditions for a BPOIR mechanism to exist. Qualifications required for this result are that each V_i be convex as well as assumptions (A.1) and (A.2) on V. Since $V_i = [0, K]$, it is evidently convex. By putting $v_i = 0, i \in B$, or $v_i = K, i \in S$, we can evidently satisfy the requirement (A.1) that there exists for each i a v_i^0 exhibiting no complementarities with others.

To verify (A.2), let $\Pi(v)$ be the set of Walrasian prices for the population v. It is easily seen that $\Pi(v)$ is an interval and we denote its length by $|\Pi(v)|$. Also, let $q(v)$ be the *minimum* number of units bought (or sold) in some y-efficient allocation for v.

Proposition 1 $\forall v,\ \sum MP_i(v) - g(v) = q(v)|\Pi(v)| \geq 0.$

Since $q(v)$ and $|\Pi(v)|$ are non-negative, the condition (A.2) of overfull appropriation is satisfied. Proposition 1 also provides the conditions for full appropriation, the key requirement for BPOIR. This will occur either in the trivial case that there are no gains from trade and therefore $q(v)$ is zero or in the case that $|\Pi(v)| = 0$, i.e., there is only one Walrasian price for the population v.

We give an informal diagrammatic proof of Proposition 1, illustrated in Figure 1, using an example with two buyers and two sellers.

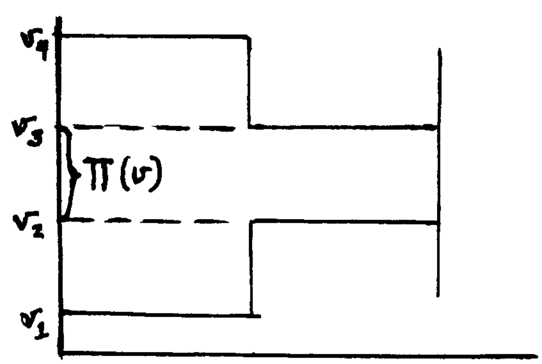

FIGURE 1

The value of $g(v)$ is given by the area between the demand and supply schedules to the left of the equilibrium quantity (here equal to 2) where they intersect. The value of MP_i is the difference between this area and the area $g^i(v)$ between the two schedules after the trader with reservation value v_i drops out. Assume the values of v_i are given in ascending order, $v_1 < v_2 < v_3 < v_4$. In this example $S = \{1,2\}$ and $B = \{3,4\}$.

It is readily seen that $\text{MP}_1 = v_3 - v_1$, $\text{MP}_2 = v_3 - v_2$, $\text{MP}_3 = v_3 - v_2$, and $\text{MP}_4 = v_4 - v_2$. Note that $\Pi(v) = [v_2, v_3]$ and $g(v) = (v_3 - v_1) + (v_4 - v_2)$. The reader may wish to verify the following interpretation of Proposition 1: each seller (buyer) will receive his $\text{MP}_i(v)$ if the price at which he sells (buys) is the highest (lowest) value in $\Pi(v)$. (There are no inactive traders in this example and if there were the rule would not apply to them.)

Adding up the MP's, we have

$$\begin{aligned}\sum MP_i &= (v_3 - v_1) + (v_3 - v_2) + (v_3 - v_2) + (v_4 - v_2) \\ &= (v_3 - v_1) + 2\Pi(v) + (v_4 - v_2) \\ &= g(v) + 2\Pi(v),\end{aligned}$$

as was to be shown.

Whenever $|\Pi(v)| > 0$, it is evident that prices are manipulable.[2] Either a buyer or a seller can, by reporting a different reservation value, change the size of the interval in his favor while continuing to trade. For example, if $|\Pi(v)| = [a, b]$, and $y_i > 0$, then by reporting $v_i' \in [a, b)$, i can change the interval to $[a, v_i']$. This is not true when $|\Pi(v)| = 0$: a buyer may be able to raise the price or a seller lower the price, but no individual can *favorably* manipulate the price by misrepresenting his preferences. Since $|\Pi(v)|$ is positive only when individuals can influence prices and since the size of $|\Pi(v)|$ is indicative of the extent of the influence, we may use $|\Pi(v)|$ as a measure of the potential monopoly power in v. (This characterization of monopoly power and particularly the association of its absence with uniqueness of Walrasian price in no way carries over to more general models.)

Define

$$V^* = \{v \in V : |\Pi(v)| = 0\},$$

[2]Trivially, prices are non-manipulable when there are no gains from trade, yet $\Pi(v)$ can be a non-degenerate interval. We ignore this as an inessential exception to the characterization of a perfectly competitive population as $|\Pi(v)| = 0$.

as the subset of populations with unique prices, or the environment on which there is perfect competition. Let f^W be a Walrasian mechanism on V. There is no ambiguity (in terms of utilities) about a Walrasian mechanism on V^* because the utility of an allocation is completely determined by the price. On $V \backslash V^*$, let the Walrasian price be the midpoint of $\Pi(v)$.

Suppose each i can announce any $v_i \in [0, K]$ but he is given the prior information that $P(V^*) = 1$. While the choice of a $v_i' \neq v_i$ by a single individual may lead to a v that lies outside of V^* — which, nevertheless, leads to a well-defined outcome — the expected gain from such a choice is non-positive because the individual knows that any misrepresentation he makes, if it changes price at all, changes it unfavorably. Therefore,

Proposition 2 *Provided $P(V^*) = 1$, the Walrasian mechanism f^W is BPOIR on V.*

The essential qualification is the proviso that $P(V^*) = 1$ rather than the hypothesis of the previous sections that $P(V) = 1$. Note that V^* is not only a subset of V but is further restricted by the fact that it cannot be written as a Cartesian product of sets V_i^*. Hence, individuals' types cannot be independently distributed. Thus, Proposition 2 provides only sufficient conditions rather than necessary ones as in our Theorems 2 and 3.

With this possibility result as background, we can draw upon our main characterization results to make some observations about impossibility. Since the model of this section satisfies assumptions (A.1) and (A.2), we can apply Theorems 2 and 3 to show that for a mechanism to be in BPOIR[V] the family of populations in V must satisfy full appropriation. But by Proposition 1, full appropriation occurs only on V^* a set for which $P(V^*) < 1$. Therefore, we have

Proposition 3 *If individuals' types are independently distributed and* supp $P = V$, *then* BPOIR[V] $= \emptyset$.

Given the current interest in investigating incentives under bilateral exchange, we point out that BPOIR is never possible in two-person models with positive gains from trade. While such a result already appears in

Myerson and Satterthwaite [1983], our more general framework and characterization provides an intuition for this conclusion: two-person economies with positive gains from trade necessarily involve monopoly or monopsony power since neither the seller nor the buyer competes against other sellers or buyers; i.e., such economies are never perfectly competitive. More succinctly,

$$\text{if } n = 2, g(v) > 0 \iff |\Pi(v)| > 0.$$

Two-person economies represent the worst case scenario for BPOIR because (a) the existence of perfect competition and (b) the presence of gains from trade occur on mutually disjoint sets of V, whereas when $n \geq 3$ there are instances of compatibility. We conclude this section by showing that as $n \nearrow \infty$, instances of approximate compatibility predominate.

To make the dependence on the number of individuals explicit, let $V(n)$ replace V. We wish to show that as n increases there is some suitable metric such that the size of the set $V(n)\backslash V^*(n)$ goes to zero. There are two problems: (i) the dimension of $V(n)$ is increasing with n and (ii) there are instances in which increasing numbers does not lead to diminution in the level of monopoly power.

Problem (ii) is illustrated by the following: (Example) Let $n = 2k$, where k = number of buyers = number of sellers and let $v_i = b$, $\forall i \in B$, and $v_i = a$, $\forall i \in S$, where $b > a$. Evidently, for all k, $|\Pi(v)| = b - a$, and therefore each individual has monopoly power. This shows that we shall have to incorporate a genericity element in our metric if we are to admit this possibility while regarding it as exceptional.

We shall deal with problem (i) by peforming an experiment that makes the metric scalar-valued for all n. Suppose $V(n)$ *did* coincide with $V^*(n)$. Then we could choose *the* Walrasian mechanism f and it would satisfy for each $v \in V(n)$, $u_i(f_i(v); v_i) = MP_i(v)$. By Theorems 2 and 3 this mechanism would satisfy BPOIR.

Suppose we pretend that $V(n) = V^*(n)$ by choosing an $f \in \text{EMP}^0$ such that each type v_i gets not only $\text{EMP}_i(v_i)$, but his exact MP in every instance, i.e., $f = (y, m)$ satisfies

(MP.1) $\forall v,\ y(v) \in \text{PO}_Y(v)$

(MP.2) $\forall\, i \forall\, v_i,\ m_i(v) = MP_i(v) - v_i(y_i)$.

By construction,

$$\sum m_i(v) = \sum MP_i(v) - g(v),$$

and by Proposition 1 this will be non-negative. In the population v, the scalar $\sum m_i(v)$ measures the size of the error associated with our pretense that v is in V^*.

If $\sum m_i(v)$ can be used as a measure of how far the population v is from being perfectly competitive, then

$$Em(n) \equiv E\sum_{i=1}^{n} m_i(v) = \sum_{i=1}^{n} \int_{V(n)} m_i(v)dP(v),$$

can be used as a measure of how far the environment $V(n)$ is from being perfectly competitive. The number $Em(n)$ can also be interpreted as the expected subsidy the market participants would have to contribute so that the market mechanism could operate on $V(n)$ according to the perfectly competitive principle of rewarding each individual with their MP. In the case of one buyer and one seller, Myerson and Satterthwaite [1983] show that this is the minimum subsidy which must be added to make a Bayesian incentive compatible mechanism efficient.

To say that increasing the number of individuals causes the environment $V(n)$ to become perfectly competitive, i.e., $V(n)\backslash V^*(n)$ is effectively shrinking to zero, might be described by one of the following:

(1) $\lim n^{-1}Em(n) = 0$.

(2) $n^{-1}Em(n) \sim O(1/n)$.

(3) $n^{-1}Em(n) \sim o(1/n)$.

They are listed in increasing order of stringency. The first requires that the *per capita* expected subsidy to make the market perfectly competitive go to zero, but it says nothing about the rate of convergence. The second requires that $n^{-1}Em(n)$ converge at the rate $O(1/n)$, which is to say that the *per capita* subsidy goes to zero sufficiently rapidly that a market with large but finite numbers would be a good approximation to the assumption that $V(n) = V^*(n)$. The third criterion implies that since the per capita subsidy is going to zero faster than n^{-1}, we have $\lim Em(n) = 0$.

With the assumption of independence of types, it will be readily verified that (1) holds and it will also be clear that (3) will not. We shall demonstrate (2) for the special case of a uniform probability distribution. A similar order of convergence result on the asymptotic efficiency of a certain trading mechanism is established by Satterthwaite and Williams [1988] under more general assumptions on the probability distribution. (See Section 6.2, below.)

For computational convenience we normalize by setting $K = 1$. Let λ be Lebesgue measure on $[0,1]$ and $\lambda^n = \lambda \times \cdots \times \lambda$ be the population measure on $V(n) = [0,1]^n$ *before* individuals are assigned to be buyers or sellers. That assignment is made for each individual by flipping a fair coin.

Proposition 4 *Let* $P(n) = \lambda^n$; *then* $\lim n^{-1}Em(n) \sim 0(1/n)$.

To prove this result, let us first apply Proposition 1 to the definition of $Em(n)$ to obtain,

$$Em(n) = \int_{V(n)} q(v)|\Pi(v)|d\lambda^n(v).$$

Recalling that $q(v)$ is the (minimum) number of units bought or sold in a Walrasian equilibrium, $0 \leq q(v) \leq n/2$. We may therefore conclude that the expected number of trades will vary proportionally with n. Thus, the crucial component of the formula above is the size of $|\Pi(v)|$.

If, as is readily shown, $E|\Pi(v(n))| \to 0$, then since $n^{-1}q(v) \leq 1/2$, we are assured that $Em(n) \to 0$. It remains to show that $E|\Pi(v(n))| \sim O(1/n)$ to demonstrate the Proposition.

Let $v(n) = (v_1, \ldots, v_n)$ represent a sequence of n random draws from $[0,1]$. Rearranging the order if necessary, let us suppose that $v_{(1)} \leq v_{(2)} \leq \cdots \leq v_{(n)}$. Call

$$\sigma_i[v(n)] = v_{(i+1)} - v_{(i)},$$

the *space* between the i^{th} and the $(i+1)$ ordered value. In the following argument, we shall ignore the zero probability event that there are ties in the reservation values, i.e., we shall only deal with those realizations in which the spaces are positive.

The spaces $\sigma_i[v(n)]$ are of interest because

Lemma 4 *If there is trade in equilibrium, then for some* $i = 1, \ldots, n-1$,

$$\Pi(v(n)) = \sigma_i[v(n)] + v_{(i)}.$$

To demonstrate, let $v_{(k)}$ ($v_{(h)}$) be the highest (lowest) reservation value of an active seller (buyer). Obviously, $h > k$. If $h = (k+1)$, then any price larger (smaller) than $v_{(h)}$ ($v_{(k)}$) would cause aggregate demand to be less (greater) than aggregate supply, while any price within this interval is market-clearing. If $h > (k+1)$, then it would contradict the definitions of $v_{(k)}$ and $v_{(h)}$ for there to be two reservation values between them, one belonging to a buyer and one to a seller, such that the buyer's exceeded the seller's. Thus, the lowest value of the sellers' reservation values in this interval, call it $v_{(i+1)}$, must exceed the highest of the buyers' reservation values which will be $v_{(i)}$. It is readily verified that in this case $\Pi(v(n)) = \sigma_i + v_{(i)}$.

It is well-known that for the uniform distribution (see David [1981, p.50])

Lemma 5

$$\begin{aligned} E\sigma_i[v(n)] &= \binom{n}{i} \int_0^1 (z)^i (1-z)^{(n-i)} dz \\ &= \frac{n!}{i!(n-i)!} \cdot \frac{i!(n-i)!}{(n+1)!} \\ &= \frac{1}{(n+1)}, \end{aligned}$$

Finally, Proposition 4 follows from the observation that the assignment of buyers and sellers is made independently of the value of $v(n)$ and therefore

$$E|\Pi(v(n))| = \sum_i \text{Prob}\{\Pi(v(n)) \in \sigma_i[v(n)]\} E\sigma_i[v(n)] = (n+1)^{-1},$$

which yields the desired conclusion on the rate of convergence. From the argument above, it is evident that the same conclusion can be shown to hold for probability distributions other than the uniform one.

6 Concluding Remarks

In this section we shall relate the results of this paper to the work of others.

6.1 BPOIR with Underfull Appropriation

While most private or collective good models will satisfy (A.1) and (A.2), there is a much studied collective good model in which they do not hold. This is the costless collective good model studied, for example, in Laffont and Maskin [1979]. In this model society must choose exactly one collective good project from a set of possible alternative projects. There are no costs of production, except for the opportunity cost of choice, so the cost-sharing advantages in models with costly collective goods — that lead such models to exhibit increasing returns over individuals — are absent here. It is easy to show that in this model that if any individual is pivotal for the choice of a particular project, then $\sum MP_i(v) < g(v)$; i.e., the model will exhibit decreasing returns over individuals (and will also have an empty core).

Assuming differentiability of the mechanism, Laffont and Maskin prove that there exists no BPOIR mechanism for this model. We show that our MP approach leads to the same conclusion, without any differentiability assumption.

Let the technology Y be a fixed set of costless alternative projects and $v_i : Y \to \mathbf{R}$. Normalize tastes so that for some $y_0 \in Y$, $v_i(y_0) = 0$, $\forall v_i$. Now redefine $\mathrm{MP}_i(v) = g(v) - g(v^i, v_i^0)$, where $g(v) = \max\{\sum v_i(y) : y \in Y\}$ and v_i^0 is the null function on Y. Thus $g(v^i, v_i^0)$ plays the role of $g^i(v)$ in the sense that it measures the maximum gains in the economy when i is effectively absent.

Now assume, instead of (A.1), that:

$$(A.1') \qquad \forall i,\ \exists \underline{v}_i \in V_i \text{ such that } Eg(\underline{v}_i) < \frac{(n-1)}{n} Eg.$$

Since the right-hand side equals $Eg-(1/n)Eg$, (A.1′) can be interpreted as saying that individual i may dislike the collective good more than it is like by the others on average. (Laffont and Maskin use a weaker assumption that individuals may simply dislike the collective good choice on average.)

Proposition 5 *For the costless, alternative collective good model, suppose that V is convex and satisfies* (A.1′) *and individuals types are independently distributed. Then,* BPOIR $= \emptyset$.

To sketch the proof, notice that

$$f \in BPOIR[V] \Rightarrow f \in EMP[V] \cap F[V] \cap EIR[V].$$

And further, (A.1′) plus $f \in$ EMP[V]∩EIR[V] implies that $\forall i$,

$$H_i \equiv \int h_i(v^i)dP^i(v^i) > Eg(v_i^0) - \frac{(n-1)}{n}Eg.$$

(The proof is exactly analogous to that of Lemma 2.)

Now, as in the proof of Theorem 2 (the argument leading to (*)

$$Eg = \sum EMP_i + \sum H_i.$$

Substituting the definition of EMP$_i$ = $\int MP_i(v)dP(v)$, where MP$_i(v)$ = $g(v) - g(v^i, v_i^0)$, and using the above inequality yields:

$$\begin{aligned} Eg = \int g(v)dP(v) &= \sum_i \int g(v)dP(v) - \sum_i \int g(v^i, v_i^0)dP^i(v^i) + \sum_i H_i \\ &> \sum_i \int g(v)dP(v) - \sum_i \frac{(n-1)}{n} \int g(v)dP(v) \\ &= nEg - \frac{(n-1)}{n}(nEg) \\ &= Eg, \end{aligned}$$

a contradiction. So, BPOIR[V] $= \emptyset$.

6.2 The Assumption of Independently Distributed Types

Our characterization result assumes independence, which may the strike the reader as a restrictive, although commonly used, assumption. We give it the following defense.

We are looking for conditions under which the market mechanism will work efficiently despite the presence of decentralized knowledge. When will the market be robust? To formalize this question, we might ask when will it be Pareto optimal on a set of economies V *no matter from what distribution* P the economy is drawn. The strongest standard of robustness in this context is dominant strategy equilibrium, explored in our VCG. A weaker standard is Bayesian incentive compatibility for *any* given distribution P, including the independent distributions examined here. While dependence might be the rule in certain instances such as common value auctions, the hypothesis of independence of types does seem to reflect the probabilistic interpretation of decentralized knowledge. Our conclusion is that with independence, the market will be robust if and only if it is perfectly competitive throughout V, which is also the conclusion reached under the stronger dominant strategy standard of robustness.

Ledyard [1978,1979] shows a result related to ours: any Bayesian mechanism under sufficient uncertainty about agents' types must have the dominant strategy property. But he restricts his attention to so-called "nonparametric" Bayesian mechanisms, i.e., ones whose outcome functions are independent of any agent's probability beliefs about others' types. This incentive compatibility requirement puts it somewhere between the Bayesian and dominant strategy standards. By contrast, we have stayed with the confines of the Bayesian concept in which individuals may rely on their common knowledge of probabilities and the inferences to be drawn from them.

6.3 BPOIR in Economies with Large Numbers

In Proposition 4 we showed that in the special case of double-auction models the impossibility results for BPOIR converge to possibility results as the number of individuals increases. This is entirely in keeping with our characterization result which says that perfectly competitive outcomes (or something expected utility equivalent to it) on perfectly competitive environments is exactly what is required for a mechanism to satisfy BPOIR. Note that this characterization, the subject of Section 4, applies equally well to models with small as well as large numbers of individuals. The reason for introducing large numbers is to provide (and confirm) an environment where perfectly competitive populations are ubiquitous.

On the sealed-bid double auction model, there have been several studies of its incentive/efficiency properties. In addition to Myerson and Satterthwaite cited above, Chaterjee and Samuelson [1983], Leininger, Linhart and Radner [1986], and Satterthwaite and Williams [1987] have investigated the efficiency and remarkable multiplicity of Bayesian Nash equilibria for the case of one buyer and one seller. For the small numbers case our main point of contact is in showing why any trading arrangement will fail to satisfy BPOIR: the environment would have to be perfectly competitive which, of course, it is not.

For the large numbers case, Gresik and Satterthwaite [1986] and Satterthwaite and Williams [1988] have established rates of convergence for Walrasian-like trading arrangements in which individuals can misrepresent their reservation values. The lack of any incentive to misrepresent implies the efficiency of their trading arrangement. The latter paper demonstrates an $O(1/n)$ rate of convergence on the maximum amount that a trader misrepresents. Proposition 4 establishes a $O(1/n)$ rate based on a different trading arrangement: the rate of convergence is on the amount of the subsidy required to implement a mechanism which encourages individuals not to misrepresent at all. These results appear to be two sides of the same coin.

Wilson [1985a,1985b] has investigated the case of a moderately large number of individuals. We comment on his interpretation of the features of the sealed-bid double auction as a desirable trading arrangement. Our position is that it is not so much the *mechanism*, but rather the *environment*, which is responsible for the efficiency of its outcomes.

Wilson argues that the sealed-bid double auction is worthy of our attention because it operates without knowledge of individuals' characteristics, knowledge that each is unlikely to have about the others. Further, its workings do not depend on the particular prior from which the types are chosen. He shows that for sufficiently large, but finite numbers of individuals, the mechanism is *interim efficient*, a qualified notion of incentive efficiency for dealing with small numbers, non-perfectly competitive models due to Holmström and Myerson [1983].

We heartily subscribe to the reasons Wilson points to for studying this mechanism. In a sense we wish to go further by saying that these incomplete information features of a sealed-bid double auction are central to an appreciation of the unique efficiency properties of perfect competition. Where we differ is in the attribution of efficiency in the large numbers case: is it the mechanism or the environment? We can make our point by saying that the interim efficiency Wilson finds seems to understate the mechanism's success. Given the rate of convergence $O(1/n)$ established for $Em(n)$ in Section 4, the mechanism is close to achieving BPOIR. Alternatively put, if the participants would make a very small contribution to cover the shortfall, with very high probability they could achieve BPOIR.

We attribute the mechanism's success with large numbers to the properties of the environment, specifically the full appropriation property that for most populations $v \in V(n), \sum MP_i(v) - g(v)$ is small compared to the number of individuals. Not only can the mechanism be said to be approximately BPOIR but, unavoidably by Theorems 2 and 3, it is approximately a dominant strategy mechanism. Hence, it is not sensitive to the prior distribution on types. Given these strong properties of the environment when n is "moderately" large, a trading arrangement that did not perform nearly efficiently (in the *ex post* sense) would be seriously flawed.

The double-auction model has also been studied in cooperative game theory as a simple version of a market-game. See, for example, Shapley and Shubik [1972]. Unlike other market-games, in this one the core always coincides with the Walrasian equilibria. Thus, our $|\Pi(v)|$ measures the width of the core of the economy with population v. Our characterization of a perfectly competitive v can be identified with a one element core, $|\Pi(v)| = 0$. Also, Proposition 4 can be interpreted as a result on the rate of convergence of the size of the core.

Our main result that there is little to choose between dominant strategy and Bayesian concepts of equilibrium when the mechanism must also satisfy PO and IR suggests that our analysis should have much in common with the dominant strategy approach, a suggestion which is confirmed throughout. Here we want to point out the connections between Proposition 4 and the work of Green and Laffont [1979], Rob [1982], and Mitsui [1983] on the asymptotic efficiency of Groves, or demand-revealing, mechanisms applied to the costless public goods model described in Section 6.1. In that model or one with private goods, paying individuals their MP's provides the proper incentive for individuals to report their characteristics truthfully.[3] Similarly, the departure of such a mechanism from PO is measured by $|\sum m_i = MP_i - g|$, the amount by which the sum of the money payments fail to balance. These authors "pretend" that they can pay everyone his MP and examine the expected value of the discrepancy as the number of individuals increases. The model they work with is distinguished by the fact that the strongest mode of convergence described above, $\lim Em(n) \to 0$, is established.

There seems little doubt that theorems on the asymptotic existence of BPOIR and DSPOIR mechanisms can be demonstrated for more general models with quasi-linear preferences than the double-auction or the costless public goods, although we have not shown this. However, elsewhere (Makowski and Ostroy [1988]) we have shown, via the concept of an individual's MP, how the theory of Groves mechanisms can be extended to continuum economies. One advantage of the continuum model is that the restrictive assumption of quasi-linear preferences can be discarded. In our [1987b], results on Groves-MP mechanisms and their properties in the continuum are given in models without quasi-linearity.

[3] Although in the public goods case the strength of the incentive to report one's characteristics truthfully is minimal when the number of individuals is large.

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