THE WELFARE EFFECTS OF INFLATION The Consequences of Price Instability on Search Markets

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The Consequences of Price Instability on Search Markets

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ABSTRACT

There is considerable disparity between popular opinion about the welfare loss due to inflation and formal estimates of this loss. This paper goes part of the way in filling that gap, by taking account of the association of higher inflation with higher relative price variability that previous studies have ignored.

I model a market for a homogeneous good, in which buyers search for low prices and sellers set prices to maximize profits, to analyze the effects of relative price variability on information acquisition, price setting, and equilibrium price distributions. Buyers react to instability by holding smaller information stocks, that is, by accepting higher real prices. This induces firms to increase their markups and allows inefficient firms to increase their sales. Production gets reallocated towards higher-cost producers. I show that the causality runs from inflation to market structure and performance.

Thus, the main consequences of relative price variability are higher real prices (a real income loss for consumers) and a lesser ability of the price system to screen out inefficient competitors. Combining these findings with the previously studied effects of inflation level and variability, we get a much larger welfare loss than the previous literature found.

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I. INTRODUCTION

The gap between economists' views and their professional justification has been nowhere greater than in the area of inflation. Fischer (1986, p.3)

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It appears to remain the case that the man-in-the-street notions of the costs of inflation have not been formalized in rigorous theoretical models. Handbook of Monetary Economics (1990, p. 1046)

There are two possible explanations for the considerable disparity between popular opinion and formal estimates of the welfare consequences of inflation. The first is that the man-in-the-street notion of inflation is incorrect; the second is that formal work has not captured all relevant dimensions of the problem. This research takes the latter view and develops a model to study the welfare effects of one of the pervasive characteristics of inflationary processes -- relative price variability.

There is extensive evidence that inflation is positively correlated with the variability of prices across markets. More recently, inflation has been shown to be positively related to the variability of prices across sellers of the same good. 2

¹See Parks (1978) for the US and the Netherlands, Fischer (1981) for the US, Domberger (1987) for the UK, Van Hoomissen (1988) and Lach and Tsiddon (1990) for Israel, and Palerm (1991) for Mexico.

²Van Hoomissen (1988), Lach and Tsiddon (1990) and Tommasi (1992).

Furthermore, Tommasi (1992) shows that the duration of real prices greatly diminishes at high inflation, i.e., that inflation lowers the informativeness of current prices about future prices. In a highly inflationary environment, it is hard to establish who the low-price sellers are, since the price observed today is not a good predictor of future prices.

In this paper, I show that price instability moves the economy away from perfect competition, generating the loss of many of its efficiency properties. I analyze a market for a homogeneous good being sold by atomistic firms. Buyers purchase the good every period. Each period they can visit as many stores as they like. Each visit entails a (search) cost, which is different for different consumers. This heterogeneity maps into downward sloping demands for individual sellers, who set prices to maximize expected profits.

Price instability derives from inflation-induced cost variability. The information that current relative prices convey about future relative prices is depreciated. Buyers react by holding smaller information stocks. Within this sequential-search model, this translates into higher reservation (acceptance) prices; that is, consumers become less choosy. The total amount of resources spent on search may either increase or decrease -- as is well known in capital theory, a higher depreciation rate implies smaller stocks, but investment flows may move in either direction.

In this model informational imperfections generate market imperfections. Sellers face downward sloping demand curves because buyers are not fully informed; if they were, all output would be produced by the lowest cost firm. In such a world, inflation

exacerbates the informational problem and hence increases market power. The increase in consumers' acceptance prices shifts upward the demand curve faced by the individual seller. This has two effects. First, sellers charge higher prices (they increase their markups). Second, high-cost producers become able to charge prices high enough to cover their costs. That is, there are real resource costs on top of the redistribution against consumers. If individual buyers' demands are elastic, we get the standard deadweight loss. Additionally, production gets allocated toward high-cost producers, increasing overall production costs. Contrary to the "administered inflation" hypothesis that establishes causality from markups to higher inflation, my model predicts the causal relationship to run from inflation to market structure and performance.

This paper is organized as follows. Section II reviews the related literature. Section III presents the model, and Section IV contains the main results. Section V summarizes and concludes the paper.

³See Domberger (1983) for a description of that hypothesis.

II. THE LITERATURE ON MICROECONOMIC CONSEQUENCES OF INFLATION

Inflation affects the economic system through many avenues, and different studies have concentrated on different aspects of the phenomenon⁴. Classical work has focused on the effects of anticipated inflation that operate through higher nominal interest rates. The role of the uncertainty and variability of the inflation rate over time has been treated by Lucas (1972, 1973), Cukierman (1984), Ball (1988) and many other authors. Inflation and relative price variability have been discussed by Vinig and Elwertowski (1976), Parks (1978), and Jaffee and Kleiman (1977). While the previous literature recognizes the positive relationship between inflation and relative price variability, it does not formalize the welfare implications. It is a folk theorem that inflation-induced "excess" price variability generates inefficiencies in resource allocation. I formalize that result here in the context of an imperfect information framework.

The findings about relative price variability and uncertainty in inflationary situations motivated Sheshinski and Weiss (1977, 1983) to introduce the notion of a "menu cost," i.e., a cost of changing nominal prices. Such a cost makes the continuous adjustment of nominal prices (to maintain real ones constant) a suboptimal

⁴See the first chapters of Fischer (1986) for a listing of the real effects of inflation present in the literature. Also (for a "micro" treatment of a traditionally "macro" topic) see Casella and Feinstein (1990), the pioneering articles in Phelps et al. (1970), and Carlton (1983).

strategy. The optimal thing to do is to follow an (S,s) rule, i.e., to allow the real price to fall to s before adjusting the nominal price to reach the real level S. They show that the range (S-s) is increasing in inflation. Their welfare analysis concentrates on sellers (they just postulate a demand curve), and their conclusion is that firms are hurt by being away from their profit maximizing point due to inflation.

Benabou (1988) closes their analysis by adding an explicit search- theoretic analysis of the consumer problem. The previously mentioned increased dispersion induces consumers to be more informed in equilibrium. This is a standard result in search theory: a spread is beneficial given the possibility of truncating the undesirable part of the distribution. The increased search intensity reduces prices on average (a fall in the price of this commodity relative to labor, the numeraire) and increases consumer welfare. ⁵

Consumers in Benabou's world are short lived; they are in the market just once. It is for that reason that one important effect of inflation is missing in that model -- its effect on relative price variability. As inflation increases, rates of change of prices over time are more dispersed across markets and across sellers within a market. I present a model of repeated purchase in which price variability induces buyers to be less informed in equilibrium. The main prediction of the model is the opposite of

⁵Models that reach similar conclusions include Fershtman and Fishman (1989), Benabou and Gertner (1990), Benabou (1991) and Diamond (1990).

Benabou's: real prices are increasing and consumer welfare decreasing in the inflation rate.

Another point that is clarified in this paper, relates to the "search cost" of price instability. Fischer (1981, p. 391) mentions that "excessive search is believed to be the mechanism through which monetary disturbances produce misallocations of resources." This paper shows that such an assertion is inaccurate. The total amount of resources devoted to search does not necessarily increase with inflation; as in any investment problem, the higher depreciation rate may be associated with a smaller or higher flow demand for information. What does decrease is the stock of information available when making a decision. This loss of information causes less efficient decisions to be made. This is what causes a welfare loss.

III. THE MODEL: THE EFFECTS OF PRICE INSTABILITY ON SEARCH MARKETS

This paper studies market interaction and the endogenous determination of equilibrium price distributions. The analysis is phrased in terms of a consumer good, but it is applicable to firms minimizing the cost of acquiring inputs or to any other market interaction between buyers that search and sellers that set prices.

There is heterogeneity on both sides of the market. Consumers belong to a continuum [0,1] and are characterized by their search cost, c. Potential sellers are also a continuum [0,1], and are

characterized by their unit production cost, θ . Consumers go from store to store and observe the price tags -- the process of information transmission is such that quotations are received instantaneously. Sequential search is optimal in such case. The model can be seen as an extension of models with heterogeneity and sequential search (as MacMinn 1980 and Carlson and McAfee 1983) to intertemporal consumption (repeated purchase).

The model is set up in real terms. I assume that aggregate inflation is either deterministic or known to every agent. I do so to concentrate on the uncertainty about relative prices. The inclusion of uncertainty or incomplete information about the price index could only reinforce the predictions of the model. It is interesting that these predictions will be obtained from relative price instability alone.

The stochastic process for relative prices over time is going to be a transformation of the process for input prices (θ_{it} , the unit cost of store i at time t). A very stylized AR(1) process is assumed,

$$\theta_{it} = \begin{cases} \theta_{it-1} & \text{with probability } \rho \\ & \text{a drawing from } \Psi(\theta) & \text{with probability } (1-\rho), \end{cases}$$

where $\Psi(\theta)$ is the cumulative density function of unit costs with domain $[\underline{\theta}, \overline{\theta}]$. It is shown below that, in equilibrium, the cost process (1) maps into a similar process for output prices.

The correlation of real costs over time, ρ , is assumed to be a decreasing function of the inflation rate, π ; that is,

This is based on the aforementioned empirical findings that inflation is positively correlated with relative price variability across sellers of the same good⁶ and that inflation is negatively related to the autocorrelation coefficient of the deviation-from-mean real price.

A more general specification of the cost process would allow some interaction between "real-real" and "inflation-induced" shocks, as well as allowing for a permanent firm-specific component of cost. In such a case, there will be a learning component to the search strategy of buyers, tending toward a long-run matching. The simple specification in (1) allows us to derive very clear comparative static results and is in the spirit of the "uncertain recall" models of Landsberger and Peled (1977) and Karni and Schwartz (1977).

III.1. THE CONSUMER PROBLEM

Consumers are infinitely lived. Each consumer has a real cost per search of c. He purchases one unit of the good per period, and his objective is to minimize the discounted present value of expected total expenditure per period (price plus search cost), or

⁶Notice that $Var_{i}(\theta_{it+1}/\theta_{it})$ is decreasing in ρ (increasing in π) in this formulation.

min
$$E \sum_{t=1}^{\infty} \beta^{t-1} (p_t + N_t c)$$
,

where β is a discount factor and N_t is the number of stores visited at t. The known and time invariant distribution of real prices has cumulative density function F(p). The location of each individual seller on that distribution follows the stochastic process

$$\mathbf{p_{t+1}} = \left\{ \begin{array}{ll} \mathbf{p_t} & \text{with probability } \rho \\ \\ \mathbf{a} \text{ drawing from } \mathbf{F(p)} & \text{with probability } (1-\rho). \end{array} \right. \tag{1'}$$

The equilibrium distribution F and the stochastic process for individual prices will be derived later. I will show that in the simplified setting of this paper, the probability of the real price being unchanged equals the parameter ρ of the cost process.

What distinguishes this model from the standard consumer search problem, as described in Sargent (1987), is that consumption takes place every period. Consumers can visit several stores per period. It is assumed that real prices follow the process (1') over time. The probability of finding any given store charging the same real price next period is equal to ρ . In terms of the "recall problem" in the search literature, recall is "uncertain" over time. 7

Given the recursive structure of the problem, it can be solved using dynamic programming techniques. The equivalence between the

⁷It is well known (see Sargent 1987) that in the standard model the possibility of recall is never exercised, due to the stationarity of the reservation price rule. But when one allows for non-stationarity (as in Landsberger and Peléd 1977 and Karni and Schwartz 1977) or for intertemporal consumption (as in this paper), recall becomes an important issue.

sequence problem (as expressed in the objective function above) and the Bellman's equation formulation will be exploited. Let V(x) be the value function reflecting the present value of expected total expenditure when the consumer is at a store offering price x. At that point, there are two possible choices: accept the current offer and go home to consume this period's unit (with expected cost $V^{K}(x)$) or keep searching (with expected cost $V^{K}(x)$). Therefore,

$$V(x) = \min \{V^{A}(x), V^{K}(x)\}.$$
 (2)

The value of accepting the current offer is

$$V^{A}(x) = x + \beta \min\{V^{N}, V^{R}\}$$
 (3a)

Acceptance implies paying the price x today and behaving optimally starting tomorrow. Tomorrow, the consumer will face two alternatives at the start of the day. He can again visit the store from which he purchased today (with expected cost V^R , the value of recalling) or he can go directly to a new store (with expected cost V^R). We get that

$$V^N = c + EV$$

and $V^R = \rho V^A(x) + (1-\rho) EV$,

where $EV = \int_0^\infty V(\mu) \, dF(\mu)$ and μ is a generic price to be found in the new store visited (in V^N) or in the old store if the real price has changed (second term in V^R). Free recall from last period's store is assumed for simplicity.

The expected value of continued search equals

$$V^{K}(x) = c + EV.$$
 (3b)

It is assumed that there is no intra-period recall. This is done to

simplify the proof but is completely harmless, since, even if available, intra-period recall will not be exercised given the stationarity of the reservation price.

Bringing all of the above together:

$$V(x) = \min \left\{ x + \beta \min \left\{ c + EV, \rho V^{A}(x) + (1-\rho) EV \right\}, c + EV \right\}.$$
 (2')

The two appearances of the "min" operator reflect the two generic decision points: after each quotation and at the beginning of each period. Proposition 1 characterizes the optimal search strategy. 8

Proposition 1.

The unique solution to the functional equation (2') is

$$V(x) = \begin{cases} \frac{x}{1-\rho\beta} + A & \text{if } x \le p \\ \frac{p}{1-\rho\beta} + A & \text{if } x \ge p, \end{cases}$$
 (4)

where

$$A = \frac{\beta(1-\rho)}{(1-\beta)(1-\rho\beta)} [p-(1-\rho\beta)c]$$

and p solves

$$\frac{p}{1-\rho\beta} = c + \int_0^p \frac{x}{1-\rho\beta} dF(x) + \int_p^\infty \frac{p}{1-\rho\beta} dF(x)$$
 (5)

In order to behave optimally, the agent must (1) start each period by visiting the store at which last period's unit was purchased $(V^R \triangleleft V^N)$, and (2) accept the first offer below the reservation price

The first recall is free in this model; that is, there is no cost of coming back to the previous store. This assumption simplifies the decision rule to a unique reservation price; otherwise there would be both a "purchase" reservation price and a "recall" reservation price.

p that comes along.

Proof. See the Appendix.

Equation (5), which defines the indifference (reservation) price, can be rewritten as

$$c = \frac{1}{1-\rho\beta} \int_0^{\rho} (p-x) dF(x). \qquad (6)$$

This is a familiar condition. The marginal cost of search is equated to the marginal benefit, which in this case equals the discounted present value of the expected price reduction due to an additional search.

The Duration of Search

Under a sequential decision procedure, the total number of observations that are taken before a decision (stopping in this case) is made is a random variable. The expected number of stores visited, n_t, will be different in the initial period -- analyzed for the sake of comparison with the standard one-period case -- than in any subsequent one (steady state). We have that

$$n_1 = \sum_{i=1}^{\infty} i [1-F(p)]^{i-1} F(p)$$

= $\frac{1}{F(p)}$ and

$$n_t = \rho + (1-\rho) F(p) \sum_{i=1}^{n} [1-F(p)]^{i-1}$$

$$= \rho + (1-\rho) - \frac{1}{F(p)}$$
 for t>1 (7)

Equation (7) reflects the possibility of stopping after a successful recall plus the possibility of having to rebuild the stock of information in case recall is unsuccessful.

Differentiating the above expressions with respect to the reservation price, we obtain

$$\frac{\partial \mathbf{n}_1}{\partial \mathbf{p}} = -\frac{\mathbf{f}(\mathbf{p})}{\left[\mathbf{f}(\mathbf{p})\right]^2} \le 0 \quad \text{and} \quad$$

$$\frac{\partial n_t}{\partial p} = -(1-\rho) \frac{f(p)}{[F(p)]^2} \le 0 \qquad \text{for t>1.}$$

The expected number of quotations asked is inversely related to the reservation price. For instance, a buyer with high cost of search will have a high acceptance price and hence will be likely to find an acceptable price early.

Comparative Statics

Differentiating (6), the implicit definition of reservation price, with respect to the discount factor gives

$$\frac{\partial \mathbf{p}}{\partial \beta} = -\frac{\mathbf{c} \, \rho}{\mathbf{F}(\mathbf{p})} < 0 \quad \Rightarrow \quad \frac{\partial \mathbf{n}}{\partial \beta} > 0 \quad \text{for } \rho > 0$$

The higher the discount factor (the lower the discount rate), the more important the investment component of search becomes and thus, more search is undertaken.

Differentiating (6) with respect to ρ gives

$$\frac{\partial \mathbf{p}}{\partial \rho} = -\frac{\mathbf{c} \, \boldsymbol{\beta}}{\mathbf{F}(\mathbf{p})} \quad < 0. \tag{8}$$

This is a crucial result in our analysis: reservation prices are decreasing in the correlation coefficient ρ . We can interpret $(1-\rho)$ as a measure of (inflation-induced) price variability. The more unstable the environment is, the higher real reservation prices are (the less choosy consumers become). If we think of $(1-\rho)$ as a depreciation coefficient on the stock of information (Van Hoomissen 1988), we predict consumers holding smaller stocks (measured by the inverse of p). In this vein, $n_{\rm t}$ can be interpreted as the "flow demand" for information, i.e., investment. The effect of ρ on the steady state demand for information $(n_{\rm t})$ is uncertain, since

$$\frac{\partial n_1}{\partial \rho} = c\beta f/F^3 \ge 0 \quad \text{and}$$
 (9)

$$\frac{\partial n_{\mathsf{t}}}{\partial \rho} = 1 - 1/F + (1-\rho) \ c\beta f/F^3. \tag{10}$$

There are two forces at work in (10). On the one hand, a higher ρ increases the likelihood of a successful recall and hence of an early stopping. On the other hand, if recall happens to be unsuccessful, the lower reservation price (from (8)) makes the consumer more likely to search longer. As in capital theory, faster depreciation can either increase or decrease steady state investment. This establishes my earlier claim that the total amount of resources spent on search (n_tc) may either increase or decrease as a response to (inflation-induced) price instability. Moreover,

⁹Little more can be said without further specification of F. If the

I have established that buyers end up accepting higher real prices than they otherwise would.

The above analysis referred to the interior case of $\rho \in (0,1)$. From equation (7), we see that

$$\rho=0 \Rightarrow n_t=1/F$$
 and $\rho=1 \Rightarrow n_+=1$.

In the case of $\rho=0$, there is no intertemporal link, and each period's search behavior is as described in the standard (static) search model. When $\rho=1$, all search is undertaken in the initial period; so, in all subsequent periods, the consumer makes only one trip, to purchase from the same store (confirming the intuition in Stigler 1961).

III.2. DEMAND AND FIRM OPTIMIZATION

The purpose of this and the next section is to show (in a simplified setting where equilibrium can be explicitly solved) how changes in the stock of information held by consumers affect the equilibrium price distribution once sellers' behavior is taken into account. The strong assumption being made, to gain mathematical

distribution of prices is uniform, search will be a concave function of inflation: increasing at low inflation (the Fischer intuition) and decreasing at very high inflation (as observed in the Argentine episodes of hyperinflation).

tractability, is that firms are myopic; they just maximize per period profits, ignoring intertemporal links. The dynamic behavior of sellers who do take those links into account is discussed in Benabou (1990) and Tommasi (1991). An explicit account of such behavior would reinforce the predictions here by strengthening the dependence of equilibrium prices on ρ , through the complementarity between sales in different periods due to "customer relationships" (goodwill).

Each seller observes his unit cost (which happens to follow the stochastic process (1)). He also knows the economy-wide ρ and F() and thus knows how well-informed consumers are. He will base his pricing on that information, trying to maximize expected profits.

The set of consumers is [0,1]. Each type is characterized by a search cost c, distributed uniformly on [0,C]. Potential sellers (also [0,1]) have unit cost θ , a drawing from $\Psi(\theta)$, on $[\underline{\theta},\overline{\theta}]$.

Let q(p) be the expected quantity sold by a firm charging price p. Note the absence of time subscript due to the myopia assumption plus the stationarity of the problem. This expected quantity equals

$$q(p) = \frac{1}{\delta C} \int_{c(p)}^{C} \sum_{n=1}^{n=1} [1-F(p(c))]^{n-1} dc$$
 (11)

where:

(1) c(p) is the search cost of buyers with reservation price p, defined in equation (6). Only buyers with search cost greater or equal to c(p) will purchase if they come to a store charging p.
(2) [1-F(p(c))]ⁿ⁻¹ is the probability that a customer of type c will get to realize the nth visit. In the non-myopic case this should be

multiplied by the probability of being involved in search (i.e., of having found the new price of the old supplier unacceptable).

Here, I set this probability to unity.

(3) δ represents the measure of sellers that are in operation this period. If all potential sellers were in operation, δ would equal one. But, as we will see, high-cost sellers may fail to make any sale, generating a truncation at the upper tail of the cost distribution (δ <1).

I can rewrite (11) as

$$q(p) = \frac{1}{\delta C} \int_{C(p)}^{C} 1/F(p(c)) dc.$$

From (6), I get

$$\frac{dc}{dp} = \frac{F(p)}{1-\rho\beta}.$$

Hence (by the implicit function theorem),

$$\frac{1}{F(p)} = \frac{1}{1-\rho\beta} \frac{dp}{dc}, \quad \text{which implies that}$$

$$q(p) = \frac{1}{\delta C} \frac{1}{1-\rho\beta} (\overline{p} - p), \qquad (12)$$

where \overline{p} is the reservation price of the buyer with the highest cost of search.

Each seller's objective is to maximize:

$$(p-\theta)q(p) = \frac{(p-\theta)(\overline{p}-p)}{\delta(1-\rho\beta)C}$$
.

Optimizing this quadratic profit function gives the pricing rule

$$p(\theta) = \frac{\overline{p} + \theta}{2}. \tag{13}$$

III.3. EQUILIBRIUM

An equilibrium is defined as a price distribution F such that

- (1) Consumers follow the reservation price rule (6) and
- (2) Firms price according to (13).

Let [m,M] be the range of equilibrium prices. From the demand curve (12), it is clear that M, the highest price in the market, should satisfy

$$\overline{p} \geq M.$$
 (14)

The upper bound of the price distribution is the reservation price of the consumers with the highest marginal cost of search. If a firm charges more than that, it won't make any sales, so, we can assume that such a firm is out of the market. There are therefore, two possible situations which depend on the relation between \bar{p} and $\bar{\theta}$. When $\bar{\theta} \leq \bar{p}$, all potential sellers are indeed in the market every period $(\delta=\Psi(\bar{p})=1)$. Otherwise, there will be truncation and only firms with $\theta \leq \bar{p}$ will be in the market $(\delta=\Psi(\bar{p})<1)$.

Applying the definition of a reservation price (6) to consumers with the highest cost, C, gives

$$\vec{p} = (1-\rho\beta)C + Ep$$

where Ep stands for the expectation of the price. 10 From (13), we

¹⁰It is worth mentioning that Ep is an unweighted average of the prices i the market. Weighting by quantities would result in a lower price. This relates to another dimension of the "index number problem." The price of one item usually used to construct indexes is, in general, an unweighted average

get

$$Ep = \frac{\overline{p} + e(\theta)}{2},$$

where $e(\theta)=E(\theta|\theta \le \overline{p})$ includes the truncation case. In such a case, not all potential firms will be in operation, so the expectation will be taken conditional on being below \overline{p} .

It follows that

$$\overline{p} = 2(1-\rho\beta)C + e(\theta). \tag{6'}$$

Notice that \bar{p} is decreasing in ρ . Using (6'), the pricing function (13) can be rewritten as

$$p(\theta) = (1-\rho\beta)C + \frac{e(\theta)+\theta}{2}. \tag{13'}$$

Each firm prices as a function of a weighted average of its unit cost and the average unit cost in the market (as in MacMinn 1980 and Carlson and McAfee 1983).

The pricing function (13') is a mapping from unit cost θ to output price p. Such a mapping will exist and be unique as long as the same is true for the conditional expectation $e(\theta)$. Conditions under which this is true are discussed in the Appendix. Together with the cost process (1), (13') determines the process for output price, which is stated in (1').

The change of variable formula can be used to express the distribution of prices f(p), as a transformation of the probability density function of unit costs, $\psi(\theta)$. We get

of several observations. From published statistics, it is impossible to distinguish a 10% increase in all prices from a mix 0-20% in different stores. However, from a welfare perspective, these are different experiments.

$$f(p) = \psi(\theta) \frac{\partial \theta}{\partial p} = 2 \psi(\theta(p)),$$

where $\theta(p) = 2p-2(1-\rho\beta)C-e$. In the truncation case, f(p) will need to be normalized by the proportion of firms in operation, $\Psi(\overline{p})$. All of the above is summarized in the following proposition.

Proposition 2.

The equilibrium distribution of prices, f(p), is equal to

$$f(p) = \begin{cases} \frac{2\psi(2p-2(1-\rho\beta)C-e(\theta))}{\Psi(2(1-\rho\beta)C+e(\theta))} & \text{for } p \in [m,M] \\ 0 & \text{elsewhere,} \end{cases}$$
(15)

where ψ is the density function of θ and

$$m = (1-\rho\beta)C + \frac{e(\theta)+\underline{\theta}}{2}$$

$$M = \min \left\{ (1-\rho\beta)C + \frac{E\theta+\overline{\theta}}{2}, \overline{p} \right\}.$$

The distribution f(p) always exists. It will be unique if $\forall x \in (\overline{\theta}, \underline{\theta})$, either $\frac{\psi(x)}{\Psi(x)}[x-e(x)] < 10r \frac{\psi(x)}{\Psi(x)}[x-e(x)] \ge 1$ Proof. See the Appendix.

IV. THE EFFECTS OF PRICE INSTABILITY

We know from the analysis of the consumer problem that acceptance prices are decreasing in ρ (increasing in price instability). Said comparative statics was performed in the partial equilibrium sense of maintaining constant the distribution F. Here, I study the effects on the distribution as a whole. It is still true that

reservation prices are decreasing in ρ (even more so now, given that the whole distribution is less favorable as ρ decreases).

The equilibrium effects of a lower ρ (higher inflation or price instability) ¹¹ can be summarized in Figure 1. There is a clockwise rotation in the demand curve for the individual firm. This movement contains several effects. First, the upward movement in the whole locus induces an increase in the real price charged by any firm (markups increase, real wages decrease). Second, the steepening of demand reflects the fact that now high-cost (high-price) firms expand their expected sales at the expense of the more "efficient" firms. Third, the intercept being higher, firms that otherwise would be out of the market can charge profitable prices and make positive sales.

Below, I address these and other effects in more detail.

Markups

From the pricing function (13'), we see that

$$\frac{\partial p(\theta)}{\partial \rho} = -\beta C + \frac{1}{2} \frac{\partial e}{\partial \rho} < 0;$$

that is, all prices increase as ρ falls. This is a differentiating prediction of this model with respect to, say, Benabou (1988). All real prices in the market increase, and consumer welfare decreases at higher inflation rates (at lower ρ). In particular, even the lowest price in the market (m in this model, s in menu-cost models)

Bear in mind that this is an exercise in comparative statics that purports to compare situations of high inflation (high price instability) with more stable situations. It should be interpreted in a cross-sectional or low-frequency time-series sense. I am not speaking about the effects of a short run (business cycle) increase in the inflation rate.

does increase, which is consistent with the empirical finding of Sheshinski et al. (1981). That finding represented a puzzle to the authors, in terms of menu cost models. One of the essential implications is, then, that the mean of the price distribution increases in an unstable environment ($\frac{\partial Ep}{\partial \rho}$ < 0).

Intercept

As we already know, the intercept of the expected sales curve is given by the acceptance price (\overline{p}) of the individuals with the highest cost of search (C). From $\overline{p}=2(1-\rho\beta)$ C+e, we get

$$\frac{\partial \overline{p}}{\partial \rho} = -2\beta C + \frac{\partial e}{\partial \rho} < 0.$$

The term $\frac{\partial e}{\partial \rho}$ will be zero if the movement is within the non-truncation range, and negative otherwise. In any case, a lower ρ could allow the operation of "marginal" firms. This agrees with the conventional wisdom about chronic inflation countries -- that inefficient producers can survive in this environment.

Slope

The price-expected sales locus can be written as $q(p) = \frac{(p - \bar{p})}{\delta C(1 - \rho \beta)}$ or $p = \bar{p} - (1 - \rho \beta) \delta Cq$, so absolute value of its slope is $(1 - \rho \beta) \delta C$. A lower ρ makes the locus steeper. In economic terms, such steepening shows the reallocation of sales toward high-price (cost) firms. Replacing \bar{p} and $p(\theta)$ in $(16)^{12}$ gives

 $^{^{12}}$ In what follows, I assume no truncation, that is why E rather than eappears.

$$q(\theta) = 1 + \frac{(E\theta - \theta)}{2C(1 - \rho\beta)}.$$
 (17)

Equation (17) shows that producers with cost equal to the mean $E\theta$, will produce one unit (remember that the size of buyers and sellers is unity). High-cost producers will produce less than one, and low-cost producers will produce more than one.

In highly unstable environments, production will get redistributed towards high-cost producers because

$$\frac{\partial q(\theta)}{\partial \rho} = \beta \frac{(E\theta - \theta)}{2C(1 - \rho\beta)^2}.$$

The change in quantity is positive for firms with $\theta < E\theta$ and negative for firms with unit cost above average. This means that a higher intertemporal correlation of shocks (
ho) will imply a more efficient output distribution. Conversely, a low ρ (high π), will redistribute output in the wrong direction. To see intuitively why this reallocation takes place, imagine a buyer walking down a road (a circular road, where each buyer starts search at a random location). He asks the price in the first store and either accepts it (if it is below his reservation price) or keeps walking. way, the buyer will get matched to a store with a price low enough for him. Low-c consumers will end up purchasing from low-price (cost) stores for sure. High-c consumers may be lucky and end up in a low-price store or may not be so fortunate and end up paying a high price (still below their reserve price). For this reason, sales will (on average) be higher for lower-price stores; they get all the low-c customers and some of the high-c ones. Now, with a lower ρ , every consumer becomes less choosy. They are likely to

stop search at a high-price place. What is not acceptable at a high ρ may be acceptable at a low ρ . This increases expected sales for high- θ firms at the expense of the low- θ ones.

Total Cost of the Industry

As a consequence of such redistribution of output, total production costs in the industry will be higher -- they are decreasing in ρ . Total cost equals

$$TC = \int_{\underline{\theta}}^{\overline{\theta}} \theta \ q(\theta) \ d\Psi(\theta) = \int_{\underline{\theta}}^{\overline{\theta}} \left[\theta + \theta \ \frac{(E\theta - \theta)}{2C(1 - \rho\beta)} \right] \ d\Psi(\theta)$$
$$= E\theta + \frac{(E\theta)^2 - E(\theta^2)}{2C(1 - \rho\beta)} = E\theta - \frac{Var(\theta)}{2C(1 - \rho\beta)};$$

so

$$\frac{\partial TC}{\partial \rho} = -\beta \frac{Var(\theta)}{2C(1-\rho\beta)^2} < 0$$

as claimed.

Aggregate Output

The analysis of output effects is somewhat constrained by the limitations of the model. The result that I do obtain is an increase in industry total cost. In a general equilibrium sense, this must be taking resources away from other sectors.

In terms of aggregate output, application of the model to input or intermediate goods markets can give us differential effects on markups, generating further distortions of relative prices with respect to marginal costs. This may be an additional reason for the negative effects of persistent inflation on aggregate output (Harberger 1991).

Price dispersion

The effect of price variability (and hence, of the degree of consumer information) on dispersion, is more subtle. Contrary to common belief, smaller information stocks (or reduced search intensity in a static model), do not necessarily imply higher price dispersions. There is, however, one important case in which it does so — when there is truncation. I look at that case first, since it is particularly relevant as a description of the effects of inflation.

Define dispersion as the difference between the maximum (M) and the minimum (m) price in the market. In the truncation case $(M=\overline{p})$,

$$\frac{\partial (M-m)}{\partial \rho} = -\beta C < 0 .$$

Note that dispersion is a function of ρ through truncation on the upper tail of the distribution of production costs. Price instability, by decreasing the amount of information buyers have, deprives them of their most powerful weapon -- their veto power ("I will not purchase here") -- and hence allows the existence of inefficient producers. In this simple model, an inefficient producer is one who drew a high θ this period. In a more general model, we can have $\theta_{it} = \theta_i + \varepsilon_{it}$ and price variability (variance of ε) would permit the subsistence of high- θ_i (permanent component of cost) producers.

In the non-truncation case, there is a problem with the commonly assumed link between information and price dispersion. It seems that price dispersion is not necessarily a measure of ignorance in

the market. 13 In this model, the price distribution is a transformation of the production cost distribution. The transformation takes place through the aggregation of consumer search strategies, as reflected by the demand curve (12). Demand here is linear because the costs of search are uniformly distributed. In such a case, the shift in the demand curve induced by a smaller ρ will have no effect on dispersion (for a given distribution of production costs) if there is no truncation because

$$\frac{\partial (M-m)}{\partial \rho} = -\beta C + \beta C = 0 .$$

More generally, the effect on dispersion will depend on the shape of the demand curve, which is equal to the cumulative density function of c (with the axis changed). For example, if the shift implies moving from a dense region (many people with similar cost of search) to a less-elastic region of demand (fewer people), dispersion will increase. If the movement is toward more-elastic regions, price dispersion will decrease. In summary, unless there is truncation, the effect of ρ on dispersion is uncertain in the general case and nil in the case of uniformly distributed search cost. This proves my previous claims of dispersion not being univocally related to "ignorance" and of dispersion being a secondary prediction of the theory.

Summary

 $^{^{13}}$ The finding here is already present (for a single purchase case) in MacMinn (1980).

In terms of the inflation interpretation of the model, the most interesting result is that price instability enables high-cost producers to charge prices high enough to cover their costs. If inflation is zero, ρ will be at its maximum. In that case, the atomistic firms will be almost perfectly competitive. They will be perfectly competitive if $\rho=\beta=1$. Inflation, by lowering ρ , increases their market power (shifts upward their demands). The structure of the industry becomes more imperfectly competitive. This generates the results that people in high-inflation countries associate with inflation: existence of inefficient producers, inflated markups, etc.

To summarize, there are two implications for market structure. First, all firms have higher markups, and second, less-efficient firms increase their market share (from zero in some cases), increasing aggregate production costs. This approach reverses the causality of the "administered inflation" hypothesis. ¹⁴ I predict market structure and performance as consequences of inflation-induced price instability.

¹⁴See the excellent summary in Domberger (1983). The hypothesis makes market power a proximate cause of inflation by making it the source of cost-push inflationary forces.

V. CONCLUSION

I introduced price variability into a market for a homogeneous good, in which repeat consumers search for low prices. The main consequences of price instability (parameterized by a smaller ρ) are the following:

- (1) Buyers become less informed in equilibrium. The total amount of resources spent on search may either increase or decrease. As is well known in capital theory, a higher depreciation rate implies smaller stocks, but investment flows may move in either direction.
- (2) Prices increase on average, relative to both production and search costs. In the case of production costs, this increase can be interpreted as an increase in "markup." If we interpret search costs as a proxy for the value of time, the increase in prices may be thought of as a drop in purchasing power of wages.

The reason for higher prices lies in the smaller stock of information held by consumers, which translates into an upward shift in the individual seller's demand curve. These demand curves are downward sloping as a consequence of informational imperfections (positive search cost for some customers). If information were costless, all output would be produced by the lowest-cost firm.

¹⁵From the sellers' perspective, inflation provides the coordination necessary to get "randomized" prices as in the "noisy monopolist" case (Salop 1977 and Benabou 1989). Without inflation, it is impossible to get this coordination in the case of a large number of sellers.

- (3) Price instability may allow less-efficient firms (high θ or, in a more general model, high permanent component of cost) to be in operation. This provides a theoretical foundation for the commonly held belief that inflationary environments are natural habitats for inefficiencies. When signals are noisier, it is more difficult to separate the chaff from the grain.
- (4) Output gets redistributed from the more- to the less- efficient producers, increasing overall production costs and reducing efficiency in the economy.

I have analyzed the case in which the products sold by different firms are perfect substitutes in consumption, and proved that in that case consumer welfare decreases with inflation. This model could be generalized to study the effect of interproduct price variability whenever information is costly to acquire. The idea is that even though in a full (costless) information case, consumers welcome relative price changes due to the quasi-convexity of their indirect utility functions, when costs must be incurred to learn the new set of prices, the net effect might be welfare decreasing. Lach and Tsiddon (1990) and Tommasi (1992) find that most price variability in high inflation is due to intragood rather than intergood variance. This leads us to believe that the results of this paper do capture an essential component of the welfare effects of inflation.

APPENDIX

Proof of Proposition 1

First, I will prove that $V^R < V^N$, i.e., that the consumer must start each period by recalling the offer from the last store visited. This will simplify (2) to

$$V(x) = \min \left\{ \frac{x}{1-\rho\beta} + \frac{\beta(1-\rho)}{1-\rho\beta} \text{ EV, } c + \text{EV} \right\}$$
 (2'')

Second, I will demonstrate that (4)-(5) is indeed a solution to (2''). Finally, I will prove that (2'') has a unique solution.

(i) The Recall Decision

Claim: $\rho V^{A}(x) + (1-\rho) EV < c + EV$.

Proof

The fact that the price x was admitted last period implies that $V^{A}(x) \le c + EV$. Hence,

$$\rho V^{A}(x) + (1-\rho) EV \leq \rho C + EV < C + EV .$$

(ii) Solution of (2")

Claim: The value function defined by (4)-(5) solves functional equation (2'').

Proof

From (4),

$$EV = \frac{1}{1-\rho\beta} \left[\int_0^p x \ dF(x) + \int_p^\infty p \ dF(x) \right] + A.$$

Using (5), EV = $\frac{p}{1-\rho\beta}$ - c + A. Substituting in (2'') we get, after some manipulation,

$$V(x) = \min \left\{ \frac{x}{1-\rho\beta} + A, \frac{p}{1-\rho\beta} + A \right\}$$

as claimed.

(iii) Uniqueness of the solution to (2")

Claim: The functional equation (2'') admits a unique solution.

Proof

It is easy to verify that for very high search cost c, the unique optimal strategy consists in accepting any quotation. Below, I prove uniqueness in the more interesting case in which search is possible.

Any solution to (2'') should belong to the set:

$$\Gamma = \left\{ z : \mathbb{R}_0^+ \rightarrow \mathbb{R} \mid z(x) = \min \left[bx + hv, c + v \right] \right\}$$

$$b = \frac{1}{1 - \rho \beta} , \quad h = \frac{\beta (1 - \rho)}{1 - \rho \beta} , \quad v \in \mathbb{R}$$

Define the distance between two elements of Γ as $d(z,w) = \sup_{x \in [0,p]} |z(x)-w(x)|$. Define the operator $T:\Gamma \to \Gamma$

$$Tz = min [bx + hEz, c + Ez]$$

From $0 \le h \le 1$, it is clear that d(Tz,Tw) = |Ez-Ew|. Hence, for any two different elements of Γ , $d(Tz,Tw) < \sup = |z(x)-w(x)| = d(z,w)$. Now, suppose that V(x) and V'(x) are two solutions to (2''), i.e., V(x)=TV and V'(x)=TV'. Then, d(V,V')=d(TV,TV'). But we have already established that if $V \ne V'$, d(TV,TV') < d(V,V'). Hence we arrive at a contradiction. \square

Proof of Proposition 2

(i) Existence of an Equilibrium

Claim: There is always an equilibrium as defined by equation (15).

Proof

A) Suppose first that $2(1-\rho\beta)C + E\theta \ge \overline{\theta}$.

In this case, the maximum price consumers are willing to pay (if all firms are in the market) is higher than the highest production cost. Hence, an equilibrium with full participation or non-truncated equilibrium is

possible. It will be characterized by $e(\theta)=E(\theta)$ and $M=\bar{p}=2(1-\rho\beta)C+E\theta$. Both expressions are functions of parameters, so they exist. Hence, f(p), m and M as defined in (15), exist.

B) Suppose now that $2(1-\rho\beta)C + E\theta < \overline{\theta}$.

In this case, an equilibrium with full participation is not possible. To get a truncated equilibrium, we need a value \bar{p} such that all the producers with cost equal or lower than \bar{p} enter the market, and \bar{p} is the maximum price that the consumers are willing to pay for the product. Hence, we need to find \bar{p} such that:

$$e = E(\theta | \theta \le \overline{p})$$
 and $\overline{p} = 2(1-\rho\beta)C + e$

That is equivalent to finding a fixed point to the expression:

$$g(x) = 2(1-\rho\beta)C + E(\theta|\theta \le x)$$

Notice that, if ψ has no mass points (this is not necessary but certainly sufficient), then g is continuous. Notice also that: ¹⁶

$$g(\underline{\theta}) = 2(1-\rho\beta)C + \theta > \theta \tag{A1}$$

$$g(\overline{\theta}) = 2(1-\rho\beta)C + E\theta < \overline{\theta}$$
 (A2)

From continuity of g and the above conditions, it follows that g(x) has at least one fixed point. Therefore, we can obtain e and \bar{p} . Hence, f(p), m and M as defined in (15) exist.

(ii) Sufficient Conditions for Uniqueness

In general we may obtain multiple truncated equilibria and at most one non-truncated equilibrium. If $g'(x) \ge 1 \ \forall x$, the equilibrium will be unique and non truncated. If $g'(x) < 1 \ \forall x$, the equilibrium will be unique, being truncated or not depending on $2(1-\rho\beta)+E\theta$ being greater or smaller than $\overline{\theta}$. Finally,

$$g'(x) = \frac{\partial}{\partial x} e(\theta) = \frac{\partial}{\partial x} \int_{\underline{\theta}}^{x} \frac{\theta}{\Psi(x)} d\Psi(\theta)$$
$$= \frac{\psi(x)}{\Psi(x)} [x-e(x)]$$

 $^{^{16}}$ I am indebted to R. Benabou for pointing out that, to be completely rigorous we should replace (A1) by a limit argument, since it involves the ratio of two expressions that converge to 0 as $x \rightarrow \theta$.

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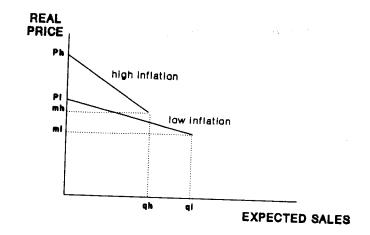
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FIGURE 1

Demand Curve
for an Individual Firm



NOTE

Ph: maximum acceptance price (p(C)) when inflation is high.

Pl: maximum acceptance price (p(C)) when inflation is low.

mh: lowest real price in the market at high inflation.

ml: lowest real price in the market at low inflation.

qh: expected sales of firms with the lowest cost at high inflation.

ql: expected sales of firms with the lowest cost at low inflation.