STICKY PRICES*

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ABSTRACT

The real world is characterized by sticky prices in the sense that prices do not respond rapidly to innovations in other variables. Many authors have tried to explain this fact by constructing model economies that contain artificial barriers to price flexibility. For some time we have known of models where prices are perfectly free to move that contain an equilibrium in which they do not. Previously known examples that exhibit this feature have generally been dismissed as serious explanations of business cycle phenomena because they make counterfactual predictions. The contribution of this paper is to present a simple example of an economy in which prices are 'sticky', in the above sense, that mimics many of the features that characterize business cycles.

1 Introduction

It is a fact that prices are 'sticky'. By this I mean that the impulse response functions that are predicted to characterize the data, if one believes in representative agent market clearing models, do not do a very good job of explaining the real world. In the literature the term sticky prices is usually applied to economic models that exhibit some kind of friction or barrier to the smooth adjustment of prices although this is not the only possible usage of the term. Whatever explanation we choose to adopt for the facts that we observe, I believe that Sims' results on estimated VAR's summarize quite succinctly just what it is we are trying to explain.[1]

Sims takes a small set of macroeconomic time series that includes the variables that one would normally talk about in an intermediate course on macroeconomics. He compares the results of simulating a representative agent economy, in the style of Kydland and Prescott, with an impulse response function that he estimates from post-war U.S. time-series data. Sims finds that:

In the simulated data, price responds sharply to every kind of innovation; in the actual data it responds much more slowly and weakly to interest rates and money, and not at all to real output.

For some time we have known of examples of market clearing models with the property that prices each period are set in advance. In models of this kind prices are 'sticky' in the sense that they do not respond to contemporaneous disturbances, but there is no artificial impediment to prevent them from freely adjusting to clear markets. The first work that systematically explores this idea is by Azariadis.[2] The Azariadis example is interesting but it is not a serious contender as a model of the business cycle. Although the model does contain a stationary rational expectations

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[1]"Models and their Uses" [6].

[2]My first introduction to this concept was in 1982 when Costas Azariadis pointed out to me that the well known Lucas 'island model' contained non-stationary equilibria in which prices are predetermined. Azariadis [1] provides a stripped down version of the Lucas model that is explored in more depth in a set of unpublished lecture notes [2].

equilibrium with prices set one period in advance[3] this equilibrium is associated with counterfactual comovements between fiscal deficits and inflation; that is, a monetized deficit is predicted to lower the inflation rate. For this reason, the implications of predetermined price equilibria in equilibrium models have not been taken seriously by many economists.

In this paper I explore a related model that does not suffer from the above defect. I shall argue that equilibrium models with an indeterminate steady state deserve to be taken seriously as models of the business cycle *because* they are capable of explaining 'sticky prices' in a rather elegant way.

2   Flexible Price Economies May Display "Price Stickiness"

The idea is to construct a model economy where the price of money is determined one period in advance. In response to a monetary disturbance, quantities adjust in the short run and prices respond assymptotically. These features characterize the Keynesian economies that we teach in our intermediate textbooks but the theory that underlies the textbook approach does not explain why there may be nominal rigidities. Actors in the textbook approach do not behave in ways that are easy to understand with the theory of rational choice.

In simple overlapping generations economies there are equilibria with predetermined prices in which all agents respond rationally to market signals. All markets clear and all prices are equilibrium prices. The key to these equilibria is that it is possible, in a world in which agents care about the future, for *future* prices to clear *current* markets. The more familiar examples of these models are thought by some critics to be curiosities because the examples that we know of contain anomalous features. For example; the Cagan hyperinflation model that I focus on in my paper "The Lucas Critique, Policy Invariance and Multiple Equilibria" [4] has an equilibrium with predetermined prices but it also happens, in this equilibrium, that increases in government expenditure cause reductions in the steady state inflation

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[3]This equilibrium is discussed in Farmer and Woodford [5] and it forms the basis for my discussion of the implications of indeterminacy for the Lucas Critique [4].

rate. This feature is not a general property of predetermined price equilibria as the following model illustrates.

3 A Simple Perfect Foresight Example

In examples like the Cagan model there are two stationary equilibria. One of these equilibria is indeterminate; but the indeterminate equilibrium is not particularly interesting because it is associated with an anomalous response of inflation to increases in government expenditure. This response is generated by a mechanism in which output fluctuates over the business cycle because *labor supply* moves in response to intertemporal prices.[4]

The model in this paper shifts the burden of output response in the face of intertemporal price fluctuations from labor supply to labor demand. The main effect of this modification is to alter the stability of the monetary and non-monetary equilibria whilst preserving the comparative static predictions of the effect of government spending on inflation at the *monetary* steady state. The model has a monetary steady state which is locally indeterminate; a property which is important in allowing one to construct equilibria that mimic 'Keynesian' impulse response functions. Unlike previously known examples, the inflation rate in this equilibrium will increase if the government engages in a money financed fiscal expansion.

The economy has a single nonstorable commodity y_t that is produced using the following technology:

$$(1) \qquad\qquad y_t = \alpha n_{t-1} + \phi n_t, \quad \alpha > 0, \quad \phi > 0,$$

where n_t represents labor input. Notice that outputs in periods t-1 and t are joint products. Other technologies will deliver similar results to those reported below although this is the simplest that I have found which generates an equilibrium with the properties that I will describe. The time delay in production is needed

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[4]I have in mind versions of Cagan's [3] model which are generated from simple overlapping generations economies such as that discussed in my joint paper with Michael Woodford "Self-fulfilling Prophecies and the Business Cycle."

to make the demand for labor sensitive to intertemporal prices. The inclusion of current output in the technology is important since it allows quantities to respond contemporaneously to demand shocks.[5]

Agents in this economy live for two periods and have perfect foresight of future prices. They work when young but consume in both periods of life. The opportunity set of an individual in the generation that is born in period t is represented by a pair of lifecycle budget constraints:[6]

$$(2) \qquad \omega_t(n_t^s - n_t^d) + \phi n_t^d - c_t^{yd} - m_t^d \geq 0,$$

$$(3) \qquad \alpha n_t^d + m_t/\pi_{t+1} - c_{t+1}^{od} \geq 0.$$

Putting these two inequalities together, one arrives at the lifecycle constraint of a representative individual:

$$(4) \qquad \omega_t(n_t^s - n_t^d) + \phi n_t^d + \alpha n_t^d \pi_{t+1} \geq c_t^{yd} + \pi_{t+1} c_{t+1}^{od}.$$

In the above inequalities, ω_t is the real wage, n_t^s is labor supply, n_t^d is labor demand, c_t^{yd} is consumption demand of the young, m_t^d is the real demand for money, p_t is the price of goods in terms of money and $\pi_t \equiv p_t/p_{t-1}$ is the inflation factor between periods t-1 and t.

Rather than model individual utility functions directly I assume that aggregate demand functions in period t are given by the folowing expressions:

$$(5) \qquad n_t^s = n^s(\omega_t),$$

$$(6) \qquad c_t^{yd} = c^d(\omega_t n_t^s),$$

$$(7) \qquad m_t^d = \omega_t n_t^s - \omega_t n_t^d - c_t^{yd} + \phi n_t^d,$$

$$(8) \qquad c_t^{od} = m_{t-1}/\pi_t + \alpha n_{t-1}^d,$$

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[5] In an equilibrium in which prices do not respond immediately there must be some way for *quantities* to increase contemporaneously to meet an increase in demand. I have found examples of economies where agents store inventories of goods and where one uses the simpler technology: $y_t = \alpha n_{t-1}$, that display equilibria with similar properties to the example that I describe here. However, these examples lead to equilibria that are characterised by difference equations of order two or more.

[6] I have suppressed a symbol that would differentiate the individual agent from the aggregate in order to cut down on notation.

where the functions $n^s(\cdot)$ and $c^d(\cdot)$ are continuous, increasing and differentiable, and the first derivative of $c^d(\cdot)$ is between zero and one. These functions are continuous and they obey Walras law. It follows from the Debreu Sonnenschein theorem that there exists an economy with at most four consumers in each generation which generates these demands as the outcome of maximizing behavior on the part of rational individuals.[7]

Young agents in this economy must choose how much labor to supply, how much to consume when young and how much to consume when old. The aggregate demands and supplies that are generated by their rational choices are described above in equations 5, 6, 7 and 8. They must also choose whether to store their wealth between periods in the form of money or by setting up a firm and holding stocks of goods in process. If money is held as a store of wealth in equilibrium then holding money must appear equally attractive as demanding labor and turning it into inventories. Since the lifecycle budget constraint is linear in labor demand it follows that in a monetary equilibrium:

(9) $$\omega_t = \phi + \alpha\pi_{t+1}.$$

Notice from equation 9 that expected inflation is positively related to the real wage and from equation 5 that the real wage is positively related to labor supply. These features allow one to find a stationary state in which increases in government spending lead to increases in inflation, real wages and employment. In more familiar versions of this model real wages and inflation are inversely related and as a result many comparative static results are of the opposite sign from those that hold in my example.

To close the model I assume that real government spending is financed by printing money and that there is no alternative outside asset. The government budget

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[7] The preferences $U = log(c_t - n_t^2) + log(c_{t+1})$ will generate demand functions which satisfy all of these conditions for an economy with a representative agent in each generation. I have avoided this example however because I will need an additional restriction which it violates. This restriction is discussed in footnote 8.
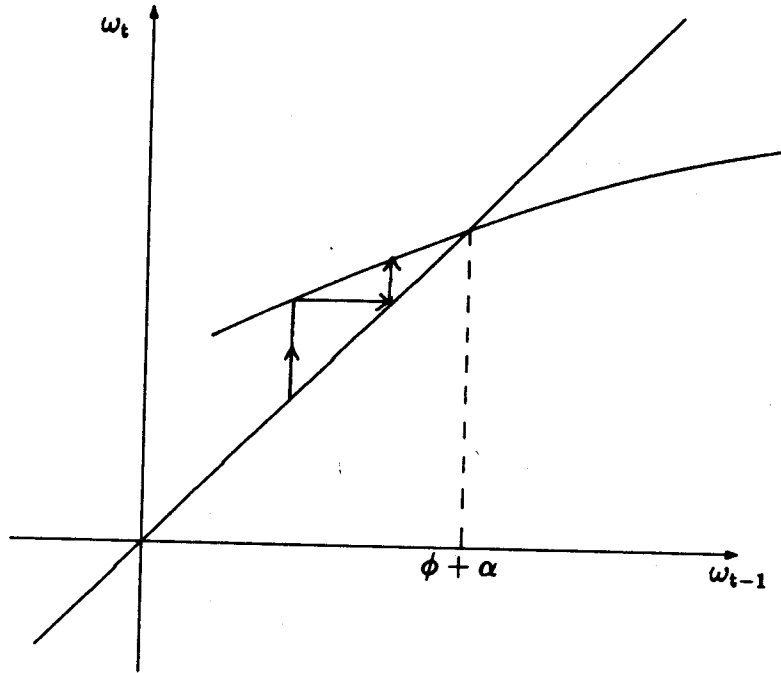
Figure 1: The Dynamics of the example

identity then implies that:

(10)
$$p_t/p_{t+1} = \frac{m_{t+1} - g}{m_t},$$

where g represents government spending and m_t is the supply of real balances. Substituting equations 5, 6 and 7 into equation 10 to write m_t in terms of ω_t and using equation 9 to write p_t/p_{t+1} in terms of ω_t one arrives at the following difference equation which characterizes equilibria for this economy:

(11)
$$\frac{\alpha}{\omega_t - \phi} = \frac{m(\omega_{t+1}) - g}{m(\omega_t)}.$$

When $g = 0$ the model has a steady state in which $\omega = \phi + \alpha$. This steady state will have positive valued fiat money if:

(12)
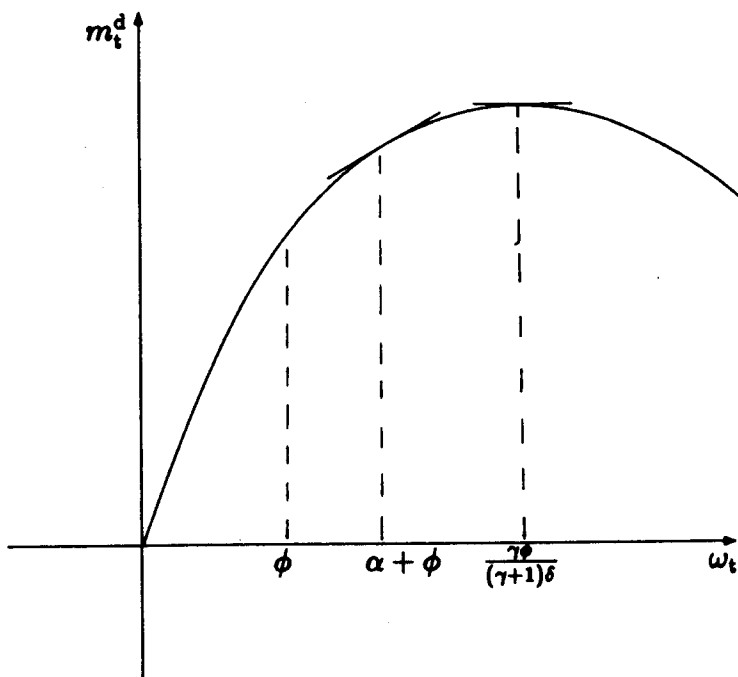$$\phi n^s(\phi + \alpha) > c^{yd}((\phi + \alpha)n^s(\phi + \alpha)).$$

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Figure 2: The Demand for Money as a Function of the Real Wage

An example of an economy that satisfies this inequality is provided by the following set of demand and supply functions:

$$(13) \qquad n_t^s \; = \; \omega_t^\gamma;$$

$$(14) \qquad c_t^{yd} \; = \; \delta n_t^s \omega_t;$$

for $\phi > \delta(\phi + \alpha)$. These functions imply that consumption and the demand for money can be described in terms of the real wage:

$$(15) \qquad c_t^{yd} \; = \; \delta \omega_t^{1+\gamma};$$

$$(16) \qquad m_t^d \; = \; \phi \omega_t^\gamma - \delta \omega_t^{1+\gamma};$$

where equation 16 follows from 7 after substitutiting in the labor market equilibrium condition, $n_t^s = n_t^d$. Equation 16 is depicted in figure 2.

Notice from figure 2 that I have chosen α in such a way that $\phi < \phi + \alpha < \frac{\phi\gamma}{\delta(\gamma+1)}$.[8]

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[8] This construction is not possible for the preferences discussed in footnote 7. There may well

Since the model has a monetary steady state at $\omega = \phi + \alpha$, and since the demand for money achieves a maximum at $\omega = \frac{\gamma\phi}{(1+\gamma)\delta}$, it follows for the above parameter configuration that the economy has a steady state at which the demand for money is both positive and increasing. These properties are important since they are sufficient conditions for one to demonstrate the existence of a monetary steady state at which an increase in the level of government spending generates an increase in the steady state inflation rate. It is also possible to demonstrate, if $0 < \frac{m}{\alpha m'} < 1$, that this steady state is indeterminate.

To find out how the model behaves close to the steady state one may linearize equation 11. Appealing to the implicit function theorem one can describe dynamic equilibria around this steady state by the solutions to a first order difference equation of the form:

$$(17) \qquad \omega_t = f(\omega_{t-1}),$$

where the first derivative of the function f is given by:

$$(18) \qquad f' = \frac{\alpha}{\omega - \phi} - \frac{m}{m'(\omega - \phi)} = 1 - \frac{m}{\alpha m'}.$$

In the above expression, ω and m are evaluated at their steady state values and m' is the derivative of m also evaluated at the steady state. The slope of the demand for money function is given by:

$$(19) \qquad m' \equiv \phi n' - c'n - c'\omega n';$$

which for the example given in equation 16 generates the expression:

$$(20) \qquad m' = \gamma\phi\omega^{\gamma-1} - \delta(\gamma + 1)\omega^{\gamma}.$$

In order for the first derivative of the function f described in equation 17 to be between zero and one, the economy must be parameterized in such a way that both m and m' are positive at the steady state. It must also be true that $\frac{m}{\alpha m'} < 1$. One example that satisfies these conditions is provided by the parameterization of

be simple examples with one agent per generation that allow one to construct a stationary state in which both m and m' are positive but I have not found one.

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equation 16 for values of $\gamma = 7$, $\phi = 3/4$, $\alpha = 1/4$ and $\delta = 1/2$. In this case there is a monetary steady state at $\omega = 1$. At this steady state the demand for money is positive and equal to $1/4$, the slope of the demand for money function is equal to $5/4$ and slope of the dynamic equation linking ω_t to ω_{t-1} is given by the expression:

$$f' = 1 - \frac{m}{\alpha m'} = \frac{1}{5}.$$

It folows that the *monetary* steady state is indeterminate. That is; there is a non-stationary equilibrium of this economy for any level of initial real balances in the neighbourhood of $m_0 = 1/4$. Furthermore, members of the set of non-stationary equilibria all converge back to the monetary steady state. In more familiar examples of the overlapping generations model it is the steady state in which money has no value which is locally indeterminate. Consequently not much interest has been directed towards the set of non-stationary monetary equilibria since they all converge to a demonetized economy.

Some interest has been shown in versions of the standard overlapping generations model in which the government finances expenditure by printing money. This model is potentially interesting since one may show that the set of non-stationary equilibria converges to a *monetary* steady state. However, this steady state is on "the wrong side of the inflationary Laffer curve" in the sense that increases in government expenditure lead to *lower* inflation. My model does not suffer from this counter factual implication as the following analysis demonstrates. Totally differentiating 11 at the monetary stationary state it follows that:

$$(21) \qquad \frac{\partial \omega}{\partial g}\Big|_{g=0} = \frac{(\omega - \phi)/\alpha}{(m/\alpha) + m'([(\omega - \phi)/\alpha] - 1)} = \frac{\alpha}{m} > 0,$$

that is, increases in the level of government purchases cause output to expand, people hold more money, and the inflation (tax) rate increases.

4 The Economics of the Example

The economy that I have described is one in which agents have perfect foresight of future prices. However, one may use the techniques described in Farmer and

Woodford [5] to construct stochastic equilibria in this economy which are characterized by first order stochastic difference equations. There are analogies between the equilibria of the perfect foresight economy and the stochastic equilibria of the more complicated example. In this paper, however, I concentrate on the perfect foresight case.[9]

The experiment that I describe below is equivalent, in a stochastic model, to an unanticipated switch in policy regime. In this experiment, one assumes that for all recorded history the level of government purchases has remained constant at zero. At date T there is an unanticipated increase in the stock of money which is used to purchase commodities but for all dates later than T the money stock is (rationally) anticipated to remain at its new higher level. Since the monetary stationary state is indeterminate there is a range of possible values of the initial level of prices all of which are consistent with a perfect foresight equilibrium. As there are many possible equilibrium paths for prices, there are also many possible ways in which the economy could respond to a regime change.

At one extreme, it is possible that prices might jump in such a way that the economy remains at the stationary state $\omega_t = \alpha + \phi$. In this scenario the jump in prices would cause the holders of money balances to be taxed exactly enough to pay for the government purchases in period T. This inflation tax would have been unforeseen in the sense that the agents who had chosen to hold money did so in the belief that the distribution of prices had point mass at its historically determined level. The unforeseen shock that occurs in this scenario is inconsistent with a single rational expectations equilibrium, that characterizes the economy both before and after date T, because at the date of the policy switch agents act on beliefs about the *distribution* of prices which later turns out to have been incorrect. However, the behavior that I have just described is not the only possible way in which this economy might react to an unforeseen event.

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[9]In my discussion I shall use the terms perfect foresight and rational expectations interchangeably in recognition of the fact that a perfect foresight equilibrium is a special case of a rational expectations equilibrium in which the distributions of the variables of the model are degenerate with mass points at particular values.
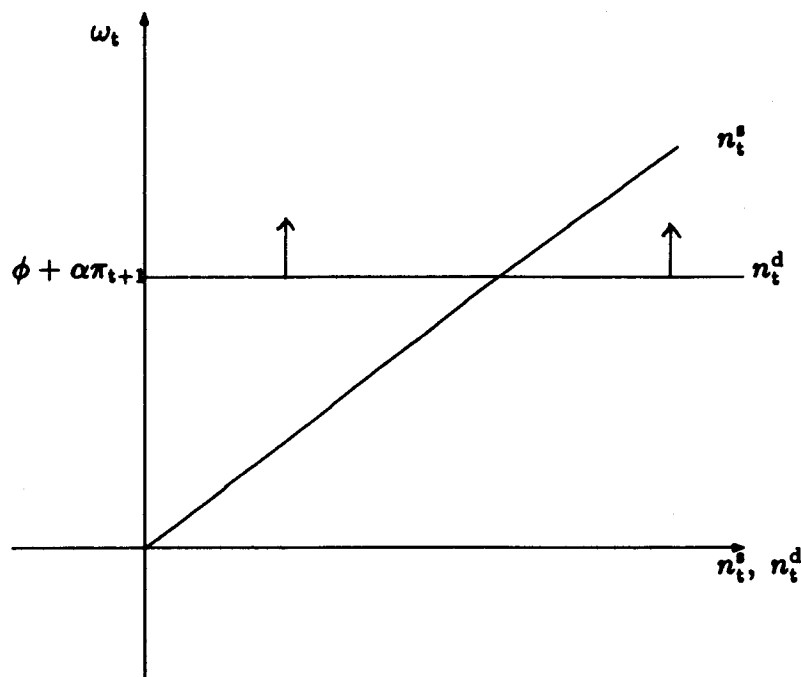
Figure 3: The Labor Market.

Suppose instead that the price level at date T does not respond to a contemporaneous increase in the stock of money. In this alternative equilibrium, firms respond to an increase in demand by increasing output and hiring more labor. Since part of the output that is produced will be sold in period T+1 it is possible for this expansion in economic activity to be a rational response if firms (correctly) anticipate that prices will increase. This situation is depicted in figure 3. Firms expect that p_{t+1}/p_t will be positive and this expectation implies that, at the level of real wages that has existed historically, they should switch from holding money as a store of wealth into holding goods in process. Acting on this belief, firms hire more labor and drive up wages to the point at which $\omega_t = \phi + \alpha\pi_{t+1}$.

In period T+1 the nominal money stock remains constant at its new higher level but output, employment and real wages contract back towards the stationary state in which $\omega = \phi + \alpha$; this scenario is depicted in figure 4. Since the demand for (real) money is locally increasing in real wages, the real demand for money in
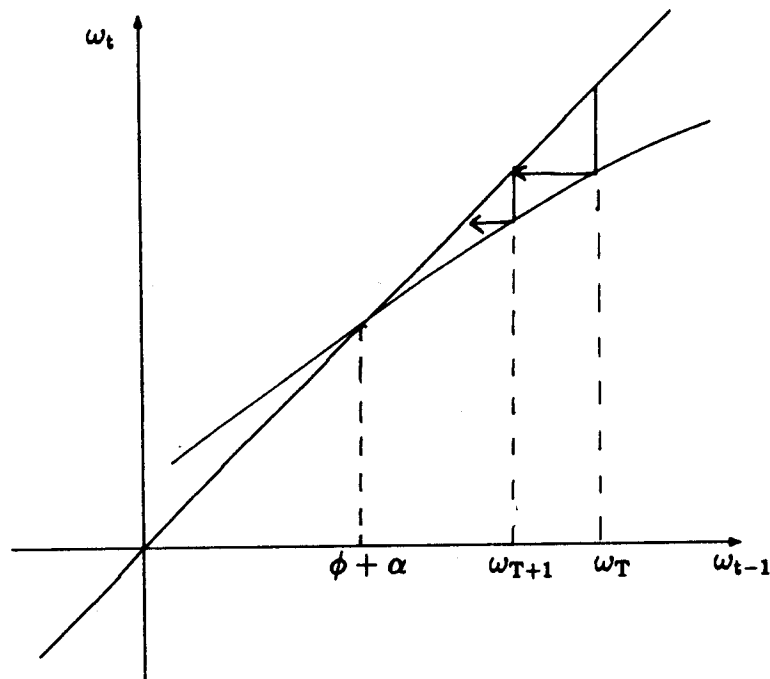
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Figure 4: The Response to a Monetary Shock.

period T+1 will be *lower* than in period T. To equate demand and supply, the price level in period T+1 must exceed the price level in period T. This is the rationally anticipated increase in the price level that triggered the expansion in economic activity in the first place.

5 Which is the Appropriate Equilibrium?

The two different perfect foresight equilibria that I have described above represent alternative ways of modelling rational expectations responses to a regime switch. They differ in the implicit assumptions that are made about the mechanism that agents use to forecast the future. In any rational expectations model with multiple equilibria there are many ways in which agents could form beliefs, each of which may be self fulfilling. This observation suggests that one should ask why one method of forecasting might become adopted over another.

My own answer to this question is explained in more depth in [4]. It comes down to a comparison of the properties of alternative forecasting rules in the face of an unforseeable switch in policy regime of the kind that I discussed above for the perfect foresight example. In this economy there is one very important difference between an equilibrium in which prices jump and an equilibrium in which they do not. This difference is most forcefully illustrated by comparing the forecast rules that agents use to support equilibrium in the two alternative situations.

Consider first, an economy in which agents have learnt that the perfect foresight price level is given by the expression;

$$(22) \qquad\qquad p_{t+1} = p^*(M_{t+1});$$

where p^* is the rational expectations pricing function associated with a stationary perfect foresight equilibrium in which money has value and M_{t+1} is the nominal quantity of money. In this economy agents in period T-1 will forecast p_T on the mistaken assumption that the distribution of M_T has point mass at its historically given level. Ex post, they will turn out to have been mistaken and they must update their beliefs about the distribution of the forcing variables to learn about the new monetary regime. A forecast rule of the kind described in equation 22 has the disadvantage that it requires knowledge of the future distribution of policy variables. But this is not true of all forecast rules!

Suppose instead that agents forecast future prices using the function:

$$(23) \qquad\qquad \pi_{t+1} = f(\omega_{t-1})\alpha + \phi.$$

This forecasting rule coincides with the equation that describes the properties of a non-stationary perfect foresight equilibrium. If ω_{T-1} is equal to its stationary value then the price level at date T that is forecast by this equation will be the same as that which is forecast using 22. The difference in these two rules occurs in the way that they behave in the face of a regime switch. If the government conducts an unanticipated regime change at date T and if agents continue to forecast prices with equation 23 at the point of the switch, then the effect of the regime change will be felt on quantities rather than prices.

Although both of the forecasting rules that I have described will support a rational expectations equilibrium for any given policy regime, they are not alike in the way in which they perform in the face of a *change* in regime. Whereas agents who use equation 22 must know the distribution of future values of monetary policy, no such information is required in the case of an equilibrium with predetermined prices since equation 23 does not depend upon the values of M_t. If all agents in all generations continue to forecast using equation 23, then nobody will be fooled by a change in regime. Another way of saying this is that, in some economies in which there are multiple equilibria, one can sometimes find an equilibrium which is immune to the 'Lucas critique'. The economy that I have presented above is an example of such an economy. In my model the 'Lucas proof' equilibrium is one in which prices are predetermined one period in advance and in which the impulse response functions of the economy are 'Keynesian' in the sense that quantities rather than prices respond contemporaneously to clear markets in the face of demand shocks.

6 Is there a Stable Phillips Curve?

Rational expectations methodology was not developed in a vacuum and one of the cornerstones of early accceptance of the theory was the explanation that it provided for the disappearance of an apparently stable trade-off between inflation and unemployment. According to this explanation, the trade off was illusory in the sense that it applied only to unanticipated inflations. Systematic attempts to exploit the Phillips curve by making use of inflation finance are understood as regime switches which eventually lead to a new rational expectations equilibrium at a higher inflation rate but at the same 'natural rate' of unemployment.

If the alternative account that I have provided is to prove successful it must also be able to explain the simultaneous occurrence of inflation and unemployment. The explanation that I propose is related to the method whereby new money enters the economy. Certain types of monetary policy that involve the payment of interest on money, will be neutral even in an economy with prices set in advance. But monetary

expansions that are used to purchase commodities may, in some environments, lead to non-superneutralities that persist for long periods of time. Consider, for example, what would hapen in the model that I have described if the government were to pay interest on money in proportion to existing balances. In this situation there would be an equilibrium in which the real wage remained constant at $\phi + \alpha$ but the inflation rate increased by exactly the amount of the interest payments. The relevant comparison of the return on money with the return on goods in process leads, in this more complex economy, to an arbitrage condition of the form:

$$\omega_t = \phi + \alpha \frac{p_{t+1}}{R_t p_t};$$

where R_t is the nominal interest factor on money. In this economy, a regression of the rate of inflation on the level of employment would uncover an apparently stable trade off for the reason that the real rate of interest will fall as inflation rises if the inflation is due to a non-neutral fiscal expansion. But monetary expansions that are used to pay interest on outstanding money balances will increase both R_t and p_{t+1}/p_t simultaneously. An expansion of this nature preserves the long run vertical Phillips curve for a reason similar to that provided by the Phelps-Friedman account; there is a missing variable. According the Phelps-Friedman theory this variable is expectations. According to the account that I have provided it is the money rate of interest.[10]

7   Conclusion

A predetermined price equilibrium has very different time series implications from those of the equilibria that arise in standard rational expectations models. It is

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[10]There is however a sense in which the two theories differ substantially. I have chosen to embed my account in a general equilibrium model in which the real rate of interest may be influenced by fiscal policy. For this reason there is a *long run* Phillips curve in my model since a permanent monetary expansion that is financed by inflation is capable of driving down real interest rates, raising real wages and expanding employment. At best, this may be true at the level of the world as a whole, but I do not think that it is a good description of any modern economy even one as large as the United States. A more appropriate assumption, which I do not have the space to explore here, is that of an open economy in which the real rate of return is provided by the world.

my contention that these implications are likely to allow us to construct model economies with equilibria which are in much closer accord with the data than those which follow from representative agent economies. In a world where the future is continually changing in unknown (and unknowable) ways, there are clearly advantages to forecasting schemes with the properties that I describe. Whether agents do indeed use such schemes is an empirically falsifiable proposition. My current research agenda involves the construction of more realistic economies that exhibit the kinds of features of the model described in this paper. By comparing the impulse response functions of these economies with those of time series data one can rank the performance of alternative models. It is too early to tell if this research agenda will prove successful but my preliminary investigations look promising.

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