INDIVIDUAL AND COLLECTIVE WAGE BARGAINING

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ABSTRACT

Wage negotiation is modeled as an "oceanic" game. The employer and the union or unions (if any) are represented as atomic players while the unorganized workers are represented as a non-atomic continuum. For simplicity, the workers are assumed to be homogeneous in the employer's production function, but heterogeneous in their outside opportunities. The total surplus that each coalition of players is capable of generating serves as a measure of its bargaining power. (Thus, any coalition that does not include the employer is powerless.) A cooperative game in characteristic-function form is thereby defined, and its Shapley-value solution, which averages the marginal surpluses of the players in all possible alignments, distributes the maximum available surplus in a way that reflects these coalitional potentials and yields a plausible "negotiated" wage settlement. Several different levels of unionization are examined and contrasted.

It is noteworthy that the present approach is not tied to any particular version of the negotiatory process, because it derives solely from the underlying economic data and the inherent bargaining power wilded by the participants.

1. Introduction

The standard models of wage determination are generally concerned with the technological or organizational aspects of the labor market and tend to obscure the fact that, in the absence of monopoly on either side or governmental controls, both labor and management are in position to bring significant bargaining power to bear. Recent attempts to use game theory to study the bargaining aspects of wage determination have mostly considered only simple bilateral bargaining -- a game pitting one employer against one worker (or a sequence of such games) or one employer against one union. Since real labor markets are seldom that simple, a methodology for dealing with a richer class of institutional structures would appear to be a worthwhile adjunct to such investigations. Ideally, both actual and potential employees should be allowed into the game, perhaps represented at the bargaining table by a union or union(s), or perhaps representing themselves individually but gaining bargaining power from the possibility of *ad hoc* coalitional action. On the other side of the table there might be one or several employers, in various postures of association or competition. Our focus in the present paper, however, will be on the labor model.

In particular, our models allow for a certain amount of heterogeneity among the workers. Though equally productive on the job in question they may have different capabilities and opportunities in other occupations. We represent this differentiation by a parameter called the "alternative wage"; it is sufficient for our purposes since it gives individual workers a variety of different incentives and fall-back positions in the bargaining game and similarly makes a union's power depend not only on its size but also on the composition of its membership.

An interesting problem of scaling arises when one tries to accommodate in the same model both "big" players (e.g. employers or unions) and "little" players (e.g. employees and potential employees). While the latter ought to have a significant effect on the outcomes due to their large numbers, as individual actors they must be regarded as infinitesimal. The modeler must steer between two extremes: 1) insisting on equal negotiating rights for all players, to the extent that the influence of the major players is diluted to the vanishing point, and 2) allowing the minor players to be reduced to passive price-taking dummies with no status at all as negotiators. This problem seems first to have been addressed by Milnor, Shapiro and Shapley in three Rand Corporation reports (1960-61) in which they borrowed from measure theory the idea of a mass distribution consisting of a finite set of "atoms" of positive mass together with a density function defined over an infinite continuum of other points, the latter called an "ocean" to suggest the lack of any order or cohesion among its members. The original application was to weighted-majority voting games, as in a large publicly owned corporation[1].

An important feature of "oceanic game" theory is its focus on *capabilities* rather than *strategies*. It begins by constructing a "characteristic function" that describes what each subset of players can accomplish by joint action. But the characteristic function does not incorporate any assumption about the procedures for coalition forming and for the subsequent bargaining. Such a cooperative-game model (which is especially suitable when there are many players) contrasts sharply with the strategic-form or extensive-form models that are used in Nash equilibrium analyses, where the interactive moves (offers, responses, compromises, etc.) must all be spelled out explicitly. Such models are inevitably sensitive to the details of protocol, and this restricts their application. Of course, a price is also paid for omitting detail. Broader assumptions yield less specific answers. So our models do not deal with specific wage disputes and their resolution. But hopefully they can give some indication of the long- or middle-run resolution of the forces at work in the labor market, based on the underlying economic and institutional data and the inherent bargaining power wielded by the participants.[2]

Let us sketch some of the results that our models do provide. 1) As different institutional structures are compared we find that the members of a partial union will under certain conditions, but not always, do better by bargaining collectively than as individuals. But when *all* workers are organized and bargain as a unit, their total payoff is quite generally higher than what they would get bargaining as

[1] The published papers of Shapiro and Shapley (1978) and Milnor and Shapley (1978) reproduce almost verbatim the still-available Rand reports. For related work see Hart (1973), Guesnerie (1977), and Fogelman and Quinzii (1980). Models with continuum of non-atomic agents are now commonplace in mathematical economics, following the lead of Aumann (1964, 1966); see also Aumann and Shapley (1974) for value theory and Hildenbrand (1974) for core theory. The term "oceanic game" however has come to denote the case that mixes atomic players with the non-atomic continuum.

[2] Our work may be compared in its level of abstraction with the classical equilibrium model of exchange and production, which takes for its data only preferences, endowments and production possibilities, not the particulars of buying and selling.

individuals, though whether each worker is better off depends on how freely the union can distribute its total gain. 2) The ability to replace incumbent workers with potential workers definitely enhances the employer's bargaining position; how effective this will be will naturally depend on the number of potential employees and their alternative wages. 3) Finally, the individually negotiated wage of a worker correlate positively to his alternative wage and correlate negatively to the size of the total labor force. While there are no big surprises in the above, it is nevertheless worth noting that our models do capture these effects and to a certain extent quantify them.

The paper is organized as follows: Section 2 describes the game-theoretic tools we employ. Section 3 presents the no-union case in which the only atom is the employer; two examples of this are worked out in Section 4. Section 5 then introduces partially-organized and fully-organized labor force; the solutions are illustrated in Section 6 with the aid of supply and demand functions of a simple linear form. Concluding remarks are given in Section 7.

2. Cooperative Games in Coalitional Form

The coalitional form of a game (N,v) is based on the *player set* N and *characteristic function* v defined on the subsets of N, where $v(S)$ is meant to indicate in some way what the players in S can do if they agree to act as a coalition. There are several different ways to formulate this, but the essential ingredient is a description of the *assurable S-payoffs*, i.e., utility levels that S can obtain for its members against "worst-case" behavior by the members of $N \backslash S$. The cooperative solutions of the game are generally assumed to be Pareto optimal in the set of assurable N-payoffs, which is simply the set of all payoff vectors that are feasible under full cooperation -- the presumption being that however bitter the dispute over the distribution of surplus, the "grand coalition" will ultimately form and achieve Pareto optimality.

When there is a common unit of utility (e.g., $$$) as well as a way to transfer it among the players more or less freely, then the various assurable sets may be so "flat" in the utility space that $v(S)$ can be defined simply as the maximum sum of utilities in S that can be assured in the worst case.[3] This

[3] A detailed account of the players' strategies is often unnecessary in the cooperative theory when the game enjoys the so-called *fixed threat* property, namely, that the worst case occurs

is called a "TU" characteristic function (for "transferable utility"), and when its use can be justified it represents a great simplification of the models, both analytical and conceptual, and even brings games with a continuum of players within reach of elementary mathematical tools.

In our current application the monetary utilities, though interpersonally commensurable, are only imperfectly transferable, and so the "TU" assumption must be used with care. In particular, we may find that the solution of the game calls for payments to be made to workers in the labor pool who influence the wage settlement by their presence but who do not in the end get hired at the production level necessary to attain the efficient outcome $v(N)$. These side payments are typically quite small, but it is not clear how or whether they might be implemented in practice. One possible interpretation might be to regard them not as actual payments but as a measure of unrealized bargaining power, expressed in terms of what the available but unhired workers *could* claim at the bargaining table *if* utility were freely transferable[4]. For the present, however, our position will be that the TU assumption in this paper is only a simplifying approximation, similar in degree and effect to various other idealizations that are commonplace in abstract microeconomic theory.[5]

Given the TU characteristic function, our solution concept will be the Shapley value as it applies to oceanic games. This solution has a well-known axiomatic basis in the finite case as well as the purely non-atomic case (see Shapley 1953; Aumann and Shapley 1974). But for oceanic games, the axiomatic approach is not so effective (see Hart 1973), and a better formal definition is obtained through approximations by finite games -- the so-called asymptotic approach (see Shapiro and Shapley

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for S when $N \backslash S$ just leaves it alone. This type of "no-externalities" condition is familiar from the classical Walras/Edgeworth model, but is also satisfied in bargaining situations when the "disagreement payoff" is unique.

[4] Imperfect transferability might also be treated by using the more general "NTU" value theory applied to a model in which certain forms of indirect or imperfect transfer are provided as strategic options for employer. Thus, the basic model could be elaborated by allowing the firm to allocate some of its surplus to programs that indirectly benefit the population of potential employees, e.g., scholarship awards, contribution to community amenities, travel expenses for job interviews, etc. But such elaborations would not be well-matched to the abstract, non-strategic character of the basic model.

[5] In this connection, we might point out that the NTU value, though more complex in its definition, is nevertheless a *continuous* extension of the TU value. This makes it permissible to regard the TU solution as an approximation to the former provided the deviations from perfect transferability are small.

1961, 1978; Fogelman and Quinzii 1980). For actual calculations, however, a third approach -- the "random order" model -- is by far the most useful; it comes from the observation that in finite games the Shapley value of a player is his *expected marginal contribution* when the players enter the grand coalition in a random order[6].

Unfortunately it is theoretically impossible to randomly order a measurable continuum of players in such a way that measurability is preserved.[7] For oceanic games, however, a procedure is available that has much the same effect: Imagine the continuum of minor players to be ordered initially in some definite way -- say spread out uniformly in the unit interval [0,1] representing a period of time during which the ocean is "flowing" at a constant rate into the grand coalition. Then insert the major players $A_1, A_2, \cdots, A_p$ at times $t_1, t_2, \cdots, t_p$ respectively, where the latter are random variables chosen independently according to the uniform distribution on [0,1]. By this device we suitably randomize the entry of the major players.

So far so good, but how do we randomize the entry of the minor players? If the ocean happens to be homogeneous there is no problem, since the value solution does not discriminate among identical players, anyway, but even an inhomogeneous ocean can be handled if each player is fully described by a *profile* consisting of a finite list of parameters (e.g, reservation wage, productivity, seniority, ...). In that case, if the ocean is sufficiently well mixed[8] then the minor players in any subinterval of [0,1] of positive length will, with probability approaching 1, be as close as we please to a "faithful sample" of the whole population of minor players.

Accordingly, only a certain class of orders will have to be considered. Without loss of generality we may assume that the random numbers t_j are all different from each other and from 0 and 1. Let $t_{j_1}, \cdots, t_{j_p}$ be these numbers arranged in increasing order. Then the only orderings we shall need have the following form: First a minor-player block of size t_{j_1} with a mix of profiles in exact proportion to

[6] The interplay between these three mutually supportive approaches to the value is a theme that recurs throughout Aumann and Shapley (1974).

[7] Op. cit., Chapter 2.

[8] Imagine [0,1] chopped up into a large finite number of intervals, which are then shuffled like a deck of cards and returned to [0,1] in the new order.

the total mix of profiles -- i.e., a faithful sample of the ocean as a whole. Denote this block by "$samp(t_{j_1})$". Then add the first atomic player A_{j_1} and pay him an amount equal to his contribution to the worth of the growing coalition, namely,

$$v\left[samp(t_{j_1}) \cup \bar{A}_{j_1}\right] - v\left[samp(t_{j_1})\right].$$

Next add another faithful sample of the ocean, of size $t_{j_2}-t_{j_1}$, followed by the second atom A_{j_2}, and pay him

$$v\left[samp(t_{j_1}) \cup \bar{A}_{j_1} \cup samp(t_{j_2}-t_{j_1}) \cup \bar{A}_{j_2}\right] - v\left[samp(t_{j_1}) \cup \bar{A}_{j_1} \cup samp(t_{j_2}-t_{j_1})\right]$$

and so on, until all the atoms are accounted for.

The Shapley value for any given atomic player is his expected payment under this scheme. To calculate it, integrate that player's contributions over the p-dimensional unit cube of the t_j's, taking note of the fact that for any k, the number l such that $j_l=k$ is a random variable that depends on the relative positions of the t_j's in [0,1].

Finally, to obtain the *value density* for any given profile, introduce a small set of new players of size $\delta > 0$, endow each of them with that profile, then merge them together into a single, $(p+1)$st atom for which we can determine the Shapley value as above. To get the desired density, merely divide this value by δ and let δ go to zero.

This whole procedure will be amply illustrated in the sequel.

3. Model 1 - Unorganized Labor

Consider a market consisting of an employer E and an ocean $[0, n]$ of workers. Let f denote the labor demand function and g the labor supply function, both functions defined on $[0, n]$ or perhaps some larger interval in $\mathbb{R}$. We assume f to be continuous and strictly decreasing up to some point x_0 where $f(x_0) = 0$, and identically 0 thereafter, and we assume g to be nonnegative and nondecreasing but not necessarily continuous. Without loss of generality we may assume that $g(n) \leq f(0)$, since workers x with $g(x) > f(0)$ always turn out to be dummies in the game.

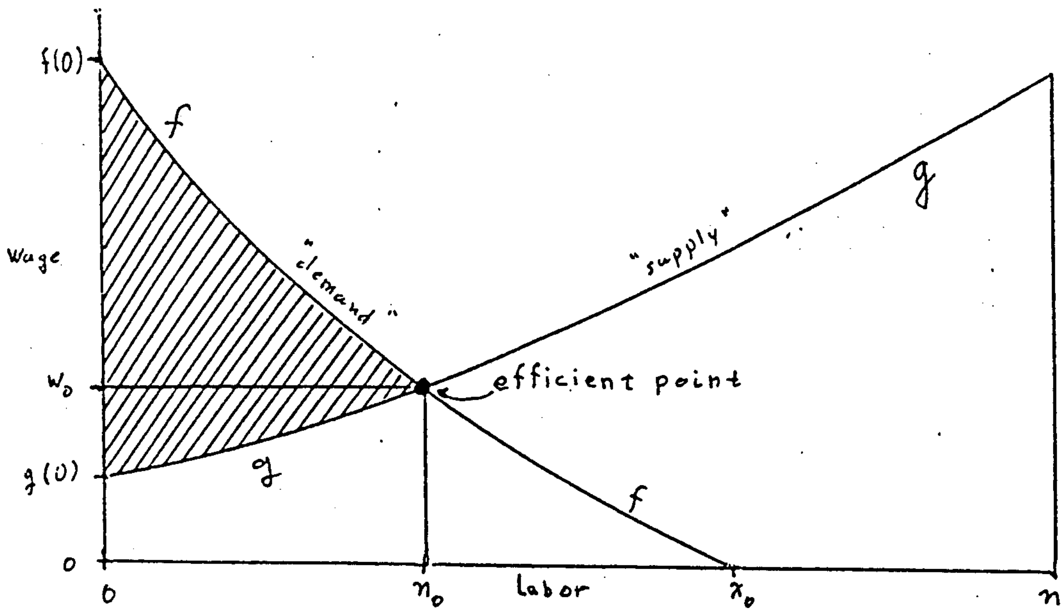

Figure 3.1

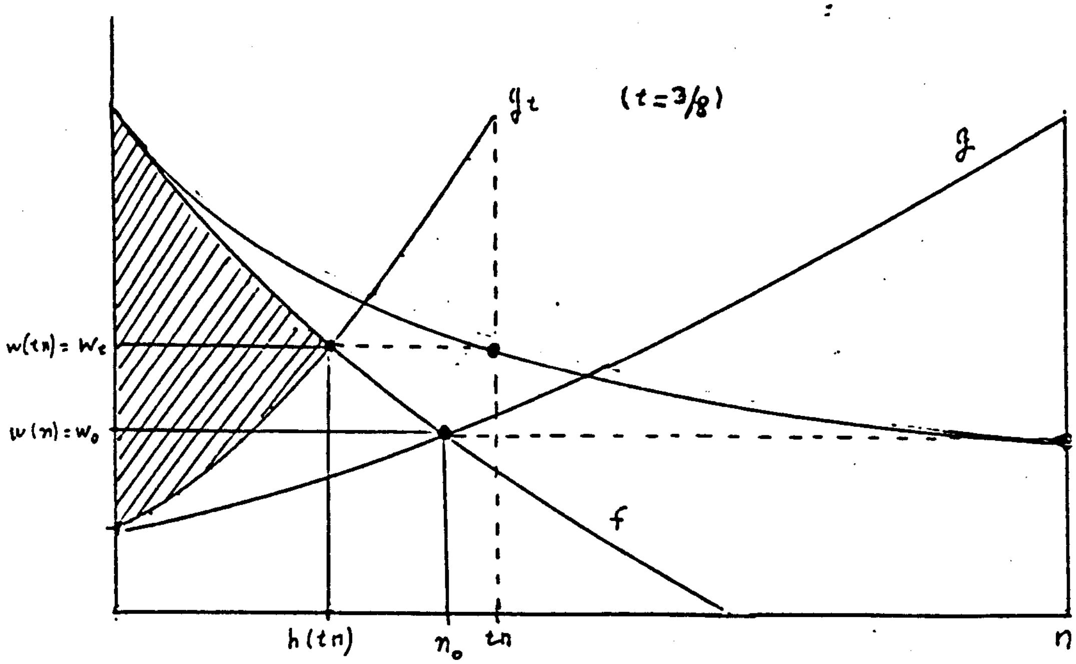

Figure 3.2

The characteristic function v defines the worth $v(S)$ of any coalition S to be the total surplus it is capable of achieving by its own efforts. In particular, $v(S) = 0$ for any S not containing E, and if S is the grand coalition $E \cup [0, n]$ then $v(S)$ is the shaded area in Figure 3.1.

Consider now a faithful sample of $[0, n]$ of size tn, $0 \leq t \leq 1$. Since we only deal with these samples as blocks of players, their internal order is irrelevant, so we merely replace g by g_t defined on $[0, tn]$ by $g_t(x) \equiv g(x/t)$, as shown in Figure 3.2. Figure 3.2 also shows the curve $w(tn)$, the equilibrium wage associated with the intersection of f and g_t; we denote the number of workers employed at that point by $k(tn)$. Note that $w(x) > f(x)$ and $k(x) < x$ over the whole domain $[0, n]$ except for x=0.

We can now express the worths of the types of coalitions needed in our calculations. Specifically, the surplus available to a coalition consisting of E and a faithful sample of size tn is given by

$$S(t) = \int_{x=0}^{k(tn)} [f(x) - g_t(x)]dx = \int_{x=0}^{k(tn)} [f(x) - g(x/t)]dx \tag{3.1}$$

This is indicated by the shaded area in Figure 3.2. Of course, as already mentioned, any coalition that does not include E has worth 0.

Thus, to obtain the Shapley value Φ_E of the employer we merely select a random number $t_E \in [0,1]$ representing the time at which he enters the coalition and award him the expected value of $S(t_E)$ -- this being his incremental contribution. Then we have

$$\Phi_E = \int_0^1 S(t)dt = \int_0^1 \int_0^{k(tn)} [f(x) - g(x/t)]dxdt. \tag{3.2}$$

This double integral may be visualized as the volume of a solid whose base is the region in the plane bounded by f and g and the axis x=0 (Figure 3.3), and whose height above the plane varies from 1 (on the left boundary) to 0 (on the curve g), the level sets being given by the curves g_t. In this representation, we can think of the height of the solid above a given point is the probability that E will obtain the bit of revenue represented by that point.

The Shapley values of the oceanic players will be represented by a cumulative distribution func-

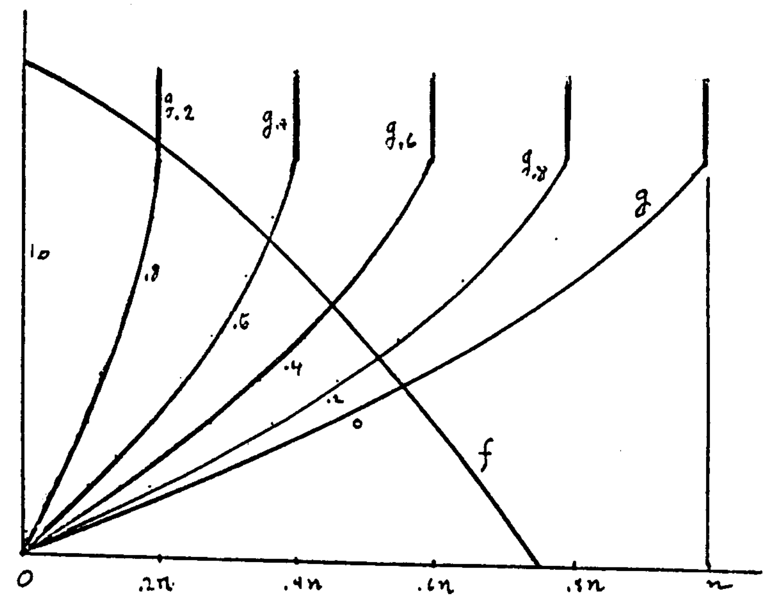

Figure 3.3

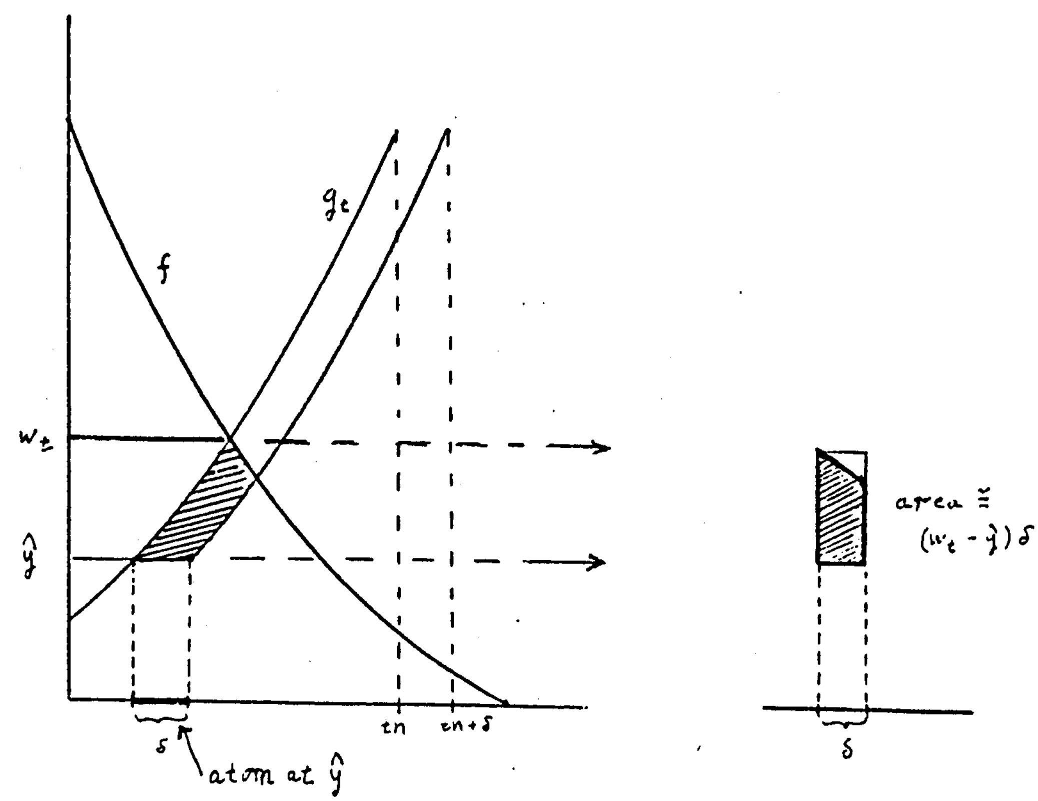

Figure 3.4

tion on $[0, n]$. Thus, let $\Phi(x)$ be the total value payoff to the interval $[0, x]$ in the domain of g; the derivative $\phi(x) = \frac{d\Phi(x)}{dx}$ is then the "payoff density" to a worker whose alternative wage is $g(x)$.

Let t be fixed and let $\hat{y}$ be a fixed alternative wage. We must determine the increment to the surplus when we add a small atom $\Delta_{\hat{y}}$ consisting of additional workers having alternative wage $\hat{y}$, to the set of workers who are on hand at the time t, i.e., the workers represented by the domain $[0, tn]$ of the function g_t. Let $\delta>0$ denote the size of $\Delta_{\hat{y}}$. There are two possible configurations. The first is illustrated in Figure 3.4; it assumes $g(0) \le \hat{y} < w(tn)$. The increment due to $\Delta_{\hat{y}}$ is the shaded area on the left. Its area, however, is equal by translation to the simpler shaded area on the right, or $[w(tn) - \hat{y}]\delta + o(\delta)$. The second configuration (not illustrated) assumes $w(tn) \le \hat{y} \le g(n)$; in this case, the increment due to $\Delta_{\hat{y}}$ is nil.

In order to obtain the value-density ϕ, let $\hat{x}$ be the worker in $[0,n]$ for whom we wish to evaluate ϕ and let t_E be the time of E's arrival. Then we have[9]

$$\phi(\hat{x}) = \int_{t_E=0}^{1} \int_{x=t_E n}^{\bar{u}} \frac{1}{n}[w(x) - g(\hat{x})]dxdt \tag{3.3}$$

where the upper limit $\bar{u}$ breaks into three cases. To present this, let $\hat{t}$ be given by $w(\hat{t}n) = g(\hat{x})$, i.e., $\hat{t} = w^{-1}(g(\hat{x}))/n$ (see Figure 3.2). (If $g(\hat{x})$ is less than $w(n)$ we formally set $\hat{t} = +\infty$.) Case I: If $g(\hat{x}) \le w(n)$, then $\bar{u} = n$. Case II: If $w(n) \le g(\hat{x}) \le w(t_E n)$, then $\bar{u} = \hat{t}n$. Case III: If $g(\hat{x}) \ge w(t_E n)$, then $\bar{u} = t_E n$ (so $\delta = 0$). The three cases are shown in Figure 3.5.

We summarize all this by writing $\bar{u}=med\{n, \hat{t}n, t_E n\}$, where $med\{a,b,c\}$ denotes the median of the three numbers a, b and c. Thus, the general formula is

$$\begin{aligned}\phi(\hat{x}) &= \int_0^1 \int_{t_E n}^{med\{n,\hat{t}n,t_E n\}} \frac{1}{n}[w(x)-g(\hat{x})]dxdt_E \\ &= \int_0^1 \int_{t_E n}^{med\{n,\hat{t}n,t_E n\}} \frac{1}{n}w(x)dxdt_E - \frac{1}{n}g(\hat{x})\int_0^1(med\{n,\hat{t}n,t_E n\}-t_E n)dt_E .\end{aligned} \tag{3.4}$$

In case I, where $\hat{x}$ does not get hired in the end, (3.4) simplifies considerably.

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[9] The variable of integration x identifies the time at which $\Delta_{g(t)}$ joins the grand coalition, i.e., $t = x/n$. The condition $x \ge t_E n$ ensures that E is already on hand when this occurs.

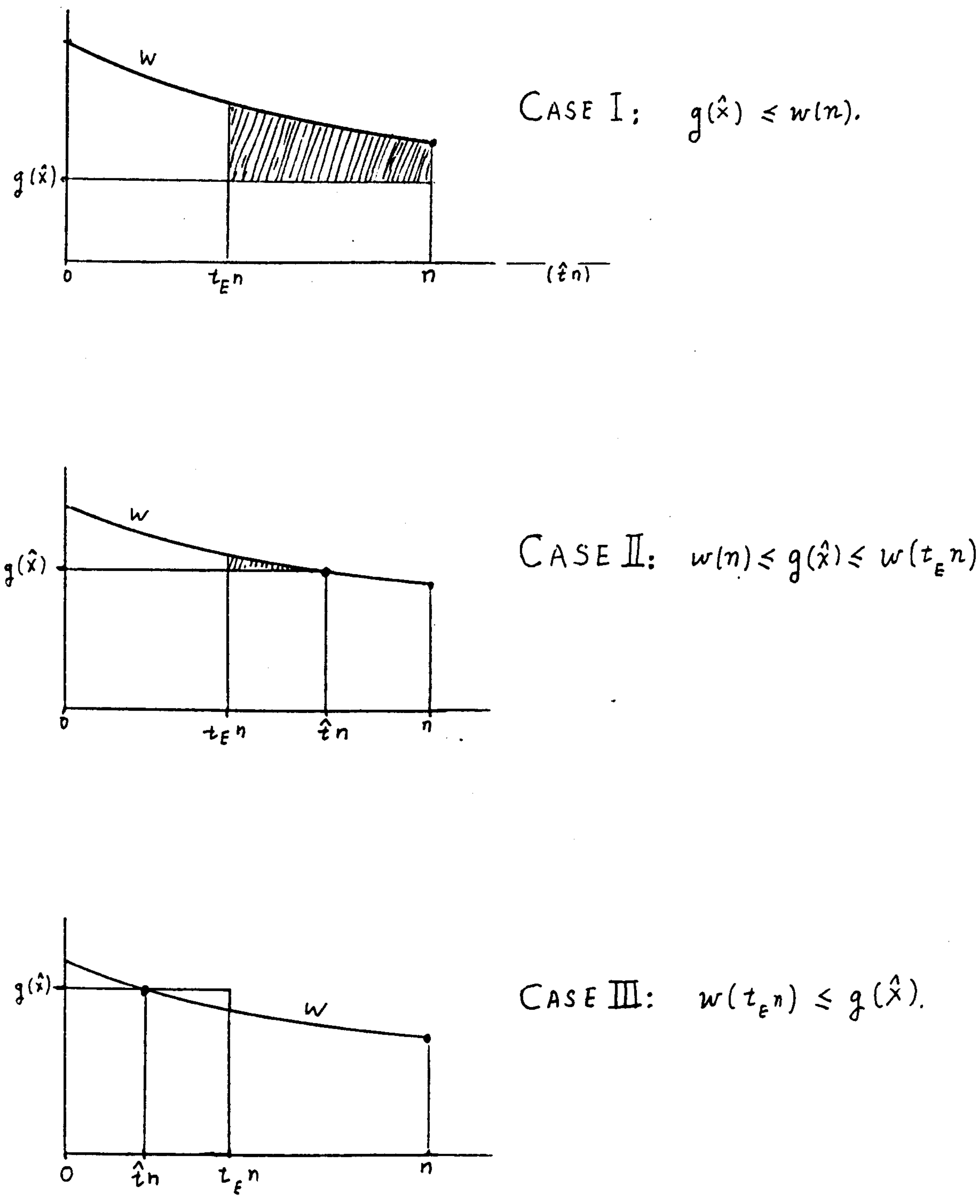
w
CASE I: $g(\hat{x}) \leq w(n)$.
$g(\hat{x})$
0
$t_E n$
n
$(\hat{t} n)$
w
CASE II: $w(n) \leq g(\hat{x}) \leq w(t_E n)$.
$g(\hat{x})$
0
$t_E n$
$\hat{t} n$
n
$g(\hat{x})$
w
CASE III: $w(t_E n) \leq g(\hat{x})$.
0
$\hat{t} n$
$t_E n$
n

Figure 3.5

$$\phi(\hat{x}) = \int_0^1 \int_{t_E n}^{n} \frac{1}{n} w(x) dx dt_E \; - \; g(\hat{x}) \int_0^1 (1-t_E) dt_E$$

$$= \frac{1}{n} \int_0^1 [W(n) - W(t_E n)] dt_E - \frac{1}{2} g(\hat{x}), \tag{3.5}$$

where $W(x)$ is the integral of w from 0 to x. Note that the first term is independent of $\hat{x}$. The second term shows that the Shapley values of the workers decrease as their alternative wages increase, as one would expect.

Cases II and III we handle together, since the case distinction depends on t_E, which is a variable. We have:

$$\phi(\hat{x}) = \frac{1}{n} \int_0^1 \int_{t_E n}^{\max\{t_E n, \hat{t} n\}} w(x) dx dt_E \; - g(\hat{x}) \int_0^1 (\max\{t_E, \hat{t}\} - t_E) dt_E$$

$$= \frac{1}{n} \int_0^1 (W(\max\{t_E, \hat{t}\}) - W(t_E n)) dt_E \; - \; g(\hat{x}) \int_0^{\hat{t}} (\hat{t} - t_E) dt_E$$

$$= \frac{1}{n} \int_0^{\hat{t}} (W(\hat{t} n) - W(t_E n)) dt_E \; - \; \frac{1}{2} g(\hat{x}) \hat{t}^2. \tag{3.6}$$

4. Model 1 - Examples

In the first example the workers are fully heterogeneous in opportunities, with g as well as f linear. In the second example there are only two types of workers: the ones hired at the efficient production level having one alternative wage and the ones not hired have higher alternative wage.

EXAMPLE 1: Let $f(x) = 1 - x$ $(0 \le x \le 1)$, $g(x) = x/2$ $(0 \le x \le n)$, and $n = 2$. Recall that $g_t(x) \equiv g(x/t)$. Also $w(x), k(x)$ are the equilibrium wage and the number of workers employed, respectively, associated with the intersection of f and g_t where $t = x/n$. Thus:

$$g_t(x) = \frac{x}{2t} \; , \quad w(x) = \frac{1}{1+x} \; , \quad k(x) = \frac{x}{1+x} .$$

From equation (3.1) the surplus is:

$$S(t) = \int_0^{k(2t)} [1 - x - x/2t] dx = \left[x - \frac{x^2}{2} - \frac{x^2}{4t} \right]_0^{\frac{2t}{1+2t}} = \frac{t}{1+2t} ,$$

and so the value payoff for the employer is:

$$\Phi_E = \int_0^1 S(t)dt = \int_0^1 \frac{t}{1+2t}dt = \frac{1}{2} - \frac{1}{4}\ln 3 = 0.22535 .$$

Note that the value of the grand coalition is $S(1) = 1/3$, so E is getting about 2/3 of the surplus. Also, note that if all workers were to unionize and behave as a monopolist, the employment level would be 0.4, the union wage would be 0.6 and E would get 0.08, which is much less than Φ_E. In contrast, under the competitive solution E would get 0.222, only slightly less than Φ_E.

Now we compute the value density to the workers. We have

$$W(x) = \int_0^x w(u)du = \ln(1+x)$$

Also,

$$\hat{t} = h^{-1}(g(\hat{x}))/n = h^{-1}(\frac{\hat{x}}{2}) \cdot \frac{1}{2} = \frac{1}{\hat{x}} - \frac{1}{2} \quad , \qquad \hat{x} = \frac{2}{2\hat{t}+1}$$

In case I we have $g(\hat{x}) \le w(n)$, that is, $\hat{x} \le 2/3$. Using equation (3.5):

$$\phi(\hat{x}) = \frac{1}{n}\int_0^1 \left[W(n) - W(t_E n)\right] dt_E - \frac{g(\hat{x})}{2}$$

$$= \frac{1}{2}\int_0^1 \left[\ln 3 - \ln(1+2t_E)\right] dt_E - \frac{\hat{x}}{4}$$

$$= \frac{1}{2} - \frac{1}{4}\ln 3 - \frac{\hat{x}}{4} .$$

In the remaining case (combining the previous cases II and III) we have $2/3 \le \hat{x} \le 2$. So, from equation (3.6)

$$\phi(\hat{x}) = \int_0^{\hat{t}} \left[W(\hat{t}n) - W(t_E n)\right] dt_E - \frac{n}{2} g(\hat{x})\hat{t}^2 = \int_0^{\hat{t}} \left[\ln(1+2\hat{t}) - \ln(1+2t_E)\right] dt_E - \frac{\hat{x}}{2}\hat{t}^2$$

$$= \frac{1}{2\hat{x}} - \frac{\hat{x}}{8} - \frac{1}{2}\ln(2/\hat{x}) .$$

In Figure 4.1, we plot both the supply curve $g(x)$ and the total wage $\phi(x)+g(x)$. The difference between the two is the oceanic value $\phi(x)$, which is a decreasing but positive function of x. In

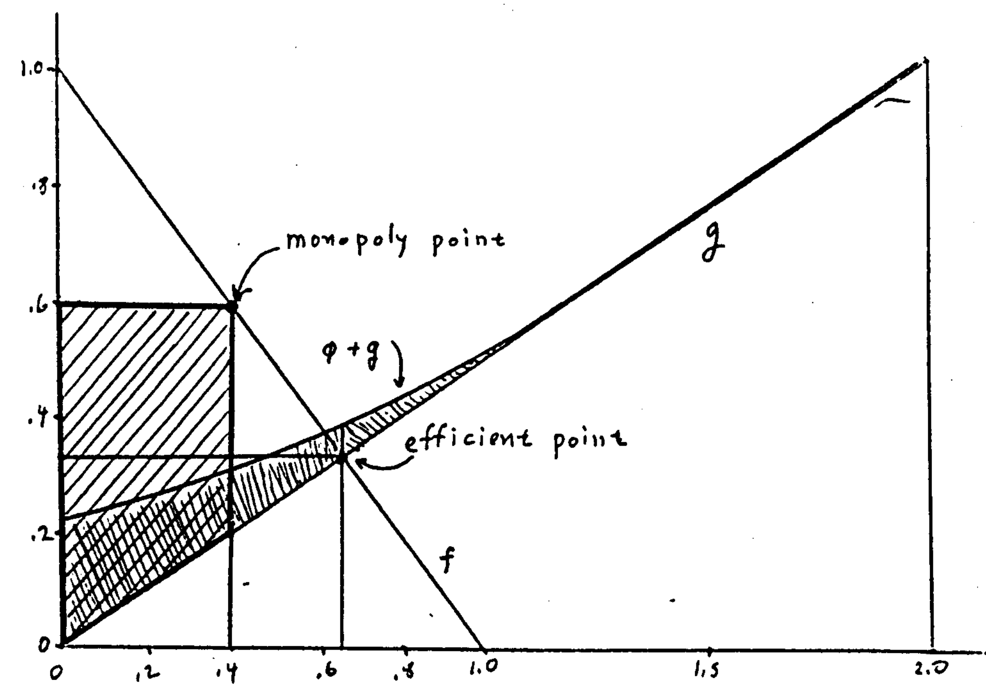
1.0
.8
.6
.4
.2
0
0
.2
.4
.6
.8
1.0
1.5
2.0
monopoly point
$\varphi + g$
efficient point
g
f

Figure 4.1

addition, the competitive wage is plotted, which in this case is 0.333 for $x \leq 2/3$. Also is plotted the monopolistic solution. In this case, the union wage is 0.6 for $0<x<0.4$. Further discussion of these results is given at the end of this section.

EXAMPLE 2: Let $f(x) = 1 - x$ $(0 \leq x \leq 1)$, let $c < 0.6$, let $n \geq 0.5$, and let

$$g(x) = \begin{cases} c & \text{if} \quad 0 < x < 1/2 \\ 0.6 & \text{if} \quad 1/2 < x < n \\ \infty & \text{if} \quad n < x \end{cases}$$

First, calculate the employer's payoff. There are three types of configuration that enter the calculation; as indicated in Figure 4.2: call the shaded areas A_I, A_{II} and A_{III} respectively. Then we have

$$\Phi_E = \int_0^{.4/n} A_I \, dt_E + \int_{.4/n}^{.8} A_{II} \, dt_E + \int_{.8}^{1} A_{III} \, dt_E$$

where

$$A_I = \frac{(t_E n)^2}{2} + (1-t_E n-c)\frac{t_E}{2} + (1-t_E n-0.6)(t_E n-\frac{t_E}{2}).$$

$$A_{II} = 0.08+\frac{t_E}{2}(0.6-c).$$

$$A_{III} = \frac{(t_E/2)^2}{2} -(1-t_E/2-c)\frac{t_E}{2}.$$

Substitution for A_I, A_{II}, A_{III} and integration yields the employer's payoff:

$$\Phi_E = 0.23 - 0.25c - \frac{0.01}{n}.$$

Now, for the workers who are hired we calculate

$$w(\hat{x}) = \begin{cases} 1-\hat{x} & 0 \leq \hat{x} \leq 0.4 \\ 0.6 & 0.4 < \hat{x} \leq 0.8n \\ 1-\dfrac{\hat{x}}{2n} & 0.8n < \hat{x} \leq n \end{cases}$$

and

$$W(\hat{x}) = \begin{cases} \hat{x}-\dfrac{\hat{x}^2}{2} & 0 \leq \hat{x} \leq 0.4 \\ 0.08+0.6\hat{x} & 0.4 < \hat{x} \leq 0.8n \\ 0.08+0.6\hat{x}-\dfrac{(xh-0.8n)^2}{4}n & 0.8n < \hat{x} \leq n \end{cases}$$

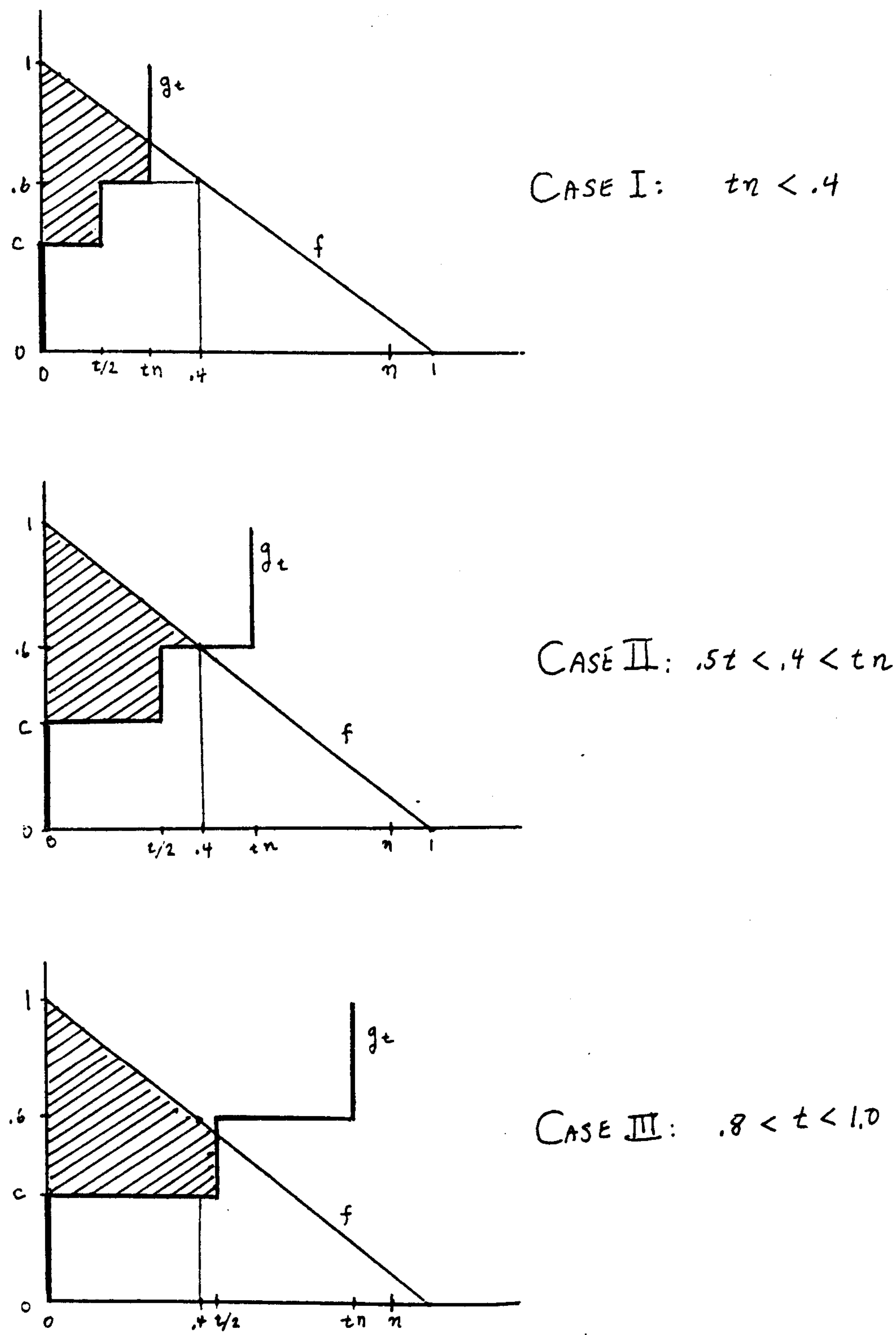

Figure 4.2

Thus:

$$\int_0^1 W(t_E n)dt_E = \frac{1}{n}[-0.01 + 0.08n + 0.3n^2]$$

and

$$W(n) = 0.08 + 0.59n .$$

Substituting these expressions into (3.5), we get the payoff density:

$$\phi_h(\hat{x}) = 0.29 - \frac{c}{2} + \frac{0.01}{n^2}$$

for the hired workers. Next, from (3.4), the payoff density to the workers who are not hired is

$$\phi_{nh}(\hat{x}) = \int_{t_E=0}^{1} \int_{x=nt_E}^{0.4} \frac{1}{n}[w(x)-g(\hat{x})]\, dx\, dt_E ;$$

clearly, the latter will not enter unless $x < 0.4$. Thus, $t_E < 0.4/n$. So,

$$\phi_{nh}(\hat{x}) = \int_{t_E=0}^{0.4/n} \int_{x=nt_E}^{0.4} \frac{1}{n}[w(x)-g(\hat{x})]\, dx\, dt_E = \frac{0.01}{n^2}$$

As a check, we note that the sum of all the players' payoffs is

$$(0.23 - 0.25c - \frac{0.01}{n}) + \frac{1}{2}(0.29 - \frac{c}{2} + \frac{0.01}{n^2}) + (n - \frac{1}{2})(\frac{0.01}{n^2}) = 0.375 - \frac{1}{2}c ,$$

which is equal to the total surplus $S(1)$.

4.1. On the Results

These calculations reveal that the individual worker has bargaining power, which despite being infinitesimal is not negligible. Though workers are unorganized, the results still indicate variations in wages which do not correspond to variations in the workers' productivity on their current job.

Specifically, Example I demonstrates the positive relationships between the value of the workers' outside opportunities and their wage (although the Shapley values of the workers decrease as their alternative wage increase, the wage, which includes the value of the opportunities, increases with opportunities). We analyzed a case where workers are relatively more heterogeneous in outside opportunities than on the present job. For such instances, the theory predicts that the wage variations will overstate variations in productivity (which in this case are zero). For the more general case where workers may have different productivity on the current job, and are homogeneous in outside opportunities, we conjecture that wage variations will understate productivities variations. The model provides a consistent way for analyzing wage outcome as function of outside opportunities. It also captures the observation that in certain occupations wages are determined on individual basis as an outcome of negotiations affected by the party's bargaining powers.

From Example II we learn that the relationships between the employer's payoff and the institutional structure ("oceanic" game vs. competitive), depend on the size of the labor force n and on the distribution of the workers' opportunities c. In particular, if n is small and c large, the worse is the employer's bargaining payoff relative to the competitive solution. Thus, the stronger is the employer's incentive to keep a competitive structure. This is clearly reversed as we examine the workers' incentives. The workers' wage is higher under the bargaining structure if n is small and c large. (Note that in the classical model, as long as n is greater then the equilibrium employment level, its size does not have any effect on the wage outcome. Also, the distribution of the workers' opportunities doesn't affect the competitive wage; only the opportunity of the marginal worker matters.) Actual wages were shown to be a direct function of outside opportunities (for a given employer and a given on-the-job productivity). Note that although our model recognizes the bargaining power of the workers (in contrast to the competitive model), the wages are not necessarily higher than in the competitive model; indeed they will be lower when the labor pool is large and the alternative wage is low.

In both examples the workers not hired are able to extract some benefit from the situation, which is not unreasonable because their existence keeps the actual wage below what it would otherwise be. This effect is usually very small. In Equation 3.4 we see that it goes to zero as n (the total number of workers) goes to infinity. We have already addressed this point in Section 2.

Finally, we emphasize that our model is not a competitive structure. The wage solution obtained is not stable in the competitive sense: some workers are getting more than the value of their marginal productivity. The employer is persuaded to agree to such a contract, by the threat of coalitional action by the various possible subsets of the worker pool.

5. Model 2: Labor Partially Organized

Model 2 is an extension of Model 1. We assume that a certain subset U of the workers have already formed a union in order to bargain as a unit with the employer. If some but not all of the workers are so organized we still have an oceanic game, but now there are two atoms: E and U.

For simplicity, we assume that U consists of the workers who occupy a certain interval $[a, b]$ of the alternative-wage scale. This is admittedly a special assumption, but it is not entirely unreasonable since union formation is likely to be a highly selective process. We shall also assume (most of the time) that

$$a < b < w_0 \tag{5.1}$$

where $w_0 = g(n_0)$ is the equilibrium wage rate. Thus U consists entirely of people who would be employed at equilibrium if there were no union.

Figure 5.1 displays some further notation. Thus, u is the size of the union; $\hat{g}$ is the supply function that characterizes the unorganized workers -- i.e.,

$$\hat{g}(x) = \begin{cases} g(x) & \text{if } 0 \le x \le d \\ g(x+u) & \text{if } d < x \le \hat{n} \end{cases} \tag{5.2}$$

and $\hat{w}_0$ is the corresponding equilibrium wage rate.

To determine the Shapley value of this two-atom oceanic game by the "random order" method, we shall require two independent uniformly-distributed random variables, say t_E and t_U, representing the "times of entry" of the atoms E and U into the ordered continuum of unorganized workers. Our probability space is therefore a unit square, as shown in Figure 5.2. The quantity of unorganized ("oceanic") workers who are present when E arrives on the scene is $t_E \hat{n}$ and their alternative wage distribution is given by the compressed curve $\hat{g}_{t_E}$, defined like the g_t of Section 3 (Figure 3.2). Note the discontinuity along the diagonal. If $t_U > t_E$, the union's entry is responsible for a substantial increase in the surplus, but if $t_U < t_E$, U brings in nothing. The boundary case $t_U = t_E$ can be ignored, since it has probability 0.

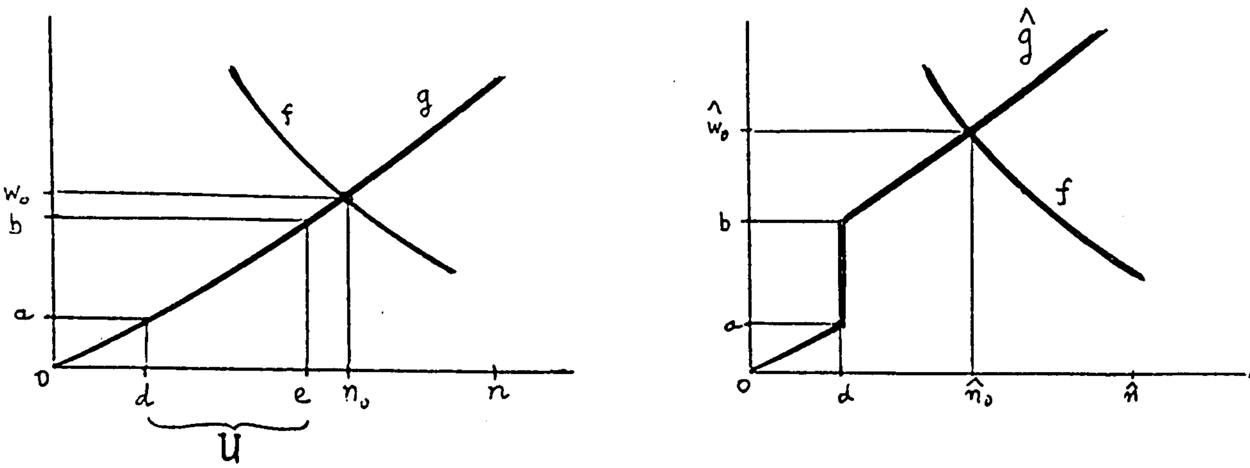

Figure 5.1

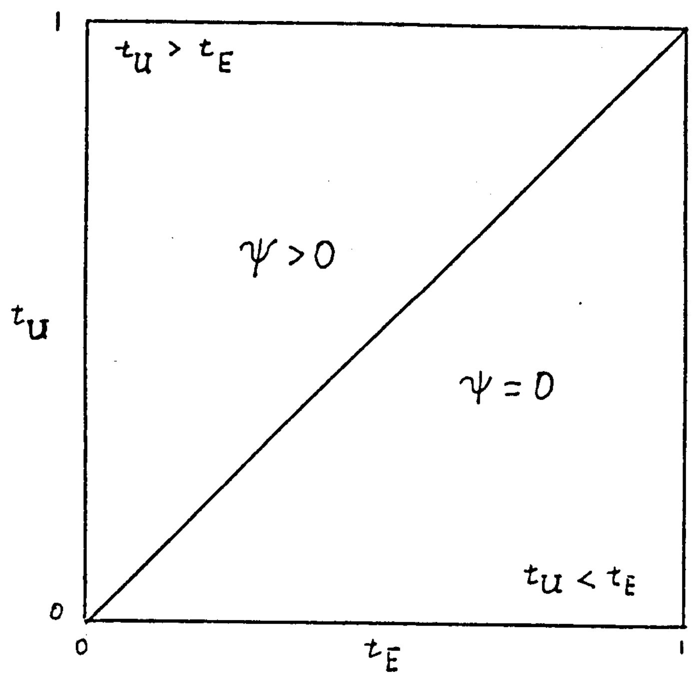

Figure 5.2

Let us now develop a formula for the union's value Φ_U, extending to the two-atom case the geometrical techniques we used in Model 1 (see the discussion in Section 2). To reduce notational clutter we shall write t for t_U until further notice.

Let $\Psi(t)$ denote U's contribution to the surplus if it enters at time t and E is already present, i.e., $t > t_E$. (Note that $\Psi(t)$ in this case is independent of t_E.) Then U's value is given by

$$\Phi_U = \int_{t=0}^{1} \int_{t_E=0}^{t} \Psi(t) dt_E\, dt = \int_{t=0}^{1} \Psi(t) t dt. \tag{5.3}$$

Figure 5.3 provides a geometric representation of the function $\Psi(t)$, namely, the area bounded by $\overline{ABCDFA}$. Here, $\overline{ABC}$ is a portion of the compressed version of $\hat{g}$, defined by $\hat{g}_t(x) = \hat{g}(x/t)$ (cf. (5.2)); $\overline{CD}$ is a portion of the graph of f; and $\overline{AFD}$ is a portion of the graph of a function we shall call $\hat{g}'_t$, defined by

$$\hat{g}'_t = \begin{cases} g_t(x) & \text{if } 0 \le x \le td \\ g(x+(1-t)d) & \text{if } td \le x \le td+u \\ \hat{g}_t(x-u) & \text{if } td+u \le x \le t\hat{n}+u \end{cases} \tag{5.4}$$

which represents the labor supply at time t with *all* the members of U included. (Thus, the segment $\overline{AF}$ is *not* compressed by a factor of t.) From the wage level b on up the graphs of $\hat{g}_t$ and $\hat{g}'_t$ are parallel, with a horizontal separation of u.[10] In order to obtain an analytical expression, we have divided the area representing $\Psi(t)$ into three parts, as shown, whose separate areas are easily written down:

$$\begin{aligned} \Psi_I &= \int_{td}^{td+u} [b-\hat{g}'_t(x)]dx = bu - \int_{d}^{d+u} g(x)dx, \\ \Psi_{II} &= u(\hat{w}'_t - b), \\ \Psi_{III} &= \int_{\hat{w}'_t}^{\hat{w}_t} [f^{-1}(y) - \hat{g}_t^{-1}(y)]dy. \end{aligned} \tag{5.5}$$

Here $\hat{w}_t$ is the equilibrium wage for $\hat{g}_t$ and $\hat{w}'_t$ is the equilibrium wage for $\hat{g}'_t$. Combining (5.3) and (5.5), we obtain

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[10] By our assumption (5.1) we ensure that F lies below D in the figure, whatever the value of t. Without this assumption, additional case distinctions would appear as t approaches 1.

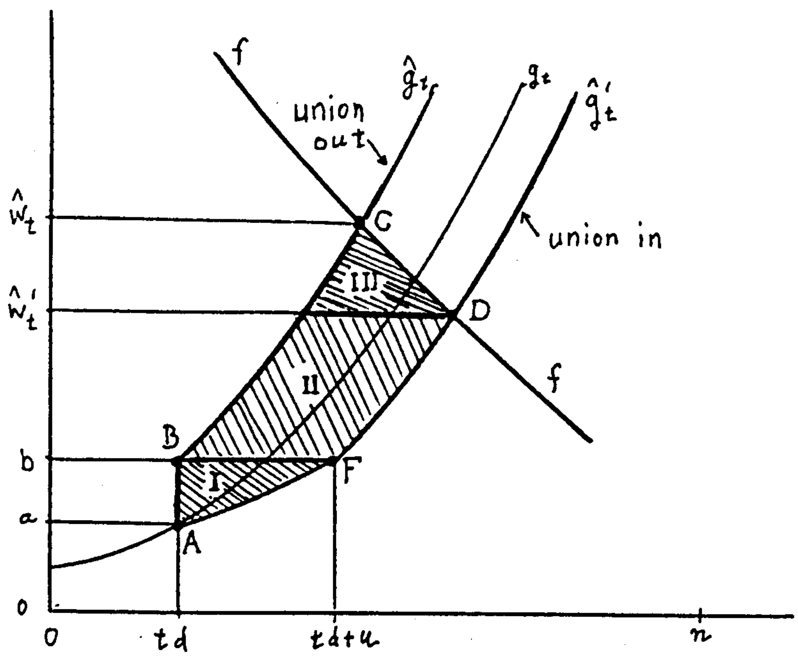


Figure 5.3

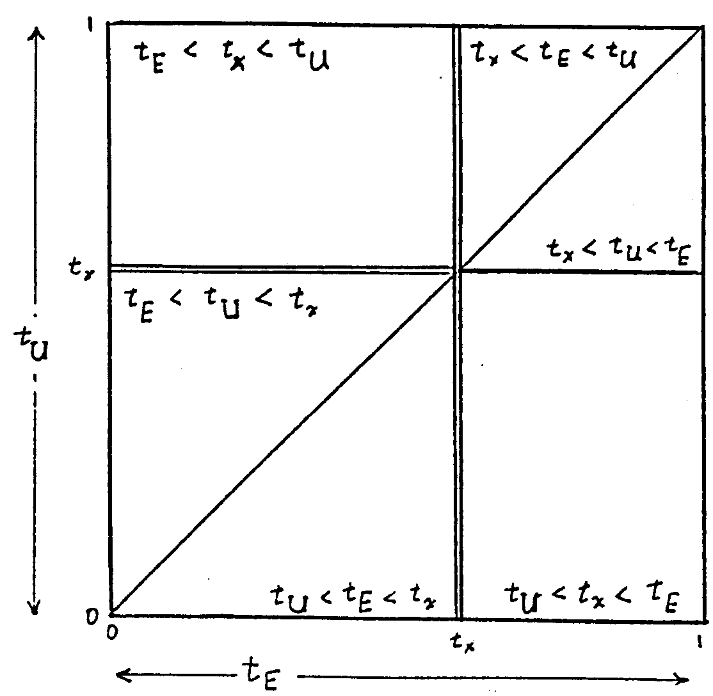


Figure 5.4

$$\Phi_U = \int_0^1 \Psi(t_U) t_U \, dt_U = \int_0^1 (\Psi_I + \Psi_{II} + \Psi_{III}) t_U \, dt_U$$

$$= \int_0^1 \left\{ u\hat{w}'_{t_U} - \int_d^e g(x)dx + \int_{w'_{t_U}}^{\hat{w}_{t_U}} [f^{-1}(y) - \hat{g}_{t_U}^{-1}(y)]dy \right\} t_U \, dt_U .$$

where we now restore the subscript to "t_U".

By a similar calculation, which we omit, it can be shown that the corresponding expression for the employer is

$$\Phi_E = \int_{t_U=0}^{1} \left\{ \int_{t_E=0}^{t_U} \int_0^{\hat{n}_{t_E}} [f(x) - \hat{g}_{t_E}(x)]dx dt_E + \int_{t_E=t_U}^{1} \int_0^{\hat{n}'_{t_E}} [f(x) - \hat{g}'_{t_E}(x)]dx dt_E \right\} dt_U ,$$

where $\hat{n}_t = f^{-1}(\hat{w}_t)$ and $\hat{n}'_t = f^{-1}(\hat{w}'_t)$.

Finally we calculate $\phi(x)$, the value-density function for the unorganized workers (compare section 3). The probability space is now a cube, but we can represent it easily in two dimensions by treating t_x, the arrival time of a typical infinitesimal oceanic player x, as a variable marker on the t_E and t_U scales, as shown in Figure 5.4. The six possible order of entry of E, U and the infinitesimal player x are conveniently grouped into three cases (heavy double lines in Figure 5.4). If $t_x < t_E$ (at the right -- total probability $1-t_x$), the oceanic player contributes nothing. If $t_x > t_E$ but $t_x < t_U$ (upper left -- total probability $t_x(1-t_x)$) he contributes $\max\{0, \hat{w}_{t_x} - g(x)\}dt_x$. If $t_x > t_E$ and $t_x > t_U$ (lower left -- total probability t_x^2) he contributes $\max\{0, \hat{w}'_{t_x} - g(x)\}dt_x$. So we obtain, writing "t" for "t_x" and integrating,

$$\phi(x) = \int_0^1 [(t - t^2)\max\{0, \hat{w}_t - g(x)\} + t^2 \max\{0, \hat{w}'_t - g(x)\}]dt.$$

5.1. Collective vs. Individual Bargaining -- I

As an application of this analysis we shall show that it is better for the members of U to bargain as a union than as individuals – at least if the functions f and g are linear. Thus, we shall be comparing Φ_U (above) with

$$\int_{x \in U} \tilde{\phi}(x)dx, \tag{5.6}$$

where $\tilde{\phi}$ is the value-density function for the "ocean" of Model 1.

Figure 5.5 shows the comparison. As previously, the integrand $\Psi(t)$ in the unionized case (Model 2) is given by the area of $\overline{ABCDFA}$. The corresponding integrand for an infinitesimal set "dx" of unorganized workers in Model 1 is given by a narrow strip along the g_t curve (see Figure 3.4). Its vertical extent is from $g_t(tx)$ to w_t while its horizontal extent is everywhere dx, so the area (disregarding second-order infinitesimals) is given by $(w_t-g_t(tx))dx$. Since $g_t(tx)$ is just $g(x)$, the combined contributions of all the members of U is

$$\int_d^e (w_t-g(x))dx,$$

as shown in inset #1, which has the same area as $\overline{ABC'D'FA}$ in the main diagram of Figure 5.5; call this area $\tilde{\Psi}(t)$. We must therefore examine the difference[11] $\Psi(t)-\tilde{\Psi}(t)$. But this is just the difference between the two triangles IV_t and V_t. Our claim is that IV_t is *larger on average* than V_t when all values of t_E and t with $t_E \le t$ are taken into account.

Note first that IV_t and V_t are similar triangles, with bases tu and $(1-t)u$ respectively. Let

$$g(x) = \alpha x \quad , \qquad f(x) = \beta - \gamma x,$$

where α, β, γ are positive constants. Then, as shown in inset #2,

$$IV_t = \frac{\alpha\gamma t^2 u^2}{2(\alpha+\gamma t)} \quad , \qquad V_t = \frac{\alpha\gamma(1-t)^2 u^2}{2(\alpha+\gamma t)}. \tag{5.7}$$

Our claim is that

$$\int_{t=0}^{1} \int_{t_E=0}^{t} [IV_t - V_t] dt_E \, dt > 0.$$

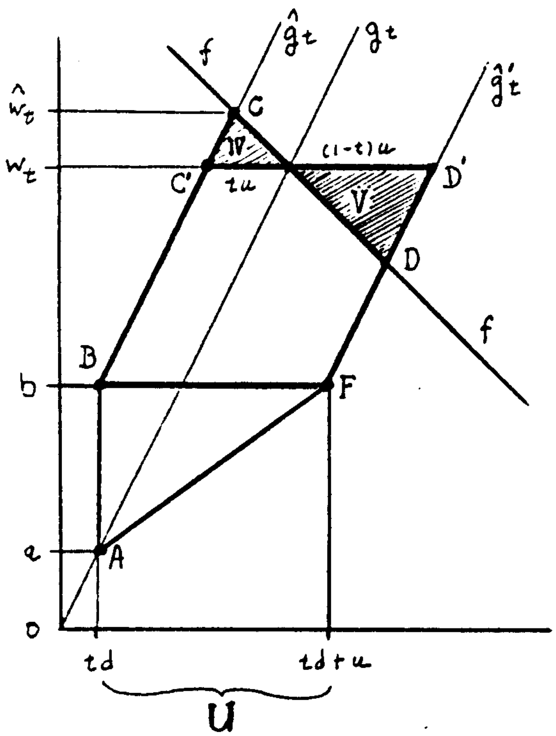
From (5.7) we have

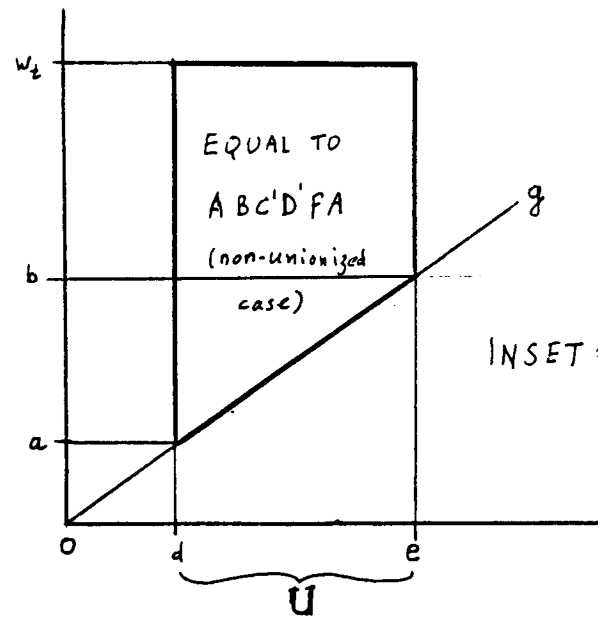
$$IV_t - V_t = \frac{\alpha\gamma u^2(2t-1)}{2(\alpha+\gamma t)}$$

In particular, for t between 0 and 1/2 we have,

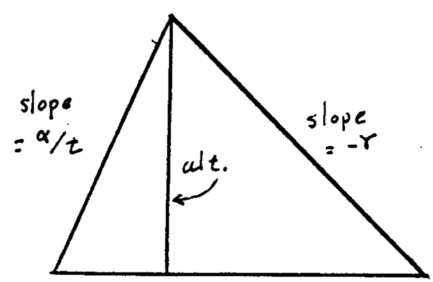
$$\int_0^t [IV_t - V_t] dt_E = \frac{\alpha\gamma u^2(2t-1)t}{2(\alpha+\gamma t)},$$

[11] In order to set up this comparison we have changed the order of integration, bringing the $\int dx$ integral inside the double integral $\iint dt_U dt_E$.





INSET # 1



INSET #2

<u>Calculation of areas:</u>

We have-- $\dfrac{\text{alt}_{IV}}{\alpha/t} + \dfrac{\text{alt}_{IV}}{\gamma} = tu.$

So -- $\text{alt}_{IV} = \dfrac{\alpha\gamma t u}{\gamma t + \alpha}$

and--- $\text{area}_{IV} = \dfrac{\alpha\gamma t^2 u^2}{2(\gamma t + \alpha)} = IV_t.$

By similarity, $V_t = \left(\dfrac{1-t}{t}\right)^2 IV_t.$

Figure 5.5

$$\int_{0}^{1-t} [IV_{1-t} - V_{1-t}]dt_E = \frac{\alpha\gamma u^2(1-2t)(1-t)}{2(\alpha+\gamma(1-t))}.$$

The sum of these two expressions is

$$\frac{\alpha\gamma u^2(2t-1)}{2}\left[\frac{t}{\alpha+\gamma t} - \frac{1-t}{\alpha+\gamma(1-t)}\right] = \frac{\alpha^2\gamma u^2(2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))},$$

which we see is always nonnegative, and in fact is positive everywhere except at $t=1/2$. So we conclude:

$$\Phi_U - \int_U \tilde{\phi}(x)dx = \left(\int_{t=0}^{1/2} + \int_{1/2}^{1}\right) \int_{t_E=0}^{t} [IV_t - V_t]dt_E\,dt = \int_{t=0}^{1/2} \int_{t_E=0}^{t} [IV_t - V_t + IV_{1-t} - V_{1-t}]dt_E\,dt$$
$$= \int_{0}^{1/2} \frac{\alpha^2\gamma u^2(2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))}dt > 0.$$

This completes the proof that in the linear case the members of U are better off organized than unorganized.[12]

5.2. Collective vs. Individual Bargaining -- II

We shall now prove that when *all* workers (hired and unhired) are unionized and bargain as a unit, their total payoff is higher than what they get in Model 1, where none are unionized and the employer E is the only atom. This result does not require that g and f be linear functions.

Consider Figure 5.6. The total surplus is

$$S = \int_{0}^{n_0} [f(x) - g(x)]dx, \tag{5.8}$$

and the value of the game for the employer (as derived in Section 3) is

$$\Phi_E = \int_{t=0}^{1} \int_{x=0}^{n_t} [f(x) - g_t(x)]dx\,dt = \int_{0}^{1/2}\left\{\int_{0}^{n_t} [f(x) - g_t(x)]dx + \int_{0}^{n_{1-t}} [f(x) - g_{1-t}(x)]dx\right\}dt$$

Taking y in place of x as the independent variable, we can rewrite this as

---

12 On the basis of several examples we have calculated, we conjecture that this remains true for all monotonic functions g and f and for any measurable set U consisting of hired workers. But we have found that it is not true in general if at the efficient production levels some of the workers in U are not hired, even if g and f are linear. Indeed, forming such a mixed set into a union is inherently inefficient since it would result in the employer either hiring some who should not have been hired, or not hiring some who should have been hired. Such inefficiencies - reducing the total surplus - can easily diminish the value payoffs to the workers in U as well as to the other players.

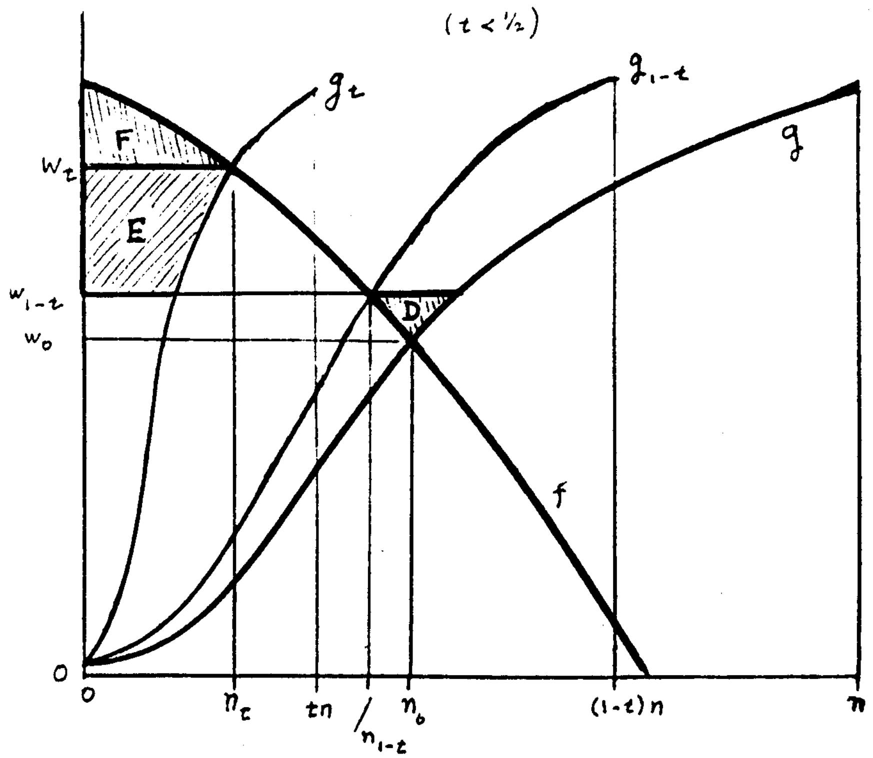


Figure 5.6

$$\Phi_E = \int_0^{1/2}\left\{\int_{y=0}^{f(0)}[A_1+A_2]dy\right\}dt$$

where

$$A_1 = \min\{g_t^{-1}(y), f^{-1}(y)\}$$
$$A_2 = \min\{g_{1-t}^{-1}(y), f^{-1}(y)\}.$$

There are then three cases:

$$\begin{aligned} &\text{for } 0 \le y \le w_{1-t}: && A_1 = g_t^{-1}(y),\ A_2 = g_{1-t}^{-1}(y),\\ &\text{for } w_{1-t} \le y \le w_t: && A_1 = g_t^{-1}(y),\ A_2 = f^{-1}(y),\\ &\text{for } w_t \le y \le f(0): && A_1 = f^{-1}(y),\ A_2 = f^{-1}(y), \end{aligned}$$

and we observe that

$$\int_0^{w_{1-t}} g_t^{-1}(y)dy = \int_0^{w_{1-t}} (g^{-1}(y) - g_{1-t}^{-1}(y))dy,$$

as indicated by the horizontal dotted lines in Figure 5.5. So the employer's value payoff may be calculated as follows:

$$\Phi_E = \int_{t=0}^{1/2}\left\{S + \int_{w_0}^{w_{1-t}} [g^{-1}(y) - f^{-1}(y)]dy + \int_{w_{1-t}}^{w_t} g_t^{-1}(y)dy + \int_{w_t}^{f(0)} f^{-1}(y)dy\right\}dt$$
$$= \int_0^{1/2} Sdt + \int_0^{1/2} Kdt = \frac{S}{2}+\frac{K}{2},$$

where K denotes the sum of the areas of D, E and F in Figure 5.5. Since the total value to all players is the surplus S, the value to the workers must be

$$S - \Phi_E = S/2 - K/2.$$

On the other hand, if all the workers get together and bargain as a unit, then it is just a two-player simple bargaining game, and the value payoff to each side is just $S/2$. So if the entire labor force is unionized, their total wage is $K/2$ greater than if they had no union at all.

6. Model 2: Examples

Here the objective is to demonstrate the computation and behavior of the oceanic game solutions in specific examples. In Section 6.1 we apply the general solution to the case where workers not hired are identical -- i.e., homogeneous with respect to their alternative wage. This is followed in section 6.2 by an example of specific functions where the payoffs are computed and compared under several different institutional structures.

6.1. General Results for Homogeneous Pool of Unhired Workers

Let $g(x)$ be monotonically increasing until the equilibrium employment level, and constant thereafter: $g(x) = w^*$ for $x > n_0$. Consider Figure 6.1: it will be seen that this is a specialization of the situation depicted in Figure 5.1. Assume that the union consists of all the hired workers: $U = [0, n_0]$. The number of workers that would be effectively employed if U were absent is denoted by n_2, while n^* will denote the total number of non-union workers; we assume that $n^* \geq n_2$.

The players in this oceanic game are E, U, and the continuum $[0, n^*]$. In view of the homogeneity of the ocean, the possible coalitions reduce essentially to the following four cases, where t represents the fraction of the ocean present:

$S_1(t)$ - A coalition consisting only of the set $[0, tn^*]$.

$S_2(t)$ - A coalition consisting of E and $[0, tn^*]$.

$S_3(t)$ - A coalition consisting of U and $[0, tn^*]$.

$S_4(t)$ - A coalition consisting of E and U and $[0, tn^*]$.

The characteristic function v is then given by

$$v(S_1(t)) = 0, \quad v(S_2(t)) = \int_0^{min(tn^*, n_2)} [f(x) - w^*]dx, \quad v(S_3(t)) = 0, \quad v(S_4(t)) = \int_0^{n_0} [f(x) - g(x)]dx \quad (6.1)$$

The value payoff to the union, Φ_U, is derived as follows: With probability 1/2, U enters before E; in this case the marginal contribution of U is zero. With the remaining probability 1/2, U enters after E; in this case U's marginal contribution is $v(S_4(t_U)) - v(S_2(t_U)) = S - v(S_2(t_U))$, where S denotes the total surplus (see equation 5.8). Thus, the value payoff to the union is

$$\Phi_U = \int_{t_U=0}^{1} \int_{t_E=0}^{t_U} [v(S_4(t_U)) - v(S_2(t_U))]dt_E\, dt_U = \int_0^1 [v(S_4(t_U)) - v(S_2(t_U))]t_U\, dt_U$$

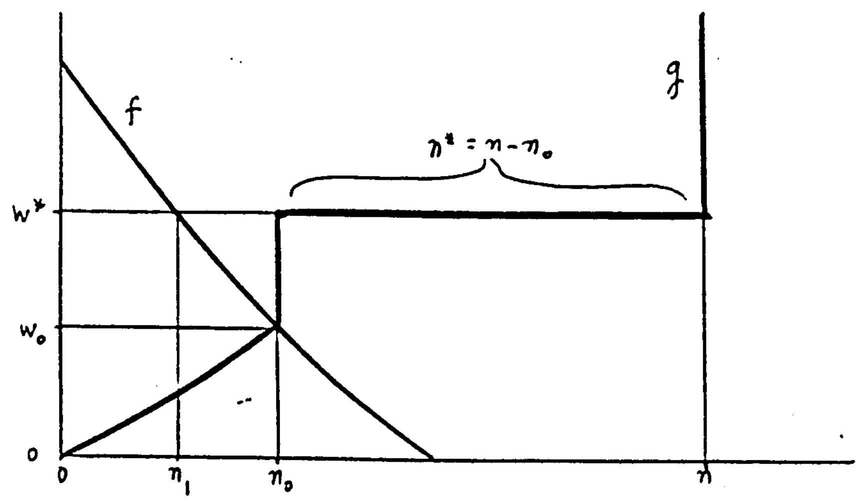


Figure 6.1

$$= \int_0^1 v(S_4(t_U))t_U\,dt_U - \int_0^{n_2/n^*} v(S_2(t_U))t_U\,dt_U - \int_{n_2/n^*}^1 v(S_2(\frac{n_2}{n^*}))t_U\,dt_U$$

$$= \frac{1}{2}S - \int_0^{n_2/n^*} v(S_2(t_U))t_U\,dt_U - \int_{n_2/n^*}^1 v(S_2(\frac{n_2}{n^*}))t_U\,dt_U \tag{6.2}$$

The value payoff to the employer is derived as follows: When the employer enters before the union, his marginal contribution is $v(S_2(t_E))$. When he enters after the union, it is $v(S_4(t_E))$. Thus,

$$\Phi_E = \int_{t_E=0}^1 \int_{t_U=t_E}^1 v(S_2(t_E))dt_U\,dt_E + \int_{t_E=0}^1 \int_{t_U=0}^{t_E} v(S_4(t_E))dt_U\,dt_E]$$

$$= \int_0^1 v(S_2(t_E))(1 - t_E)dt_E + \int_0^1 v(S_4(t_E))t_E\,dt_E$$

$$= \frac{1}{2}S + \int_0^1 v(S_2(t_E))(1 - t_E)dt_E \tag{6.3}$$

Finally, since the total value of all the players is S, the value to the oceanic players is

$$\Phi_{\text{ocean}} = S - \Phi_U - \Phi_E = \int_0^{n_2/n^*} v(S_2(t))t\,dt + \int_{n_2/n^*}^1 v(S_2(n_2/n^*))t\,dt - \int_0^1 v(S_2(t))(1-t)dt.$$

6.2. A Comparison of Different Institutional Structures

Let n_0, n_2, n^*, w_0 and w^* be as in Section 6.1, and for ease of calculation let us linearize f and g as follows:

$$f(x) = \alpha-\beta x$$
$$g(x) = \begin{cases} \gamma x & \text{for } 0 \le x \le n_0 \\ w^* & \text{for } n_0 < x \le n_0+n^* \\ \infty & \text{for } x > n_0+n^* \end{cases} \tag{6.4}$$

where α, β and γ are positive numbers. Note the identities $\alpha-\beta n_0 = \gamma n_0$ and $\alpha-\beta n_2 = w^*$. The total available surplus is easily seen to be $S{=}\alpha n_0/2$.

I. Let the union U consist of the hired workers, while the rest form an unorganized ocean. From (6.1), the characteristic function is given by

$$v(S_1(t)) = 0,$$

$$v(S_2(t)) = \alpha M_t - \beta M_t^2/2 - w^* M_t = \beta n_2 M_t - \beta M_t^2/2,$$
$$v(S_3(t)) = 0,$$
$$v(S_4(t)) = \alpha n_0 - \beta n_0^2/2 - \gamma n_0^2 = \beta n_0^2/2,$$

where M_t denotes $\min(tn^*, n_2)$ and we have made use of the identities mentioned above.

From (6.2) the value payoff to U is

$$\Phi_U = \frac{S}{2} - \int_0^{n_2/n^*} (\beta n_2 t n^* - \beta (tn^*)^2/2) t dt - \int_{n_2/n^*}^{1} (\beta n_2^2/2) t dt$$

$$= \frac{S}{2} - \frac{\beta n_2^2}{4} + \frac{\beta n_2^4}{24 n^{*2}}, \tag{6.5}$$

by straightforward calculus. From (6.3), the value payoff to E is

$$\Phi_E = \frac{S}{2} + \int_0^{n_2/n^*} (\beta n_2 t n^* - \beta (tn^*)^2/2)(1-t) dt + \int_{n_2/n^*}^{1} (\beta n_2^2/2)(1-t) dt$$

$$= \frac{S}{2} + \frac{\beta n_2^2}{4} - \frac{\beta n_2^3}{6 n^*} + \frac{\beta n_2^4}{24 n^{*2}}. \tag{6.6}$$

Finally, the total value payoff to the oceanic players is

$$\Phi_{\text{ocean}} = S - \Phi_U - \Phi_E = \frac{\beta n_2^3}{6 n^*} - \frac{\beta n_2^4}{12 n^{*2}}; \tag{6.7}$$

this would of course have to be realized by a side payment, not through wages (see the discussion of this point in Section 2). We see that if $n^* \to \infty$ this payment goes to zero, leaving us in the limit with

$$\Phi_E = \frac{S}{2} + \frac{\beta n_2^2}{4}, \quad \Phi_U = \frac{S}{2} - \frac{\beta n_2^2}{4}.$$

The term $\frac{\beta n_2^2}{4}$ is half of the area in Figure 6.1 that lies above the wage level w^*. Thus, in the presence of an infinite pool of workers willing to work for w^*, the value solution for the negotiation game awards all the "high" surplus to E and divides the rest equally between E and U, with nothing to the unhired workers.

It is interesting (but not necessary) to express the union value payoff in terms of an equivalent wage rate w_U, on the assumption that the union changes to have all its members paid an equal amount, regardless of their alternative wages:

$$w_U = \frac{\Phi_U + \gamma n_0^2/2}{n_0} = \frac{w_0}{2} + \frac{\alpha}{4} - \frac{\beta n_2^2}{4n_0} + O\left(\frac{1}{n^{*2}}\right).$$

Note that it is entirely possible here that $w_U < w_0$. In other words, the union members may be worse off than if they did not bargain collectively or as individuals. This could happen, for example, if α were low (i.e., near w_0) or if n_2 were high (i.e., near n_0). In the former case there would not be much employer surplus for the union to go after, and in the latter case w^* is only slightly above w_0, so the union's bargaining threats are weak.

II. Now let $\bar{U}$ be a union of all available workers. The bargaining game is reduced to just two players E and $\bar{U}$, and the Shapley value divides the surplus equally:

$$\Phi_E = \Phi_{\bar{U}} = \frac{S}{2} = \frac{\alpha n_0}{4}.$$

Comparing with the previous case, we see that the full union $\bar{U}$ is better for all workers combined than the partial union U.

$$\Phi_{\bar{U}} \geq \Phi_U + \Phi_{\text{ocean}}.$$

In fact, this still holds without the linearity (6.4), since from (6.3) we have

$$\Phi_E = \frac{S}{2} + \int_0^1 \int_0^{M_t} [f(x) - w^*](1-t)\,dx\,dt,$$

which shows that the employer does better in the presence of U than $\bar{U}$ -- and hence the workers collectively do worse. However, this does not mean that union of the employed workers would necessarily find it advantageous to admit the ones that would not be hired at the efficient production level if its objective is to maximize *per capita* gain to its members.

III. Finally, let there be two unions: U_1 consisting of the n_0 employed workers and U_2 consisting of the rest of the pool. There are now three atoms and no ocean. The characteristic function is:

$$v(\varnothing) = 0 \ , \quad v(\{E\}) = 0 \ , \quad v(\{U_1\}) = 0 \ , \quad v(\{U_2\}) = 0 \ , \quad v(\{U_1, U_2\}) = 0$$

$$v(\{E, U_1\}) = \int_0^{n_0} [f(x) - g(x)]\,dx = S \ , \quad v(\{E, U_2\}) = \int_0^{n_2} [f(x) - w^*]\,dx \ ,$$

$$v(\{E, U_1, U_2\}) = \int_0^{n_0} [f(x) - g(x)]\,dx = S \ ,$$

where we have assumed as before that $n_2 \leq n^*$. The Shapley value to a player P may be determined by taking a random permutation of the players and calculating P's expected contribution. For player U_1 only the orders EU_1U_2, EU_2U_1 and U_2EU_1 matter, because of all the zeros in the characteristic function, and we have

$$\Phi_{U_1} = \frac{1}{6}[v(\{E,U_1\}) - v(\{E\})] + \frac{1}{3}[v(\{E,U_1,U_2\}) - v(\{E,U_2\})]$$

$$= \frac{S}{6} + \frac{1}{3}\left[S - \int_0^{n_2}[f(x) - w^*]dx\right] = \frac{S}{2} - \frac{1}{3}\int_0^{n_2}[f(x) - w^*]dx .$$

Similarly,

$$\Phi_{U_2} = \frac{1}{6}[v(\{E,U_2\}) - v(\{E\})] + \frac{1}{3}[v(\{E,U_1,U_2\}) - v(\{E,U_1\})]$$

$$= \frac{1}{6}\int_0^{n_2}(f(x) - w^*)dx + \frac{1}{3}[S - S] = \frac{1}{6}\int_0^{n_2}(f(x) - w^*)dx .$$

So

$$\Phi_E = S - \Phi_{U_1} - \Phi_{U_2} = \frac{S}{2} + \frac{1}{6}\int_0^{n_2}(f(x) - w^*)dx ,$$

and we see immediately that "one union is better than two", since in the present structure they are again getting less than $\Phi_E = S/2$.

Plugging in our linear functions (6.4), we obtain without difficulty

$$\Phi_{U_1} = \frac{S}{2} - \frac{\beta n_2^2}{6} , \quad \Phi_{U_2} = \frac{\beta n_2^2}{12} , \quad \Phi_E = \frac{S}{2} + \frac{\beta n_2^2}{12}$$

This may be compared with the corresponding solution obtained for structure I. It is noteworthy that the size of n^* does not affect the present case, except through the assumption that $n^* \geq n_2$.

7. Concluding Remarks

Models of wage determination tend to ignore the fact that wages are directly or indirectly the outcome of negotiated contracts between the workers (individually or collectively) and the employers. As one recognizes the potential losses in terminating employment relationships, it becomes clear that bargaining power generated by the ability of each side to inflict costs on the other should be explicitly considered in the analysis of wage determination. Studies that do account for bargaining potentials are mainly in the framework of single employer and single worker. The complex problem of n-person bargaining has not yet explicitly modeled.

This is in spite the fact that the structures of labor markets are not uniform: they vary from the extreme situation where a single employer faces a single worker to the other extreme where many workers face a single large firm or many small firms. Since most real structures are at neither extreme but rather something in between, we have developed a framework for wage determination which can deal effectively with many intermediate structures on a consistent basis. In particular, the use of "oceanic" games allows us to consider labor contract bargaining even when the setting includes a continuum of unorganized workers.

Our approach in this paper has stressed the role of the workers' alternative opportunities; as a result, our models are rather abstract. We do not attempt to represent the bargaining *process* as a game in strategic form, with proposals and counter proposals following according to some fixed protocol. Such process-specific models are suitable for formalized bargaining situations (e.g., auctions), but are too restrictive to do justice to the free-wheeling and essentially cooperative[13] nature of labor-management negotiations. Instead, we adopt the viewpoint that the wage determined will be the result of the underlying bargaining powers of the participants, irrespective of tactical considerations, and use a cooperative game in characteristic function form as the basic model with the Shapley value as the solution concept.

In the first model we dealt with unorganized labor. It was shown that for a given employers' structure and a given on-the-job productivity, the workers wage is inversely related to the size of the labor force and is positively affected by the workers' outside opportunities. The model is capable of predicting wage variations among workers that are independent of productivity variations, even when workers bargain multilaterally as individuals without a formal union. In the second model such a formal union was introduced, acting as a single agent to represent at least some of the workers. The Shapley value payoff to all the participants was derived, and the question of the wage to the union members under different institutional structures was addressed. The major predictions of the first model hold in

[13] "Cooperative" in the game theory sense; i.e., when the contenders come to terms, they can write any contract they please -- and then they are bound by it.

the presence of a union as well. In addition it was shown, among other results, that when *all* available workers negotiate through a single union (bilateral monopoly), their total payoff is higher than what they would get as individual negotiators.

Another feature of our approach is that some payments are made to workers in the labor pool who do not in the end get hired. The reason is that their presence influences the wage settlement. These side payments are usually small (see for example Equations 3.4 and 6.5-6.7). It is not clear how these payments might be implemented in practice, but a possible interpretation might regard them as a measure of unrealized bargaining power, expressed in terms of what the recipients could get if utility were freely transferable. Using the NTU value theory would be another way to deal with the imperfect transferability, as discussed in Section 2.

At a more general level, there is an important distinction between the classical model and the present bargaining model in the factors they capture as affecting the wage outcome: In the classical models only *local* properties are important in affecting the outcome. For example, the size of the labor pool capable of working in a particular occupation does not affect the classical wage outcome provided that its reservation wages are above the equilibrium employment level. In contrast, the *size* and the *whole distribution* of *outside opportunities* of the labor force (not only around the equilibrium level) play a significant role in determining the outcome under the present bargaining model.

Finally, in a subsequent research, we wish to extend the present model in two directions: to allow for more than one type of labor, and to investigate in more detail the various institutional structures that might be adopted on the employer's side, there being a somewhat delicate balance to be struck between cooperation in wage negotiation and competition in the product market.

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