

# THE AGGREGATE EFFECTS OF MONETARY EXTERNALITIES \*

By  
Jess Benhabib  
and  
Roger E. A. Farmer

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Address for Correspondence  
Department of Economics  
University of California  
Los Angeles  
California 90024

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\*Jess Benhabib is a Professor of Economics at NYU and Roger Farmer is an Associate Professor of Economics at UCLA. Farmer's research was supported by NSF award number SES 87 2243. An earlier version of this paper was circulated under the title "An Equilibrium Model of a Monetary Economy." The authors wish to thank Roberto Chang, Mark Gertler, Ken Matheny, Tom Sargent and Charles Wilson for their comments on the earlier draft, and the CV Starr Center at NYU for providing an environment in which the work could be completed.

## ABSTRACT

We construct a theoretical model of a monetary economy in which money is used to facilitate exchange. Our model differs from existing approaches by allowing an important role for an externality in transacting. We calibrate the model and argue that the calibrated model has rational expectations equilibria in which beliefs may influence allocations independently of fundamentals. One of these equilibria provides a way of understanding the covariance of prices and money in small vector autoregressions.

## 1 Introduction

This paper introduces money into a real business cycle model. A number of authors have conducted similar exercises, using the cash-in-advance model, building on papers by Lucas and Stokey [1987] and Svensson [1985]. Notable examples are those of Cooley and Hansen [1989] and Greenwood and Huffman [1987]. Our work is similar in spirit to this literature and we share their concern with a parametric specification that is calibrated to accord with data from the U.S. economy. But rather than restrict ourselves to the rigid payment patterns that characterize cash-in-advance models, we choose a more flexible approach. While goods and services are produced with labor and capital, the flow of final goods and services in our model also depends on the level of real balances. Our approach captures the role of money as a means of reducing transactions costs in a flexible way. Indeed a special case of our specification reduces to the cash in advance technology (see Section 5).

Our most significant departure from existing formulations of monetary economies is the idea that there is an important externality in the exchange technology. This idea has been widely discussed in literature on the microfoundations of exchange<sup>1</sup> but its implications for the equilibria of a macroeconomy have not previously been studied. We capture externalities at the aggregate level by introducing the social stock of money as an argument of the exchange technology. Our approach borrows much from the literature on endogenous growth and, although we are addressing a different set of issues, it is closely linked to work by Lucas [1988], Romer [1986], King Plosser Rebello [1988] and Baxter King [1990].

Keynesian economists have been critical of market clearing rational expectations models for a number of reasons. One point that leads many economists to be skeptical of the market clearing approach is the apparent inflexibility of prices in U.S. data. Sims [1989] points out that prices do not respond in the data in the

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<sup>1</sup>The literature of which we are aware includes the papers by Diamond [1984], Jones [1976], Ostroy [1973], Oh [1989], and Kiyotaki and Wright [1989]. David Laidler's article [1978] is the only work, of which we are aware, to trace out the macroeconomic effects of exchange externalities. Laidler is concerned with the welfare costs of inflation although the spirit of his analysis is close to our own.

way that most market clearing models predict that they should. We will show that one may reconcile the behavior of the data with the market clearing approach by adopting an equilibrium model in which externalities are important.<sup>2</sup>

We follow the standard methodology of real business cycle studies and calibrate the parameters of our model with independent estimates obtained for the U.S. economy. In particular, to obtain parameter values associated with monetary variables, we use empirical estimates of income and interest elasticities of money demand as well as estimates of the costs of inflationary finance, all of which are widely available in the literature. We find that our calibrated economy exhibits a steady state that is indeterminate in the sense that equilibrium is not uniquely pinned down by fundamentals. We resolve this indeterminacy in a way suggested by Sargent and Wallace [1985] who point out that, in models with multiple rational expectations equilibria, each of the equilibria has a different implication for the time series properties of the data. We parameterize expectations and we argue that the parameter which indexes beliefs should be treated as a 'deep' parameter in the same way as the parameters of the utility function and the production function.

Each of the equilibria in our economy is associated with a different behavior for the co-movements of prices, money, interest rates and output. One of these equilibria is able to track the impulse response functions in the data. We present the qualitative properties of the impulse response functions for an equilibrium of this kind and we show that it replicates the features that are typically associated with text-book Keynesian models. In response to a money shock: (i) output rises temporarily and returns asymptotically to its steady state, (ii) the real and nominal interest rates both fall temporarily and return asymptotically to steady state levels and (iii) the price level increases slowly to a new higher steady state associated with an unaltered level of stationary real balances.

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<sup>2</sup>An earlier attempt to achieve the same goal in an overlapping generations framework is contained in the paper by Farmer [1991].

## 2 A Formal Model

Our economy consists of a large number of identical producer-consumers each of whom solves the following problem:

$$(1) \quad \max \sum_{t=1}^{\infty} \beta^t E_t [U(c_t, 1 - \ell_t) \mid \Omega_t], \quad \beta \in [0, 1),$$

where  $c_t$  represents consumption,  $\ell_t$  is the agent's supply of labor,  $\beta$  is the discount factor and  $E_t$  is the expectation operator conditional on the information set  $\Omega_t$ . The agent chooses current consumption and leisure,  $c_1$  and  $1 - \ell_1$ , and a contingent plan  $\{c_t(\Omega_t), \ell_t(\Omega_t)\}_{t=2}^{\infty}$  for consumption and leisure taking into account the sequence of budget constraints,

$$(2) \quad c_t \leq y_t + \frac{M_t^D - M_{t+1}^D}{p_t} + \tau_t, \quad t = 1, \dots, \infty,$$

and the non-negativity constraints:

$$(3) \quad M_t^D \geq 0, \quad \text{for all } t.$$

The term  $M_{t+1}^D$  represents the nominal demand for money that the agent plans to carry between periods  $t$  and  $t+1$ ,  $\tau_t$  is a real lump-sum transfer,  $y_t$  is net output and  $p_t$  is the money price of commodities. Although money is the only asset that appears explicitly in the agents' budget constraints we are not imposing exogenous restrictions on the set of markets. Our formulation of the problem exploits the assumption that all agents in our economy are identical and that the only asset that is in non-zero net supply is fiat money. Underlying this specification is the assumption that agents may freely trade a complete set of contingent claims, but equilibrium asset prices will adjust to cause agents to voluntarily choose zero net positions in these claims.

To distinguish demand and supply of money we use the term  $M^D$  to refer to money demand and  $M$  to refer to money supply. The money supply process is governed by the rule,

$$(4) \quad M_{t+1} = \sigma_t M_t,$$

where:

$$(5) \quad \sigma_t \equiv \sigma e_t^1,$$

and,

$$(6) \quad \sigma(L) \log(e_t^1) = \log(u_t^1).$$

$\sigma(L)$  is a polynomial in the lag operator with roots outside the unit circle and  $u_t^1$  is an iid white noise disturbance. We choose the unconditional mean of  $u_t^1$  in such a way that the (gross) monetary growth rate is a stationary stochastic process with an unconditional mean of sigma.<sup>3</sup>

The monetary policy rule generates an endogenous sequence of real transfer payments  $\{\tau_t\}_{t=1}^{\infty}$  from the definition of the government budget constraint,

$$(7) \quad \frac{M_{t+1} - M_t}{p_t} - \tau_t = 0,$$

which may be written, using the definition  $m_t \equiv M_{t+1}/p_t$ , in the form:

$$(8) \quad \tau_t = \frac{\sigma_t - 1}{\sigma_t} m_t.$$

Our definition of real balances  $m_t \equiv M_{t+1}/p_t$  is a beginning of period concept which represents the real money balances that are available to facilitate transactions in period  $t$ . Notice from our definition of  $\sigma$  that  $M_{t+1}$  is known at date  $t$ .

We model production in two stages. The first stage combines labor and capital to produce an intermediate good  $x_t$ . We model capital as endowed<sup>4</sup> and we normalize the endowment of capital to unity in each period. To represent the effect of economic growth on the technology we multiply the production function by a time dependent parameter  $a_t$ ,

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<sup>3</sup>If  $e^1$  itself is white noise then this choice implies that the mean of  $e^1$  is equal to unity. If  $e^1$  is autocorrelated then we can always choose a process for  $u^1$  with an unconditional mean of unity. In view of Jensen's inequality, the mean of  $e^1$  in this case will not equal one but we can always redefine  $\sigma$  to incorporate the additional constant term.

<sup>4</sup>It would be more satisfactory if we were to endogenise the investment decision. We have chosen to explore a simpler model first mainly because we can solve it. The model with endogenous capital leads to a third order dynamical system on which we have as yet been unable to make significant progress.

$$(9) \quad x_t = a_t \phi(\ell_t).$$

The term  $x_t$  represents final output that has not yet been delivered to consumers,  $\ell_t$  represents labor input and  $\phi(\cdot)$  is assumed to display diminishing returns to the variable factor labor. We assume that technical progress,  $a_t$ , grows exogenously according to the process:

$$(10) \quad a_t \equiv \gamma^t e_t^2,$$

where  $\gamma$  is an exogenous growth parameter and  $e_t^2$  is a technology shock that is governed by the stochastic process,

$$(11) \quad \Gamma(L) \log(e_t^2) = \log(u_t^2).$$

The term  $\Gamma(L)$  is a polynomial in the lag operator with all roots outside the unit circle and  $u_t^2$  is a serially uncorrelated white noise disturbance that is independent of  $u_t^1$ .<sup>5</sup> This formalization is reasonably flexible and is capable of capturing the stylized fact that output has a near unit root. We choose to model this feature of the data in the same way as most real business cycle models by choosing a highly autocorrelated process to describe the evolution of  $e_t^2$ . To model the process of exchange we assume that the intermediate good  $x_t$  must be delivered to consumers by an exchange technology,

$$(12) \quad y_t = \psi \left( x_t, \frac{M_t^D}{p_t} + \tau_t; \tilde{v}_t \right),$$

$$(13) \quad \tilde{v}_t \equiv \left( \frac{\tilde{M}_t^D}{p_t} + \tilde{\tau}_t \right) / \tilde{y}_t,$$

and we write the *combined* technology,  $\phi(\psi, \cdot)$  as,

$$(14) \quad y_t = F \left( \ell_t, a_t, \frac{M_t^D}{p_t} + \tau_t; \tilde{v}_t \right).$$

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<sup>5</sup>As with the money supply process we define  $\gamma$  in such a way that the unconditional mean of  $a_t$  equals  $\gamma^t$ .

The agent chooses  $M_t^D$  in period  $t-1$ . He arrives in period  $t$  carrying real balances  $M_t^D/p_t$  which are immediately supplemented by a real lump-sum transfer  $\tau_t$ . It is these beginning of period real balances,  $(M_t^D/p_t) + \tau_t$ , which are used to facilitate exchange during period  $t$ .

Our most significant departure from conventional monetary models involves the inclusion of the effect of externalities in the process of exchange. Our strategy is to parameterize the exchange technology directly by choosing a functional form for  $\psi$ . We argue that other agents' real balances,  $(\tilde{M}_t^D/p_t) + \tilde{\tau}_t$ , will have external effects on the exchange costs of the firm but these effects must be weighted by the volume of transactions that are serviced by these balances in the rest of the economy. We use the variable  $\tilde{y}$  to represent the average volume of transactions and we define the inverse velocity of circulation,  $\tilde{v}$ , as the ratio of end of period social balances to social output. In equilibrium  $\tilde{M}_t^D$  will equal  $M_t^D$  and  $\tilde{y}_t$  will equal  $y_t$ . But the individual agent does not take this dependence into account when making his decisions.

There are alternative ways to explain the high correlation of output and money without relying on externalities. A strict cash-in-advance specification will deliver such a result as a response to technology disturbances. In fact, most models of this kind explain the correlation between real balances and output by assuming that output fluctuations are driven either by productivity disturbances or taste shocks. But the standard cash-in-advance model will not explain the impulse responses to disturbances in nominal money that are uncovered by vector autoregressions on time series data. In a cash-in-advance model, the equilibrium response of the system to a money shock requires an instantaneous price response that leaves real balances unaltered. This explanation is internally consistent but it cannot account for the response of the economy to nominal disturbances documented in Sims [1989].



### 3 Some Properties of the Technology

We assume that the function,

$$y = \psi \left( x, \frac{M^D}{p} + \tau; \tilde{v} \right),$$

has the following properties:

1.  $\psi$  is continuous, increasing and twice continuously differentiable in  $x$  and  $(M^D/p) + \tau$ .
2.  $\psi$  is concave in  $x$  and  $(M^D/p) + \tau$ .
3.  $\psi$  is homogenous of degree one in  $x$  and  $(M^D/p) + \tau$ .
4.  $\psi$  is continuous, increasing and twice continuously differentiable in  $\tilde{v}$ .

Properties (1), (2) and (3) are standard assumptions in production theory when one is discussing non-monetary inputs. We will explore the implications of extending these assumption to a technology of exchange. There are good theoretical reasons for imposing the constant returns assumption (3) with respect to the intermediate input  $x$ , and real money balances  $(M^D/p) + \tau$ . It implies that the exchange technology can be replicated at any scale. We have implicitly extended this property to the effect of externalities by writing the external effects of money in the form of a ratio,

$$\left( \frac{\tilde{M}^D}{p} + \tau \right) / \tilde{y}.$$

To make this point explicitly it may help to define two 'social' production functions:

$$(15) \quad y = \tilde{\psi} \left( x, \frac{M^D}{p} + \tau \right),$$

$$(16) \quad y = \tilde{F} \left( \ell, a, \frac{M^D}{p} + \tau \right).$$

These functions are defined by imposing the assumptions  $\tilde{y} = y$  and  $(\tilde{M}^D/p) + \tilde{\tau} = (M^D/p) + \tau$ , on equations (12) and (14) and appealing to the implicit function

theorem. Since  $\psi$  depends only on the ratio of  $(\tilde{M}^D/p) + \tilde{\tau}$  to  $\tilde{y}$  it follows that the function  $\tilde{\psi}$  is homogenous of degree one. Although the assumption of constant returns with respect to private inputs is reasonably easy to defend it is less clear that the effects of externalities should display this property. Indeed there is some evidence that they should not since the property of unit homogeneity in all inputs, private and social, may be shown to imply that the demand for money should display a unit income elasticity. The evidence reported by Laidler [1985] is mixed and for the post-war period there is evidence of a less than unit income elasticity. For the pre-war period there is evidence of a greater than unit elasticity. We have nevertheless chosen to impose unit homogeneity because we want to describe the behavior of a growing economy. Our model will display a steady state growth path only in the special case of a unit income elasticity for money demand.

#### 4 The Solution to the Agent's Problem and the Definition of a Competitive Equilibrium

This section is notation intensive but the basic idea is easy to explain. The representative agent makes two independent decisions in each period. The first of these is captured by a static first order condition that we invert to obtain an expression that relates the level of labor effort in each period to the stock of real balances. The second decision is intertemporal and it leads to a first order condition that relates the level of real balances between periods. By using both of these conditions we are able to derive a single equation that must be satisfied by equilibrium stochastic processes governing the evolution of real money.

To simplify the agent's optimization problem we will make use of some accounting identities. In any optimal plan the consumer's budget constraint (2) will hold with equality:

$$(17) \quad c_t = F \left( l_t, a_t, \frac{M_t^D}{p_t} + \tau_t; \tilde{v}_t \right) + \frac{M_t^D - M_{t+1}^D}{p_t} + \tau_t.$$

Substituting this expression into the objective function and maximizing with respect to  $l_t$  and  $M_{t+1}^D$  one attains two first order conditions that must be obeyed by the

optimal plan. The first of these conditions comes from the optimal choice of  $\ell_t$  and leads to a static equation that holds at each period:

$$(18) \quad U_1(c_t, 1 - \ell_t) F_1 \left( \ell_t, a_t, \frac{M_t^D}{p_t} + \tau_t; \tilde{v}_t \right) = U_2(c_t, 1 - \ell_t).$$

The following algebra exploits the properties of an equilibrium. Our goal is to invert equation (18) and obtain an expression for the supply of labor as a function of a single state variable. In equilibrium  $M^D = M$  and consequently the term:

$$\frac{M_t^D - M_{t+1}^D}{p_t} + \tau_t,$$

is identically equal to zero from the government budget constraint. It also follows that social balances,  $(\tilde{M}_t^D/p_t) + \tilde{\tau}_t$ , and private balances,  $(M_t^D/p_t) + \tau_t$  are both equal to the real beginning of period money supply  $m_t$ :

$$(19) \quad \frac{\tilde{M}_t^D}{p_t} + \tilde{\tau}_t = \frac{M_t^D}{p_t} + \tau_t = m_t.$$

Using these equalities together with the definition of the social production function one may simplify the expression that defines consumption,

$$(20) \quad c_t = \tilde{F}(\ell_t, a_t, m_t),$$

and the expression for inverse velocity  $\tilde{v}$ ,

$$(21) \quad \tilde{v}_t = \frac{m_t}{\tilde{F}(\ell_t, a_t, m_t)}.$$

Using equations (20) and (21) it follows from the implicit function theorem that equation (18) can be inverted to define the optimal labor supply in period  $t$  as a function of  $m_t$  and  $a_t$ ,

$$(22) \quad \ell_t = \ell(m_t, a_t).$$

Equation (22) defines the function that we referred to above. We now proceed to derive a second first-order condition which dictates the relationship of the state

variable  $m_t$  between periods. The optimal choice of  $M_{t+1}^D$  generates the dynamic functional equation:

$$(23) \quad \frac{1}{p_t} U_1^t = \beta E_t \left[ \frac{1}{p_{t+1}} U_1^{t+1} \left[ 1 + F_3 \left( \ell_{t+1}, a_{t+1}, \frac{M_{t+1}^D}{p_{t+1}} + \tau_{t+1}; \tilde{v}_{t+1} \right) \right] \mid \Omega_t \right],$$

where,

$$U_1^t \equiv U_1(c_t, 1 - \ell_t).$$

Multiplying both sides of this expression by  $M_t$ , using the definition of  $\sigma_t$ , and eliminating  $\ell_t$  by substituting the function  $\ell_t = \ell(m_t, a_t)$  this dynamic equilibrium condition may be written as a non-autonomous functional difference equation in the single endogenous state variable  $m_t$ :

$$(24) \quad G(m_t, a_t) = E_t \left[ \frac{\beta}{\sigma_{t+1}} G(m_{t+1}, a_{t+1}) X(m_{t+1}, a_{t+1}) \mid \Omega_t \right],$$

where the functions  $G$  and  $X$  are defined as follows:

$$(25) \quad G(m, a) \equiv m U_1(\tilde{F}[\ell(m, a), a, m], 1 - \ell(m, a))$$

$$(26) \quad X(m, a) \equiv 1 + F_3 \left[ \ell(m, a), a, m; \frac{m}{\tilde{F}[\ell(m, a), a, m]} \right].$$

A competitive equilibrium is a sequence of distributions  $\{P_t(m)\}_{t=1}^{\infty}$  which defines the evolution of the random variables  $\{m_t\}_{t=1}^{\infty}$  that satisfies equation (24).

## 5 Parameterizing Technology

Our specification of the exchange technology is designed with two ends in mind. We need the specification to be tractable and to be capable of explaining some simple facts related to estimates of demand for money functions. Tractability dictates that we choose a class of constant elasticity functions which display unit homogeneity. Our choice takes the form,

$$(27) \quad y = \psi(x, m; \tilde{v}) = A(x^\chi + bm^\chi)^{\frac{1}{\chi}} \tilde{v}^\theta, \quad \chi < 1,$$

which is associated with a social exchange technology:

$$(28) \quad y = \tilde{\psi}(x, m) = A(x^\chi + bm^\chi)^{\frac{1}{\chi(1+\theta)}} m^{\frac{\theta}{1+\theta}}.$$

To model the intermediate technology,  $\phi$ , we use a standard Cobb-Douglas production function,

$$(29) \quad x = a\phi(\ell) = a\ell^\alpha,$$

where we have normalized the endowment of capital to unity in every period. Combining equations (29) and (27) leads to our specification of the private and social technologies,  $F$  and  $\tilde{F}$ :

$$(30) \quad y = F(\ell, a, m, \bar{v}) = A(a^\chi \ell^{\alpha\chi} + bm^\chi)^{\frac{1}{\chi}} \bar{v}^\theta,$$

$$(31) \quad y = \tilde{F}(\ell, a, m) = A^{\frac{1}{1+\theta}} (a^\chi \ell^{\alpha\chi} + bm^\chi)^{\frac{1}{\chi(1+\theta)}} m^{\frac{\theta}{1+\theta}}.$$

Within the class of unit homogenous exchange technologies we have picked the CES class rather than the analytically simpler Cobb-Douglas class because data on the demand for money indicates a very low interest rate elasticity. Since Cobb-Douglas exchange technologies display a unit interest rate elasticity they are not rich enough to explain the facts. When the parameter  $\chi$  approaches minus infinity our technology collapses to a generalized cash-in-advance model similar to that studied by Lucas and Stokey [1987], Svensson [1985] and Cooley and Hansen [1989]. The most significant way in which our approach generalizes cash-in-advance is through the effect of externalities on transactions possibilities. This effect is critical in allowing us to generate equilibria that explain predetermined prices for reasonable parameter values.

For the intermediate technology we have chosen a Cobb-Douglas function to accord with the observation that the shares of national income going to labor and capital have been approximately constant. Our calibration of the technology implies that the parameter  $b$  in (30) is small and that  $F$  itself is close to Cobb-Douglas.

## 6 Parameterizing Preferences

To parameterize utility we have adopted a specification first suggested by Kydland and Prescott in their "Time to Build..." paper [1982]:

$$(32) \quad U(c, 1 - \ell) = \frac{[c^\lambda(1 - \ell)^{1-\lambda}]^{1-\rho} - 1}{1 - \rho},$$

for  $0 < \lambda < 1$  and  $\rho > 1$ . This form separates the parameter  $\rho$  that governs intertemporal substitutability from the parameter  $\lambda$  that governs the relative shares of time devoted to leisure and work. The choice of a Cobb-Douglas aggregator between consumption and leisure is suggested by the observation that real wages have grown substantially over the last century but the fraction of time devoted to market activities has stayed roughly constant.

Given our choice of functional forms for preferences and technologies the function  $\ell(\cdot)$  given in equation (22) is homogenous of degree zero in  $m$  and  $a$ . The first order condition (18) has the following structure:

$$(33) \quad \frac{\alpha a^x \ell^{\alpha x}}{a^x \ell^{\alpha x} + b m^x} = \mu \frac{\ell}{1 - \ell}, \quad \mu \equiv \frac{\lambda}{1 - \lambda}.$$

Dividing the top and bottom of the left-hand-side of this expression by  $a^x$  allows one implicitly to define a function  $h : R_+ \mapsto R_+$ ,

$$(34) \quad \ell = h(q),$$

where the variable  $q$ , defined as

$$q \equiv m/a,$$

will be referred to as normalized real balances. Notice that our assumption of exogenous growth implies that  $a_t$  will be growing with a mean rate of growth  $\gamma$ . If we can show that our model has a steady state in terms of normalized variables like  $q$  then we will be able to identify the parameters of our model by drawing inferences from data which has been detrended by regressing the logarithm of the appropriate variable on a time trend. This is the strategy that we will pursue below in our discussion of calibration.

## 7 Analyzing the Properties of Equilibria

By choosing constant elasticity preferences and a CES exchange technology we are able to find a normalized representation of equation (24) that must be satisfied by equilibrium sequences of distributions over  $q$ .

$$(35) \quad G(q_t) = E_t \left[ \frac{\beta \Theta_{t+1}}{\sigma_{t+1}} G(q_{t+1}) X(q_{t+1}) \mid \Omega_t \right],$$

where:

$$\Theta_{t+1} \equiv \left( \frac{a_{t+1}}{a_t} \right)^{\lambda(1-\rho)},$$

$G(q) \equiv G(q, 1)$  and  $X(q) \equiv X(q, 1)$ . This normalization exploits the homogeneity of preferences which allows us to pull out a term in powers of the technology shock from each of the marginal utility expressions.

To understand what happens in the stochastic economy it is helpful to examine a perfect foresight model by setting  $\sigma_t = \sigma$  for all  $t$  and  $a_t = \gamma^t$ . In this case equation (35) takes the form:

$$(36) \quad G(q_t) = \frac{\hat{\beta}}{\sigma} G(q_{t+1}) X(q_{t+1}),$$

where

$$\hat{\beta} \equiv \beta \gamma^{\lambda(1-\rho)}.$$

Our strategy is to find a stationary state of this difference equation and to examine the behavior of non-stationary sequences that satisfy this equation in the neighborhood of the steady state. We will show that the stability of this difference equation depends on the magnitude of externalities. When the steady state is stable there exist multiple perfect foresight equilibria since any sequence that begins in the neighborhood of the steady state will converge back towards it.

Let  $\bar{q}$  be the unique positive solution to the equation,<sup>6</sup>

$$(37) \quad G(\bar{q}) = \frac{\hat{\beta}}{\sigma} G(\bar{q}) X(\bar{q}).$$

<sup>6</sup>The standard optimum quantity of money rule, which sets the monetary contraction rate to equal the discount rate, or  $\hat{\beta}$  equal to  $\sigma$ , induces the representative agent to be satiated in real balances: to see this note that at the steady state for  $\hat{\beta} = \sigma$ ,  $X(\bar{q}) = 1$  and therefore  $F_3 = 0$ . With externalities on the other hand, the optimal quantity of stationary balances is obtained when  $F_3 + F_4$

The local behavior of perfect foresight sequences in the neighborhood of  $\bar{q}$  is governed by the difference equation:

$$(38) \quad \sigma G' dq_t = \hat{\beta} [XG' + GX'] dq_{t+1},$$

where the partial derivatives of the functions  $G$  and  $X$  are evaluated at the stationary state  $\bar{q}$ . The implicit function theorem implies that in the neighborhood of  $\bar{q}$  there will exist a function  $f$  such that:

$$(39) \quad dq_{t+1} = f(dq_t),$$

where:

$$(40) \quad f'|_{q=\bar{q}} = \frac{1}{1 + \frac{\epsilon_X}{\epsilon_G}},$$

and the terms:

$$\epsilon_X \equiv \frac{X'q}{X}$$

$$\epsilon_G \equiv \frac{G'q}{G}$$

represent the elasticities of the functions  $X$  and  $G$ . The variable  $q$  is non-predetermined, in the terminology of Blanchard and Kahn [1980], since it depends on the price level  $p_t$ . It follows that, if  $|f'| < 1$ , then there is a continuum of perfect foresight equilibria in the neighborhood of  $\bar{q}$  since any initial value of  $q$  will generate a sequence  $\{q_t\}$  that converges back to  $\bar{q}$  which satisfies the definition of a competitive equilibrium.

At this point a few heuristic remarks about the reasons for the indeterminacy of the steady state are in order. In models where the (private or social) productivity of money is relatively small, a sudden increase in real balances will generate an excess supply of money. It would be possible for the increased supply to be willingly held if the rate of return on money, on the equilibrium path, were to be increased by

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is zero. Thus equation (37) would require  $\sigma/\hat{\beta} = X(q_0)$  where  $q_0$  sets  $F_2(q_0) + F_4(q_0) = 0$ . However, our choice of the exchange technology given by (31) does not allow satiation at finite values of  $q$  so that under the optimal quantity rule steady state balances will be infinite. Nevertheless, we may achieve private and social satiation of money by appropriately bending the exchange technology  $F$  at a high value of  $q$  so that  $F_3(q) + F_4(q) = 0$  has a lowest root  $q_0$  with  $F_3(q) = F_4(q) = 0$  for  $q \geq q_0$  and  $F_3(q), F_4(q) \geq 0$  for  $q < q_0$ .



an appropriate decline of the price level in the subsequent period. This response, however, would cause real balances to move away from their stationary level in an ever increasing deflationary spiral. When money is highly productive, however, either privately or through externalities, an increase in real balances leads to a higher flow of net output and may result in an excess demand for money. In equilibrium, it may be possible for prices not to respond instantaneously and for the excess demand for money to be corrected by a fall in the rate of return on money, or an expected rise in the price level in the subsequent period and for real money balances to decline slowly back towards their stationary steady state levels.

### 8 Approximate Rational Expectations Equilibria

We are interested not only in the perfect foresight economy but also in what happens in the presence of uncertainty. To analyze the set of rational expectations equilibria we will look at approximate equilibria that hold when the support of the distributions of the error terms is small. Our assumptions about the processes governing the evolution of  $\sigma_t$  and  $a_t$  imply that the unconditional means of these variables are equal to  $\sigma$  and  $\gamma^t$ . We will linearize equation (35) around the point  $\{\bar{q}, \sigma, \gamma^{\lambda(1-\rho)}\}$ , by using a Taylor series expansion. The importance of assuming that the support of the distribution of the driving uncertainty is small is that it keeps the state variable in the neighborhood of its unconditional mean, a region in which the Taylor series expansion is approximately valid.

Linearizing equation (35) generates the linear functional difference equation:

$$(41) \quad \epsilon_G \hat{q}_t = E_t \left[ (\epsilon_G + \epsilon_X) \hat{q}_{t+1} + \hat{\Theta}_{t+1} - \hat{\sigma}_{t+1} \right],$$

where the variables  $\hat{q}_t$ ,  $\hat{\Theta}_{t+1}$  and  $\hat{\sigma}_{t+1}$  are defined as percentage deviations from their unconditional means

$$\hat{q}_t \equiv \frac{q_t - \bar{q}}{\bar{q}},$$

$$\hat{\sigma}_{t+1} \equiv \frac{\sigma_{t+1} - \sigma}{\sigma},$$

$$\hat{\Theta}_{t+1} \equiv \frac{\Theta_{t+1} - \bar{\Theta}}{\bar{\Theta}},$$

and,

$$\bar{\Theta} \equiv \gamma^{\lambda(1-\rho)}.$$

This equation may also be written in the form:

$$(42) \quad \hat{q}_t = E_t [\nu \hat{q}_{t+1} + \delta w_{t+1}],$$

where:

$$\nu \equiv \frac{\epsilon_G + \epsilon_X}{\epsilon_G},$$

$$\delta \equiv \frac{1}{\epsilon_G},$$

and,

$$w_{t+1} \equiv \hat{\Theta}_{t+1} - \hat{\sigma}_{t+1}$$

is a combination of the fundamental disturbance terms of the model. If the coefficient  $\nu$  is less than one in absolute value then this model has a unique rational expectations equilibrium which is approximated by the linear function:

$$(43) \quad \hat{q}_t = \delta E_t \left[ \sum_{s=1}^{\infty} \nu^{s-1} w_{t+s} \right].$$

In this case the price level depends on the fundamental disturbances  $u_t^1$  and  $u_t^2$  through a function that is found by substituting the parameters of the polynomial  $\sigma(L)$  and  $\Gamma(L)$  into equation (43) and solving for the values of  $E_t w_{t+s}$ .

The situation in which  $\nu$  is less than one is generated by most models that have been analyzed to date. The situation when  $\nu$  is greater than one has been examined theoretically but is generally dismissed as a curiosity. The reason is that if  $\nu$  is greater than one there exist many possible rational expectations equilibria each of which is consistent with the assumptions of rationality and market clearing. This shows up in the mathematics in the observation that any process in the class:

$$(44) \quad \hat{q}_{t+1} = \frac{1}{\nu} \hat{q}_t - \frac{\delta}{\nu} w_{t+1} + \frac{1}{\nu} z_{t+1},$$

is a rational expectations equilibrium, where  $z_{t+1}$  is an arbitrary stochastic process with mean zero conditional on the information set  $\Omega_t$ . This assertion may be verified by plugging the proposed solution back into the functional equation (41) that defines equilibrium sequences of probability measures.

We will avoid the path that is usually taken in the rational expectations literature in which one parameterizes one's model in a way that tries to avoid a situation in which multiple equilibria may arise. We will argue instead that the data suggests the parameter  $\nu$  should be positive and greater than one and we will exploit this idea to explain the behavior of prices in U.S. time series.

## 9 Calibrating the Economy

In this section we present some rough estimates of the elasticities  $\epsilon_X$  and  $\epsilon_G$  which together determine the key magnitude  $\nu$ . For our economy, some algebra shows that  $\epsilon_X$  is negative. Since  $\nu \equiv (\epsilon_G + \epsilon_X)/\epsilon_G$ , a sufficient condition for the existence of multiple indeterminate equilibria is that the elasticity of  $G$  should be negative. The function  $G$  is given by the identity  $G(q) \equiv qU_1(y/a, 1 - \ell)$ . We will show below that  $y/a$  and  $\ell$  are functions of  $q$  and we will draw on evidence from time series data to draw inferences about the magnitudes of the elasticities of these functions.

### 9.1 Evidence from Money Demand Studies

To gain some insight into the probable magnitude of the parameters of the exchange technology we turn to evidence from studies of the demand for money, which we identify with the first order condition (23). Our model of exogenous growth allows us to write the first order conditions of the model in terms of levels and to identify the parameters of these first order conditions with estimates from data that have been detrended.

Our first observation is that the first order condition (23) represents a demand

for money function since the term,

$$\beta E_t \frac{p_t U_1^{t+1}}{p_{t+1} U_1^t}$$

is, by definition, equal to  $1/(1+i_t)$  where  $i_t$  is the rate of interest on a one period nominal bond. The representative agent equates the opportunity cost of holding money,  $1+i$ , to its marginal benefit in the following period,  $\psi_2$ . For our particular choice of functional forms the demand for money equation that arises in our model has the structure,

$$(45) \quad \ln m = \ln y - \frac{1}{[1 - \chi(1 + \theta)]} \ln(1 + i) + \text{constant.}$$

Notice that our specification imposes a unit income elasticity and it is fairly easy to show that this property is a direct implication of the assumption of linear homogeneity that we have imposed on the transactions technology. In U.S. data the interest elasticity of money demand for M1 is of the order of -0.1 which suggests that the calibrated value of the compound parameter  $\chi(1 + \theta)$  should be set at:

$$(46) \quad \chi(1 + \theta) = -9.$$

We will argue shortly that  $\theta$  is of the order of 3.5 which sets  $\chi$  at -2. For the remainder of this exercise we will take  $\theta = 3.5$  and  $\chi = -2$ . We will show below that this assumption has implications for the correlation of money and output and we will argue that 3.5 is about the right magnitude if one wants this correlation to fit the facts.

To identify the exponent  $\alpha$  of the production function given by (48) below, we will require an additional piece of evidence that is related to the welfare costs of inflation. We need an estimate of the magnitude of the parameter  $\delta_m$  which we define as:

$$\frac{bm^x}{a^x \ell^{\alpha x} + bm^x},$$

evaluated at the steady state. Dividing top and bottom of this expression by  $a^x$  and evaluating at  $\bar{q}$  allows us to define the parameter:

$$\delta_m \equiv \frac{b\bar{q}^x}{h(\bar{q})^{\alpha x} + b\bar{q}^x}.$$

Exploiting the CES functional form for  $\psi$  we may write the demand for money function as:

$$(47) \quad \frac{m\psi_2}{y} = \delta_m = \frac{im}{y}.$$

Taking the opportunity cost of holding money,  $i$ , to be around 3.5% for M1 and velocity to be around 5 puts the value of,

$$\delta_m = 0.007.$$

Literature on the welfare costs of inflation argues that money is relatively unimportant and agrees with the magnitude of  $\delta_m$  obtained above. This argument is based on equation (47) which fails however to account for the potential social benefits of holding money.<sup>7</sup>

## 9.2 Calibrating the Production Function $\phi(\cdot)$

We have chosen a functional form for the technology  $\phi(\cdot)$  in the class:

$$(48) \quad \phi(\ell) = a\ell^\alpha.$$

In standard real business cycle theory it is typical to choose the parameter  $\alpha$  to be around 0.7 since  $\alpha$  may be shown to represent the share of national income that goes to labor. In an economy in which exchange is costly we must be a little more careful since there are three factors of production and the combined technology takes the form:

$$(49) \quad y = \psi(\phi(\ell), m; \bar{v}) = F(\ell, a, m; \bar{v}).$$

We will pick out the share of national income that goes to labor from the first order condition for labor evaluated at the stationary state  $\bar{q}$ :

$$(50) \quad \frac{\ell F_1(\ell, a, m; \bar{F}(\ell, a, m))}{y} = \text{labor's share.}$$

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<sup>7</sup>See for example the paper by Lucas [1981] for an analysis of the private costs of inflation. See also Fischer [1981]. Laidler [1978] argues that these costs are underestimated due to the effects of exchange externalities but we are unaware of any comprehensive attempt to estimate the magnitude of such effects.

For the CES exchange technology this condition takes the form:

$$(51) \quad \alpha\delta_\ell = \text{labor's share} = 0.7. \quad (\text{approximately}),$$

where

$$\delta_\ell \equiv \frac{h(\bar{q})^{\alpha x}}{h(\bar{q})^{\alpha x} + b\bar{q}^x}.$$

But we have already identified the value of  $\delta_\ell$  since:

$$\delta_\ell \equiv 1 - \delta_m,$$

and our evidence from demand for money studies, which indicates that  $\delta_m$  is of the order of 1%, implies that  $\delta_\ell$  is approximately equal to 0.99. Since the private importance of money is so small our private technology is approximately Cobb-Douglas. It follows that we will not go far wrong by choosing

$$\alpha = 0.7.$$

### 9.3 Calibrating Preferences

We begin with the preference parameter  $\lambda$  which identifies the consumer's relative preference for leisure. We get a fix on this parameter from the first order condition (33) which may be normalized by dividing through by  $a^x$  to give:

$$(52) \quad \frac{\alpha \ell^{\alpha x}}{\ell^{\alpha x} + b q^x} = \left( \frac{\lambda}{1 - \lambda} \right) \left( \frac{\ell}{1 - \ell} \right).$$

Evaluating this expression at  $\bar{q}$  gives:

$$\alpha\delta_\ell = \left( \frac{\lambda}{1 - \lambda} \right) \left( \frac{h(\bar{q})}{1 - h(\bar{q})} \right).$$

We take the ratio of hours worked to leisure, that is  $h(\bar{q})/(1 - h(\bar{q}))$  to be about 1/3. Since we have set  $\alpha$  to 0.7 and  $\delta_\ell$  to 0.99 the implied value of  $\lambda$  is approximately .67:

$$\lambda = .67.$$

Evidence concerning the value of  $\rho$  is more mixed. Estimates in the literature are usually conducted in the context of a model with fixed labor supply and constant relative risk aversion preferences. In this context researchers attempt to estimate the value of the co-efficient of relative risk aversion,  $ra$ , where:

$$ra \equiv \frac{c \partial^2 U}{\partial c^2} \bigg/ \frac{\partial U}{\partial c}.$$

For the preferences that we are using  $ra$  is identified with the compound parameter,

$$\lambda(1 - \rho) - 1.$$

Reported estimates vary widely depending on whether one measures the covariance of consumption with a safe asset or with a risky asset. However it seems to have become conventional wisdom that a value of the coefficient of relative risk aversion in the range  $(-1, -4)$  is acceptable in the calibration literature. Our results are highly sensitive to the choice of  $ra$  and we have chosen to report results for three different cases,  $ra = -1$ ,  $ra = -2$ , and  $ra = -4$ . For these values, given our choice of  $\lambda = 0.67$  the value of  $\rho$  is tabulated below:

$$\begin{array}{ll} ra = -1, & \rho = 1.0, \\ ra = -2, & \rho = 2.5, \\ ra = -4, & \rho = 5.5. \end{array}$$

#### 9.4 The Effect of Money on Economic Activity

Before deriving estimates of the magnitudes of the parameters  $\epsilon_G$  and  $\epsilon_X$  it is useful to investigate the correlations that our model implies between money and employment and money and output. We have already shown that our parameterization allows us to write  $\ell$  as a function of  $q$ ,

$$\ell = h(q).$$

The elasticity of the function  $h$  which is given by:

$$(53) \quad \epsilon_h \equiv \frac{\partial \ell / \ell}{\partial q / q} = \frac{\chi \delta_m}{\alpha \chi (1 - \delta_\ell) - [1 + (\ell / 1 - \ell)]},$$

represents the correlation that our model implies between money and employment. Evaluating this expression at  $\bar{q}$ , taking the ratio of hours spent working to hours spent in leisure,  $h(\bar{q})/(1 - h(\bar{q}))$  to be 1/3, and taking  $\chi = -2$  gives an estimate of  $\epsilon_h$  of approximately 0.0049,

$$\epsilon_h = 0.0049.$$

Our parameterization also allows us to write normalized output,  $y/a$ , as a function of  $q$ ,

$$(54) \quad \frac{y}{a} = H(q) \equiv \tilde{F}(h(q), 1, q),$$

where the function  $H(\cdot)$  is equal to,

$$A^{\frac{1}{1+\theta}} [h(q)^{\alpha\chi} + bq^{\chi}]^{\frac{1}{\chi(1+\theta)}} q^{\frac{\theta}{1+\theta}}.$$

Taking logarithmic derivatives of  $H$  yields the following expression for the elasticity of normalized output with respect to  $q$ ,

$$\epsilon_H \equiv \frac{\partial(y/a)/(y/a)}{\partial q/q} = \frac{\theta}{1+\theta} + \frac{\alpha}{1+\theta} \delta_e \epsilon_h + \frac{1}{1+\theta} \delta_m.$$

This expression is dominated by the value of  $\theta/(1+\theta)$  which represents the external effects of real balances on the exchange technology. This term is two orders of magnitude bigger than either of the second two terms which represent the indirect effect of money through increasing private labor effort and the direct private effect of money on exchange. We have chosen a value for  $\theta$  of 3.5 because this value implies that our model predicts that the raw correlation of real money with real G.N.P. should be approximately equal to 0.77,

$$\epsilon_H = 0.77.$$

Regressions of various detrended measures of the logarithm of real M1 on the logarithm of G.N.P., or of the logarithm of G.N.P. on real M1 put this magnitude between 0.5 and 1.0 in U.S. time series.<sup>8</sup>

<sup>8</sup>Data from 1960 to 1985 taken from the Economic Report of the President. Estimates differ depending on which variable is chosen as the dependent variable, on the detrending method used and on the sample period.



An alternative way of capturing the high correlation between money and output without relying on monetary externalities is to parameterize our transaction technology so that it is much closer to a strict cash-in-advance specification. This requires setting the value of  $\chi$  to be large and negative. Even with a low or zero value of the externality parameter  $\theta$ ,  $\epsilon_H$  could be in the appropriate range because the elasticity of hours worked with respect to real balances, that is  $\epsilon_h$ , would then be sufficiently large. (As  $\chi \rightarrow -\infty$ ,  $\epsilon_h \rightarrow (1/\alpha)$ ). But, too large a value of  $|\chi|$  with  $\theta \geq 0$ , would contradict the money demand studies by implying too low an interest elasticity for the demand for money (see equation (46)). We have chosen, instead, to rely on a positive monetary externality to explain the high correlation between money and output.

### 9.5 Deriving Expressions for the Elasticities of $X$ and $G$ .

Recall that the function  $X$  is given by the expression:

$$X \equiv 1 + F_3 \left( h(q), 1, q, \frac{q}{\bar{F}(h(q), 1, q)} \right),$$

which can be written, for our parameterization, in the form:

$$X = 1 + \frac{H(q)^{1-\chi(1+\theta)}}{q^{1-\chi(1+\theta)}}.$$

The elasticity of the function  $X$  is given by the expression,

$$\epsilon_X \equiv \frac{\partial X/X}{\partial q/q} = [1 - \chi(1 + \theta)]\epsilon_H - [1 - \chi(1 + \theta)],$$

which for our calibrated values of  $\chi$ ,  $\theta$  and  $\epsilon_H$  takes the value,

$$\epsilon_X = -2.3.$$

A similar analysis applies to the function  $G(q)$  which is defined by the expression,

$$G(q) \equiv U_1(H(q), 1 - h(q))q,$$

The elasticity of  $G$  is given by,

$$\epsilon_G \equiv \frac{\partial G/G}{\partial q/q} = 1 + \frac{U_{11}c}{U_1} \epsilon_H - \frac{U_{12}(1-h(\bar{q}))}{U_1} \epsilon_h \frac{h(\bar{q})}{1-h(\bar{q})},$$

where the expressions  $U_{11}c/U_1$  and  $U_{12}(1-h(\bar{q}))/U_1$  are, for the preferences that we are working with, simple functions of the parameters  $\rho$  and  $\lambda$ ,

$$\frac{U_{11}c}{U_1} = \lambda(1-\rho) - 1,$$

$$\frac{U_{12}(1-h(\bar{q}))}{U_1} = (1-\lambda)(1-\rho).$$

We are now in a position to report the values of the key parameter  $\nu$  for alternative choices of the co-efficient of relative risk aversion. For logarithmic preferences,  $ra = -1$ , the value of  $\nu$  is negative and the perfect foresight economy displays multiple equilibria that cycle into the steady state. As  $ra$  is increased, the value of  $\nu$  converges to minus infinity. It attains this value when  $ra$  is approximately equal to 1.3. For values of  $ra$  greater than 1.3 the economy displays multiple paths that converge monotonically to the steady state. The following table presents the approximate values of  $\nu$  that are implied by the different values of risk aversion, -1, -2 and -4.

$ra = -1,$	$\epsilon_G = +0.33,$	$\nu = -6,$
$ra = -2,$	$\epsilon_G = -0.54,$	$\nu = 5.3,$
$ra = -4,$	$\epsilon_G = -2.08,$	$\nu = 2.1.$

For more modest values of the externality parameter  $\theta$  it is still possible to obtain a theoretical model in which multiple equilibria can arise but typically one requires a very high degree of relative risk aversion.

## 10 An Equilibrium with Pre-determined Prices

So far we have established that our model may have many equilibria and we have argued that the region of the parameter space in which multiple equilibria occur

is consistent with other evidence that concerns the behavior of money and output. In this section we are going to push this idea a little further by observing that equilibria in which beliefs may independently influence allocations can help us to explain the apparently anomalous behavior of prices in U.S. data.

Limiting ourselves to the linearized system we have argued that an equilibrium is a sequence of probability distributions that satisfies the functional equation,

$$(55) \quad \hat{q}_t = E_t \left[ \nu \hat{q}_{t+1} + \delta \hat{\Theta}_{t+1} - \delta \hat{\sigma}_{t+1} \right].$$

But in writing down this equation there is a sense in which we have dropped some information. The way in which people behave in period  $t$  depends on the beliefs that they form about prices in period  $t+1$ . In a standard rational expectations equilibrium there is a unique way of forming beliefs about future prices that is consistent with the rational expectations assumption. But in our model there are many ways of forming beliefs each of which will pick out a different fundamental equilibrium. To focus on beliefs it will be useful to define a stationary transformation of prices that we will use to index equilibria,

$$(56) \quad Q_t = \frac{M_t}{p_t a_{t-1}} \equiv \frac{q_t \Theta_t^{1/\lambda(1-\rho)}}{\sigma_t}.$$

Since  $M_t$  and  $a_{t-1}$  are known at date  $t-1$  all deviations of the variable  $Q_t$  from its conditional mean result from differences of prices from their one period ahead expectation. Using hats over variables to denote deviations from unconditional means the variable  $Q$  is related to  $q$  by the equation,

$$(57) \quad \hat{Q}_t = \hat{q}_t - \hat{\sigma}_t + \frac{1}{\lambda(1-\rho)} \hat{\Theta}_t.$$

In U.S. data prices are very slow to respond to fluctuations in contemporaneous variables; this is the evidence documented by Chris Sims [1989] which has been used by "Keynesians" for some time to criticize rational expectations models that display instantaneous price adjustment. We propose to capture this feature of the data by looking for an equilibrium in which agents *believe* that prices move slowly. In terms of stationary variables we will capture this by finding an equilibrium in

which the variable  $Q_t$  is set one period in advance. Given our definition of  $Q$  and given the equation that defines equilibria, (equation (55)), the solution we seek takes the form:

$$(58) \quad \hat{q}_{t+1} = \frac{1}{\nu} \hat{q}_t + [\hat{\sigma}_{t+1} - E_t(\hat{\sigma}_{t+1}|\Omega_t)] - \frac{1}{\lambda(1-\rho)} [\hat{\Theta}_{t+1} - E_t(\hat{\Theta}_{t+1}|\Omega_t)].$$

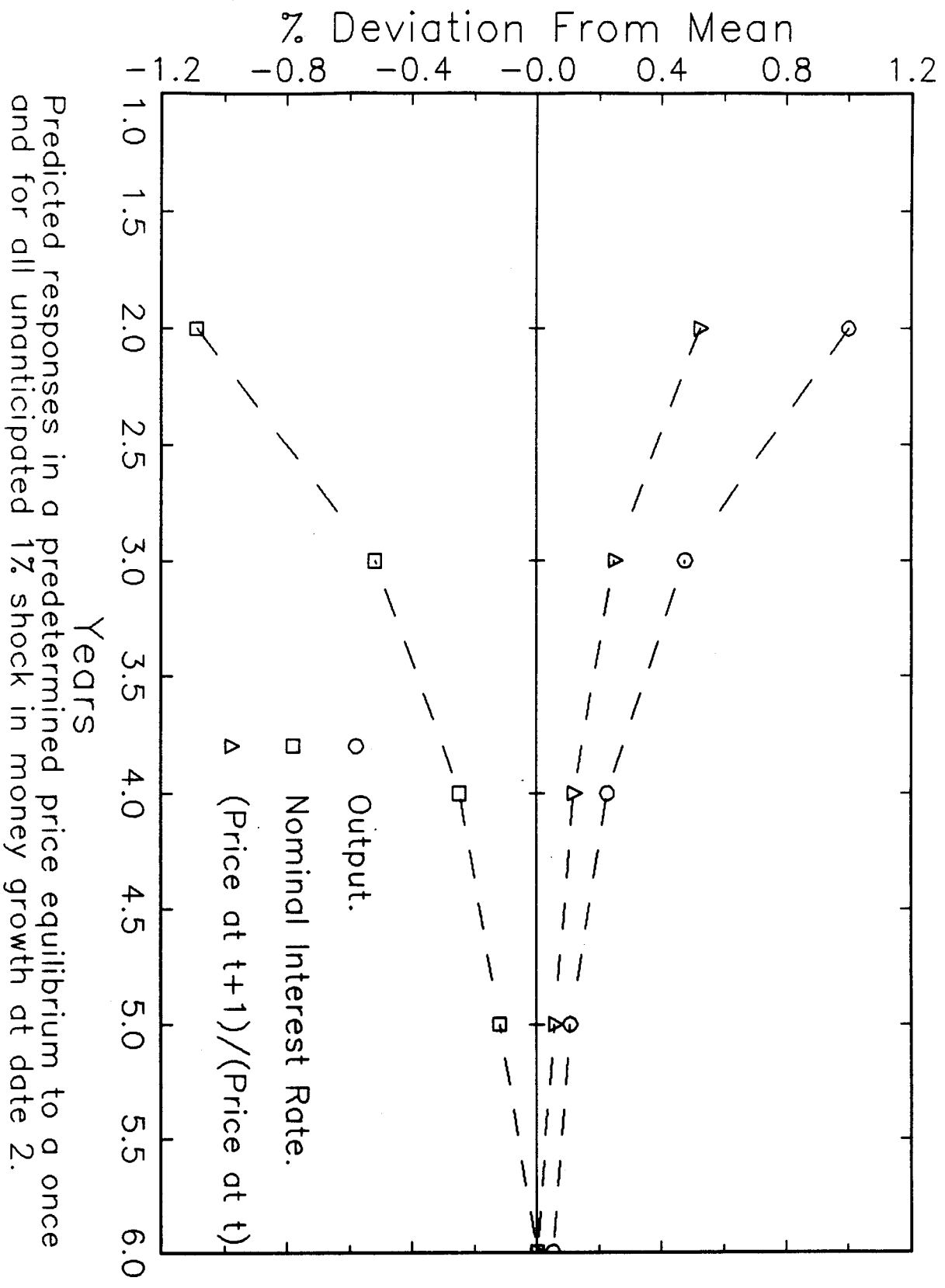
The expectations that appear in this equation are conditional expectations which may be different from zero since the variables  $\hat{\Theta}$  and  $\hat{\sigma}$  refer to deviations of the growth rates of real and nominal shocks from their *unconditional* means. By replacing this solution into the functional equation (55) one verifies that equation (58) defines a rational expectations equilibrium. But the variable  $\hat{Q}_{t+1}$ , in this solution, is defined by the equation:

$$(59) \quad \hat{Q}_{t+1} = \frac{1}{\nu} \hat{q}_t - E_t(\hat{\sigma}_{t+1}|\Omega_t) + \frac{1}{\lambda(1-\rho)} E_t(\hat{\Theta}_{t+1}|\Omega_t),$$

which is predetermined in period  $t$ . But  $Q_{t+1}$  is just the price level normalized by a growth factor and by the money stock at date  $t$ . It follows that, in this solution prices are perfectly predictable one period in advance and are unresponsive to contemporaneous disturbances in the stock of money.

Figure 1 makes this point explicitly by depicting the impulse responses of output, inflation and money rates of interest in the predetermined price equilibrium for the case of  $ra = -4$ . Lower values of  $ra$  lead to qualitatively similar pictures but to less persistent shocks. To draw figure 1 we calculated the response of the linearized system to a once and for all unit shock to the proportional deviation of the money growth rate at date 2. Money growth and real growth were held equal to their unconditional means for all other periods and we took the benchmark case in which money growth and real growth were white noise. Notice that, in this equilibrium, our model captures the stylized feature of the data that prices are slow to respond fully to a money growth shock. The movement in the variable  $p_{t+1}/p_t$  that is featured in figure 1 is achieved by a movement in the future price  $p_{t+1}$  since the variable  $p_t$  is set in advance and is always equal to its one period ahead conditional expectation. Notice that our model predicts a *fall* in nominal and real interest

FIGURE 1



Predicted responses in a predetermined price equilibrium to a once and for all unanticipated 1% shock in money growth at date 2.

rates in response to an unanticipated shock to the money growth rate. Both of these variables drop initially and return asymptotically to their long run levels.

## 11 Conclusion

We have argued that models in which beliefs may independently influence allocations deserve to be taken seriously as models of the business cycle. Our model is rather simple but the basic ideas should carry over to more realistic models that allow capital to be chosen endogenously and which model the effects of government spending and taxes. We think that the most important part of our work is that it allows one to reconcile the rational expectations equilibrium modelling strategy with the sluggish behavior of prices in the real world. We believe that models in this class have a good chance of performing 'better' than existing alternatives when confronted with data from the U.S. economy and although we have not yet formalized this idea we are planning to estimate a model that incorporates the idea of self-fulfilling beliefs and to compare it with existing alternatives.

There are other dimensions in which our work differs from more traditional rational expectations models. One departure, that is implied by the important role that we attribute to externalities, arises in an analysis of the welfare implications of expansionary monetary policy. Our work suggests a more traditional interpretation of the distinction between temporary and permanent increases in the rate of monetary growth that is more in line with monetarist theories of the inflation output trade-off than with rational expectations explanations based on 'nominal-real-confusion'.

In our model, if agents believe that prices are sticky, policymakers face a particularly difficult choice. A once and for all *unanticipated* increase in the rate of money growth is associated with an initial increase in real balances which subsequently decline towards their stationary distribution. The increase in real balances is accompanied by increased output and unambiguously higher welfare.<sup>9</sup> But if the policy maker attempts to exploit this situation repeatedly he will generate an

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<sup>9</sup>The analysis of welfare in a deterministic model is treated in a paper by Benhabib and Bull [2].

increase in the conditional mean of the monetary growth rate which is equivalent to an *anticipated* change in monetary policy. The beneficial effects of a monetary expansion do not carry over to this case since the agent will be induced to economize on his holdings of money, resulting in a stationary distribution for real balances that has an unambiguously lower mean and unambiguously lower welfare. In view of long-run considerations it may therefore be desirable to set a low monetary growth rate and stick with it.

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