ANALYSIS OF U.S. REAL GNP AND UNEMPLOYMENT INTERACTIONS: STATE SPACE APPROACH

by

Masanao Aoki[+]

and

Riccardo Fiorito[++]

UCLA Department of Economics
Working Paper #647
February 1992

ABSTRACT

Instead of the more common unemployment rate, the time series data of the level of U.S. unemployment is used together with U.S. Real GNP, and later also with the U.S. money stock, to identify a bivariate and trivariate structural model. The model is then used to examine the interaction of the unemployment level of real GNP in the business cycle frequencies. The commonly perceived Okun's law is shown to disappear in the trivariate model.

[+] 2263 Bunche Hall, Department of Economics, UCLA, Los Angeles, CA 90024-1477 USA

[++] Piazza San Francisco 7, Department of Economics, University of Siena, Siena 53100, Italy

1. Introduction

Okun claims that returns to labor can be inferred by regressing changes in unemployment on percentage changes in the real GNP. This procedure, however, overestimates the returns to labor if labor supply is increasing over time. A support for increasing returns to labor can be made by finding a smaller response of employment to output changes which could be explained, for example, by labor hoarding. Labor hoarding, however, seems to be more likely in Europe than in the USA since employment in the U.S. has been growing at about the same rate as the real GNP over the last thirty years.

The purpose of this paper is to investigate whether unemployment in the U.S. may be explained by co-movement in output only, for example, as in Okun (1962), Evans (1989) and Blanchard-Quah (1988), or also is affected by monetary shocks. Moreover, we want to ascertain whether output and unemployment respond symmetrically to demand and supply disturbances i.e., whether Okun's law still holds in the U.S. when supply factors and money are introduced. For this purpose we build a three-variable structural model for the levels of unemployment (not the rate), of the real GNP and M1, and identify the model using trivariate time series after suitable detrending them as we later explain. In our model the unemployment level changes reflect shocks in both labor supply and productivity. Unlike Beveridge-Nelson (1981), cycle and trends are not treated separately but are related through a two-step procedure (Aoki, 1989) where short-run fluctuations can affect long-run behavior. Cycles here are obtained as residuals from the first stage of the state space modeling, which lets the algorithm to select common stochastic trend for the three macroeconomic series. Residuals of the first stage display cross and serial correlation, which is modeled in the second stage to produce a vector-valued innovation process. The residuals obtained in the second stage should be serially uncorrelated but possibly mutually correlated. Instead of adopting -- as in standard VAR practice -- an arbitrary, i.e., non-unique Choleski decomposition, we utilize identifying restrictions of the contemporaneous shocks to evaluate a structural model and its dynamic response to structural disturbances.

The paper is organized as follows. Section 2 posits the structural model used in this paper to identify structural innovation to be used in multiplier analysis. Responses of the estimated model to various structural shocks are discussed in Section 3. The paper concludes with Section 4. To be self-contained, the Appendix contains a brief but complete description of the detrending and state space model procedure used in the paper.

2. The Structural Model

In traditional VAR time series modeling, a (suitably detrended) weakly-stationary data vector y is modeled by

$$\phi_0 y_t = \phi(L) y_t + \eta_t \tag{1}$$

where η_t is a vector of structural disturbances. The matrix ϕ_0 is assumed to be nonsingular. This structural model is decomposed into a contemporaneous part, ϕ_0, and an unrestricted dynamic part, $\phi(L) = \phi_1 L + \phi_2 L^2 + \ldots$, where $L^j y_t = y_{t-j}$. Usually ϕ_0 is a sparse matrix whose zero restrictions are guided by economic theory. The model can be identified by the procedure used in Bernanke (1986) and Sims (1986), for example.

In (1), the innovation vector of y_t with respect to its own past is $e_t = y_t - E(y_t | y_{t-1}, y_{t-2} \ldots)$. On the assumption that the σ-field generated by past y's is the same as that generated by past structural shocks, we have

$$\eta_t = \phi_0 e_t, \tag{2}$$

Assuming that structural disturbances are uncorrelated, the covariance matrix $E(\eta\eta') = \Sigma$ is diagonal while the covariance matrix of the unrestricted state space model $E(ee') = \Omega$ will generally be non-diagonal. The covariance Σ is related to the innovation covariance matrix by

$$\Sigma = \phi_0 \Omega \phi_0'. \tag{3}$$

Of the $p(p+1)/2$ independent relations in ϕ where $p = \dim y_t$, p of them are used to estimate the diagonal elements in the covariance matrix Σ. We may impose $p(p-1)/2$ zero restrictions on the off-diagonal terms of ϕ_0 since its diagonal terms may be taken to be one without loss of generality. We will return to the specification of matrix ϕ_0 later.

Instead of VAR modeling, we use a state space representation for the data series and estimate the innovation vector time series $\{e_t\}$ and its covariance matrix, Ω, to identify ϕ_0 and Σ. The specifics of this procedure is similar to the one in Aoki (1990, Sec. 7.4).

Let us now formulate our structural model for the short-run dynamics as follows:

$$U_t = \alpha GNP_t + lags + \eta_{1t} \quad \text{(Aggregate Supply)} \tag{4}$$

$$GNP_t = \beta U_t + \gamma M_t + lags + \eta_{2t} \quad \text{(Aggregate Demand)} \tag{5}$$

$$M_t = lags + \eta_{3t} \quad \text{(Money Feedback Rule)} \tag{6}$$

where U, GNP and M denote respectively logarithms of the unemployment level, real GNP and nominal money stock (M_t) in the U.S. Here, $p = 3$ and the model is just identified. The level of unemployment U_t measures the excess supply of labor in unit of workers:

$$U_t = L_t^s - L_t^d.$$

Labor supply is approximated for statistical measurement by labor force data and is allowed to vary over time

$$L_t^s = \epsilon_{1t},$$

where ϵ_{1t} reflects stochastic factors affecting both population and participation rates (approximated by civilian employment). Once labor demand is extracted from the linear, short-run, production function posited as

$$L_t^d = \alpha GNP_t + \epsilon_{2t},$$

it is apparent that η_{1t} in (1) is equal to

$$\eta_{1t} \equiv \epsilon_{1t} - \epsilon_{2t}$$

which combines both labor supply and productivity shocks. Since these variables have opposite effects on the real wage, we can consider it to be approximately constant. Thus price and wage equations are ignored. These assumptions are made, of course, for simpler analysis, yet may be expected to hold reasonably well for the U.S. data which show that the real wage has a much smaller variability than those of labor force and employment.

Equation (5) is a condensed version of the IS-LM model. Aggregate demand is determined by consumption and investment components: consumption expenditure is affected mostly by the employment level -- thus negatively by unemployment -- while investment expenditure depends inter alia on interest rate, thus on the nominal money stock. Fiscal and monetary shocks are captured by η_{2t} and η_{3t}, where the latter denotes unpredictable money stock innovations.

Because of information lags, monetary policy, (6), is assumed not to react to contemporaneous shocks in GNP or unemployment. Money does not affect contemporaneously the supply equation because of time lags.

Lacking an interest rate equation, we cannot separate out money supply from money demand disturbances.[1] Similarly, participation and productivity shocks cannot be disentangled in the supply side. Despite this, we decided to keep the model in the present form in order to have a direct comparison with the output/unemployment studies in the literature where typically no money is introduced and only casual attention is paid to detrending. The data of the model come from OECD sources (Main Economic Indicators, Paris, several years), are quarterly, seasonally adjusted, and range from 68:1 to 88:4.

After detrending (as discussed in the Appendix), U, GNP, and M in (4)-(6) are interpreted as deviational variables from trend. They are then put in the form of (1) with matrices

$$\phi_0 = \begin{bmatrix} 1 & -\alpha & 0 \\ -\beta & 1 & -\gamma \\ 0 & 0 & 1 \end{bmatrix}, \quad \Sigma = \mathrm{diag}(\sigma_1^2, \sigma_2^2, \sigma_3^2).$$

Then (3) is solved uniquely for the parameters α, β, γ, and σ_i^2, $i = 1,2,3$. It is of some interest to compare this model with a bivariate model in which M is dropped:

$$U_t = \theta GNP_t + \nu_{1t} + \text{lags}$$

$$GNP_t = \nu_{2t} + \text{lags}$$

with

$$\phi_0 = \begin{bmatrix} 1 & -\theta \\ 0 & 1 \end{bmatrix}, \quad \Sigma = \mathrm{diag}(\sigma_1^2, \sigma_2^2).$$

We can also identify

$$U_t = \theta GNP_t + \nu_{1t} + \text{lags}$$

$$GNP_t = \mu U_t + \nu_{2t} + \text{lags}$$

---

[1] This issue is specifically addressed by Gali (1989).

by minimizing the sum $\sigma_1^2 + \sigma_2^2$. This procedure, however, produces nearly zero μ, i.e., the same model as above.

Panel I of Figure 1 (with the horizontal axis from 1 to 45) shows for the response of the bivariate model to structural shock to U (supply shock). Panel II (with the horizontal axis from 46 to 90) are the responses to the structural shock to GNP (demand shock). Note that GNP and -U respond approximately the same even though demand shocks produce larger fluctuations. This symmetric response disappears when money is introduced. The matrix Ω is:

$$\Omega = \begin{bmatrix} .159 & -.0196 \\ -.0196 & .0099 \end{bmatrix} \times 10^{-2}$$

We estimate $\theta = 1.96$, $\sigma_1^2 = .12 \times 10^{-2}$ and $\sigma_2^2 = .01 \times 10.^{12}$

With the three variables, we estimate

$$\Omega = \begin{bmatrix} .172 & -.0196 & -.0135 \\ -.0194 & .019 & -.009 \\ -.0135 & .009 & .019 \end{bmatrix} \times 10^{-2}.$$

Using the estimated Ω (from the second stage of the state space modeling procedure described in Appendix), we obtain[2]

$$\alpha = 1.01 \qquad \sigma_1^2 = .20 \times 10^{-2}$$

$$\beta = .20 \qquad \sigma_2^2 = .01 \times 10^{-2}$$

$$\gamma = -.63 \qquad \sigma_3^2 = .02 \times 10^{-2}$$

---

[2] An alternative structural model with $\phi_0 = \begin{bmatrix} 1 & -\alpha & -\delta \\ -\beta & 1 & -\gamma \\ 0 & 0 & 1 \end{bmatrix}$ can be identified by minimizing $\sigma_1^2 + \sigma_2^2$. This result is:

$$\alpha = 1.43,\ \beta = .13,\ \delta = 1.32,\ \gamma = -.59,$$

$$\sigma_1^2 = .12 \times 10^{-2},\ \sigma_3^2 = .01 \times 10^{-2},\ \sigma_3^2 = .02 \times 10^{-2}$$

The impulse responses of these two models are qualitatively very similar and for this reason this model is dropped from further consideration.

Figures 2 through 4 show impulse responses to one-standard deviation shocks to U, GNP and M respectively. The model is estimated with a five dimensional state vector. The model has eigenvalue .97, .91, $.7 \pm j.27$ and .23. Of these five, .97 represents a dynamic mode which is longer than the usual business cycle time span of 20 quarter or so. (Note: $.97^{20} = .54$, $.91^{20} = .15$). The pair of complex eigenvalues has magnitude .74 and the period of 8.5 quarters. We then deflate A to eliminate the eigenvalue .97. The resulting dynamic multipliers are the ones shown in Figs. 2-4.

3. Effects of Structural Shocks

In this section we describe structural responses of the identified model to demand, supply and monetary shocks. Unlike the bivariate model output and unemployment levels respond asymmetrically. The level of unemployment reacts more to all the shocks than the level of real GNP. Unlike Blanchard and Quah (1988), we do not assume that supply shocks are permanent even though the effects of all shocks are in fact long-lasting. Monetary shocks have smaller impact on real output than on unemployment.

Supply Shocks

A supply shock has a much stronger impact on unemployment than on real GNP, i.e., there is no behavior consistent with Okun's law. This is hardly surprising in view of the fact that a shift in labor supply may raise unemployment without increasing productivity. With increased productivity, labor demand is also reduced and unemployment has two reasons to be larger. Since increases in productivity cannot entirely account for the gap between unemployment and real GNP responses, it appears that labor supply shocks are much more relevant in explaining unemployment than recognized by Okun's law. Since we cannot disentangle in this model shifts in supply for labor from productivity shifts, the transmission of the overall supply shock can only be tentatively interpreted. By comparing, however, Figures 2 and 3 it appears that real GNP is driven more by demand rather than by supply shocks, even though this could reflect the fact that our combined supply shock might be characterized by a large labor supply component rather than by exogenous shifts to the production function which are not explicitly introduced in our model. Another advantage of introducing money in the output/unemployment relationship is that the difference between the responses of nominal money stock and real GNP can be approximately interpreted as the implied price response to demand and supply shocks (if velocity of money remains approximately constant). In this case one can see that a supply shock tends, as expected, to reduce prices. Prices move countercyclically and absorb rather slowly the initial shock.

Demand Shocks

Unlike previous studies (Blanchard-Quah, 1988) the main difference between supply and demand shocks is not their duration since both tend to decay rather slowly over time. Also in this case, the main effect of introducing money to the bivariate model is that the symmetry between output and unemployment is lost in a way that is incomparable with Okun's law. Unemployment response is still higher than that of real GNP. The latter is affected more by demand than by supply shocks. In the meantime expenditure innovations raise GNP more in the very short-run, increasing inflation, and to an extent which again is not compatible with Okun's law. Indeed, most of the increase in GNP happens when the shock starts. Price reaction is not immediate but is fairly strong if we compare nominal money and real GNP responses. By ignoring again changes in velocity, one can see that most of the inflationary pressures occur in about 2 years and tend to be absorbed rather slowly over time. As expected, we find that prices move in this case procyclically or that demand shocks produce a positive covariance between output and prices (Blanchard, 1989). What is remarkable in our results is also that much stronger reduction of unemployment has to be attributed to cyclical expansion since labor supply should not be reduced by a demand shock.[3] Thus if we compare the implied employment and output responses, decreasing rather than increasing labor returns are found by our analysis. This result seems to fit well enough some of the typical features of the U.S. labor market where labor force is very heterogeneous in terms of skill and seniority, and most of the increase in employment over the sample period did not occur in manufacturing but in small or labor intensive services (see Table 1). Combining all these factors, it is reasonable to argue that a large part of the spectacular increase in labor supply has been absorbed by non-manufacturing firms. This implies in some cases a selective demand for labor that makes productivity countercyclical and that it is not compatible with Okun's law since secondary workers (who have lesser skill and productivity) are typically hired in boom phases and fired in recession times. In others, labor hoarding may occur which makes productivity procyclical but which does not imply *per se* that Okun's law holds since labor supply also increases over time.

Monetary Shocks

Monetary innovations have a small impact on real variables and seem to not be sustained over time. However, also in this case, unemployment and not real GNP has the larger response. The monetary shock has a higher

---

[3] The empirical evidence shows that labor supply is procyclical, i.e., that "discouragement" effect dominates the "additional" worker effect (Bowen-Finegan, 1969).

TABLE 1

Variability and Growth of Sectoral Employment and GNP

	Employment(*)		GNP		
	Variability	Growth	Variability	Growth	$\dot{\pi}$(**)
Agriculture(+)	0.04	-0.9	0.13	2.0	2.9
Mining	0.21	1.2	0.05	-2.0	-1.4
Construction	0.12	2.4	0.08	-2.0	-2.6
Manufacturing	0.04	0.0	0.17	2.7	2.7
Durable Goods	0.05	0.0	0.18	2.8	2.8
Nondurable Goods	0.03	0.0	0.15	2.6	2.6
Transportation	0.07	1.2	0.18	1.3	0.1
Trade	0.17	3.0	0.19	3.5	0.5
Wholesale	0.14	2.4	-	-	-
Retail	0.18	3.2	-	-	-
Finance and Insurance	0.20	3.5	0.19	3.4	-0.1
Services	0.26	4.5	0.25	4.2	-0.3
Government	0.11	2.0	0.07	1.2	-0.8

(*)Employees on non-agricultural payrolls

(+)Labor force data

(**)Productivity growth

Source: Economic Report of the President, January 1989.

inflationary impact at the beginning but has a smaller rate than that implied by demand shock. After the initial negative surprise, the effect on real GNP is negligible even though one has to consider that all types of shocks seem to produce little deviations of real GNP from its equilibrium path.

4. Conclusions

In this paper we apply a two-stage state space approach (Aoki, 1988) to estimate a structural model for unemployment, real GNP and M_1 in the U.S. While the model is in the spirit of the so-called structural VAR approach as in Bernanke (1986), Sims (1986), the state space model is utilized to extract stochastic trends from non-stationary data, without constraining the series to have one or more unit roots. Cyclical variables, which are modeled in the second stage, do not have persistent response to innovations.

A few constraints introduced in the model are economic rather than statistical in nature, which are designed to relate estimated innovations to corresponding set of structural disturbances.

Okun's law never holds in the output-unemployment relationship when money stock is introduced. Unemployment response is always larger than real GNP response to any type of shock. Real GNP is affected more by demand than by supply shocks and shows a little response to monetary innovations in terms of M_1.

REFERENCES

Aoki, M., State Space Modeling of Time Series, Springer-Verlag, Heidelberg, 1987.

__________, "On Alternative State Space Representation of Time Series Models," Journal of Economic Dynamics and Control, (1988): 595-607.

__________, "State Space Models for Vector-Valued Time Series with Real and Complex Random Walk Components," mimeo, UCLA, 1988.

__________, "A Two-Step State Space Time Series Modeling Method," Computers Math Applications, 1989.

__________, State Space Modeling of Time Series, Second Revised and Expanded Edition, New York and Heidelberg: Springer-Verlag, 1990.

Bernanke, B.S., "Alternative Explanations of the Money-Income Correlation," Carnegie-Rochester Conference on Public Policy, 1986.

Beveridge, S. and C.R. Nelson, "A New Approach to Decomposition of Economic Time Series into Permanent and Transitory Components with Particular Attention to Measurement of the 'Business Cycle'," Journal of Monetary Economics, 1981.

Blanchard, O.J., "A Traditional Interpretation of Macroeconomics Fluctuations," American Economic Review, (September 1989): 655-673.

__________ and D. Quah, "The Dynamic Effects of Aggregate Demand and Supply Disturbances," American Economic Review, (September 1989): 655-673.

Bowen, W and T.A. Finegan, The Economics of Labor Force Participation, Princeton: Princeton University Press, 1969.

Evans, G., "Output and Unemployment Dynamics in the United States: 1950-1985," Journal of Applied Econometrics, (1989): 213-237.

Gali, J., "How Well is the IS-LM Model Fit Postwar U.S. Data?," mimeo, MIT, June 1989.

Okun, A.M., "Potential GNP: Its Measurement and Significance," American Statistical Association, Proceedings of the Business and Economics Statistics Section, 1962.

Park, J.Y. and P.C.B. Phillips, "Statistical Inference in Regression with Integrated Processes: Part 1 and 2," Econometric Theory, 4 (1988): 468-497; 5 (1989): 95-132.

Phillips, P.C.B. and B. Hansen, "Statistical Inference in Instrumental Variables Regression and I(1) Processes," Rev. Econ. Stud., 57 (1990): 99-125.

Sargan, J.D., "Wages and Prices in the United Kingdom: A Study in Econometric Methodology," in Hart et. al. (eds.), Econometric Analysis for National Economic Planning, London: Butterworths, 1964.

Sims, C.A., "Are Policy Models Usable for Policy Analysis," Quarterly Review, Federal Reserve Bank of Minneapolis (1986).

APPENDIX

A.1 State Space Representation for Trends and Cycles[4]

Suppose that the data series have rational spectral density and that the dynamics of linear time-invariant data-generating process is such that all the eigenvalues of the dynamic matrix A are not greater than one in magnitude. We collect all eigenvalues of magnitude greater than some critical value ρ into one class, C_1 as representing slower or longer-run dynamic modes, and the rest into the other class C_2. When there is a clear gap between these two classes of eigenvalues, the exact value of ρ does not matter. For example, suppose that there is no eigenvalue of magnitude between .89 and .97. Then any value of ρ between .89 and .97 will classify all the eigenvalues into two mutually exclusive classes.

We next construct aggregate state vectors and their dynamic equations by introducing a set of basis vectors which spans the right invariant subspace of matrix A associated with the eigenvalues in C_1, and another set of basis vectors to span the left invariant subspace of A associated with the eigenvalues in C_2. This particular association yields what we call common mode dynamic representation †[5] of longer-run or slower-moving components of the data series. Suppose that there are k eigenvalues of Class 1. Then let P be $n \times k$ such that

$$AP = P\Lambda$$

where

---

[4]This section is based in part on Aoki (1988, 1989a).

[5]If the opposite assignment is made, we obtain an error-correction model representation in the sense of Sargan (1964) of the original state space model. See Aoki (Sec 11.4, 1990).

$$P'P = I_k$$

and S be $n \times (n\text{-}k)$ such that

$$S'A = NS'$$

where

$$S'S = I_{n-k}.$$

Note that $S'P = 0$. Then, the original state transition equation for an n-dimensional state vector, z_t, becomes in the new coordinate system

$$z_t = Ps_t + Sf_t$$

i.e., $s_t = P'z_t$ and $f_t = S'z_t$ are the two aggregate (sub-) state vectors; the vector s_t representing slow modes and f_t representing fast modes. From the dynamic equation $z_{t+1} = Az_t + Be_t$, they evolve with time according to

$$\begin{bmatrix} s_{t+1} \\ f_{t+1} \end{bmatrix} = [P,S]^{-1} A[P,S] \begin{bmatrix} s_t \\ f_t \end{bmatrix} + [P,S]^{-1} Be_t$$

noting that

$$[P,S]^{-1} = \begin{bmatrix} P' \\ S' \end{bmatrix}$$

the dynamic equation becomes

$$\begin{bmatrix} s_{t+1} \\ f_{t+1} \end{bmatrix} = \begin{bmatrix} \Lambda & P'AS \\ 0 & N \end{bmatrix} \begin{bmatrix} s_t \\ f_t \end{bmatrix} + \begin{bmatrix} P' \\ S' \end{bmatrix} Be_t \tag{1}$$

which is related to the data vector by

$$y_t = CPs_t + CSf_t + e_t \tag{2}$$

Equation (2) expresses the data vector as the sum of the slow modes, fast modes, and the innovation vector, and (1) is the recursive representation of the dynamics. If there is only one eigenvalue close to one, then Λ is that eigenvalue. Then s_t is a scalar-valued "trend" term. If $\lambda = 1$, then s_t is a random walk term. (The residuals are weakly dependent.)

A practical procedure to produce the representation (2) is to use the algorithm described in Aoki (1987) in two steps.[6] We describe the procedure for the case in which there is one eigenvalue λ near 1. To eliminate the effects of nonzero initial condition, we normalize each component of the data vector by its initial value and take its logarithms, if all components of the data vector seem to be trending. If not, only those trending components are treated this way.

Let y_t be the data vector, thus properly scaled. The first stage of the algorithm builds a model:

$$y_t = C\tau_1 + w_t$$

$$\tau_{t+1} = \lambda\tau_t + r_t$$

where τ_t is a common "trend" term (the s_t variable in (2)), and w_t is the residuals. The series r_t is the residual $\tau_{t+1} - \lambda\tau_t$, where λ is estimated from $\{\tau_t\}$.

Since the variable τ_t removes the effects of the slower modes which contribute to "nonstationary" appearance of the series, the residuals w_t should be weakly stationary.

Let $\bar{w}$ be the sample mean of w_t. The $w_t - \bar{w}$ is modeled again by the algorithm which produces

$$w_t - \bar{w} = H\chi_t + e_t \tag{3}$$

$$\chi_{t+1} = F\chi_t + Ge_t$$

and where e_t is the innovation in w_t. Using L^{-1} as the lead operator,

[6]An improved way of extracting a common trend series is mentioned in Aoki (1990). Alternatives are discussed in Phillips and Hansen (1990) or Park and Phillips (1988,1989).

as in $L^{-1}\tau_t = \tau_{t+1}$, the transfer function from e_t to w_t is given by (ignoring the $\bar{w}$ term) $I + H(L^{-1}I-F)^{-1}G$. This is the closed form expression for the shorter-run multiplier. The infinite series in L is the Wold decomposition for w_t.

The variable τ_t captures the slow dynamic modes (trend) (if such exists) common to the component series in the data vector y_t. The matrix C distributes or disaggregate these modes (trends) to individual series. Therefore the residuals $r_t = \tau_{t+1} - \lambda\tau_t$ is related to the innovation vector e_t in w_t by $r_t = \phi x_t + be_t$ with some ϕ and b.

A.2 Stochastic Trends and Related Cycles

The modeling procedures described in Aoki (1990, chapter 9 and 11) are used. The first stage of the model has been estimated taking one-dimensional state vector (n=1) and selecting a Hankel matrix (J=K=1) to describe the autocovariance of the observation vector. The one dimensional state was selected because of the ratio between the first and second singular value is large enough to justify modeling of the largest eigenvalue (.988) separately as the nearly-integrated root of the state vector. The residuals from this step are used in the second stage modeling as weakly stationary.

The first stage parameters are:

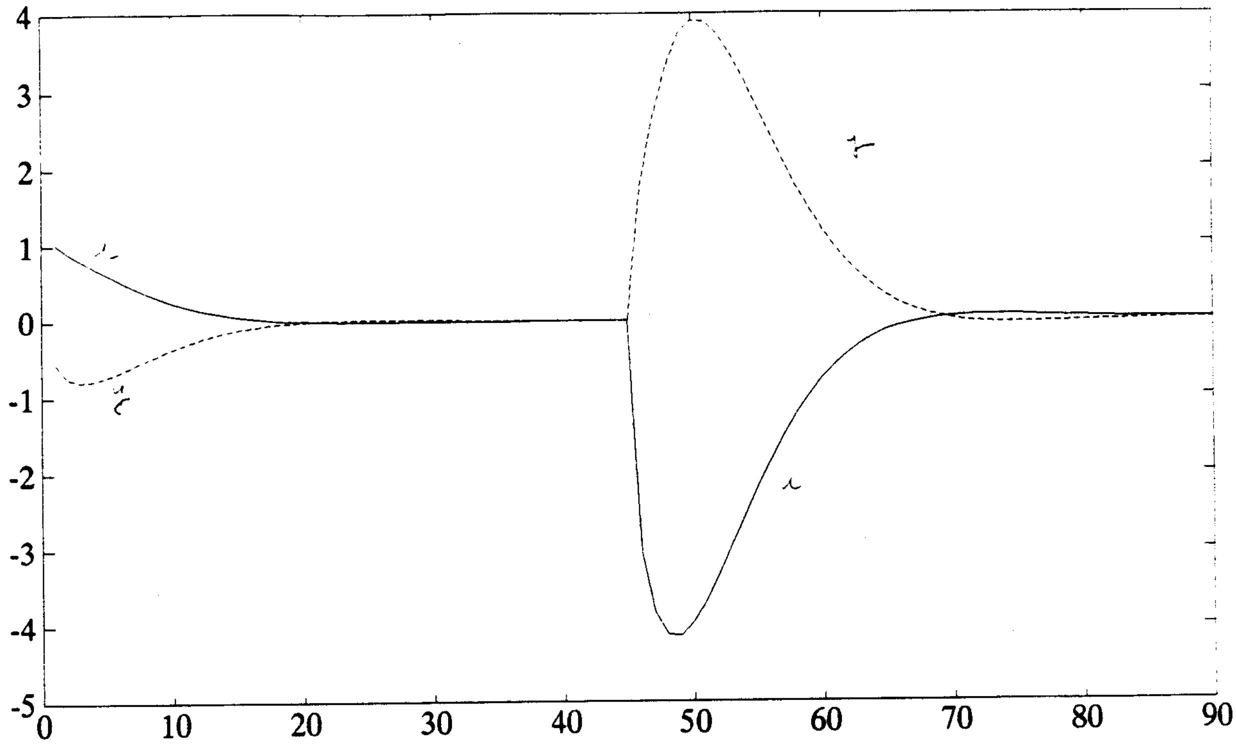
$$\lambda = .988, \quad C = \begin{bmatrix} .44 \\ .45 \\ .78 \end{bmatrix}$$

The stochastic trend dynamics is disaggregated by the elements of matrix C

When a 3×3 matrix Λ is estimated from

$$y_{t+1} = \Lambda y_t + \eta_t$$

using the least squares, the eigenvalues of the matrix Λ are .985, .977 $\pm$ j.015. This pair of complex eigenvalues has a period of over 400 quarters and this pair of complex roots could be spurious due to statistical errors in data. This pair will be difficult to distinguish from two real eigenvalues near .97 when sample errors are considered. In the state space modeling, in addition to eigenvalue .988, .966, .908 are the eigenvalues with magnitude. The roughly correspond to .9 or larger. We extract $.70 \pm .27j$, with a period of 17.3 quarters as the eigenvalue of magnitude .74.



I

II

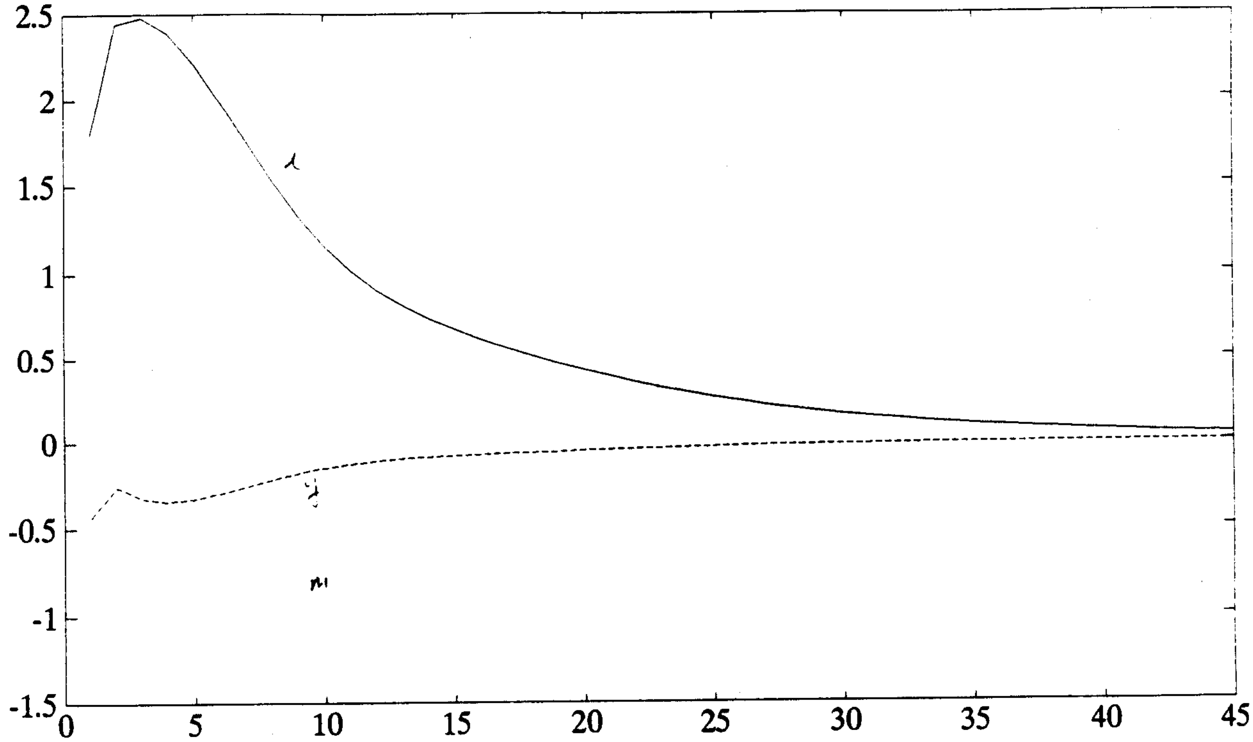


Fig. 2

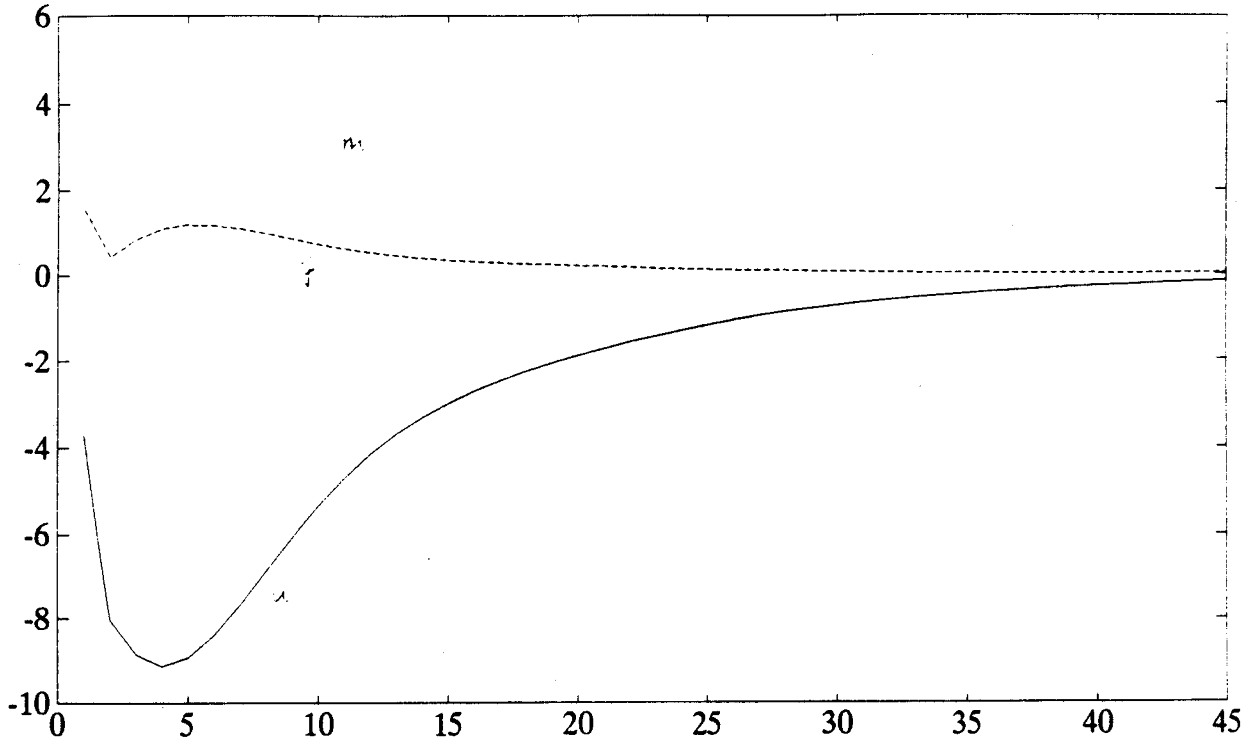


Fig 3

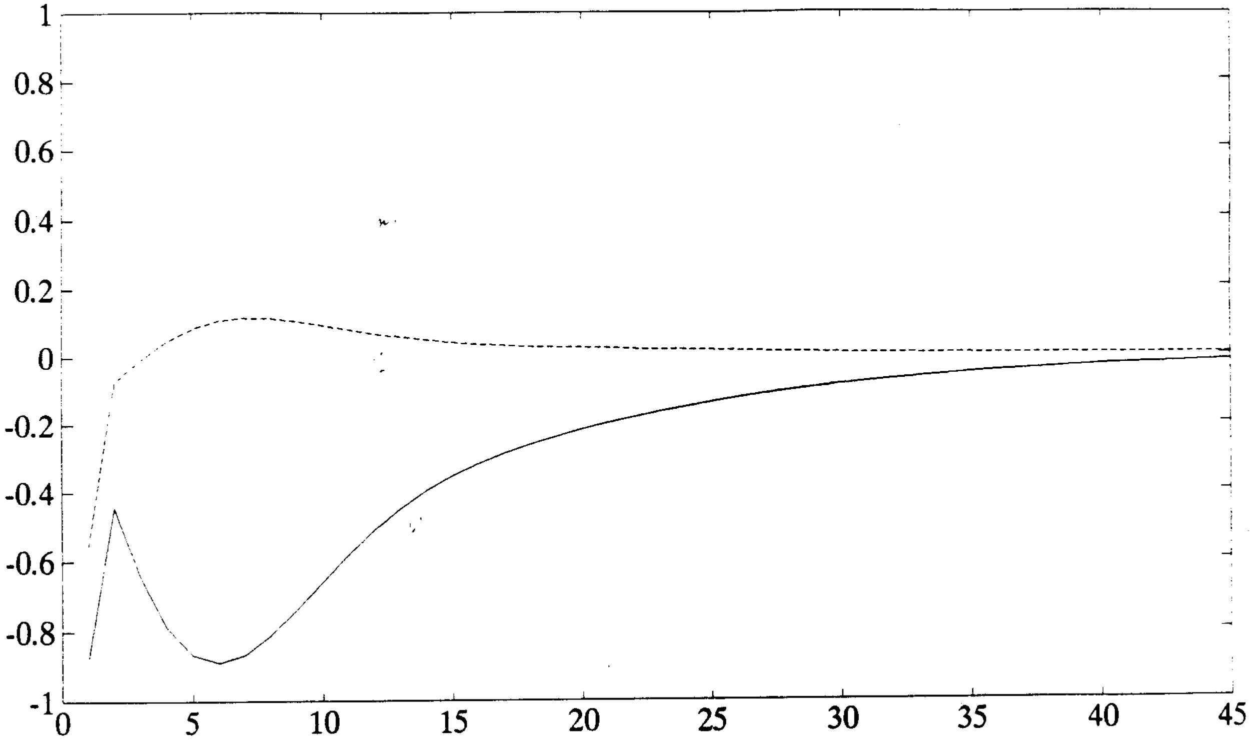


Fig 4