THE DETERMINANTS OF POWER

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Abstract

Power is the ability to achieve one's ends in the presence of rivals. The struggle for power is universal; it occurs not only among nations and political factions within nations but also among and within firms, families, and animal communities. In the steady-state model analyzed here, each contender strikes an optimal balance between productive activity and conflictual activity. The result is a continuing interaction having elements of both "war" and "peace."

The factors determining power include: (1) Capabilities: Resources and the efficiencies with which they are utilized. (2) Payoff functions: The equations translating productive efforts into income and conflictual efforts into distributive shares. (3) Protocol: The "rules of the game" -- whether, for example, the Cournot or Stackelberg or other solution concept is applicable.

It might be thought that in a continuing power struggle the stronger side will grow ever stronger still. On the contrary, the weaker side in terms of resources has a comparative advantage in conflictual activity (the Power Equalization Principle). Thus, while conflict necessarily reduces the aggregate of income produced, it tends to bring about a more equal division of whatever income remains. As a corollary, any group suffering an exogenous impoverishment shock will rationally redirect some of its energies toward the distributive struggle. This explains why governments subject to political pressures are observed to "lean against the wind" so as to moderate the impact of such shocks.

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THE DETERMINANTS OF POWER[1]

Power, for the purpose of this analysis, is the ability to achieve one's ends in the presence of rivals. Contests for power are a near-universal of life: they occur not only between nations on the world scene and social classes in domestic politics, but also within and among animal communities, firms, and even families. The intent here is to provide a framework for the analysis of struggles for power at all levels.

I will assume that decision-makers are rational and interested solely in maximizing income. To this end, each contender strikes an optimal balance between productive activity (aimed at generating income through economic cooperation with other parties) versus conflictual activity (aimed at appropriating the income produced by others, or else defending against opponents' efforts to do the same).[2] For simplicity here, the analysis is limited to two-sided interactions.

Conflict and settlement are usually regarded as mutually exclusive. Rival nations are said to be at war or else at peace. And similarly for the ordinarily nonviolent struggles sharing some common features with war -- strikes and lockouts, lawsuits, political campaigns, etc. In contrast, in the model employed here the antagonists do not choose between polar extremes of "going to war" versus "making peace." Instead, they arrive at a steady-state equilibrium typically involving elements of both struggle and accommodation.[3]

Two broad, almost self-evident generalizations apply to mixed interactions in which the parties are simultaneously cooperating yet competing with one another:

(i) The resources devoted to productive activity mainly determine the aggregate income available for the two sides together.

(ii) The commitments to conflictual activity mainly determine how the aggregate income will be divided between them.

The factors determining success in appropriative struggles can be grouped under three main headings:

1. Capabilities: The resources available to each side, and the efficiency with which they can be utilized for productive or for conflictual ends.

2. Technologies of production and conflict: The equations that translate (i) productive efforts into income and (ii) conflictual efforts into distributive shares.

3. Protocol: The "rules of the game" that specify, for example, whether the Cournot or Stackelberg or other solution concept is applicable.

The analysis here provides the basis for answering questions such as:

(1) To what extent can the richer side in a contest translate its larger initial endowment of resources into higher consumption income? A contender with greater resources is in a position to devote more effort to fighting activity and productive activity both, so it might seem obvious that the richer side must end up better off than its opponent. Remarkably, as will be seen, this is not the general result.

(2) If the yield from productive efforts rises, owing to exogenous technological improvements, will the parties be motivated to shift their resource allocations toward fighting more or fighting less? While arguments could be made either way, we will see that the normal result is to leave those allocations exactly as they were before.

(3) Considering another type of exogenous technological change, what if productive complementarity between the parties increases? In the first

decade of the twentieth century, increasingly close productive linkages among the national economies of Europe led to a widespread belief that large-scale war had become out of the question.[4] The first World War dashed these hopes. Since we are now seeing something of a revival of such beliefs,[5] it will be important to understand why productive complementarity is so often ineffective as a force for peace.

(4) <u>What consequences can be anticipated from changes in military technology -- or, more generally, in the technology of conflict?</u> As will be seen, when the "decisiveness" of fighting effort increases, the intensity of struggle rises.

(5) <u>When one side has an intrinsic advantage at fighting, and the other at producing, which tends to do better?</u> If "one of us is worth two of them" when it comes to producing goods, but the other way around when it comes to fighting, whose chances look more promising?

That somewhat counterintuitive results sometimes emerge from the analysis, as hinted above, is due to the <u>interacting</u> nature of the two sides' decisions. An improvement of productive capabilities on one side may, for example, induce greater efforts by the opponent to seize a share of that improvement.

I. ELEMENTS OF THE MODEL

The basic equation system has four classes of logical elements.

First, each side must divide its resources R_i between productive effort E_i and fighting effort F_i, leading to accounting identities in the form of Resource Partition Functions:

$$(1) \qquad \begin{aligned} E_1 + F_1 = R_1 \\ E_2 + F_2 = R_2 \end{aligned} \qquad \text{Resource Partition Functions}$$

Second, the productive technology is summarized by an Aggregate Production Function (APF) showing how the productive efforts E_1 and E_2 combine to determine "contestable income" I -- the social total available for division between the two parties:

$$(2) \qquad I = A(E_1^{1/s} + E_2^{1/s})^s \qquad \text{Aggregate Production Function}$$

Equation (2) is an APF characterized by constant returns to scale and constant elasticity of substitution. The multiplicative parameter A is an <u>efficiency index</u> that scales overall productivity. The parameter s is a <u>productive complementarity index</u>. As can be seen from the typical productive isoquants displayed in Figure 1, when $s = 1$ the APF takes on a linear (additive) form; as s rises above 1, the increasingly convex curvature represents higher and higher degrees of synergy between the parties' productive commitments. (Values of s below 1 are ruled out, as they imply increasing marginal product throughout to each side's productive input.)[6]

The third element, relatively novel in economic analysis yet an essential feature of all interactions where conflict and struggle play a role (and they almost always do!), is the Contest Success Function (CSF). Where the APF represents the familiar <u>technology of production</u>, the CSF summarizes <u>the technology of conflict</u> -- whose inputs are the fighting efforts F_1 and F_2 and whose outputs are the distributive shares p_1 and p_2. By assumption here the outcome of the struggle depends only upon the ratio of the parties' conflictual efforts F_1 and F_2.[7] In equation (3) the CSF is indexed by a single mass effect parameter m. As illustrated in Figure 2, the mass effect parameter m determines the "decisiveness" of conflict, that is the degree to which a superior input ratio F_1/F_2 translates into a superior success ratio p_1/p_2.

$$p_1 = F_1^m/(F_1^m + F_2^m)$$

(3) Contest Success Functions

$$p_2 = F_2^m/(F_1^m + F_2^m)$$

Finally, there are Income Distribution Functions defining the achieved income levels I_1 and I_2:

$$I_1 = p_1 I$$

(4) Income Distribution Functions

$$I_2 = p_2 I$$

In the model of equations (1) through (4), all income falls into a common pool available for capture by either side. More generally, the contenders might also have opportunities for generating autonomous or invulnerable income, as suggested by equations (4') in which each side's total income Y_i is shown as the sum of its "invulnerable income" X_i and its share I_i of the "contestable income" I:

$$Y_1 = X_1 + I_1 = X_1 + p_1 I$$

(4') Income Distribution Functions (generalized)

$$Y_2 = X_2 + I_2 = X_2 + p_2 I$$

Employing equations (4'), each side would decide among three rather than only two options: resources R_i could be allocated to conflictual effort, to productive effort generating contestable income I, or to productive effort aimed instead to produce invulnerable income X_i. In the interests of tractability, however, only the simplified form represented by equations (4) will be used in this paper. All income is assumed to be contestable; there are no protected enclaves invulnerable to capture.[8] Thus each contender faces only two options: fighting or producing.

The overall model is illustrated by the four-way diagram of Figure 3. The upper-right quadrant shows the range of choices for contender #1, within his initial resource endowment R_1, between productive effort E_1 and conflictual effort F_1. The diagonally opposite quadrant shows the corresponding options for contender #2. The upper-left quadrant shows how the respective fighting efforts F_1 and F_2 determine p_1, the share of aggregate income won by #1, where of course $p_2 = 1 - p_1$. (The p_1 contours are straight lines emerging from the origin, which follows from the assumption stated above that the distributive shares are functions only of the ratio F_1/F_2.) And finally, the lower-right quadrant shows how the productive efforts E_1 and E_2 combine to generate different overall totals of income I.

The dashed rectangle in Figure 3 illustrates one possible outcome, given initial choices E_1, F_1 on the part of decision-maker #1 and E_2, F_2 on the part of #2. The productive activity levels E_1 and E_2 determine aggregate income I, while the conflictual commitments F_1 and F_2 determine the respective shares p_1 and p_2. The dotted rectangle shows what happens when the two sides both choose to devote more effort than before to fighting. As drawn here, the increases in F_1 and F_2 have cancelled one another out so that p_1 and p_2 remain unchanged. Thus, the only effect of symmetrically increased fighting efforts may be to reduce the amount of income available to be divided between the parties.

II. REACTION CURVES AND COURNOT EQUILIBRIUM -- BASIC SOLUTION

We now need to show how the two sides' optimizing choices and the consequent power equilibrium are determined. On the assumption that the underlying strategic situation justifies the Cournot solution concept,[9] the Reaction Curves RC_1 and RC_2 show each side's optimal fighting effort given the corresponding choice on the part of the opponent. The Cournot solution occurs at the intersection of the Reaction Curves, where each party's decision is a best response to the opponent's action.

Decision-maker #1's optimizing problem can be expressed:

(5) $$\text{Max } I_1 = p_1(F_1|F_2) \times I(E_1|E_2) \quad \text{subject to} \quad E_1 + F_1 = R_1$$

For expository simplicity, it will be convenient initially to use the simplest possible parametrizations of the Aggregate Production Function (APF) and the Conflict Success Function (CSF). With regard to production, the multiplicative index A and the productive complementarity index s of equation (2) are set at unity. Then:

(2') $$I = E_1 + E_2 \qquad \text{APF (for } A = s = 1)$$

With regard to conflictual effort, the mass effect parameter m of equations (3) is also set at unity:

(3') $$p_1 = F_1/(F_1 + F_2)$$
$$p_2 = F_2/(F_1 + F_2) \qquad \text{CSF } (m = 1)$$

Then, by standard constrained-optimization techniques[10] the Reaction Curves RC_1 and RC_2 may be expressed as:

(6) Reaction Curves
$$F_1/F_2 = (E_1 + E_2)/(F_1 + F_2) \qquad RC_1$$
$$F_2/F_1 = (E_1 + E_2)/(F_1 + F_2) \qquad RC_2$$

As an important qualification, however, these Reaction Curves are simultaneously valid only when each side's choices are in the <u>interior</u> range, with $F_i < R_i$. If, say, the F_1 indicated by (6) exceeds R_1, then side #1 will be in its range of <u>corner</u> solutions, where RC_1 simply becomes $F_1 = R_1$.

Assuming an interior solution, the Reaction Curves (6) summarize the entire system of equations. In this simple case an explicit solution is obtained:

(7) $$F_1 = F_2 = E_1 = E_2 = (R_1 + R_2)/4$$ Cournot solution (interior)

This solution is illustrated by the intersection of RC_1^o and RC_2^o in Figure 4.[11] And since $F_1 = F_2$, it follows that $p_1 = p_2$, so that:

(8) $$I_1 = I_2 = (R_1 + R_2)/4$$ Incomes at Cournot solution (interior)

This completes the simplest version of the general equilibrium model.

<u>Numerical Example 1</u>: Suppose the resources are $R_1 = R_2 = 100$. Then, in accordance with equations (7) and (8), half of the social aggregate of resources is dissipated in conflictual effort $(F_1 = F_2 = 50)$, the other half going to productive efforts -- generating incomes equally divided between the contenders $(I_1 = I_2 = 50)$.

III. RESOURCE DISPARITIES AND ACHIEVABLE INCOME: THE POWER EQUALIZATION PRINCIPLE

When equal initial resource endowments are assumed, that the contenders end up with identical incomes at the Cournot equilibrium is hardly surprising. Power differences can only emerge given some asymmetry in the postulated conditions. However, equation (8) tells us more generally that the incomes I_1 and I_2 will depend symmetrically only upon the aggregate sum of the resources $R_1 + R_2$. Regardless of the distribution of the initial endowments, therefore, the achieved incomes will still be exactly equal at any interior solution. Thus we are led to a remarkable result I shall call the Power Equalization Principle (PEP):

> The Power Equalization Principle: Greater resource availability does not generally imply greater achieved income.

This rather startling proposition can be expressed in a strong and a weak form:

> PEP (strong form): Regardless of the initial resource distribution, contending parties in mixed conflict-cooperation interactions will end up with exactly identical incomes.

> PEP (weak form): In mixed conflict-cooperation interactions, the final distribution of income will always have lesser dispersion than the initial distribution of resources.

Much of what follows is aimed at exploring the ranges of applicability of the strong and the weak forms of the Power Equalization Principle.

Resource differentials -- interior vs. corner solutions

Retaining the simplified versions of the Aggregate Production Function and the Contest Success Function of the previous section, what happens when the parties have unequal resource endowments? Figure 5 indicates the nature of the forces at work. Here the Reaction Curves pictured in the previous diagram, based upon the equal resource endowments of Numerical Example 1, are shown again as the solid curves RC_1^o and RC_2^o. The dashed curves RC_1' and RC_2' represent a situation where, with #2's resources held constant, #1's resources have doubled. As can be seen in the diagram, at the new intersection the equilibrium fighting efforts F_1 and F_2 -- though both larger than before -- remain equal to one another. (It follows, of course, that the richer party must now be devoting absolutely and relatively more resources to production.) The equality of F_1 and F_2, implying that $p_1 = p_2 = 1/2$, also dictates that the parties end up with equal incomes once again. Thus, the strong form of the Power Equalization Principle applies.

Numerical Example 2: Let the initial resources be $R_1 = 200$, $R_2 = 100$. Then, in accordance with equations (7) and (8), half of the enlarged social aggregate of resources is still dissipated in conflictual effort: $F_1 = F_2 = 75$. It follows of course that the wealthier side is devoting more effort to production: $E_1 = 125$, $E_2 = 25$. Since the fractional shares remain one-half each, the final incomes are $I_1 = I_2 = 75$.

An intuitive interpretation is as follows. The better-endowed contender, #1, can afford to spend more on each of the two types of activity and so will be dividing his increment of resources between E_1 and F_1. The opponent, although no richer than before, now has both offensive and defensive incentives to shift toward spending more than before on fighting. The offensive incentive for making F_2 larger is that there is more social income available to be seized. The defensive incentive is that, facing a bigger F_1, decision-maker #2 must make his F_2 larger even if only to maintain his previous level of income.

Underlying the Power Equalization Principle is the fact that, when a contender's resources are small relative to the opponent's, the marginal yield of fighting activity is higher to begin with than the marginal yield of productive activity. Specifically, supposing that #2 is the poorer-endowed side, straightforward differentiation of $I_2 = p_2 I$ leads to:

(9)
$$\frac{\partial I_2}{\partial E_2} = \frac{F_2}{F_1+F_2} \quad \text{and} \quad \frac{\partial I_2}{\partial F_2} = \frac{F_2(E_1+E_2)}{(F_1+F_2)^2}$$

When R_2 is very small then of course E_2 and F_2 must be small as well, meaning that the marginal product of fighting $\partial I_2/\partial F_2$ is initially larger than the marginal product of productive effort $\partial I_2/\partial E_2$.

So conflict is a relatively more attractive option for the less well-endowed side.[12] Fighting effort permits you to "tax" the opponent's production, while your own production is "taxed" by his fighting effort. When your rival is richer it becomes relatively more profitable to tax him (to capture part of his larger production) and relatively more burdensome to be taxed by him (to devote effort to production that will be largely captured by him anyway).

Thus rational behavior in a conflict interaction is for the poorer side to specialize more in fighting, the richer side more in production. In ancient times, cities or empires with relatively advanced productive industry were regularly raided by nomadic tribes who specialized in developing their fighting prowess. The effect was to moderate the initial wealth disparity between rich city and poor nomads.

But there is a limit to this process. Eventually, at some critical resource ratio $R_1/R_2 = r^*$, the poorer side runs into its resource constraint -- where <u>all</u> of its resources are optimally devoted to fighting. (It becomes a pure predator, so to speak.) For resource ratios more extreme than this we are in the range of corner solutions, so that only the weak form of the PEP applies: attained incomes will no longer be exactly equalized, though remaining much more equal than the initial resource endowments.

> <u>Numerical Example 3</u>: Suppose the resource endowments are $R_1 = 400$, $R_2 = 100$. At such an extreme resource ratio, side #2 optimally devotes all its resources to fighting ($F_2 = 100$). Side #2 will then set $F_1 = 123.6$, in accordance with RC_1. The better-endowed side now obtains a larger proportionate share, $p_1 = .553$, the achieved incomes being $I_1 = 152.8$ and $I_2 = 123.6$.

Another point of interest: at corner solutions there is less overall wastage of resources in fighting.

Figure 6a provides a more general picture of how the aggregate and the separate incomes respond to the resource ratio R_1/R_2, given the parameter assumptions of equations (2′) and (3′), and assuming for concreteness that $R_2 = 1$. The critical resource ratio is $r^* = 3$.[13] The $\hat{I}$ curve represents the potential income achievable in the absence of fighting ($F_1 = F_2 = 0$). As can be seen, the incomes I_1 and I_2 remain exactly equal in the range of interior solutions before the critical ratio is reached; only in the range of corner solutions does the better-endowed side gain an advantage.

Figure 6b illustrates these results from a different angle. In the range of interior solutions the proportionate shares remain equal ($p_1 = p_2 = 1/2$), but the better-endowed side gains an advantage once the critical resource ratio r* is passed. The other curves, I_1/R_1 and I_2/R_2, might be called "conversion ratios"; they measure success in translating resources into income. For the poorer side I_2/R_2 rises throughout, reaching unity at r* and continuing to rise (though a little less rapidly than before) further without limit. For the better-endowed side, I_1/R_1 falls until the critical ratio is reached, and then begins to recover -- slowly approaching its limit at unity.

Summarizing for the simplest base-case solution, the strong form of the Power Equalization Principle (PEP) holds when the resource ratio is only moderately unequal, so that the equilibrium falls within the interior-solution range. When the resource asymmetry becomes sufficiently great, however, the parties enter a corner-solution range where only the weak form of the PEP applies. The ability of the poorer side to convert its resources into income rises without bound as the opponent's resource endowment increases; the wealthier side, in contrast, can never go beyond 100%. The source of the improvement for both sides is the fact that, once the contenders are in the corner-solution range, an increasingly extreme resource ratio implies a smaller and smaller proportionate wastage of resources in fighting.

IV. WHEN CONFLICT BECOMES MORE DECISIVE

Going beyond the base-case solutions of the preceding section, the outcomes of mixed conflict-cooperation interactions will of course depend upon the assumptions made as to the technology of production (the Aggregate Production Function) and the technology of struggle (the Contest Success Function). Here we deal with the latter. For the illustrative base case, the mass effect parameter in the CSF of equations (3) was fixed at $m = 1$. When m increases so that fighting becomes "more decisive" (see Figure 2), at all resource levels each side is inclined to devote somewhat more resources to the struggle. So the Reaction Curves of Figure 5 would all be displaced outward.

Without repeating the entire analytical development, when m changes parametrically the Reaction Curve equations at interior solutions become:

(10)
$$\frac{F_1}{F_2^m} = \frac{m(E_1+E_2)}{F_1^m+F_2^m}$$
$$\frac{F_2}{F_1^m} = \frac{m(E_1+E_2)}{F_1^m+F_2^m}$$
Reaction Curves (general m, with $A = s = 1$)

And the condition for the critical ratio is:

(11) $$r^* = (2 + m)/m$$ Critical resource ratio

Figure 7 is analogous to Figure 6, except for a change in the mass effect parameter from $m = 1$ to $m = 2$. The critical resource ratio has fallen from 3 to 2, so that the range of corner solutions commences earlier. More significant, owing to the increased fighting efforts the aggregate income I is substantially lower throughout. Note that in the range of corner solutions where $F_1 > F_2$, the more affluent side does relatively better for larger m -- fighting having become more decisive. And indeed, for a sufficiently extreme resource ratio, the wealthier contender can end up absolutely better off.

Numerical Example 4: In Numerical Example 3, with $m = 1$, for initial resources $R_1 = 400$, $R_2 = 100$ the solution was in the regime of corner equilibria (F_2=100). When $m = 3$ instead, both contenders will want to devote more effort to fighting. But side #2 is already at its limit, so F_2 remains equal to 100. However, #1 will raise its fighting effort to $F_1 = 155.1$, sharply increasing its fractional share to $p_1 = .789$. The attained incomes become $I_1 = 193.1$ and $I_2 = 51.7$. So here the increase in the mass effect parameter has made the more affluent side absolutely better off in comparison with the results for $m = 1$, despite the smaller social total of income.

Summarizing, as the mass effect parameter rises above $m = 1$: (i) The marginal product of fighting effort rises,[14] increasing the incentive to commit more resources to conflict, with the evident consequence of reduced social income in aggregate. (ii) At interior solutions, the two contenders still end up with equal incomes; the strong form of the PEP holds as before. (iii) However, with higher values of m the poorer-endowed side reaches its upper-bound constraint at a smaller resource disparity. In consequence, the strong form of the Power Equalization Principle does not hold over as wide a range. (iv) In the range of corner solutions, where the upper constraint on the poorer-endowed player is binding, as before the weak form of the PEP holds: that is, I_1/I_2 always exceeds R_1/R_2. (v) With increasing m, any given resource ratio translates into a larger p_1. So the more amply endowed side always gains relatively as m increases, and may even gain absolutely.

While the sweep of the Power Equalization Principle is therefore somewhat attenuated as the mass effect parameter of fighting increases, it remains true that resource disparities are always converted only in a diluted way into power differences. The underlying reason remains the same: <u>the poorer side is always motivated to invest relatively more heavily in fighting effort</u>.

What are the circumstances that make fighting more decisive? According to Lanchester [1916], under "ancient" conditions fighting was man-for-man, so the larger army could not bring all its forces to bear -- hence military strength was linear in the forces committed ($m=1$). Under "modern" conditions, long-range weapons allow concentration of fire, making strength proportional to the square of the forces engaged ($m=2$). Turning to political contests within nations, majority control of the executive government machinery is the less decisive to the extent that checks and balances, civil rights, and other constitutional provisions limit the ability to use that machinery. Super-majority provisions like those required for amending the U.S. constitution would seem to be a deliberate attempt to make majority control less decisive, and hence to limit the intensity of the domestic political struggle.

V. WHEN PRODUCTIVE TECHNOLOGY CHANGES

Turning now to the technology of production, the Aggregate Production Function (APF) of equation (2) has two parameters: a multiplicative <u>efficiency index</u> A and a <u>productive complementarity</u> index s.

The forces at work are immediately revealed by setting down the generalized Reaction Curves:[15]

$$\frac{F_1}{F_2} E_1^{(1-s)/s} = \frac{E_1^{1/s}+E_2^{1/2}}{F_1+F_2}$$

(12) Reaction Curves (general A and s, with $m = 1$)

$$\frac{F_2}{F_1} E_2^{(1-s)/s} = \frac{E_1^{1/s}+E_2^{1/s}}{F_1+F_2}$$

Since A does not appear in the Reaction Curves at all, a change in the efficiency index has no effect whatsoever upon the chosen fighting efforts F_1 and F_2. (Reason: an increase in A raises the marginal products of E_1 and of F_1 in exactly the same proportion.) On the other hand, it will be evident, in a diagram like Figure 6a the income levels I_1, I_2, and I would all rise proportionately with the efficiency index A.

Turning to the productive complementarity index, as s rises the total product (aggregate income I) will also increase, for given productive resource commitments E_1 and E_2. Also, it is easy to verify, for either side the marginal product of productive effort E_1 will rise in comparison with the marginal product of fighting effort F_1. Thus, we would be inclined to anticipate "an era of better feeling" in which each side redirects its activities so as to devote more effort to increasing the size of the pie, and less to grabbing a bigger slice thereof. However, this is one of those cases where somewhat counterintuitive results are obtained when the <u>interaction</u> of the two sides' decisions is taken into account.

Figure 8 illustrates the effect when the productive complementarity index rises from 1 to 1.5 -- the other parameters m and A being held at unity.[16] Since the poorer side is now motivated to shift somewhat away from fighting and toward production, we would expect the critical resource ratio separating the ranges of interior and corner solutions to rise. Actually, the form of equations (12) indicates, more specifically, that there can only be interior solutions when $s > 1$. In these interior solutions the distributive shares p_1 and p_2 and therefore also the incomes I_1 and I_2 do now come to favor the better-endowed side. When there is productive complementarity the wealthier contender can tilt a bit more toward fighting, since his poorer opponent will be devoting more effort to production. Nevertheless, the effect is not so great as to overcome the Power Equalization Principle in its weak form: it remains true that $I_1/I_2 < R_1/R_2$.

<u>Numerical Example 5</u>: In Numerical Example 3 with initial resources $R_1 = 400$, $R_2 = 100$ and $s = 1$, the solution was in the range of corner equilibria. Holding all conditions the same except for an increase in s to 1.5, the fighting efforts become $F_1 = 125.6$ and $F_2 = 80.7$. The proportionate share of the better-endowed side rises to $p_1 = .609$, the attained incomes rising to $I_1 = 211.5$ and $I_2 = 135.9$.

VI. DISCUSSION AND SUMMARY

There are many kinds of conflict. The models here deal with struggles involving an element of cooperation, in that the productive efforts on each side generate a common pool of income. The two sides have a joint interest in making the pool as large as possible, despite their discordant interests over how to divide it between them.

Management and labor, for example, ordinarily cooperate in production even while struggling over the factor shares. Much the same can be said for contests among members of families, clubs, or committees united by a common goal. Looked at in another way, the analysis applies when the _resource bases_ of the competitors are not in contention, only the distribution of _income_ being at issue. Thus, within the firm, workers do not ordinarily try to seize the factory or machinery, nor does management aim at enslaving the employees. (Although under abnormal or revolutionary conditions such escalation may occur.)

In interactions among nations, once again there are cooperative aspects: the advantages of trade, and the mutual incentive to avoid destructive warfare. But of course nations do often engage in escalated struggles over resources that do not square so well with the model here. Domestic politics of the familiar type represent a somewhat better fit. Such contests mainly take the form of struggles over the distribution of income; massive seizures of property are not usually on the agenda. (Once again, however, under revolutionary circumstances such constitutional constraints may no longer apply.)

Methodologically, the model here demonstrated the interacting roles of the _technology of production_ and the _technology of conflict_ in determining the outcome of a steady-state process in which contenders divide their efforts between productive activity and appropriative struggle. Substantively, the key result -- holding under the Cournot solution protocol -- was the Power Equalization Principle (PEP). When the strong form of the PEP applies, in equilibrium the antagonists' terminal incomes are exactly equal despite initial resource disparities. When only the weak form holds, it remains true that the distribution of achieved income is more equal than the initial distribution of resources. At least in its weak form, the PEP proved to be a remarkably robust generalization, applying under a wide range of assumptions about the technology of production and the technology of conflict. The underlying explanation derives from a comparison of the marginal products of productive versus conflictual activities, which reveals that the less well-endowed side has a comparative advantage in fighting, the richer side in producing. Appropriative effort allows you to place a tax upon your opponent's productive effort, and it is more profitable to tax a rich opponent than a poor one. Hence the conflict process, while of course it dissipates income in aggregate, also tends to bring about a more equal distribution of whatever income remains.

As in all attempts to model complex phenomena, a variety of simplifying assumptions had to be employed in this analysis. To comment further on a few of these: (i) Only two-party interactions were examined, ruling out issues like alliances and the balance of power.[17] (ii) Full information was assumed throughout, so that factors like deception have been set aside.[18] (iii) The simplified mathematical form of the Contest Success Function does not allow for differences between offensive and defensive weapons, between ground and naval forces, between battle-seeking and Fabian tactics, and so on. For the economist, special interest attaches to the distinction between capital-intensive and labor-intensive modes of warfare (Stockfisch [1976]).

(iv) The steady-state assumption rules out issues involving timing, such as arms races, economic growth, or (on a smaller time-scale) signalling resolve through escalation. (v) In the model here, all income fell into a common pool available for capture. More generally, each side might have some income secure from capture, and in fact would be making an optimizing choice between resources devoted in that way versus resources generating income in a common pool. (vi) The underlying resources on each side were assumed invulnerable to seizure, and in addition, fighting was assumed non-destructive (apart from the opportunity cost in the form of foregone production). (vii) The effects of distance and other geographical factors were not considered. (viii) Only Cournot solutions were obtained here. Among the important alternative possibilities are the Stackelberg condition (one side has the first move) or else protocols under which one or the other contender can issue a credible threat so as to influence the opponent's action.[19]

Even when generalized in the various ways indicated by the list above, the steady-state model will remain inappropriate for the analysis of conflicts dominated by a single overwhelming or irreversible event like a thermonuclear attack. In the military domain it is more applicable to "cold wars" or to continuing low-level struggles -- such as those taking place among the small states of pre-imperial China (as described in Sun Tze's The Art of War), or among colonial powers contesting for territories. Outside the military domain, the model is particularly relevant for analyzing the ongoing cooperative-conflictual processes observed in labor-management relations, in politics, and within families.

I will expand briefly on only one application, political redistribution of income. In modern polities, at least, redistribution is overwhelmingly from the rich to the poor. This might seem surprising. After all, starting from their initial resource advantage the rich could, it appears, make themselves richer still by appropriating what others have produced. Or looking at this another way, why doesn't an initially poor group, if able to redistribute income toward itself, go on further so as to end up changing places with the rich?

Among the reasons that might be offered are: (1) what makes such redistributions politically possible are altruism[20] on the one side and envy on the other -- both of which are moderated as the losers and gainers approach equality; or (2) the deadweight costs of the redistributive process set limits upon whatever the poor might hope to attain thereby.[21] Without necessarily denying some validity to these reasons, the analysis here suggests another explanation. The Power Equalization Principle (PEP) indicates that, for the less well-endowed side, the marginal product of conflictual effort aimed at taxing the remainder of society tends to be greater than the marginal product of directly productive effort. So popularist politics will always be profitable for the poor.

Finally, it follows more generally that any group -- whether initially rich or poor -- suffering an exogenous impoverishment will predictably shift its energies from the productive toward the conflictual end of the activity spectrum. City-dwellers in India riot when bus fares go up, disaster-impacted areas demand assistance from the rest of the community, and depressed industries enter the political arena to achieve tariff protection. So the observed tendency of big-brother governments to lean against the wind,[22] to shelter the shorn lambs, is largely a result of pressure groups' changing marginal products of productive effort versus distributive struggle. When agricultural prices fall, Kansas farmers will find it advantageous "to raise less corn and raise more hell."[23]

APPENDIX

This Appendix summarizes the equations underlying the curves in the diagrams.

Figure 1: $I = A(E_1^{1/s} + E_2^{1/s})^s$, for $I=A=1$ and $s = 1, 1.5, 2$

Figure 2: $p_1 = F_1^m/(F_1^m + F_2^m)$, for $m = 0.5, 1, 3$ and $F_2 = 1$

Figure 3: Plot is suggestive only.

Figure 4: $F_1/F_2 = (R_1-F_1+R_2-F_2)/(F_1+F_2)$ [RC_1, interior range]

$F_2/F_1 = (R_1-F_1+R_2-F_2)/(F_1 + F_2)$ [RC_2, interior range]

Figure 5: $F_1/F_2^m = m(R_1-F_1+R_2-F_2)/(F_1^m + F_2^m)$ [RC_1, interior range]

$F_2/F_1^m = m(R_1-F_1+R_2-F_2)/(F_1^m + F_2^m)$ [RC_2, interior range]

for $m = 0.5, 1, 3$

Figure 6: [$R_2 = 1$ throughout]

$\bar{I} = R_1 + R_2$

$I = R_1 - F_1 + R_2 - F_2$

$I_1 = p_1 I$ and $I_2 = p_2 I$, where:

In range of interior solutions ($R_1/R_2 \leq 3$):

$F_1 = F_2 = (R_1 + R_2)/4$

$p_1 = p_2 = 1/2$

In range of corner solutions ($R_1/R_2 > 3$):

$F_1 = \text{sqrt}[R_2(R_1 + R_2)] - R_2$

$F_2 = R_2$

$p_1 = F_1/(F_1 + R_2)$

$p_2 = R_2/(F_1 + R_2)$

Figure 7: [$R_2 = 1$ throughout]

$\bar{I} = R_1 + R_2$

$I = R_1 - F_1 + R_2 - F_2$

$I_1 = p_1 I$ and $I_2 = p_2 I$, where:

In range of interior solutions ($R_1/R_2 \le 2$):

$$F_1 = F_2 = (R_1 + R_2)/3$$

$$p_1 = p_2 = 1/2$$

In range of corner solutions ($R_1/R_2 > 2$):

$$F_1 = \frac{(2R_2)^{2/3}(\mathrm{sqrt}[R_1^2+4R_2^2]+R_1)^{2/3} - (2R_2)^{2/3}}{2(\mathrm{sqrt}[R_1^2+4R_2^2)+R_1)^{1/3}}$$

$$F_2 = R_2$$

$$p_1 = F_1^2/(F_1^2 + R_2^2) \quad \text{and} \quad p_2 = R_2^2/(F_1^2 + R_2^2)$$

<u>Figure 8</u>: [$R_2 = 1$ and $s = 1.5$ throughout.]

All solutions are in the interior range.

$$\bar{I} = (R_1^{1/s} + R_2^{1/s})^s$$

I, I_1, and I_2 were calculated numerically over the range $1 \le R_1/R_2 \le 6$:

$$I = [(R_1 - F_1)^{1/s} + (R_2 - F_2)^{1/s}]^s$$

$I_1 = p_1 I$ and $I_2 = p_2 I$, where:

F_1 and F_2 were computed using the Reaction Curve equations (12) of the text, and as before:

$$p_1 = F_1/(F_1 + F_2) \quad \text{and} \quad p_2 = F_2/(F_1 + F_2)$$

Endnotes

[1]An earlier version of this paper was titled The Dimensions of Power as Illustrated by a Steady-state Model of Conflict, The RAND Corporation, N-2889-PCT (July 1989). For helpful assistance and comments I thank Malcolm Fisher, G. Hildebrandt, D. A. Hirshleifer, J. A. Stockfisch, and K. Terasawa. This research was supported by the Pew Charitable Trusts.

[2]"The efforts of men are utilized in two different ways: they are directed to the production or transformation of economic goods, or else to the appropriation of goods produced by others." -- Vilfredo Pareto.

[3]To some extent this distinction is one of perspective. At any single moment, members of a union are either on strike or at work. Adopting the long view, however, we can regard the union as having adopted a strategy of striking a certain proportion of the time. Either/or choices between war and peace, as polar alternatives, are analyzed in, for example, Wittman [1979] and Bueno de Mesquita [1981].

[4]A famous work propounding this thesis was Angell [1911].

[5]For example, Mueller [1989].

[6]The equations underlying the plotted diagrams are summarized in the Appendix to the paper.

[7]There are other significantly different ways of formulating the Contest Success Function. I have explored elsewhere some of the implications of making fighting success a function of the numerical difference

between the magnitude of commitments (Hirshleifer [1988, 1989]). Another approach is to think in terms of a winner-take-all contest, where it is only the rank order of the commitments that counts (see, for example, Hillman and Riley [1988]).

[8] In the empire-building period between 1500 and 1800, the main contenders -- England, France, and Spain -- had essentially invulnerable home bases. The struggle was over the contestable income derivable from the new territories of Asia and the Americas. (A fourth contender, Holland, was eventually reduced to minor status owing mainly to the vulnerability of its home base to military occupation.) With the increasing geographical scope of military technology, all nations' zones of invulnerability have drastically shrunk.

[9] The Cournot equilibrium concept has been used by Brock and Magee (1978) and Becker (1983) to analyze pressure-group politics, a special case of conflict as defined here.

[10] Decision-maker #1 maximizes $I_1 = p_1(F_1|F_2) \times I(E_1|E_2)$ subject to the constraint $E_1 + F_1 = R_1$, with the opponent's E_2 and F_2 taken as constant, and similarly for decision-maker #2. Forming the Lagrangian and differentiating leads directly to the equations in the text.

[11] The Reaction Curves also intersect at $F_1 = F_2 = 0$, but the zero-zero intersection is not a Cournot equilibrium. Owing to the ratio form of the CSF, p_1 and p_2 are indeterminate when $F_1 = F_2 = 0$. It might at first seem that, at the origin, $p_1 = p_2 = 1/2$ -- since that is the value approached as F_1 and F_2 go to zero together. But if (say) player #1 chooses $F_1 = 0$, then player #2 would rationally respond by setting F_2 equal to any

small positive magnitude. (Since doing so discontinuously improves his fighting success from 50% to 100%.) It follows, therefore, that the respective Reaction Curves are defined only over the open interval that does <u>not</u> include the singular point at the origin.

[12] Becker obtains a somewhat analogous proposition: "Politically successful groups tend to be small relative to the size of the groups taxed to pay their subsidies" (1983, p. 385).

[13] From (7) we have seen that, in an interior solution, $F_1 = F_2 = (R_1+R_2)/4$. Inserting the corner condition that $F_2 = R_2$ determines the critical ratio: $R_1/R_2 = r* = 3$.

[14] Generalizing equations (9) for any m, the marginal products for player #2 become:

$$\partial Y_2/\partial E_2 = F_2^m/F_1^m+F_2^m \quad \text{and} \quad \partial Y_2/\partial F_2 = (E_1+E_2)\ mF_1^m F_2^{m-1}/(F_1^m+F_2^m)^2$$

Note that for $m > 1$ there will be an initial range of increasing returns in $\partial Y_2/\partial F_2$, the marginal product of fighting effort. This is reflected in the shapes of the CSF curves of Figure 2.

[15] Obtained by reducing the system of equations (1), (2), (3), and (4) for $m = 1$.

[16] For $s > 1$, the equations for I_1 and I_2 do not reduce to convenient closed forms. The curves in the diagrams were computed numerically.

[17] Of the vast literature on these questions, I shall cite here only Blainey (1973) and Bernholz (1985).

[18]See, for example, Tullock (1974, Ch. 10) and Brams (1977).

[19]See Hirshleifer (1988).

[20]See, for example, Roberts [1984].

[21]This factor is emphasized by Becker [1983].

[22]On this see Peltzman [1976] and Hirshleifer [1976].

[23]Mary Lease (populist agitator in 1920s and 1930s).

REFERENCES

Angell, Norman, The Great Illusion, G.P. Putnam's Sons, New York (1911).

Becker, Gary S., "A theory of competition among pressure groups for political influence," Quarterly Journal of Economics, v. 98, pp. 370-400 (August 1983).

Bernholz, Peter, The International Game of Power. Mouton Publishers, Berlin (1985).

Blainey, Geoffrey, The Causes of War. The Free Press, New York (1973).

Brams, Steven J. and Donald Wittman, "Nonmyopic equilibria in 2x2 games," Conflict Management and Peace Science, v. 6 (Fall 1981).

Brock, William A. and Stephen P. Magee, "The economics of special interest politics: The case of the tariff," American Economic Review, v. 68, pp. 246-250 (1978).

Bueno de Mesquita, Bruce, The War Trap, Yale Univ. Press, New Haven (1981).

Hillman, Arye L. and John G. Riley, "Politically contestable rents and transfers," UCLA Dept. of Economics Working Paper 451 (rev. March 1988).

Hirshleifer, J., "Toward a more general theory of regulation: Comment," Journal of Law and Economics, v. 19, pp. 241-244 (1976).

__________, "The analytics of continuing conflict," Synthese, v. 76, pp. 201-233 (1988).

__________, "Conflict and rent-seeking success functions: Ratio vs. difference models of relative success," Public Choice, v. 63, pp. 101-112 (1989).

Lanchester, Frederick William, Aircraft in Warfare: The Dawn of the Fourth Arm. Constable (1916). Extract reprinted in James R. Newman, ed., The World of Mathematics, v. 4, pp. 2138-2157. New York, Simon and Schuster (1956).

Mueller, John, Retreat From Doomsday: The Obsolescence of Major War, Basic Books, New York (1989).

Peltzman, Sam, "Toward a more general theory of regulation," Journal of Law and Economics, v. 19, pp. 211-240 (1976).

Roberts, Russell D., "A positive model of private charity and public transfers," Journal of Political Economy, v. 92, pp. 136-148 (Feb. 1984).

Stockfisch, J.A., "The capital-intensive military production process and perverse incentives," in Peter de Leon and James Digby, Workshop on Asymmetries in Exploiting Technology as Related to the U.S.-Soviet Competition: Unclassified Supporting Papers, The Rand Corporation Report R-2061/1 (Nov. 1976).

Wittman, Donald, "How a war ends," Journal of Conflict Resolution, v. 23, pp. 743-763 (1979).

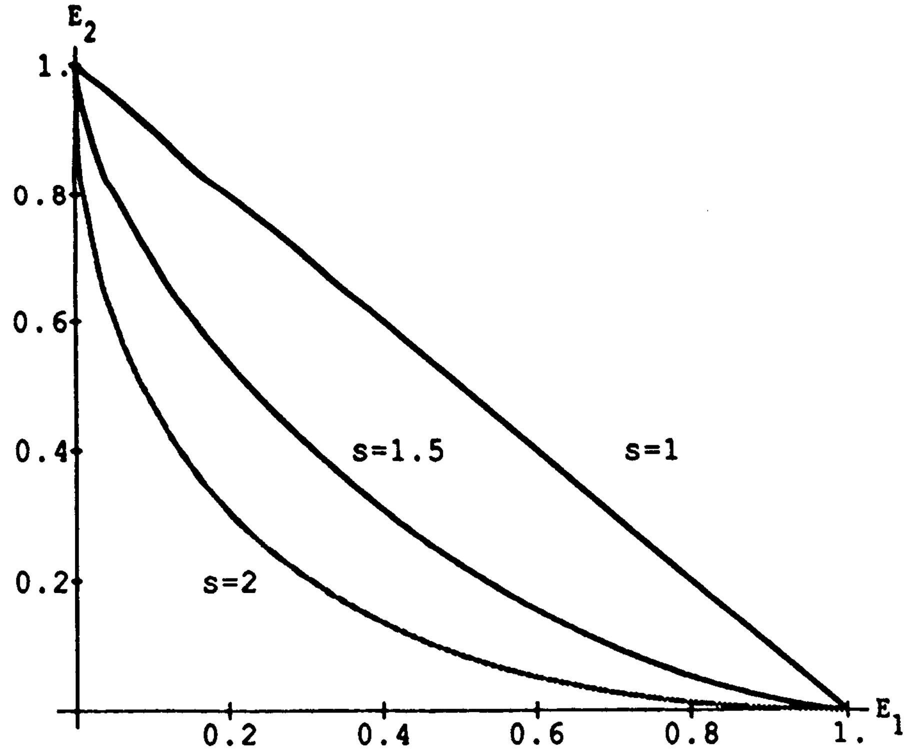

Fig.1 — Unit isoquant as complementarity parameter *s* varies

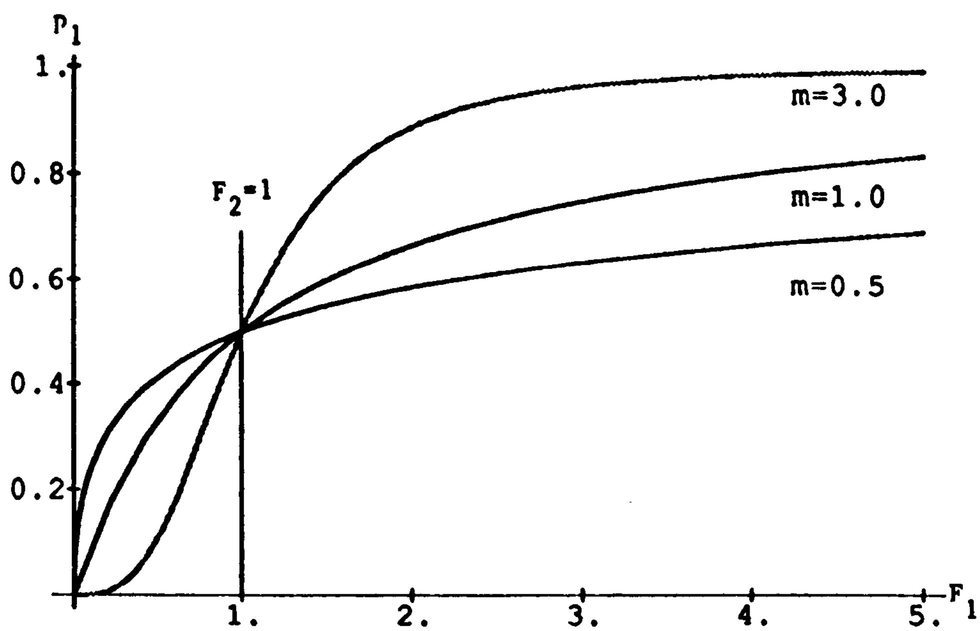

Fig.2 — Contest Success Function (CSF) as mass effect parameter *m* varies

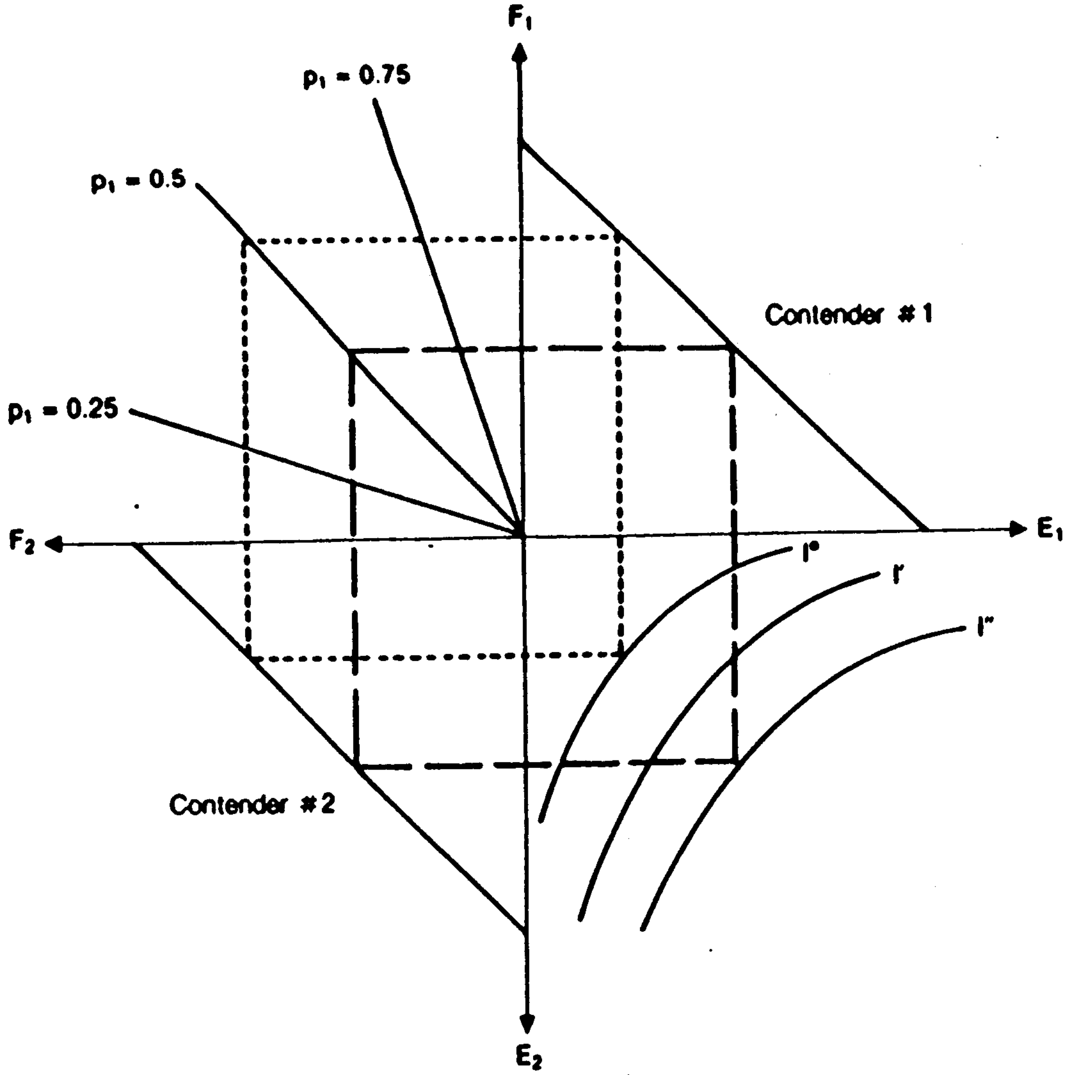

Fig.3 — Productive technology determines income I and conflict technology determines fractional division p_1

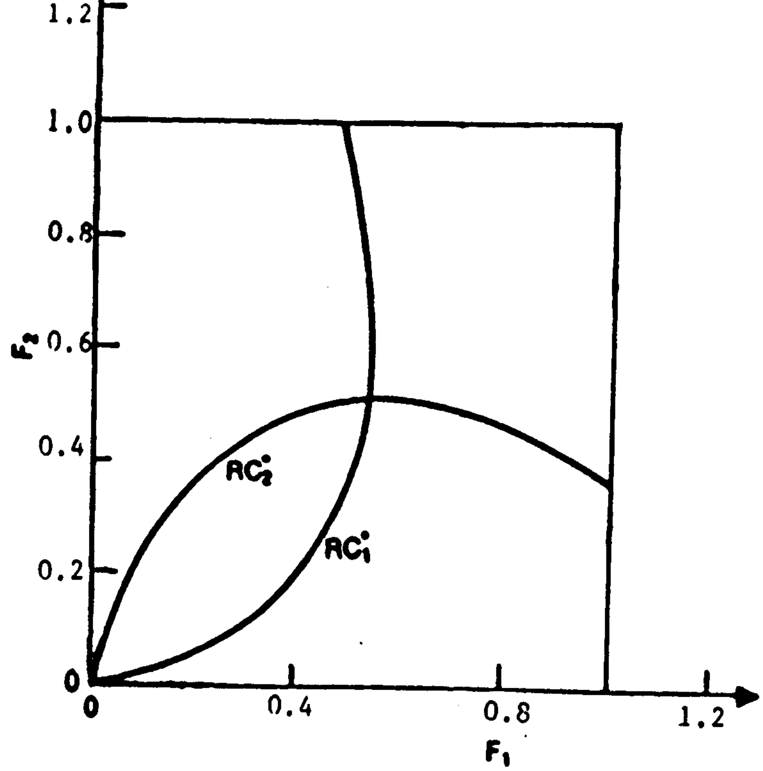

Fig.4 — Reaction Curves and Cournot equilibrium – basic solution

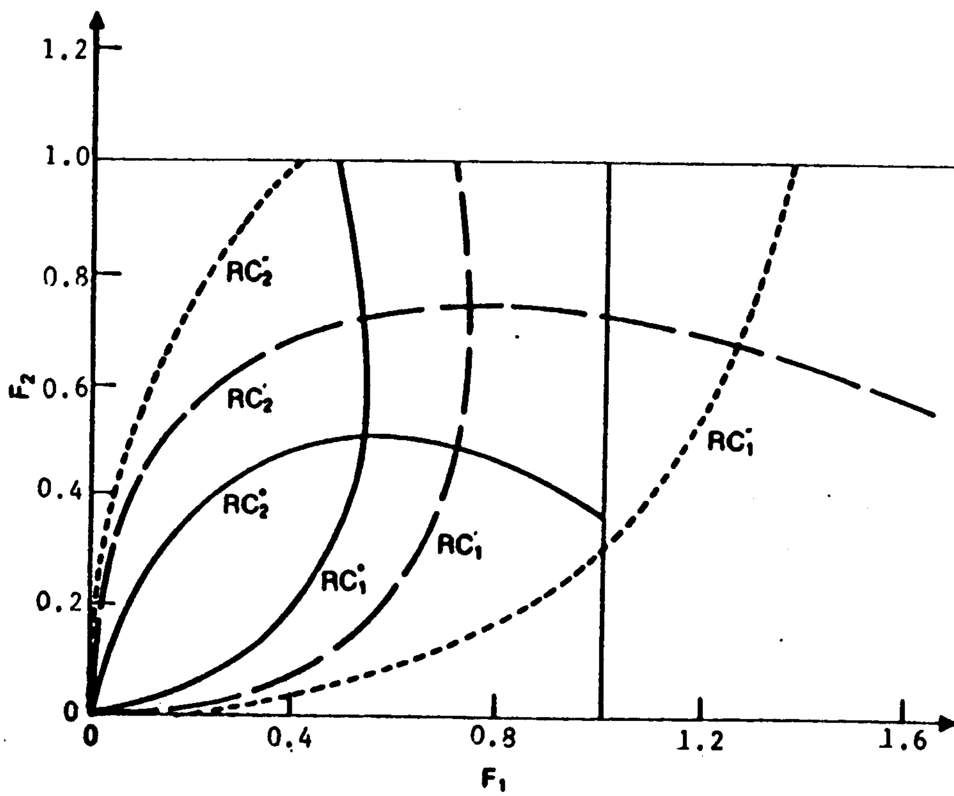

Fig.5 — Reaction Curves and Cournot equilibrium: Interior and corner solutions as resource ratio varies

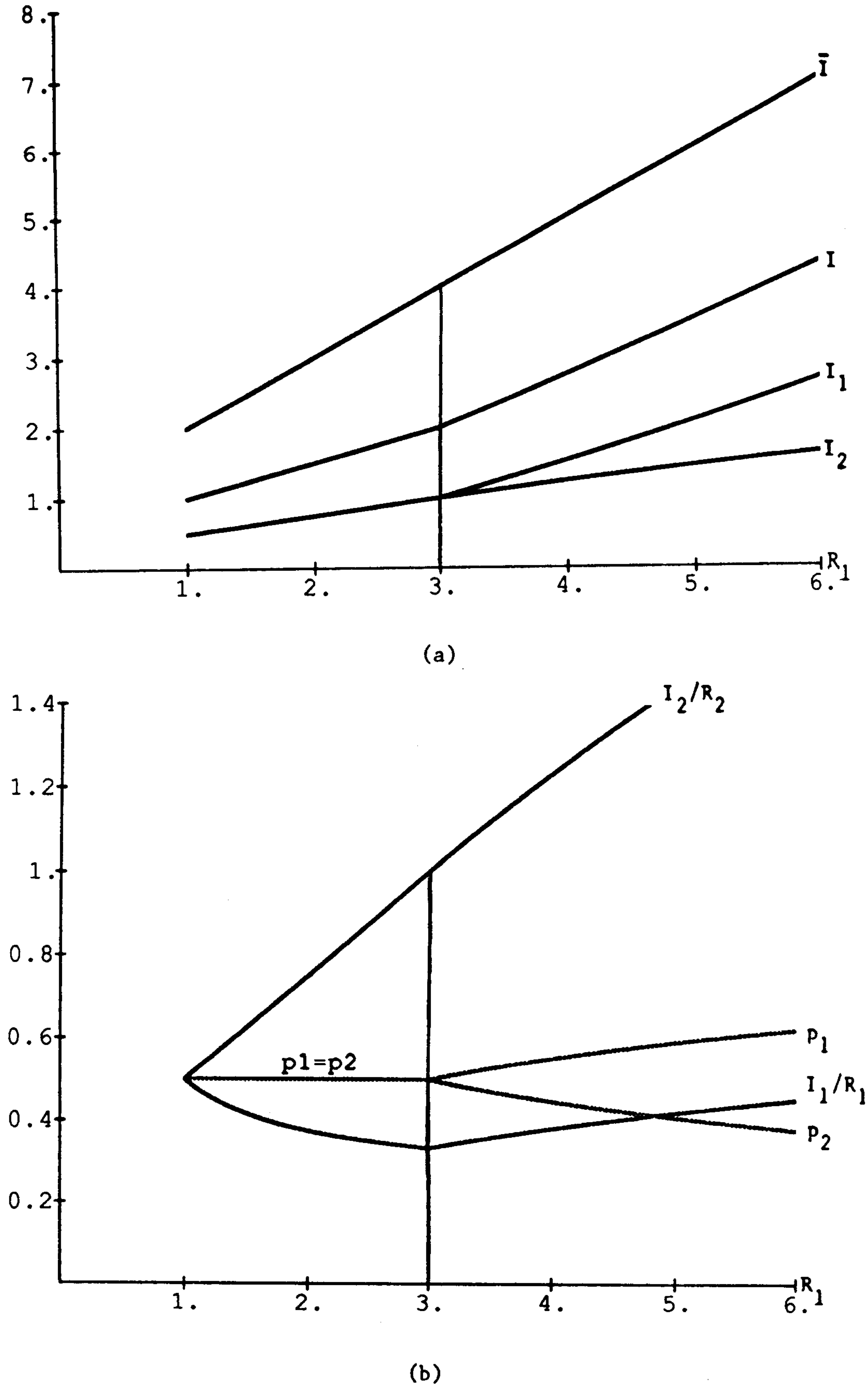

Fig.6 — Incomes, fractional divisions, and conversion ratios – base case

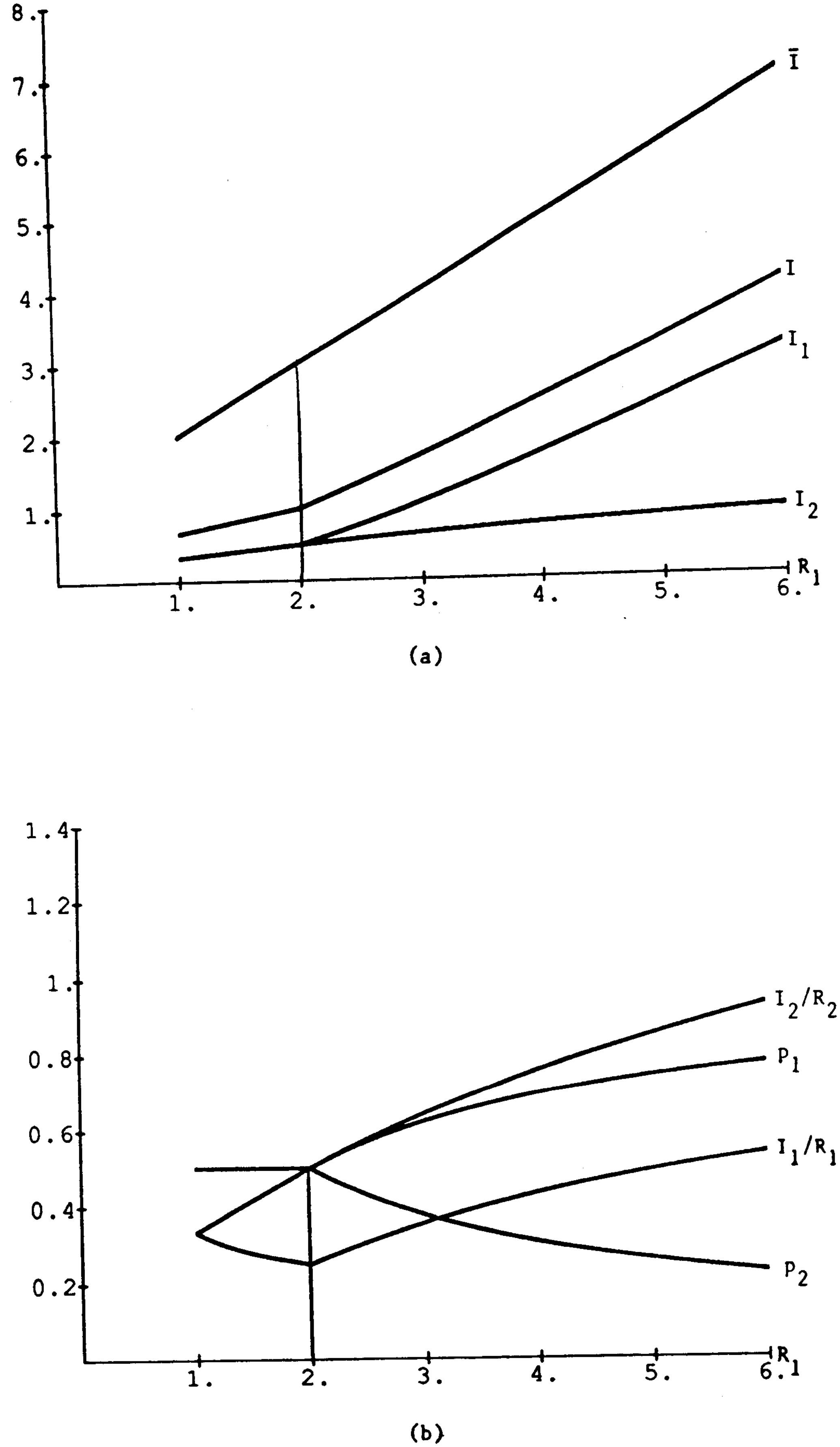

Fig.7 — Incomes, fractional divisions, and conversion ratios – when fighting becomes more decisive ($m = 2$)

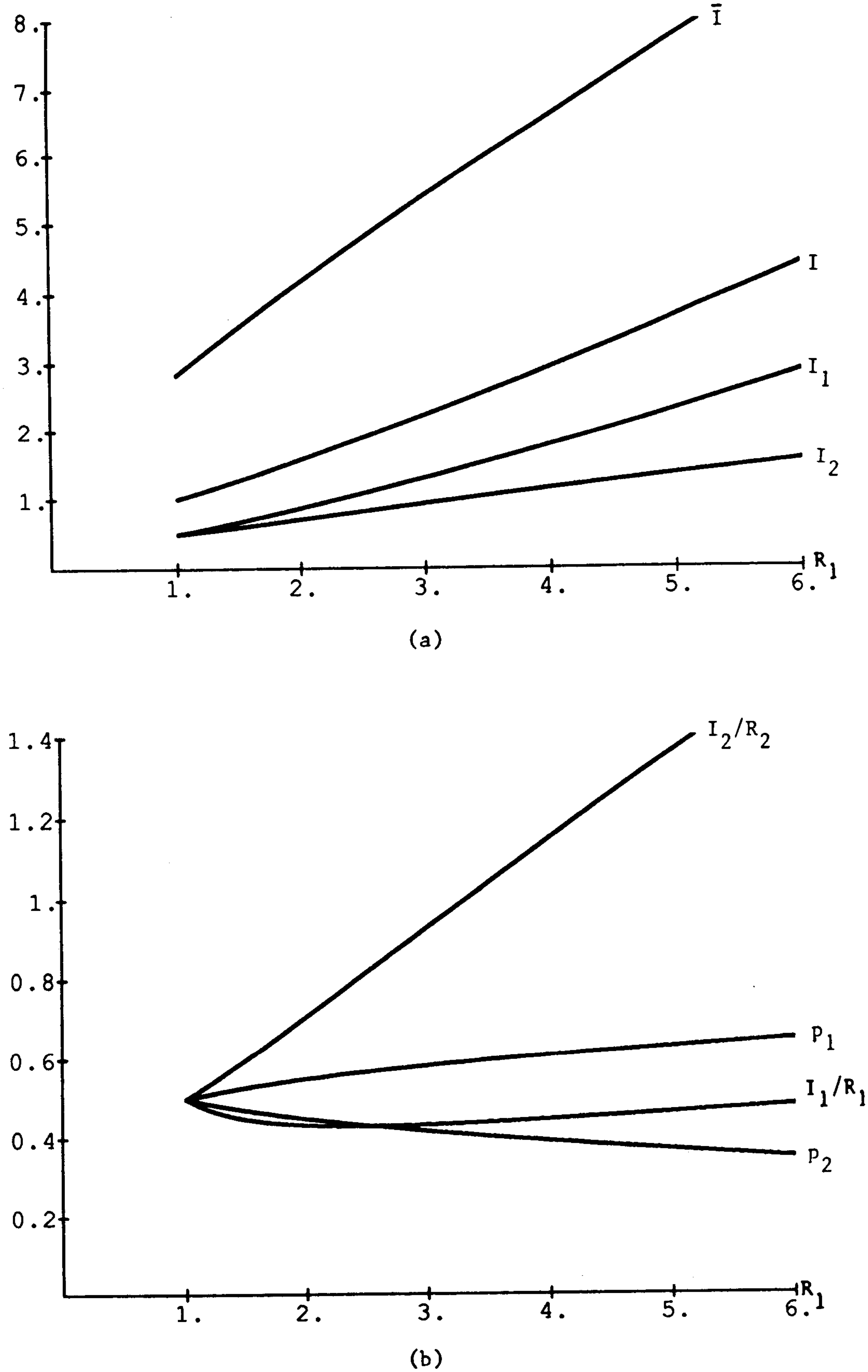

Fig.8 — Incomes, fractional divisions, and conversion ratios – with productive complementarity ($s = 1.5$)

Straight Time and Overtime in Equilibrium*

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