

Inflation and Social Welfare in a Model  
with Endogenous Financial Adaptation

by

Federico A. Sturzenegger

Working Paper 658  
Department of Economics  
University of California, Los Angeles  
April 1992



## Abstract

This paper develops a model with endogenous financial adaptation. With a representative agent, inflation and welfare increase upon introduction of financial adaptation. Once we allow for agents' heterogeneity, we can show that inflation still increases and that the "poor" are hurt, while the "rich" benefit from the process of financial adaptation. Finally, we consider the optimal level of seigniorage collection. With a representative agent, financial adaptation increases both the optimal level of government spending and the inflation rate. With heterogeneous agents, if the government cares for the low income group, the optimal amount of government spending falls even though the rate of inflation increases. The model accounts for many stylized facts of high inflation economies and explains the incentives behind many policy actions.



# Inflation and Social Welfare in a Model with Endogenous Financial Adaptation

Federico Sturzenegger \*

## Abstract

This paper develops a model with endogenous financial adaptation. With a representative agent, inflation and welfare increase upon introduction of financial adaptation. Once we allow for agents' heterogeneity, we can show that inflation still increases and that the "poor" are hurt, while the "rich" benefit from the process of financial adaptation. Finally, we consider the optimal level of seigniorage collection. With a representative agent, financial adaptation increases both the optimal level of government spending and the inflation rate. With heterogeneous agents, if the government cares for the low income group, the optimal amount of government spending falls even though the rate of inflation increases. The model accounts for many stylized facts of high inflation economies and explains the incentives behind many policy actions.

---

\*Assistant Professor, Department of Economics, University of California, Los Angeles. Mailing Address: 405 Hilgard Ave., Los Angeles, California 90024- 1477. [sturzene@sscnet.econ.ucla.edu](mailto:sturzene@sscnet.econ.ucla.edu). I want to thank Roland Benabou, José De Gregorio, Rudiger Dornbusch, Stanley Fischer, Paul Krugman, Raúl Labán, Seonghwan Oh, Roberto Perotti, Tom Sargent, Kazimierz Stanczak, and David Wilcox for useful comments. Special thanks to Olivier Blanchard. The usual caveat applies.



With agent heterogeneity the results depend upon, among other things, on the political preferences of the central planner. We show that if the central planner cares strongly about the utility of low-income groups, the optimal level of government spending will fall with financial adaptation. The model explains why the increase in the inflation rate induced by the process of financial adaptation is accompanied by a fall in government spending.

Finally, Section 4 contains some concluding remarks.

## 1 The Economy

### 1.1 Preferences

The economy is composed of a continuum of identical agents who produce and consume a continuum of commodities defined over  $\mathbf{R}^+$ . The utility function follows that in Prescott (1987). The agent obtains utility from the consumption of each good, indexed by  $\theta \in \Theta = \mathbf{R}^+$ . The agent is constrained to buy one and only one unit of each commodity he decides to purchase every period. The utility function,  $v(\theta)$ , is increasing, twice continuously differentiable, strictly concave, and satisfies the Inada conditions. High- $\theta$  commodities are goods that give higher utility. Total utility is additive in individual commodities, or

$$V(\theta) = \int_{\Theta} v(\theta) i(\theta) d\theta, \quad (1)$$

where  $i(\theta)$  is an indicator function that equals one if the good is purchased.<sup>2</sup>

While high- $\theta$  commodities give higher utility, we also assume that they are more expensive. Taking labor as the numeraire, the price of commodity  $\theta$  equals  $p(\theta)$ , where  $p(\theta)$  is a continuous, increasing, and convex function of  $\theta$  with  $p(0) = 0$ .

The fixed-purchase requirement reduces the consumption decision to the choice of whether or not to purchase a particular good. While these characteristics may seem somewhat restrictive, they are realistic as they do properly capture some critical features of the real world namely, that goods are to some extent indivisible, that, as will be shown below,

---

<sup>2</sup>Each commodity in our setup corresponds to the equivalence classes in Prescott (1987). Our model generalizes Prescott (1987) in that we make the consumption-leisure choice endogenous further below.

a set of commodities that are purchased with domestic currency and one operating with foreign currency.

The purpose of this paper is to evaluate the welfare implications for the society as a whole of the process of monetary adaptation to high inflation. The process has implications for social welfare because in the presence of a government that finances itself through seigniorage, the reduction in the demand for domestic currency will change the equilibrium rate of inflation. It will also affect the relative prices and implicit taxes across commodities and affect income distribution.

In Section 1 we analyze the general equilibrium of an economy with endogenous dollarization and a representative consumer-producer. It is shown that while financial adaptation increases inflation, it unambiguously increases welfare, because it redirects taxes towards relatively inelastic commodities.

In Section 2 we extend the previous model by allowing for agent heterogeneity. Agents have different productivities and therefore consume different sets of goods. Therefore, the process of financial adaptation will affect agents differently. In the model, the inflation tax is proportional to income for low-productivity agents. Eventually, if the wage increases to the point at which the agent starts consuming goods in the dollarized sector, the inflation tax falls as a percentage of income. We also show that in equilibrium, poor agents are hurt by the process of financial adaptation, while rich agents benefit. This happens because the poor have a consumption pattern that is biased towards goods that are always purchased with domestic currency. Financial adaptation is equivalent to a change in the tax structure that increases the tax on these commodities; therefore, with financial adaptation, the poor pay a higher inflation tax on their purchases and are made worse off.

In Section 3 we consider the issue of the optimal level of government spending when the government cares not only about its own consumption but also private utility and income distribution. The government will spend until the marginal utility of its consumption equals the marginal disutility of private agents. In the context of a representative-agent model, financial adaptation increases the level of government spending. This follows from the fact that financial adaptation decreases the costs of inflation; that is, when a government learns to live with inflation, it will end up inflating more.



as income increases, the set of consumption goods purchased by an agent expands, and that the purchase of *basic* (low- $\theta$ ) goods does not differ very strongly across agents.

Finally, we assume a perfectly elastic supply of labor, or a constant marginal disutility of labor. Together with the additivity of the utility function this assumption separates the consumption problem into infinitely many independent submaximization problems. These subproblems take the form of deciding on the purchase of any particular good  $\theta$ .

## 1.2 Technology

Time is discrete. Consumers purchase many different goods at different locations. At each location, there is precisely one good sold, meaning that bunching of purchases is not feasible.<sup>3</sup> Agents face a cash-in-advance constraint but can choose the currency in which to do transactions. The reason why, the entire monetary system does not switch to the use of foreign currency if domestic inflation increases above that of the foreign country, will be shown to depend basically on a technological restriction. This paper therefore develops a theory of the imperfect substitutability of currencies.<sup>4</sup>

The basic feature which drives the imperfect substitutability of both currencies in our model is the existence of a transaction cost for operations in foreign currency. There are two alternative interpretations of this cost that give a similar formal structure. First, consider the costs of introducing a foreign currency into the domestic system. This is done basically through trade and is rather costless for large denominations. For small denominations, such an automatic mechanism is no longer available, and these costs may not be negligible.<sup>5</sup> Second, consider a transactions technology in which transactions in an indexed currency are allowed but at a cost. This cost may arise because there is a cost of

---

<sup>3</sup>This assumption is made only for simplicity. Alternatively, in order to rule out the bunching of purchases, one could argue that there exist diseconomies in the transactions of different commodities. In the real world, we actually do see some bunching of transactions, eg., when we buy in supermarkets or in department stores. But bunching is far from being complete, so this setup remains realistic, and the assumption should not be very costly.

<sup>4</sup>There is an extensive literature on currency substitution. A small subset includes Calvo and Rodriguez (1977), Calvo (1985), Liviatan (1981), Vegh (1989), and Weil (1990).

<sup>5</sup>Just think of delivering the cash payment of exports in nickels or dimes!

changing the units of account or because agents use financial instruments, such as credit cards or checks, which entail fees.

The relevant property that these costs must exhibit is that they should fall as a proportion of the value of the transaction or the denomination. This imposes only a very loose restriction. Declining costs (as with the costs of introducing foreign currency in the system) or fixed costs (as with the Brazilian checkable interest-bearing accounts) will do. But the result will hold even for increasing costs as long as they do not increase very rapidly with the denomination.

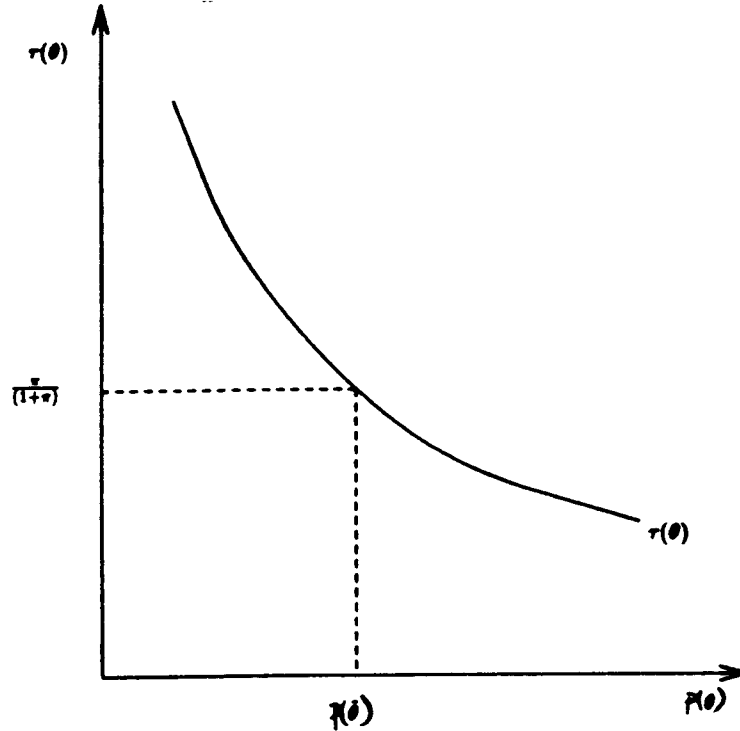
If foreign currency denominations are continuous,<sup>6</sup> we can think of a locus that maps the size of the foreign denomination,  $p(\theta)$ , to the transaction cost per dollar value of purchase,  $\tau(p(\theta))$ . This is presented in Figure 1. For big denominations, or equivalently, large purchases, this transaction cost is negligible, but it increases as we approach smaller denominations. For small purchases this cost is almost prohibitive given the small value of the transaction.

In this setup, the rule for optimal monetary holdings is simple enough. The agent minimizes the inflation tax he pays by holding the foreign currency denomination that exactly matches the corresponding price of each good he wants to buy. However, for purchases made using very small denominations, this may not be optimal because the “transaction” cost of using the foreign currency may be higher than the corresponding inflation tax. Therefore, he will use foreign currency only for those purchases that are large enough that the transaction cost is smaller than the inflation rate. This characterization of money holdings is true even under the “transportation cost” interpretation for the transaction cost, which is paid only once. The representative agent will only be willing to incur the transportation cost if it is below the implicit tax rate that he incurs by holding domestic currency for one period. The reason for this is that since he is a negligible fraction of the market, he will be able to use that currency (and save the inflation tax) only once if others have not also switched to the foreign currency. But everybody behaves the same way so that only when this condition is satisfied, does the market for this good get dollarized. After this market

---

<sup>6</sup>The assumption of a continuum is certainly excessive and unrealistic. On the other hand, it adds computational tractability without altering the conceptual issues at hand.

Figure 1: Transaction Costs of Foreign Denominations



becomes dollarized, everybody uses the foreign currency for this commodity forever after. This is the unique equilibrium, since if foreign currency had been introduced earlier, every agent would have wanted to deviate, free ride, and not pay the transaction cost.

In Figure 1, those goods that are purchased using denominations smaller than  $p(\tilde{\theta})$ , will be purchased with domestic currency, while those goods purchased using denominations above  $p(\tilde{\theta})$  will be purchased with foreign currency. This optimal pattern of monetary holdings implies that as inflation increases, foreign currency will progressively make its way toward cheaper commodities. Total money demand will therefore depend both on agents' consumption patterns and the transaction cost schedule.

Each agent produces goods with a constant-returns-to-scale technology. We assume that the unit labor requirement is an increasing and convex function of  $\theta$ . In equilibrium, the price of each commodity will equal its labor requirement times the wage.

### 1.3 Equilibrium

**Definition 1:** A *stationary nominal equilibrium* is defined as a sequence of commodity

prices, wages, consumption and production decisions, real government consumption, and nominal money stock,  $\{p(\theta)_t, w_t, c(\theta)_t, n_t, g_t, M_{t+1}\}_{t=0}^{\infty}$ , which given the initial money stock,  $M_0$ , and constant real government spending,  $g$ , satisfy conditions 1(i)-1(iii).

1(i) The government budget constraint,  $g = \frac{\pi}{1+\pi} m^d$ , is satisfied for all  $t \geq 0$ , where  $\pi$  is the inflation rate and  $m^d$  is real money demand.

1(ii) The stochastic processes for  $\{c(\theta)_t, n_t\}_{t=0}^{\infty}$  solve the agent's maximization problem given the price and wage sequence  $\{p(\theta)_t, w_t\}_{t=0}^{\infty}$  for all  $t \geq 0$ .

1(iii) The markets for all commodities  $\theta$ , labor, and money clear at each period  $t \geq 0$ .

Under seigniorage financing of a constant level of government spending, the inflation tax is equivalent to a tax on all those goods subject to a cash-in-advance constraint in domestic currency. As the economy is nonstochastic and stationary, the optimal solution for the real variables will repeat itself period after period. There are no true intertemporal links except those imposed through the cash-in-advance constraint. While nominal variables will be increasing each period, real variables will not move in the stationary nominal equilibrium. We can therefore work with an equivalent alternative equilibrium definition.<sup>7</sup>

**Definition 2:** A *stationary real equilibrium* is defined as a sequence of commodity prices, wages, consumption and production decisions, real government consumption, and tax rates,  $\{p(\theta)_t, w_t, c(\theta)_t, n_t, g_t, \pi_t\}_{t=0}^{\infty}$  which given constant real government spending,  $g$ , satisfy conditions 2(i)-2(iii).

2(i) The government budget constraint,  $g = \frac{\pi}{1+\pi} m^d$ , is satisfied for all  $t \geq 0$ .

2(ii) The stochastic processes for  $\{c(\theta)_t, n_t\}_{t=0}^{\infty}$  solve the agent's maximization problem given a fixed price and wage sequence  $\{p(\theta), 1\}_{t=0}^{\infty}$  for all  $t \geq 0$ .

---

<sup>7</sup>In concentrating on the stationary equilibria, we omit the issue of transitional dynamics. During the transition, agents will want to sell the goods abroad to obtain the foreign currency required to do transactions in the future. This corresponds to the notion of running a current account surplus in order to purchase the foreign currency money stock. We consider this cost when computing optimal government spending below. Unfortunately, the dynamics are not simple. When some agents want to sell good  $\theta$  abroad, say because the rate of inflation has increased to the point at which now it is cheaper to transact in foreign currency, we still would have to consider what happens to the buyers of that commodity, who, coming from last period with domestic currency, want to buy the same commodity.

2(iii) The markets for all commodities  $\theta$  and labor clear at each period  $t \geq 0$ .

Labor has been chosen as the numeraire, and real prices are stationary. To characterize the stationary real equilibrium, we solve the maximization problem of each individual agent in each period, namely,

$$\text{Max} \int_{\Theta} v(\theta)i(\theta)d\theta - n, \quad (2)$$

subject to

$$\int_{\Theta} p(\theta)i(\theta)d\theta = n. \quad (3)$$

The characterization of  $i(\theta)$  will give the solution to the problem. If there is no inflation, i.e., if  $\pi_t = 0 \quad \forall t$ , the optimal consumption-production decisions are given by Proposition 1. For positive government consumption, the optimal consumption-production decisions are presented in Proposition 2.

**Proposition 1.** With producers prices normalized to one and no inflation, the optimal  $i(\theta)$  is characterized by a cutoff property of the form

$$i(\theta) = \begin{cases} 1 & \text{for } \theta \leq \theta^* \\ 0 & \text{otherwise,} \end{cases}$$

where  $\theta^*$  solves the maximization problem above subject to the characterization proposed for  $i(\theta)$ , and represents the upper bound on the consumption set of the agent.

*Proof.* The proof consists of two steps. First, maximize (2) subject to (3) imposing the characterization of  $i(\theta)$  given in the proposition. Second, show that no other  $i(\theta)$  gives a higher level of utility.

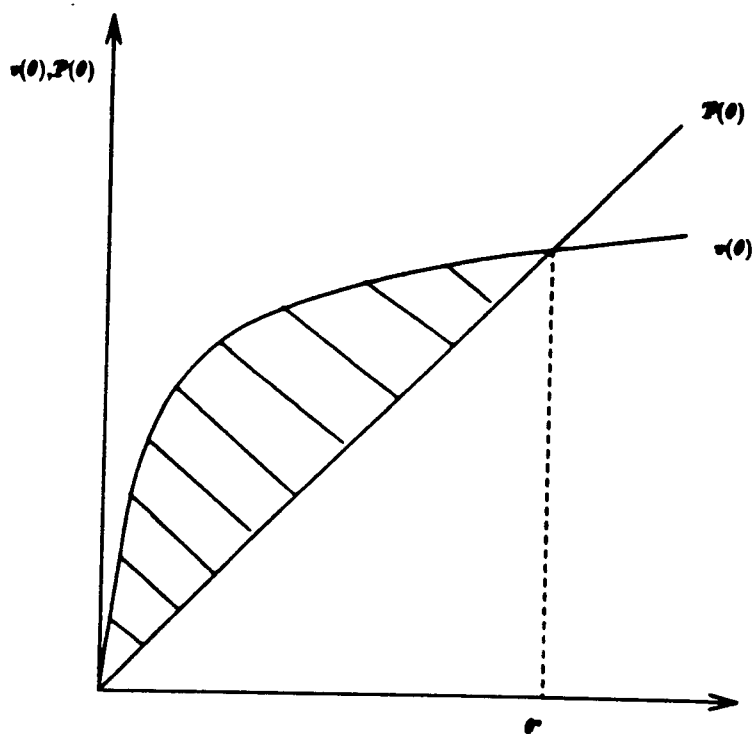
Eliminating  $n$  and substituting in the utility function the problem can be rewritten as

$$\max_{\theta^*} \int_0^{\theta^*} v(\theta)d\theta - \int_0^{\theta^*} p(\theta)d\theta. \quad (4)$$

If  $i(\theta)$  is optimal, it solves (4), and the first order condition is:

$$v(\theta^*) = p(\theta^*). \quad (5)$$

Figure 2: Optimal Consumption Pattern Without Inflation



The first-order condition is expressed in terms of total utility because marginal decisions have been eliminated by the purchase constraint. The individual has to compare the total level of utility obtained from consuming this good with the total disutility of supplying the labor required to purchase it. The constant marginal disutility of labor plays a critical role here by making this decision independent of the purchase of the other goods.

The second-order condition is satisfied by the concavity of  $v$  and the convexity of  $p$ . As these functions are globally concave and convex, the equilibrium is unique.

The graphical solution is presented in Figure 2.  $v(\theta)$  is an increasing, concave function that takes a value of zero at zero and gives the total utility from consuming good  $\theta$ .  $p(\theta)$  is increasing and convex. It represents the total disutility of labor from consuming good  $\theta$ . The agent maximizes total utility, which is depicted by the shaded area between the two curves. The optimal solution is as given by the first-order condition, where the value of the utility equals the disutility cost of providing the labor required to purchase  $\theta$ .

But any other decision criteria would give an inferior result because for  $\theta < \theta^*$ ,  $v(\theta) > p(\theta)$  and for  $\theta > \theta^*$ ,  $v(\theta) < p(\theta)$ ; so, the strategy described is the unique best pol-

icy. Q.E.D.

We can immediately see that the consumption-production pattern described in Proposition 1 is a stationary real equilibrium. Condition 2(i) is trivially satisfied as government consumption is zero. Condition 2(ii) follows from the solution of the proposition for every period plus the stationarity of the setup. Finally, for 2(iii), we notice that each agent decides on an optimal supply of labor that is exactly enough to produce his consumption pattern. Perfect competition and constant returns to scale make profits equal to zero given that prices are fixed at the unit labor requirement. Demand determines the allocation of labor across commodities.

To characterize consumption in an inflationary environment, we need to know how inflation affects the structure of relative consumer prices. Inflation is equivalent to a tax on purchases made with domestic currency. In Figure 1, there is a cutoff good  $\tilde{\theta} = \tau^{-1}(\frac{\pi}{1+\pi})$  such that all goods beyond  $\tilde{\theta}$  are transacted in foreign currency. Now, consider the transportation cost interpretation of the transaction cost of using foreign currency. After the introduction of foreign currency, for a certain denomination and for some good, all future purchases take place with the new currency. Given that we consider the steady state, we disregard this sunk cost, which implies that consumer and producer prices will equal each other and those corresponding to the noninflation case for these commodities.<sup>8</sup> But for low- $\theta$  commodities, the implicit tax is equal to the inflation rate. We can now characterize the pattern of consumption in the inflationary equilibrium.

**Proposition 2.** The optimal consumption pattern of the representative agent will have one of the two following properties (i) The agent consumes all goods below a cutoff  $\theta = \theta^*$ . (ii) The consumption pattern of the representative agent may have one and only one discontinuity.

*Proof.* For the proof of Proposition 2, we use Figure 3. Figure 3 depicts the utility function,  $v(\theta)$ , and the original purchase constraint,  $p(\theta)$ . From Proposition 1, we know

---

<sup>8</sup>All the results extend with small quantitative but not qualitative modifications, to the case in which there is a cost of transacting in foreign currency that has to be paid every period. We treat the above case for simplicity.

case, the agent consumes all goods until  $\theta_1^*$ . Now consider those goods beyond  $\tilde{\theta}_1$ . They will be consumed as long as  $p(\theta)$  lies below  $v(\theta)$ , which necessarily holds until  $\theta_u^*$ . The consumption pattern is discontinuous with the goods between  $\theta_1^*$  and  $\tilde{\theta}_1$  not consumed. No other characterization to the consumption pattern is possible. **Q.E.D.**

Again, it is easy to see that the solution in Proposition 2 is a real stationary equilibrium. Condition 2(i) is satisfied, as the inflation rate, or implicit tax, adjusts to finance government consumption in each period. Condition 2(ii) follows from the solution of the proposition for each agent in every period together with the stationarity of the setup. Finally for 2(iii) notice that by perfect competition, producer prices are once again determined by the unit labor requirements, and total production of each good will be demand determined. In addition to private demand, public demand uses the resources provided by inflation taxation. The higher consumer prices induce agents to supply a larger amount of labor every period. This excess production satisfies government consumption.

Domestic monetary holdings,  $m^d$ , equal

$$m^d = \int_0^{\min(\tilde{\theta}, \theta^*)} p(\theta) d\theta, \quad (6)$$

where  $\theta^*$  represents the lowest cutoff consumption point, corresponding to either statement (i) or (ii) of Proposition 2. The demand for foreign money,  $f$ , equals

$$f = \int_{\tilde{\theta}}^{\theta_u^*} p(\theta) d\theta, \quad (7)$$

if  $\theta_u^* \geq \tilde{\theta}$  and 0 otherwise. These two demand functions depend on  $\pi$  through the relationship of  $\tilde{\theta}$  and  $\theta^*$  to inflation. The properties of  $\tau(\theta)$  are critical for the determination of money demand, as it determines the value of  $\tilde{\theta}$ .

At the beginning of the paper, we noticed that there are two kinds of movements in money demand. One shift takes place *along* the money demand schedule and measures how an increase in inflation reduces money demand. In our model, this shift is represented by movements in  $\theta^*$ . The second kind of movement is a change in the *money demand function* itself, which in this model, is represented by changes in  $\tilde{\theta}$ . Each time an additional currency is introduced, domestic money demand is reduced at each rate of inflation. The model



is successful in identifying and separating these two distinct changes in money demand. Under the transportation cost interpretation, the model also suggests strong permanent effects of inflation on money demand, since there is no reason in the model why, once a foreign currency is introduced, people should discontinue its use, regardless of how low the inflation rate falls. Evidence in support of hysteresis in domestic money demand is presented in Dornbusch, Sturzenegger, and Wolf (1990).<sup>9</sup>

Having derived the optimal consumption behavior and the endogenous cash-in-advance constraints on domestic currency, we can now look for the equilibrium inflation rate given government spending. The government's budget constraint is

$$g = \frac{\pi}{1 + \pi} m^d = \frac{\pi}{1 + \pi} \int_0^{\min(\tilde{\theta}, \theta^*)} p(\theta) d\theta. \quad (8)$$

If  $g$  is exogenous, the equilibrium rate of inflation increases when we introduce financial adaptation because the tax base on which the inflation tax is levied is reduced. Notice also that this government revenue curve has the usual Laffer curve properties. (To see this, just notice that if inflation goes to infinity, tax collection falls to zero.) To compute how inflation is affected by changes in government spending, we implicitly differentiate (8)<sup>10</sup> to obtain

$$\frac{dg}{d\pi} = \int_0^{\min(\tilde{\theta}, \theta^*)} \frac{p(\theta)}{(1 + \pi)^2} d\theta + \frac{\pi}{(1 + \pi)} p(\min(\tilde{\theta}, \theta^*)) \frac{d\min(\tilde{\theta}, \theta^*)}{d\pi}. \quad (9)$$

For the region in which dollarization is a relevant constraint,  $\frac{d\min(\tilde{\theta}, \theta^*)}{d\pi}$  is the inverse of the slope of  $\tau(\theta)$ . Equation (9) is positive on the right portion of the Laffer curve and approaches zero as the government reaches the seigniorage maximizing point. The inflation rate that maximizes government revenue is defined as that for which (9) equals zero.

For dollarization to be a binding constraint on government financing, it must be true that as the inflation rate increases,  $\tilde{\theta}$  eventually falls below  $\theta^*$ . This reduces the amount of goods transacted in domestic currency and, therefore, reduces the amount of seigniorage collected at each rate of inflation. The amount of government spending that it is possible

---

<sup>9</sup>Notice that under the alternative interpretation for the transaction cost, there is no hysteresis.

<sup>10</sup>The function  $\min(\tilde{\theta}, \theta^*)$  is not differentiable at  $\tilde{\theta} = \theta^*$ . But this is a measure zero event, so we need not worry about it.

to finance through inflationary financing is therefore reduced as compared with the case in which there is no dollarization.

The welfare implications may be easily understood by looking at Figure 4. Without dollarization and inflation, consumption takes place until  $\theta_u^*$ , with total consumer surplus being equal to the area between the curves  $v(\theta)$  and  $p(\theta)$ . With seigniorage financing and no dollarization, the consumption set reduces to  $\theta_l^*$ , where the triangle  $0ab$  represents the inflation tax. Now, consider a level of currency penetration given by  $\tilde{\theta}_1$ . The rate of inflation does not increase, as the demand for domestic money is unchanged. The benefit of dollarization is unambiguously positive and stems from the reduction in the cost of the cash-in-advance constraint for high- $\theta$  commodities (all those between  $\tilde{\theta}$  and  $\theta_u^*$ ). Finally, consider the case of a strong level of currency penetration, which implies a  $\tilde{\theta}$  to the left of  $\theta_l^*$ . In this case, the equilibrium looks like that corresponding to  $\tilde{\theta}_0$  in the figure.<sup>11</sup> In this case, the areas of A (the dotted area) and B (the cross-hatched area) are equal, and the agent experiences an unambiguous welfare gain ( $abc$ ).

In summary, we have shown that a representative agent is made unambiguously better off by the process of financial adaptation. The result is nothing but an application of the Ramsey taxation rule. Small bundle commodities have a lower demand elasticity, so welfare improves if they are taxed more heavily. In our model, low- $\theta$  commodities are completely inelastic. This is certainly sufficient but not necessary for our result. Nevertheless, it captures the general idea that inflation may improve the tax system by taxing more heavily the basic commodities, which are relatively inelastic compared to other commodities in the economy.

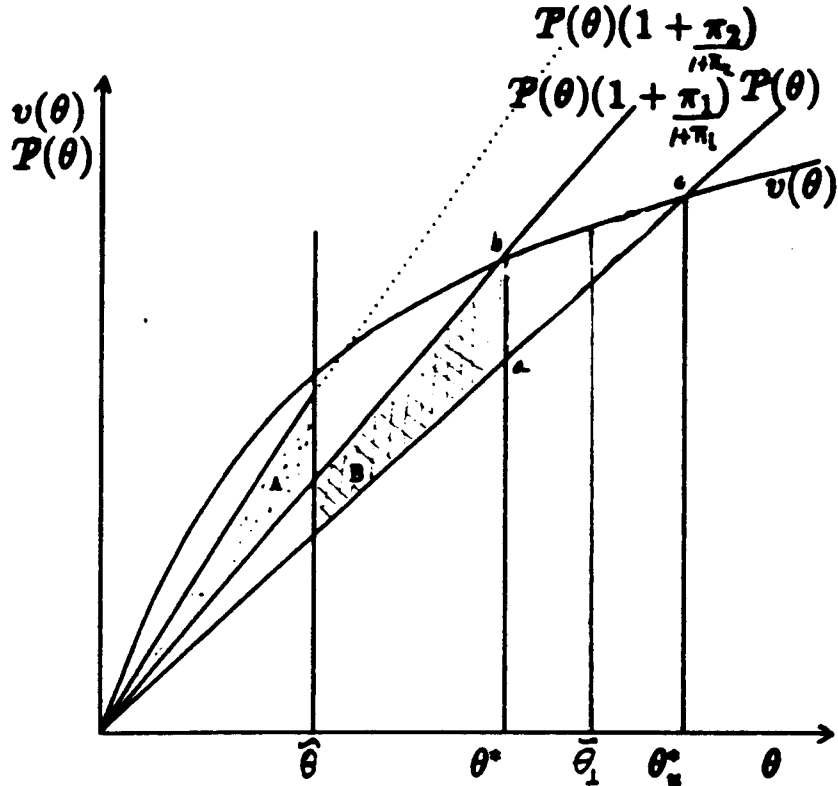
## 2 Income Distribution and Dollarization

In this section we consider the implications of assuming that agents are heterogeneous on the issues considered in the previous section, i.e., the equilibrium inflation rate and welfare. We assume that there is a continuum of agents with support  $H$ , which differ in regard to

---

<sup>11</sup> $p(\theta)$  cannot intersect  $v(\theta)$  at  $\tilde{\theta}_0$  because the level of government spending is fixed and we are on the correct side of the Laffer curve.

Figure 4: Welfare Effects of Financial Adaptation



their productivities or real wages. This affects the budget constraint, which now must include a wage term. The budget constraint for each individual  $h$  becomes

$$\int_{\theta} p(\theta) i(\theta) = w^h n, \quad (10)$$

therefore effectively changing the relevant  $p(\theta)$ , measured in terms of labor units, for each individual. The consumption pattern without inflation is the same as before. It is straightforward to show that the cutoff good  $\theta^*$  increases with the wage rate.

Government spending,  $g$ , is financed through inflation as in the previous section. In particular, we have that in the case of heterogeneous agents,

$$g = \frac{\pi}{1 + \pi} \int_H \int_0^{\min(\hat{\theta}, \theta^{*h})} p(\theta) d\theta dh, \quad (11)$$

where  $\theta^{*h}$  is the optimal cutoff good for individual  $h$ . As before, the equilibrium rate of inflation with an exogenous level of government spending will increase in the presence of financial adaptation as domestic money demand is reduced.

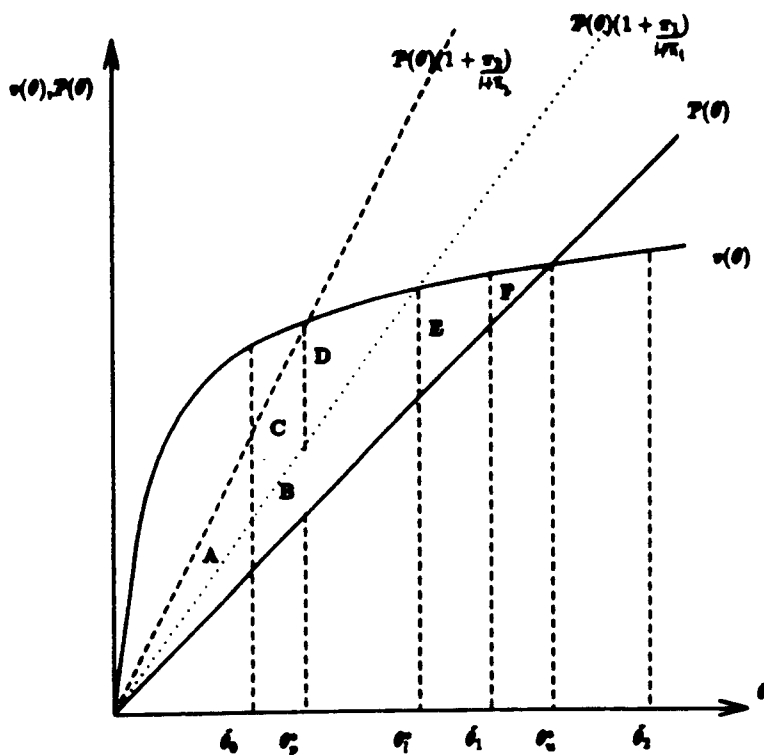
The total inflation tax paid by each agent depends on his consumption of goods trans-

acted in domestic currency. It therefore increases with the wage until the agent consumes the complete set of goods transacted with domestic currency, say at wage  $\bar{w}$ . For levels of income larger than  $\bar{w}$ , the inflation tax remains constant at the  $\bar{w}$  level, as spending in goods purchased with domestic currency does not increase any further. In this model, the inflation tax is proportional to income at low levels of income and declines as a proportion of income at high levels of income.

In this economy, the results regarding the welfare effects of financial adaptation are richer than those of the representative agent economy. Consider Figure 5. Figure 5 shows three curves:  $p^h(\theta)$ ,  $p^h(\theta)(1 + \frac{\pi_1}{1+\pi_1})$  and  $p^h(\theta)(1 + \frac{\pi_2}{1+\pi_2})$ . The first measures the disutility of labor required to purchase a particular commodity without inflation. The second incorporates the inflation tax in an economy without financial adaptation. Finally, the last considers the relevant tax with dollarization. For the economy as a whole, there is a unique cutoff commodity,  $\tilde{\theta}$ , but where this cutoff commodity lies for each agent relative to his consumption pattern depends on his relative wage. If he has low productivity, he may not get to consume the dollarized commodities at all. (Graphically,  $\tilde{\theta}$  is to the right of his consumption set). If he has high productivity, he may consume most of his income in dollarized commodities. ( $\tilde{\theta}$  is in the left end of his consumption set). Graphically, the three curves will rotate downward together as the wage increases; equivalently, for a set of curves, we can move the relevant  $\tilde{\theta}$  for each group. For compactness this is how we carry our analysis in Figure 5. Therefore, the graph shows three cutoff goods:  $\tilde{\theta}_0$ ,  $\tilde{\theta}_1$ , and  $\tilde{\theta}_2$ . If the wage is very low, the relevant cutoff good in the graph is  $\tilde{\theta}_2$ . In this case, with financial adaptation, the agent has to pay a higher inflation tax (measured by areas A+C) on the goods purchased plus the surplus cost, D, without any compensating welfare gain. When financial adaptation is introduced, the equilibrium rate of inflation increases because the tax base of the inflation tax falls. Poor agents consume goods that are only purchased with domestic currency. Therefore, the only effect on them is that they must pay a higher tax for the goods they purchase; they are clearly made worse off.

For the middle class, the relevant cutoff good is between  $\theta_u^*$  and  $\theta_p^*$ , say  $\tilde{\theta}_1$ . This case presents the particular feature that there will now be a discontinuity in the consumption pattern of the agent, as was characterized in Proposition 2. The welfare effects of financial

Figure 5: Welfare Effects of Financial Adaptation with Heterogeneity



innovation are ambiguous and will depend on the relative loss, areas A + C (the increase in the inflation tax paid) and D (loss of consumption surplus), with the relative gains, area F, which is the increased consumer surplus from the reduction in the inflation tax paid on the cash in advance constraint for all goods above  $\tilde{\theta}_1$ .

Finally, if the wage is sufficiently high, the relevant cutoff good is below  $\theta_p^*$ , say at  $\tilde{\theta}_0$ . The gains is now area B+E+F and the losses equal area A. As can be seen from the figure, the costs of financial adaptation approach zero as the wage increases, and the benefits approach a strictly positive number. Therefore, there exists a wage such that for wages above it, the individual gains unambiguously from the financial adaptation process.

The process of financial adaptation has therefore been shown to have very clear-cut welfare effects. The poor are hurt by the possibility that the rich can engage in the process of financial adaptation. This result carries a very strong policy message: the inflation tax is very regressive once we realize that access to the financial sector differs between individuals. We study the implications of this on the optimal level of government spending next.

### 3 Endogenous government spending

Now, we endogenize  $g$  by assuming that the government maximizes a utility function that depends both on the level of government spending and on private utility. The government evaluates private utility through a social welfare function defined over the utilities of the agents in the economy. These utilities are the solution for each private agent to the problem discussed in Section 2. In the case of a representative agent, an alternative but equivalent interpretation is that the agent maximizes utility over private and public consumption. The problem below is the solution to the optimal level of public consumption through a two-step budgeting process.

At this point, one may ask how restrictive is the assumption that the government relies solely on inflationary taxation to finance government spending; that is, how would the results of the paper change if optimal commodity or labor taxation was also allowed? Even if some government consumption were financed through an optimal Ramsey rule (i.e., taxing the low  $\theta$  commodities more heavily), inflation without financial adaptation would still imply a flat tax structure across all commodities that could be improved upon by reducing the tax on the most elastic commodities. The welfare implications would therefore be the same.

For the representative-agent case, we assume that the agent maximizes

$$U = u(g) + V(g) - \int_{\tilde{\theta}}^{\theta_u^*} p(\theta) d\theta, \quad (12)$$

where  $V(g)$ , represents the indirect utility function derived in Section 2. The last term represents the seigniorage cost of purchasing the stock of foreign currency, which represents a net loss for the economy. The first order condition is

$$u'(g) = -\frac{dV}{dg} - 1(\theta_u^* \geq \tilde{\theta}) p(\tilde{\theta}) \frac{d\tilde{\theta}}{d\pi} \frac{d\pi}{dg}, \quad (13)$$

where  $1(\theta_u^* \geq \tilde{\theta})$  is an indicator function that equals one when  $\theta_u^* \geq \tilde{\theta}$ . Equation (13) defines the optimal amount of government spending,  $g$ , as that for which marginal utility equals marginal cost. The marginal cost includes both the utility cost in terms of privately provided commodities and the marginal cost of purchasing the stock of foreign currency.

The second order condition

$$u''(g) + \frac{d^2V}{dg^2} + 1(\theta_u^* \geq \tilde{\theta})p(\tilde{\theta})\frac{d\tilde{\theta}}{d\pi}\frac{d^2\pi}{dg^2} < 0 \quad (14)$$

will also be satisfied, since  $u$  and  $V$  are concave and the marginal cost of purchasing the stock of foreign currency is increasing in  $g$ .

Changes in optimal  $g$  will take place with financial adaptation if the right-hand side of (13) changes with the financial setup. Without dollarization, the optimized value of private consumption equals

$$V^1 = \int_0^{\theta^*} [v(\theta) - (1 + \frac{\pi}{1+\pi})p(\theta)]d\theta, \quad (15)$$

which implies, differentiating and using the envelope theorem that,

$$\left(\frac{dV}{dg}\right)^1 = -\left[\int_0^{\theta^*} \frac{p(\theta)}{(1+\pi)^2}d\theta\right]\frac{d\pi}{dg}. \quad (16)$$

With dollarization, the value of private consumption equals

$$V^2 = \int_0^{\tilde{\theta}} [v(\theta) - (1 + \frac{\pi}{1+\pi})p(\theta)]d\theta + \int_{\tilde{\theta}}^{\theta_u^*} [v(\theta) - p(\theta)]d\theta, \quad (17)$$

which implies that<sup>12</sup>

$$\left(\frac{dV}{dg}\right)^2 = -\left[\int_0^{\tilde{\theta}} \frac{p(\theta)}{(1+\pi)^2}d\theta\right]\frac{d\pi}{dg} - \frac{\pi}{1+\pi}p(\tilde{\theta})\frac{d\tilde{\theta}}{d\pi}\frac{d\pi}{dg}. \quad (18)$$

But, for  $p(\theta) = \theta$ , it can be shown that

$$\left(\frac{dV}{dg}\right)^1 = -\frac{1}{1 + \frac{2\epsilon_{\theta\pi}}{1+\pi}}, \quad (19)$$

where  $\epsilon_{\theta\pi} < 0$  is the elasticity of  $\theta$  with respect to the inflation rate. For the case in which financial adaptation is present

$$\left(\frac{dV}{dg}\right)^2 = -1. \quad (20)$$

Equations (19) and (20) show that the cost of inflation is higher if the economy does not have access to financial adaptation. If the financial adaptation process becomes very easy,

---

<sup>12</sup>Notice that the envelope theorem no longer applies.

the marginal cost of inflation becomes very small. This increases inflation as compared with the equilibrium without financial adaptation because the inflation rate necessary to finance a given  $g$  increases and because the optimal  $g$  increases. This result reflects the fact that when economies learn to live with inflation, the government has a bigger incentive to use inflationary financing of the deficit.

In the case of heterogeneous agents, we assume the government maximizes

$$L^g = u(g) + \int_H \omega_h V_h(g) dh - \int_H \int_{\bar{\theta}}^{\theta_u^*} 1(\theta_u^* > \bar{\theta}) p(\theta) d\theta dh, \quad (21)$$

where  $\omega_h$  is the weight assigned to individual  $h$  and the last term accounts for the cost of purchasing the stock of foreign currency. The first-order condition for the government's maximization problem is

$$u'(g) = - \int_H \omega_h \frac{dV_h}{dg} dh - \left[ \int_H 1(\theta_u^* > \bar{\theta}) p(\bar{\theta}) dh \right] \frac{d\bar{\theta}}{d\pi} \frac{d\pi}{dg}. \quad (22)$$

This condition states that the marginal utility of government consumption has to be equal to the weighted marginal disutility of private agents plus the marginal cost of purchasing the stock of currency that will displace domestic currency due to the increase in inflation generated by government spending. The solution will depend on, among other things, the weights attached to different income groups.

Consider the case in which there are two kinds of agents, poor and rich, and  $p(\theta) = \theta$ . Agents differ in their productivity, the poor having lower productivity and therefore consuming a smaller set of commodities. Without financial adaptation, the maximized value of utility for agent  $i$  with wage  $w^i$  is equal to

$$V^i = \int_0^{\theta^{*i}} \left[ v(\theta) - \frac{1}{w^i} \left( 1 + \frac{\pi}{1 + \pi} \right) p(\theta) \right] d\theta, \quad (23)$$

where  $i = p, r$ . The derivative of the indirect utility function with respect to government spending is the same as in (16) with different  $\theta^*$ 's for each agent, or

$$\left( \frac{dV^i}{dg} \right)^1 = - \left[ \int_0^{\theta^{*i}} \frac{p(\theta)}{w^i (1 + \pi)^2} d\theta \right] \frac{d\pi}{dg}; \quad (24)$$

where  $i = p, r$ .



From (24), we observe that there are two effects at work when comparing this derivative for  $i = p$  and  $i = r$ . First, rich people consume more goods purchased with domestic currency ( $\theta^{*r} > \theta^{*p}$ ). This tends to increase the cost of inflation for the rich. Second, rich agents will have a higher wage and consume more goods, and their utility is not substantially affected by changes in the inflation rate. This effect is captured by the wage rate in the denominator of (24). It can be shown that the value of the integral in equation (24) increases with the wage. Therefore, in the economy without financial adaptation, the optimal degree of government spending will increase as the weight on low-income groups is increased.

With financial adaptation, the low productivity agent consumes only goods purchased with domestic currency, so his problem is as above. The agent with the higher wage has a set of consumption goods that includes “dollarized” commodities; so, his utility level is

$$V^r = \int_0^{\tilde{\theta}} [v(\theta) - \frac{1}{w^r} (1 + \frac{\pi}{1 + \pi}) p(\theta)] d\theta + \int_{\tilde{\theta}}^{\theta^{*r}} [v(\theta) - \frac{1}{w^r} p(\theta)] d\theta. \quad (25)$$

Differentiating (25) respect to  $g$ , we obtain

$$\frac{dV^r}{dg} = -[\int_0^{\tilde{\theta}} \frac{1}{w^r} \frac{p(\theta)}{(1 + \pi)^2} d\theta + \frac{\pi}{(1 + \pi)} p(\tilde{\theta}) \frac{d\tilde{\theta}}{d\pi}] \frac{d\pi}{dg}. \quad (26)$$

In addition to the two effects discussed above, in the case of dollarization, the rich derive an additional benefit from financial adaptation, given by the shift of the cutoff commodity,  $\tilde{\theta}$ , to the left. This increases the range of goods they can buy with foreign currency. Now, it is not longer clear that the optimal amount of government spending increases as the weight on the poor increases; if the elasticity of  $\tilde{\theta}$  is high enough, the perceived cost of inflation may be lower for the rich.

To discuss the implications of financial adaptation on the level of government spending, we consider the case in which the poor receive a weight equal to one in (21). Substituting for  $\frac{d\pi}{dg}$  from (11), we obtain for (22)<sup>13</sup>

$$u'(g)^{-1} = w^p [1 + \left(\frac{\theta^{*r}}{\theta^{*p}}\right)^2 + (1 + \pi_1)(\epsilon_{\pi\theta^{*p}} + \left(\frac{\theta^{*r}}{\theta^{*p}}\right)^2 \epsilon_{\pi\theta^{*r}})], \quad (27)$$

<sup>13</sup>In what follows we disregard the last term in (22). Since the cost of additional currency penetration increases with inflation, the arguments would be reinforced if we considered this term.

$$u'(g)^{-1} = w^p \left[ 1 + \left( \frac{\tilde{\theta}}{\theta^{*p}} \right)^2 + (1 + \pi_2)(\epsilon_{\pi\theta^{*p}} + \left( \frac{\tilde{\theta}}{\theta^{*p}} \right)^2 \epsilon_{\pi\tilde{\theta}}) \right]. \quad (28)$$

Equation (27) is derived for the case when there is no financial adaptation. Equation (28) corresponds to the case in which financial adaptation is present. For the level of government spending to be lower with financial adaptation, (27) must be bigger than (28). Comparing the two equations, we see that almost all of the terms are different: the ratio of cutoff commodities, the inflation rates, and the elasticities. We expect the ratio of cutoff commodities to decline with the introduction of financial adaptation. Even though the rate of inflation increases with financial adaptation, so  $\theta^{*p}$  falls, we know that  $\tilde{\theta}$  will be substantially lower than  $\theta^{*r}$ . In addition, the inflation rate will increase with financial adaptation, so the second negative term will be larger in (28). If the elasticities do not change much (and if they change, we will probably have that  $\epsilon_{\pi\tilde{\theta}} > \epsilon_{\pi\theta^{*r}}$ , reinforcing the previous effects), it will be the case that financial adaptation induces a lower level of government spending. Once again, the intuition relies on the fact that the costs of inflation for the poor are very large at high rates of inflation.

The implications for the optimal level of government spending are once again clear. If the government cares more for the poorest agents in the economy, then it will want to reduce the level of government spending as inflation increases with financial adaptation because the marginal cost of financing a certain level of government spending increases.

While the effect of financial adaptation on the level of government spending is clear, it is not so for the inflation rate. Even though the optimal  $g$  falls in the presence of financial adaptation, the rate of inflation may increase, as the fall in domestic money demand generates the need to increase the inflation rate even more to finance a lower level of government spending. Many countries have experienced strong increases in the rate of inflation together with sharp contractions in the total level of government spending (eg., Argentina and Peru in the late 1980s). This model explains why both things take place at the same time.

## 4 Conclusions

This paper has tried to cover the welfare implications and describe monetary equilibria in economies in which endogenous financial adaptation occurs. We discussed three issues. First, we endogenized the monetary holdings of a representative agent. The model accounts for the fact, that as inflation increases, there is process of currency penetration and that there are two different kinds of changes in money demand, one generated by an increase in the inflation rate and the other induced by a process of financial adaptation that actually lowers domestic money demand at each rate of inflation. Second, we discussed the welfare effects of financial adaptation. It was shown that with a representative agent, financial adaptation is welfare improving, but once we allow for heterogeneity between agents, we find that the poor are hurt and the rich benefit. Third, we showed how the model suggests variables on which government spending depends and how this decision is affected by the degree of financial liberalization.

The model accounts for the stylized observation that there are “currency circuits” in which some commodities are purchased with domestic currency and some with an inflation-proof technology. The model is very vague as to what this other technology may be. The current model may be understood as one that explains not only the role of “dollarization”, but also of interest-bearing checkable accounts, alternative commodity monies, or credit cards. The motivation for the transaction cost seems appealing because an alternative, say a fixed cost (to open an account, to get to know a currency dealer, or to get your credit card) would imply that the currency circuits take place at the level of people and not at the level of commodities. This is certainly not the pattern that arises.

The adoption of financial adaptation increases the rate of inflation for a constant level of seigniorage financing. In this paper, we considered the steady state, but the introduction of new techniques or foreign currency takes time. The path of money demand therefore should be declining, and the path of inflation rising through time. This may be an important component in explaining the dynamics of the inflation process. Inflation is a regressive tax. Baumol-Tobin-Barro money demands have an income elasticity equal to one half. This model gives additional reasons why inflation taxation may be regressive. Some goods

do not require domestic currency for transactions, and rich agents have a consumption pattern strongly biased toward these commodities. Furthermore, as rich agents substitute out of domestic currency, the inflation rate paid by those who still purchase with domestic currency increases. Financial adaptation works as a redistributive mechanism from poor to rich.

Many countries have found that tough stabilization programs have become very popular. Argentina, in June 1985 and in February 1991, implemented very strict fiscal programs that rendered stunning electoral successes months later. In Brazil, the Cruzado plan of January 1986 raised Sarney's popularity to the highest peak in Brazilian history. These facts are not puzzling, when we consider the strong regressivity of the inflation tax in these economies, where a long history of inflation has led to an expansion of financial institutions and pervasive dollarization of the economy.

With respect to the effect of financial adaptation on the inflation rate, our results are rather ambiguous. A deepening of the financial adaptation process generates an increase in the rate of inflation but also induces a decline in the optimal level of government spending. In general, the decrease will not compensate for the increased inflation, so we will observe economies with increasing inflation and smaller government spending. This has certainly been the experience of Argentina, Bolivia, Brazil, and Peru in the 1980s.

Finally, we reach the question of policy implications. Does our model suggest a positive or a negative role for financial adaptation? Should the government allow and encourage this process, or should it try by all its means to reduce its importance? The answer is not clear. Financial adaptation is certainly effective, as the representative agent model showed, in enhancing the transactions efficiency of the economy and inducing a more optimal tax system. However, the income distribution implications may be appalling. On this count, a government very concerned with income distribution should be expected to impose strong capital and financial controls. Again, the model gives a result which is clearly in line with the real world. Very "efficiency oriented" governments that do not care about income distribution are likely to implement sweeping financial liberalization. "Populist" governments, or those very concerned with income distribution, will nationalize the banking system, impose exchange rate controls, and forbid capital flows. The model shows that both are right,

and that their policies are helpful in attaining the objectives pursued.

## References

- [1] Bomberger, W. A. and Makinen, G.; (1980) "Indexation, Inflationary Finance, and Hyperinflation: the 1945-1946 Hungarian Experience". *Journal of Political Economy*, Vol.88, no.31.
- [2] Bomberger, W. A. and Makinen, G.; (1983) "The Hungarian Hyperinflation and Stabilization of 1945-1946". *Journal of Political Economy*, vol. 91, No.51.
- [3] Calvo, G. A. and Rodriguez C.; (1977) "A Model of Exchange Rate Determination under Currency Substitution and Rational Expectations" *Journal of Political Economy*, Vol. 85, No. 3.
- [4] Calvo, G. A.; (1985) "Currency Substitution and the Real Exchange Rate: The Utility Maximization Approach", *Journal of International Money and Finance*, 2:175-188.
- [5] Dornbusch, R., Sturzenegger, F. and Wolf, H.; 1991, "Extreme Inflation: Dynamics and Stabilization", *Brookings Papers on Economic Activity*.
- [6] Kaldor, N.; (1946a) "A study in inflation, I-Hungary's Classical Example", published in *Manchester Guardian*, 21st. November.
- [7] Kaldor, N.; (1946b) "A study in inflation, II-Stabilization", published in *Manchester Guardian*, 22nd. November.
- [8] Katzenellenbaum, S.; (1925) *Russian Currency and Banking 1914-1924*. London:P.S. King&Son, LTD.
- [9] Keller, A.; (1958) *Das Notegeld*, Munich Battenberg: Verlag.
- [10] Liviatan, N.; (1981) "Monetary Expansion and Real Exchange Rate Dynamics", *Journal of Political Economy*, vol 89, No. 6.

- [11] Prescott, E.; (1987) "A Multiple Means-of-Payment Model" in *New Approaches to Monetary Economics* eds. W. A. Barnett and K. J. Singleton.
- [12] Vegh, C.; (1989), "The Optimal Inflation Tax in the Presence of Currency Substitution", *Journal of Monetary Economics*, Vol. 24.
- [13] Weil, P.; (1990), *Currency Competition and the Evolution of Multi-Currency Regions*, Harvard University, Mimeo.
- [14] Yurovsky, L.; (1925) *Currency Problems and Policy of the Soviet Union*. London: Leonard Parsons.