

VICO AND de FINETTI: AN ECONOMIC PERSPECTIVE*

by

John J. McCall

University of California, Los Angeles

December 1987
UCLA Dept. of Economics
Working Paper #630
October 1991

*I am indebted to A. Alchian, H. Demsetz, A. Director, M. Friedman, J. Hirshleifer, B. Lehman, A. Leijonhufvud, S. Zambelli, and especially K. Velupillai for their inspiration and comments. The first version of this paper was written while the author was a Visiting Scholar at the Hoover Institution, 1987-88. It is a pleasure to acknowledge the financial and intellectual support of this research haven.

In my discourse *On the Study Method of Our Time*, I upheld the position that difficulties in the study of physics can be overcome by the cultivation of ingenuity. This caused some people, wholly infatuated with the problem of method, to raise their eyebrows; but I stand on that position. In its concern for facility, method obstructs inventive minds; in its care for truth it destroys inquisitiveness. Geometry sharpens the mind not when it is taught methodically, but when it is discovered and applied by ingenuity amid many diverse and disparate experiences. Thus I advocated that it be taught not analytically, but synthetically; just as we demonstrate by relating elements, so let us not discover truths, but make them, that is, construct them with the mind.

Vico [The New Science]

Caveat Emptor

The following warning by Berlin (1976) is pertinent to this essay:

One can readily understand that in the case of a thinker so rich and so confused, and above all so genuinely seminal -- the forerunner of so many of the boldest ideas of later, more celebrated, thinkers -- there is a permanent temptation to read too much into him, especially to sense intimations, perceive embryonic forms and prefigured contours of notions dear to the interpreter himself. Michelet, Dilthey, Croce, Collingwood (and less certainly Herder and Hegel) are among his progeny, and some among them, notably Michelet and Croce, consciously or unconsciously tried to repay their debt by attributing too many of their own most characteristic ideas and attitudes, sometimes at the cost of patent anachronism, to Vico's writings. To attribute one's own opinions to an earlier thinker is doubtless a sincere form of admiration. It is one of the attributes of intellectual depth that very different minds fancy that they find their own reflection in it. But this characteristic is purchased at a price, and has rendered Vico a disservice.

Actually, to abide by this warning strictly would mean that Vico was not worth reading. Selectivity is the essence of induction. If Vico fell through my subjective net, this would be a brief essay. If my distorted clay statue of Vico casts dull and hackneyed shadows, then both Vico and the reader have been abused.

Summary

It may be slightly extravagant, given the perceived equivalence between evolution and Darwin, to claim that the evolution of mind and society was Vico's basic idea. Yet almost all of his seminal ideas are compatible with this assertion. These include: an adaptive theory of natural law, the proposition that comprehension of "modern" society is impossible without serious contemplation of the history of man, the emergence of a civilized society was contingent on the evolution of an institutional matrix that shielded individuals from the raw uncertainty confronting early man, the development of institutions was based on a theory of connections among individuals. These connections or relations were basic to the formation of societies.

Invention was a synthetic activity of the mind again based on a connectionist theory. Vico's inductive process showed how seemingly unrelated ideas were intimately linked. The search for these connections comprised the learning process, while memory is essentially the storage of these insights (templates). The structure of society and mind were neither fixed nor homogeneous. The structure of society depended on its own responses to a unique set of Nature's sample paths that together comprised the historical process. Similarly, the uniqueness of the individual was produced by the interaction among genetic endowment, experience, and the peculiar sequence of choices among the alternatives presented by nature.

Both society and mind were idiosyncratic entities formed by historical processes. Their response to novelty was conditioned by this vast history. Any "science" must be founded on the study of history. Perhaps the constructivist approach to mathematics will be Vico's most attractive

innovation. The synthetic mathematics exhibited in Vico's spirited attack on what was then the conventional wisdom of Descartes is now espoused by many modern mathematicians.

Finally, it is Vico's subjective philosophy, with its emphasis on heterogeneity, change, and synthesis that forges an extraordinarily tight bond with another Italian genius, Bruno de Finetti.

I. Background

The purpose of this paper is to elaborate on several ideas¹ that include: (1) The importance of the legal system; (2) The evolution and significance of institutional economics; (3) The burgeoning interest by economists in rhetoric; (4) The continued growth of mathematical economics and its divergence from concrete economic problems; (5) The legacy of de Finetti and Savage and their ability to unify mathematics and the practical decisionmaking of businessmen; (6) The role of exchangeability² in the de Finetti-Savage theory; and finally (7) The isolation of economics from the other social sciences and humanities and, hence, the significance of the flourishing discipline: law and economics.

¹These ideas were rekindled at lunch with Aaron Director and Milton Friedman.

²We shall say that $X_1, X_2, \dots, X_n, \dots$ are exchangeable random quantities if they play a symmetrical role in relation to all problems of probability, or, in other words, if the probability that $X_{i_1}, X_{i_2}, \dots, X_{i_n}$ satisfy a given condition is always the same however the distinct indices i_1, \dots, i_n are chosen. As is the case for exchangeable events, any problem of probability is perfectly determined when it has been stated for generic random quantities; in particular if $X_1, X_2, \dots, X_n, \dots$ are exchangeable random quantities, the events $E_i = (X_i \leq x)$ (where x is any fixed number) or more generally $E_i = (X_i \in I)$ (I being any set of numbers) are exchangeable. This property will be very useful to us, as in the following case: the mathematical expectation $[M]$ of any function of n exchangeable random quantities does not change when we change the n -tuple chosen; in particular there will be values $m_1, m_2, \dots, m_k, \dots$ such that $M(X_i) = m_1$, whatever i may be; $M(X_i, X_j) = m_2$, whatever be i and j ($i \neq j$), and in general $M(X_{i_1}, X_{i_2}, \dots, X_{i_k}) = m_k$ whatever be the distinct i_1, i_2, \dots, i_k .

Reflection on these ideas reminded me of a philosopher I had been introduced to many years ago -- Giambattista Vico (1668-1744).³

Surprisingly, the questions raised at lunch were addressed either directly or indirectly by Vico.

2. A Brief Review of Vico's Philosophy

The design, and evolution of social institutions sometimes are overlooked by modern economists. However the last few years have witnessed a renewed interest in the formation, function and demise of economic institutions.⁴ Matthews observes that this new theory of institutions is based on two premises: "institutions do matter" and they can be studied using modern economic analysis. As anticipated, economic historians have been at the forefront of this revival.⁵ The research stimulated by this renewal has not been limited to historians and applied economists. It also includes economic theorists: mathematical economists and theoretical econometricians. The interest by economic theorists in history and institutions may seem surprising until one reflects on the relation between the formation of institutions and the reduction of uncertainty. Any serious study of the economics of uncertainty must consider the evolution of institutions as a major source of uncertainty reduction.

³I was introduced to Vico in a course by R. Caponigri at the University of Notre Dame. In his preface to Vico (1990), Verene observes that Caponigri's penetrating book (1953) kept "Vico alive for English-speaking scholars."

⁴An excellent portrayal of this rebirth is contained in Matthews (1986). An economic version of the evolution of institutions is presented in Hicks (1969). The definitive papers are Alchian (1950,1984).

⁵For example, see the classic study by North and Thomas (1973) and the subsequent literature.

Institutions include the customs, language and legal system of a society.⁶ In addition, insurance, capital markets, information networks, the education system and industrial organization are institutions. The manner in which they adapt to changing circumstances affects the well-being of each member of society.

The reduction of uncertainty occurs because institutions guide information flows and improve induction by the mind and society, with a corresponding enhancement of predictability.

Institutions not only reduce uncertainty and generate specialization, regularity and civility. They also harbor the ethical qualities distinctive to humanity. Velupillai (private correspondence) says it best:

Our sense of justice, goodness, etc., call forth institutions that are repositories of them. Laws, moral codes, ethical conduct, etc. These must have a relative permanence so that behaviour can be in accordance with those senses that were the backdrop for the emergence of these institutions. Hence the reduction of uncertainties and the enhancing of predictabilities. In this way symmetry plays the fundamental role.... The manner in which institutions adapt to changing circumstances must, somehow, be compatible with our sense of justice, goodness, beauty, truth and kindness. When the adaptive process violates any of these entities then one will detect asymmetries.... Till now one was always theorizing about the symmetric-asymmetric state. Thus the importance of exchangeability.

Properly functioning institutions are invisible. Individuals convert complex and risky behavior into regular routines that are virtually riskless. This permits each of society's members to specialize in those activities yielding the greatest personal rewards. If society is organized smoothly, the welfare of its members coincides with society's welfare.⁷

⁶Recall that the Latin root for "institution" means "to teach". More specifically: L. instituere means to teach or arrange and L. institutum means design.

⁷But then there is Buchanan et al. See Cornes and Sandler (1986).

The genetic evolution that was necessary for this emergence is now fairly well understood and provides a satisfactory explanation of physical evolution. Fright and the quest for certainty were the driving forces of social evolution. Clearly, nature would not endow man with consciousness until the mind had devised methods for increasing the predictability of his behavior above some threshold. That is, consciousness would survive only if inherent, "raw" risk was reduced by the development of "insurance" situations. They enabled individuals to rely on routines or habits for resolving recurring problems, while they concentrated on specialized activities. A highly conscious organism would be unable to accomplish this resolution if it were thrust into a dangerous, uncertain and unpredictable environment. Dread would induce inaction and extinction. Thus society, mind, and the brain evolved in tandem.

The theses proposed here are (1) that Vico was to social evolution what Darwin was to physical evolution and (2) the evolution of civilized society was dependent on both the Viconian and Darwinian theories. (3) Vico was a pragmatist par excellence⁸ and belongs to that distinguished group that includes: Hume, Smith, Darwin, Peirce, Dewey, James, and de

⁸ Ironically, it was the Dominican G.F. Finetti who pierced the Catholic verbiage and saw the New Science in its true light. Religion was an integral part of Vico's theory, but Catholicism was not essential. See page 22. The part played by Finetti in the intellectual history of Vico is multifaceted. First, it reminds us of the influence of scholasticism and especially Thomas Aquinas on Vico. Aquinas' theory of aesthetics are remarkably similar to Vico's. Surprisingly, the eminent Vico scholar, Verene, maintains that Thomism had little impact on Vico. St. Thomas also was a neopolitan and, of course, his philosophy is nourished by Aristotle. The name Finetti points to the key message of this essay: Vico's constructive approach to cultural evolution unveiled an inductive process closely connected to the inductive process (exchangeability and the related stochastic and number-theoretic processes) developed by Bruno de Finetti. This observation provides the nexus between science and the humanities that is personified by James Joyce. Joyce extends the aesthetics of Aquinas and Vico and poetically depicts the recurrent theme crucial to exchangeable processes. For superb discussions of this background see Eco (1988), Kenner (1984), Verene (1987) and Feyerabend (1987). The link with economics is elaborated in McCall and Velupillai (1991).

Finetti.⁹ And finally, (4) there is a natural connection between the philosophy of Vico and the subjective probabilistic decisionmaking of de Finetti. This connection is fortified by exchangeability.

In recent years economists have applied their methods to other disciplines especially law, psychology, sociology, anthropology and biology.

A simple application of exchangeability demonstrates what is obvious to the intuition. These intellectual excursions will not be successful unless the intruder's discipline adapts to the intellectual content of the host discipline. Without this contractual arrangement in which both parties acknowledge the gains from trade, there will be little chance of continuous commerce. Even though there are many untied threads that could unify these disciplines, imperialistic probes by one discipline will never weave the interdisciplinary tapestry that appears so natural in Vico's philosophy. There may be ephemeral links as techniques are transferred and absorbed by the host disciplines.¹⁰ But as long as imperialism is practiced, heterogeneity will block the exchangeability required for productive interdisciplinary research.¹¹

The prime mover in Vico's evolutionary theory of institutions was the quest to control uncertainty. Uncertainty manifested itself in "sheer fright" of the inexplicable behavior of natural phenomena. According to

⁹ In a brief comment on Vico, Hayek (1978) who had just written: "I do not intend to pitch my claim on behalf of Mandeville higher than to say that he made Hume possible. It is indeed my estimate of Hume as perhaps the greatest of all modern students of mind and society [This is exactly de Finetti's appraisal of Hume] which makes Mandeville appear to me so important," continues in a footnote: "It may deserve notice that J.G. Herder seems to have been the earliest instance where the influence of Mandeville joined with that of the somewhat similar ideas of G. Vico."

¹⁰ This problem is nicely articulated by Coase (1985).

¹¹ For a lucid and perceptive discussion of imperialistic economics see Hirshleifer (1985).

Vico, the response to this uncertainty was the formation of marriage, monogamy, family, tribes, and, finally, nations. The evolution of language was a necessary prerequisite to this nation-building. Vico's theory of linguistics was highly original. Communication was first by sign language and grunting, then by a guttural singing refined into Homeric poetry. Rhetoric, eloquence and jurisprudence emerged from this primitive poetry and were absolutely essential to the diminution of uncertainty and the formation of a civilized society. Just as primitive man was certain of the sunrise, so civilized man must be certain of a stable legal system: a system that defined the sphere in which he could operate without fear of violence by either individuals or government. Religion was the force that explained nature's erratic behavior. Language was the cement that held groups together. Eloquence was the method used to persuade society to undertake new and risky enterprises like forming a treaty with contiguous strangers or adopting a technological change like farming. Eloquence merged into jurisprudence that "protected" the individual and group during these enterprises.

Vico is frequently and mistakenly associated with the Platonists and idealists. In fact, he was a vehement critic of Descartes' idealism. His emphasis was on the real, the tangible, and the frailties and flux of human nature. Most Vico scholars seem to agree that Bacon, Plato, and Tacitus were the strongest forces in Vico's philosophy.

Paradoxically, the most decisive influence on Vico was Descartes.¹² He provided the stimulus that ignited Vico's creative genius. Frequently, the provocative function of conflicting ideas are undervalued in their genesis of novelty. The sparks generated by the collision of the Cartesian

¹²As one might guess, Vico was a devout Cartesian before building his own New Science.

and Vicovian systems may be the most significant contribution to the intellectual evolutionary process. The tension between scientists and humanists has been depicted by Tolstoy ("analysis destroys") and several arrogant scientists ("humanism is a tangled web of imprecision") as a complete separation. C.P. Snow's characterization (1959) remains lucid. We contend that once Francis Bacon's remark¹³ is considered seriously, the rich intersection between these "two cultures" can be visualized together with their manifold connections.

Vico repeatedly attacked the abstract concept that was never enriched by concrete experience. Descartes' profound mistake was to abstract from concrete reality before perceiving the relationships among facts. Vico anticipated the limitations of a purely mathematical social science. By withdrawing from the richness of experience one was left with a distorted and desiccated view of social phenomena.

The decline of altruism as one moved away from the family was one manifestation of Vico's non-Utopian view of human nature. Another was his clear recognition of the primitive aggressiveness that reigned before the formation of institutions and the onset of civilization. Only the simultaneous growth of mind and civility could keep destructive instincts in check. These instincts persisted in civilized societies and, hence the return to chaos was always a threat. He was painfully aware of the Hereclitian flux and, indeed, it is essential to his theory of a recurrent civilized society.

¹³"We cannot command nature except by obeying her."

3. The Well-Known Contributions of Vico

Vico is best known for his trenchant critique of Cartesian philosophy. He refused to acknowledge the bifurcation of man into mind and matter, the supremacy of mathematics as a science, and the deterministic system that behaved with clockwork precision. This position was very popular among 19th century philosophers who regarded Descartes' philosophy as the humanistic counterpart of the scientific discoveries of Newton and Kepler. But Vico would have none of this! Concrete reality was too unpredictable to be captured by any deductive system. This is especially true of the social sciences.

His writings were well-known by Comte and Marx, but not by Smith or Hume. The most famous expositor of Vico's philosophy was Benedetto Croce who, of course, selected those parts that were agreeable with his philosophy.¹⁴ As Mooney (1985)¹⁵ sagely observes:

Croce could only be embarrassed by Vico's link with the rhetorical tradition. For nothing is quite so foreign to romanticism as any thought of utility, and nothing is further from the spirit of oratory than the romantic idea of self-expression.

Although poetry represents, as Croce saw, a form of knowledge that is imaginative and not intellectual, it does not mark, as he also held, the boundary between arts and sciences. Not only does Vico maintain, with Bacon his mentor, the traditional understanding of "arts," but he also assigns a "poetic" origin to the entire cycle of learning -- sciences as well as arts. Poetry -- or "poetic wisdom," as Vico more commonly called it -- designated that earliest phase of human culture which was spontaneous, synthetic, and constructive, in contrast to a later phase, which was reflective, discursive, and critical. Vico had no interest, as Croce so vainly hoped, in "forms of the spirit" independent of the course of concrete history. He was totally absorbed in the question of

¹⁴Some may claim that this is exactly what I am doing. See the discussion on p. ii.

¹⁵This is a splendid book containing much wisdom and a very fair and complete appraisal of Vico. For economists interested in Vico and/or the evolution of economic and legal institutions, there is no better introduction to the New Science.

how humanity arose, and once established, how it advanced from its crude beginning to an age of critical intelligence. Within his theory of culture, poetry is the crucial reality. It is the clarion call of civilization, the first, clear sign of humanity's birth. More than its sign, however, it is also its cause. For poetry is not simply language; it is *effective* language, wisdom that establishes humanity. Myth, in Vico's account, is its principal form, but it has other forms as well -- hieroglyphs, sign languages, coats of arms, medals, currencies, and the like. Whatever form it takes, however, its effect is one and the same: Through it, the *giganti* are transposed from aimlessly wandering individuals into rough, but nonetheless true, citizens. What Croce considered as excrescence, therefore, a "dark nook in the general blaze of light," occupies the center of Vico's position. Poetry, like eloquence, has intrinsically a civil end. It addresses creatures in their isolation, reverses the habit of independent action, and shapes them into a people. Poetry, in fact, is eloquence. It is the language that constitutes humanity.

The power, rhetorical splendor, and in some cases, strident assertiveness of Vico's writing would not attract most economists and certainly not the Scotch. In his attempt to mold the wisdom of Homer, Tacitus and Bacon into a language promoting his own deep insights, Homer dominated. This mythical "Irish stew" was savored by Joyce and Yeats, but proved repugnant to many "rational" economists.

The main point is that the social sciences sans economics were influenced directly by Vico. Indeed many attractive ideas that are attributed to Comte, Marx, Durkheim, or Mauss are traced easily to Vico. These include: the network of contractual relationships that comprise the individual, influence individual decisionmaking,¹⁶ and reduce uncertainty; the basic institutions -- family, community, state, nation that create homogeneity (exchangeability) when combined with a common legal system that protects and enforces contracts, a common moral code that has become instinctual by reason of parental educational and religious training; the process orientation of many social scientists manifested by an evolutionary

¹⁶ Again it was Peirce who formalized these networks in his contributions to Graph Theory.

approach to social problems; an affinity for classification that is shared by many scientists, especially biologists. Indeed, as we will see later, all the physical sciences and social sciences are characterized by adaptive classification processes. The motivation for this is practical. Decisionmaking will be defective if the agent does not respond to the changing environment.¹⁷

4. Vico and de Finetti

It is impossible in this short paper to articulate the linkage between the philosophies of Vico and de Finetti. de Finetti's search for subtle connections among diverse phenomena, his fastidious use of language indicating a Viconian respect for this miraculous mechanism, his attachment to pragmatism from Hume to Pierce to James, his disdain for the abstract divorced

¹⁷ This classification process will be formalized in a subsequent paper. It is the key to M. Friedman's methodology.

Viewed as a language, theory has no substantive content; it is a set of tautologies. Its function is to serve as a filing system for organizing empirical material and facilitating our understanding of it; and the criteria by which it is to be judged are those appropriate to a filing system. Are the categories clearly and precisely defined? Are they exhaustive? Do we know where to file each individual item, or is there ambiguity?... The Canons of logic alone can show whether a particular language is complete and consistent, that is, whether propositions in the language are "right" or "wrong." Factual evidence alone can show whether the categories of the "analytical filing system" have a meaningful empirical counterpart, that is, whether they are useful in analyzing a particular class of concrete problems.

M. Friedman (1952)

This is a rather harsh evaluation of theory and classification. An axiomatic system can aid the scientific process by implying new and surprising consequences. In many ways classification is the essence of science. For example, an insurance company must classify its heterogeneous clients in a proper manner if it is to avoid bankruptcy. This activity requires ingenuity. The validity of any economic process must be checked periodically. There are several powerful nonparametric methods for detecting structural change. In the insurance example, individuals who were misclassified or whose behavior changed can be reclassified and charged the appropriate premiums. Thus the process is never deemed acceptable. The company is always alert to changing circumstances and modifies its portfolio to achieve conditional independence and avoid bankruptcy.

from concrete reality, his respect for the uniqueness of the individual, for history, for institutions, and his overwhelming ambition that his fundamental insights have practical significance. The reader must confirm these assertions for himself, but may find the following quotes useful. We begin with de Finetti's disdain for the frequency approach to probability:

In probability, reference is usually made to the notion of frequency. Why? ...

My answer is a very simple and natural one, in my opinion, although most people would consider it paradoxical: *The persistency of the frequency interpretation of probability hangs on its being the worst possible one.*

It is the worst because there are so many distinct connections between two notions so reciprocally alien in their essence as probability and frequency are, that, since a confusion has occurred, any effort to clarify the situation escaping the unnatural identification risks being ineffective, like in the farces where identical twins are continuously interchanged giving rise to funny and absurd misunderstandings.

Such confusion is so much more difficult to overcome because of the terminological trick, unfortunately so widespread, of calling "events" not the single events but some vaguely defined "species" of which the single events are "trials". ...

In contrast with such objectivistic jargon, and according to the subjectivistic views, every probability one may be interested to appraise (as well as every probability altogether) refers to a single, well specified, event: the shipwreck of the vessel we are considering to insure for its next voyage; the diagnosis and prognosis for this particular patient under such treatment or another; the success of a given candidate in an election -- of a particular student in passing a specified examination -- of a given football team in the next match; and so on. This remark is no way intended to deny the usefulness of considering and confronting any single event of interest in connection and comparison with others, more or less similar under any of the possibly relevant aspects and usually called with a common name (like "a head by coin tossing"); but every relevant circumstance must be realistically pondered case by case.

Frequencies may enter into this framework only incidentally, although in several different ways. Observed frequencies of past outcomes in events more or less similar to the one (or ones) of present interest must be taken into account because they concur to the present assessment through Bayes' theorem, as every other piece of information. (This point becomes significantly illuminated when related to the notion of exchangeability.)¹⁸

¹⁸ For a discussion of exchangeability, see the Appendix.

The similarities between Vico and de Finetti are uncanny. We are concentrating on those ideas that display an almost genetic resemblance. Some might dismiss the comparison because of this selectivity. This would overlook the basic constraint imposed on the selection process. We are comparing the most fundamental feature of their methodological positions, namely, their appraisal of induction vis-a-vis deduction. One could search until two like-minded intellectuals were discovered. This search need not be conducted by the same individual. The others are silent because they failed in their quest. Choice and change are always intermingled in ways so subtle they are impossible to disentangle. The reader must judge the relative importance of selection every day of his life. The following quote permeates de Finetti's methodology: "To reason by induction means nothing other than to learn from experience."

... In connection with induction, the tendency to overestimate reason -- often in an exclusive spirit -- is particularly harmful. Reason, to my mind, is invaluable as a supplement to the other psycho-intuitive faculties, but never a substitute for them. Figuratively, reason is a pole that may keep the plant of intuitive thought from growing crooked, but it is not itself either a plant or a valid substitute for a plant. A consequence of this distortion is the elevation of deductive reasoning to the status of a standard, though actually all non-tautological truths are based on something else. Thus inductive reasoning is generally considered as something on a lower level, warranting caution and suspicion. Worse still, attempts to give it dignity try to change its nature making it seem like something that could almost be included under deductive reasoning.

De Finetti also emphasizes the ability to perceive connections among apparently unrelated entities:

... As has been clearly understood -- in its main outlines -- since Hume's (1748) analysis of the idea of cause, inductive reasoning derives primarily from the association of ideas linked with the impression of "analogy" between certain facts, and, therefore, from the impression of an association of diverse facts (cause and effect in earlier terminology). It is at this point, in order to make precise what can be made precise in such seeming vagueness, that mathematics has to be brought in.

What is the contribution of mathematics? Can it bring clarification to conceptual questions? Can it furnish tools for logical formulations? Can it provide general or particular methods for applications or for certain types of applications?

Let us begin by pointing out that although mathematics is the realm of tautological truths, probabilistic thinking is not only not excluded but even necessary in mathematical work itself. Inductive reasoning is necessary at the creative moment; for no one would try to prove a theorem if he did not attribute a certain likelihood to it. As Levy (1954, Preface) says, one who wants to get to a certain point must first see it with his eyes (intuition) before he reaches it with his feet (logic). Examples illustrating this point are given by Poincaré (1906, 1908) and Polya (1954). But these are merely incidental considerations, useful only to illustrate the range of the field of application of inductive reasoning; what really interests us is the role of mathematics in inductive reasoning, that is, in the theoretical and exact formulation of inductive reasoning.

He also is vehemently opposed to an educational process that stifles the child's curiosity and reduces learning to a series of multiple-choice exams.¹⁹

5. Vico on Order, Processes, Insurance, Contracts, Evolution, Ingenuity, Physics and Induction

Vico recognized clearly that the quest for order and stability in everyday affairs was a genetic thirst first quenched by the Homeric myths, then by Religion and, finally, by Science. These "mysterious" forces were used to explain nature's disorderly conduct, while at the same time, provided the impetus for the evolution of civilized societies.

Economists sometimes ignore the "magical" aspects of their creative models. Friedman (1952), however, seems to be aware of it.

The construction of hypotheses is a creative act of inspiration, intuition, invention; its essence is the vision of something new in familiar material. The process must be discussed in psychological, not logical, categories; studied in autobiographies and

¹⁹ See French (1987) for an insightful discussion of de Finetti's inductive pedagogical position. There is a close tie among the Vico-de Finetti foundations of pedagogy and the seminal research of Piaget. See Chapman (1988).

biographies, not treatises on scientific method; and promoted by maxim and example, not syllogism or theorem.

Feynman's Path "theorem" has defied axiomatization for a generation.

Yet it is one of the most powerful explanations in the natural sciences.²⁰

Vico's "forecast" of the "Feynman phenomenon" is astounding:

... Descartes's method is useful in geometry because geometry is adapted to it, in that the defining of names and the postulation of possible [constructions] is allowed. But when it is taken from the discussion of three measures and of numbers, and is imported into physics, the method is not useful for making new discoveries so much as for setting in order the discoveries we have already made.

... Let us conclude finally that demonstration, and not the geometrical method, ought to be introduced into physics. The greatest geometers saw physical principles in the principles of mathematics. Among the ancients, there were Plato and Pythagoras; among the moderns, Galileo. In this view, we have to explain the particular effects of nature by the special type of experiments which are the distinctive results of geometry. In Italy, this has been the concern of the great Galileo and other outstanding physicists who explained countless natural phenomena of great importance in this way (*ratio*) before the geometrical method was introduced in physics.

... in my essay *On the Method of Studies of Our Time* I argued that it is possible to avoid the pitfalls of physics through the cultivation of *ingenium*. This may be surprising to anyone who is concerned with method. Since method inhibits intuitive wit while aiding facility, it dissolves curiosity while providing for truth. Geometry does not sharpen the wit when it is taught by method only, but when it is employed with creative wit upon diverse complicated, different, and disparate [problems]. Therefore, I wanted it to be taught in the synthetic rather than the analytical way, that is, through demonstration by composition, so that we do not just discover the truth, but make it.

Evolution: The evolution progressed from marriage to family to community to nation. At the same time, language and jurisprudence were

²⁰ A fine technical appraisal of the search for the axioms of the Feynman system is contained in Zambrini (1988). Roughly put, an axiomatic stochastic process with all the properties of the Feynman integral has not been discovered. Just as Dirac's function led to L. Schwartz's profound theory of generalized functions, so too the formalization of the Feynman integral could produce major innovations in mathematics and across all the sciences. See Schwartz (1991).

evolving. Language began with pointed grunts and evolved through a poetic-like singing to a simpler more conventional form. The key to Vico's philosophy is the development of this common language. Rhetoric merged into eloquence and these culminated in jurisprudence.

Contracts: Thus, language gave rise to the legal system which protected individuals from one another, guaranteed property rights, and enforced contracts. Contracts (treaties) with other nations were enforced by the threat of war. Vico considered war as a contractual sanction which reduced its likelihood. Preparedness for war was thus a deterrent to its occurrence. "The origins of war and peace, whereby all commonwealths were brought into being by force of arms and then composed by laws. From the nature of these human institutions derives their external property: that wars are waged so that peoples may live secure in peace."

The symmetry that Newton and Bacon discovered in nature was also present in civilized societies.²¹ While uncertainty was not eliminated, it was reduced substantially. There were costs associated with these homogeneous societies. The "auslander" (outsider) was disdained, innovations were not encouraged, conformity was applauded,²² and the reflective, solitary individual was suspect.

Cooperation and Exchangeability: Gary Becker (1976) has observed that cooperative behavior is found within families, whereas competitive behavior characterizes non-family relationships. As usual he devises an ingenious

²¹Martin Gardner (1980) notes that: "wherever there is symmetry there is a group." The permutation is the group that we are focusing on. That is, exchangeability is closely related to the permutation groups. An excellent discussion of the manifold applications of group theory and graph theory to the social sciences is presented in Hage and Harary (1983). The classic study of symmetry is Weyl (1952).

²²"The human kind is naturally impelled to take delight in uniformity."

economic argument that is compatible with the Hamiltonian-genetic explanation of this phenomenon.²³

Both of these explanations can be augmented by an exchangeability approach which underlies de Finetti's attempts to study social phenomena. The homogeneity that resides within families has been promulgated by nature, common culture, familiarity with one another's idiosyncracies, that is, a smoothing of heterogeneity. This homogeneity is mathematically equivalent to exchangeability and is the major source of cooperation.

But let us stop and listen to Vico:

We thereby establish the fact that man in the bestial state desires only *his own welfare*; having taken wife and begotten children, he desires *his own welfare* along with that of his family; having entered upon civil life, he desires *his own welfare along with that of his city*; when its rule is extended over several peoples, he desires *his own welfare* along with that of the nation; when nations are united by wars treaties of peace, alliances, commerce, he desires *his own welfare* along with that of the entire human race. In all these circumstances man desires principally *his own utility* ... Unable to attain all the utilities he wishes, he is constrained by these institutions to seek those which are his due; and this is called just.

As one moves from the family to the "tribe", many of these homogeneous forces remain and hence a "hardball, tit-for-tat, competitive" environment need not dominate tribal relations. Common moral codes, common language, common legal systems engender homogeneity and cooperative relationships. As the group enlarges, heterogeneous factors tend to outweigh the commonalities and cooperation is less likely. But one need not remain in this anonymous, alien land. One learns quickly how to discriminate and classify individuals, given key background variables. This gives rise to exchangeability and instills the confidence required for cooperative arrangements. In short,

²³The brilliant contributions of W. Hamilton are exposted with wit and wisdom in Dawkins (1976). An incisive and subtle critique of the sociobiological industry, spawned by Hamilton and Wilson, is contained in the last chapter of the monumental study by Eisenberg (1986).

there is a hierarchy of exchangeable environments with the family at the center and unexplored regions at the outer boundary. As the cost of achieving contractual relations increases uniformly, the outer boundary shrinks and the hierarchy begins to collapse onto the family. Vico would interpret this collapse as the decline of the society. In the limit, not even the family is an exchangeable entity -- the initial feral state has recurred.

An excellent example of this phenomenon is provided by the "Current Insurance Crisis.." ²⁴ Priest (1987) correctly dismisses the current popular explanations of this "crisis" ²⁵ and strikes at the heart of the matter: "the current crisis is the judicial compulsion of greater and greater levels of provider third-party insurance for victims." This third-party provision violates the principles of insurance. In particular, insurance companies are virtually incapable of partitioning their markets into homogeneous subsets,, calculating, via the law of large numbers, the competitive premium for the each subset, and then charging premiums according to subset membership. The insurance premium has been designed to cope with moral hazard and adverse selection - it is composed of a fixed fee, a deductible, and a coinsurance rate. Inability to partition means that these contracts cannot be adapted to client characteristics. Potential clients in low risk categories can not be identified by the insurance company and will be overcharged. Self-insurance is this category's rational response. I have always maintained that the Akerlof lemon model was a flawed description of a healthy insurance industry. But when the industry has been wounded by

²⁴The partial title of an article by Priest (1987). I am indebted to A. Alchian for alerting me to this essay.

²⁵They are: (1) Price fixing by insurance companies, (2) Fluctuating interest rates, and (3) the Justice Departments' theory that "attributes the crisis to modern tort law's expansion of corporate liability exposure.."

government intervention, Akerlof applies and the insurance market begins to unravel. This is, of course, only a single albeit significant contractual market. Nevertheless, Vico's language mixed with de Finetti's is apt: the "unnatural" reduction in exchangeability marks the initiation of a collapsing society.²⁶ In this setting, self-insurance is a sign that society is tending toward the uncivilized "feral" state.

Olson (1982) argues that the decline of nations is accompanied ("caused by") the formation of self-interest groups "that have access to selective incentives will be more likely to act collectively to obtain collective goods than those who do not and that smaller groups will have a greater likelihood of engaging in collective action than larger ones." This division of society into "exclusive rent seeking clubs" increases transactions costs, reduces trade, interferes with the efficient operation of the economy, induces conflict, and, in short, undoes the homogeneity process brought about by common culture, common laws, and common language. It remains to be seen how powerful this Balkanization process is relative to the forces of homogenization that are remarkably potent in the United States. The main point here is that a book whose title is close to that of Vico's classic and whose methodology is clearly Viconian is innocent of Vico's thought and the de Finetti probabilistic and inductive procedures that provide the natural setting for testing its conjectures.

By now the reader may have concluded that few non-Italian economists have ever heard of Vico. Schumpeter (1954) is a prominent exception. He regards Vico as "one of the greatest thinkers to be found in any age in the field of the social sciences." He also sagaciously observes that "part of

²⁶ Inflation is the prototype of this collapsing phenomenon. See Heymann and Leijonhufvud (forthcoming).

the extensive Vico literature is impaired by attempts of authors to claim the great name for tendencies of their own..." Finally, he claims that "Vico's achievement is best described by the phrase 'An evolutionary *science of mind and society*'" [italics added].²⁷

Processes: Perhaps Vico's most important contribution was abhorrence of static systems. Of course, this Heraclitian realism was not novel in itself. The novelty manifested itself in two ways. First, societies are processes and are subject to cyclical decline and renewal. "Men first feel necessity, then look for utility, next attend to comfort, still later amuse themselves with pleasure, thence grow dissolute in luxury, and finally go mad and waste their substance." More succinctly, "the nature of peoples is first crude, then severe, then benign, then delicate, finally dissolute." Thus no civilization can persist without adapting to these "natural phenomena". So too, no mathematical model of nature is invariant to the flux of concrete phenomena. Unless a science adapted to these fluctuations, it would soon be obsolete. Just as a static society was doomed, so too was any mathematical model that remained remote from the concrete and, indeed, was not permeated with the concrete.

This coincides with the probabilistic causality promulgated by Peirce and articulated by Suppes (1985) in his recent display of scholarship and insight. Suppes uses exchangeability to demonstrate several important theorems about "inferences from phenomenological correlations to common causes". While Suppes applies them to the hidden variable controversy in quantum mechanics, they also are pertinent to the burgeoning economic

²⁷I am indebted to A. Leijonhufvud for showing me Schumpeter's appraisal of Vico. Unfortunately, Schumpeter demeans Adam Smith thereby flaunting his own fallibility. Selectivity strikes again.

literature on mechanisms.²⁸

Many economists have interpreted the myriad asymmetries in economic processes as proclamations of market failure with some sort of mending required by non-market interventions. Closer scrutiny reveals that many of these asymmetries have been symmetrized by natural forces.²⁹ These forces have produced a variety of institutions that are sometimes quite subtle and difficult to observe. Nevertheless, they may be sufficiently strong and durable to eliminate the asymmetric gaps. This, of course, does not mean that all is well and no interventions are justified. However, asymmetries should only be mended after a diligent search for symmetrizing mechanisms comes up empty. The absence of these natural institutions suggests that their implementation is difficult and perhaps more costly than doing nothing. A journal article proclaiming the absence of a market mechanism means that many businessmen, whose livelihood depended on obtaining a solution, probably have tried and failed.

Insurance: Another uncertainty reducing institution that Vico advocates is insurance.

In the very birth of [domestic] economy, they fulfilled it in its best idea, which is that the fathers by labor and industry should leave a patrimony to their sons, so that they may have an easy and comfortable and secure subsistence, even if foreign commerce should fail, or even all the fruits of civil life, or even the cities themselves, so that in such last emergencies the families at least may be preserved, from which there is hope that the nations may rise again.

This description shows Vico's concern for renewal when societies collapse.

²⁸ There are several fine surveys of this research. For example, see Arrow (1974) and Levinthal (1984). The article by Saari (1987) is very important.

²⁹ Just as advances in medical science have accompanied the study of the sick, so too many insights have been gleaned from economists' study of asymmetric behavior. This rewarding research activity should be encouraged.

A stable society is much less vulnerable to disintegration than is a fluctuating society.³⁰ There is a balance between innovation and tradition that resides in all surviving institutions. This natural range of flexibility permits the institution or organism to adapt to changes that would extinguish a brittle entity, while at the same time controls endogenous change by preserving the "wisdom of the past." Thus, internal heterogeneity never becomes self-destructive. Observe that a wildly fluctuating heterogeneity increases the costs of partitioning groups into homogeneous subsets. For example, it becomes more and more difficult for firms to rank their workforce, for contracts to be formed and sustained, and for the associated institutions to survive.

Vico's primitive society is characterized by three principles:

And in no nation, however savage and crude, are any human actions performed with more elaborate ceremonies and more sacred solemnity than the rites of religion, marriage, and burial. For, by the axiom that "uniform ideas, born among peoples unknown to each other, must have a common ground of truth", it must have been dictated to all nations that from these three institutions humanity began among them all, and therefore they must be most devoutly guarded by them all, so that the world should not again become a bestial wilderness. For this reason we have taken these three external and universal customs as three first principles of this Science.

Many anthropologists discount Vico because he denies diffusion -- each civilized society began in isolation -- and he ignores the polyandry and other aspects of primitive life that are now well-known. Vico might claim that these primitive "societies" were still in the feral state or, more likely, he would adapt his theory to these concrete findings. On the other

³⁰The tradeoff between stress and durability has received much attention in the ecological literature. In a recent study, Carnevali and Patarnello (1989) apply neural networks to this problem. Their findings are compatible with the earlier work by Minnis (1985) et al. Organisms raised in low-stress environments are less likely to survive severe changes than those raised in a high-stress milieu. For an extensive discussion, see Lippman and McCall (1985).

hand, Martin and May (1982) conclude: "Much work on birds, mammals and other creatures indicates that monogamy often emerges when large investments in offspring necessitate cooperation between the parents. It is possible that the increasing dependence of the human infant, associated with progressively increasing brain size and cultural complexity, has favored culturally determined monogamous tendencies in various human societies, even though these have not led to the complete suppression of biological indicators of a polygynous society." There is also much truth in Vico's burial principle. C.J.S. Scarre (1984) observed that the genetic composition of bones found in ancient graves show that families were buried in the same grave site.

Ingenuity: Vico's notion of ingenuity is "the faculty of bringing together things that are disparate and widely separated". Mooney notes that "ingenuity" is used consistently throughout his work and "is a lodestone of his thought, drawing to itself so many of his original ideas...."

One does not need to call on ingenuity; one either has it or does not, sees connections or misses them utterly. Vico was a child of acute ingenuity, he claimed, and so, too, are children generally, if only we will recognize it and train them accordingly.

Vico's passionate concern for education anticipates much of Dewey's philosophy as does his attitude toward uncertainty. His emphasis on the concrete and the richness of experience that must replace the abstract mathematics of the Cartesian curriculum reminds us of Mach.

the object of science is the connection of phenomena; but the theories are like dry leaves which fall away when they have ceased to be the lungs of the tree of science. E. Mach (1872)

Consider the following quotations:

In the main, if a phenomenon is important for our welfare, it interests and excites us..Dangerous things fill us with involuntary fear; poisonous things with distaste; indispensable things with appetite. Mind and world in short have been evolved together, and in consequence are something of a mutual fit.

However man may have acquired his faculty for divining the ways of nature, it has certainly not been by a self-controlled and critical logic... It resembles instinct too in its small liability to error, for though it goes wrong oftener than right, yet the relative frequency with which it is right is on the whole the most wonderful thing in our constitution.

They certainly could have been said by Vico, but the first was by William James in 1890 whereas C.S. Peirce authored the second in 1903. Both quotes appear in a basic paper by J.A. Anderson (1983) on neural networks. Peirce finally is receiving the attention he deserves. This is evident in the monograph by Holland et.al. (1986).

Poetry and Number Theory³¹

The influence of number theory on modern physics, and vice versa, exemplifies the threshold phenomenon like the sudden appearance of a large firm in a competitive industry.³²

The choice of the word "quark" from James Joyce's Finnegan's Wake by Murray Gell-Mann was a harbinger of more profound connections. The inability to axiomatize the Feynman integral has been discussed. Kenner (1984) observes the ubiquity of the number 11 in Ulysses.

A peculiarity present from the beginning is the book's awareness of numbers: how many people in a room, shillings in a ledger, even words in a sentence. A recurrent number is 11; the first sentence has 2×11 words, the third one (from "He" to "dei") has 11: as we become skilled navigators we recognize 11 as a Ulyssean seamark. For in this book 11, the fresh start after a decade, is the number for the two primary kinds of events, beginnings and endings; and while sometimes it is specified, it sometimes lurks behind a count of episodes or paragraphs or words. A sampling of elevens: Rudy Bloom died 11 years ago, aged 11 days; Stephen's age is 2×11 ; "Marion Bloom" has 11 letters and so has "Hugh E. Boylan," and the hour of their tryst is set in the 11th episode; 11 paragraphs of entry and 11 of exit precede and follow the 40 paragraphs of gestation in "Oxen of the Sun"; as

³¹This section is based on joint work with K. Velupillai.

³²The economic applications of graphs, their spectral representation, and the significance of the threshold phenomenon are presented in McCall (forthcoming).

late as 1919, when he conceived "Wandering Rocks," Joyce had meant the center of the book to consist of 11 episodes.

We conjecture that the rhythm, harmony, pattern, and recurrence used by the creative poet in his inductive art have the same number theoretic foundations as the birth and death process. We also conjecture that the Feynman integral is closely related to the birth and death process. This process is formally connected to the LaPlace operator and exchangeability. The invariant structure of a particular process is determined by its spectral representation.³³

These birth and death processes that may unlock the mystery of the Feynman integral are also crucial to the process approach to economics. They generate the Jackson networks used in queuing, and communication engineering and the Ramanujan graphs that have great promise in computer science and neuroscience. Networks have great potential in vitalizing the economics of information which is central to both the intra and interdisciplinary unification of economic science.

The sensuous aspects of language enhance its ability to convey information and to perform induction, that is, extract symmetry, pattern, and meaning from the vast variety of waves emanating from concrete phenomena. Vico was the first to see poetry as the primal source of human intelligence. The evolution of language fortified induction, not deduction. Induction is the basis of both Bacon's physical science and Vico's "NEW" science. In his attack on the rigid Cartesian formalism, Vico overlooked the practical beauty inherent in mathematics. Just as poetry is the language of Vico's New Science, so too is mathematics the language of Bacon's Physical Science. The "no arbitrage principle" is common to both of

³³ See McCall (forthcoming), Sarnak (1991), and, of course, Kac (1966).

these languages. Exchangeability exhibits the practical and formal qualities of this principle. Hence the transmission and interpretation of information -- induction -- began with grunting and counting. These basic activities evolved into poetry and mathematics. Both processes are refinements of their original functions -- learning and trading.

While poetry and mathematics are similar they are not identical. Creativity is common to both. The pathos and ethos characteristic of poetry is quite different from the mechanical and perfunctory aspects of mathematics. The human spirit can not be captured by a mathematical process.

Induction

Berlin considers Vico's reconstruction of the past one of his most important contributions. He refers to Vico as the "true father both of the modern concept of culture and ... cultural pluralism, according to which each authentic culture has its own unique vision, its own scale of values ...". Furthermore, these values and visions change -- culture is a process. With hard work and imagination, individuals living in period $t_0 + t$ can infer the culture of their forefathers at t_0 . Berlin sees this backward induction as similar to the studies of modern anthropologists, who search for meaning in the unfamiliar and, frequently, grotesque myths, rituals, visions and "natural responses of extant primitives". This spatial induction is quite difficult. Decoding observed primitive behavior requires insight, tenacity and luck. The attempt by Vico to reconstruct the behavior of his early ancestors and explain the myths and other features of their reconstructed behavior was truly breathtaking in its novelty and precariousness. It shattered the static concept of human nature. Yet Vico thought the poetry of these early barbarians possessed a simplicity and linguistic purity superior to that of Vico's contemporaries. How could Vico contempl-

ate a process when the 17th century notion of time was completely wrong?³⁴

In the midst of the Inquisition, how was Vico able to evade persecution?

Rossi claims that:

The tortuous, difficult path that had led humanity out of barbarism to civil order was, for Vico, decisive proof of the presence in history of a Providence that "disposes to a universal end" the particular ends of human actions and that uses those narrow ends, beyond the conscious purposes of individuals and of peoples "to preserve the human race upon this earth." To acknowledge that brutish state of barbarism and to assert a generalized dehumanizing of men seemed to Finetti, on the contrary, an impious denial of the work of Providence:

What could be harder to accept, more difficult to harmonize with the idea that we have of Providence, than to imagine that the Lord, after having made the supreme gesture of preserving mankind from total destruction through the Ark, should then abandon it so completely that men reached the point where they became like beasts, all religion lost, the use of reason lost, all language lost, all companionship lost, in few words, humanity lost, falling into a bestial state, rather, worse than that of the beasts themselves?

Vico's preference for induction is explicit in his discussion of medicine.

In medicine it would be safer to adhere to concrete particulars, and to refrain from assigning to the sorites a role more important than that which, in this field, it deserves to play; it would be safest to lean firmly on induction.

and

... we should pay a great deal of attention to symptoms and diagnostics. Let us cultivate the practices of preventive medicine as applied by the Ancients, chiefly gymnastics and dietetics, along with the curative procedures devised by us.

Vico reiterates his belief in the uniqueness of the individual the difficulty of learning in a fluctuating heterogeneous environment. No two sick people are ever the same and the diseases are always novel.

³⁴An excellent discussion of these points is contained in the scholarly monograph by Rossi (1984). The role of the "other" Finetti receives the delicate and incisive attention it deserves. Only time revealed the irony. See footnote 5 on page 3.

Those using the syllogistic method in this milieu almost never come close to the real source. They "are led astray by their fondness for that strictly deductive form of reason which the Greeks called sorites. The person who uses the syllogism brings no new element, since the conclusion is already implied in the initial proposition or assumption." The most we can hope for in our assiduous application of inference to a longitudinal data base is a "probable approximation to truth."

On Probability

Vico's notion of probability or "luck" is seminal. It differs substantially from earlier concepts.³⁵

... A private evil may be a public good; and just as in the republic founded by men the safety of a nation is the supreme law, so in the universe established by God luck would be the queen of all. To put it another way, luck is God's will by which He dominates over the private goods of all men and their peculiar natures, while He looks to the safety of the universe. And just as private safety yields to public safety, so the good peculiar to each person must take second place to the conservation of the universe, and in this way the adversities of nature are its goods.

This is very close to the idea of fairness presented in McCall (1987) and Parthasarathy (1988).

The environment where prudence is essential resembles the landscape of economic decisionmaking. Indeed, it is apparent that Vico's prudential behavior is almost identical to Friedman's economic decisionmaking.³⁶

... When it comes to the matter of prudential behavior in life, it is well for us to keep in mind that human events are dominated by Chance and Choice, which are extremely subject to change and which are strongly influenced by simulation and dissimulation (both pre-eminently deceptive things). As a consequence, those whose only concern is abstract truth experience great difficulty in achieving their means, and greater difficulty in attaining their ends. Frustrated in their own plans, deceived by the plans of others, they often throw up the game. Since, then, the course of action

³⁵Palmer has a nice discussion in Vico (1988).

³⁶See Friedman (1953).

in life must consider the importance of the single events and their circumstances, it may happen that many of these circumstances are extraneous and trivial, some of them bad, some even contrary to one's goal. It is therefore impossible to assess human affairs by the inflexible standard of abstract right; we must rather gauge them by the pliant Lesbian rule, which does not conform bodies to itself, but adjusts itself to their contours.

On Myth, Language and Art

According to Ernst Cassirer (1955):

... modern philosophy has in this point turned more and more to man's subjectivity. Myth became a problem of philosophy insofar as it expresses an original direction of the human spirit, an independent configuration of man's consciousness. Anyone aiming at a comprehensive system of human culture has, of necessity, turned back to myth. In this sense, Giambattista Vico, founder of the modern philosophy of language, also founded a completely new philosophy of mythology. For Vico the true unity of human culture is represented in the triad of language, art, and myth.

6. Methods of Our Time

The economic relevance of Vico's combinatoric vision of science reaches its zenith in On the Study Methods of Our Time (1990). The power of his constructive approach to wisdom is hazardous to both friend and foe. What one previously considered as novel and seminal in his own constructive methods must be recognized as at least three hundred years old. However, there is exhilaration in this experience of humility. If one's method of achieving wisdom relies on an axiomatic, deductive approach, then Vico must be subverted or dismissed in order to avoid humiliation. This attack on the abstract is too withering to be true. There certainly is a deductive niche in the creative process. But never again can the serious reader of Vico identify creativity with abstract deductions.

Vico's aesthetics is drenched by the practical. A constitution, an invention, a language, a legal defense is beautiful insofar as it reflects

the natural flux, the heterogeneous, and the vulnerable³⁷ in an economical manner.

The idiosyncratic response of Croce verifies Vico's assertion that each individual is unique.

... The fact is that his philosophy of history, his *Scienza nuova d'intorno alla communa natura delle nazioni*, does not concern the concrete empirical history which unfolds itself in time: it is not history, it is a science of the ideal, a Philosophy of the Spirit.

and

... In conclusion, we remark that fundamental terms are not always used by Vico in the same sense: it is not always clear how far "sensation," "memory," "imagination," "wit" are synonymous or different. Sometimes "sensation" seems outside the spirit, at others one of its chief moments; poets are sometimes the organ of "imagination," sometimes the "sensation" of humanity; and imagination is described as "dilated memory." These are the aberrations of a thought so virgin and original that it was not easy to regulate.

As we show³⁸ "dilated memory" is a measure of creativity compatible with a constructive approach to knowledge.

³⁷ ... it is an error to apply to the prudent conduct of life the abstract criterion of reasoning that obtains in the domain of science. A correct judgment deems that men -- who are, for the most part, but fools -- are ruled, not by forethought, but by whim or chance. The doctrinaires judge human actions as they ought to be, not as they actually are (i.e., performed more or less at random). Satisfied with abstract truth alone, and not being gifted with common sense, unused to following probability, those doctrinaires do not bother to find out whether their opinion is held by the generality and whether the things that are truths to them are also such to other people.

³⁸ See McCall and Velupillai (1991a), where the individual's creativity is measured by a Ramanujan graph. As more ideas (nodes) enter memory and are connected by the imagination with the current inventory, creativity is enhanced. This process continues as long as the individual resides in a stable environment and can specialize in that combination of "jobs" most agreeable with his tastes and talents. A deep analysis of Ramanujan graphs is contained in Sarnak (1990).

It is not only consistent with the most recent advances in neuroscience and number theory, but also satisfies the practical (economical) criteria of mechanical design.

In his perceptive monograph on design, French (1988) uses similar criteria for measuring creativity:

Perhaps the most important evidence about the nature of invention has come from mathematicians and physicists, and it supports two principal ideas. The first is that the essential process is one of choice, and that the choice is made on principles which are best described as aesthetic. The mathematician Poincare wrote of aesthetic sensibility as "the delicate sieve" with which the most useful combinations of ideas are separated from all the others which might be studied. (There is an interesting parallel here with the crude sieve, natural selection, with which nature separates the most useful combinations of genes from all the others which sexual reproduction generates.

The second is that the process of invention usually falls into stages, of which the first three may be described as preparation, incubation, and illumination.

The inventive man is marked out by the range of elements he considers combining, and the skill and judgement with which he selects or converges upon his preferred combination.

Vico agrees with French's assessment of the invention of the printing press,³⁹ but adds a caveat:

... when books were written by hand, only works composed by authors of tested and well-established reputation were reproduced, since they were the only ones worth the labor spent on the task of copying them; and since the price of such works was, at times, very high, students were obliged to copy them in their own handwriting. Now, there is no better exercise than this; we meditate on the text and write without haste or interruption, calmly and with continuous order. By copying, we gain, not a perfunctory knowledge, but an intimate familiarity with the original, and we are, so to speak, transformed into the author's very self. That is why second-rate authors palled in the copying; but superior ones were held in great esteem and celebrated, to the great profit of the general public.

In his insightful monograph on the importance of being "fit", Calder (1984) notes that nature's inventions must satisfy Stahl's (1962)

³⁹See French (1988) pp. 249ff.

conservation principles: (1) volume must be preserved and (2) time must be synchronized.

The Ramanujan graph is a modular form and satisfies (1).⁴⁰ The timing constraint is a sequential decision rule and is satisfied when the process is "fair". Vico is cognizant of both of these constraints on inventiveness: "We need to train young minds for the practice of mechanics by means of a close study of visual geometrical figures ..." and

... But the sage who, through all the obliquities and uncertainties of human actions and events, keeps his eye steadily focused on eternal truth, manages to follow a roundabout way whenever he cannot travel in a straight line, and makes decisions, in the field of action, which, in the course of time, prove to be as profitable as the nature of things permits.

Language is a pillar of creativity. Sensuous languages increase the mind's inductive prowess. These languages prepare the mind for its contact with reality via sight, hearing, taste and touch. The thinking, writing, and conversation that precede and coincide with sensual contact are less likely to overlook crucial, but subtle, connections when they are guided by a sensuous language. The reconstruction process (induction) will "see" the invariants associated with recurring patterns when it is conducted with a sensuous instrument -- language. This process is unlikely to be corrupted by noise when its instruments possess an adaptive symmetry akin to that inhering in nature's recurring patterns. Finally, individuals who are able to extract nature's signals from the apparent turbulence of reality achieve satisfactory rewards. Hence language, aesthetics, and utility conspire to decode nature's mysteries.⁴¹ Vico says it best.

⁴⁰This conservation principle was emphasized by Mach. We call it the economy of nature. See McCall and Velupillai (1991a).

⁴¹This is a glimpse of the process whereby nature's recurring patterns are imbedded into the form and structure of survivors. It is well-known that some seeds will not germinate unless they have passed through the dig-

... We Italians, instead, are endowed with a language which constantly evokes images. We stand far above other nations by our achievements in the fields of painting, sculpture, architecture, and music. Our language, thanks to its perpetual dynamism, forces the attention of the listeners by means of metaphorical expressions, and prompts it to move back and forth between ideas which are far apart.

In his jingoistic passage (the interested reader should read the entire section in Vico (1990) to appreciate the label jingoism),⁴² Vico spontaneously confirms his theory! It is too instinctive to be a contrivance. After reading this ode to the Italian language one is tempted to insert Q.E.D.....E. Mach is a brilliant and persuasive advocate of the sensuous foundations of knowledge. See Mach (1906).⁴³

7. Vico and Joyce

The influence of Vico on James Joyce is substantial.⁴⁴ Ulysses is Homeric and contains many of Vico's ideas. The renewal theme is evident in Finnegan's Wake whose last sentence is completed by part of a sentence contained in the first paragraph: "A way a lone a last a loved a long the reverrun, past Eve and Adam's, from a swerve of shore to bend of bay, brings us by a commodious viscus of recirculation back to Howth Castle and Environs." Viscus is the Latin form of Vico meaning street. These displays of Vico's philosophy are well-known and quite superficial, cocktail talk at best.

estive tract of predators. Other seeds require the intense heat of a "forest fire" to trigger their reproductive mechanism. Hence, induction or learning is nature's sculpting process. The selected statues are symmetric. They "fit" in their habitat. In short, they are economical.

⁴²In his cogent essay, E. Gianturco disputes the "jingoistic" interpretation. The essay also contains many other perceptive remarks about Vico's contributions. See Vico (1990).

⁴³While Mach overlooks Vico, he does identify two extraordinary Italians, Cavallieri and Saccheri. See McCall and Velupillai (1991a). Turnbull (1951) has a deep appreciation of Cavallieri's contributions.

⁴⁴For a glimpse see the essays in Verene (1987).

The common thread running through Vico, Joyce and de Finetti is their profound interest in communication, language, and the encoding, transmission, and decoding of information. In short, INDUCTION is their primary concern -- how humans learn nature's secrets and the various devices employed for conveying this information among themselves. In her splendid essay, Mary Reynolds⁴⁵ observed that Joyce may have found Vico's notion of cycles amusing, but certainly not original. The attempt to reconstruct the history of a nation from its language was the fundamental attraction. This ambitious inductive process was emulated by Joyce in his literary effort to convey the essence of the Irish in his writings. After reading Joyce, a visit to Ireland would convey little, if any, additional information. This method of showing readers the soul of Ireland reached its zenith in Ulysses. Joyce claimed that if Dublin were destroyed it could be rebuilt by those who read Ulysses. "He meant this, obviously, not only for the streets and public structures but also -- more fundamentally and centrally -- the customs, manners, beliefs, attitudes, frailties and strengths of the people who lived in his native city, Dublin."

The mythic element in Joyce's novels revealed the ancient history of the Dubliners. Both Vico and Joyce were influenced by Dante's Divine Comedy. Vico "saw" how Dante perceived language as the instrument for inferring the historical process.

Reynolds claims that Vico was a poet living in a society that valued the "cold science of mathematical logic." But Vico realized that the creative spark that warmed and illuminated the mathematician was identical to poetic inspiration. The source of both was "to see" a novel connection, and then convey it with the enthusiasm of the child observing the magic of a

⁴⁵ See Verene (1987).

hummingbird. Of course, these modes of communication will, for the most part, only reach those who have retained their childlike wonder as they reflect on the constantly changing landscape.

8. Vico's Impact on the Social Sciences

Marx discovered Vico and transformed him into an historical materialist. Trotsky quotes Vico on the first page of the History of the Russian Revolution. E. Wilson's To the Finland Station begins by referring to Michelet's discovery of Vico. Michelet translated an abridged version of Vico into French, where it achieved great popularity. Weber translated Vico into German in 1822, but without Michelet's success.

Given this background, it is not surprising that most American economists are unfamiliar with Vico's work. For example, in his influential article on Rhetoric, McCloskey (1983) does not reference Vico. The sparkling combination of Vico, de Finetti and modern law and economics in Posner (1981) is unaware of its debt to two of Italy's greatest philosophers and mathematicians. In the superb Viconian essays by Hayek (1967), there is no mention of Vico. On the other hand, in 1878 F.B. Sanborn, one of the founders of the American Social Science Association referred to Vico, along with Smith and Comte, as comprising the intellectual foundation of the Association.

In his Opus Magnus, Mayr (1982) does mention Vico's devastating attack on Descartes and observes that "For Vico, the various periods of human history were not different aspects of essentially the same story; rather they were successive stages of a continuing process, a process of necessary evolution". Toulmin (1982) also notes that: "In biology as in the human sciences structural perspectives -- in the spirit of Descartes and Newton -- can at most complement, never displace, the temporal perspectives of Vico or Darwin."

9. Random Reflections

There are of course flaws in Vico's philosophy.⁴⁶ His creative and spontaneous writing is truly extraordinary but does lend itself to misinterpretation relative to a more disciplined, less exciting prose. Nevertheless, many of his positions are crystal clear. The attack on Descartes' abstract analysis and his pedagogical thesis that the concrete should illuminate analysis retain their vitality. However, the durability of a society depends on a subtle mixture of uniformity and diversity. It would seem that a common language and a common legal system are essential. The individuality that is the hallmark of de Finetti's philosophy is conspicuous in Vico, but not really embraced. The formal equivalence between certainty and uncertainty is not emphasized by either of these great men.

Vico would probably not agree that abstraction in one discipline when sufficiently deep encounters the concrete base unifying the arts, social sciences and physical sciences.⁴⁷

Beyond this there lies a different role and a different impact less obvious and more difficult to capture. It has much to do with borrowing, filtering, reinterpreting, processing - even mutilating - ideas and concepts and then ejecting them back into an ever swelling stream. It has much to do with allowing under one roof the extremes of the concrete and of the excesses of the abstract.

M. Kac, Probability Theory: Its Role and Its Impact, 1962

⁴⁶This portrait of Vico ignores the contributions of the "analytic mind" to society's pool of wisdom. Any balanced appraisal of epistemology must weight both synthetic and analytic contributions. Vico's lack of balance is necessary to capture the attention of a society dominated by Descartes. Similarly, our position with respect to economics emphasizes synthesis because of the reign of the deductive. McMullin (1979) gives a balanced and lucid account of historicism (induction) and analysis (deduction).

⁴⁷An attempt at unifying these diverse disciplines was begun in McCall (1985).

His position does seem compatible with the view promulgated by Peirce that the symmetries present in nature are reflected in the composition of the evolving minds of all animals.

Recent neurological research has confirmed Peirce's hypotheses, while augmenting it in a fundamental way. The symmetry persists up to a threshold, at which point the neural behavior becomes erratic.

This sort of symmetry-breaking is analogous to the maintenance of a contractual arrangement. As long as the behavior of both parties remains within a critical region, the contract is symmetrically followed. If a boundary is broken, the contract disintegrates. Replace contract by treaty, interpret boundary literally; and replace broken by invasion. The result is the transition from the symmetry of peace to the chaos of war.

10. Conclusion

Certainly there has been much written about Vico in the history of economic thought. The lack of citation reflects my ignorance not oversight.

The main point of this essay is to show that Vico anticipated many of the modern developments in economics and the other social sciences. Exchangeability clarifies these remarkable insights. When one combines Vico's theory of renewal with his network analysis, a whole new circle of ideas is revealed. They begin with Peirce's combinatorial analysis, to Markov's stochastic networks, and then to the seminal work of Fisher and Slutsky on business cycles, of Hebb (1949) and Hopfield and Tank (1986) on neural networks, of MacArthur on ecology, of Lorentz on animal behavior, and to the fundamental insights of Leibniz, Turing, and artificial intelligence. The key to these relations is some form of symmetry like exchangeability with birth and death Markov processes and no-arbitrage conditions being major actors. These relationships will be the subject of a subsequent essay.

For now note that the structure of networks is characterized by their spectral properties. The sensual roots of the inductive process is nicely conveyed by spectral analysis. In his classic article, Kac (1966) captured this delicate and intricate intertwining of the abstract and the concrete. The senses are able to learn novel information from the process emitting signals, merge this information with the accumulated pool (memory) and make wiser decisions. In particular, one can detect structural change and mend the methodology accordingly. When properly posed, the validity of the economic process is always questioned. For example, a pronounced environmental shock may induce a phase transition, that is, a radical change in behavior. The analytical power of this inductive process resides in group theory and its symmetric patterns, recurrences, and modularity. The synthetic strength is in the sensual contacts with concrete phenomena.⁴⁸

The combined wisdom of Vico and de Finetti has led us to this rich inductive process. The "no arbitrage principle" is at the center of this interdisciplinary process. Thus, the practical (trade) the beautiful (poetry), and the abstract (mathematics) are united in the Economy of Nature.⁴⁹

We conclude with a quote from Mooney (1985):

...the fundamental novelty of all in his work [is that] mind does not precede language, but arises with it, and both in turn are the necessary outcome of social urgency, the result of a spontaneous attempt, gradually made conscious, to grasp a startling experience through images that are familiar. At first mere gestures, or actions taken in common, the ordering images become in time articulate and complex, even as the institutions from which they are initially indistinguishable grow large and diverse. This elaborate universe of meaning, always restless and changing, remains intact so long as those who inhabit it continue to have a common sense of things; indeed whole areas of experience may in the process achieve the status of refined, quasi-independent arts and sciences. But let the founding sense of things be overturned

⁴⁸For a beautiful extension of Kac's insights, see Berard (1986).

⁴⁹See McCall and Velupillai (1991).

by events, or let the struggle for a public consensus give way to a tyranny of private opinions, and the structure will weaken and totter, and possible collapse....Often crude in its statement and underdeveloped in its parts, Vico's work was nonetheless right in its essential assumptions, a monument to his age if not a legend in its time. That language has primacy in human life; that poetry is prior to prose, and image to concept; that society takes form as a growth of human senses; that human actions and arrangements are the first statements of ideas, and that mind and society, with language as a means, share a common history -- these and other lead ideas of Vico's science, startling in his own day, have lost none of their luster in ours.

Appendix⁵⁰

Exchangeability⁵¹ is a powerful and elegant concept that permeates and unifies economic processes. It is a major conceptual link between de Finetti and Vico, and we refer to it repeatedly. Hence, this brief survey may be helpful.

Consider the following experiment. A coin is flipped 5 times. If a head (tail) occurs a 1(0) is recorded. What probability shall we assign to the $2^5 = 32$ possible outcomes? De Finetti answers this question by assuming that sequences of length 5 having the same number of 1's are equally likely. Hence, the sequence 11000 has the same probability as the sequence 00101. The precise position of the ones is irrelevant; only their sum enters the probability calculation. This type of symmetry was christened "exchangeability" by de Finetti.

⁵⁰ A more complete elementary treatment of exchangeability and its relevance for economics is presented in McCall (1991). The definitive survey is the elegant paper by Aldous (1985).

⁵¹ Exchangeable, symmetric dependence and interchangeable are synonyms. There is much to be said for "interchangeable". It emphasizes the homogeneous aspects of "exchangeability." One of its most important applications in economics and elsewhere is decomposing heterogeneous populations into homogeneous subpopulations. However, "exchangeable" is used most frequently. For example, if an insurance company were unable to partition its customers into "homogeneous" subsets by applying exchangeability, it would be ruined.

Definition:⁵² A sequence of random variables is said to be exchangeable if for each n the distribution of (X_1, X_2, \dots, X_n) is invariant to permutations of the n subscripts, that is, (X_1, X_2, \dots, X_n) and $(X_{\pi(1)}, X_{\pi(2)}, \dots, X_{\pi(n)})$ have the same joint distribution for all permutations π .

Properties a) Identically Distributed.

Suppose $n = 2$. Then exchangeability means that

$$F_{X,Y}(x,y) = F_{Y,X}(x,y)$$

for all x and $y \in \mathbb{R}$. Letting $y \rightarrow \infty$ gives

$$F_X(x) = \lim_{y \rightarrow \infty} F_{X,Y}(x,y) = \lim_{y \rightarrow \infty} F_{Y,X}(x,y) = F_Y(x).$$

Therefore, exchangeable random variables are identically distributed.

b) Stationarity

Furthermore, it can be shown that for any j , $1 \leq j \leq n$, the distribution of the subset $(X_{i_1}, \dots, X_{i_j})$ of the set of exchangeable r.v.'s (X_1, \dots, X_n) is given by $P(X_{i_1} \in B_1, \dots, X_{i_j} \in B_j) = P(X_1 \in B_1, \dots, X_j \in B_j)$ for all $B_1, \dots, B_j \in \mathcal{B}(\mathbb{R})$, the Borel subsets of the \mathcal{L} -algebra generated by the open subsets of \mathbb{R} .

⁵²There are many ways of defining exchangeability. This is probably the standard textbook version. We will present other definitions and other variations of exchangeability when we examine de Finetti's theorem in I.2. Of course, every Bayesian knows that a Bayesian statistical analysis entailing repetitions of a fixed experiment (like sampling from an urn or tossing a coin) generates sequences of exchangeable random variables. De Finetti's theorem is a general converse to this well-known result. That is, an infinite sequence of exchangeable r.v.'s is distributed as if the probability function F were chosen at random, and then iid rv's were drawn from F . Briefly stated, every exchangeable sequence is a mixture of coin tossing experiments.

c) Correlation of Exchangeable Random Variables

Suppose (X_1, \dots, X_n) is an n -exchangeable sequence. The correlation coefficient $\rho(X_i, X_j)$, $i \neq j$ is given by:

$$(1) \quad \rho \geq \frac{-1}{n-1}$$

with equality when sampling without replacement from an urn containing n balls, i.e., $\sum_{i=1}^n X_i = \text{constant}$.

As $n \rightarrow \infty$, the RHS of (1) approaches 0. Thus in the limit

$$\rho \geq 0.$$

Aldous (1985) also shows that for an infinite exchangeable sequence ρ satisfies

$$0 \leq \rho < 1.$$

Theorem 1: de Finetti Theorem

Let $P_n(r)$ be the probability of a sequence of r heads and $n-r$ tails, where the order is irrelevant, by exchangeability. Then

$$(2) \quad P_n(r) = \int_0^1 \theta^r (1-\theta)^{n-r} P(\theta) d\theta$$

for some "prior" probability, $P(\theta) \geq 0$ and $\int_0^1 P(\theta) d\theta = 1$.

Note that exchangeable sequences are mixtures of Bernoulli sequences where the mixture is by a distribution over θ , the probability of a head. Recall that probability assessments are altered as new information unfolds. Let A be the event of interest and suppose our initial probability assignment is $P(A)$. An event C occurs and we wish to modify $P(A)$ to incorporate this information. This is accomplished by a mechanism that is fundamental to probability theory -- conditional probability. It is

represented as $P(A|C)$ and defined by:

$$P(A|C) = \frac{P(AC)}{P(C)}, \quad P(C) > 0$$

The conditional probability $P(\cdot|C)$ is a revision of the original or prior probability measure $P(A)$ and contains all the information in C that affects the occurrence of A . If this information is null, we say that A and C are independent events. This is represented by: $P(A|C) = P(A)$. Hence, $P(AC) = P(A)P(C)$ iff A and C are independent events.

Independence is the cornerstone of probability theory. Indeed, some have claimed that its absence would render probability theory indistinguishable from measure theory. Any method that "creates" independence enhances the significance of probability theory. It is remarkable that independence can be generated by using the methods of conditional probability.

Definition: Let A , B and C denote three events. The two events A and B are said to be conditionally independent given C iff

$$P(AB|C) = P(A|C)P(B|C).$$

Note that conditional independence is independence with respect to the conditional measure $P(\cdot|C)$.

The relation between conditional independence, identical distributions and exchangeability is contained in the following important theorem:

Theorem 2: Chow and Teicher

The random variables $(X_n, n \geq 1)$ are exchangeable iff they are conditionally independent and identically distributed given the appropriate sigma algebra ("information") of events.

Example 1: In econometrics many estimators have the form:

$$(3) \quad S_n = \sum_{i=1}^n a_{ni} X_{ni}$$

where (a_{ni}) and (X_{ni}) are sequences of constants and random variables, respectively. It is important to show that these estimators are weakly or strongly consistent, that is, converge to the appropriate parameter (weakly) in probability or (strongly) with probability one.

The asymptotic properties of estimators can be derived if S_n also converges in distribution.

It is almost always reasonable to assume that, for each n , the sequence X_{n1}, X_{n2}, \dots is exchangeable, whereas the same can not be said for independence.

For example, consider the sample mean \bar{Z}_n of the random sample, Z_1, \dots, Z_n . Clearly, \bar{Z}_n can be put in the form of (??) by letting $a_{ni} = 1/n$ and $X_{ni} = Z_i$. Then

$$(4) \quad S_n = \bar{Z}_n = \sum_{i=1}^n \left(\frac{1}{n} \right) (Z_i).$$

In similar fashion, the sample variance $s^2 = 1/(n-1) \sum_{i=1}^n (Z_i - \bar{Z}_n)^2$ can be transformed to (1) by letting $a_{ni} = 1/(n-1)$ and $X_{ni} = (Z_i - \bar{Z}_n)^2$, $1 \leq i \leq n$. The X_{ni} are not independent, but are exchangeable.

Example 2: Regression and Conditional Independence

A firm wishes to explain its annual income Y . The key explanatory variables are believed to be: X_1, \dots, X_T . We wish to choose estimators of the $T+1$ parameters $\beta_0, \beta_1, \dots, \beta_T$ such that the L^2 norm of $Y - (\beta_0 + \beta_1 x_1 + \dots + \beta_T x_T)$ is minimized. Geometrically, we project Y on $[X] = [1, X_1, X_2, \dots, X_T]$. The "goodness of fit" is measured by $E[Y - \hat{Y}]^2$, where $\hat{Y} = \hat{\beta}_0 + \hat{\beta}_1 X_1 + \dots + \hat{\beta}_T X_T$ is the solution to the minimization problem.

Given these "least squares" estimators, the difference between Y and \hat{Y} is independent of (X_1, \dots, X_T) .

Example 3: The U-Statistics

Let $(X_n)_{n=1}^{\infty}$ be a sequence of exchangeable random variables and $f(\bullet)$ a symmetric Borel function on R^m such that $E|f(X_1, \dots, X_m)| < \infty$. A sequence of U-statistics is given by:⁵³

$$U_{m,n} = \binom{n}{m}^{-1} \sum_{1 \leq i_1, \dots, i_m \leq n} f(X_{i_1}, \dots, X_{i_m}), \quad n > m \geq 1$$

Let $F_n = \sigma(U_{m,j} \mid j \geq n)$. If $1 \leq i_1 \leq \dots < i_m \leq n+1$ and $B \in F_{n+1}$,

$$\int f(X_{i_1}, \dots, X_{i_m}) dP = \int_B f(X_1, \dots, X_m) dP$$

by symmetry and exchangeability.

Hence

$$E[f(X_{i_1}, \dots, X_{i_m} \mid F_{n+1})] = E[f(X_1, \dots, X_m \mid F_{n+1})] \quad \text{a.s.}$$

and

$$E(U_{m,n} \mid F_{n+1}) = E[V_{m,n+1} \mid F_{n+1}] = U_{m,n+1}, \quad \text{a.s.}$$

An easy method for generating infinite and finite exchangeable sequences is to sample from an urn containing n distinct balls. Let Y_1, Y_2, \dots, Y_n be the numbers painted on the n balls. Sampling with (without) replacement yields an infinite (finite) exchangeable sequence, $X_1, X_2, \dots, (X_1, X_2, \dots, X_n)$.

The finite sequence X_1, \dots, X_n of random variables has the same joint distribution as any other n -sequence obtained by permuting the n subscripts. Any n -sequence of random variables with this property (invariance under permutation) is said to be n -exchangeable.

⁵³Clearly, Example 1 is a special case of the U-statistic.

Similarly, the infinite sequence of random variables is called exchangeable if its joint distribution is invariant to permutations of the subscripts, that is,

$$X_1, X_2, X_3, \dots, \stackrel{d}{=} X_{\pi(1)}, X_{\pi(2)}, X_{\pi(3)}, \dots$$

for any finite permutation π of $(1, 2, 3, \dots)$, where $\stackrel{d}{=}$ is read "has the same distribution as".

de Finetti's Theorem (1)

Suppose the urn is composed of balls labelled either 0 or 1 and let sampling with replacement proceed. The ensuing infinite sequence of 0-1 random variables has a probability distribution F with support $[0, 1]$, such that

$$P(Y_1 = 1, Y_2 = 1, \dots, Y_j = 1, Y_{j+1} = 0, \dots, Y_n = 0) = \int_0^1 \theta^j (1-\theta)^{n-j} F(d\theta),$$

for all n and $0 \leq j \leq n$.

The Bayesian interpretation of this theorem is immediate. The distribution F is the prior distribution of the random parameter θ . The observed sequence of 0's and 1's is a Bernoulli sequence given that $\theta = \theta$. Another more formal statement of de Finetti's theorem is:

Theorem 3: Suppose X_1, X_2, \dots is an exchangeable sequence of random variables. Then there exists a σ -algebra such that the sequence is i.i.d. given the σ -algebra.

Symbolically, we have

$$P(X_1 \leq x_1, \dots, X_n \leq x_n | F) = E(P(X_1 \leq x_1, \dots, X_n \leq x_n | F_\infty) | F) = \prod_{i=1}^n F(x_i), \quad \text{a.s.,}$$

where for each x , F is an F_∞ -measurable r.v.

Kingman notes that by letting F be a r.v., the randomizing kills the independence of a sequence of i.i.d. r.v.'s, but preserves exchangeability.

Kendall proved the following

Theorem 4: Let X_1, X_2, \dots be an exchangeable sequence of 0-1 r.v.s. Then, there exists a r.v. $Y \sim G$, where $[0,1]$ is the support of G such that

$$P \left(\sum_{j=1}^n X_j = i \mid Y \right) = \binom{n}{i} Y^i (1-Y)^{n-i} \quad \text{a.s.}$$

and

$$\lim_{n \rightarrow \infty} \frac{1}{n} \sum_{j=1}^n X_j = Y \quad \text{a.s.}$$

Return to the Polya urn. Instead of adding just one ball when its color is drawn, add k balls of that color. Then,

$$P(X_1=1, \dots, X_n=1) = \frac{b(b+k) \dots (b+(n-1)k)}{(b+w)(b+2+k) \dots (b+w+(n-1)k)} = \frac{\Gamma\left(\frac{n+b}{k}\right) \Gamma\left(\frac{b+w}{k}\right)}{\Gamma(n+(b+w)/k) \Gamma\left(\frac{b}{k}\right)}$$

= EY^n , $n = 1, 2, \dots$, by Theorem 4.

This is Hausdorff's moment problem.

Exchangeability has been applied in almost every discipline. These concrete uses of exchangeability have illuminated the discipline and indeed are indispensable to many like insurance and economics. On the other hand, it is exactly this pragmatic concept that has been generalized by mathematicians beginning with Hewitt and Savage through the recent work of Diaconis and Freedman, Aldous, Kallenberg, and Ressel. Just as there is a fundamental duality between uncertain and deterministic processes so to are the abstract and the practical intertwined like heads and tails of the same coin. The source of these remarkable dualities is the symmetry infusing all of nature and perpetuated by the evolutionary process.

REFERENCES

- Alchian, A.A., "Uncertainty, Evolution and Economic Theory," Journal of Political Science, 58 (1950): 211-221.
- _____, "Specificity, Specialization and Coalitions," J. of Instit. and Theor. Econ., 140 (1984): 34-49.
- Aldous, D.J., Exchangeability and Related Topics, Springer-Verlag, 1985.
- Anderson, J.A., "Cognitive and Psychological Computation with Neural Networks," IEEE TRANS. on Syst., Man, and Cybern., SMC-13(1983) 799-815.
- Arrow, K.J., The Limits of Organization, Norton, 1974.
- Becker, G.S., The Economic Approach to Human Behavior, University of Chicago Press, 1976.
- Berard, P.H., Spectral Geometry: Direct and Inverse Problems, Springer-Verlag, 1986.
- Berlin, I., Vico and Herder, Hogarth Press, 1976.
- Berlin, I., The Crooked Timber of Humanity, Alfred A. Knopf, 1991.
- Burke, P., VICO, Oxford University Press, 1985.
- Burks, A.W., Change. Cause. Reason, University of Chicago Press, 1977.
- Calder, W.A. III, Size, Function, and Life History, 1984.
- Caponigri, R., Time and Idea: The Theory of History in Giambattista Vico, 1953.
- Carneval, P., and S. Patarnello, Neural Networks "Living" in an External Environment, Rome: IBM ECSEC, 1989.
- Cassirer, E., The Philosophy of Symbolic Forms: Mythical Thought, Yale University Press, 1955.
- Cavalli-Sforza, L.L., and M.W. Feldman, Cultural Transmission and Evolution, Princeton University Press, 1981.

- Chapman, M., Constructive Evolution: Origins and Development of Piaget's Thought, Cambridge University Press, 1988.
- Coase, R.H., "The Nature of the Firm," Economica, 16 (1937), 386-405.
- _____, "The Problem of Social Cost," Journal of Law and Economics, 3 (1960), 1-44.
- Cohen, M.A. and S. Grossberg, "Absolute Stability of Global Pattern Formation and Parallel Memory Storage by Competitive Neural Networks," IEEE Trans. on Syst., Man, and Cybern., SMC-13 (1983), 815-825.
- Cornes, R., and T. Sandler, The Theory of Externalities, Public Goods, and Club Goods, Cambridge University Press, 1986.
- Croce, B., Aesthetic, Macmillan, 1909.
- De Finetti, B., "Foresight: Its Logical Laws, Its Subjective Sources," in Kyburg and Smokler (eds.), Studies in Subjective Probability, John Wiley & Sons, 1964.
- _____, Theory of Probability I and II (1974), Wiley and Sons.
- Dewey, J., The Quest For Certainty, G.P. Putnam, 1929.
- Eco, U., The Aesthetics of Thomas Aquinas (translation by H. Bredin), Harvard University Press, 1988.
- Eisenberg, J.F., The Mammalian Radiations, University of Chicago Press, 1981.
- French, M.J., Invention and Evolution: Design in Nature and Engineering, Cambridge University Press, 1988.
- French, S., "The Analysis of Multiple Choice Tests in Educational Assessment," in R. Viert (ed.), Probability and Bayesian Statistics, Plenum Press, 1987.
- Friedman, M., Essays in Positive Economics, University of Chicago Press, 1953.

- _____, "Choice, Chance, and the Personal Distribution of Income,"
Journal of Political Economics, 61 (1953b), 277-90.
- Hage, P., and F. Harary, Structural Models in Anthropology, Cambridge
 University Press, 1983.
- Hayek, F.A., Studies in Philosophy Politics and Economics, University of
 Chicago Press, 1967.
- Hayek, F.A., New Studies (1978), 264-265, University of Chicago Press.
- Hebb, D.O., Organization of Behavior, J. Wiley and Sons, 1949.
- Heymann, D., and A. Leijonhufvud, (forthcoming).
- Hicks, J.R., A Theory of Economic History, Oxford, 1969.
- Hirshleifer, J., "The Expanding Domain of Economics," American Economic
 Review, (Dec. 1985), 53-68.
- Hobbes, T., Leviathan, Penguin Books, 1651.
- Holland, J.H., K.J. Holyoak, R.E. Nisbett, and P.R. Thagard, Induction
 (1986), M.I.T. Press.
- Hoppfield, J.J. and D.W. Tank, "Computing with Neural Circuits: A Model,"
Science 233 (1986), 625-633.
- Joyce, J., Ulysses, Random House, 1961.
- Kac, M., "Can One Hear the Shape of a Drum," American Mathematical Monthly,
 73 (1966).
- Kenner, H., A Colder eye, Penguin Books, 1984.
- Kimura, M., The Neutral Theory of Molecular Evolution, Cambridge University
 Press, 1983.
- Levinthal, D., A Survey of Agency Models of Organizations, (1984) Tech. Rep.
 443 Department of Economics, Stanford.
- Lippman, S.A., and J.J. McCall, Ecological Decisionmaking and Optimal
 Stopping Rules, UCLA, 1984.

- Martin, R.D. and R.M. May, "Outward Signs of Breeding," in Evolution Now, Edited by J. Maynard Smith (1982), W.H. Freeman.
- Matthews, R.C.O., "The Economics of Institutions and the Sources of Growth," Economic Journal, 96 (1986), 903-18.
- Mayr, E., The Growth of Biological Thought, Harvard University Press, 1982.
- McCall, J.J., "A Primer on Exchangeability" Hoover Working Paper (1987a).
- _____, "The Unifying Power of Exchangeability," Hoover Working Paper (1986).
- _____, The Bridge Across the Disciplines, UCLA, (1986).
- _____, The Smithian Self and Its Bayesian Brain, UCLA Working Paper (1989).
- _____, "Exchangeability and Its Economic Applications," Journal of Econ. Dyn. and Cont., 15 (1991), 549-68.
- _____, "The Unifying Power of Exchangeability," UCLA Working Paper, 1991.
- _____, "Exchangeability and the Structure of the Economy: A Preliminary Process Analysis," in H. Brink (ed.), Advances in Business Cycle Theory, Macmillan, 1991.
- _____, "The Random Foundations of Economic Processes I," in K. Velupillai, (ed.) Proceedings of the International Economic Association, Macmillan, 1991.
- _____, "Graphs and Their Economic Applications: A Survey," (forthcoming).
- _____ and K. Velupillai, The Economy of Nature: I. A Survey, UCLA Working Paper, 1991.
- _____ and K. Velupillai, "Constructive Foundations For Economics: The Emperor's Old Clothes," resubmitted to Economic Journal, 1991a.

- _____ and K. Velupillai, Infinite Divisibility, UCLA Working Paper, 1991b.
- _____ and K. Velupillai, The Modular Economy, UCLA Working Paper, 1991c.
- _____ and K. Velupillai, "A Neuroscientific Approach to Economics," Abstract Math. Soc. Sciences, 22 (1991d).
- McCloskey, D.N., "The Rhetoric of Economics," Journal of Economic Literature, XXI (1983), 481-517.
- McMullin, E., "The Ambiguity of 'Historicism'," in P.D. Asquith and H.E. Kyburg, Jr. (eds.), Current Research in Philosophy of Science, Philosophy of Science Association, 1979.
- Minnis, P.E., Social Adaptation to Food Stress, University of Chicago Press, 1985.
- Mooney, M., Vico in the Tradition of Rhetoric, Princeton University Press, 1985.
- Parthasarathy, K.R., "A Unified Approach to Classical, Bosonic and Fermionic Brownian Motions," Asterisque, 1988, 157-58, 303-20.
- Pompa, L., VICO: A Study of the 'New Science'," 2nd ed., Cambridge University Press, 1990.
- Posner, R.A., The Economics of Justice, Harvard University Press, 1981.
- Priest, G.L., "The Current Insurance Crisis and Modern Tort Law," The Yale Law Journal 96 (1987), 1521-1590.
- Rossi, P., The Dark Abyss of Time, translated by L.G. Cochrane, University of Chicago Press, 1984.
- Saari, D.G., "The Source of Some Paradoxes From Social Choice and Probability," J. Economic Theory, 41 (1987), 1-22.

- Salmon, W.C., Scientific Explanation and the Causal Structure of the World, Princeton University Press, 1984.
- Sarnak, P., Some Applications of Modular Forms, Cambridge University Press, 1990.
- Scarre, C.J., "Kin-Groups in Megalithic Burials," Nature, 311 (1984), 512-13.
- Schotter, A., The Economic Theory of Social Institutions, Cambridge University Press, 1981.
- Schumpeter, J.A., History of Economic Analysis (1954), Oxford University Press.
- Schwartz, L., "Review of Stochastic Calculus on Manifolds, by M. Emery," Bull. Amer. Math. Soc., 24 (1991), 451-66.
- Shils, E., Tradition, University of Chicago Press, 1983.
- Smith, A., The Theory of Moral Sentiments (1853), Liberty Classic, 1976.
- _____, Lectures on Jurisprudence, Oxford University Press, 1978.
- _____, Lectures on Rhetoric and Belles Lettres, Oxford University Press, 1983.
- Snow, C.P., The Two Cultures and the Scientific Revolution, Cambridge University Press, 1959.
- Suppes, P., Probabilistic Metaphysics, Basil Blackwell, 1984.
- Toulmin, S., The Return to Cosmology, University of California Press, 1982.
- _____, Human Understanding, Princeton University Press, 1972.
- Turnbull, H.W., The Great Mathematicians, 4th ed., Methuen and Co. Ltd., 1951.
- Verene, D.P., (editor) Vico and Joyce (1987), State University of New York Press.

Vico, G., Autobiography, translated by M.H. Fisch and T.G. Bergin, Cornell University Press, 1944.

_____, The New Science, translated by M.H. Fisch and T.G. Bergin, Cornell University Press, 1948.

_____, On the Most Ancient Wisdom of the Italians, translated by L.M. Palmer, Cornell University Press, 1988.

_____, On the Study Methods of Our Time, translated by E. Gianturco, Cornell University Press, 1990.

Weyl, H., Symmetry, (1952) Princeton University Press.

Williamson, O.E., The Economic Institutions of Capitalism, The Free Press, 1985.

Wilson, E.O., Sociobiology, Harvard University Press, 1975.

Wittgenstein, L., On Certainty,

Zambrini, J.C., "Euclidian Quantum Mechanics," in R. Gielerak and W.

Karwowski (eds.), Stochastic Methods in Mathematics and Physics, World Scientific, 1988.