STOCHASTIC AGGREGATION AND DYNAMIC FIELD EFFECTS*

by

Masanao Aoki+

and

Yoshio Miyahara++

UCLA Department of Economics
Working Paper #637
November 1991

ABSTRACT

Two macroeconomic variables are introduced to represent aggregate effects of decisions by firms in the goods and labor markets. They are called field effect variables and are shown to produce two co-integrating relations and two unit roots among the macroeconomic variables of the model due to fluctuations of goods and labor market conditions near (stochastic) equilibrium. These effects are distinct from stochastic (trending) effects caused by productivity factor.

---

*Discussion with T. Cooley, K. Matheny and S. Potter have helped clarify the paper.

+2263 Bunche Hall, Department of Economics, UCLA, Los Angeles, CA 90024-1477 USA

++Faculty of Economics, Nagoya City University, Nagoya 467 Japan

Key words: aggregation, field effects, co-integration, unit roots

Introduction

When we wish to dispense with the assumption of representative agents and deal directly with a (large) collection of heterogeneous agents, we face a number of modeling difficulties, especially when the interactions are dynamic or nonlinear.[1] When characteristics of agents are not identical but parameterized by a finite dimensional vector with a known distribution or similar in some sense, there is some hope to characterize the dynamic behavior of the collection of such agents in macroscopic or global way.[2] This paper presents one such illustration in a way that appears not to be discussed in the literature. This paper goes further than earlier attempts[3] by adopting a class of models in which explicit aggregation of micro relations into macro ones are possible. This is accomplished by introducing a key notion of "field" effects and "field" variables, which summarize for micro agents aggregate (macroeconomic) effects of actions by all the microeconomic units. This notion originates in statistical mechanics and has recently been borrowed successfully in the neural network literature.[4] Stated in the language of economics the notion of field effects is very natural: Interactions among economic agents are mediated through field variables. It is assumed that there is no direct interaction among economic agents, that economic agents are subject to or affected by the same macroeconomic (field) effects, and that the macroeconomic effects

---

[1]Some progress has been made in neural network literature in nonlinear dynamic interactions. Characteristics of individual units are, however, extremely simple.

[2]An earlier example is Kelejian (1980).

[3]See Aoki (1976, 1979a,b, 1980), and Miyahara (1990).

[4]See Peterson and Anderson (1987), Hertz et al. (1991) or Dawson (1983), for example. See Example 1 in Aoki (1979b) for an economic example. For somewhat related works, see Lippi (1988).

are, in turn, determined by the aggregate actions of microeconomic agents (in previous time epoch). In discrete time formulation we adopt in this paper, the key assumption is that the current values of the field effects which are functions of macroeconomic variables are functions of the microeconomic (state) variables in the previous period. (Or, several lags can be easily introduced.) More specifically in the model we construct firms face three types of uncertainty: productivity shock; number of workers responding to announced wage rates, or labor supply schedule; and demand conditions for the good. We focus on the latter two effects. There are thus two field variables, A_t and B_t, one to express the good market condition and the second to express the labor market conditions. They are in general stochastic processes and reduce to constants or "invariants" in the idealized case of stable deterministic representative agent model. These two "invariants" or field (effect) variables give rise to two co-integrating relations among underlying macroeconomic variables and two associated unit roots when we relax the representative agent assumption and introduce heterogeneity of agent characteristics or the diversity of stochastic environments facing individual agents. Autoregressive technical progress factors are additional sources of unit roots in the outputs, and are not needed for that purpose in our model.

Micro-model

We construct a model of economy consisting of n firms somewhat along the line discussed in Iwai (1981). We suppose that firm i produces a perishable good i with one period production lag

$$q_i(t+1) = j_i(t)\ n_i(t)^{\gamma}, \quad 0 < \gamma < 1 \tag{1}$$

where $n_i(t)$ is the number of workers employed by firm i during period t

and $j_i(t)$ is possibly firm i specific productivity parameter relevant to period t.

Labor supply is assumed to respond to real wage rate which we take to be $w_i(t)$ deflated by $\hat{p}_i(t+1)$

$$\ell_i(t) = b_i(t)[w_i(t)/\hat{p}_i(t+1)]^{\epsilon}, \quad 0 < \epsilon \tag{2}$$

where $\hat{p}_i(t+1)$ is the price firm i uses in planning period t + 1 production defined later in (6), and $b_i(t)$ is a random variable unobservable at the beginning of period t which represents "tightness" of labor market. Changes in $b_i(t)$ shift the labor supply schedule. Firm i's estimate of it at the beginning of period t is denoted by $\hat{b}_i(t)$. Firm i employs workers up to the estimated labor supply, assuming that firms can always fulfill their labor demands,

$$n_i(t) = \hat{b}_i(t)w_i(t)^{\epsilon}/\hat{p}_i(t+1)^{\epsilon}. \tag{3}$$

Demand for good i is postulated to be

$$d_i(t) = a_i(t)\ p_i(t)^{-\eta}, \quad \eta > 1 \tag{4}$$

where $p_i(t)$ is the good price prevailing in the market at end of period t, and the random variable $a_i(t)$, not observable at the beginning of period t, is an indicator of how active demand conditions are for good i. The demand schedule shifts as $a_i(t)$ changes. Its estimate by firm i is denoted by $\hat{a}_i(t)$.

Firm i maximizes one-period expected profit

$$\hat{\pi}_i(t) = \hat{p}_i(t)\ \hat{s}_i(t) - w_i(t-1)\ n_i(t-1) \tag{5}$$

where defining $\hat{d}_i(t)$ by (4) with $\hat{a}_i(t)$ instead of $a_i(t)$, and

$$\hat{s}_i(t) = \min(\hat{d}_i(t), q_i(t)).$$

Since $w_i(t-1)n_i(t-1)$ is fixed at the beginning of period t, firm i plans production by maximizing the first term, i.e., by setting price to achieve

$$\hat{d}_i(t) = q_i(t)$$

i.e.,

$$\hat{p}_i(t) = [\hat{a}_i(t)/q_i(t)]^{1/\eta}. \qquad (6)$$

This is the price firm i uses in planning period t production. From (6), (1), and (3), the estimated price is rewritten as

$$\hat{p}_i(t) = [a_i(t)/j_i(t-1)\hat{b}_i(t-1)^{\gamma} w_i(t-1)^{\epsilon\gamma}]^{1/(\eta-\epsilon\gamma)}.$$

Substituting this into (5) with $s_i(t)$ equal to $q_i(t)$, $\hat{\pi}_i(t)$ becomes a function of $w_i(t-1)$ only. Firm i chooses it to maximize $\hat{\pi}_i(t)$.

The necessary condition of maximizing expected one-period profit $0 = d\hat{\pi}_i(t+1)/dw_i(t)$ produces

$$w_i(t) = \kappa^{(1-\epsilon\gamma/\eta)/c}\ \hat{a}_i(t+1)^{1/\eta}\ j_i(t)^{(\eta-1-\epsilon)/c\eta}\ \hat{b}_i(t)^{-(1-\gamma\delta)/c\mu} \qquad (7)$$

with

$$c = 1 + \epsilon(1-\gamma), \quad \kappa = \epsilon\gamma\delta/(\mu+\epsilon), \quad \delta = 1 - 1/\eta.$$

Substitute (7) into (3) to obtain the number of labor firm i hires

$$n_i(t) = \kappa^{\epsilon/c}\ j_i(t)^{\epsilon/c}\ \hat{b}_i(t)^{1/c}. \qquad (8)$$

Note that $n_i(t)$ is independent of $\hat{a}_i(t)$. From (1) and (8), good i is produced in the amount of

$$q_i(t) = \kappa^{\epsilon\gamma/c} j_i(t-1)^{(1+\epsilon)/c}\ \hat{b}_i(t-1)^{\gamma/c}. \qquad (9)$$

By the assumed perishability of goods, the market clearing price of good i is obtained by equating the <u>actual</u> demand to $q_i(t)$

$$d_i(t) = a_i(t)\ p_i(t)^{-\eta} = q_i(t),$$

or from (9)

$$p_i(t) = \kappa^{-\epsilon\gamma/c\eta} j_i(t-1)^{-(1+\epsilon)/c\eta} \; a_i(t)^{1-\delta} \; \hat{b}_i(t-1)^{-\gamma/c\eta}. \tag{10}$$

This is the market clearing price of good i at the end of period t.

We next describe the expectation formation by firms. Consider first a model where all firms are identical, no random changes in demand or supply schedules occur, and perfect information prevails. Thus, $q_i(t)$ is the same as the (average) output $\bar{Q}(t)$ in macroeconomy and $p_i(t)$ is the same as the price $P(t)$ which is the price (level) in macroeconomy. Thus, when there is no uncertainty, the parameter that fixes the demand schedule is given from (4) by $a_i(t) = \bar{Q}_{t-1} P^{\eta}_{t-1}$. (We choose $t - 1$ since they are known at the beginning of period t.) We therefore introduce a field variable by

$$A_{t-1} = \bar{Q}_{t-1} P^{\eta}_{t-1}, \tag{11}$$

to fix a "reference" demand condition, and we measure relative shifts from it by defining $\alpha_i(t)$

$$a_i(t) = \alpha_i(t) A_{t-1}. \tag{12}$$

Variable A_t represents the macroeconomic demand condition in the goods market and plays an important role in the evolution of the economy. Under the same set of assumptions, the parameter that fixes the labor supply schedule, i.e., the labor market condition is expressible from (2) by $b_i(t) = \bar{N}_{t-1} W^{-\epsilon}_{t-1} P^{\epsilon}_{t}$, where $\bar{N}_t$ is the average number of workers per firm, and W_{t-1} is the wage rate in a macromodel. Introduce a second field variable by

$$B_{t-1} = \bar{N}_{t-1} W^{-\epsilon}_{t-1} P^{\epsilon}_{t}, \tag{13}$$

and measure relative shifts in the labor supply schedule from this reference by defining

$$b_i(t) = \beta_i(t)\ B_{t-1}. \tag{14}$$

In the steady state, the demand and the labor market conditions remain the same and hence A_t and B_t will be constants, i.e., A_t and B_t are the invariants in this idealized situation. Firms are not actually identical, and they face different demand and labor market conditions so we treat $\alpha_i(t)$ and $\beta_i(t)$ as random variables. In other words, we measure the random variables $a_i(t)$ and $b_i(t)$ normalized by A_{t-1} and B_{t-1}.

To reflect that information of firms is incomplete, we posit that firm i estimate $a_i(t)$ by

$$\hat{a}_i(t) = \alpha_i(t-1)\ A_{t-1},$$

and

$$\hat{b}_i(t) = \beta_i(t-1)\ B_{t-1}.$$

For example, if $\alpha_i(t)$ is a martingale, then $\alpha_i(t-1)$ is its rational expectation. Note that a_i, b_i, α_i and β_i are all positive valued random variables. With these assumptions, (7) - (10) completely describes the microeconomic model.

Ex post profit of firm i is given by

$$\begin{aligned}\pi_i(t) &= q_i(t)\ p_i(t) - n_i(t-1)\ w_i(t-1)\\ &= \kappa^{\epsilon\gamma\delta/c}\ j_i(t-1)^{(1+\epsilon)\delta/c}\ \hat{b}_i(t-1)^{\gamma\delta/c}\ \Delta_i(t)\end{aligned}$$

where

$$\begin{aligned}\Delta_i(t) &= a_i(t)^{1/\eta} - \kappa\hat{a}_i(t)^{1/\eta}\\ &= A_{t-1}^{1/\eta}[\alpha_i(t)^{1/\eta} - \kappa\alpha_i(t-1)^{1/\eta}].\end{aligned}$$

If $E(\alpha_i(t)|\mathcal{I}_{t-1}) = \alpha_i(t-1)$ as in the case of martingale, where $\mathcal{I}_{t-1}$ is the information set of firm i at time $t-1$,

$$E(\alpha_i(t)^{1/\eta} | \mathcal{I}_{t-1}) \le \alpha_i(t-1)^{1/\eta}$$

by the conditional expectation version of the Jensen's inequality since $\eta > 1$. Here $\kappa < 1$. Thus the sign of $E(\Delta_i(t) | \mathcal{I}_{t-1})$ is ambiguous. There may or may not be a set of parameters for which the conditional expected ex post profit is zero. (When $\eta = 1$ and for η large, the expected profit is positive.) A positive expected profit may be used by firm i to invest in capital stock by modifying the production function to include capital stock explicitly.[5] Alternatively $\hat{a}_i(t)$ can be redefined by

$$\hat{a}_i(t)^{1/\eta} = \kappa^{-1} E(a_i(t)^{1/\eta} | \mathcal{I}_{t-1})$$

to ensure zero profit condition.

Macro-model

We now produce the macroeconomic model by aggregating the microeconomic units. Assume that the industry classification is such that all outputs of the firms are simply summed to produce the total output and the average output per firm is defined by

$$\bar{Q}_t^{(n)} = \frac{1}{n} \Sigma_{i=1}^n q_i(t)$$

where n is the number of firms. Similarly, the average number of workers per firm is defined by

$$\bar{N}_t^{(n)} = \frac{1}{n} \Sigma_{i=1}^n n_i(t).$$

To preserve the accounting identity we define the macroeconomic price level $P_t^{(n)}$ by

$$\bar{Q}_t^{(n)} P_t^{(n)} = \frac{1}{n} \Sigma_{i=1}^n q_i(t)\, p_i(t)$$

and $W_t^{(n)}$ by

[5]This would also cure the defect of the model that the marginal product of labor goes to zero as the number of workers increase indefinitely.

$$\bar{N}_t^{(n)} \; W_t^{(n)} = \frac{1}{n} \; \Sigma_{i=1}^n \; w_i(t) \; n_i(t).$$

To simplify the resulting expression assume that the productivity random variables are the same for all firms, i.e., $j_i(t) = j_t$ for all i. This assumption is easily dropped and the consequences are briefly discussed at the end of this paper. Then, using (9) the average output is

$$\bar{Q}_t^{(n)} = C_{1,t-1}^{(n)} \; \kappa^{\epsilon\gamma/c} \; j_{t-1}^{(1+\epsilon)/c} \; B_{t-2}^{\gamma/c}, \tag{15}$$

with the random constant

$$C_{1,t-1}^{(n)} = \frac{1}{n} \; \Sigma_{i=1}^n \; \beta_i(t-2)^{\gamma/c}.$$

From (9) and (10), the price level is

$$P_t^{(n)} = C_{2,t-1}^{(n)} \; \kappa^{-\epsilon\gamma/c\eta} \; j_{t-1}^{-(1+\epsilon)/c\eta} \; A_{t-1}^{1/\eta} \; B_{t-2}^{-\gamma/c\eta}, \tag{16}$$

with

$$C_{1,t-1}^{(n)} \; C_{2,t-1}^{(n)} = \frac{1}{n} \; \Sigma_{i=1}^n \; \beta_i(t-2)^{\gamma\delta/c} \; \alpha_i(t)^{1/\eta}.$$

Proceeding analogously, we derive from (8) the average number of workers to be

$$\bar{N}_t^{(n)} = C_{3,t}^{(n)} \; \kappa^{\epsilon/c} \; j_t^{\epsilon/c} \; B_{t-1}^{1/c}, \tag{17}$$

with

$$C_{3,t}^{(n)} = \frac{1}{n} \; \Sigma_{i=1}^n \; \beta_i(t-1)^{1/c},$$

and from (7), (8) and (17) the aggregate wage rate to be

$$W_t^{(n)} = C_{4,t}^{(n)} \; \kappa^{\mu/c} \; j^{(\delta-\epsilon/\eta)/c} \; A_t^{1/\eta} \; B_{t-1}^{-(1-\gamma\delta)/c}, \tag{18}$$

with

$$C_{3,t}^{(n)} \; C_{4,t}^{(n)} = \frac{1}{n} \; \Sigma_{i=1}^n \; \alpha_i(t)^{1/\eta} \; \beta_i(t-1)^{\gamma\delta/c}.$$

We note from (15) and (17) that the aggregate production function is

$$\bar{Q}_t^{(n)} = C_{1,t-1}^{(n)} (C_{3,t-1}^{(n)})^{-\gamma} \; j_{t-1} (\bar{N}_t^{(n)})^{\gamma}. \tag{19}$$

This corresponds to (1) which is the micro-production function. We later show that $\ell nQ_t^{(n)} - \gamma\ell n\,\bar{N}_t^{(n)}$ and another expression given in (24) are co-integrated in the sense of Granger (1983). We note here that $C_{i,t}^{(n)}$, $i = 1,\ldots,4$ are weakly stationary if $\alpha_i(t)$ is.

Macrodynamics

The time evolutions of the field variables are implicit in (15)-(18). To make them explicit note (11). Then (suppressing superscript (n)) from (15) and (16), we derive

$$A_t = C_{1,t-1}(C_{2,t-1})^{\eta}A_{t-1}.$$

Denoting the logarithm by ~, this becomes

$$\tilde{A}_t = \tilde{A}_{t-1} + \tilde{C}_{1,t-1} + \eta\tilde{C}_{2,t-1}, \qquad (20)$$

i.e., $(\tilde{A}_t)$ is a random walk unless (the expected value of) $\tilde{C}_{1,t-1} + \eta\tilde{C}_{2,t-1}$ is 0. Similarly, from the definition in (13), B_t evolves with time according to

$$B_t = C_{3,t}(C_{4,t})^{-\epsilon}\,C_{2t}^{\epsilon}\,B_{t-1},$$

or taking logarithms

$$\tilde{B}_t = \tilde{B}_{t-1} + \tilde{C}_{3,t} - \epsilon\tilde{C}_{4,t} + \epsilon\tilde{C}_{2,t} \qquad (21)$$

is another random walk, unless the rest of terms sum to zero in expected values. Equation (20) and (21) show that $\ln A_t$ and $\ln B_t$ are no longer invariants in the face of changing demand and labor market conditions. They are, however, still nearly so in a stochastic sense if the additive disturbance are "small".

In Appendix we show that when $\alpha_i(t)$ and $\beta_i(t)$ are lognormal random variables $\tilde{A}_t = \tilde{A}_{t-1}$ is likely to hold if conditions in the labor market is less volatile than in the goods market, while $\tilde{B}_t = \tilde{B}_{t-1}$ holds only when the labor market condition is non random.

The relations of these field variables to the macro-variables become clear when we take logarithms of (15)-(18) and collect them as

$$\begin{bmatrix} \tilde{Q}_t^{(n)} \\ \tilde{P}_t^{(n)} \\ \tilde{N}_{t-1}^{(n)} \\ \tilde{W}_{t-1}^{(n)} \end{bmatrix} = H_0 \begin{bmatrix} \tilde{A}_{t-1} \\ \tilde{B}_{t-2} \end{bmatrix} + H_1\tilde{\kappa} + H_2\tilde{j}_{t-1} + \tilde{u}_{t-1}^{(n)} \tag{22}$$

where

$$H_0 = \begin{bmatrix} 0 & \gamma/c \\ 1/\eta & -\gamma/c\eta \\ 0 & 1/c \\ 1/\eta & -(1-\gamma\delta)/c \end{bmatrix}$$

$$H_1 = \frac{1}{c}\begin{bmatrix} \epsilon\gamma \\ -\epsilon\gamma/\eta \\ \epsilon \\ \mu \end{bmatrix}, \quad H_2 = \frac{1}{c}\begin{bmatrix} (1+\epsilon) \\ -(1+\epsilon)/\eta \\ \epsilon \\ (\delta\eta-\epsilon)/\eta \end{bmatrix}, \quad \text{and} \quad u_{t-1}^{(n)} = \begin{bmatrix} \tilde{C}_{1,t-1}^{(n)} \\ \tilde{C}_{2,t-1}^{(n)} \\ \tilde{C}_{3,t-1}^{(n)} \\ \tilde{C}_{4,t-1}^{(n)} \end{bmatrix}.$$

The null subspace of H_0' is two dimensional and is spanned, for example, by $[1\ 0\ -\gamma\ 0]'$ and $[1,1,-1,-1]'$. It means that there are two co-integrating relations. For example, $\tilde{Q}_t - \gamma\tilde{N}_{t-1}$ and $\tilde{Q}_t - \tilde{N}_{t-1} + \tilde{P}_t - \tilde{W}_{t-1}$ are co-integrated if $\tilde{j}_t$ is stationary. The former is the macro-production relation, and the latter shows that per capita output is proportional to the real wage rate in stochastic sense.

Note that the demand field variable A's affects only the price variables P's and W's. If B's produce stochastic trends, then the $\tilde{Q}_t$ and $\tilde{N}_t$ tends to infinity. In this model the labor population is also tending to infinity which drives the field variable B's.

The average profit is defined by

$$\bar{\Pi}_t = \bar{Q}_t P_t - \bar{N}_{t-1} W_{t-1},$$

which is obtained from aggregating $\pi_i(t)$ to be

$$\bar{\Pi}_t = [C_{1,t-1} C_{2,t-1} - \kappa C_{3,t-1} C_{4,t-1}] \bar{\Pi}_t^0$$

where

$$\bar{\Pi}_t^0 = \kappa^{\epsilon\gamma\delta/c} j_{t-1}^{(1+\epsilon)\delta/c} A_{t-1}^{1/\eta} B_{t-2}^{\gamma\delta/c}.$$

Using the notation $<\bullet>$ to express arithmetic average over firms, the expression in the square bracket is[6]

$$<\beta(t-2)^{\gamma\delta/c} \alpha(t)^{1/\eta}> - \kappa<\beta(t-2)^{\gamma\delta/c} \alpha(t-1)^{1/\eta}>.$$

Microdynamics

With these macro-dynamics spelled out, we can now describe micro-dynamics and co-integrating behavior of some microvariables. Taking logarithms of (9)-(10) and noting (11) and (12), we have

$$\begin{bmatrix} \tilde{q}_i(t) \\ \tilde{p}_i(t) \\ \tilde{n}_i(t-1) \\ \tilde{w}_i(t-1) \end{bmatrix} = H_0 \begin{bmatrix} \tilde{A}_{t-1} \\ \tilde{B}_{t-2} \end{bmatrix} + H_1 \tilde{\kappa} + H_2 \tilde{j}_{t-1}$$

$$+ H_0 \begin{bmatrix} \tilde{\alpha}_i(t-1) \\ \tilde{\beta}_i(t-2) \end{bmatrix} + (1-\delta) \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} [\tilde{\alpha}_i(t) - \tilde{\alpha}_i(t-1)]. \qquad (25)$$

At the micro-level $\tilde{p}_i(t)$ is the only variable which exhibits serial correlation if α's, β's and h's are serially uncorrelated. In the limit

---

[6] If all firms form the same expectation and if α's and β's are independent, the expressions in braces is proportional to $1 - \kappa$. Then the expected profit rate is positive. This expected positive profit may be invested or taxed away or capital cost can be introduced in the profit equation so that the expected profit becomes zero. To treat them explicitly the model needs be expanded suitably.

of $n \to \infty$, $\mathrm{cov}(\tilde{p}_i(t), \tilde{p}_i(t-1))$ is positive if $\{\tilde{j}_t\}$ is uncorrelated. The contemporaneous correlation between $\tilde{w}_i(t)$ and $\tilde{p}_i(t)$ is positive, so is the correlation between $\tilde{q}_i(t)$ and $\tilde{p}_i(t-1)$.

Correlation Relations

From (22) we can derive expressions for the differenced macroeconomic variables such as $\Delta\tilde{Q}_t$ and $\Delta\tilde{W}_{t-1}$ (dropping the superscript (n) from (22)). For example

$$\Delta\tilde{Q}_t = \frac{\gamma}{c}\,\zeta_{t-2} + \frac{1+\epsilon}{c}\,\Delta\tilde{j}_{t-1} + \Delta\tilde{C}_{1,t-1}$$

where from (21)

$$\zeta_{t-2} = \Delta\tilde{B}_{t-2} = \Delta\tilde{C}_{3,t-2} - \epsilon\Delta\tilde{C}_{4,t-2} + \epsilon\Delta C_{2,t-2}.$$

Similarly,

$$\Delta\tilde{P}_t = \frac{1}{\eta}\,\xi_{t-2} - \frac{\gamma}{c\eta}\,\zeta_{t-2} - \frac{1+\epsilon}{c\eta}\,\Delta\tilde{j}_{t-1} + \Delta\tilde{C}_{2,t-1},$$

where from (20)

$$\xi_{t-2} = \Delta\tilde{C}_{1,t-2} + \eta\Delta\tilde{C}_{3,t-1},$$

$$\Delta\tilde{N}_{t-1} = \frac{1}{c}\,\zeta_{t-2} + \frac{\epsilon}{c}\,\Delta\tilde{j}_{t-1} + \Delta\tilde{c}_{3,t-1},$$

and

$$\Delta\tilde{W}_{t-1} = \frac{1}{\eta}\xi_{t-2} - \frac{(1-\gamma\delta)}{c}\,\zeta_{t-2} - \frac{\hat{\delta}}{c}\,\Delta\tilde{j}_{t-1} + \Delta\tilde{C}_{4,t-1}.$$

We note that $\mathrm{var}\,\Delta\tilde{p}_t \gtrless \mathrm{var}\,\Delta\tilde{Q}_t$ according to $\sigma_A^2/\mathrm{Var}\,\Delta\tilde{Q}_t \gtrless \eta^2 - 1$ if $\mathrm{var}\,\Delta C_1 = \mathrm{var}\,\Delta C_2$.

Assuming that ξ_{t-2}, ζ_{t-2}, $\Delta\tilde{j}_{t-1}$, $\Delta C_{i,\tau}$, $i = 1 \sim 4$ are all stationary, let

$$\sigma_A^2 = \mathrm{var}\,\zeta_{t-2},$$

$$\sigma_B^2 = \mathrm{var}\zeta_{t-2},$$

$$\sigma_i^2 = \mathrm{var}\Delta\tilde{j}_{t-1},$$

and

$$s_i^2 = \mathrm{var}\Delta C_{i,\tau}, \quad i = 1 \sim 4.$$

On the assumption that s_i^2 are must smaller than the other variances and that s_i^2/σ_j^2 is negligible, the correlation coefficients of the (differenced) macroeconomic variables are simpler to state.

The correlation of output with wage is approximately given by

$$\mathrm{Corr}(\Delta\tilde{Q}_t, \Delta\tilde{W}_{t-1}) \cong \frac{-\gamma(1-\gamma\delta)\,\sigma_B^2 + (1+\epsilon)\hat{\delta}\sigma_j^2}{\sqrt{(\gamma^2\,\sigma_B^2 + (1+\epsilon)^2\,\sigma_j^2)}\ \sqrt{[(1-\gamma\delta)^2\,\sigma_B^2 + \hat{\delta}^2\,\sigma_j^2]}}$$

Empirically $\mathrm{corr}\Delta(\tilde{Q}_t, \Delta\tilde{W}_{t-1})$ is negative. This indicates that (since δ and $\hat{\delta} \approx 1$)

$$(1+\epsilon)\sigma_j^2 < \gamma(1-\gamma)\sigma_B^2. \tag{26}$$

If $\mathrm{var}\ \Delta C_3 = \mathrm{var}\ \Delta C_4$, then $\mathrm{var}\ \Delta\tilde{N}_{t-1} \gtrless \mathrm{var}\ \Delta\tilde{Q}_t$ according to $(1-\gamma^2)\sigma_{B^2} \gtrless (1+2\epsilon)\sigma_j^2$. Thus, if (26) holds, then $\Delta\tilde{N}_{t-1}$ is likely to have greater variance than $\Delta\tilde{Q}_t$. When σ_{AB} is neglected, we have

$$\mathrm{Corr}\Delta(\tilde{Q}_t, \Delta\tilde{P}_t) \cong -\left(1 + \frac{\left(\frac{\gamma}{1+\epsilon}\right)^2 \frac{\sigma_A^2}{\sigma_B^2}}{1 + \left[\left(\frac{\gamma}{1+\epsilon}\right)^2 \frac{\sigma_B^2}{\sigma_j^2}\right]^{-1}}\right)^{-1/2} < 0.$$

As $\gamma^2\sigma_A^2$ gets smaller than $\gamma^2\sigma_B^2 + (1+\epsilon)^2\sigma_j^2$, this latter correlation approaches minus one. Correlation coefficients with the productivity shock are[7]

$$\mathrm{Corr}(\tilde{j}_{t-1}, \tilde{Q}_t) = \frac{(1+\epsilon)\,\sigma_j}{\sqrt{\gamma^2\,\sigma_B^2 + (1+\epsilon)^2\,\sigma_j^2}} > 0,$$

and

$$\mathrm{Corr}(\tilde{j}_{t-1}, \tilde{N}_{t-1}) = \frac{\epsilon\sigma_j}{\sqrt{\sigma_B^2 + (\epsilon\delta)^2\,\sigma_j^2}} > 0.$$

On the assumption that σ_A^2/σ_j^2 is smaller than σ_B^2/σ_j^2 the model implies

---

[7]If $\tilde{j}_t$ is uncorrelated with $\tilde{j}_{t-1}$, then $\mathrm{Cov}(\tilde{j}_t, \tilde{Q}_t)$ and $\mathrm{cov}(\tilde{j}_t, \tilde{N}_{t-1})$ are both zero.

that the correlation coefficients satisfy the following inequalities:

$$-\mathrm{Corr}(\Delta\tilde{Q}_t, \Delta\tilde{P}_t) \geq -\mathrm{Corr}(\Delta\tilde{Q}_t, \Delta\tilde{W}_{t-1}) \geq \mathrm{Corr}(\Delta\tilde{P}_t, \Delta\tilde{W}_{t-1}) > 0$$

where $\mathrm{Corr}(\tilde{Q}_t, \tilde{P}_t)$ and $\mathrm{Corr}(\tilde{P}_t, \tilde{W}_{t-1})$ are negative. These inequalities follow since

$$\frac{\mathrm{Corr}(\Delta\tilde{Q}_t, \Delta\tilde{W}_{t-1})}{\mathrm{Corr}(\Delta P_t, \Delta W_{t-1})} \simeq -\left[1 + \frac{(\frac{c}{1+\epsilon})^2 \frac{\sigma_A^2}{\sigma_j^2}}{1 + (\frac{\gamma}{1+\epsilon})^2 \frac{\sigma_B^2}{\sigma_j^2}}\right]^{1/2} \leq -1,$$

which approaches -1 as σ_A^2 approaches zero, and

$$\frac{\mathrm{Corr}(\Delta\tilde{Q}_t, \Delta\tilde{W}_{t-1})}{\mathrm{Corr}(\Delta Q_t, \Delta P_t)} \simeq \frac{\left\{1 - \frac{\gamma(1-\gamma\delta)}{(1+\epsilon)\delta} \frac{\sigma_B^2}{\sigma_j^2}\right\}\left[1 + \frac{(\frac{c}{1+\epsilon})^2 \frac{\sigma_A^2}{\sigma_j^2}}{1 + (\frac{\gamma}{1+\epsilon})^2 \frac{\sigma_B^2}{\sigma_j^2}}\right]^{1/2}}{\left[\left\{1 + (\frac{\gamma}{1+\epsilon})^2 \frac{\sigma_B^2}{\sigma_j^2}\right\}\left\{1 + \left(\frac{1-\gamma\delta}{\delta}\right)^2 \frac{\sigma_B^2}{\sigma_j^2}\right\}\right]^{1/2}}$$

≤ 1 for σ_A^2/σ_j^2 small.

The signs of these correlations and relative magnitude change as we change the assumptions on the relative magnitudes of σ_A^2, σ_B^2, σ_{AB} and σ_j^2. These expressions are given as possible illustrations.

Limiting Behavior

As the number of firms increase, the $C_i^{(n)}$ $i = 1 \sim 4$ converges to numbers by the strong law of large numbers, under suitable technical conditions described in the Appendix. Thus

$$C_i^{(n)} \to C_i, \quad i = 1 \sim 4 \quad \text{a.e.} \quad \text{as} \quad n \to \infty.$$

Instead of random walks we have deterministic time trends.

The only random vector affecting the macrodynamics is $H_2\bar{j}_{t-1}$ in (22). The covariance of $\tilde{Q}_t$ and $\tilde{P}_t$ are negative in this limiting case. Appendix contains technical discussions on the convergence.

Some Extensions

We can treat $j(t)$ as a random variable, or more generally let $j_i(t)$ to be $j_c(t)\zeta_i(t)$ where $j_c(t)$ is a random variable common to all firms and $\zeta_i(t)$ is firm i specific random effects with no substantial change in the limiting process. We can also relax the "i.i.d" assumption under which the micro and macro model convergence behavior has been proved. The random variables α's and β's can also be decomposed into common parts and idiosyncratic parts. These generalizations are useful since the macrodynamics are no longer deterministic but become random. For example, we can assume some or all of random variables indexed by i as exchangeable processes. This generalization is an attractive way to allow for positive correlations of shocks among firms to embody an idea of firm complementarity.

Concluding Remarks

By adopting a form for microeconomic units that is suited to incorporate macroeconomic effects as "mean field effects", we showed how the corresponding model for macroeconomic dynamics can be derived. The macro-dynamics naturally possess the co-integration effects without having them exogenously grafted by technical progress, for example. Alternative assumptions on stochastic processes such as exchangeability are yet to be examined but expected to present no serious technical difficulties.

As the number of micro units goes to infinity, macrodynamics become deterministic, if j_t is deterministic. Otherwise macrodynamics generate stochastic processes.

APPENDIX

We make the following assumptions on the stochastic processes:

1) $\alpha_i(t)$, $\beta_i(t)$, $q_i(0)$, $p_i(0)$, $n_i(0)$, $w_i(0)$ are all positive random variables.

2) $\{\alpha_i(t), \beta_i(t), t = -1,0,1,\ldots\}$, $i = 1,2,\ldots$ are i.i.d. with respect to i.

3) $\alpha_i(t)$ and $\beta_i(t)$ possess all the moments $E(|\alpha_i(t)|^\theta)$, $-\infty < \theta < \infty$.

4) The sequence $\{q_i(0), p_i(0), n_i(0), w_i(0);\ i = 1,2,\ldots\}$ is given and i.i.d. with respect to i. These random variables have all the moments.

On the assumption that the sequence of microvariables start with the same initial conditions

$$q_i^{(n)}(0) = q_i(0)$$

$$p_i^{(n)}(0) = p_i(0)$$

$$n_i^{(n)}(0) = n_i(0)$$

$$w_i^{(n)}(0) = w_i(0), \quad n = 1,2,\ldots$$

the stochastic process $x_i^n(t) = (q_i^{(n)}(t), p_i^{(n)}(t), n_i^{(n)}(t), w_i^{(n)}(t))'$ is uniquely specified.

<u>Claim 1</u>: For each t, there exists limiting microvariables $x_i^*(t)$ such that $x_i^{(n)}(t) \overset{P}{\to} x_i^*(t)$, $i = 1,2,\ldots.$

<u>Claim 2</u>: As $n \to \infty$,

$$\bar{Q}^{(n)}(t) \overset{P}{\to} \bar{Q}^*(t) = E(q_i^*(t)),$$

$$P^{(n)}(t) \overset{P}{\to} P^*(t) = E(p_i^*(t)q_i^*(t))/\bar{Q}^*(t),$$

$$\bar{N}^{(n)}(t) \overset{P}{\to} \bar{N}^*(t) = E(n_i^*(t)),$$

$$W^{(n)}(t) \overset{p}{\rightarrow} W^*(t) = E[w_i^*(t)n_i^*(t)]/\overline{N}^*(t).$$

<u>Claim 3</u>: The limiting sequence $\{x_i^*(t)\}$ $i = 1,2,\ldots$ is i.i.d. and satisfies the set of equations analogous to (25) with $A^*(t) = \overline{Q}^*(t)P^*(t)^{1/\eta}$, and $B^*(t) = \overline{N}^*(t)\ W^*(t)^{-\epsilon}\ P^*(t)^{\epsilon}$.

<u>Claim 4</u>: The limiting macrovariables satisfy the same set of equations as (20), (21) and (22) with the exception that $C_{i,t}^{(n)}$ is replaced by $C_i(t)$. Using $< >$ to denote expectation, they are

$$C_1(t-1) = < \beta_i(t-2)^{\gamma/c} >$$

$$C_2(t-1) = < \alpha_i(t)^{1-\delta}\ \beta_i(t-2)^{\gamma\delta/c} >/C_1(t)$$

$$C_3(t) = < \beta_i(t-1)^{1/c} >$$

and

$$C_4(t) = < \alpha_i(t)^{1/\eta}\ \beta_i(t-1)^{\gamma\delta/c} >/\ C_3(t).$$

Note that these $C_i(t)$'s are non-random. Now if we add a reasonable assumption:

5) $\{\alpha_i(t)\}$ and $\{\beta_i(t)\}$ are independent for each i and t.

Then dropping subscript i, since they are identically distributed by assumption

$$C_1(t) = < \beta(t-2)^{\gamma/c} >,$$

$$C_2(t) = < \alpha(t)^{1-\delta} >< \beta(t-2)^{\gamma\delta/c} >/\ C_1(t),$$

$$C_3(t) = < \beta(t-1)^{1/c} >$$

and

$$C_4(t) = < \alpha(t)^{1/\eta} >< \beta(t-1)^{\gamma\delta/c} >/\ C_3(t).$$

<u>Condition for $\tilde{C}_1 + \eta\tilde{C}_2 = 0$</u>

Suppose that $\tilde{\alpha} \sim N(0,\sigma_1^2)$ and $\tilde{\beta} \sim N(0,\sigma_2^2)$, i.e., $\alpha(t)$ and $\beta(t)$ have lognormal distributions.

Recall the relation

$$E\alpha^\theta = Ee^{\theta\tilde{\alpha}} = e^{\sigma_1^2\theta^2/2}.$$

Then

$$C_1 = \exp(\gamma^2\sigma_2^2/2c^2)$$

and

$$C_1C_2 = \exp(\sigma_1^2/2\eta^2 + \gamma^2\delta^2\sigma_2^2/2c^2),$$

and

$$\tilde{C}_1 + \eta\tilde{C}_2 = \sigma_1^2/2\eta - \gamma^2\delta\sigma_2^2/2c^2.$$

Hence the condition $\tilde{C}_1 + \eta\tilde{C}_2 = 0$ becomes

$$[c^2 - \epsilon^2\gamma^2\delta(1-\delta)]\sigma_1^2/\eta = \gamma^2\delta\sigma_2^2$$

or

$$\sigma_2^2/\sigma_1^2 = c^2/\gamma^2(\eta-1).$$

This condition is likely to be satisfied by a variety of parameter values.

<u>Condition for $\tilde{C}_3 - \epsilon\tilde{C}_4 + \epsilon\tilde{C}_2 = 0$</u>

Under the same set of assumptions, we obtain

$$C_3 = \exp \frac{1}{2} [\sigma_2^2/2c^2]$$

and

$$C_3C_4 = \exp \frac{1}{2} [\sigma_1^2/\eta^2 + \gamma^2\delta^2\sigma_2^2/c^2]$$

and

$$\tilde{C}_3 - \epsilon\tilde{C}_4 + \epsilon\tilde{C}_2 = (1+\epsilon - \epsilon\gamma^2)\sigma_2^2/2c^2.$$

This condition is met if and only if $\sigma_2 = 0$ since $\gamma < 1$.

Only when $\sigma_1 = \sigma_2 = 0$, both conditions are simultaneously met with lognormal α and β.

REFERENCES

Aoki, M., "Dynamics and Control of a System Composed of a Large Number of Similar Subsystems," in P-T Lin (ed.), Dynamic Optimization and Mathematical Economics, New York and London: Plenum Press, 1980.

__________, "Interaction among Economic Agents under Imperfect Information: an Example," in M. Aoki and A. Marzollo (eds.), New Trends in Dynamic System Theory and Economics, New York: Academic Press, 1979a: 57-103.

__________, "On Aggregation of Similar Dynamic Systems," in M. Singh and A. Titli (eds.), Large Scale Systems Engineering Applications, Amsterdam: North Holland, 1979b: 7-19.

__________, "On Fluctuations in Microscopic States of a Large System," in Y.C. Ho and S.K. Mitten (eds.), Directions in Large Scale Systems, New York: Plenum Press, 1976.

Dawson, D.A., "Critical Dynamics and Fluctuations for a Mean-Field Model of Cooperative Behavior," J. Stat. Physics, 31 (1983): 29-85.

Granger, C.W.J., "Co-integrated Variables and Error-Correcting Models," Univ. of California-San Diego Discussion Paper 83-13, 1983.

Hertz, J., A. Krogli, and R.G. Palmer, Introduction to the Theory of Neural Computation, Addison-Wesley, 1991.

Iwai, K., Disequilibrium Dynamics, Monograph 27, Cowles Foundation, Yale University, 1981.

Kelejian, H., "Aggregation and Disaggregation of Nonlinear Equations," in J. Kmenta and J.B. Ramsey (eds.), Evaluation of Econometric Models, New York: Academic Press, 1980, pp. 135-52.

Lippi, M., "On the Dynamic Shape of Aggregated Error Correction Models," J. Econ. Dyn. Control, 12 (1988): 561-85.

Miyahara, Y., "An Essay on the Models of Disequilibrium Dynamics: From Microdynamics to Macrodynamics," Oikonomika (in Japanese), 27 (1990): 115-23.

Peterson, C., and J.R. Anderson, "A Mean Field Theory Learning Algorithm for Neural Networks," Complex Systems, 1 (1987): 995-1019.