ESTIMATING LIMITED-DEPENDENT RATIONAL EXPECTATIONS MODELS:* WITH AN APPLICATION TO EXCHANGE RATE DETERMINATION IN A TARGET ZONE

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ABSTRACT

This paper is concerned with the solution and estimation of a simple class of linear rational expectations models with current expectations of the endogenous variables when there are *a priori* bounds on the dependent variable. We show that for plausible values of the parameters, the model has a unique RE solution. We first consider the exact maximum likelihood estimation of such a limited-dependent rational expectations (LD-RE) model and perform a number of Monte Carlo experiments to shed light on the small sample properties of a number of alternative estimators. The results clearly illustrate the importance of taking proper account of the limited nature of the dependent variable and its expectations in the estimation of the parameters of the LD-RE models. We then extend the analysis to a two-limit situation where the dependent variable is within a band, prove the existence and uniqueness of the RE equilibrium for this case and present an empirical application to the Deutsche mark/French franc exchange rate within the Exchange Rate Mechanism of the European Monetary System.

1. Introduction

Government interventions to regulate commodity or foreign currency markets often take the form of policies that aim at preventing the price level to fall below or rise above certain levels. The observed price, therefore, becomes a censored variable. In many such situations, however, the agents' decisions, and thus market's supply and demand are influenced by expectations and it would be unreasonable to ignore the impact of government policy announcements or regulations on the way expectations are formed and the resultant outcomes. This means that government's policy, as long as it is credible, may not only operate through the bounds it directly sets on the variable of interest, but also through the effect that the policy announcements may have on expectations. Thus even when the variable of interest remains within the bounds (apparently removing the need for taking account of limited-dependence in estimating the model), government regulations will still influence the market via expectations and ignoring this may introduce important errors into the analysis.

There is a large literature on the econometric estimation of models with censored or truncated variables (see Maddala, 1983 for an extensive discussion of various models) but, despite their obvious relevance, very little has been done on the estimation of limited-dependent models when expectations matter. Chanda and Maddala (1983, 1984) discuss a rational expectations model with bounded price variations and briefly comment on the estimation of such a model. Shonkwiler and Maddala (1985) further discuss the model and present an interesting application to the U.S. corn market. The model allows for disequilibrium caused by government intervention in a manner that when the market price is below the support price, the producers receive the support price while the consumers pay the market price. A review

of this earlier work can be found in Maddala (1990). Holt and Johnson (1989) further consider the estimation problem in the one limit case and utilize the numerical algorithm proposed by Fair and Taylor (1983) for the solution of the non-linear RE models. This solution method relies on the certainty equivalence assumption and in general can only provide an approximate solution. By contrast, the solution we give in this paper is exact.[1] Also, the estimation method adopted by Holt and Johnson does not seem to take full account of the cross equation parametric restrictions in the computation of the asymptotic standard errors. These authors do, however, conjecture the uniqueness of the rational expectations solution and support this by some simulation exercises. Pesaran (1990) discusses the properties of the solution and provides an analytical proof of its uniqueness.

These studies provide a very interesting and useful framework to analyze government-regulated markets. In this paper we further analyze the issues that have been raised in the literature by discussing the solution and estimation of a single equation limited-dependent rational expectations (LD-RE) model where the dependent variable is bounded below. By taking proper account of how expectations are formed and how they affect estimation procedures, we analyze the properties of the rational expectations solution and discuss the exact maximum likelihood estimation of the model. A number of Monte Carlo results on the small sample properties of the alternative estimators of the parameters of the model are also presented. These results further highlight the importance of making proper allowances for the direct and indirect (via expectations) influences of the bounds in econometric estimation of such models. We then extend the model to a two-limit case

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[1] See Fair and Taylor (1990, p. 383) for a further discussion.

where the dependent variable is bounded both above and below, prove the uniqueness of the RE solution in this more general case and present an application using data on exchange rates in the Exchange Rate Mechanism of the European Monetary System (EMS).

The model studied in the paper is similar to that discussed in the literature in that only current expectations matter. Limited-dependence, however, is of the tobit variety so that the dependent variable cannot fall below a certain exogenously given level. We concentrate on the estimation of the reduced-form equation of the variable which is limited-dependent, rather than the structural equations that may underlie it. Our empirical application to models of exchange-determination under a target-zone will illustrate the relevance of the reduced-form estimation. Analysis of the structural LD-RE models can be carried out along a similar lines, but will not be attempted here. Extensions to models with future expectations of the dependent variable, on the other hand, is not a straightforward matter and requires further investigation.

The plan of the paper is as follows. Section 2 presents the model and discusses the properties of the rational expectations solution. Section 3 outlines the various methods of estimation. Section 4 presents the results of the Monte Carlo experiments, and section 5 extends the analysis to the two-limit case and presents the empirical application.

2. The Model In The One-Limit Case

Consider the linear rational expectations (RE) equation

$$P_t = \gamma P_t^e + \beta' x_t + u_t, \quad u_t \sim N(0,\sigma_u^2) \tag{1}$$

where P_t is the market price, P_t^e is its expectations given the information at $t-1$, and x_t is a vector of exogenous variables. In the absence

of any a priori restrictions on the range of variations of P_t, this equation has the equilibrium RE solution

$$P_t = (\frac{\gamma}{1-\gamma})\ \beta' x_t^e + \beta' x_t + u_t, \tag{2}$$

where x_t^e represents the rational expectations of x_t formed at time t-1. For the derivation of x_t^e we assume that x_t is generated according to the general linear process

$$x_t = Rz_t + v_t \tag{3}$$

where z_t represents the predetermined variables that are known at time t-1, R is a matrix of parameters and v_t is a vector of disturbances distributed independently of u_t with mean zero and the non-singular covariance matrix Σ. Then $x_t^e = Rz_t$. When P_t is unconstrained, consistent and asymptotically efficient estimates of the structural parameters can be obtained by the joint maximum likelihood estimation of (2) and (3) subject to the cross-equation parametric restrictions implied by $x_t^e = Rz_t$.

Suppose now that P_t is subject to the bound $P_t \geq P_{tL}$, where P_{tL} is the lower limit on P_t announced prior to the determination of P_t. Further assume that the announcement is fully credible and that P_{tL} is taken to be a part of the agent's information set at time t-1.[2] The price equation in this case will be given by:

$$P_t = \begin{cases} \gamma P_t^e + \beta' x_t + u_t, & \text{if } u_t > P_{tL} - \gamma P_t^e - \beta' x_t, \\ P_{tL}, & \text{otherwise,} \end{cases} \tag{4}$$

and the RE solution (2) obtained in the absence of market intervention will

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[2]The econometric analysis of the LD-RE models in the case where the policy announcements are not fully credible is beyond the scope of the present paper.

no longer be valid. To derive the rational expectations solution in this case, we need to make explicit assumptions concerning the distribution of the disturbances, u_t and $\mathbf{v}_t$. Assume that

$$\begin{bmatrix} u_t \\ \mathbf{v}_t \end{bmatrix} \sim N\left\{ \mathbf{0}, \begin{bmatrix} \sigma_u^2 & 0 \\ 0 & \Sigma \end{bmatrix} \right\}, \tag{5}$$

and let

$$C_{tL} = (P_{tL} - \gamma P_t^e - \boldsymbol{\beta}' \mathbf{x}_t^e)/\sigma, \tag{6}$$

$$W_t = (\boldsymbol{\beta}' \mathbf{v}_t + u_t)/\sigma, \tag{7}$$

where

$$\sigma^2 = \sigma_u^2 + \boldsymbol{\beta}' \Sigma \boldsymbol{\beta}. \tag{8}$$

Then (4) may be rewritten as:

$$P_t = \begin{cases} \gamma P_t^e + \boldsymbol{\beta}' \mathbf{x}_t^e + \sigma W_t, & \text{if } W_t > C_{tL} \\ P_{tL}, & \text{otherwise} \end{cases} \tag{9}$$

where $W_t \sim N(0,1)$. The expression for P_t^e may now be written as[3]

$$P_t^e = P_{tL}\Phi(C_{tL}) + P_{1t}^e[1-\Phi(C_{tL})], \tag{10}$$

where Φ is the distribution function of the standard normal variable, P_{1t}^e is the expectations of P_t conditional on $u_t > P_t^s - \gamma P_t^e - \boldsymbol{\beta}' \mathbf{x}_t$ (or equivalently conditional on $C_{tL} > W_t$) and is given by

$$P_{1t}^e = (1-\gamma)^{-1} (\boldsymbol{\beta}' \mathbf{x}_t^e + \sigma\phi(C_{tL})/[1-\Phi(C_{tL})]). \tag{11}$$

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[3] See Shonkwiler and Maddala (1985), Maddala (1990) and Pesaran (1990) for more details. The expressions given for C_{tL} in Shonkwiler and Maddala (1985) and in Holt and Johnson (1989, p. 606) implicitly assume that $\mathbf{v}_t = 0$ and are, therefore, not appropriate for the case where $\mathbf{x}_t$ are not perfectly predictable. On this also see footnote 3 in Maddala (1990).

The first term in this expression is the price expectations in the absence of the bound, the second term enters due to the truncation of u_t, and ϕ is the density function of the standard normal variable. Substituting (11) in (10) now yields

$$P_t^e = (1-\gamma)^{-1} \{\beta' x_t^e + \sigma\phi(C_{tL})\} + \{P_{tL} - (1-\gamma)^{-1}\beta' x_t^e\} \Phi(C_{tL}). \quad (12)$$

Due to the dependence of C_{tL} on P_t and the complicated forms of ϕ and Φ, it is not possible to write the functions P_t^e in terms of the parameters of the model explicitly.

In dealing with this problem Shonkwiler and Maddala (1985) suggest that P_t^e be estimated as the maximum of P_{tL} and some quadratic function of the exogenous variables, with the switch depending on whether P_t is above or below P_{tL}. Apart from the error involved in the quadratic approximation, this specification implicitly assumes that at the time expectations are made (namely at time t-1) agents in fact know whether P_t is above or below P_{tL}. This incorrectly introduces a kink in the expected price function, which as will be seen below is unnecessary and can be avoided.

Proposition 1: For values of $\gamma < 1$, equation (12) has a unique solution for P_t^e that is above the support price.

Proof: See Pesaran (1990) and the proof of Proposition 2 below.

3. Exact Maximum Likelihood and Other Estimation Procedures

The log likelihood function of the price-equation (4) is similar to the log-likelihood of the standard Tobit model and is given by

$$\ell_p(\gamma,\beta,\sigma_u^2;R,\Sigma) = \sum_0 \log(\Phi_t) - \frac{1}{2}\sum_1 \log(2\pi\sigma_u^2) - \frac{1}{2\sigma_u^2}\sum_1 (P_t - \gamma P_t^e - \beta' x_t)^2. \quad (13)$$

where the indices 0 and 1 respectively refer to observations below and

above the limit and $\Phi_t = \Phi(C_{tL})$. This is not, however, a complete specification of the likelihood function of the model and ignores the cross-equation parameter restrictions implied by the dependence of P_t^e on the parameters of the x_t-equation. The complete log-likelihood function of the model is given by[4]

$$\ell(\gamma,\beta,\sigma_u^2;R,\Sigma) = \ell_p(\gamma,\beta,\sigma_u^2;R,\Sigma) + \ell_x(R,\Sigma), \tag{14}$$

where $\ell_x(\cdot)$ is the log-likelihood function of the x_t-equation, (3):

$$\ell_x(R,\Sigma) = -\frac{1}{2}\sum_{0,1}\log(2\pi|\Sigma|) - \frac{1}{2}\sum_{0,1}(x_t-Rz_t)'\Sigma^{-1}(x_t-Rz_t). \tag{15}$$

The exact ML estimators of $(\gamma,\beta,\sigma_u^2;R,\Sigma)$ can now be obtained by maximization of (14), noting that P_t^e is defined uniquely as a function of these parameters (see relation (12) and the proposition 1).

What differentiates the estimation problem here from that encountered in estimating the standard Tobit model is that in the present case not only $\Phi(C_{tL})$, but also P_t^e is a function of the unknown parameters. Furthermore, given the complicated nature of the dependence of P_t^e on the parameters, in evaluating the derivatives of the log-likelihood function one needs to solve for P_t^e iteratively at each stage in the computation of the ML estimators.[5] Note also that although the likelihood function for the Tobit model without expectations is known to be globally concave in parameters and thus has a unique maximum (see Olsen, 1978), it does not seem that one can make a similar claim here. This is again due to the dependence of P_t^e on the unknown parameters.

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[4]Notice that u_t and v_t are assumed to be independently distributed.

[5]The relevant expressions for the analytical derivatives of P_t^e and the log-likelihood function are given in the Appendix.

The above ML procedure imposes the cross-equation parametric restrictions implied by the REH and yields the Full Information Maximum Likelihood (FIML) estimates of all the parameters. Like Tobit ML estimators, these estimators are asymptotically efficient and have a multivariate normal distribution in large samples, assuming, of course, that certain regularity conditions are satisfied.[6] These conditions are the same as those assumed in the literature on the standard Tobit model and in particular requires the disturbances u_t and v_t to be homoskedastic and be normally distributed.

The FIML estimators, despite their obvious desirability, can be computationally time-consuming. Somewhat less cumbersome are what may be called the 2-step maximum-likelihood (2SML) estimators. These are obtained by separating the estimation of the parameters of (4) from those of (3). The parameters of (3), namely $\mathbf{R}$ and Σ are estimated by applying OLS to the equations for $\mathbf{x}_t$ while the coefficients in the equation for P_t are estimated by maximizing $\ell_p(\cdot)$ given in (14) conditional on the first stage (step) estimates of $\mathbf{R}$ and Σ. This is the analogue of the two-step procedure discussed in Pagan (1984) and analyzed in some detail in Pesaran (1987, Ch. 7). These estimators will be asymptotically less efficient than the FIML estimators. This is because the estimators of $\mathbf{R}$ and Σ do not take account of the dependence of $\ell_p(\cdot)$ on the parameters of the $\mathbf{x}_t$ process. The 2SML estimators are, however, somewhat easier to compute. We shall compare the small sample properties of these two estimators below. The 2SML estimators will be used as initial values in the computation of the

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[6] The proof of consistency and asymptotic normality of the Tobit ML estimators can be easily extended to cover the present problem by noting that P_t^e is a unique continuous function of the unknown parameters.

FIML estimators.

Finally, one may also be interested in the degree of bias or inconsistency generated in the parameter estimates when the bound on the dependent variable is not taken into account. The degree of bias present will obviously depend on the proportion of the observations that are censored. If this proportion is zero then the first term in the log-likelihood function, (13), will disappear. However, as mentioned above, it is important to note that unlike the usual Tobit model (where if no observation is below the limit, then OLS can be applied) here the mere announcement of the bound will, by influencing expectations, complicates the model and introduces bias in the estimation procedures that ignore the effect of (credible) announcements on expectations. To estimate the misspecified RE model obtained by ignoring the bound one may apply the two-step procedure discussed in Pagan (1984) to (2). Such an estimator can be obtained by first estimating R (say $\hat{R}$) by applying the OLS procedure to (3), and then regressing P_t on $\hat{x}^e_t - \hat{R}z_t$ and x_t to obtain estimates of γ and β. In what follows these estimators will be referred to as the 2-step estimators.[7] An alternative possibility would be to apply the above method only to the non-censored part of the sample. This procedure may be expected to give estimates that are on average better than those obtained by using the whole data. In the following we shall refer to these latter estimators as the 2-step non-censored (2SNC) estimators and will use them as initial estimates for the 2SML procedure.

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[7]Note that the computation of the standard errors of these two-step estimators by the usual OLS formula is not appropriate and needs to be done along the lines suggested in Pagan (1984) and Murphy and Topel (1985).

4. Monte Carlo Simulations

In order to shed light on the small sample properties of the various estimators discussed above, we carried out a number of Monte Carlo experiments. Two sets of experiments were conducted, assuming alternatively a univariate and a bivariate process for x_t. The latter is more general and allows for an extra set of cross-equation parameter restrictions.

4.1 The Univariate Case

In this case x_t is assumed to be a scalar and generated according to the first-order autoregressive scheme $x_t = 4 + \rho x_{t-1} + v_t$, $v_t \sim N(0,1)$.[8] To generate the values of P_t we first generated P_{tL} and P_t^e by controlling for the proportion of censored observations.[9] As to be expected, this parameter plays a key role in the relative performance of the alternative estimators. Denoting this proportion by π we have $\pi = \text{prob}(P_t \leq P_{tL}) = \Phi(C_{tL})$, and hence $C_{tL} = \Phi^{-1}(\pi)$, where $\Phi^{-1}(\cdot)$ is the inverse of the distribution function of the standard normal variable. Now using (6) we have

$$P_{tL} = \gamma P_t^e + \beta x_t^e + \sigma \Phi^{-1}(\pi), \tag{16}$$

where $\sigma^2 = \sigma_u^2 + \beta^2$, and $x_t^e = 4 + \rho x_{t-1}$. Also using (12) we have:

$$P_t^e = \left\{\frac{\beta x_t^e + \sigma \phi[\Phi^{-1}(\pi)]}{1-\gamma}\right\} + \left[P_{tL} - \frac{\beta x_t^e}{1-\gamma}\right]\pi. \tag{17}$$

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[8] The random-number generation and all the estimations were carried out using a computer program written in Gauss (version 2.0). In order to reduce the influence of initial values on x_t twice the number of observations needed were generated and the first half were dropped. For numerical optimization the quadratic hill-climbing method was employed with analytical first derivatives given in the Appendix and numerical second derivatives.

[9] Note that the alternative procedure for generating P_t would be to control the values of P_{tL} directly. We also did some experiments using this procedure and the results were similar to those presented below. The relevant tables are available from the authors on request.

Equations (16) and (17) then simultaneously determine P_t^e and P_{tL}.[10] These can now be used in (4) to generate P_t for a given value of u_t generated as $N(0,\sigma_u^2)$. The value of σ_u^2 was fixed by controlling the coefficient of determination of the equation for P_t in the absence of the bounds [see equation (2)]. Denoting this coefficient by R_p^2 we have[11]

$$\sigma_u^2 = \psi(1-R_p^2)/R_p^2 , \quad \psi = \beta^2 \left\{1 + \left[\frac{\rho}{1-\rho^2}\right]\left[\frac{1}{1-\gamma}\right]\right\} .$$

We fixed the values of γ and β at -0.8 and 2.0, respectively, and carried out 24 sets of experiments corresponding to the values of the control parameters $R_x^2 = (0.50, 0.90)$, $R_p^2 = (0.50, 0.95)$, $n = (40, 80)$, and $\pi = (0.10. 0.25, 0.50)$.[12] All the experiments were based on 500 replications.

4.2 The Bivariate Case

Here the two variables in $\mathbf{x}_t$ were generated according to the following stationary vector autoregressive process

$$x_{1t} = 1 + \rho_{11}x_{1,t-1} + \rho_{12}x_{2,t-1} + v_{1t}, \quad v_{1t} \sim N(0,1), \tag{18}$$

$$x_{2t} = 1 + \rho_{21}x_{1,t-1} + \rho_{22}x_{2,t-1} + v_{2t}, \quad v_{2t} \sim N(0,1), \tag{19}$$

where ρ_{ij}'s are determined once R_1^2, R_2^2, the coefficients of

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[10] Notice that since we are controlling the value of C_{tL}, and thus that of $\phi(C_{tL})$ and $\Phi(C_{tL})$, P_t^e is obtained without any need for numerical iterations. The same is not, however, true of the alternative procedure of generating P_t^e for a given series of price bounds.

[11] This result is derived by first using (2) and (3) to obtain $V(P_t) = \psi + \sigma_u^2$, and then noting that by definition $R_p^2 = 1 - (\sigma_u^2/V(P_t))$.

[12] It is worth noting that the likelihood function of the LD-RE model is well defined only for values of $\gamma < 1$. This restriction was imposed at the estimation stage. Although the estimate of γ was close to one on a few occasions for high values of σ_u^2, the restriction was never actually binding. However, convergence of the equation for P_t^e towards its solution, and also the likelihood function towards its maximum became increasingly more time-consuming as the estimate of γ moved towards unity.

determination of equations (18) and (19) and, r, the correlation coefficient between x_{1t} and x_{2t}, are known. Having considered various possibilities in the univariate case, in these experiments we focus our attention only on the effect of varying the correlation between x_{1t} and x_{2t}. R_1^2 and R_2^2, were, therefore both set equal to 0.80, while r was allowed to take the values of 0.20, 0.50 and 0.70. Under this parameterization the variance matrix of $\mathbf{x}_t = (x_{1t}, x_{2t})'$ is given by[13]

$$\Sigma_{xx} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{12} & \sigma_{22} \end{pmatrix} = 5\begin{pmatrix} 1 & r \\ r & 1 \end{pmatrix}.$$

Three of the four autoregressive parameters, ρ_{ij}, $i,j = 1,2$ in (18) and (19) may now be derived from the following relations:

$$\sigma_{11} = \rho_{11}^2\sigma_{11} + \rho_{12}^2\sigma_{22} + 2\rho_{11}\rho_{12}\sigma_{12} + 1,$$

$$\sigma_{22} = \rho_{21}^2\sigma_{11} + \rho_{22}^2\sigma_{22} + 2\rho_{21}\rho_{22}\sigma_{12} + 1,$$

$$\sigma_{12} = \rho_{11}\rho_{21}\sigma_{11} + \rho_{12}\rho_{22}\sigma_{22} + (\rho_{11}\rho_{22}+\rho_{12}\rho_{21})\sigma_{12}.$$

In the experiments we set $\rho_{12} = 0.30$ and obtained the other three parameters from the above relations.

To generate P_t we followed the same procedure as before. We generated the values of P_t^e and P_{tL} by setting the proportion of the censored observations equal to 0.25, and draw the values of u_t from $N(0,\sigma_u^2)$. These series were then used in (4) to generate the values of P_t. The variance of u_t is fixed by controlling R_p^2. In this case the parameter ψ in the relationship between σ_u^2 and R_p^2 is given by

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[13]Notice that $V(x_{it}) = 1/(1-R_i^2)$, $i = 1,2$ and $cov(x_{1t},x_{2t}) = r/((1-R_1^2)(1-R_2^2))^{\frac{1}{2}}$,

$$\psi = (1-(\frac{1}{1-\gamma})^2)\ \beta'\beta + (\frac{1}{1-\gamma})^2\ \beta'\Sigma_{xx}\beta.$$

The remaining parameters were set as follows: $\beta = (1.0,1.0)'$, $\gamma = -0.8$, and $R_p^2 = 0.85$. The results for the values of $r = (0.2,0.7)$ and the two sample sizes 40 and 80 are presented in Tables 3 and 4.

4.3 Summary of the Monte Carlo Results

To save space we only report the results for six of the 24 experiments carried out for the univariate case. (see Tables 1 and 2). These correspond to three different values of π (0.50, 0.25, 0.10), the proportion of censored observations, and two sample sizes (40 and 80). Other results are available from the authors on request. Each table gives the simulation results for four different estimators of the structural parameters of the P_t equation. The four estimators covered are the two-step estimator, $\hat{\theta}_{2S}$, the two-step non-censored estimator, $\hat{\theta}_{2SNC}$, the two-step maximum-likelihood estimator, $\hat{\theta}_{2SML}$, and the full-information maximum-likelihood estimator, $\hat{\theta}_{FIML}$. The tables also give type I errors of testing the null that β and γ take their true values, namely $\gamma = -0.80$ and $\beta = 2.0$. The nominal size of the test is set at the 5 percent level. The simulation standard errors of the estimates are in brackets.

The results show clearly that the two-step estimators that do not take account of the bound ($\hat{\theta}_{2S}$ and $\hat{\theta}_{2SNC}$) perform very poorly and inferences based on them tend to over-reject the null by a wide margin.[14] Not surprisingly these estimators do particularly badly when the proportion of censored

[14]The standard errors of the two-step estimators are computed using the consistent estimators discussed, for example, in Pagan (1984) and Pesaran (1987, section 7.2.2). The relevant expressions for the variance matrix of the two-step estimators, both for the univariate and the bivariate cases are available from the authors on request.

observations is high. For example, when this proportion is 0.50, the two-step estimates of γ have a wrong sign and the null hypothesis that γ is equal to its true value is incorrectly rejected in the majority of cases. In the case of these experiments ($\pi = 0.50$), the two-step non-censored estimators do generally better than the standard two-step estimators, but they are still unsatisfactory and inferences based on them can be very misleading. The same is also true of the two-step estimates of β. Overall, the two-step estimators tend to underestimate β and overestimate γ. The bias of the two-step estimators can still be quite large even when the proportion of censored observations is low. Also, increasing the sample size is not helpful in the case of these estimators and makes the over-rejection problem even worse. This is not surprising and is due to the fact that when $\pi > 0$, the two-step estimators which do not allow for the price bound are inconsistent.

Turning our attention to the ML estimators, $\hat{\theta}_{2SML}$ and $\hat{\theta}_{FIML}$, we first note that both estimators perform reasonably well irrespective of whether the proportion of censored observations is high or low. Even for $\pi = 0.50$, the estimated type I errors are generally close to their nominal size. The effect of increasing the sample size on the ML estimators is as predicted by the asymptotic theory, and results in significant improvements both in the parameter estimates and in the rejection probabilities. A comparison of the two-step ML estimators with the FIML estimators also reveals that based on their small sample properties there is little to chose between them; but on computational grounds the 2SML estimators are preferable.

The above findings seem to be fairly robust to the inclusion of a second regressor to the RE equation. As can be seen from Tables 3 and 4, the two-step least squares estimators continue to perform very poorly, and

the ML estimators do extremely well, especially when the larger sample size is considered. The parameter estimates are very close to their true values and the estimates of the rejection probabilities are well within the simulation confidence intervals. Also, increasing the correlation coefficient between x_1 and x_2 seems to have little effect on the small sample performance of the ML estimators. Once again, there is little to choose between the two ML estimators. The 2SML estimator is, however, much simpler to compute and is therefore to be recommended.

5. The Two-Limit Case With An Empirical Application

In this section we extent the analysis of the previous section to a situation where the dependent variable is bounded both above and below. An important empirical example in this context is the Exchange Rate Mechanism of the European Monetary System (EMS), where all the bilateral exchange rates have to remain within a band agreed by the member countries. There is a recent theoretical literature which analyzes this problem in a continuous-time stochastic setting (see, for example, Krugman, 1990). There has not, however, been much empirical work in this area.[15] We first illustrate how the analysis of the previous section may be modified to take account of two bounds and then present an empirical application in the case of the Deutsche mark/French franc during the period when the target-zone regime has been in operation.

Suppose P_t, the observed price or exchange rate, be determined as follows:

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[15] But, see Meese and Rose (1989, 1990), Diebold and Nason (1990) and Smith and Spencer (1990). The approach adopted in these papers is, however, different from ours and focus on testing for non-linear effects in exchange rate models.

$$P_t = \begin{cases} P_{tU} & \text{if } P_t^* \geq P_{tU} \\ P_t^* & \text{if } P_{tL} < P_t^* < P_{tU} \\ P_{tL} & \text{if } P_t^* \leq P_{tL} \end{cases} \tag{20}$$

where P_{tU} and P_{tL} are the upper and lower bounds on the exchange rate (or the price level) and P_t^* is determined by:

$$P_t^* = \gamma P_t^e + \boldsymbol{\beta}' \mathbf{x}_t + u_t, \quad u_t \sim N(0,\sigma_u^2) \tag{21}$$

where P_t^e is the expectations of P_t given the information at time $t-1$ and $\mathbf{x}_t$ is generated according to (3). The expectations of P_t can be obtained following a similar procedure as in the one-limit case. We have

$$P_t^e = [\Phi(C_{tU}) - \Phi(C_{tL})]\, P_{1t}^e + \Phi(C_{tL})\, P_{tL} + [1-\Phi(C_{tU})]\, P_{tU} \tag{22}$$

where:

$$P_{1t}^e = \frac{\boldsymbol{\beta}' \mathbf{x}_t^e}{1-\gamma} - \left(\frac{\sigma}{1-\gamma}\right) \left\{\frac{\phi(C_{tU})-\phi(C_{tL})}{\Phi(C_{tU})-\Phi(C_{tL})}\right\}, \tag{23}$$

is the expectations of P_t conditional on P_t being within the band, and $C_{tU} = (P_{tU}-\gamma P_t^e-\boldsymbol{\beta}' \mathbf{x}_t^e)/\sigma$. The other notations are as before. We now prove the following generalization of the Proposition 1.

Proposition 2: As long as γ is less than unity there exists a unique solution for P_t^e that lies within the band, $P_{tL} < P_t^e < P_{tU}$. If γ is greater than unity a solution may still exist but this is not guaranteed. For γ equal to one no solution will exist.

Proof: Substitute P_{1t}^e from (23) in (22), and let the right-hand-side of the resultant equation be $F(P_t^e)$. To prove the proposition we first demonstrate that for $\gamma < 1$, and P_t^e inside the band, $F(\cdot)$ is a monotonically decreasing function of P_t^e. We then show that for $\gamma < 1$, $F(\cdot)$ does in

fact lie within the band.

(i) Using (22) and differentiating its right-hand-side with respect to P^e_t, we have (suppressing the time subscripts for notational convenience),

$$F'(P^e) = -\gamma(\phi_U-\phi_L)\,\beta'x^e/\sigma(1-\gamma) - \gamma(C_U\phi_U-C_L\phi_L)/(1-\gamma)$$

$$+ \gamma(P_U\phi_U-P_L\phi_L)/\sigma,$$

where $\phi_i = \phi(C_i)$, for $i = L,U$. Now substituting for C_U and C_L we have

$$F'(P^e) = \frac{-\gamma^2}{\sigma(1-\gamma)}\,((P_U-P^e)\phi_U + (P^e-P_L)\phi_L),$$

which establishes that $F'(P^e) < 0$, for $\gamma < 1$ and for P^e in the band.

(ii) To prove that for $\gamma < 1$, $F(\cdot)$ does in fact lie within the band it is sufficient to show that $F(P_L) > P_L$ and $F(P_U) < P_U$.[16] For this purpose we introduce the following notations:

$$w_1 = \theta\left(P_L - \frac{\beta'x^e}{1-\gamma}\right)$$

$$w_2 = (P_U-P_L)/\sigma > 0, \quad \theta = (1-\gamma)/\sigma > 0.$$

Evaluating the right-hand-side of (22) at $P^e = P_L$, we now have

$$\theta[F(P_L)-P_L] = [\Phi(w_2-w_1)-\Phi(-w_1)]w_1 - [\phi(w_2-w_1)-\phi(-w_1)]$$

$$+ (1-\gamma)\,[1-\Phi(w_2-w_1)]w_2.$$

The last term of this expression is always positive. Consider now the sum of the first two terms which can also be written as

$$H(w_1,w_2) = [\Phi(w_1)-\Phi(w_1-w_2)]w_1 + [\phi(w_1)-\phi(w_1-w_2)].$$

---

[16] Recall that for $\gamma < 1$, $F(\cdot)$ is a decreasing function of P^e over the range $P_L < P^e < P_U$.

We now show that $H(\cdot)$ is positive for $w_2 > 0$ and for any arbitrary value of w_1. First notice that

$$\partial H/\partial w_2 = w_2\, \phi(w_1-w_2) > 0.$$

Thus for any given value of w_1, H is monotonically increasing in w_2. Also $H(w_1,0) = 0$, and $H(w_1,\infty) = w_1\Phi(w_1) + \phi(w_1)$ which is positive for all w_1. [See, for example, Amemiya (1985, p. 274)]. Therefore, H will be positive for all w_1 and for $w_2 > 0$. This in turn implies that $\theta[F(P_L)-P_L] > 0$ and since $\theta > 0$ we have $F(P_L) > P_L$.[17] A similar argument can be made to establish the inequality $F(P_U) < P_U$.

Consequently, for $\gamma < 1$, $F(\cdot)$ (being a continuous mapping) will cross the 45° line once and at most once between P_L and P_U. In the case where $\gamma > 1$, for $P_L < P^e < P_U$, $F(\cdot)$ is an increasing function of P^e and since $F(\cdot)$ may or may not be within the band, there may or may not exist an intersection with the 45° line. The condition $\gamma < 1$ is therefore sufficient for existence and uniqueness of the RE solution in the band, but it is not necessary. This is in contrast with the one-limit case where $\gamma < 1$ is both necessary and sufficient for the existence and uniqueness of the RE solution above the support price.[18]

The log-likelihood function in this case has the same form as in (14), with the difference that $\ell_P(\cdot)$ is now given by

$$\ell_P(\cdot) = \sum_{0} \log(\Phi_{tL}) - \frac{n_1}{2}\log(2\pi\sigma_u^2) - \frac{1}{2\sigma_u^2}\sum_{1}(P_t - \gamma P_t^e - \beta' x_t)^2 + \sum_{2}\log(1-\Phi_{tU}), \tag{24}$$

---

[17] Notice that when $\gamma > 1$, the sum of the first two terms in $\theta[F(P_L)-P_L]$ continues to remain positive but the third term will now be negative. The sign of $\theta[F(P_L)-P_L]$ is ambiguous and depends on the size of the other parameters.

[18] The proof in the one-limit case may be obtained as a special case of Proposition 2 by letting $P_U \to \infty$.

where $\Phi_{ti} = \Phi(C_{ti})$, $i = L,U$, n_1 is the number of non-censored observations in the sample, and the indices 0, 1 and 2 respectively refer to observations below, within and above the band.

As an empirical application of the above two-limit LD-RE model we estimated the following simple two-country monetary model of the exchange rate using monthly data on the Deutsche mark/French franc bilateral exchange rate over the period May 1979 to May 1989, inclusive:

$$P_t^* - P_{t-1} = \gamma(P_t^e - P_{t-1}) + \beta'\mathbf{x}_t + \delta'\mathbf{h}_t + u_t, \tag{25}$$

where P_t is the log of the exchange rate and is determined by (20). We assume that the pre-announced target zone (P_{tL}, P_{tU}) is fully credible. The analysis of stochastically varying target zones is beyond the scope of the present paper and is discussed elsewhere. [See Pesaran and Samiei (1991)]. In the simple monetary model $\mathbf{x}_t = (m_t, y_t)'$, where m_t is the log of relative money supplies, and y_t is the log of relative GDPs. As elements of the vector of predetermined variables we tried the 1-month and 12-month lagged values of m_t, y_t, P_t, and the interest rate differentials. These were included because of the possibility of lagged adjustment (for example in the demand for money) and in order to account for the possible seasonality in the monthly data [see, for example, Haache and Townend (1981)]. The GDP series were proxied by industrial output for which monthly data is available. The money supply figures are M1 plus the quasi-money, the interest rate differential is the difference between the rate on 3-month loans (Frankfurt) and the short term interest rate in France. All the data are from OECD Main Economic Indicators, except for the data on the upper and lower bounds on exchange rates that are calculated using bilateral central rates obtained from Eurostat, Money and Finance and the fixed

maximum deviation from the central rates (2.25%). The exchange rate movements and the bands around them are displayed in Figure 1.

We computed three sets of estimates for the exchange rate equation. These are presented in Table 5. The first panel of this table gives the two-step ML estimates ignoring the band.[19] The middle panel gives the two-step ML estimates that allow for the band. This latter estimates are obtained by maximizing $\ell_p(\cdot)$ defined by (24), taking the estimates of the parameters of the x_t process as given.[20] Finally, the third panel in Table 5 gives the FIML estimates which are computed by maximizing the joint log-likelihood function in (14) with $\ell_p(\cdot)$ given by (24).

The contrast between the estimates that do take account of the band and those that do not is very striking. None of the coefficients estimated without allowing for the target zone are statistically significant, and in line with other results in the literature on linear exchange rate models, it is not possible to reject the random walk hypothesis. By contrast the estimates that allow for the band explain as much as 75 percent of the variations in the exchange rates. The coefficient of the expected exchange rate is significantly different from zero (and from one), and there is strong evidence of a positive effect of the differential interest rate variable on the exchange rate. Given that only four observations lie on the boundaries, the results indicate the importance of a proper treatment of expectations formation in estimating LD-RE models. One must, however, be careful in drawing conclusions from these results since the above procedure

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[19] Notice that due to the presence of over-identifying restrictions in the exchange rate equation, the two-step ML estimation method is more appropriate than the simple two-step procedure discussed in Pagan (1984).

[20] The auxiliary equations for the exogenous variables, $x_t = (m_t, y_t)'$, were assumed to have a vector autoregressive process of order 12.

assumes that the band at time t is known to the agents at t-1. The literature on the market for corn referred to earlier makes a similar assumption with regards to the price support. This is a strong assumption since the exchange rate bands are not fully credible. Ideally, one should model the determination of the band and thus the expectations of it as formed by the private sector. Also lack of any diagnostic tests for these models make the results very preliminary. Extensions along these lines should be the purpose of further research.

APPENDIX

Derivatives of the Log-Likelihood Function in the One-Limit Case

Here we give the first-order derivatives of $\ell(\gamma,\beta,\sigma_u^2;\mathbf{R},\Sigma)$ with respect to the structural parameters of the model in the one-limit case. Similar expressions can also be derived for the two-limit case and are obtainable from the authors on request. Using (14) we have:

$$\partial\ell/\partial\gamma = \sum_0 [(\partial\Phi_t/\partial\gamma)/\Phi_t] + \sum_1 (P_t-\gamma P_t^e-\beta'\mathbf{x}_t)\ [P_t^e+\gamma(\partial P_t^e/\partial\gamma)]/\sigma_u^2,$$

$$\partial\ell/\partial\beta = \sum_0 [(\partial\Phi_t/\partial\beta)/\Phi_t] + \sum_1 (P_t-\gamma P_t^e-\beta'\mathbf{x}_t)\ [\mathbf{x}_t+\gamma(\partial P_t^e/\partial\beta)]/\sigma_u^2,$$

$$\partial\ell/\partial\sigma_u^2 = \sum_0 [(\partial\Phi_t/\partial\sigma_u^2)/\Phi_t] - n_1/2\sigma_u^2$$

$$+ \sum_1 [(P_t-\gamma P_t^e-\beta'\mathbf{x}_t)^2]/\sigma_u^4 + [\gamma(\partial P_t^e/\partial\sigma_u^2)(P_t-\gamma P_t^e-\beta'\mathbf{x}_t)]/\sigma_u^2,$$

$$\partial\ell/\partial\mathbf{R} = \sum_0 [(\Phi_t/\partial\mathbf{R})/\Phi_t]$$

$$+ \sum_1 [\gamma(\partial P_t^e/\partial\mathbf{R})(P_t-\gamma P_t^e-\beta'\mathbf{x}_t) + \gamma(\partial P_t^e/\partial\mathbf{R})(P_t-\gamma P_t^e-\beta'\mathbf{x}_t)]/\sigma_u^2$$

$$+ \sum_{0,1} [(\mathbf{x}_t-\mathbf{R}\mathbf{x}_t)\mathbf{z}_t']\Sigma^{-1},$$

$$\partial\ell/\partial\Sigma = \sum_0 [(\partial\Phi_t/\partial\Sigma)/\Phi_t] - \frac{n}{2}\Sigma^{-1}$$

$$+ \sum_1 [\gamma(\partial P_t^e/\partial\Sigma)(P_t-\gamma P_t^e-\beta'\mathbf{x}_t)]/\sigma_u^2$$

$$+ \sum_{0,1} \Sigma^{-2} \left[(\mathbf{x}_t - \mathbf{R}\mathbf{z}_t)(\mathbf{x}_t - \mathbf{R}\mathbf{z}_t)'/2\right]$$

In the above expressions, n represents the total number of observations, n_1 the number of observations for which $P_t > P_{tL}$, and ϕ_t and Φ_t are $\phi(C_{tL})$ and $\Phi(C_{tL})$, respectively. The expressions for the derivatives of P_t^e and Φ_t with respect to the structural parameters can also be obtained using equations (12) and (6) in the text. We have:

$$\partial P_t^e/\partial\gamma = [\sigma\boldsymbol{\beta}'\mathbf{x}_t^e(1-\Phi_t) + \sigma^2\phi_t + \gamma(1-\gamma)\,\phi_t P_t^e(P_{tL}-P_t^e)]/$$
$$[\sigma(1-\gamma)^2 - \gamma^2(1-\gamma)\,\phi_t(P_{tL}-P_t^e)],$$

$$\partial P_t^e/\partial\boldsymbol{\beta} = [\sigma^2(1-\Phi_t)\mathbf{x}_t^e + \gamma\phi_t(C_{tL}\boldsymbol{\Sigma}\boldsymbol{\beta}+\sigma\mathbf{x}_t^e)(P_{tL}-P_t^e) + \sigma\phi_t\boldsymbol{\beta}\sigma_u^2]/$$
$$[(1-\gamma)\sigma^2 - \gamma^2\sigma\phi_t(P_{tL}-P_t^e)],$$

$$\partial P_t^e/\partial\sigma_u^2 = [\sigma\phi_t + \gamma\phi_t C_{tL}(P_{tL}-P_t^e)]/2[\sigma^2(1-\gamma) - \gamma^2\phi_t(P_{tL}-P_t^e)\sigma],$$

$$\partial P_t^e/\partial\mathbf{R} = [\sigma\boldsymbol{\beta}\mathbf{z}_t'(1-\Phi_t) + \gamma\boldsymbol{\beta}\mathbf{z}_t'\phi_t(P_{tL}-P_t^e)]/[(1-\gamma)\sigma - \gamma^2\phi_t(P_{tL}-P_t^e)],$$

$$\partial P_t^e/\partial\boldsymbol{\Sigma} = (\boldsymbol{\beta}\boldsymbol{\beta}')[\sigma\phi_t + \gamma\phi_t C_{tL}(P_{tL}-P_t^e)]/2[\sigma^2(1-\gamma) - \gamma^2\phi_t(P_{tL}-P_t^e)\sigma].$$

Similarly,

$$\partial\Phi_t/\partial\gamma = -\phi_t[P_t^e + \gamma(\partial P_t^e/\partial\gamma)]/\sigma,$$

$$\partial\Phi_t/\partial\boldsymbol{\beta} = -\phi_t[\mathbf{x}_t^e\sigma + \gamma(\partial P_t^e/\partial\boldsymbol{\beta})\sigma + \boldsymbol{\Sigma}\boldsymbol{\beta}C_{tL}]/\sigma^2,$$

$$\partial\Phi_t/\partial\sigma_u^2 = -\phi_t[C_{tL}/2 + \gamma\sigma(\partial P_t^e/\partial\sigma_u^2)]/\sigma^2,$$

$$\partial\Phi_t/\partial\mathbf{R} = -\phi_t[\boldsymbol{\beta}\mathbf{z}_t' + \gamma(\partial P_t^e/\partial\mathbf{R})]/\sigma^2,$$

$$\partial\Phi_t/\partial\boldsymbol{\Sigma} = -\phi_t[(\boldsymbol{\beta}\boldsymbol{\beta}')C_{tL}/2 + \gamma\sigma(\partial P_t^e/\partial\boldsymbol{\Sigma})]/\sigma^2.$$

TABLE 1

Monte Carlo Results For Alternatives Estimators Of The Parameters Of The RE-LD Model (Univariate x_t)*

($R_p^2 = 0.95$, $R_x^2 = 0.90$)

Sample Size = 40

	$\pi = 0.50$		$\pi = 0.25$		$\pi = 0.10$	
Alternative Estimators	γ	β	γ	β	γ	β
$\underset{\sim}{\theta}_{2S}$	0.110 (0.161)	0.994 (0.180)	-0.352 (0.159)	1.505 (0.178)	-0.622 (0.147)	1.804 (0.164)
α	1.000 (0.000)	1.000 (0.000)	0.882 (0.014)	0.878 (0.015)	0.288 (0.020)	0.282 (0.020)
$\underset{\sim}{\theta}_{2SNC}$	-0.399 (0.276)	1.558 (0.305)	-0.576 (0.191)	1.754 (0.212)	-0.688 (0.155)	1.877 (0.172)
α	0.418 (0.022)	0.418 (0.022)	0.260 (0.020)	0.258 (0.020)	0.136 (0.015)	0.130 (0.015)
$\underset{\sim}{\theta}_{2SML}$	-0.813 (0.264)	2.014 (0.292)	-0.817 (0.199)	2.019 (0.220)	-0.796 (0.156)	1.996 (0.173)
α	0.086 (0.013)	0.090 (0.013)	0.062 (0.011)	0.062 (0.011)	0.064 (0.011)	0.062 (0.011)
$\underset{\sim}{\theta}_{FIML}$	-0.766 (0.253)	1.961 (0.280)	-0.782 (0.189)	1.978 (0.208)	-0.794 (0.155)	1.992 (0.171)
α	0.094 (0.013)	0.094 (0.013)	0.078 (0.012)	0.076 (0.012)	0.058 (0.010)	0.060 (0.011)

*All experiments are based on 500 replications. Figures in brackets are simulation standard errors, $\pi = \text{Prob}(P_t \leq P_{tL})$, $\underset{\sim}{\theta} = (\gamma,\beta)' = (-.80, 2.0)'$, $\underset{\sim}{\theta}_{2S}$ is the two-step estimator ignoring the bound on P_t, $\underset{\sim}{\theta}_{2SNC}$ is the two-step estimator ignoring the bound on P_t, but using only the non-censored observations, $\underset{\sim}{\theta}_{2SML}$ is the 2-step maximum likelihood estimator, $\underset{\sim}{\theta}_{FIML}$ is the full-information maximum likelihood estimator, α is the type I error (set at the nominal value of 0.05), R_p is the coefficient of determination of the P_t-equation in the absence of the bound, and R_x is the coefficient of determination of the x_t-equation..

TABLE 2

Monte Carlo Results For Alternatives Estimators Of The Parameters Of The RE-LD Model (Univariate x_t)*

($R_p^2 = 0.95$, $R_x^2 = 0.90$)

Sample Size = 80

	$\pi = 0.50$		$\pi = 0.25$		$\pi = 0.10$	
	Alternative Estimators					
	γ	β	γ	β	γ	β
$\underset{\sim}{\theta}_{2S}$	0.097	1.010	-0.358	1.514	-0.627	1.811
	(-0.115)	(0.129)	(0.109)	(0.122)	(0.097)	(0.108)
α	1.000	1.000	0.992	0.990	0.484	0.470
	(0.000)	(0.000)	(0.004)	(0.004)	(0.022)	(0.022)
$\underset{\sim}{\theta}_{2SNC}$	-0.390	1.550	-0.572	1.751	-0.686	1.876
	(0.176)	(0.195)	(0.121)	(0.135)	(0.102)	(0.113)
α	0.688	0.680	0.468	0.460	0.172	0.168
	(0.021)	(0.021)	(0.022)	(0.022)	(0.017)	(0.017)
$\underset{\sim}{\theta}_{2SML}$	-0.801	2.002	-0.804	2.004	-0.799	1.999
	(0.163)	(0.180)	(0.121)	(0.133)	(0.103)	(0.114)
α	0.046	0.044	0.052	0.054	0.068	0.068
	(0.009)	(0.009)	(0.010)	(0.010)	(0.011)	(0.011)
$\underset{\sim}{\theta}_{FIML}$	-0.754	1.948	-0.773	1.967	-0.793	1.990
	(0.157)	(0.173)	(0.117)	(0.129)	(0.102)	(0.112)
α	0.082	0.082	0.068	0.070	0.062	0.064
	(0.012)	(0.012)	(0.011)	(0.011)	(0.011)	(0.011)

*See the notes to Table 1A.

TABLE 3

Monte Carlo Results For Alternatives Estimators Of The Parameters of the RE-LD Model (Bivariate x_t)*

(r = 0.20)

	n = 40			n = 80		
	γ	β_1	β_2	γ	β_1	β_2
True Values	-0.800	1.000	1.000	-0.800	1.000	1.000
Alternative Estimators						
$\underset{\sim}{\theta}_{2S}$	-0.289	0.773	0.755	-0.246	0.752	0.752
	(0.245)	(0.145)	(0.152)	(0.174)	(0.105)	(0.096)
α	0.710	0.412	0.438	0.934	0.724	0.710
	(0.020)	(0.022)	(0.022)	(0.011)	(0.020)	(0.020)
$\underset{\sim}{\theta}_{2SNC}$	-0.358	0.827	0.804	-0.314	0.810	0.809
	(0.355)	(0.193)	(0.197)	(0.222)	(0.130)	(0.127)
α	0.588	0.340	0.352	0.806	0.516	0.526
	(0.022)	(0.021)	(0.021)	(0.018)	(0.022)	(0.022)
$\underset{\sim}{\theta}_{2SML}$	-0.866	1.022	1.013	-0.802	1.001	1.000
	(0.333)	(0.151)	(0.173)	(0.204)	(0.098)	(0.114)
α	0.082	0.074	0.090	0.056	0.074	0.068
	(0.012)	(0.012)	(0.013)	(0.010)	(0.012)	(0.011)
$\underset{\sim}{\theta}_{FIML}$	-0.874	0.998	1.013	-0.798	0.974	0.998
	(0.339)	(0.140)	(0.168)	(0.203)	(0.090)	(0.110)
α	0.082	0.084	0.080	0.060	0.092	0.072
	(0.012)	(0.012)	(0.012)	(0.011)	(0.013)	(0.012)

*In all the experiments $R_p^2 = 0.85$, $R_1^2 = R_2^2 = 0.80$ and $\pi = 0.25$.

r = Correlation coefficient between x_1 and x_2. Other notations are as in Table 1

TABLE 4

Monte Carlo Results For Alternatives Estimators Of The Parameters Of The RE-LD Model (Bivariate x_t)*

(r = 0.70)

	n = 40			n = 80		
	γ	β_1	β_2	γ	β_1	β_2
True Values	-0.800	1.000	1.000	-0.800	1.000	1.000
Alternative Estimators						
$\underset{\sim}{\theta}_{2S}$	-0.458 (0.622)	0.774 (0.158)	0.755 (0.165)	-0.373 (0.353)	0.752 (0.114)	0.751 (0.105)
α	0.518 (0.022)	0.364 (0.022)	0.392 (0.022)	0.686 (0.021)	0.646 (0.021)	0.630 (0.022)
$\underset{\sim}{\theta}_{2SNC}$	-0.879 (3.269)	0.782 (0.211)	0.758 (0.215)	-0.437 (0.563)	0.761 (0.147)	0.762 (0.140)
α	0.516 (0.022)	0.394 (0.022)	0.398 (0.022)	0.636 (0.022)	0.566 (0.022)	0.580 (0.022)
$\underset{\sim}{\theta}_{2SML}$	-0.839 (0.321)	1.030 (0.219)	1.010 (0.226)	-0.800 (0.202)	0.999 (0.146)	1.002 (0.149)
α	0.088 (0.013)	0.074 (0.012)	0.068 (0.011)	0.062 (0.011)	0.072 (0.012)	0.076 (0.012)
$\underset{\sim}{\theta}_{FIML}$	-0.851 (0.319)	1.032 (0.216)	1.011 (0.224)	-0.799 (0.196)	0.994 (0.143)	0.996 (0.146)
α	0.098 (0.013)	0.078 (0.012)	0.086 (0.013)	0.064 (0.011)	0.076 (0.012)	0.072 (0.012)

*See footnote to Table 3.

Figure 1

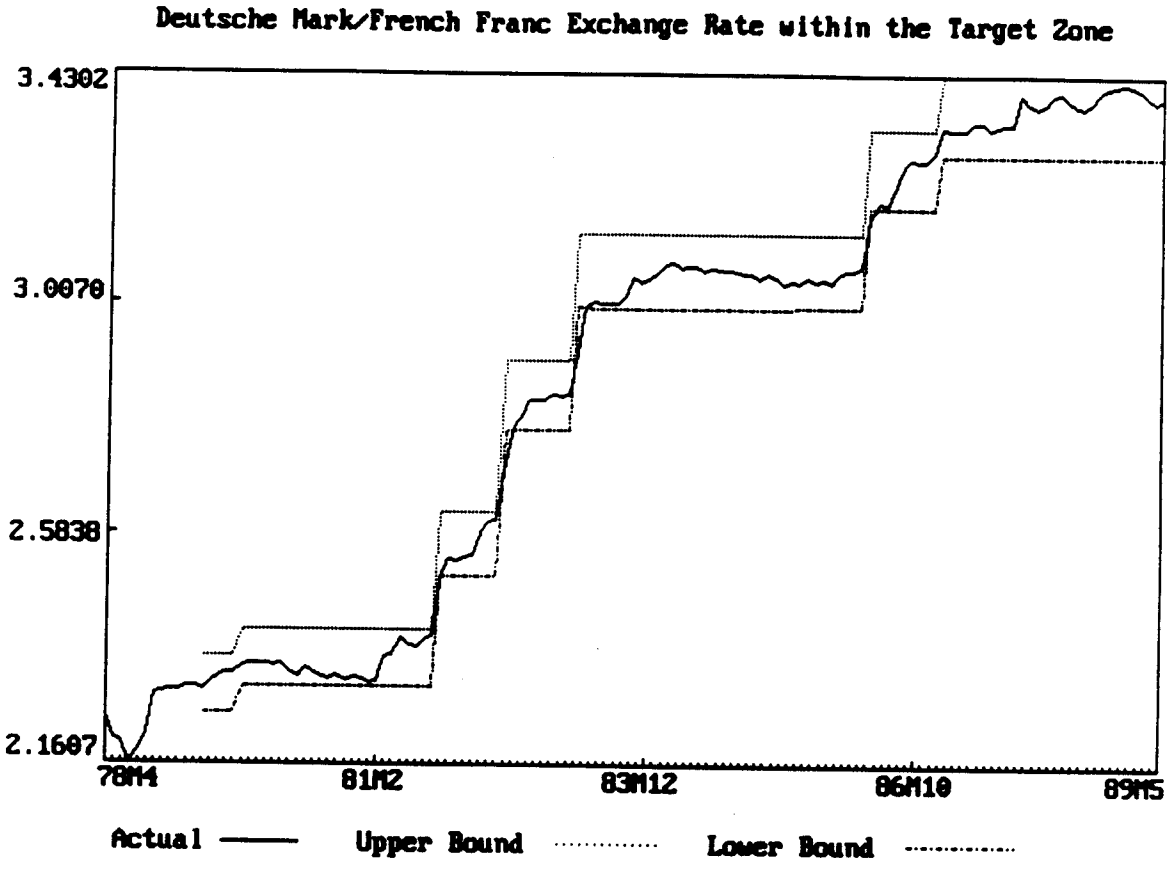
Deutsche Mark/French Franc Exchange Rate within the Target Zone
3.4302
3.0070
2.5838
2.1607
78M4
81M2
83M12
86M10
89M5
Actual
Upper Bound
Lower Bound

TABLE 5

Alternative Estimates of the Monetary Model of the Deutsche mark/French Franc Exchange Rate†

[1979(5)-1989(5)]

Independent Variables	2-Step ML Estimators Not Taking Account Of the Band	2-Step ML Estimators Taking Account of Of the Band	FSML Estimators Taking Account Of the Band
Intercept	-6.285 (18.128)	-0.496 (0.310)	-0.474 (0.305)
$P^{e}_{t}-P_{t-1}$	-0.1672 (2.890)	0.867* (0.040)	0.870* (0.039)
P_{t-1}	0.0408 (0.098)	0.0068* (0.0028)	0.0069* (0.0028)
P_{t-12}	-0.1054 (0.267)	-0.0090* (0.0036)	-0.0089* (0.0035)
m_t	5.434 (8.967)	-0.823 (0.557)	-0.819 (0.551)
m_{t-1}	-5.056 (11.913)	-0.293 (0.321)	-0.271 (0.318)
m_{t-12}	-8.101 (17.466)	0.618 (0.377)	0.611 (0.372)
y_t	-0.997 (4.323)	0.443 (0.419)	0.585 (0.485)
y_{t-1}	0.271 (1.149)	0.036 (0.065)	0.031 (0.065)
y_{t-12}	1.197 (4.739)	-0.356 (0.386)	-0.486 (0.446)
r_{t-1}	-7.674 (17.674)	2.121* (0.889)	2.100* (0.869)
r_{t-12}	-5.606 (13.699)	0.930 (0.565)	0.960 (0.559)
$\hat{\sigma}_u$	0.7465 0.1677	0.4045 0.7557	0.4019 0.7588
R^2	0.1677	0.7557	0.7588
LL	-136.32	-60.20	-59.66

Table 5 (cont.)

†The dependent variable is percentage change in the exchange rate measured as $100(P_t - P_{t-1})$, P_t is the logarithm of the spot exchange rate, m_t is log of the ratio of German to French money supplies, y_t is the log of the ratio of German to French industrial production, r_t is the difference between the three-month interest rates in Germany and in France, "*" denotes statistical significance at the 5 percent level. The bracketed figures are asymptotic standard errors, $\hat{\sigma}_u$ is the estimate of the equation's standard error, $R^2 = 1 - (\hat{\sigma}_u^2 / \hat{var}(\Sigma P_t))$, LL is the maximized value of the log-likelihood function.

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