THE "SMITHIAN" SELF AND ITS "BAYESIAN" BRAIN*

by

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*Apologies to Popper and Eccles (1977). I am indebted to Vela Velupillai for his critical comments and encouragement. Computability, the von Neumann-Morgenstern formalism, and the McCulloch-Pitts-Turing approach are studied in the companion piece McCall and Velupillai (1989).

Synopsis And A Caveat

The paper "The 'Bayesian' Brain and Its 'Smithian' Self" outlines a project which K. Velupillai and I have just begun. The purpose of this synopsis is to present a brief review of the project and to clarify some of the obscurities in the paper by summarizing its major objectives.

During the past few years the practical and scientific advances by parallel processing and the associated neural network methodology have been spectacular. This burst of creativity had its source in the interdisciplinary research of physics, neuroscience, computer science, psychology and artificial intelligence. Economics is conspicuously absent from this interdisciplinary venture even though it contains a richness of economic implications and connections. The entire enterprise may be enlightened if these economic connections are exploited.

The purpose of our research is to demonstrate that economic science is a process with its foundations in the natural sciences (technology) and neuroscience (tastes). Much of the current economic research is static with respect to tastes and technology and ignores these fundamental processes. We propose and are showing how economics can be reconstituted as a process based on the recent advances in neuroscience, computer science, psychology and biophysics. The economic insights which should flow naturally from a process-oriented economic science will edify economics and may provide guidance for computer and biological neural science.

The efficient flow of information within and across organizations, the evolution of organizations in a fluctuating environment, the surveillance of the behavior of human and physical processes ensuring that the initial classifications are not disturbed by oscillating heterogeneity; these are some of the problems considered in the economic design of flexible organizations. Our analysis begins with <u>individual</u> decisionmaking under imperfect information and then considers the modular genesis of the organization formed by nonlinear vibrating networks of adaptive individuals. The vibrations are induced by random processes, which are constantly modifying the networks as they adapt to new information.

The birth and death process plays an important role in our analysis. With respect to individuals, each is a collection of relations which are being formed and terminated continuously. The information processing required for these formation and dissolution decisions is also transmitted through a network, characterized as a birth and death process. The spatial relations of the individual also are described by a spatial birth and death process. Hence, the spatial-temporal decisions of the economic agents are intertwined processes. The pertinent networks depend on the agent's decision <u>or</u> the analyst's question. The same sort of networks constrain and enlighten the neuron's "decisionmaking". This birth and death metaphor has proved useful for studying these evolving, self-similar processes. The well-known neural networks are all founded on temporal and spatial birth and death processes. In these models it is imperative that the appropriate time scales <u>and</u> spatial scales be identified in order that the network can be applied to brain-like processes. As we will see, the prototypical neural net is the Ising model. It appears that a percolation generalization of this basic model might begin to mimic brain processes.

In the past, economic models have been grandiose, static, and beset by methodological flaws and deficient data. Their dramatic failures may explain the current status of economics among the scientific community.

Our program is ambitious and has many limitations. Some of these will be alleviated by the illumination received from the other sciences involved in neural network analysis. Indeed, we plan to test many of our models using data generated in neuroscience laboratories. The robustness of the economic implications will be assessed by simulation methods devised by computer scientists.

This interplay between economics and the other sciences will be mutually beneficial. We reemphasize that its feasibility depends on recasting economics as a dynamic science. This is our first priority. The strengthened economic science should clarify problems which are currently intractable and make conditional forecasts.

We hope the following comments clarify our project:

1. Economic science pervades reality from the structure of the crystal to the subtle gestures facilitating communication among members of a sophisticated society. The economy of nature is the unifying principle encompassing both the deterministic processes of Parmenides and the stochastic processes of Heraclitus.

2. All natural processes are guided by an information network that emerges spontaneously and provides order as the processes adapt to environmental fluctuations. The formation of novel entities and the extinction of nonadaptive entities is the essence of the evolutionary process which enforces the basic economic principle of no-arbitrage. The harmony and symmetry of the natural economy is the consequence of the no-arbitrage principle. Adam Smith discovered this principle of spontaneous order in nations. He recognized its dependence on information flows ("feedback") provided by the institutions comprising a civilized society. Charles Darwin applied this principle to the evolution of all organisms. Modern biochemical physics is applying this principle to all natural processes.

3. Our view of the no arbitrage principle encompasses maximization, convexity, and the strategies characterizing relations among individuals. These are the unifying principles used by Samuelson, Arrow-Debreu, and game

theorists respectively. The no arbitrage principle is closely related to optional stopping, the process version of the static derivative. There is also an intimate connection with (sub)modularity, the combinatoric version of convexity. Finally, the Markov property permeates our analysis from the information and decision networks, through the Ising model, the prototypical neural network, to computability and complexity theory.

4. Our first task is to revive the economics of Adam Smith as portrayed in The Wealth of Nations and The Theory of Moral Sentiments.

5. Smithian economics is a unified process of individual decisionmaking in a fluctuating and evolutionary environment. These decisions when guided by self-interest, constrained by the Impartial Spectator, society's rules and institutions usually are both rewarding and prudent. There is an element of chance which confounds these appraisals. Smith clearly enunciates this conundrum. Life is a game where the players behave according to society's institutional rules. The outcomes depend on a mixture of skill, chance and perseverance, with the rules yielding "fairness".

6. The second task is to link Smithian economics with the recent advances in neurobiology, computer science, and related disciplines, especially psychology and physics. A comprehensive review of neurobiology is presented in Shepherd (1988), who maintains that "the essence of the nervous system is organization; neural mediation of behavior requires coordination of a hierarchy of level of organization." The synapse is central to this organization. The coordination of the hierarchical levels of organization is achieved by the vast neural system as it mediates behavior. This entails a complex flow of information, a "beautifully orchestrated division of labor" within the neuron, and the neural networks. It seems clear that the immune system is an integral part of this information system. "Hormones

play essential roles in <u>the economy of the body</u> ... It is becoming increasingly difficult to make a distinction between a neuroendocrine cell and a nerve cell".

7. The traditions, rules, customs and routines comprising a civilized society are evolutionary phenomena possessing the wisdom of the ages. These institutions reside in societies, individuals, and cells. While some of these institutions may be "improved" or even discarded, such changes should be deliberate, thoughtful, and reversible. Frequently, the order which they induce within societies, individuals, and cells is incomprehensible. Yet, it is precisely this order and the corresponding organization which makes civilization and life possible.

> The process of the evolution of a system of values passed on by cultural transmission must implicitly rest on criticism of individual values in the light of their consistency, or compatibility, with all other values of society, which for this purpose must be taken as given and undoubted. The only standard by which we can judge particular values of our society is the entire body of other values of the same society. More precisely, the factually existing, but always imperfect, order of actions produced by obedience to these values provides the touchstone for evaluation. Because prevailing systems of morals or values do not always give unambiguous answers to the questions which arise, but often prove to be internally contradictory, we are forced to develop and refine such moral systems continuously. We shall sometimes be constrained to sacrifice some moral value, but always only to other moral values which we regard as superior. We cannot escape this choice, because it is part of an indispensable process. In the course of it we are certain to make many mistakes. Sometimes whole groups, and perhaps entire nations, will decline, because they chose the wrong values. (Hayek, 1978).

There is much to be learned from the study of <u>evolving</u> institutions. Legal systems, accounting, insurance and monetary systems are recognized as important components of economics.

8. The study of the cell, the hormone system and neural networks require interdisciplinary research. Ronald Coase (1978) has warned economists that:

> It is not general theories which we lack, but theories which explain the workings of our actual economic system ... our analy-

> sis of the division of labor is primitive ... We likewise have a primitive analytical system to handle the firm, the process of contracting, and property rights...

He fears that dabbling in these other sciences will be more attractive than using their findings to improve economics. On the other hand, Coase recognizes that:

> ... modern utility theory ... largely regards man as a rational utility maximizer. It tells us little about the purposes which impel people to action ... it seems unlikely that we can make much progress without a comprehensive view of man's nature.

The lessons to be learned from the hormone system, the cell, and biochemical physical processes are substantial. They will as Coase hopes, "lead us back to our founding father".

9. The transmission of information to the cerebral cortex and the behavioral response have been studied using analogies from electricity, mechanics, hydrodynamics, economics, and stochastic processes. These physical analogies are intertwined in Feynman's (1963) beautiful description of <u>Stepping On A Sharp Stone</u>.

A man unexpectedly steps on (incurs) several tacks (a tax):

> ... somehow or other the information goes from the leg up. It is interesting how that happens. In their study of nerves, the biologists have come to the conclusion that nerves are very fine tubes with a complex wall which is very thin; through this wall the cell pumps ions, so that there are positive ions on the outside and negative ions on the inside, like a capacitor. Now this membrane has an interesting property; if it "discharges" in one place, i.e., if some of the ions are able to move through one place, so that the electric voltage is reduced there, that electrical influence makes itself felt on the ions in the neighborhood, and it affects the membrane in such a way that it lets the ions through at neighboring points also. This in turn affects it farther along, etc., and so there is a wave of "penetrability" of the membrane which runs down the fiber when it is "excited" at one end by stepping on the sharp stone. This wave is somewhat analogous to a long sequence of vertical dominoes; if the end one is pushed over, that one pushes the next, etc. Of course this will transmit only one message unless the dominoes are set up again; and similarly in the nerve cell, there are processes which pump the ions slowly out again, to get the nerve ready for the next impulse. So it is that we know what we are doing (or at least where we are). Of course the electrical effects associated with this nerve impulse can be picked up with electrical instruments,

> and because there are electrical effects, obviously the physics of electrical effects has had a great deal of influence on understanding the phenomenon.
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> The opposite effect is that, from somewhere in the brain, a message is sent out along a nerve. What happens at the end of the nerve? There the nerve branches out into fine little things, connected to a structure near a muscle, called an end-plate. For reasons which are not exactly understood, when the impulse reaches the end of the nerve, little packets of a chemical called acetyl-choline are shot off (five or ten molecules at a time) and they affect the muscle fiber and make it contract -- how simple! What makes a muscle contract? A muscle is a very large number of fibers close together, containing two different substances, myosin and actomyosin, but the machinery by which the chemical reaction induced by acetyl-choline can modify the dimensions of the molecule is not yet known. Thus the fundamental processes in the muscle that make mechanical motions are not known.

In this example, Feynman uses metaphors from many disciplines to describe the entire decision process. These subsidiary processes include:

(a) a hydrodynamic process,

(b) a chemical process,

(c) a mechanical process* and

(d) an electrical process.

The corresponding tax example would rely on:

(e) a price process.

A random walk on a graph shows that these metaphors are mathematically equivalent and can be interpreted as random flows in space and time. [Samuelson (1952) employed linear programming analysis to obtain the equivalence between a spatial equilibrium model, linear programming, and electrical networks.]

The mathematical equivalences of Brownian motion <u>and</u> potential theory and Gibbs fields <u>and</u> Markov fields are the source of the Economy of Nature (no-arbitrage principle) as a Unifying Principle.

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*Dominoes are used by McCulloch (1956) to depict logical relations and thresholds.

10. It seems that symmetry resolves the induction problem. The mind comprehends reality when it is symmetrized. Indeed some would go further and maintain that nature abhors asymmetry. The observed "nearly symmetric" world reflects our fallibility. Feynman's explanation is edifying and poetic:

> We might like to turn the idea around and think that the true explanation of the near symmetry of nature is this: that God made the laws only nearly symmetrical so that we should not be jealous of His perfection!

We conjecture that questions become intractable and complex when symmetry is absent. Nature reveals its secrets when interrogated by a symmetric process. Exchangeability and infinite divisibility are useful for decoding nature's messages because they achieve symmetry by partitioning and factorization, respectively.

11. Decisionmaking is always accompanied by evaluations based on incomplete information. The path chosen is a combinatorial entity whose survival depends on the distance between the decisionmaker's and society's evaluations as manifested by prices. In some circumstances (e.g. scientific research), the path and the associated process are evaluated by society, whereas in most cases (e.g. production processes) only the output of the process receives society's attention.

Thus, economics occupies a central position in seemingly remote scientific advances. It orchestrates those spontaneous and intuitive ideas which characterize the creative process. The activity of thresholds (prices) evaluates alternative information flows and chooses those which add the greatest value to the creative enterprise. Both economics and probability live in the core of science. The latter is recognized by most scientists. The former is dimly perceived by few scientists (including economists).

12. The emphasis here on the symmetry of nature and ergodic processes is compatible with phase transitions and path dependence. These phenomena occur near thresholds or critical boundaries. The turmoil created when these boundaries are hit reflects the power of symmetry and the cost of symmetry-breaking. A naive interpretation is the following:** Given the current environment, nature is partitioned into ergodic (exchangeable) segments. All of the interesting activity occurs at the threshold boundaries separating these exchangeable partitions. During the passage from one exchangeable partition to another, the laws of large numbers are suspended with placid lakes replaced by wild rivers. Amit (1989) presents a lucid discussion of the significance of non-ergodic episodes for the genesis of artificial neural nets. The article by Wolynes (1989) is an extremely valuable discussion of non-ergodic processes and much more. For example, he has a clear discussion of tunneling and its relation to the Feynman path integral.

**We plan to analyze these processes and distinguish clearly among ergodicity, broken-symmetry, path-independence, and the influence of initial conditions in McCall and Velupillai (forthcoming).

TABLE OF CONTENT

Preface

A major thesis of this paper is that the knowledge recently acquired in neurology on learning, memory, habit formation and addiction is compatible with many of the current trends in economic theory and should be formally included in economic studies of the foundations of utility theory. Many of these cognitive processes are composed of neural networks and can be studied by analyzing complex Markov chains with parallel processors. A decisive aspect of the formation of routines is the absence of disruptive shocks. Each of these processes is undergoing intense scrutiny to determine its value and stability. Finite state reversible Markov processes converge rapidly and seem to be appropriate for studying the formation of cognitive processes on parallel processors.

If the formation of routines is conducted in an unstable environment, the processes will not achieve ergodicity and, indeed, may induce perverse behavior patterns. Obviously this is crucial in childhood development and has received considerable attention. The neural net approach should provide new insights into this crucial learning phase.

The rate of convergence is analogous to the relaxation time of an ergodic Markov process. There are several processes that warrant study. First, the evolution of institutions essential for the genesis and survival of a civilized society. The institutions include: language, a legal system, insurance mechanisms, a monetary system to facilitate trade and growth, and protection organizations to deter invasion by foreign societies and insure that law abiding citizens will be safe from those who "break the law."

The second process is the acquisition of civilized traits by children during their first twenty years. Obviously, parents play the key role on this learning. We conjecture that those processes essential for survival, have the shortest relaxation times, and are least vulnerable to disruption. However, as λ increases, the learning process is more vulnerable to shocks which may prevent convergence. Indeed, large perturbations may disrupt steady-states, increase transience and severely tax the overall learning process.

In the third learning process those talents with the highest market potential are nurtured by the individual and his parents. As the individual ages, proficiency increases in these endowments while market opportunities narrow. At the same time more information is gained about the preferred bundle through various informational networks. This information steers the individual to that bundle possessing the greatest return. When market participation occurs the search process is conditioned by the investment process. The most rewarding occupation is searched first. The chosen occupation will not be a random selection from all occupations, but will be a member of the partition pertinent to the accumulated bundle. Furthermore, any policy recommendation flowing from this search process will be relevant only for this partition. In this way, individuals are able to specialize in their choice of human capital investments and in the subsequent career, because the other institutions shield them from concerns unrelated to their specialization. The routinization of complex tasks comprises a portfolio of insurance arrangements. Individuals can disregard many of the daily concerns associated with these repetitive tasks and concentrate on improving their specialized skills.

Much of the excitement surrounding neural networks is being generated by the implications for computer and algorithm design. Many of the researchers in the neural network area are more interested in Artificial Intelligence than the actual functioning of the brain. The insights obtained in real neural biology have been extraordinary, but the serendipitous output in Artificial Intelligence has been staggering. A second major goal of this paper is to demonstrate that the design and assembly of computer systems and algorithms may be the source of profound changes in the theory and practice of production and technology.

A bond has formed among biologists, neurologists, physicists, psychologists and computer scientists. We maintain that because this research possesses great significance for economics, economists should be included in this bond. There is a wealth of economic problems embedded in the neural network research in both biological and computer neuroscience. Economic science not only can contribute to the clarification and resolution of these problems, but also could acquire the knowledge which may revolutionize economics.

If "revolutionize" strikes you as being a bit strong, consider the following. The von Neumann-Morgenstern formalism has been driving stochastic economics and game-theoretic economics for the past 40 years. This includes the general equilibrium model, the theory of rational expectations and the recent real business cycle research.

We claim that economics would be a richer discipline had it developed the seminal research of McCulloch-Pitts and Turing. As pointed out by Velupillai these innovators were the first to see the relation between computers and the brain in the sense of Crick (1989). This research exploited neuroscience in biology to design and construct the earliest computers. Hence, tastes and technology would have entered economics naturally and interdisciplinary connections with biology, psychology and computer science would have been forged. A constructive theory of economic behavior based on fallible entities blends nicely with the Bayesian subjective analysis of de Finetti which recently has been generalized to encompass a rich class of stochastic processes and the corresponding adaptive estimators.

Perhaps the most important consequence of this approach is the realization that each of us shoulders the most sophisticated economic laboratory containing tastes, technology, endowments, price processes, adaptive processes, and an endless nested sequence of secrets so profound we may never decipher them.

In developing inductive methods for extracting information about brain behavior, we begin to understand how the brain extracts information from the fluctuating environment. Positron emission tomography and quantum tunneling are two of the latest methods. They comprise an approximation to Hume's induction problem as reformulated by de Finetti. Their use in neuroscience could produce important new insights for economics, biology, computer science and cognitive psychology.

1. PRELIMINARY REMARKS

• Hume, Smith and the earlier natural philosophers perceived the scope and unity of today's non-communicating disciplines. Ironically, Smith rediscovered the unifying principle explaining the centuries of intellectual exchange among philosophy, mathematics and physics; at the same time, he also demonstrated the gains from specialization and "predicted" the disintegration of natural philosophy into its diverse branches. The growth of science demanded specialization and the formation of separate disciplines. A conundrum addressed in this paper is why economics surrendered both the study of tastes and preferences and the study of technology to psychology and engineering, respectively. The paper does not resolve this puzzle, but does comment on its profound influence on economics. The paper's major theme is the value of interdisciplinary research among biological and computer neuroscience, the philosophy of science, physics and economics. The gains from trade among these disciplines are substantial. They, no doubt, will manifest themselves in a variety of ways, which are unpredictable now. Their major source is Smith's unifying principle: the conservation of fairness. This martingale conservation law reveals that these disciplines possess similar structures. Each is an <u>evolutionary</u>, <u>adaptive</u> (Bayesian), <u>symmetric process</u> with a <u>hierarchical</u>, <u>modular</u> composition.

• The mobility of resources among alternative projects is the essence of economics. In a fluctuating economy with imperfect information, resources are constantly in motion, seeking out their best use. The economics of information describes how agents design experiments to extract information efficaciously about the pertinent aspects of the fluctuating environment. The economic process and learning are inextricably linked. The number of alternatives to consider, the stopping time when a particular alternative is chosen, the rate at which attention should be narrowed, the entry of new alternatives as tastes and technology change -- these are some of the decision processes which economic agents must continuously resolve by abandoning obsolete projects and seizing new opportunities.

• It may be useful to recall several of the insights contained in Popper and Eccles (1977), which motivated the title of this paper. Popper distinguishes three worlds: World 1 is composed of physical entities; World 2 contains the mental states: conscious, unconscious as well as psychological dispositions; all of the products of thought reside in World 3. These include myths, scientific theories, social institutions and works of art. These worlds and their interactions are in constant motion and evolving according to the Darwinian paradigm. The brain is controlled by the self.[1] Indeed, the brain is the computer and the self is the programmer. The brain's output, as designed by the self, <u>combines</u> with extrinsic stochastic processes to determine individual behavior. The person is <u>responsible</u> for his actions. The inability of society and, indeed, of the individual, to separate intentions from <u>combined</u> outcomes is at the core of the "human condition".

Popper maintains a sharp distinction between active learning and passive Pavlovian reinforced learning:

> According to the theory here defended, we learn from experience <u>by action and selection</u>. We act with certain aims or preferences, and with certain expectations or theories, especially expectations of realizing or approaching these aims: we act on the basis of action programmes. According to this view, learning by experience consists in modifying our expectations and theories and our action programmes. It is a process of modification and of selection, especially by the refutation of our expectations. Organisms can

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[1]Popper quotes Smith on the importance of social interaction for self-development (p. 111, fn. 7), but, surprisingly, does not mention the Impartial Spectator!

learn from experience, according to this view, only if they are active; if they have aims or preferences; and if they produce expectations. Since we can speak instead of the holding of expectations of the holding of theories or action programmes, all this can also be stated by saying that we learn by modifying our theories or our action programmes by selection, that is to say, by trial and by the elimination of error. (Of course, our aims or preferences may also change in the learning process, but as a rule such changes are rare and slow, although they are sometimes of the nature of a conversion.)

The theory of the learning process which I have sketched applies equally to adaptive learning on the level of animal behaviour (where my theory clashes with the old theory of the conditioned reflex), and on the level of the formation of objective knowledge, for example scientific theories. And it corresponds closely to adaptation by natural selection on the most basic level, the level of genetic adaptation.

On all three levels of adaptation (the genetic level, the behavioural level, the level of scientific theory formation) adaptive changes always start from some given structures: on the genetic level, the structure is the genome (the DNA structure). On the level of animal and of human behaviour, the structure consists of the genetically inherited repertoire of possible forms of behaviour and, in addition, of the rules of behaviour handed on by tradition. (On the human level, some of these belong to World 3.) On the scientific level, the structure consists of the dominant scientific theories, handed on by tradition, and of open problems. These structures or starting points are always transmitted by instruction: the genome is replicated qua template, and thus by instruction; the tradition is handed on by direct instruction, including imitation. But the new adaptive changes in the inherited structure happen on all three levels by way of natural selection: by way of competition, and of the elimination of unfit tentative trials. More or less accidental mutations or variations come under the selection pressure of mutual competition, or under external selection pressure which eliminates the less successful variations. Thus the conservative power is instruction; the evolutionary or revolutionary power is selection.
... Repetition does play a role in behavioural adaptation, but it does not contribute to discoveries. Rather, it helps, after the discovery is made, to make of it an unproblematic routine and therefore to make it unconscious. (This is so with the skills mentioned before, like walking, or bicycling, or piano playing.,) Repetition, or practising, is no way of acquiring new adaptations: it is a way of turning new adaptations into old ones, into unproblematic background knowledge; into unconscious dispositions.

Eccles conceptual portrait of the cerebral cortex and its modular interactions influenced our vision of the Bayesian Brain.

In summary it can be stated that the important discovery for our purpose is that there are more or less well-defined groups of cells, perhaps up to 10,000, which are locked together by mutual connectivities, and which have as a consequence some unitary existence, building up power within themselves and inhibiting the cells of columns nearby. This is the modular concept.

We propose that a module has to be regarded as a power unit. Its raison d'etre is to build up power at the expense of its neighbours. We think the nervous system always works by conflict -- in this case by conflicts between each module and the adjacent modules. Each one is trying to overcome the other one by building up its own power by all the vertical connections which Ramon y Cajal and Lorente de No first described and by the projection of inhibition out of the neighbouring modules. ... That functional discriminatory action is really what makes a module. A module is a unit because it has a system of internal power generation and around it is the delimitation secured by its inhibitory action on the adjacent modules. Of course each of these modules in turn has its own intrinsic power and it is fighting back with a counter-inhibition to its surrounding modules. Nowhere is there uncontrolled excitation. There is an immense power interaction of excitation and inhibition. It is in this continuous interaction that we have to think of the subtlety of the whole neuronal machine of the human cerebral cortex composed perhaps of one to two million modules each with up to 10,000 component neurones. We can only dimly imagine what is happening in the human cortex or indeed in the cortices of the higher mammals, but it is at a level of complexity, of dynamic complexity, immeasurably greater than anything else that has ever been discovered in the universe or created in computer technology.

The self-conscious mind ... from moment to moment is selecting modules according to its interest, the phenomenon of attention, and is itself integrating from all this diversity to give the unified conscious experience. ... The patterned neuronal activity is detectable by the self-conscious mind at the time that there is the requisite build-up of the neuronal activity. The antedating is effected ... in a compensation for the tardy development of weak neuronal spatiotemporal patterns to the threshold level of conscious recognition. In this way all experienced events have a time correction so that the experiences will have a time sequence corresponding to the initiating stimuli.

- The Bayesian approach to the Brain is attractive for several reasons. First it emphasizes the subjective aspects of decisionmaking and compels the analyst to evaluate decisions using an individualistic criterion The intuitive or constructive methods of de Finetti are founded on the mathematics of individual counting. The second advantage is the recursive structure of Bayesian analysis. The method is also hierarchical and compatible with the modular or connectionist analysis which pervades neuroscience. Learning is another key component of the de Finetti paradigm. This connects the Bayesian method with the evolutionary forces which are so prominent in any neuroscience study. Indeed, it is the evolutionary quest for survival which has forced the brain to adopt its modular form.[2] The flexibility and efficiency of this mode with respect to evolutionary forces must be the major source of its ubiquity in nature.[3] The modular structure is riveted by a constant tension between competitive and cooperative forces. The environment is primarily repetitive with only occasional surprises. The organism reflects this duality by harboring deviants within the conformity manifested by its network of routines.

• For example, the economy is composed of industries, firms, teams, and employees. Each of these entities is a node within a complicated and oscillating network of relations (implicit and explicit contracts). One immediately detects an analogy between industrial organization and the immune system. The latter must identify the geometric aspects of antigens, among which there is considerable heterogeneity (complexity). This information is transmitted to the brain which begins assembling antibodies to bind (match) the geometry of the antigens.[4] This production activity is

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[2]Evolutionary economics has a distinguished history beginning with the influence of Malthus and Smith on Darwin. Its current renaissance was initiated by Alchian (1950), with additional contributions by Hirshleifer, Riley, and many other economists. The definitive work is Nelson and Winter (1982).

[3]Simon (1962) has shown that modularity possesses exceptional resilience to environmental shocks. The hierarchical structure may collapse, but each module remain intact. See Ikeda and Siljak (1985) for a Liaponov treatment of Simon's "stratified stability".

[4]A description of this process is revealed in Kraut (1988). For a trenchant appraisal of immunology, reminiscent of the recent surge of critiques in the economic journals, see Langman (1989).

performed after the brain transmits the information to the appropriate organs. Enzymes also are assembled to quicken the binding process. The manner in which firms detect fluctuations in consumer demands, respond to this heterogeneous information, alter inputs, assemble new commodities, and distribute them to consumers, is analogous to the immune system's reaction to a new antigen. This simple analogy could prove valuable for both immunology and industrial organization. The recent work by economists on industrial organization problems has been intense and extremely fruitful, producing some of the profession's most original ideas.[5] On the other hand, Kraut concludes his article on enzymes and the immune system: "Enzymes have been perfecting their skills for more than 3 billion years and they surely have a great deal of sophisticated chemistry to teach us .... Can we understand what the enzymes are trying to tell us?" Indeed, if Vilenkin-Linde are correct, the tunneling action, which is crucial in the thresholding behavior of enzymes, was performing from the <u>very beginning</u>.[6]

Tunneling is a quantum physical phenomenon whereby particles penetrate "solids".[7] It is explicable within the Schwinger-Nakano constructive

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[5]A beautiful summary of this research is presented in Alchian (1984). Following the game-theoretic research, Alchian refers to teams as coalitions. A good example of the novelty and depth of the game-theoretic research is Fudenberg and Levine (1989). The important paper by Leijonhufvud (1986) bridges the Smithian and modern approaches to production. Leijonhufvud refers to the early Babbage computer. A delightful history of the computer including Jevons' major contribution is Gardner (1958).

[6]They claim that tunneling was fundamental to the origin of the universe. Hawking disagrees.

[7]In the microworld, matter displays its wave side and performs in a manner forbidden by the Newtonian "laws" of the macroworld. In the macroworld, the particle side of matter is manifest. <u>Tunneling</u> occurs at the microlevel. Chemical reactions, requiring the passage of atoms through "barriers," take place on a regular basis and, indeed, these reactions are essential for the genesis and persistence of life.

Euclidean-Markov field theory and is basic to stochastic quantum mechanics as formulated by Nelson, Yasue, et al. Tunneling not only has been identified as a crucial entity in enzyme action on the hormone system, a source of neurotransmitters; there also are several tunneling algorithms which satisfy the Crick criterion: they are connected with brain behavior. An elegant exposition of quantum stochastics is contained in Nelson (1985). Namsrai (1986) presents a thorough, but difficult quantum-theoretic treatment of tunneling.

• In both the biological and economic processes, pattern recognition is crucial. The key methodology for performing this classification is some form of thresholding via the application of optimal stopping rules. The complexity (heterogeneity) of consumer demands requires surviving firms to assemble modular and/or a wide variety of products. Methods used are bundling, segmentation of the consumer space, advertising and distribution of the variety of modular products through supermarkets. All of these methods facilitate matching, that is, maximize expected profits.

• There are many ways of characterizing the networks associated with hierarchical processes. Some of the most important are: the fault tree analysis used in reliability theory, influence diagrams which began in industry and have been subject to a succession of generalizations culminating in the research by Lauritzen (1988), Pearl (1988), Schacter (1988) and Smith (1989). These graphs are combinatorial, representations of the underlying symmetry in counting (partitioning)[8] and multiplication (prime number

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[8]See Andrews (1975) for the definitive piece.

decomposition).[9] The following inclusions hold: Fault Tree $\subset$ Influence Diagrams $\subset$ Matroid Analysis (general graph theory).[10]

The influence diagram is a useful technique for representing and solving Bayesian decision problems. It depicts the relations among a problem's component decision variables and random vectors. A decision tree, which reveals the relations among each possible combination of decision and outcome, is dominated by the influence diagram. The influence diagram is more compact, shows relations among variables regardless of whether they are discrete, continuous or mixtures and reveals almost immediately whether variables are conditionally independent.

The influence diagram is a network with modes and arcs representing variables and relations among variables, respectively. Pertinent information about each variable is stored within each mode. The solution of a particular problem is achieved by transforming the network so that the nodal information is modified together with the network's structure. The modified net displays the conditional independence which is the source of

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[9] The fundamentals of number decomposition into primes is contained in Borwein and Borwein (1987). Infinite divisibility is the probabilistic counterpart.

[10] These inclusions are approximate. Fault tree analysis was developed by reliability engineers to identify potential failures at the design stage thereby improving system safety. The fault tree requires the analyst to specify the undesired event, the "top" event, and all system configurations that might cause the "top" event. It is a "top-down" analysis as opposed to the "bottom-up" approach emphasized herein. An excellent discussion is contained in Barlow and Lambert (1975). The compatibility of bottom-up and top-down analyses is noted in the seminal paper by Arbib (1978), which begins:

> A truly satisfying theory of any brain would place it in an evolutionary and socio-biological context. It would build upon a careful analysis of the co-evolution of the patterns of individual and social behavior which enable the animal's species to survive, and the brain structures which enable the animal to exhibit that behavior.

the diagram's power and versatility. The solution value is invariant to these transformations. The modified network reveals the data required for calculating an optimal decision.

Shachter (1988) demonstrates that any probabilistic inference problem also is solvable through a series of simple transformations of the initial network. The transformed network also specifies the informational requirements for both inference and optimal decisionmaking and clearly reveals the conditional independence which may have been hidden in the network's original structure. Almost all paradoxes are based on hidden variables. When the process is conditionally independent (exchangeable), there is no place to hide. Some data sets and/or theoretical structures refuse to eliminate their hiding places, thereby inhibiting induction and/or promoting paradox. In the social sciences behavior is so complex it is difficult to design symmetric theoretical structures and gather appropriate data sets.

• The seminal article by Rosenthal (1979) revealed that many of these network models are computationally difficult. Indeed, some are NP hard.[11] These computational considerations are ignored by most econometricians and statisticians. They, of course, eventually would be confronted if economic theory were in touch with real economic behavior. Theories which lack concrete foundations may be immune to refutation. Not only are the empirical data and econometric methods prerequisites for these crucial

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[11] An algorithm's "goodness" is measured by polynomial time, that is, the running time is bounded by a polynomial in the "length" of the input. Class P problems are solvable in polynomial time. Class NP problems are solvable in polynomial time by a non-deterministic (N) Turing machine. Almost all combinatorial decision problems belong to the NP class. The NP complete problems are in NP and all other NP problems are reducible to them. Problems as hard as NP, but not in NP class are called NP hard. A cogent and concise description of computer jargon is presented in Dewdney (1989).

encounters, but the empirical implications must be computable. The companion piece, Velupillai and McCall (1989), contains an extended discussion of these complexity/computability issues.

- The transition from deterministic to probabilistic constructs is depicted nicely by these graphical techniques, Markov processes, infinite divisibility and exchangeability (conditional independence and identically distributed) are represented by these graphs. All of these graphs adhere to the basic conservation principle -- the martingale.

- These graphs are special cases of matroid analysis, a generalized matrix algebra. Matroid analysis fully reflects the modular nature of the decisionmaking process. Some of the key concepts are generalized versions of max flow-min cut, convexity (submodularity) optimization (combinatorial optimization), and stability.

- All of the sophisticated Artificially Intelligent algorithms -- simulated annealing, genetic, harmony, et al. -- can be characterized by graphs. We plan to illustrate this by developing a graphic algorithm of advertising diffusion.

- The Strassen dilation theorem (1965), introduced to economists as the "mean preserving spread ordering" by Rothschild and Stiglitz, can be proved using the max flow-min cut methods and has a martingale representation when analyzed as a Choquet ordering. This pathbreaking theorem is the nexus between traditional economics and economic processes.

The basic entity linking neural processes to the theory of modern martingale theory, is optional stopping. This link is also clarified by the electrical aspect of neural networks and the corresponding potential theory. The equivalence between potential theory and Brownian motion (Markov processes) is another path to the general theory of processes. Finally, the capacity measure of the electrical network is equivalent to Choquet capacity. Choquet capacity is submodular, is a simple function of the stopping time, and is the foundation of many innovations in probability and statistics.

• This connection with stochastic processes is important because it leads immediately to a sophisticated estimation methodology designed for Bayesian processes. As far as we know, this has not been exploited in any of these related disciplines. It also illustrates the tenuous distinction between deterministic and stochastic analysis. Pragmatism as practiced in economics is the final judge of this "metaphysical" distinction! If it is more economical to conduct the decisionmaking stochastically (deterministically), then the process is stochastic (deterministic), given the current subjective estimate of costs and benefits by the decisionmaker. If there is a change in perceived relative returns such that a threshold is crossed, the environment is "transformed" from deterministic to stochastic.

• Our philosophical biases strongly favor a constructive, combinatoric, and intuitive basis for economics, one that tolerates error and accepts approximation. To impose a rigid and fragile, axiomatic system on a discipline as complex, heterogeneous and "young" as economics violates not only the economic institutions, which have evolved and survived in the only test tube we have, but also ignores the wisdom of Gödel, Wittgenstein, and Peirce.[12] These foundational issues are discussed in McCall and Velupillai (forthcoming).

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[12] Peirce (1955) not only founded Pragmatism, but invented graph theory, saw that nature is stochastic, that a robust explanatory system cannot be mathematical, but must have fallibism intrinsic to its "axioms". He also made several contributions to economics, e.g., cost-benefit analysis. These are discussed in Hage and Harary (1983). His relevance to our research is clear.

• Role of Subjectivity: An individual's response to a particular event is conditioned by subjective factors. Homogeneous responses are elicited by those events affecting those factors common to humans. When offered a glass of water, a thirsty man will drink and display gratitude. But even here, the thirst must exceed some threshold before most men will consume a similar glass of water. Customs, knowledge of the one who is offering the glass, the appearance of the water, etc., will produce divergent responses given the intensity of thirst relative to cultural diversity.

The economic consequences of individuality are manifold and profound. All individuals will demand food, shelter, and clothing, but the diversity of tastes will become more manifest as real income rises and the cost of producing diversity declines. Heterogeneous utility functions combine with flexible production processes to create viable institutions like supermarkets which offer 50 different types of breakfast cereal, and automobiles are "custom designed" via modularity to meet each individual's demand.

To satisfy the complex subjective demands of consumers, firms imitate nature: by producing output containing well-matched modules; by relying on "feedback control" mechanisms, by achieving high reliability via redundancy, by conserving on informational requirements, by simplifying the organization of the firm to enhance information flow and increase flexibility as consumer demands fluctuate. As noted by French (1988), the newspaper is "perhaps man's nearest approach yet to the extravagant production processes of nature[13] ... a million copies a day and a million characters per copy,

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[13]Frequently, nature is called profligate because of the wasted seeds, salmon, etc. in the reproduction process. Similarly, the high wages of those individuals who survive in a highly competitive (risky) industry are criticized. In fact, the reproductive process is designed to achieve survi-

perhaps, and all more or less correct and with a message, however trivial, for someone." (emphasis added). French regards the printing press and the silicon chip as "two of the most profound influences of engineering on human history: and both produced from a simple template -- a pattern in the plane.

The point is that subjective forces naturally tend to generate a heterogeneous society. In primitive societies, deviation is not permitted. The society is not strong enough to cope with internal change. All of its energy is devoted to controlling the environment. Adherence to established routines is essential for survival. As more is learned about physical forces, society becomes more flexible and successful innovators are rewarded.

Subjectivity and individuality are at the core of the evolutionary process. They are also essential to Crick's "searchlight hypothesis".[14] Each person has a searchlight which focuses attention on those aspects of reality pertinent to personal aims and goals. Economically, this corresponds to specialization and comparative advantage. Simply put, it is consistent with rational decisionmaking: acquiring information to nourish those genetic endowments and acquired talents yielding the most utility. It is in this sense that one sees what one believes is much more than Wilde wordplay; it is the core of individuality, enforced by the evolutionary imperative: maximize survivability in a complex and fluctuating environment. The origin of life was nature's response to this evolutionary imperative. Matching and modularity, competition and cooperation, homogeneity and heterogeneity,

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val of the species; and the remuneration of survivors in a risky industry is required to "compensate" (in terms of expected value) those who tried and failed. The first error ignores survivability; the second ignores everything but survivors.

[14]Eccles attributes the searchlight analogy to Popper. Crick's hypothesis is much more specific and is compatible with the observed interactions between the thalamus and the cortex.

deterministic and stochastic, equilibrium and disequilibrium, birth and death are all intertwined within this subjective force.

• The word "searchlight" is felicitous. The region of individual search is illuminated by personal endowments and investments. The personal development of the neural networks necessary for attentive concentration on some specialized environmental aspect presupposes stability. It is precisely the common knowledge possessed and applied by the natural (political and physical) economy which provides a stable reliable battery for each citizen's searchlight. It is in this sense that the economy's stable equilibrium (Lucas (1986)) is analogous to the brain's stable equilibrium. The latter is based on a statistical mechanical model in which the central nervous system's tissue is homogeneous. Peretto and Niez (1986) showed that this system has a unique stationary distribution, provided the noise level is below some critical threshold. In this stable system, the stored patterns associated with specialized knowledge are retrievable on command. These patterns correspond to society's civilized routines, the stable battery.

A more complex model of Hopfield's associative memory has been designed by McEliece et al. (1989). It shows the relation between Hopfield's memory model and both coding and information theory. Asymptotic bounds are calculated for retrieval of original memories.[15] The close relations among neural networks and other dynamical systems like spin-glass and cellular automata are noted. Thresholding is a crucial ingredient of all these models, with statistical mechanics the unifying methodology.

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[15]The combinatoric structure of the brain implies that exchangeable processes are the basic mechanisms for studying the brain's spatio-temporal behavior.

Dyson's appraisal is a fine summary of these preliminary remarks:

> The concept of homeostasis can be transferred without difficulty from a molecular context to ecological, economic and cultural contexts. In each area we have the unexplained fact that complicated homeostatic mechanisms are more prevalent, and seem to be more effective, than simple ones. This is most spectacularly true in the domain of ecology, where a typical stable community, for example a few acres of woodland or a few square feet of grassland, comprises thousands of diverse species with highly specialized and interdependent functions. But a similar phenomenon is visible in economic life and in cultural evolution. The open market economy and the culturally open society, notwithstanding all their failures and deficiencies, seem to possess a robustness which centrally planned economies and culturally closed societies lack. The homeostasis provided by unified five-year economic plans and by unified political control of culture does not lead to a greater stability of economies and cultures. On the contrary, the simple homeostatic mechanisms of central control have generally proved more brittle and less able to cope with historical shocks than the complex homeostatic mechanisms of the open market and the uncensored press.
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> ... It is well known to historians of science that Charles Darwin was strongly influenced in his working-out of the theory of evolution by his readings of the political economists from Adam Smith to Malthus and McCullough. Darwin himself said of his theory: "This is the doctrine of Malthus applied to the whole animal and vegetable kingdom." What I am proposing is to apply in the same spirit the doctrines of modern ecology to the molecular processes within a primitive cell. In our present state of ignorance we have a choice between two contrasting images to represent our view of the possible structure of a creature newly emerged at the first threshold of life. One image is the hypercycle model of Eigen, that is to say a molecular structure tightly linked and centrally controlled, replicating itself with considerable precision and achieving homeostasis by strict adherence to a rigid pattern. The other image is the "tangled bank" of Darwin, an image which Darwin put at the end of his Origin of Species to make vivid his answer to the question "What is Life?", an image of grasses and flowers and bees and butterflies growing in tangled profusion without any discernible pattern, achieving homeostasis by means of a web of interdependences too complicated for us to unravel. The tangled bank is the image that I have in mind when I try to imagine what a primeval cell would look like. I imagine a collection of molecular species that are tangled and interlocking like the plants and insects in Darwin's microcosm. This was the image which led me to think of error-tolerance as the primary requirement for a model of a molecular population taking its first faltering steps toward life. Error-tolerance is the hall-mark of natural ecological communities, of free market economies and of open societies. I believe it must have been a primary quality of

life from the very beginning.[16]

[16] Dyson (1985).

2. INTRODUCTION

A major thesis of this paper is that the knowledge recently acquired in both computer-neuroscience and biological-neuroscience is enhanced when interpreted in price-theoretic terms. Furthermore, this interpretation could be of more value to economics than neuroscience. The spectacular advances in the neurobiology of learning, memory, habit and addiction formation should be known, applied, and extended by economists. The utility function has overstayed its time in the *ceteris paribus* prison. It deserves a pardon and an immediate unification with economic science. This program also stipulates a major overhaul of production theory. Production is a reassembly process guided by the interaction of tastes and technology. We maintain that the neglect of the evolutionary foundations of the taste and technology processes has been harmful to the growth and applicability of economics.

In addition to an intense economic inquiry into the source and implications of the formation and oscillation of tastes, the subjective aspects of economic decisionmaking should replace the "representative decisionmaker". This would be consistent with the recent discoveries in neurobiology and common practice in econometrics regarding heterogeneity.

The proposed enterprise is constructive and relies on a combinatoric theory which gently and almost imperceptibly mutates into the modern theory of stochastic processes. The beauty of this process theory is enhanced by its practicality in that the appropriate econometric processes are "dual" to the economic theory.

The enlightened individualism characterizing Smithian economics is compatible with the foregoing. We will show that the invisible hand is one manifestation of a natural conservation principle: the conservation of fairness. This principle pervades all of nature and is a powerful unifying force.[17] Its implications for economics are well-known but not always practiced. Flaws in the behavior of an economic system founded on individual decisionmakers, in a stable political environment must be mended at the individual level. Abandoning the Walrasian auctioneer was a realistic step in this direction. We claim that economics is a bottom-up science in which the billions of individual decisions slowly merge to give order and momentum to the genesis of a well-organized decentralized economy. This approach is based on stopping rules and is consistent with the fundamental economic principle, the conservation of fairness. Any tampering with the conservation of fairness should recognize explicitly the costs of alternative mechanisms.

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[17]This principle is discussed at length in McCall (1989). Smith's "invisible hand" as interpreted by Hayek (1978) was its first clear manifestation. Dubins and Savage (1965) is its most sophisticated presentation. Simply put, it is the "no arbitrage" principle. Rawls (1981) compares his "justice as fairness" concept with Smith's approach.

3. IN THE BEGINNING

The influence of The Wealth of Nations and the "invisible hand" has been so powerful that many of Adam Smith's other books and ideas are relatively unknown. Smith not only founded economics, but was a brilliant natural philosopher.[18] In this section we concentrate on Smith's psychological, ethical and probabilistic insights contained in The Theory of Moral Sentiments.

The following concepts are important for the subsequent discussion: the impartial spectator, the formation of habit, friendship, stochastic ethics, the subservience of utility to tastes, and the evolution of civilized behavior.

Consider the following quotes:

(1) The Impartial Spectator

> The man of real constancy and fairness ... thoroughly bred in the school of self-command ... has never dared to forget for one moment the judgement which the impartial spectator would pass upon his sentiments and conduct. He has never dared to suffer the man within the breast to be absent one moment from his attention. With the eyes of this great inmate he has always become perfectly familiar to him: he has been in the constant practice, and, indeed, under the constant necessity of modelling ... his conduct and behavior ... according to those of this awful and respectable judge.

(2) On Tastes and Utility

> The utility of these qualities, it may be thought is what first recommends them to us; and, no doubt, the consideration of this ... gives them a new value. Originally, however, we approve of another man's judgement not as something useful, but as right, as accurate, as agreeable to truth and reality .... Taste ... is originally approved of not as useful but as just .... The idea of the utility of all qualities of this kind is plainly an after-thought, and not what first recommends them to our approbation.

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[18] For immediate verification compare Smith's essay on astronomy with Feynman's first lecture in his masterful essays (1965).

(3) The Familial Bond

The general rule is established that persons related to one another in a certain degree ought always to be affected towards one another in a certain manner, and that there is always the highest impropriety and even a sort of impiety in their being affected in a different manner. A parent without parental tenderness, a child devoid of all filial reverence, appear monsters, the objects ... of horror.

(4) On Education and Learning

Do you wish to educate your children to be dutiful to their parents, to be kind and affectionate to their brothers and sisters? ... educate them in your own house ... they may ... attend public schools; but let their dwelling be always at home .... Domestic education is the institution of nature -- public education the contrivance of man. It is surely unnecessary to say which is likely to be wisest.

(5) On Business Bonds

Colleagues in office, partners in trade call one another brothers and frequently feel towards one another as if they really were so. Their good agreement is an advantage to all.

But of all attachments to an individual that which is founded altogether upon esteem and approbation of his good conduct and behavior, confirmed by much experience and long acquaintance is by far the most respectable.

(6) On Detecting Signal (Design) in Event (Signal Plus Noise)

Such is the effect of the good or bad consequence of actions upon the sentiments both of the person who performs them and of others; and thus, fortune, which governs the world has some influence where we would be least willing to allow her any .... That the world judges by the event, and not by the design, has been in all ages the complaint, and is the great discouragement of virtue.

(7) On Self-Command and Sympathy

Our sensibility to the feelings of others, so far from being inconsistent with the manhood of self-command, is the very principle upon which manhood is founded ... The man of the most perfect virtue, the man whom we naturally love and revere the most, is he who joins, to the most perfect command of his own original and selfish feelings, the most exquisite sensibility both to the original and sympathetic feelings of others.

(8) The Formation of Habits

> The man of the most exquisite humanity is naturally the most capable of acquiring the highest degree of self-command. He may not, however, always have acquired it; and it very frequently happens that he has not. He may have lived too much in ease and tranquillity. He may have never been exposed to the violence of faction, or to the hardships and hazards of war. He may have never experienced the insolence of his superiors, the jealous and malignant envy of his equals, or the pilfering injustice of his inferiors. When in an advanced age some accidental change of fortune exposes him to all these, they all make too great an impression upon him. He has the disposition which fits him for acquiring the most perfect self-command, but he has never had the opportunity of acquiring it. Exercise and practice have been wanting; and without these no habit can ever be tolerably established. Hardships, dangers, injuries, misfortunes, are the only masters under whom we can learn the exercise of this virtue. But these are all masters to whom nobody willingly puts himself to school.[19]

Adam Smith saw that the conservation of fairness would work only if citizens trusted one another. A harmonious and fair society depended on the "invisible hand" and the Impartial Spectator. Individuals responded to price because they agreed that the game was fair. If the demand for their product declined and several individuals were laid off, they would seek employment elsewhere -- realizing that they were blameless.[20] Similarly, consumption of a favorite commodity would be reduced if the price rise was sufficiently high. The rules of the economic game comprised an essential component of society's customs, ethics, and routines learned during childhood. In short, the imprinting of the conservation of fairness is one of the most important conditions for citizenship. If this principle were

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[19] Throughout these selected quotations there is a remarkable Aristotelian thread. If in (8) man is replaced by nation, suddenly the basic rationale for Vico's theory of cycles bursts forth.

[20] Smith was too practical to believe that employees cheerfully accept discharge from a long held job. Even if they were able to separate chance from choice, this type of action would test both the stability of the individual, the durability of closely connected relations, and the viability of society. This is clear from Smith's statement in (6).

abandoned, society would literally collapse. Vico's cyclical behavior of societies would commence.[21]

Briefly stated, Vico argues that the transition from solitary hunter to family to tribe to community require behavioral adjustments. Members are recognized by their routines, habits and customs. There is an intrinsic relation among civilized societies and exchange. As civilization becomes more sophisticated, transaction costs decline and exchange expands. Merchants form exchangeable groups with common interests, common rules, and common knowledge. It is possible now to trace these characteristics to the configuration of neural networks. These exchangeable partitions which promote cooperation at the same time are founded on self-interest at the neuronal level.[22]

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[21]Vico's philosophy is interpreted economically in McCall (1987).

[22]Hayek observes that the origin of the word exchangeable is the Greek word for friend. Hence, an apparent tangent from Smith is in fact an elaboration of (5).

Ancient Precursors to Smith

There were several precursors to Smith. The most ancient and, perhaps, the most germane were Democritus,[23] Protagoras, and Thucidides. In her recent book, Farrar (1988) vividly, but innocently,[24] portrays the Smithian society:

> The Athenians were concerned to identify man's freedom and his essence -- what is genuinely internal -- against the background of cosmic, social and psychological forces. Freedom was defined as both the absence of constraint and the exercise of self-government. The realization of one's purposes demanded that one be an active citizen of a self-governing polis. In the first place, it made no sense to think of leading a genuinely human life outside a political community. Membership of such a community was not merely essential for survival, but also greatly extended the range of ends which it was possible to pursue. A self-governing community enabled men to act to secure the ends they desired, to express their autonomy, and by its very operation ensured that the social order was such as to preserve the liberty of its members. The political and social interactions characteristic of self-governing community fostered those capacities and dispositions essential to the preservation of the autonomy of all citizens, their security against tyranny and exploitation.
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> In form as well, the interpretations offered by Protagoras, Thucydides and Democritus, unlike those of their successors, Plato and Aristotle, are democratic and imbedded in a particular kind of political society. They do not seek to construct an ideal city, but rather to reveal the sources of human well-being by interpreting the city of ordinary experience. The interpreter invites his audience to investigate the structure of the world as they experience it; he seeks to guide collective interpretation, not to impose an absolute truth. The theory is, therefore, itself part of the process it interprets, part of the dynamic (democratic) process of preserving order, unity and continuity, as well as autonomy and diversity, in the course of change. The theory both exemplifies and promotes reflection within political society -- not a society constructed abstractly so as to generate certain social principles, but society as perceived, experienced, and assessed by real agents in real and changing contexts.

One could claim that this elegant vision is based on an ancient statistical mechanical "theory", whereas Smith's mechanism is Newtonian. We now know that these theories are equivalent to quantum mechanics.

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[23]An excellent discussion of Democritus, statistical mechanics, and the mind-body problem is contained in Popper's section of Popper-Eccles (1977).

[24]Adam Smith is not referenced nor is a unifying principle identified.

4. THREE COMMENTARIES ON THE INVISIBLE HAND

The following passage from The Wealth of Nations is the familiar characterization of the "invisible" strength of self-interest in promoting cooperation:

> But man has almost constant occasion for the help of his brethren, and it is in vain for him to expect it from their benevolence only. He will be more likely to prevail if he can interest their self-love in his favour, and show them that it is for their own advantage to do for him what he requires of them. ... It is not from the benevolence of the butcher, the brewer, or the baker, that we can expect our dinner, but from their regard to their own interest.

Stigler (1976) acknowledges Smith's belief that men are not guided by self-interest alone; sympathy for others also affects behavior. He then gives a powerful Stiglerian statement of Smith's insight.

> Nevertheless Smith believed that the most persistent, the most universal, and therefore the most reliable of man's motives was the pursuit of his own interests.
>
> On this behavioral basis Smith constructed a theory of how markets work: how goods, once produced, are sold to the highest bidders, and how the quantities of the goods that are produced are governed by their costs and selling prices. The explanation is extended to how men choose occupations, how capitalists choose investments, how farmers choose their crops. Book I presents the central theory of the determination of all prices, and it is a tribute to Smith's insight that this explanation is to this day the center of the economists' theory of value. The proposition that the owner of a resource (how own labor, a sum of capital, or whatever) will seek to employ it where it will yield the most, and, as a result, that a resource will yield equal rates of return in all uses (unequal rates would make a reallocation profitable) is the central proposition of economic theory.

Ronald Coase's (1977) comments on Smith's quote and the Wealth of Nations in general possess a prescience matched by their profundity:

> It is very strange but I do not recall anyone who, when giving this famous quotation ... also includes what Adam Smith says in just the sentence but one before. "In civilized society [man] stands at all times in need of co-operation and assistance of great multitudes, while his whole life is scarce sufficient to gain the friendship of a few persons." This ... completely alters one's perception of Adam Smith's argument. To rely on benevolence to bring about an adequate division of labor is an impossibility.

> ... Reliance on self-interest is not simply one way in which the required division of labor is achieved; for the division of labor needed for a civilized life, it is the only way.
>
> Adam Smith's economics in no way suffers because he did not also give us the theory of diminishing marginal utility. Utility theory has always been an ornament rather than a working part of economic analysis.
>
> Adam Smith's view of competition was quite robust. He thought of competition ... as rivalry, as a process, rather than as a condition defined by a high elasticity of demand ... I ... [believe] that ultimately the Smithian view of competition will prevail.

Like Stigler, Coase finds the preeminence of the Wealth of Nations disturbing. "What have we been doing in the last 200 years? Our analysis has certainly become more sophisticated, but we display no greater insight into the working of the economic system and, in some ways, our approach is inferior to that of Adam Smith."[25]

F. Hayek's interpretation is similar to Coase's:

> Economists usually ascribe the order which competition produces as an equilibrium .... While an economic equilibrium never really exists, there is some justification for asserting that ... order ... is approached in a high degree. This order manifests itself ... in the circumstance that the expectations of transactions to be effected with other members of society ... can be mostly realized. This mutual adjustment of individual plans is brought about by what, since the physical sciences have also begun to concern themselves with spontaneous order, or self-organizing systems, we have learnt to call "negative feedback", ... long before ... Wiener developed cybernetics, Adam Smith has used ... the idea ... The "invisible hand" ... is clearly this idea. In a free market, says Smith in effect, prices are regulated by negative feedback.
>
> The market leaves the particular combination of goods, and its distribution among individuals, largely to unforeseeable circumstances - and in this sense, to accident. It is, as Adam Smith already understood as if we had agreed to play a game, partly of skill and partly of chance.

Hayek observes that a system in which each individual receives what others regard as his due reward would be inefficient. The market makes these assessments. Individuals must be adaptive, accepting lower

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[25] A trenchant expansion of this theme is presented in Clower (1989).

remuneration when they do their best, but market conditions change. He seems to underestimate the psychological impact of a lower salary and a job loss. The tensions created by the "efficient" society must be modulated by a sensible policy of social preservation.

5. MODERN ECONOMICS AND THE INVISIBLE SMITH

Why has modern economics ignored many of these Smithian concepts? There are probably many explanations. The view proposed here is based on the dominating influence of Marshall relative to Edgeworth and Fisher. In his influential masterpiece The Economic Organization, even Frank Knight severely restricts the scope of economics.

> Economics has nothing to do with the concrete processes of producing or distributing goods, or using goods to satisfy wants. The study of these matters comes under the head of technology, including engineering, business management, and home economics. Economics deals with the social organization of economic activity. ... In the science of economics the wants are largely taken for granted as facts of the time and place, and the discussion of their origin and formation is left for the most part to the distinct studies of social psychology and cultural anthropology.

Limiting the scope of economics may create mischief. Certainly Knight is not the sole cause.[26] The dominance of Marshall over the broadly gifted Edgeworth is probably another source.[27] Debreu and Scarf (1972) and Stigler (1986) contain superb intellectual portraits of Edgeworth's contributions to economic theory and statistics, respectively.

Why was Edgeworth so completely dominated by Marshall? Edgeworth was one of the very few thinkers who could have integrated statistics, probability economics, and psychology. It would then have appreciated and used the

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[26]Patinkin (1974) claims that this quote is misleading -- Knight deplored the neglect of tastes by economics.

> Knight's continued emphasis on the dynamic and progressive aspects of the real world -- and hence the inappropriateness of analyzing it in terms of a static model -- was one of the most important messages that he conveyed to his students ... he conveyed this message not only with respect to ... production, but also ... demand. Thus Knight never tired of emphasizing that wants did not remain constant.

Yet "he was not an institutionalist in the sense of advocating the historical or statistical approaches to economics".

[27]This is elaborated in McCall (1989).

McCulloch-Pitts seminal work on neuroscience. Schumpeter proposes the following solution. Edgeworth, who contributed as much or more than Marshall to economic analysis,

> ... lacked the force that produces impressive treatises and assembles adherents; amiable and generous, he never asserted himself in any claims of his own, he was over-sensitive on the one hand, over-modest on the other; he was content to take a backseat behind Marshall whom he exalted into Achilles; hesitating in conversation, absent-minded to a pathological degree, the worst speaker and lecturer imaginable, he was personally ineffective -- unleaderly is, I think, the word.

It is ironic that Stigler finds psychology receptive to statistical methods, whereas psychological quirks inhibited its entry into economics. It is also plausible that the late entry of probability and statistics into economics is another symptom of the Platonism which remains a vital force in much of economic theory.[28]

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[28] An excellent article reflecting this Platonic influence is Aumann (1984).

6. THE ECONOMICS OF NEUROSCIENCE: AN EXAMPLE

A hungry crayfish has been foraging for some time without success. Finally, it discovers a rich food patch and commences eating. At this point a small predator approaches the crayfish. The neuronal response of the crayfish would under normal circumstances have surpassed its escape threshold and rapid motion would ensue. However, the threshold has been raised by eating. The predator leaves, but is shortly replaced by the distant sound of a dangerous predator. This threat creates sufficient neuronal activity to exceed the escape threshold. The crayfish immediately stops eating, flips its tail and swims to the nearest sanctuary.

The economics of this behavior is clear. The threshold corresponds to a reservation level. The neurotransmitter is a signal which conveys environmental information. Each activity of the crayfish has an opportunity cost, determining the size of the reservation level. This level is high during eating and only high prices (threats or rewards) induce termination (and flight or attack).

This behavior was observed and is reported in the illuminating review article by Bicker and Menzel (1989). In this elegant experiment, the signals (prices) were a combination of transmitters, modulators and hormones.[29] The action of transmitters in promoting information transfer along the synaptic connections among neurons is well-known. These informational transfers are coordinated by modulators at the cell level and also by hormones. The action of the transmitters is rapid and proximate; the modulators operate on a longer time scale at greater distances; and the hormones act like modulators, but on an even longer time scale and at

[29] Neuromodulators and neurohormones are controlled by the action of neurotransmitters.

greater distances. Hence, the tail flip that initiated the escape of the crayfish was the consequence of the coordinated activity of an information and control system operated by a plethora of spatio-temporal prices. This behavior corresponds to a decentralized economy's dependence on the coordinated response of numerous agents to information conveyed by prices and interest rates in a stable environment.

The experiments reported by Bickel and Menzel were among the first to observe organisms perform routines in response to the coordinated information transfers among circuits at the cellular level. The experiments are wide-ranging including: the control of hormonal release <u>over the life cycle</u> to synchronize the timing of courtship and reproductive behavior of crickets as well as the division of labor among honeybees.

Learning behavior is also studied, "As neuromodulators confer flexibility on neuronal circuits and trigger intracellular biochemical events that outlast the signal, they have been implicated in learning, memory, and the performance of learned tasks in vertebrates and mollusc".

A variety of experiments are being conducted

> ... using neuromodulators or their analogues to show that the natural release of neuromodulators can evoke coherent behaviour. The chemical code for regulating behaviour arises from the patterns of release of the various modulatory substances. These in turn are decoded by the distribution in the target tissue of receptors specific for each type of modulator. As the receptors are linked to the intracellular biochemical signalling pathways, the neuromodulators can modify the excitability of nerve cells on a longer time scale than that of conventional rapid synaptic transmission, leading to longer lasting functional changes in neural circuitry that serve processes such as arousal, selective attention, reinforcement and retention of memory.

For example, the motion of the teeth of a lobster's gastric mill, controlled by neurons released from a nerve center in the stomach, was studied using this method.[30] The rhythmic motion of the teeth is synchronized by proctolin, a recent discovery by Heinzel and Selverston (1988).

Marder's summary of this research concludes that it has "exciting" ramifications for computational neuroscience. Using synaptic rules for linking neuron-like components, complex tasks have been performed by network models.

> The challenge is to elucidate how small neuronal networks work; how large networks of simple neuronal-like elements work; and how the complexity of neurons (conferred by their shape and ionic currents) and the richness of their connectivity define and constrain the possible functions of a neuronal network. So we need eventually to determine which properties of biological networks are not shared by networks of simple neuronal-like elements. Insights that have been gained from computer-modelled neuronal-like networks and studies of modulation in neuronal networks such as the stomatogastric ganglion should help to understand how large biological networks generate behaviours and process information.

It appears that those chemicals transmitting information among the complex assemblies comprising the brain are the unifying force in neuroscience in the same way that the price mechanism unifies economics. Indeed, our basic hypothesis stipulates an equivalence between the two processes, each of which is equivalent to Brownian motion.

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[30]Heinzel augments this traditional procedure by direct observation with an endoscope.

7. A UNIFIED APPROACH TO COMPUTER NEUROSCIENCE[31]

The design and implementation of connectionist and classifier models by computer scientists, psychologists and physicists is prodigious. Comprehending these complicated, interdisciplinary models must be challenging even for the participants in this novel and burgeoning enterprise. There are at least three unifying threads linking this myriad. The first is philosophical, the second phenomenological, and the third probabilistic.

(1) For now, we have little to say about the induction problem beyond the standard statement. Let X and Y denote the stochastic processes entering and leaving a "black box". The induction problem is: Infer the action of the "black box" from observations on these input and output processes. In particular, predict the behavior of the Y process in the interval (t_1, t_2) given observations on the X-process in this same interval. This is Hume's problem which some claim de Finetti has "solved".

The distinguishing feature of the neural network models is their ability to perform induction. They extract the underlying behavioral rule and organize their own behavior accordingly. In this sense, they are dynamical, self-organizing systems. This has led reductionists to conclude that eventually brain behavior will be replicated. We think not.

(2) The major physical sources of these models are: thermodynamics, statistical mechanics, quantum mechanics and evolution. Most of the neural network models presented in Rummelhart et al. are founded on a special model in statistical mechanics -- the Ising model. This is also true of the Hopfield-Tank models.

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[31]A remarkably thorough survey of artificial neural networks is contained in Simpson (1989).

Several versatile classifier models are based on the logic of evolutionary processes. The most famous evolutionary model is Holland's genetic algorithm.

Finally, a recent neural network is based on the tunneling phenomenon in quantum mechanics.

(3) From a mathematical perspective all of these models can be characterized as birth and death processes (and/or their continuous counter-part -- the diffusion). They all possess the strong Markov property. This mathematical thread is strong. While it is the subject of Appendix A, the treatment there is quite inadequate. We will present a more sophisticated analysis in a subsequent piece. For now, we merely note that while the processes are Markovian, they are interactive and usually defy conventional analysis. For example, Holland's genetic algorithm is a complicated multi-armed bandit process, subject to continual perturbations. On the other hand, some of the algorithms are similar to stochastic approximation, a less complicated Markov process. A physical source of these Markov processes is the Ising model, which has sparked a surge of creative mathematics. A key result is Spitzers equivalence between Gibbs fields and spatial Markov processes.

The Ising Model[32]

The theory of interacting particle systems is a recent and subtle development engaging many of the best mathematicians and probabilists. From a physicist's perspective interacting particle systems are concerned with the cooperative behavior of particles as they evolve over space and time. The mathematical views these systems as infinite collections of coupled stochastic processes. Letting S be a countable set of sites, at each time the state of a site is characterized by a point in a finite set F. In physics, each site is characterized by a spin belonging to some finite set. At t, the composite state of the process is determined by a vector $\eta_t = \{\eta_t(x); x \in S\}$, where $\eta_t(x)$ is the state of site x at time t. The interest was provoked by a physical problem: understanding how an evolutionary system of interacting particles achieves equilibrium. This problem is the heart of the Ising model of ferromagnetism. Each spatial point is characterized by a "spin" with fluctuating orientation. The evolution of the Ising process occurs at a rate dependent on the spatial configuration of all the points in the system. The evolutionary process is characterized by a Markov process with the appropriate equilibrium behavior.

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[32]For a more thorough discussion see Durrett (1981), from which this was extracted. The recent paper by Cipra (1987) is lucid and brief. The latter by Coughlin and Baran (1988) is especially pertinent. The Ising model is a special case of interacting Markov processes (infinite particle systems). An infinite particle system has the strong Markov property, that is, its behavior at t is the same as at $t + \tau$. The process is invariant to a translation by a random stopping time, τ. This is the <u>source</u> of its remarkable usefulness. For example, almost all of the ingenious artificial intelligence algorithms from simple stochastic approximation to the genetic and quantum tunneling algorithms are variations of the Ising model. The tunneling algorithm is unique in that external shocks are not required to escape local minima. The global minimum is achieved by the tunneling <u>process</u>.

To be specific let $\chi(z)$ represent the orientation of a spin at each z, and suppose it is either +1 or -1. The corresponding speed function is:

$$c(\chi,\eta) = \exp(-\beta\Sigma\chi(x)\chi(y)),$$

where $\beta \geq 0$ and the summation is over all those y which are neighbors of the vertex x.

In the Ising model spins at neighboring vertices try to conform. Note that the speed function exceeds zero and is invariant under translation. Distaste for deviant behavior of neighbors of x is reflected by large values of c whenever the orientations [up (+1) and down (-1)] of many neighbors y of x differ so that $\chi(x)\chi(y) = -1$, for a large number of y vertices. The parameter β measures the taste for conformity. In the physical version of this model β is the reciprocal of temperature. Clearly, the two states with the greatest stability have all spins pointed up or down.

An interesting mathematical question is: determine the necessary and sufficient conditions such that the set of I of invariant measures has cardinality one. In one dimension $|I| = 1$ for all β.

In two or more dimensions d the equilibrium behavior differs for "large" and "small" β. That is, there is a positive threshold value $b(d)$ with the following property:

(a) $|I| = 1$, if $\beta < b(d)$

and

(b) $|I| > 1$, if $\beta > b(d)$.

This implies that the Ising model is ergodic for (a) and nonergodic for (b), when $d \geq 2$.

<u>Physical Interpretation</u>: When the temperature is sufficiently high there is enough energy for local activity to dominate past influences; however, when energy drops below a critical level local activity cannot escape the historical influence and a stasis occurs.

Onsager (1944) determined the threshold value of β for $d = 2$: $b(d) = 1/2 \log(1+\sqrt{2})$. Little is known about the Ising model for $d \geq 3$.

8. COMPUTER NEUROSCIENCE AND ECONOMICS

Hopfield and Tank (1987) mimic neural networks to construct optimization algorithms. But how are decisions being made with respect to ranking optimization problems, designing algorithms, and assembling computers? These are basic economic problems. On the other hand, the exciting economic research on organization theory from decentralized decision making to conflict resolution can be fortified and stimulated by the network analyses currently used in computer science, information theory, and circuit analysis. These economic theories can be tested directly by the structure and behavior of neural networks as well as by the implied social behavior.

For example, conflict resolution arises in communication networks whenever congestion occurs.[33] Various algorithms have been devised for resolving these conflicts. Several exhibit both practical wisdom and mathematical ingenuity. Implicit prices and social efficiency are rarely studied. This is an obvious topic for collaboration.

The relevance of factoring algorithms for decentralization of organizations frequently is mentioned by the designers and is well-known to several economists. Thus far, there has been virtually no research on this important topic combining economics and computer science. In game theory, algorithms are certainly as appealing as "experimental methods" to test alternative conjectures.[34]

Li and Basar (1987), develop a synchronous distributed algorithm for noncooperative stochastic games, stochastic teams, and stochastic routing in networks which achieves asymptotic agreement. The problem of algorithmic

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[33] See Bertsekis and Gallagher (1986).

[34] Abreu and Rubinstein (1988) is an important step in this direction.

conflict resolution, the collision of information packets in communication networks, is surveyed and a solution proposed in Szpankowski (1987).

Algorithm network analysis of competition and cooperation has played a key role in several of the major advances in neural network theory. They contain many economic problems and develop methods which may be very useful in economics. For example, several of the articles in Amari and Arbib (1982) possess great potential.

The Two Faces of Noise

There is no question that noise contaminates the signal and when of sufficient relative size prevents information transfer. A variety of methods have been developed for controlling noise and extracting useful information from time series which may be nonstationary or nonlinear. On the other hand, the paper by Yu, et al. (1989) argues that signal processing occurs in two types of neural networks: those where the neuron is binary employs thresholding and those where the neuron's action is modeled by a continuous analog processor with the thresholds as point processes. Apparently there is no adequate explanation for the observed continuous spike-rate modulation in neurons. Yu, et al. adroitly manipulate the noise to transform the non-linear spike processes into processes which are almost linearly modulated. This result was robust against a variety of information transmission regimes.

On reflection, this result is not surprising. Most of the methods for reducing the probability of "sticking" at local optima use disturbances to promote continued exploration. At a more fundamental level, the shuffling of genes and the genesis of new organisms to survive an environmental fluctuation explains the evolution of sex. The power of crossover in Holland's genetic algorithm is due to the same shuffling phenomenon.

Skarda and Freeman (1987) go a step further and argue that chaos is produced by the brain to "ensure continual access to previously learned sensory patterns and as a means for learning new sensory patterns." They maintain that chaos is a controlled noise process manufactured by the brain in a deterministic fashion to carry the individual and his pattern recognition abilities across critical phase transitions, thereby coping with non-linear processes.

9. THE CRICK CRITIQUE

There have been many reservations expressed by distinguished scientists regarding the value of the "neural network explosion" in Artificial Intelligence. None is more penetrating nor more constructive than Crick (1989). It also has pedagogical value for those who are not specialists in either neurology or artificial intelligence.

The basic ingredients of a neural network are units possessing some of the qualities of a real neuron. Each unit has both inhibiting and exciting inputs. If the signal, contained in these multiple inputs, exceeds a given threshold, the unit sends this message to other units.[35] Crick acknowledges that since many synapses are plastic, adjusting their strength on a trial and error basis, it is important to understand this phenomenon. But this is exactly what the algorithms are attempting as they vary the architecture and strength of the connections.

One of the main tasks is to design the connections among the units so that the network contains "memories." It does this by modifying the connections among the units. Using a <u>local rule</u>[36] the network can generate a pattern. By presenting the network with partial information about the "stored pattern," the entire pattern can be reproduced. The Hebb rule is not supervised and hence receives no feedback. More complicated algorithms contain a teacher who tells the net when errors are made. Nets with

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[35]It should be noted at the outset that <u>all</u> the fancy algorithms -- simulated annealing, genetic, cellular automata, etc. -- are mathematical variations of the McCulloch-Pitts <u>optimal stopping rule</u>. This observation in no way diminishes either the remarkable ingenuity or practical significance of these algorithms. Physics, Computer Science and Biology comprise a formidable trio. If economists are familiar with auction and/or search theory, they need not be intimidated by these "hard sciences."

[36]This rule or algorithm is called Hebb's rule.

supervised algorithms can discover patterns in the correlated inputs and modify outputs accordingly. The back propagation algorithm is a sophisticated rule by which the teacher identifies those units making errors and corrects their behavior. These faulty units are hidden in the middle layer of units. The teacher, by knowing the input and output can infer not only the size of the error, but modify the behavior of the hidden unit(s) by "gradient descent." Hence, the back propagation algorithm generalizes the Hebb local algorithm by adding a "teacher" that tells the net how much its "performance" deviated from the ideal. Repeated "scoldings" by the teacher and small responses by the pupil lead eventually to the desired minimum.

Crick shows that this ingenious algorithm composed of back-drop nets is "unrealistic in almost every respect, as indeed some of their inventors have admitted". He goes on to characterize neural "models" as non-constructive existence proofs and not models at all, "because they do not correspond sufficiently closely to the real thing."[37] Just as some have urged economists to study corporations from the inside, so Crick encourages the computer oriented brain scientists to "look inside the brain, both to get new ideas and to test existing ones".[38]

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[37]His most trenchant comment is: "I also suspect that within most modelers a frustrated mathematician is trying to unfold his wings".

[38]The paragraph ending with this advice contains much wisdom: "Constructing a machine that works (such as a highly parallel computer) is an engineering problem ... The brain is given to us, the product of a long evolution. We do not want to know how it might work but how it actually does work. This has been called reverse engineering ... on the products of an alien technology. And what a technology! Natural selection is not a clean designer ... It is opportunistic: anything will do as long as it works. Naturally it is constrained by both chemistry and physics, but this does not mean that its mechanism will embody deep general principles; the structure of the genetic code is a good example of this. It may prefer a series of slick tricks to achieve its aim. Only a close inspection of the gadgetry will tell. And this brings us to the crux of the matter. "Why not look inside the brain both to get new ideas and to test existing ones?"

Crick is not anti-theoretical. He strongly opposes those linguists and psychologists who prefer simple unrealistic models of the brain which are consistent with some of the psychological data. Their position is that the brain is too complex for abstract modeling.

The eloquent arguments of Crick (1988) are relevant to the proposed interdisciplinary research. Just as neuroscience should maintain close contact with empirical brain science, so too this joint research should establish close ties with the empirical research in ecology, biology, and economics. The studies by Endler (1985) are superb examples of applied biology. Experimental economics should be encouraged to apply and expand its methods to study actual market behavior within industries and firms.

Presumably Crick is excited by the research of Bicker and Menzel (1989). They study the coordinated action of amines and peptides on the neural and metabolic processes and their coherent behavioral effects on the organism's routines. They also investigate the action of neuromodulators at the cellular level and the corresponding spatio-temporal behavioral responses. In addition to neuro-modulators, both GABA and the other standard neurotransmitters and the neurohormones were analyzed for the crayfish and insects. The crayfish and insects were chosen because the relation between the effects of neuromodulators and behavior is subject to direct study.

There were a number of interesting findings: (1) The threshold response depends on the behavioral circumstances. (2) The plasticity of the neural network is enhanced by neuromodulators. Thus learning new tasks can occur over the life of the animal. (3) The temporal action the neural

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(Italics added).

triggers is quite different with GABA evoking a quick response and neurohormones a delayed reaction. These hormones are released over the life cycle.

"The most important outcome of these experiments is that learning and retrieval can be improved or depressed by different amines acting at different locations and at different times in the learning process".

This observation is germane to the analysis of economic processes. The "correct" spatio-temporal scaling of heterogeneous economic events can have a crucial effect on the conclusions.

The Bee Analogy

The transmission of information in the brain is analogous to the signaling occurring in beehives[39]. For example, consider the "decision-making process" associated with nest choice. The scouts discover several alternatives and describe them by their waggle dance. Scouts who have found "good homes" dance with great vigor. The best sites are visited by the scouts and experienced bees. Finally, a consensus is achieved as scouts change their minds and essentially synchronize their dance for the preferred site. This information is then passed on to the other members of the new nest. Again stopping rules are coordinated as information is collected and disseminated among the hive. The classic studies of the honeybee and bumblebee are by Seeley (1985) and Heinrich (1979), respectively. Of course, the seminal research was by Frisch. (See Landa and Wallis (1986) for an economic analysis of this behavior. I am indebted to J. Hirshleifer for this reference.)

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[39] Hofstadter (1979) considers the ant as analogous to the neuron.

The classic Fable of the Bees, by Mandeville may have had an important effect on Adam Smith's thought. Hayek regards this influence as decisive. This strikes me as an overestimate.

The bee also has methodological significance. The dance of the bee, who had located a flower patch the day before, differed the next morning. Bees navigate by the sun, so the "location" transmitted by the dance had changed. The bee behaved as if he were a celestial mechanic. Recently, a mechanism has been discovered in the bee which accounts for its navigational ability. This lends support to the as if methodology of Milton Friedman. Tunneling's history is almost a "proof" of Friedman methodology.

10. THE ORIGINAL[40] THRESHOLD ANALYSIS BY McCULLOCH AND PITTS

Let a "neuron" have m inputs $x_1,\dots,x_m$ and one output y. Each x_i is weighted by w_i, $i = 1,\dots,m$. The module (neuron) fires at time n+1, $y(n+1) = 1$ iff $X(n+1) \geq \theta$, where θ is the threshold and $X(n+1) = \Sigma_i x_i$. A positive (negative) weight represents an excitatory (inhibitory) input.

Arbib (1964) generalizes this seminal model to a modular net (neural network) and then shows that a finite automaton is equivalent to the modular net.

In this remarkable book, Arbib also demonstrates the connections among ergodicity, entropy, encoding, reliability, Markov processes and capacity.

The key role of optional stopping (thresholding) and the link via submodularity[41] between network optimization and Choquet capacity was

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[40] Actually, Sherrington introduced threshold analysis. In "The Project" Freud described three types of neurones and "called 'contact-barriers' between the individual neurones -- the synapses described by C. Sherrington a few years later." Freud's "Project" had the "framework of a complex machine whose workings would explain in psychological terms almost any psychological state." His enthusiasm for "The Project" plummeted. He wrote Fliess, "I cannot conceive how I came to inflict it on you," and abandoned these seminal ideas.

Some fifteen centuries earlier St. Augustine had speculated in his "Confessions" on the fact that potential recollections would sometimes be beyond memory but would at others thrust themselves into consciousness from some unknown reservoir. -- Leibniz, with his idea of limen, or threshold of perception, carried the investigation a little further. Johann F. Herbart ... proposed a model in which strong conscious perceptions thrust weaker perceptions below the threshold into the unconscious, and began to give the mind an almost quantitative framework. See Clark (1980) for a complete discussion.

[41] As we will see, submodularity is closely related to convexity. It is the discrete version of convexity. To see this let $S \subseteq R^n$, f a set function on S, and define

$$\hat{f}(x) = \Sigma_{i=1}^{k} \lambda_i f(a_i),$$

where $f(\phi) = 0$, $x \in R_+^s$, $\lambda_i > 0$ and $a_i \geq \dots \geq a_n$ are (0,1) vectors.

unknown in 1964 and prevented Arbib from employing the martingale theory (probabilistic potential theory) of Doob (1985) and Dellacherie-Meyer (1983). Indeed, as far as I know, this will be the first application of these connections.

The renaissance of "neural networks"[42] began with the Hopfield model. In this model the computational energy of the neural network is given by

$$H(S) = \sum_{j=i+1}^{N} \sum_{i=1}^{N-1} S_i S_j T_{ij} = \sum_{i=1}^{N} S_i U_i$$

where S_i is the state of the i^{th} neuron: $S_i = 1$ if firing, $S_i = 0$ if inactive;

(T_{ij}) is the connectivity matrix denoting the power of the "synapse" from neuron i to neuron j: this connection is exciting (inhibiting) when $T_{ij} > 0$ $(T_{ij} < 0)$.

The threshold test is applied to the i^{th} "neurons" input,

$$X_i = \sum_j S_j T_{ji} + U_i,$$

to determine its state. For example, the state of the McCulloch-Pitts "Neuron" is

$$S_i = \begin{cases} 1 & \text{if } X_i > 0 \\ 0 & \text{if } X_i \le 0, \end{cases}$$

<u>This</u> network moves to local states, where H achieves a local minimum.

Then $\hat{f}$ is convex iff f is submodular. It should come as no surprise that Slutsky's theorem can be found in submodular processes. Veinott's unique analysis has located substitutes and complements in network flows.

A good survey of combinatorial optimization is presented in Pulleyblank (1984). There are several excellent texts. One of the best is Nemhauser and Wolsey (1989).

[42] An excellent comprehensive review of artificial neural systems and neural computing is contained in Rosati et al. (1988).

This assumes T is symmetric, real valued, with zero on the diagonal.

The simulated annealing algorithm adds perturbations such that local minima are no longer stable. The network continues to explore the region for smaller local minima. The perturbation permits the network to jump out of small local minima. Large perturbations correspond to more intense exploration.

Probabilistic cellular automata[43] are regular arrays of binary variables, which evolve according to some set of local rules. The asymptotic state of the "network" is the object of interest together with the universal class of phase transitions as the rules change. It has been shown that these automata belong to the same universal class as the Ising model.

The recently discovered relations between stochastic quantum theory and statistical mechanics is the source of the "equivalence" between the Ising model and the tunneling algorithm.[44] The performance of the immune system is based on tunneling. The application of the genetic algorithm to the immune system has been very successful. Improvements may be obtained by incorporating tunneling into the genetic algorithm. Recall that 's eloquent argument for the uniqueness of the individual was based on the behavior of the immune system. A source of individuality is "quantum mechanical".

In this reflective essay on the immune system, Medawar (1956) observes the ability of the self to recognize foreign cells, the combinatorial source of this uniqueness, the evolutionary motivation for this diversity in all organisms: it is nature's mechanism for anticipating unknown future

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[43]The original research is Wolfram (1987).

[44]That is, the disorder characteristic of the Ising magnet is equivalent to tunneling.

environments. The maintenance of diversity provides an evolutionary menu for matching the current environmental demands with an ample supply of diverse individuals so that at least one of the selected matches has a high probability of survival. Medawar also notes the resemblance between the immune and nervous systems.

Recent research has discovered the utility of embedding the immune system in a network. There also is a merging underway of the immune and nervous systems. Hormones and neurotransmitters are produced by the immune system. Hence neuronal behavior must include the immune network as an integral part of the neural network.

The study of artificial intelligence from the mid-1960s to the mid-1980s concentrated on serial computation, almost completely ignoring neuroscience research on neural networks. Of course, all of this has changed with the remarkable ascendancy of parallel and distributive computation and the close relation between biological and computer neuroscience. The impetus for this interdisciplinary research is the spectacular industrial growth of "networking" not only in computers, but in all of the communication and information industries. In his authoritative survey article, Arbib (1989) discusses the frontiers of computer neuroscience and anticipates the structure of computer systems for 1995-2005.

11. THE ECONOMICS OF ROUTINES

Much of what is regarded as civilized behavior is routine. The common wealth which binds a nation is composed of routines like: a common language, a common ethic, and, in short, a shared patriotism that encompasses a broad range of habitual behavior. These routines diminish uncertainty: prediction of events contingent on common behavior are almost always correct. The purpose of this section is to present a methodology explaining the acquisition of habits from economic, neurobiological, and information theoretic perspectives. The explanation seems compatible with behavior even though it is based on these three very different disciplines: economics, engineering and neurology.

The methodology, network analysis, is essentially a simple Markov chain -- the birth and death process. Network analysis has been extremely useful in describing the behavior of neural networks in neurobiology; the flow of information in computer networks, communication systems, and the reliability of systems composed of graph theory -- the so-called fault-tree and influence diagram analyses. Network analysis has potential in economics for explaining hierarchical structures -- from the individual to the entire economy. Contractual relations and efficiency are the key economic concepts which compose and regulate these stochastic networks. The resulting behavior explains partially the genesis of firms. The drive to belong and be recognized accounts for the higher productivity of teams. The manager's primary role is not to control shirking, but to transmit information and coordinate individual effort.

The formation of routines begins in early childhood: speech, walking, and a host of basic functions are learned in the first few years of life. Their acquisition is achieved at random rates, fostered by parents and once learned are performed unconsciously and effortlessly. The child builds on these basic routines to learn other habits fundamental to society. The process is hierarchical and the end product is a civilized individual. The interactions of these civilized individuals promote a regularity and predictability in society. The selfish instincts are mollified by these habits. They are the source of the trust that while not eliminating does mitigate shirking and cheating and promotes contractual relations, the basic building blocks of an efficient society. Without the common wealth of habitual behavior which also entails Smith's Impartial Spectator, the monitoring and legal costs of a contractual system would inhibit decentralization and coordination, and for all practical purposes, prevent the formation of a viable society.

The Acquisition of Routines: A Simple Markov Model

Our learning methodology for routines is elementary. Essentially a routine is acquired when the learner reaches a designated subset S_1 of the N states comprising a Markov chain, $S_1 \ll N$. The routine may be crawling, walking, level 1 speech, or any of the multitude of habits acquired during early childhood.

The rapidity of the early learning process depends on genetic endowment, parental investment, and other environmental factors. There are several reasons for adopting the Markov methodology: (1) It is compatible with a network formulation of learning and labor mobility over the life-cycle; (2) The Markov network theory of economic behavior has, roughly, the same structure as the pertinent advances in neurobiology; (3) The steady-state of the Markov chain will be identified with a stable process that once achieved is performed automatically; (4) These simple models continue to receive the attention of the best probabilists and, as a consequence, more is known about these birth and death processes than any other stochastic process. (5) A corollary of (4) is the ease with which the birth and death process can be formulated in potential theory, combinatorics, martingale theory, and related to orthogonal polynomials. This versatility, plus its exponential approach to ergodicity, spectral decomposition and graphical representation, means that the process possesses extraordinary pliability. (6) Hence, it is not only susceptible to substantial modifications as our ambitions to explain economic behavior expand, but is almost an ideal entity for quickly establishing stochastic Economics as a vital discipline linking Computer Science (Engineering) and Neurobiology (Psychology/Biology).

<u>The Markov Model</u>

Let $\{X_n;\ n=1,2,\ldots,N\}$ be a stochastic process on the finite state space $I = \{1,2,\ldots,N\}$. Whenever the process is in state i it moves to j with probability p_{ij}. The transition from i to the next state depends only on i; the previous behavior of the process is irrelevant. This is called the <u>Markov Property</u>. Clearly, $P_{i,j} \geq 0$, $i,j = 1,2,\ldots,N$ and $\Sigma_{j=1}^{N} P_{ij} = 1$. The matrix of transition probabilities is given by $\{P_n(i,j)\}$ and $\pi = \{\pi_1,\ldots,\pi_N\}$ is the stationary distribution.

It is well-known that $\{P_n(i,j)\}$ converges to π as $n \to \infty$.

<u>Definition</u>: The process X_n is called <u>reversible</u> if

$$\pi_i P_n(i,j) = \pi_j P_n(j,i), \quad i,j \in I \text{ and } n \geq 0.$$

Aldous (1982) shows that this finite Markov process converges to the stationary distribution π, "rapidly". Furthermore, this rapid convergence is equivalent to: the mean time to reach the critical set of states S_1 can be bounded by a function of $\pi(S_1)$.[45]

If S_1 represented acquisition of language level 1, the child would proceed to level 2 when S_1 was reached. Presumably language is a nested routine and can be formulated as a recursive stochastic process. Indeed, all of these learning processes interact with one another, but for now this is ignored.

However, it should be noted that the accumulation of routines can be characterized by the sojourn time through a network. There are many paths through a network and the route taken by the learner consumes less time the more frequently the network is traversed. In this model a routine is learned when sojourn time is less than T_1. This type of analysis has a close counterpart in the neural network analysis of learning.

Kennedy (1988) reports that in their research Malinow et al. (1988) "provide strong support for the general idea that persistent protein-kinase activity underlies the maintenance of long-term potentiation -- long-lasting increase in the effectiveness of synaptic transmission". This result is the first step in constructing accurate network models of memory.

The random walk in a random environment (RWRE) appears to be most suitable for modelling learning, memory, and malfunctioning of neural networks. The methods being developed by Pemantle (1988) combining the Polya-Eggenberger exchangeable urn with a version of Spitzer's random walk on infinite trees are especially attractive. Roughly speaking, this random walk is manipulated so that a transition from node a to node b along arc

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[45]Indeed, if the stopping times satisfy certain "reasonable" properties the convergence of a "learning type" of Markov process is a threshold phenomenon. See Matthews (1988).

ℓ = acdb is more probable every time it is taken. The positive feedback has a critical value (threshold) where a phase transition occurs from transience to recurrence. By exchangeability this process is shown to be equivalent to a RWRE.

12. RECENT DISCOVERIES IN BIOLOGICAL AND COMPUTER NEUROSCIENCE

The goal of this section is to demonstrate that Adam Smith's "invisible hand" is a unifying force in real-neuroscience and has significant implications for computer-neuroscience. If this assessment is correct, then to assert that economists and neuroscientists have much to learn from one another is the grandest of understatements.[46]

The competition and cooperation that exists among neurons and neural-networks[47] corresponds to those activities among agents in a free enterprise economy.

The neuron is composed of <u>dendrites</u> which receive signals from <u>axons</u>, the <u>cell body</u> (soma) which produces a <u>voltage pulse</u>, <u>the action potential</u>, the <u>axon</u> which transmits the action potential (spike) to the <u>synapse</u>. The synapse causes <u>neurotransmitters</u> to be released as action potentials arrive. The neurotransmitter is a molecular chemical signal. They cause the cell to fire (or prevent it from firing) depending upon whether they excite or inhibit.

There are two kinds of neurons. The first produces action potentials (APs) at the <u>axon hillock</u>, the place where the axon is joined to the soma, by means of <u>threshold mechanism</u>. The second kind of neuron communicates

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[46]An important lesson to be learned by many economists is the necessity of spatial and temporal scaling transformations. These are implicit in dynamic economic models, but deserve explicit consideration. When this is done, the initial claim becomes stronger: prices and neurotransmitters unify economics, neuroscience and all of neurobiology. Indeed, this unity is so strong that a new discipline is emerging: <u>econeurology</u>. The emergent discipline includes: ecology, biological neuroscience, computer neuroscience, economics, and psychology. A defense of these claims is presented in McCall (1989).

[47]Furthermore, the research by Eigen et al. has discovered similar price-like mechanisms steering the evolution of macromolecules. Adam Smith's "invisible hand" appears to be a universal law, whose conservation principle is fairness.

through its axon the composite intercellular potential (CIP) in the form of a graded potential (GP). The GPs are essentially continuous time signals generated by a variety of biophysical interactions. These GPs carry information a relatively short distance, whereas the APs convey information over long distances as they are recharged by excitable membrane tissue. The main point is that the information flow through the central nervous system is in the form of either an action or graded potential, both of which can be interpreted as prices.

The influence of computer neuroscience has tended to dominate biological neuroscience. Frequently, discoveries in the latter are overshadowed by innovations in the former. For example, computer neuroscience almost always portrays the axon as the primary (only!) informational conduit connecting neurons. Yet, biological neuroscience has been emphasizing for sometime the extraordinary communications conducted by the dendritic "bushes" and their corresponding significance in the organism's memory and neural decisionmaking.[48]

Furthermore, the cost of concentrating on neural networks is the tendency to neglect the important functions of the individual neuron and its amazingly intricate network.[49]

The ability of the neurotransmitter to depolarize and repolarize is exploited to form neural networks. The famous network models by Hodgkin and

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[48]An excellent discussion of these processes is contained in the monograph by Lynch (1986). The role of the apical dendritic spines in the brain's memory processes is elaborated in the commentary by Shepherd. Killackey in his commentary reiterates the importance which Lynch attributes to combinatorics and notes that connection patterns are spatio-temporal phenomena in learning and also in achieving idiosyncratic uniqueness.

[49]The splendid paper by Lewis (1983) is the exception, proving the value of interdisciplinary research.

Huxley (H-H) are based on electrical network theory.[50] These networks when properly informed by the neurotransmitters are the source of the body's coordinated responses to fluctuations in the external environment. Agreement among neurons at one level of the informational hierarchy pyramid is displayed by a "consensus" rhythm.[51] This rhythmic pattern increases the probability of "price" information crossing the threshold of the higher level neurons and <u>inducing</u> the same rhythmic behavior among them. The source of the rhythmic pattern and its intensity was the action of neurotransmitters on the originating neurons.[52] Gray et al. (1989) have shown that

> ... neurons in the cat visual cortex have oscillatory responses in the range 40-60 Hz which occur in synchrony for cells in a functional column and are tightly correlated with a local oscillatory field potential. This led us to hypothesize that the synchroniza-

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[50]A succinct and lucid introduction to the mathematics of neurons is contained in Hoppensteadt (1986). An informative, less technical comparison between the neuron and the logic gates in the CPU of a computer is in Churchland (1988). Those interested in random walks and electric circuits should consult Doyle and Snell (1985). The most complete and lucid analysis of the H-H experiments is contained in Cronin (1987). We have ordered, but not read the highly acclaimed volumes by Tuckwell (1988).

[51]For a lucid discussion of nature's rhythms and stopping times (clocks) see Glass and Mackey (1988). They display a rare and appropriate modesty for chaotic analysis. In particular, they observe that the difference between noise and chaos is usually impossible to measure. Furthermore, for discrete dynamical processes, the addition of measurement error (noise) converts a time-dependent process into one that is ergodic. Yu and Lewis (1989) argue that noise "converts the basically nonlinear operation of spike initiator into a nearly linear modulation process." Similar arguments have been advanced for chaos. It does seem reasonable that an adaptive organism would learn how to exploit persistent forces, like noise and chaos, and increase its survival probability. An excellent discussion of the foundations and economic pertinence of chaos is contained in Brock and Malliaris (1989). The importance of chaos for economic analysis is also being studied by other prominent economists. An overall appraisal by Arrow and a nice survey by Boldrin are contained in the volume edited by Anderson et al. (1988).

[52]This type of behavior can be described probabilistically. See Aldous (1988).

> tion of oscillatory responses of spatially distributed, feature selective cells might be a way to establish relations between features indifferent parts of the visual field. In support of this hypothesis, we demonstrate here that neurons in spatially separate columns can synchronize their oscillatory responses. The synchronization has, on average, no phase difference, depends on the spatial separation and the orientation preference of the cells and is influenced by global stimulus properties. [p. 334][53]

The flow of information among the neural networks is accomplished by a signalling mechanism comparable to the "invisible hand", competition among signals and cooperation once agreement is achieved regarding the informational content and its value to system performance. One can carry the metaphor to extremes, but based on the latest empirical findings there seems little doubt that the information and control system of mammals is strikingly similar to the corresponding system operating in capitalist economies.

These findings are compatible with Eccles' modular hypothesis. He maintains that the Central Nervous System (CNS) operates by continuous conflict among adjacent modules.

> There is an immense power interaction of excitation and inhibition. it is in this continuous interaction that we have to think of the subtlety of the whole neuronal machine of the human cerebral cortex composed of one to two million modules each with up to 10,000 component neurons. We can only dimly imagine what is happening in the human cortex or indeed in the cortexes of the higher mammals, but it is at a level of complexity, of dynamic complexity, immeasurably greater than anything else that has ever been discovered in the universe or created in computer technology. [p. 243 Eccles]

Eccles' competitive modular hypothesis has the self-conscious mind modulating and controlling the informational flows from the modules. There is consensus (cooperation) among the neurons in a given module and dictatorial control by the self-conscious mind. Here the brain corresponds

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[53] The synchronous oscillatory hypothesis is a version of Crick's neural searchlight, which simultaneously affects those neurons stimulated by the identical external object. This theory also has a price-theoretic interpretation. We return to it later.

to a competitive economy while the self-conscious mind is a proxy for the Impartial Spectator.

The neuronal assembly stops its usual activity to initiate correlated oscillations in response to the information about the external environment. This phased vibration generates a postsynaptic membrane potential in the higher order neuron such that it could fire rhythmic high frequency spikes above the threshold. This synchronization of stopping (and starting) rules demonstrates that the hierarchical arrangement of neurons corresponds to an influence diagram.

The Source of Subjectivity: The Impartial Spectator Revisited

Llinas (1988) reviews the electroresponsive characteristics of the neuron in mammalian central nervous system (CNS). Some of these cells also display rhythmic oscillations. The networks in which they reside also oscillate. In these networks the neurons behave as true oscillators or resonators. This description is similar to the fluctuating networks comprising the building blocks of our business cycle model. At any rate Llinas shows that these networks regulate the sleep-wake cycle and control "attention". Crick calls the latter function the "searchlight" mechanism. It enables the brain to focus on a single aspect of reality and ignore all other information. Several mathematical models have been developed to describe the limit-cycle behavior of these networks.

Llinas suggests that the "information-carrying properties of the brain" is related to two components. The first is "closed" and is the source of subjectivity and semantics, while the second component is "open" and generates sensory-motor transformations for relating the private component and the external environment.

In concluding this important paper, Llinas observes that:

> Especially during development, oscillation and resonance allow single elements of the CNS to be woven into functional states capable of representing and embedding external reference frames into neuronal connectivity. In addition to these embedding properties, oscillation and resonance generate global states such as sleep-wakefulness rhythms and probably emotional and attentive states. Although sensory nerve pathways deliver messages to the CNS that are quite invariant with respect to given sensory stimuli, the manner in which the CNS treats these messages depends on the functional state of each relay station. Thus, rather than a simple mirror of the external world, the CNS embodies a dialogue between the internal states generated by the intrinsic electrical activity of the nerve cells and their connectivity, which represents the internal context, and the information that reaches the brain from the senses. This latter point may also be significant to CNS pathology if one considers that alterations of this intrinsic reference frame may underlie much that is important to certain neurological and psychiatric conditions.
>
> Still more fundamental, however, is the possibility that functional organization of the CNS, based in part on the intrinsic activity of neurons, may be the key to understanding the nature of subjectivity. In principle one can see how the intrinsic activity of neurons, which reflect a closed reference system, may be the stage on which our image of the external world is ultimately generated.

Lisberger (1988) studies the vestibulo-ocular reflex (VOR), the movement used to investigate the neural basis for motor learning in monkeys. Lisberger notes that while the neural and cellular foundations of learning are well-developed, little is known about the corresponding learning in mammals. His findings modify the original work by Ito in several important ways. From an economic perspective we discover: <u>prices</u> (neurotransmitters) conveying information and coordinating behavior, <u>specialization</u> in the design of the error detection and correction pathways, producing smooth adjustment to the changing circumstances and the neural basis of <u>learning-by-doing</u>.

The generality of our findings on the VOR suggests that we can think of simple motor systems in terms of three sets of parallel pathways. One set -- is specialized, for initiating a response at short latencies, one for providing accurate movement, and one for directing and correcting errors in motor performance.

> One general role of the cerebral cortex may be to assemble an output that guides motor learning ... the output from the cerebellar cortex of the flocculus appears to be essential for immediate guidance of eye movement as well as for it long-term calibration ... This dual effect of the output from the cerebellar cortex ...

13. GAINS FROM TRADE: THE RATIONALE FOR INTERDISCIPLINARY RESEARCH WITH APPLICATIONS[54]

The paucity of joint research projects by economists, psychologists, neuroscientists and computer scientists reveals the difficulties and perceived rewards of such an enterprise. Recent research has altered the reward structure. If our appraisal is correct, the community of neuroscience and artificial intelligence can learn much from Adam Smith's disciples -- modern price theorists who are familiar with the benefits and limitations of the price mechanism. Interpreting neurotransmitters (potentials) as prices and thresholds as reservation levels suggest that the accumulated wisdom of microeconomics and its stochastic counterpart is pertinent to the neurosciences.

On the other hand, economic science also can reap substantial gains. For example, the formation of tastes, the foundations of assembly (production functions) and the underpinnings of human capital theory may be fortified. The serendipity characterizing many of the revolutionary innovations in computer science vis neural network research could accompany this interdisciplinary enterprise.

Combinatorial mathematics is the methodological glue preserving the linkage and in many instances showing the way. Combinatorics is primarily

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[54]The mutual gains reaped by both neuroscience and statistical mechanics in their interdisciplinary research are summarized by Toulouse (1987). Clearly, the physicists have brought precision to the enterprise plus a formidable tool box, the most useful tool being the thermodynamic kit (von Neumann predicted that computers would develop a theory closely related to thermodynamics). Their success was based on the ability of these models to ask the right questions plus their respect for the real neuroscientists and their research on the masterpiece -- the brain.

Comment: Those who correctly object to our neglect of von Neumann may be mollified by the revised version where they will find him almost everywhere. For now they may wish to read the superb tribute to him by Brink and Haden (1989).

concerned with flows. The power of hydrodynamic analysis throughout the sciences testifies to the clear thinking imposed by H_2O. The equivalences among hydrodynamics, electric networks, combinatorics, random walks, and the price mechanism have not been exploited by any of the sciences comprising this interdisciplinary enterprise. Indeed, it is in this area that economics may receive immediate rewards.

As we will see, reality is deterministic when viewed from one perspective and stochastic when the focus changes. The nature of the problem coupled with the abilities of the researcher influence his choice between deterministic and probabilistic paths to the solution set. When the analysis is conducted properly, the solution set is invariant to the path.

The separation of signal from noise is a serious problem in an economy undergoing a phase transition. In a real economy this might be a period of rapid inflation, while in the neural economy, it might be a period of abnormal transmitter behavior caused by some organic failure. At any rate, several methods have been devised to isolate the signal.

Applications

The following is a list of applications which could comprise the initial phase of an interdisciplinary research program.[55]

(a) The Economics of Habit Formation and Addiction

There is a small but important economic literature on habits, addictions and fluctuating tastes. Several of these articles are related to economic growth and learning-by-doing. As far as I know none of these

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[55] The current absence of communication is exemplified by the number (n) of references to economic research in Logue's (1988) survey of the self-control literature: n = 0.

papers refers to neuroscience research and its relation to Smithian "psychology".

Some of the key concepts emerging from this literature include:

(1) The evolution of preferences: "'Habit forming' means that people learn to consume by consuming and the more they learn the more they enjoy; the evolution of preferences of the consumption level and, of course, the consumption path is itself dependent upon the evolution of preferences". Boyer (1978) then analyzes the effect of habit-formation on economic growth. A sophisticated study of the growth process concludes with a stability analysis of the steady state and its sensitivity to discount rates when habits are endogenous.

Iannaccone (1986) designs a rational model of habit formation and determines when the outcome will be addiction or satiation. "A commodity is addictive if its current consumption increases as the habits derived from its previous consumption accumulate. It will be called satiating if the opposite occurs". His discussion attempts to clarify the Stigler-Becker (1977) distinction between "beneficial" and harmful addiction.

In their recent article Becker and Murphy (1988) include a wide variety of activities in their definition of addiction. They maintain that addictions are usually rational "in the sense of involuntary forward-looking maximization with stable preferences. This rational view provides new insights into addictive behavior. Their definition is so broad as to include work, music, etc.

They maintain that their model <u>explains</u>: why abrupt withdrawal is required to terminate strong addictions, binge behavior, and relate temporary stressful events to permanent addictions. The analysis is insightful but tends to ignore the recent advances in neuroscience regarding addictive behavior. Their assumption of homogeneity their description of "cold turkey", and the inclusion of strong attachments (habits) as <u>addictions</u>, which are necessary requirements for civilized behavior suggest flaws not in their economic analysis, but in their overall perspective.

The economic studies of addiction seem to overlook the effect of addiction on human capital formation, reputation, the importance of parental guidance in the educational process, and the externalities created by the behavior of addicts. Addiction shocks habits which are essential for civilized behavior and sometime the shock is sufficient to unravel these basic traits.

<u>The Neuroscience of Drug Dependence</u>

In their recent study Koob and Bloom (1988) try to discern if the "molecular, cellular, and behavioral data on acute and chronic effects of addictive drugs possess a consistent pattern in which molecular effects influence cellular effects which then after behavioral phenomena in keeping with the common features of drug dependence. Three of the most prominent addictive drugs -- opiates, psychostimulants and ethanol -- are analyzed from this triple perspective -- molecular, cellular, and behavioral.

They also review a variety of addictive theories. The focus of their study is on

> both the acute initial effects of drugs of abuse on specific neurons and the change in these effects with continued drug exposure. We then seek to link these data into generalizable features of addictive drugs and of the systems on which they product their effects. Finally, we propose a potential role for a specific limbic-extrapyramidal system that has been implicated in both the reinforcement and adaptive responses to all three drug classes.

They define addiction as follows:

> A more recent definition of psychoactive substance dependence used for diagnostic purposes is that dependence "is a cluster of cognitive, behavioral, and physiologic symptoms that indicate that

> the person has impaired control of psychoactive substance use and continue use of the substance despite adverse consequences.

They conclude:

> Across all levels of analysis, molecular and cellular mechanisms of the nervous system react to addictive drugs to initiate and maintain patterns of drug-seeking behavior. We hypothesize that these patterns of behavior emerge primarily because the drugs are able to usurp the crucial reinforcement systems and the small finite number of transmitters and response sites that operate normally to shape survival of the organism.
>
> We further hypothesize that the same neurobiological circuits involved in the acute hedonic or "positive reinforcing" actions of drugs may become modified through chronic use as the self-corrective homeostatic responses of the brain adapt to the drug actions. The opposing process may neutralize reinforcing effects and on withdrawal produce the aversive stimulus effects of the abstinence syndrome. We speculate that these "negative reinforcing" effects (for example, malaise, dysphoria, and anhedonia) are a major etiological and motivational factor in maintaining drug dependence. Thus in this conceptual framework, physical signs of abstinence per se are not necessary or sufficient for dependence but the "negative reinforcing" effects are necessary and sufficient for dependence.

The notable difference between the neurological and economic studies is the absence of neurological factors in the economic studies <u>and</u> the absence of economic factors in the neurological studies. We maintain that neuro-economic analysis of addiction could yield significant new findings.

It is imperative to distinguish between habits that are essential for civilized behavior and habits or addictions that interfere with civilized behavior. The fact that these may vary among cultures does not diminish the significance of the distinction. As emphasized by Adam Smith et al., the learning and sustenance of civilizing routines coupled with the actions of the "invisible hand" and the "impartial spectator" are vital to the smooth functioning of those institutions that comprise the free enterprise economy. The disruptive effects of addictions on these civilizing habits is their most important <u>social cost</u>. More formally, the shock of substances like heroin and cocaine frequently is sufficient to undo the ergodicity of these habits and routines essential for reliable performance in the civilized economy. Whether the source of this disintegration is the addictive substance interacting with environmental and neuronal influences or simply the effect of the substance on neuronal behavior is irrelevant. The disintegration is prominent and must be included in any analysis of addiction.[56]

(b) Learning-by-Doing

This is another area where interdisciplinary research could be valuable. The recent study by Stockey is a sophisticated economic analysis of learning by doing. This factor is identified as "the force behind sustained growth". The effect of learning on growth is analyzed in economics with and without a traditional sector where no learning occurs. The analysis reveals the importance of learning in spillover to the production of other goods.

Learning by Doing in the Neuroscience Literature

Learning by doing is precisely the manner in which habits and routines

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[56] One would surmise that the dramatic phase transitions associated with the onset of a disease, like schizophrenia or mania, would not only sunder the circuits associated with civilized behavior, but also generate chaotic neuronal behavior. Mandel (1985) studies mental illness from this perspective. The response of the psychiatric community to the recent and spectacular discoveries in neuroscience is appraised in Pardes (1986). While there is surely a broad spectrum of opinions among practicing psychiatrists, Pardes concludes:

> ... we are experiencing a dramatic and dazzling renaissance in research on the brain. This promises to have extraordinary application to our understanding of behavior and of psychiatric disease. Leaders are pointing the way for linking brain and behavior. Further, clinical psychiatry is becoming differentiated and more potent.
>
> In answer to the question, Are neuroscience and psychologically based psychiatry reconcilable?, I submit that they are increasingly and that their reconciliation is one of the cornerstones of the field. We have to take responsibility for allaying the apprehensions of the large group of psychiatrists who feel intimidated and threatened by scientific developments and help make them feel strengthened rather than endangered.

are instilled. This activity has received much attention in the neuroscience literature. The attempt to mimic learning includes several strategies. Among the most noteworthy are those inspired by Hebbs, the correlation strategies, those inspired by Newton, the gradient descent methods, and those which trace their origin to Darwin, the random mutation and crossover learning algorithms. Routines are formed by repetition. The hypothesis modeled here is the existence of a critical number (threshold) T such that after individual i has repeated the action T_i times, conscious effort becomes negligible in the performance of habitual behavior.

In their recent study Davies et al. (1989) analyze the behavior of long term potentiation (LTP). LTP increases synaptic strength markedly with repeated use -- a micro version of learning by doing.

Most neurologists consider the induction and maintenance of LTP in the hippocampus as the mechanism fundamental to learning and memory in vertebrates. While the induction of LTP is understood, the causes of its persistence are not well-known. Davies et al show that both pre- and post-synaptic activity appear to influence the maintenance of LTP.

(c) Conflict and Cooperation[57]

Algorithms have been devised to solve a host of interesting "agreement/conflict" models on a synchronous and asynchronous system of processes linked by a communications network and conducting distributed computation. Each processor has both private and public information. The public information is acquired from the other processors. For example, n computers may combine to solve a fixed point problem, where the fixed vector has n

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[57]Economists have made substantial contributions to understanding the conflict/cooperation perplex. It's impossible to mention these here. The interested reader should consult Hirshleifer (1987) and the references therein.

components, one calculated on each machine. Synchronous computation is sometimes less efficient than asynchronous because several computers may be much slower than the others and real time decisionmaking may be too costly.

Problems studied by Bertsekas (1983), Borkar and Varaiya (1982) and Li and Basar (1987), include a variety of cooperative and non-cooperative games, team problems, and stochastic versions of decentralized decisionmaking. The seminal paper was Aumann (1978).

The transmission of information through a communications network causes "conflict". Messages are lost when simultaneous transmission is attempted. The controller of the network must detect these collisions and resubmit lost messages. This type of conflict has received much attention in the last few years. A survey is contained in Kelly (1985). Recent research includes the papers by Aldous (1987) and Padmanabahn and Netravali (1987).

The general area of convergence theory in an organizational setting is surveyed by Bertsekas, et al. (1988). The seminal paper is Arrow and Hurwicz (1960). The Russian school has made considerable progress on applying stochastic automata theory to random walks on random environment. Their analysis of games of infinite automata is quite distinct from classical game theory. Automata select strategies based on the information contained in their inputs. A good survey of this research is contained in Korolyuk et al (1988). The recent paper by Abreu and Rubinstein (1988) is in this tradition.[58]

There is little doubt that games with automata can illuminate real economic behavior. However, their limitations also must be acknowledged. Automata built according to the von Neumann-Morgenstern axioms allow the

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[58]The recent book by Goles and Martinez (1990) is a superb survey and novel analysis of automata and neural networks.

researcher to alter axioms and the structure of games so that the latter corresponds more closely to "real" games as perceived by the economist, and the former improves its explanations. But this interplay will never converge to an understanding of the laws of economic science because economic reality never enters directly. Furthermore, the improved axiomatic system will always be subject to embarrassing paradoxes.

These perplexes and the paucity of realism may be mitigated by constructing neural network automata according to the brain's template and learning the behavior of the real economic environment. Here the convergence, however slow, is being guided toward the "truth". When these games are augmented by controlled experiments within the neuroscience laboratory, using actual behavior of individuals and firms to test the hypotheses devised by the economic/neuroscientist, nature may begin to reveal the laws of economic science.

The evolutionary strategies devised by the immune system[59] to defeat intruders seem pertinent to the recent work on conflict and cooperation by economists. For example, Consider an individual who poses as a Swiss citizen with all the necessary papers perfectly forged. He enters easily and remains undetected for t periods. At this point he arouses the suspicion of a co-worker by his ignorance of Swiss history. He is investigated and ejected. Now consider an individual who has just received a heart transplant. His immune system is initially "convinced" that the heart is

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[59]The evolution of the immune system, its ability to distinguish self-from non-self, thereby conferring uniqueness on the individual, Medewar (1957), and the strategies and tactics utilized by intruders and the immune system all are worthy of economic analysis and vice versa. See Langman (1989).

the same as the original. Eventually, the constant surveillance by the T cells reveals the truth, and the organ is rejected.[60]

(d) Game Theoretic Implications

The question of common knowledge has captured the attention of some of the leading game theorists. For example, in Binmore and Brandenberger (1988), the "Harsanyi doctrine" that priors should be regarded as common knowledge is critiqued along with Aumann's appendage that everyone should know that priors are common. This is interpreted as saying that homogeneity reigns supreme. The key point is: decisionmakers do not "play games" with randomly chosen individuals. If the outcome of the game has serious consequences, the choice of players will be very careful. To use Smith's terminology, only close friends will be chosen. (This assumes that serious games played daily with automobiles, etc., are guided by shared routines; and indeed are not games). Among the group of close friends, it seems reasonable to assume that there is common knowledge relative to the game being played. These games will be fair and predictable in the Doob sense. Of course, mistakes will be made and moral hazard remains, but they are being controlled by the circle of decisionmakers one chooses. The size of the expected losses is determined by the radius of the circle.[61]

(e) Neuronal Assembly and Organization Theory

The formation of assemblies of neurons to perform specific tasks

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[60]This example is not far-fetched. It has been observed that an intruding parasite mimicked the behavior of the immune system so well that it was incorporated into the system and ceased being a parasite.

[61]When contemplating a particular economic "game", individuals instinctively partition society into independent subsets with exchangeable members. Decisions are confined to the "friendliest" subset. The analogy with insurance is compelling. Crossing subsets corresponds to a phase transition, that is, suspension of the law of large numbers.

corresponds with the cooperative and competitive organizations which characterize a free enterprise society. McCulloch and Pitts (1943) and Hebb (1949) predicted these cell assemblies. Gerstein et al. (1989) define neuronal membership in an assembly on the basis of "correlated" firing, that is, the cooperative timing relations among member neurons. This associated behavior has been observed and "can be used as a defining indicator of neural assembles and assembly processes."

Palm (1982) applies the assembly method to his circuit arrangements for associative[62] memory". These assembly processes have their economic counterpart in the production processes of the firm. Indeed, all of the organization theory receiving widespread attention by the economics profession is relevant to neuroscience. This is most clearly dramatized by Grossberg et al. (1989). This broad-ranging article uses the following terminology: hierarchical resolution of uncertainty, division of labor, cooperative-competitive feedback, discounting, boundary-feature tradeoff: a new organizational principle and real time statistical decision theory and signaling. The pioneering research by Grossberg and his associates at the Center for Adaptive Systems has many implications for the design of economic organizations.

(f) <u>The Neuroscience of Foraging</u>

The economics of foraging by animals and early man is a well-developed area.[63] The basic model has a threshold structure in that stop and go decisions are determined by optimal stopping rules.

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[62]Association and correlation are dependence relations which can be defined precisely. They are then implied by infinitely divisible processes. See the <u>Connection Theorem</u>.

[63]For example, see Stephens and Krebs (1986) and Lippman and McCall (1985).

Several neuroscience studies show that foraging has its foundations in neuronal behavior.[64] Indeed, the learning and competition, cooperation, and threshold phenomena observed at the neuronal level are self-similar with the corresponding activities associated with foraging.

<u>Comment</u>: The economics of neuroscience reveals modularity and hierarchy as self-similar phenomena. Neural networks, the brain, the individual, the family, the firm, the industry and the entire economy have evolved modular and hierarchical structures for converting incoming information into decisionmaking. It has been shown recently that the individual neuron is composed of an intricate network!

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[64] One of the first papers on this topic was "What the Frog's Eye Tells the Frog's Brain," (reprinted in McCulloch (1965), by Lettvin, et al. The recent studies by Deno, et al. (1989), and Arbib, and Lara et al. in Amari and Arbib (1982) completes this title and show how the frog's brain converts incoming information (the presence of a prey) into decisionmaking behavior (attack the prey if a threshold criterion is satisfied).

14. RELATED LITERATURE: A BRIEF REVIEW

In their influential book, Holland et al. (1987) arrive at conclusions closely related to many of those expressed here. For example, their framework "demands that individuals differ in the rules they use to understand the world". This heterogeneity is vital to their conception of inference and adaptive learning. They emphasize the "permanence" of rules learned in childhood in that the "child will be found in the man -- ready to compete with him in understanding the world". The theory they espouse is applicable to economics,[65] immunology,[66] psychology, neurophysiology and other complex systems. Their goal of requiring all assertions "to be rendered computational" seems to be one source of the difference between our programs. Another being their dismissal of Bayesian inference. Presumably, many of these perplexes would be resolved or clarified as an interdisciplinary program unfolded.

In her expository book. Churchland (1986) discusses the tendency for practical neuroscientists to undervalue theory claiming that it is premature, too abstract, untestable and irrelevant to experimental neuroscience.[67] Churchland presents a vigorous defense of theory and describes three of those receiving the most attention: The Tensor Network Approach of

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[65] The unifying principle they seek may be contained in the economic interpretation of the "bucket brigade algorithm", namely, the conservation of fairness. A recent application of the genetic algorithm to the economics of liquidity is presented in the ingenious paper by Marimon et al. (1989).

[66] The genetic algorithm and its versatility is portrayed in Davis (1987). Also see the superb article by Farmer et al. (1986).

[67] Churchland contains lucid descriptions of both computational and biological neuroscience. Her discussion of the body-mind problem is too scientific. Perhaps some philosophers of science may find it insightful, but the "modern Aristotle" probably would not be convinced. See Stent (1986) for a critical review.

Pellionisz and Llinas, The Parallel Computation Approach (Connectionism) of McClelland, Rumelhart, Smolensky et al. and The Attentional (Searchlight) Theory of Crick.[68]

We will summarize the connectionist theory contained in the articles by Holyoak (1987), and Smolensky (1988) and relate it to the approach advocated here.

Connectionism and Computer Neuroscience[69]

In his incisive and comprehensive review of Rumelhart et al. (1986), Holyoak (1987) observes that computer science has influenced cognitive science (psychology) much more than neuroscience. As more is learned about the brain, this imbalance should be rectified. One of the goals of Parallel Distributed Processing is to replace the "computer metaphor" as the model of the mind with the "brain metaphor". This corresponds to the Crick position and also reflects our vision of "neural economics". The brain solves NP hard problems continuously. We conjecture that this ability flows in part from the symmetry which pervades the design organization and operation of the Central Nervous System (C.N.S.) The brain now can be analyzed directly. This analysis may resolve many of the mysteries of the C.N.S., which have pertinence for economics, psychology, and computer science.

Holyoak notes that the basic premise of parallel distributed processing is the representation of information by patterns over neuron-like elements connected by synapses (excitatory or inhibitory). Simply put, this is an input-output system with Bayesian updating by learning algorithms such that

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[68] The silence on tunneling is deafening. It provides by far the most support for reductionism and was discussed by Von Foerster in 1949. His work is described by Hoagland (1966).

[69] It is impossible to summarize the PDP research in Rumelhart et al. (1986).

the neuronal response "optimizes" a system criterion given the environmental inputs.

The competitive learning algorithms utilize a generalized delta rule to adapt to the patterns presented by the environment. A "teacher" enters the weighting process so that the proper weights are assigned ("propagated back") to hidden and input units which are only indirectly connected to output units. This back propagation is crucial to the algorithm's ability to learn basic logical operations.

Both Smolensky's harmony theory and Hinton and Sejnowski's Boltzmann machine are described in detail. Holyoak remarks that both models exploit the isomorphism between "information processing" and "statistical thermodynamics". They are, therefore, formally equivalent. The mathematical analysis of the Ising model[70] employs birth and death processes, graphical methods, Ruelle's (1979) thermodynamic formalism which is founded on the Choquet generalization of de Finetti's exchangeability theorem, and a host of novel probabilistic techniques -- coupling, reversed processes, duality, and the general theory of processes -- which generate estimation methods for approximating the complex spatial and temporal behavior of the brains. These methods take account of the different time scales associated with the stochastic brain processes.

In his articulate and scholarly treatment of Connectionism, Smolensky (1988), clarifies its scope and goals. His remarks on the distinction between individual and cultural knowledge are similar to the Smithian view advocated here. In particular consider the sentence singled out by Belew: "We can view the top-level conscious processor of individual people as a

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[70]Liggett (1985) is the definitive monograph. See Section 7.

virtual machine -- the conscious rule interpreter -- and we can view cultural knowledge as a program running on that machine". The only difference is that machine is a metaphor for brain. The significance of shared cultural knowledge in a rule structured society is precisely what Smith envisaged. However, rather than a mechanical device he emphasized the spontaneous order that emerged when individuals followed society's rules and adapted to new information provided by the price system because it was in their self-interest. To do otherwise would demonstrate the absence of common sense. A decentralized brain is an immediate consequence of this Smithian common sense.

Smolensky also has the conservation of fairness integrated with his harmonious connectionism. "The competence of the system is described by hard constraints: Ohm's Law and Kirchoff's Law -- the laws of simple circuits. It's as though the model had those laws written down inside it". This is the "no arbitrage" principle, the fair game rule that is hard to follow when one suffers a streak of bad luck. These circuits are equivalent to random walks. Hence fairness and common sense combine to imply: a decentralized Bayesian brain, a decentralized adaptive computer, and, of course, a decentralized adaptive economy.[71]

To be more specific about the operation of the Bayesian brain, the neurons and/or modules adapt to new "price" information in Bayesian fashion. They are engaged in a stochastic game with outcomes determined by both choice and chance. The self "oversees" their performance, so that self-interest is maximized. The behavior of the person in society accords with

[71]Bayesian adaptation is recursive and, hence may be computable. An excellent discussion of rationality, computability and complexity is contained in B. Rustem and K. Velupillai (1989).

this description. Decisions are optimal given constraints, that is, decisions and relations with other members of society are constrained by the civilizing routines and customs acquired before the individual is accepted as a full-fledged member. Once again the outcomes of the stochastic game are determined by choice and chance.

Just as the acquisition of routines was accomplished in a random environment so too an orderly society is the consequence of this enormous stochastic game. The only caveat is that no society-wide shock disturb this ergodic process. This could cause a disastrous phase transition, with a long period of disorder marking the transition to a less (or less likely, a more) civilized state.

The crucial insight is the realization that a healthy society is composed of equilibria and processes that are not in equilibrium. These competitive processes guided by the price mechanism create surprise and order as individuals adjust their plans and modify their expectations. The institutions comprising the equilibria must be continuously nourished so that citizens can rely on them when the stochastic competitive process jars their self-confidence. Smith recognized that bad luck could cause a weak society to unravel. Those who play the competitive game and lose must be sustained by society's equilibrium institutions.

For example, one of the costs imposed by the conservation of fairness is its indifference to both good and bad luck. Over long periods of time extinction is certain for organisms that have not developed some kind of insurance mechanism. The operation of the "invisible hand" also is ruthless. Fair games frequently punish the innocent. For example, a middle-aged worker may discover that the demand for his skills has vanished overnight in spite of his diligence and productivity. In the course of his life he has developed many local attachments and is reluctant to migrate to areas with job vacancies. If he has prepared for this contingency, and also receives unemployment benefits, it is likely that he will remain unemployed (in the accounting sense) rather than sever relations, try to acquire new skills and move to a "foreign" location. Any theory of unemployment which ignored these impediments to job search probably would be rejected by empirical analysis.

In his stimulating book, Buss (1987) argues convincingly that hierarchies emerged as shelters from the violent oscillations of environment which threatened extinction for non-hierarchial organisms.

The obvious economic analogy to biological extinction is bankruptcy. The mergers phenomenon depicted by Buss is one arrangement for reducing this risk of failure. The synergism and competition attending mergers is precisely what Buss associates with his hierarchial "Dawkins" vehicles. The evolutionary insurance policy operates in a subtle manner. "Whereas the appearance of a new unit, selection is imposed by the external environment alone, after the unit arise, selection by the external environment acts only upon the higher unit." The lower unit is not a passive partner in this selection activity. When the merger is "optimal", the lower unit enables the ensemble to exploit aspects of the environment that were previously irrelevant. The end product is a hierarchical entity less vulnerable to external change. Of course, most of these attempts to survive by merger will fail. The survivors, however, will be more robust. Buss' reversed reductionism not only depicts gaps in the accepted evolutionary wisdom, but also suggests that the ability to isolate the genome in cells like the neuron signals the onset of an exciting intellectual period in the life sciences. This innovation is the history of the life sciences. The evolution of the spatial relations among the occupants of a particular shelter are present in this monumental history. Furthermore, we now possess the technological devices for "reading" this history.

15. THE STRUCTURE OF THE OSCILLATING ECONOMY[72]

The economy has a modular and hierarchical structure, which resembles the design of the brain (see Figure 1). The firm and the individual are the two basic entities. The individual has a fluctuating utility function, whereas the firm possesses an assembly process which converts inputs into outputs demanded by individuals. The individual is a collection of contracts (relationships) which are continuously modified as circumstances changes. Job, marriage and location are three of the basic relationships. The formation and dissolution of contracts is characterized by birth and death processes which give rise to a vibrating spatio/temporal network.

The match between the firm and the individual is a key contract and has received much attention in the economic literature.[73] The match is based on thresholds and persists as long as both parties remain in the jointly acceptable region.

The formation and demise of firms and industries also follow birth and death processes.

Some industries are positively related, while others are relatively independent. The motion of the exchangeable set of industries is positively correlated. The independent sets should show no correlation in their fluctuations as a consequence of internal factors.

Of course, the entire economy -- industries, firms, teams, and individuals, are all influenced by external shocks generated by nature and government policy. For example, by enforcing contracts and pursuing stable

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[72]This is a brief summary of McCall (1985, 1988) which the interested reader may wish to consult.

[73]Mortensen (1985) and McMillan and Rothschild (1989) are fine surveys of this enormous literature.

monetary and fiscal policies, the government mollifies the internal, natural fluctuations.

The Modular Economy

As soon as the utility function is freed, the subjective forces are recognized. While there are many basic wants demanded by most consumers there is a remarkable range of commodities which attract some while being neutral or even repugnant to others. Technology responds to this diversity by diversifying its output. Heterogeneous tastes are satisfied by modular production processes and modular outputs. For example, consider the personal computer. If after one year the originally identical computers sold to n individuals were dismantled, the range of differences due to subjective variability would be quite large. These plastic modular products are molded by the individual to satisfy his peculiar wants. The analogy with the brain and the genes is obvious. Many products are not amenable to subjective adjustment after purchase. The supermarket and shopping malls are two important institutions which satisfy this hard wire constraint while maximizing diversity. Flexible production processes, the search for new products and processes which yield heterogeneous output, and the testing of deviant ideas in controlled environments are all responses to the uniqueness of the individual consumer. The firm's production process is in constant flux adjusting to spatio/temporal variations in tastes, while at the same time adhering to those common values shared by most of the population. The survival of the firm hinges on its ability to adapt to the delicate balance between conformity and deviance.

16. CONCLUSION

It is appropriate to conclude this preliminary paper with a brief review of its economic content. Our position is not novel and merely reiterates and expands on the Smithian foundations. Economic science is a process pervading reality. Tastes and technology are the interacting dynamic forces reflecting individual preferences and environmental constraints, respectively. The evolutionary price mechanism is the consequence of these interactions. It is the basic source of information creating orderly resource allocation. By focusing on the spatio-temporal aspects of economic processes and their hierarchical architecture, we can accommodate several economic phenomena which have eluded formal models. These include: the genesis of teams, firms and industries all manifesting fluctuating behavior, increasing returns to scale, the formation and dissolution of implicit and explicit contracts, the presence and significance of indivisibilities, the fundamental and manifold effects of institutions on the viability of any economic system, the intimate connection between economic behavior and neuroscientific phenomena <u>and</u> the ability to measure these nexi thereby converting economics into an experimental science, and, finally, the design of experiments can be conducted as an integral part of the adaptive economic theory, that is, the economic processes envisaged here contain the statistical methods for confronting their implications with real economic behavior. This is in sharp contrast with many current economic models which are immune, and seemingly indifferent, to real economic behavior. The scope of economics is enlarged substantially by activating the links with the other social and physical sciences. This interdisciplinary enterprise can be studied using sophisticated parallel processors to analyze the neural network most appropriate for the specific problem.

The mutual gains from interdisciplinary research by neuroscientists and economists are manifold. On the one hand, many economic experiments can be performed by robots and neuroscience. The feasibility of various decentralization schemes can be tested by algorithms and real neurological systems. The ability to transmit information by price mechanisms when complex spatio-temporal coordination is the required output, can be tested. The value of information could be measured explicitly. Problems of conflict and cooperation, agreement and disagreement, and detection of cheating are amenable to analysis by robots, information networks, immune systems, and neural networks. The validation of econometric methods and the creation of new techniques are almost certain to occur. Of greatest significance is the freedom given to tastes and technology. Their evolution could vitalize economic science and stimulate completely unexpected developments in neuroscience.

On the other hand, economics strongest field, microeconomics, is applicable immediately to these disciplines. Much of the current research in experimental economics is pertinent. In the last few years neurobiologists have learned much about the behavior of neurons and neural networks. They are able to relate changes at the neurological level with behavioral fluctuations. Neurons of mammals exhibit a dynamism in their broad range of electro-responsive properties and possess a <u>greater percentage</u> of the genome than any other cell. Their web of communication links with other cells and their reproductive ability[74] are all contrary to the earlier conceptions of neuroscience.

The acquisition of routines, habits and addictions by the mammal are reflected so finely in the recent experimental research that, for example, the cellular and molecular mechanisms associated with cocaine, opioid, and ethanol addiction are distinguished. Furthermore, the cellular and molecular distinctions are correlated with the corresponding behavior patterns.

Even when one discounts for exaggerated claims of zealots, the advances in artificial intelligence and their implications for computer science neurology and economics are notable. Perhaps the most decisive evidence for joint economic-neurological-computer science research is the remarkable discoveries of the neurological correlates of learning by trial and error, the formation of expectations, and the genesis of a subjective view of reality.

The heart and soul of economics resides in the utility function. Let us not abandon this precious entity to the computer scientists and electrical engineers while we become intoxicated with static VNM paradoxes and Lysenko/Skinner experiments. If we surrender our soul, we risk extinction or, perhaps a fate worse than extinction, equivalence with Turing machines.[75]

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[74]This has been observed in vertebrates.

[75]The possibility of such an equivalence was noticed by Velupillai (1987).

17. APPLICATIONS

This is a tentative list of applications which will test the validity of our methodology and, at the same time, perhaps benefit from this interdisciplinary enterprise.

1. We have designed a crude model of the economy[76] and plan to program various portions of it using some form of the Ising model. The program will be augmented on a parallel processor and the output compared with the most appropriate data set.

2. We are currently studying the signal detection problem in an options market setting. The evolutionary aspects of this problem as the signal/noise ratio declines are of major concern. In particular, we have designed learning algorithm which can respond to low signal/noise ratios by changing the institutional setting.[77]

3. Once we have established the credibility of the dynamic economy model, the power of parallel processing will enable us to expand it so that international economic forces can be considered explicitly.

4. As soon as possible, we hope to apply our model in an area of neuroscience which is fairly well-developed and possesses substantial economic content. The research on vision and foraging is a likely candidate. Obviously, we would require the active participation of a neuroscientist to conduct this application.

5. We have concentrated on neuroscience (tastes) and have been inattentive to the assembly process (production function). This neglect will be corrected when this paper is merged with McCall and Velupillai

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[76] A refined version will appear in McCall and Velupillai (forthcoming).

[77] We are indebted to Arthur DeVany for suggesting this application.

(1989). It also is imperative that we apply these methods to a well-known problem in production. The research on reliability is well-developed and it is from this urn that we will draw our assembly problem. Once again the success of this application would require the full-time collaboration of a system's engineer.

6. The learning process is another important problem we plan to address. Like the other problems, it is multi-faceted. Several significant components are:

(a) The reduction of complex behavior patterns to routines.

(b) The response of routine formation to external shocks.

(c) The stability and adaptability of learning to gradual and sudden environmental change.

7. In 1. a subsidiary, but by no means unimportant, problem is: The resilience of the economy to environmental disturbances. These external shocks can take many forms and may affect only a few firms or the entire economy. We plan to study the economy's response to local and global changes first when the change is gradual and then when it occurs suddenly. We think these four problems can be addressed with the parallel processing technology. These are exceedingly complex questions. The main ingredient for properly posing these questions and providing conditional answers is not technical expertise, but common sense, the scarcest input.

8. The design of experiments: Suppose the combinatorial structure of a sensing mechanism utilized by the brain is estimated. Does the design of the mechanism and the ancillary induction process correspond to the methods we use to extract information from nature? In particular, is there a rough similarity between the way in which we order information processes (Choquet

ordering), design equipment (Position Emission Tomography), and conduct statistical inference and the corresponding methods used by the brain?

9. The design of hierarchies: Does the architecture used by the brain to transmit information reflect decentralized decisionmaking? If the hierarchy is decentralized how does the brain resolve (or exploit) the tension between conflict and cooperation among the modules and within each module? How is coordination achieved? Conjecture: The brain mimics the free enterprise economy.

10. Production decisions: If the immune system classifies heterogeneous invaders too finely, it imposes a heavy burden on the assembly system that "manufactures" antigen covers. The covers must be made-to-order for each of the finely-detailed antigen. On the other hand, if the classification is too crude, the invasion may succeed. How is this trade-off resolved?

APPENDIX

A. The Theory of Connections[78]

The unifying force which commands us to engage in interdisciplinary research can be described in several equivalent ways. The basic conservation principle is the "no arbitrage" condition. The key deterministic concept is convexity, whereas the fundamental stochastic concept is the semimartingale process.

The connections are manifest in many different environments but we focus on three. The first is combinatorial optimization of flow problems. The polynomial solvability of these problems hinges on the notion of unimodularity.

Definition: The second circle of connections revolve around a simple concept: unimodality. A real random variable X is said to be unimodal about a mode m if its c.d.f. F is convex on $(-\infty, m)$ and concave on (m,∞).

Definition: Unimodality is closely related to infinite divisibility and characteristic functions. Let ϕ be the characteristic function (c.f.) of X. The c.d.f. of X, F is said to be infinitely divisible if for every $n \geq 1$, there is a c.f. ϕ_n such that $[\phi_n(t)]^n = \phi(t)$, for all $t \in \mathbb{R}$.

Suppose $\{X_n\ n\geq 1\}$ is a sequence of independent random variables and $S_n = \Sigma_{j=1}^n X_j$. If there are sequences $\{a_n\}$ and $\{b_n\}$ such that G_n, the c.d.f of $S_n^* = S_n - a_n/b_n$, converges weakly to G and

$$\text{for every } \epsilon > 0, \quad \sup_{1\leq j\leq n} P(|Y_{nj} = \frac{X_j}{b_n}| \geq \epsilon) \to 0$$

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[78]This puny theory will be fortified in the revised version when von Neumann algebra is applied to the unimodular concept. When this is done the shift operator and the dilation operator unify probability theory, dynamical systems and economic processes.

as $n \to \infty$, then G is infinitely divisible. The random variables $Y_{n_j} = X_j/b_n$, $1 \le j \le n < \infty$ are said to be infinitesimal. The class of limit distributions obtained as X_n, a_n, and b_n vary is called Class L.

The c.d.f. G is strongly unimodal if the convolution G*F is unimodal for every unimodal F.

Two fundamental results are:

1. Every distribution in Class L is unimodal.
2. A (nondegenerate) c.d.f. G is strongly unimodal iff G is continuous and $G' = g$ is logconcave (log g is concave).

Many of the ordering concepts are related to unimodality. The recurrence of random walks depends on subtle arguments based on symmetry and unimodality. This links unimodality with electric networks, potential theory and the no arbitrage principle. The graphical underpinnings for both unimodularity and unimodality are similar.

Many original theories were formulated as hydrodynamic processes. A random walk on a graph gives rise to a hydrodynamic process. Similarly, specific conformal maps of complex analysis yield hydrodynamic processes.

The explicit introduction of complex analysis reminds us that Brownian motion is an analytic function are connected. This brings us to the third basic concept: univalence.

Definition: A function defined on a set D is called univalent in D if it is one-to-one in D.

Typically, D is the open unit disc $\Delta = \{z \in C: |z| < 1\}$ and functions on D are univalent and analytic.

These concepts open the door to geometric function theory. Complex applications of convexity yield powerful results which are useful in both real and complex analysis.

Theorem (The Connection Theory for Convexity)

The basic relations between convex deterministic and stochastic processes and the fundamental connections among convex stochastic processes are contained in three concepts: unimodality, unimodularity and univalence.

B. Combinatorics, Graphic Methods and Thresholds

Kolcin (1986) has shown how to convert difficult combinatorial problems into sums of independent random variables and then apply characteristic functions to obtain weak convergence of sequences of distributions. Random combinatorial objects usually are dependent, but by clever use of exchangeability, the joint distribution is representable as a conditional distribution of independent variables. Kolchin et al. exploited the similarity between random mappings and particle allocation processes (urns) and between random trees and branching processes. This unified approach to seemingly diverse areas also is covered by the Connection Theorem and frequently yields threshold phenomena.

In contingency table analysis and related estimation procedures, Simpson's Paradox is a recurring problem. Graphical analysis as contained in Lauritzen's contingency tables and Pearl's influence diagram in which conditional independence is the critical concept usually used to resolve these problems. Influence diagrams are optimal estimators and correspond to the analysis of Gotz and McCall (1984). One of the methods used is to infer neuron behavior is positron emission tomography and is another application of the Connection Theorem via the dilation operator.

C. Birth and Death Processes

In addition to its uses in specifying the number of contacts a neuron has with its neighbors and analyzing the formation of routines, the birth and death process also is applicable to threshold analysis. Recall that by shrinking the interval between states and "normalizing" the time scale, any diffusion process can be formulated as the limit of a birth and death process. Furthermore, the distributions of first passage times and sojourn times of the limiting diffusion are characterized by the converging birth and death process. It is well-known that its first passage distribution is unimodal (and this property is invariant under weak convergence). That is, if $Y(t)$ denotes the Markov process, the sample path is a jump process (with size of each jump being ± 1). Let

$$s^{mn} = \inf\{t: Y(t) = n | Y(0) = m\},$$

be the 1^{st} time the "population" reaches size n, given an initial size of m.

Let τ_i^{mn} be the length of time spent in state i before the 1^{st} passage time, s^{mn}. Then the random variable

$$T = \int_0^{s^{mn}} f(Y(t))dt$$

is called the sojourn time at state i.

Sato (1987) notes that if $m < n$, T is given by

$$T = \sum_{i=0}^{n-1} f(i)\tau_i^{mn}$$

and is the weighted sojourn time. The state-dependent weights may be positive, negative, or zero. Sato calls T the generalized sojourn time and specifies the conditions such that the distribution of T is unimodal and infinitely divisible.

Our substantive interest is in the time till neurotransmitter exceeds a threshold value. Hence, the pertinent first passage time is the first time of this exceedence. Unimodality implies that the passage times of a given neuron or neural network are associated.[79] If the threshold activity being studied is some type of learning, it becomes less difficult with repetition.

D. The Economic Interpretation of an Electric Network[80]

The potential at a node corresponds to a price while the tension (or potential difference) across arc j represents the price differential across j. The conservation of fairness requires that the commodity flow in j should be a maximum where the tension across j exceeds the cost per unit flow in j. On the other hand, the commodity flow is less than or equal to zero if the price difference is negative. This is simply a statement of the no arbitrage condition.

E. Positron Emission Tomography (PET)

Positron Emission Tomography[81] is one of the methods used to observe the behavior of neurons. It is unsurprising that this computer science/ statistical technique is based on symmetry. "Watching the brain" is an inductive process belonging to the class of inverse problems. The

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[79] Two random variables S and T are associated if $\mathrm{cov}[f(S,T),g(S,T)] > 0$, for all f and g increasing in each argument.

[80] A clear discussion of electrical networks and neuronal networks is presented in Appendix IIIA 879-894 of Kandel and Schwartz (1985).

[81] The problem entails inversion of the Radon Transform. A direct approach to the inversion with approximations for incomplete data is discussed in Vardi, et al. (1985).

estimation process closely resembles the inverse scattering techniques of physics, the realization methods of systems science and the identification problem of econometrics. These estimation processes can be conducted using the dilation operator. Recently, this operator has been applied to problems in quantum mechanics.

In QED, Feynman observes that quantum theory not only is the jewel of physics, but also illuminates chemistry which in turn is at the core of biology. In a recent experiment reported by Kinoshita (1989), quantum tunneling by hydrogen is increased dramatically by an enzyme, thereby causing a particular reaction to occur much more rapidly. Roughly speaking, tunneling lowers the energy barrier[82] (threshold) that must be jumped before the reaction can occur. R. Landauer suggested that quantum tunneling is essentially a scattering problem. See Bruinsma and Bak (1986). This implies that quantum biological phenomena (like photosynthesis) also can be analyzed by dilation operators.

The dilation operator is closely related to de Finetti's inductive methods, which, in turn, comprise the modern solution to Hume's induction problem. This operator is exchangeable and unifies probability and quantum mechanics.

The discovery of tunneling in the brain could be a major breakthrough. There seems little doubt that a tunneling algorithm can be constructed using the McCulloch-Pitts methodology as exploited and modernized by the connectionists. This algorithm will be unique in that it would be based not on the Ising model or genetics, but on a quantum mechanical phenomenon crucial to the functioning of the brain. It, therefore, satisfies simultaneously

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[82]This barrier is a potential and should have a price-theoretic interpretation.

the Crick critique and the Feynman prediction. While consistent with the reductionist program, it also is compatible with an emergent and constructive philosophy. These philosophical implications as well as the details of an algorithmic construction will be discussed in McCall and Velupillai (forthcoming).

F. Mind-Body Problem[83]

The final remark is on the mind-body problem. It is fitting that this presumptuous piece terminate with a brief foray into the philosophy of science. Here, as elsewhere, we hope the reader will not be offended by critical comments based on superficial scholarship. We hope to remedy this ignorance in subsequent papers. Obviously, improvements in this future work depend critically on readers' comments.

The distinguished philosopher, Donald Davidson, claims that there are no strict laws linking the mental and physical. The basic ideas of mental phenomena bear no systematic relation to ideas in physics. In particular, rationality, consistency and coherence have no "echo" in physics. He concludes that the social sciences can never be hard sciences. Searle (1984) agrees with the conclusion, but maintains that biology has no "echo" in physics, but, nevertheless, is a hard science. Searle supports Davidson's conclusion by arguing that the social sciences are concerned with various aspects of intentionality.

"Since economics is grounded not in systematic facts about physical properties such as molecular structure ... but rather in facts about intentionality, about desires, practices, states of technology, and states of knowledge, it follows that economics cannot be free of history of

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[83]An excellent discussion is in Popper and Eccles (1977).

context. Economics as a science presupposes certain historical facts about people and societies that are not themselves part of economics, and when those facts change, economics has to change. Economics is ... not independent of context or free of history. It is grounded in human practices, but those practices are not themselves timeless, external or inevitable .... The fact that the social sciences are powered by the mind is the source of their weakness vis a vis the natural sciences ... What we want from the social sciences at their best are theories of pure and applied intentionality".

But the tunneling phenomenon is crucial to the operation of the mind. Clearly, this quantum mechanical entity links social science, biology, and physics. It does not free them from historical context. They are all equally soft in this respect. Echos from the Platonic cave are no longer pertinent to mathematics, science and the philosophy of science.

Curiously enough, the tunneling phenomenon was discovered in 1928 by George Gamow, et al.[84] This discovery coupled with the new quantum theory supported Eddington's hypothesis that nuclear processes were the source of a star's energy. This is the same George Gamow who wrote the popular book, One, Two, Three .. Infinity (1953). No doubt, the San Diego Imperialist Reductionists would claim that the combinatorial program outlined here joins with tunneling to validate their materialistic philosophy: "One, Two, Three, ... Consciousness". This is completely contrary to our realistic philosophy. The uniqueness of the individual undoes reductionism. To

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[84] This is mentioned in Pagels (1985). Bate (1988) estimates that a semiconductor device based on quantum tunneling principles can be designed so that the entire circuitry of a supercomputer occupies a single chip.

Tunneling has also provoked philosophical comments on the "reversibility issue" (see Bohm et al. (1989)).

discover tunneling in the brain and in the stars is the <u>source</u> of wonder not its <u>sink</u>.

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