The Implications of Convex Arbitrage Costs for International Macroeconomics

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Abstract

I study the implications of convex arbitrage costs in a two-country dynamic general equilibrium model. The presence of the selling/buying costs implies that the real exchange rate (the relative price of the foreign-market goods in terms of the home-market goods) in a trade surplus (deficit) country is above (below) unity. I demonstrate why and how the long run real exchange rate depends on the initial net foreign asset position. I analyze the relationships between the real exchange rate, net exports, output, employment, and consumption. In particular, I derive an upward (downward) sloping aggregate supply (demand) curve in a trade surplus (deficit) country. I also discuss the Harberger-Laursen-Metzler effect. I subsequently introduce the nontraded goods and argue that the real exchange rate and the relative price of the traded in terms of nontraded goods are inversely related. The stochastic properties of the data implied by the model have a reasonable success in matching the available empirical evidence on the international business cycles.

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1. Introduction

This paper develops a simple two-country general equilibrium model to study the implications of convex arbitrage costs for international macroeconomics. Over the years, the assumption of a costless price arbitrage and its implication - the Law of One Price have been made increasingly questionable by both the direct evidence and the variability of the real exchange rates (see e.g., Isard (1977), Kravis and Lipsey (1978), Engel (1991)). I recognize the empirical failure of the Law of One Price by explicitly introducing costs involved in an international exchange of goods.

I consider a case when the arbitrage costs are convex. The intuition when the arbitrage/transaction costs are fixed or linear is relatively well-known (see e.g., Dumas (1988)). In a recent paper Backus and Kehoe (1991) attempt to improve the performance of the simulated two-country real business cycle model by assuming the quadratic (and thus convex) costs of trade. They study the centralized equilibrium and the exact intuition behind the effects of the convex trading costs is not discussed. In contrast, I study the decentralized model in which sellers and buyers optimally choose the geography of sales/purchases. As expected, at the optimum agents are - on the margin - indifferent between selling (buying) on the expensive (cheap) market. One of the insights of the paper is an observation that, by the convexity of the costs, the marginal transaction costs are above the average costs. Hence rents are earned in selling (buying) the intramarginal units on the expensive (cheap) market. This allows me to derive either an aggregate supply or an aggregate demand as, respectively, an increasing or decreasing function of the real exchange rate (i.e., the relative price of the foreign-market goods in terms of the home-market goods) in an environment in which the domestic and foreign goods are perfect substitutes in the utility function. I discuss the Harberger-Laursen-Metzler effect (see Harberger (1950), Laursen and Metzler (1950)). The selling/buying costs trivially imply that the real exchange rates of the net seller (buyer) countries are above (below) unity. I formally analyze a relationship between the initial net foreign asset position and the long run real exchange rate. I also discuss how the elasticities of labor supply and consumption demand affect the dependence of consumption and output on the real exchange rate. Subsequently, I introduce the nontraded goods and argue that the real exchange rate and the relative price of the traded in terms of nontraded goods are inversely related. The stochastic properties of the data implied by the model have a reasonable success in matching the available empirical evidence on the international business cycles.

The remainder of the paper is organized as follows. In section 2 I lay down and study a two-country model. The standard distinction between traded and nontraded goods is introduced and briefly discussed in section 3. In section 4 I discuss the implications of the model for the stochastic properties of the data, as well as their relevance in the light of the available evidence. Section 5 concludes.

2. The Model[1]

Consider the following perfect foresight two-country model set up in a continuous time. The home and the foreign countries are symmetric (all the foreign country-related variables are starred). Given the symmetry, I will only describe the home economy. It is populated by a large number of immortal and identical households. Each household has three members: a manager, a shopper, and a worker. They all act independently. The manager runs the family-owned firm. It produces a perishable tradeable good using hired labor and a linear technology. The firm can sell its output either on the domestic market (for the domestic money) at price p, or on the foreign market (for the foreign money) at price p^*. The only role of money performed in these transactions is that of a unit of account (i.e., it is not held as an asset). The nominal exchange rate is E units of domestic money for one unit of foreign money. E is assumed to have a differentiable path. Denote by h the productive employment (= output), and let a be the fraction of h that is sold on the world market. Foreign sales are costly in terms of the domestic labor. One may think of these selling costs as arising due to a preparation of sales documents, labels, and catalogs in foreign languages. I assume that selling ah units of the good abroad requires hiring $h\alpha(a)$ hours of domestic labor at the competitive nominal wage w units of domestic money per hour (i.e., $(\alpha/a)/a\ w$ is the average selling cost). α is a positive, increasing,

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1 The model generalizes and applies the idea of the trading costs which I developed in my (1992) paper.

strictly convex and twice-differentiable function. The presence of some fixed factor (e.g., a stock of human capital in foreign languages) can explain the convexity of the selling costs. The shopper's job is to buy the consumption good, c. Buying goods abroad is costly (e.g., there is a time cost of translating the foreign labels, etc.). Let e be the fraction of c that is purchased on the foreign market. A purchase of ec units of the foreign-produced good requires $c\alpha(e)$ hours of domestic labor, where the function α has already been introduced. The last family member, the worker, sells n hours of his labor to the domestic employers.

I assume that the home-currency denominated interest-bearing assets are not available to foreign citizens. Given that all the domestic families are alike and there is no government borrowing, it is necessarily the case that in equilibrium the domestic bonds are in zero net supply. Therefore such bonds can be abstracted from. The domestic household holds b units of the foreign-currency-denominated consol-type bonds (the foreign-currency denominated assets are available abroad). They sell for p^* and, at any instant of time, pay δp^* units of foreign money (i.e., the instantenous real rate of interest equals δ). I assume that the path of p^* is differentiable. The initial stock of the foreign bonds is denoted by b_0.

Households maximize a life-time integral of discounted intraperiod utilities subject to a life-time wealth constraint. The momentary utility is a separable function of consumption and labor. The utility of consumption has standard properties (including the Inada conditions), and - to facilitate the algebra - the disutility of labor is linear (which, obviously, implies a perfectly elastic labor supply). This completes the description of the model.[2]

Formally, each domestic household solves the following problem (a symmetric problem is solved by the foreign households):

$$(1) \qquad \max_{n,h,c,a,e} \int_0^\infty \{u(c) - n\}\exp(-\delta t)dt$$

subject to:

$$(2) \quad A_0 + \int_0^\infty \Big[wn + h[aEp^* + (1-a)p - \alpha(a)w] - wh - c[eEp^* + (1-e)p + \alpha(e)w]\Big]\exp\{-\int_0^t (id\tau)\}dt = 0$$

where: $\delta > 0$ is the subjective rate of time preference; in deriving (2) I integrated the appropriate flow budget constraint using the no Ponzi

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2 As pointed out by Roger Farmer, the model developed here has an isomorphic representation in which the assumption about the convexity of trading costs is replaced by the assumption about the concavity (diminishing returns) in the production/consumption of the foreign goods. One such equivalent representation can be described as follows. Let the domestic producers produce two types of goods: a domestically-sold good and an export good. Suppose further that while the technology in the production of the home-sold good exhibits constant returns, there are diminishing returns in the production of an export good. On the demand side, let let each household produce the final consumption good out of two market goods, a home-bought good and an import good. The home production function exhibits constant returns in the amount of the domestically-bought good but decreasing returns in the amount of the import good.

games condition; $A = bEp^*$; A_0 is an initial nominal stock of foreign assets; $i = \delta+\varepsilon+v$ = the nominal interest rate on foreign bonds; $\varepsilon = \dot{E}/E$ = the rate of exchange rate depreciation; $v = \dot{p}^*/p^*$ = the rate of foreign inflation; in addition to the already mentioned properties, the function α satisfies $\alpha(0) = 0$; time subscripts are suppressed to economize on notation.

(1)-(2) define an optimal control problem that simultaneously captures two independent optimizations: profit maximization by firms (choices of h and a), and utility maximization by consumers/workers (choices of n, c, and e). This together with the subsequently assumed market clearing implies that its solution defines a competitive equilibrium. Given the convexity of the maximization problem, (1)-(2), the necessary conditions for its solution are also sufficient. A straightforward argument shows that a (e) is positive if and only if $Ep^* > p$ ($Ep^* < p$). (Note, however, that a^* (e^*) is positive if and only if $Ep^* < p$ ($Ep^* > p$).) It follows that a and e cannot be jointly positive. Accordingly, in the solution I simultaneously account for a case of a positive a and a case of a positive e. The primed equations refer to the latter case.

Denoting by μ_0 the multiplier on (2) (the date zero marginal utility of the nominal life-time wealth), the conditions for an optimum consist of (2) and:

$$(3)\quad 1 = \mu_0 \exp[\delta t - \int_0^t i d\tau] w$$

$$(4)\quad aEp^* + (1-a)p = w(1 + \alpha)$$

(4') $w = p$

(5) $$u'(c) = \mu_0 \exp[\delta t - \int_0^t i d\tau] p$$

(5') $$u'(c) = \mu_0 \exp[\delta t - \int_0^t i d\tau][eEp^* + (1-e)p + \alpha w]$$

(6) $$Ep^* - p = \alpha'(a)w$$

(6') $$Ep^* - p = -\alpha'(e)w$$

where a prime - "'" - indicates the derivative.

(3) through (5') have standard interpretations. As usual, (3) and (4)-(4') define the optimal labor supply and demand, respectively. (5)-(5') define the optimal demand for the consumption good. The "new" conditions, (6)-(6'), say that sellers (buyers) are - on the margin - indifferent between selling (buying) the traded good on the domestic or the foreign market. Such indifference occurs only if the geography of transactions is chosen to equalize the marginal cost of selling (buying) on the high (low) price location to the price differential between the two markets. It will turn out to be important for a subsequent derivation of an aggregate supply and demand that since the function α is strictly convex and $\alpha(0) = 0$, the marginal trading cost is

necessarily higher than the average trading cost. Therefore the sellers (buyers) on an expensive (cheap) market earn rents on the intramarginal units.

It is easy to show that at the optimum all the real variables are constant. To do that first use (4)-(4') and (6)-(6') to solve for the real exchange rate (Ep^*/p) and the real wage:

$$(7)\quad Ep^*/p = [1+\alpha'(1-a)+\alpha]/(1-\alpha' a+\alpha) \geq 1$$

$$(7')\quad Ep^*/p = 1-\alpha' \leq 1$$

$$(8)\quad w/p = 1/(1+\alpha-\alpha' a) \geq 1$$

where the inequalities in (7)-(8) follow from the strict convexity of α and are strict for positive a and e; below I will make appropriate assumptions to ensure that a and e are bounded between zero and one and that $1-\alpha' > 0$;

It is seen from the above that the real exchange rate is rising (falling) in a (e). I has been established earlier, that if e is positive then the real wage is equal to one (see (4')). (8) says that it is rising in a. In particular, it is above one for any positive a.

Now, solving (4') and (8) for w, (7)-(7') for p and substituting the resulting expressions into (3) and (5)-(5') yields:

$$(9) \quad 1 = \mu_0 \exp[\delta t - \int_0^t i d\tau]\{Ep^* / [1+\alpha+\alpha'(1-a)]\}$$

$$(9') \quad 1 = \mu_0 \exp[\delta t - \int_0^t i d\tau][Ep^* / (1-\alpha')]$$

$$(10) \quad u'(c) = \mu_0 \exp[\delta t - \int_0^t i d\tau]\{Ep^* (1+\alpha-\alpha' a) / [1+\alpha+\alpha'(1-a)]\}$$

$$(10') \quad u'(c) = \mu_0 \exp[\delta t - \int_0^t i d\tau][Ep^* (1+\alpha-\alpha' e) / (1-\alpha')]$$

It is immediate from (9)-(10') that:

$$(11) \quad u'(c) = 1+\alpha-\alpha' a$$

$$(11') \quad u'(c) = 1+\alpha-\alpha' e$$

Thus consumption is a function of a (e) alone. The monotonicity of the marginal utility and the convexity of α together imply that c is rising in a <u>and</u> e. It follows that the domestic purchases on the foreign market (ce) are an increasing function of e. The negative dependence of e and the real exchange rate implies that ce increases as the real exchange rate falls. The remaining step is to show that at the optimum a (e) must be constant. Substituting into (3) the expression for w in terms of a and e (from (4') and (7)-(8)), differentiating with respect to time, and using the Fundamental Theorem of Calculus yields:

(12) $0 = \dot{a}\alpha''(1-a)/[1+\alpha+\alpha'(1-\alpha')]$

(12') $0 = \dot{e}\alpha''/(1-\alpha')$

Hence a and e - and subsequently - all the real variables are constant. Recall now that if a (e) is positive, so is e^* (a). In a two-country equilibrium it is true that:

(13) $c + c^*e^* = h(1-a)$

(13') $ce + c^* = h^*(1-a^*)$

(14) $ha + h^* = c^*(1-e^*)$

(14') $h^*a^* + h = c(1-e)$

(15) $n = h(1+\alpha)$

(15') $n = h + \alpha c$

(16) $n^* = h^* + \alpha c^*$

(16') $n^* = h^*(1+\alpha(a^*))$

(17) $b_0 = - b_0^*$

(13)-(14') say that the domestic and foreign goods market clear. That means the the sum of the domestic-(foreign)-currency-denominated purchases by the domestic (foreign) residents, $c(1-e)$ ($c^*(1-e^*)$), and the foreign (domestic) residents, c^*e^* (ce), equals the domestic-(foreign)-currency denominated sales by the domestic (foreign) and foreign (domestic) producers, $h(1-a) + h^*a^*$ ($h^*(1-a^*) + ha$). It is clear from the previous discussion that countries would never simultaneously sell and buy abroad. This explains the precise formulation of (13)-(14'). Labor is internationally immobile. The equilibrium conditions (15)-(16') say that the labor markets clear internally, i.e., domestic (foreign) demand for domestic (foreign) labor equals the domestic (foreign) labor supply. Finally, (17) formalizes the fact that in a two-country world one country's asset is another country's liability.

It is easy to demonstrate that a (e) and e^* (a^*) are positively related. As already discussed, in equilibrium neither country would simultaneously sell and buy on the other country's market. Suppose that the domestic country is a net exporter, i.e., $a > 0$. Then the domestic real exchange rate is determined by (7). By definition, the real exchange rate in the foreign country is equal to the inverse of the domestic rate (i.e., it equals p/Ep^*). Clearly, the foreign real exchange rate is determined by (7') with e^* replacing e. It follows from the convexity of the function α that e^* is a strictly increasing function of a. An analogous argument shows that a^* is a strictly increasing function of e.

The positive relationship between a (e) and e^* (a^*), (11)-(11') and (13)-(13') jointly imply that the total domestic (foreign) output, h (h^*), is an increasing function of a (a^*). Thus the home (foreign) country's sales on the other country's market, ha (h^*a^*), are an increasing function of a (a^*). Given that a (a^*) and the domestic (foreign) real exchange rate are positively (negatively) related, this is equivalent to saying that the domestic (foreign) net foreign sales increase (decrease) in the domestic real exchange rate.

It can be seen from (11)-(11') and (14)-(14') that there is no presumption on the relation between h and e. An increase in e raises the domestic consumption as it lowers its effective price (see (11')). At the same time it lowers the fraction of the consumption demand that falls on the domestically-produced goods. Also, higher e implies higher a^* and thus higher sales of the foreign country on the domestic market, h^*a^*. Therefore whether or not h is rising in e depends on the effective price elasticity of the consumption demand (or, equivalently, on the elasticity of marginal utility) and the relative-price elasticity of the foreign sales on the domestic market, h^*a^*.[3] When the consumption demand is elastic (i.e., the elasticity of the marginal utility of consumption - u''c/u' - is small in the absolute value) and/or the foreign country supply of goods on the domestic market is

[3] Since the labor supply is infinitely elastic the elasticity of h^*a^* depends solely on the elasticity of the trading cost function.

inelastic, then the total domestic output is increasing in e, and vice versa. One can see from (14) that the total domestic employment (i.e., in production and trading) is increasing in a. It is easy to show that, in general, it is nonmonotonic in e. However, if the total output is increasing in e so is the total employment.

One of the most debated questions in the international economics is concerned with the relationship between the trade balance and the relative price of foreign goods in terms of the domestic goods. A case when this relationship is negative is called the Harberger-Laursen-Metzler effect. The more recent work on the Harberger-Laursen-Metzler effect often stresses the roles played by the dependence of the discount factor on the future utility and the persistence of shocks (see e.g., Obstfeld (1982), Svensson and Razin (1983)). Backus (1991) shows that in his stochastic two-country model with complete asset markets the Harberger-Laursen-Metzler effect remains true if the elasticity of substitution (in the utility function) between foreign and domestic goods is low.

The previously derived results imply that the volume of the domestic country's net exports (imports), $ha + c^*e^*$, $(ce + h^*a^*)$ is unambiguously increasing (decreasing) in the real exchange rate. The the Harberger-Laursen-Metzler effect was originally stated as a negative relationship between the terms of trade and the nominal trade balance (see Harberger (1950)). In the current model the money demand/supply considerations are absent and, consequently, the nominal price levels and the nominal exchange rate cannot be separately determined. Therefore I can only study the behavior of the trade balance in the real value terms. Recall that as long as there is trade in equilibrium, the relative price of the home-market goods in terms of the foreign-market goods differs from 1. Thus the geographical composition of exports (i.e., sales (purchases) abroad vs. domestic sales (purchases) to (from) foreign agents) matters for the real value of the trade balance. The trade balance valued in the home-market goods (TBH) is given by:

$$(18H) \quad TBH = (Ep^*/p)ha + c^*e^*$$

$$(18H') \quad TBH = -\{(Ep^*/p)ce + h^*a^*\}$$

The trade balance expressed in the foreign-market goods is:

$$(18F) \quad TBF = ha + (c^*e^*)/(Ep^*/p)$$

$$(18F') \quad TBF = -\{ce + (h^*a^*)/(Ep^*/p)\}$$

It follows from the previous findings that when $a > 0$ ($e > 0$) then an increase in the real exchange rate (i.e., a deterioration of the terms of trade) unambiguously improves TBH (TBF) but has an ambiguous effect on TBF (TBH). The source of the ambiguity is easily found. While an increase in a (the real exchange rate) increases both ha and c^*e^*, the value of c^*e^* in terms of the foreign-market goods may go down or up depending on the relative price elasticity of the foreign demand for the home-market goods. It can be seen from (18F) that if c^*e^* and ha are very inelastic, then TBF and the real exchange rate may move in the opposite directions (i.e., the Harberger-Laursen-Metzler effect occurs). Similarly for $e > 0$. Clearly, the Marshall-Lerner-Robinson-Harberger-type of elasticities condition is sufficient to rule out the Harberger-Laursen-Metzler effect.

The remaining step in solving the model is to determine the equilibrium a and e. Exploiting the constancy of a, e and c and h, and the Fundamental Theorem of Calculus I can rewrite (2) as:

$$(19) \quad (b_0\delta + ha - ce)E_0p_0^* + [h(1-a) - c(1-e)]p_0 = 0$$

that is, the (possibly negative) net exports equal the (possibly negative) interest payment of the foreign debt or, equivalently, the equilibrium current account of the balance of payments is zero.

I will now show that (19) implicitly determines a (e) - and consequently all the real variables - as functions of the home country initial asset position. To do that, it is useful to distinguish three cases:

(i) when $b_0 = 0$, then $a = e = a^* = e^* = 0$;

(ii) when $b_0 < 0$, then $a > 0$ and $e^* > 0$;

(iii) when $b_0 > 0$, then $e > 0$ and $a^* > 0$;

In case (i) PPP does hold (i.e., $Ep^*/p = 1$) but there is no trade in equilibrium. The remaining real variables can be solved for by setting a (e) equal to zero in the relevant expressions.

Consider now the case (ii). Substituting from (13) an expression for h(1-a) - c one can rewrite (19) as:

$$(19ii)\quad -b_0\delta = ha + [c^* e^* /(Ep^* /p)] \equiv TBF$$

that is, the domestic country's foreign-currency-denominated debt service equals the trade surplus expressed in terms of the foreign-market goods. The analog of (19ii) in the case (iii) reads:

$$(19iii)\quad b_0\delta = ce + [h^* a^* /(Ep^* /p)] \equiv -TBF$$

It is now clear that all the endogenous real variables in the model are determined by the home country's initial asset position. In particular, when the home country is a (the present value) debtor (creditor) then its real exchange rate is above (below) unity. At this point I am making implicit assumptions on b_0 which ensure that a and e are bounded between zero and one and that $1-\alpha'(e)$ is positive.

Given the previous discussion about the dependence of TBF on Ep^*/p the following comparative statics results are not surprising:

$$(20ii)\quad da/db_0 \begin{matrix} < \\ = \\ > \end{matrix} 0$$

$$(20iii)\quad de/db_0 > 0$$

Recalling (7)-(7'), (20ii)-(20iii) say that a decrease of the foreign debts (assets) in a debtor (creditor) nation has an ambiguous effect on (increases) the real exchange rate. The negative dependence of the real exchange rate on the stock of (positive) foreign assets has been previously derived by Dornbusch and Fischer (1980). They study a descriptive portfolio balance model of a small open economy which stresses the imperfect substitutability between the domestic and foreign goods (see also e.g., Frenkel and Mussa (1985)). Dornbusch and Fischer (1980) argue that an increase in the foreign assets leads via the wealth effect to an excess demand for the domestic goods that has to be alleviated by an increase in their price in terms of the foreign goods. As I shall elaborate in a moment, the argument advanced here is different. Finally, the ambiguity in (20ii) is clearly reminiscent of a discussion about the "perverse valuation effects" of the negative asset position in the portfolio balance literature (see e.g., Boyer (1977), and Henderson and Rogoff (1982)).

The intuition behind the results derived so far is rather simple. Barring default, a debtor country has to service its foreign debt. In equilibrium the only way it could do it is by selling goods abroad and/or domestically to foreign agents. If prices abroad and at home were the same (i.e., $p = Ep^*$), then no profit-maximizing domestic seller would ever sell abroad and no utility-maximizing foreign buyer would ever buy on the home market as doing that involves transaction costs. Thus, if the foreign debts are to be serviced, then it must be the case that the real exchange rate is above one. The other important prediction of the model, i.e., a positive dependence of the total output on a - and thus on the domestic real exchange rate - is implied by the convexity of the selling costs and can be established in the following way. Consider the labor market. When a is positive, then there are two offsetting effects on the firm's demand for labor. On one hand there is (effectively) a subsidy to the marginal value product of labor. Namely, the term $aEp^* + (1-a)p = a(Ep^*-p) + p$ is larger than p. On the other, foreign sales (ah) are (effectively) taxed at the (average) rate $\alpha w/ha$. It follows from the strict convexity of the function α and the envelope condition (6) that the subsidy is always larger than the tax. Precisely, using (7) and (8) one can show that the effective price of output equals $p\{1 + [\alpha' a-\alpha/(1+\alpha-\alpha' a)]\}$, where the second term in a square bracket is the net subsidy, s. One can show that there is a following relationship between s and the real exchange rate:

$$(21)\quad s = (Ep^*/p)/\{-1 + [(1+\alpha')/(\alpha' a-\alpha)]\} \equiv (Ep^*/p)F$$

The factor of proportionality, F, is positive and increasing in a (and hence Ep^*/p). A positive net subsidy to the marginal value product of labor shifts up the labor demand thus increasing employment, the real wage and output (the latter follows from the fact that manufacturing of an export good is characterized by the Leontief technology that - for a given a - combines in fixed proportions labor employed in production and marketing). Given (21) and the fact that F is increasing in a, both the aggregate quantity supplied and the relative-price elasticity of the supply are increasing in the real exchange rate (i.e., the slope of the aggregate supply curve is positive and increasing).

When $Ep^*/p > 1$ ($a > 0$), then the domestic consumers shop exclusively at home. Therefore there is a wedge between prices faced by them and by the producers. It is clear from (3), (5) and (8) that an increase in the real exchange rate increases the domestic consumption due to a a substitution effect arising from the higher real wage.

The reasoning in the case (iii) is as follows. A creditor country collects the interest income on its foreign currency denominated assets only if the domestic consumers choose to shop abroad and/or the foreign agents choose to sell their goods on the domestic market. Since the transactions across the border are costly, they would never occur unless the real exchange rate is below one (i.e., money-wise the foreign goods are cheaper). As already said, c and e (Ep^*/p) are positively (negatively) related. To derive the downward sloping aggregate demand curve notice that the convexity of the function α implies that when e is positive, then consumption is *effectively subsidized (i.e., the* effective real price of consumption is lower that the marginal utility of real wealth). The subsidy is earned on the intramarginal units acquired abroad. Precisely, it is easy to show using (4') and (7') that the term $eEp^* + (1-e)p + \alpha w$ is equal to $p(1+\alpha-\alpha' e)$, where $\alpha-\alpha' e$ is negative and thus $-(\alpha-\alpha' e)$ is the rate of the subsidy, r. Further, as $\alpha'' > 0$, r is increasing in e or, by (7') decreasing in Ep^*/p. For the same level of the real wealth, a subsidy increases the demand for consumption. This explains the negative relationship between the latter and Ep^*/p. As discussed above, depending on the relative price elasticity of demand for the domestic consumption and the relative price elasticity of the foreign exports, the demand for the domestically-produced goods - and thus output - either decreases (increases) or increases (decreases) in Ep^*/p (e). Lastly, since for positive e, the domestic producers are selling exclusively on the domestic market, there is a wedge between the consumers' and producers' prices.

It is easy to show what would happen if the trading costs were linear (i.e., $\alpha = ka$; for the technical reasons which will be apparent in a moment, k must be less than one). First, the real exchange rate will then be equal to 1-k, 1, and 1+k when b_0 is, respectively, positive, zero, and negative. Second, there would be no export (import) subsidy, and therefore the aggregate supply (demand) would be independent of the real exchange rate. It follows from (19ii)-(19iii) that changes in the debt service (interest income) would be reflected solely in changes of a (e) and the domestic (foreign) consumptions.

Notice also that in a current model - apart from the differentiability requirement - the equilibrium does not depend in any way on the path of the nominal exchange rate. That is, whether the nominal exchange rate is fixed forever or changes does not matter. This replicates Helpman's (1981) original finding about the irrelevance of the exchange rate regimes for the real side of the economy. As in Helpman (1981), it obtains due to three assumptions: the infinite horizon, perfect capital mobility, and the neutrality of money.[4]

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4 Helpman (1981) studied a cash-in-advance economy with endowments. In the current model money is neutral because it serves only as a unit of account.

3. A Model with Traded and Nontraded Goods

How would the results change if I introduced a standard distinction between the traded and nontraded goods and the usual concave transformation schedule? First of all, notice that, by definition, spending (servicing) the foreign-currency denominated interest income (debt service) cannot fall on (be financed from the sales of) the nontraded goods. A rather striking new result is a negative relationship between the relative price of traded goods in terms of nontraded goods and the real exchange rate. The argument is straightforward. Denote the domestic price of home goods by q. I take the case with $a > 0$ first. In that case, the relative price of traded goods in terms of the nontraded goods faced by the domestic consumers equals p/q, while the one faced by the domestic producers equals $p(1+s)/q$, where s is the export subsidy discussed above. An increase in the real exchange rate, Ep^*/p, increases s. A subsidy to the production of the traded goods is equivalent to a tax on the production of the home goods. Therefore as a and s increase, the quantity of the home goods supplied at any relative price of the home goods in terms of the traded goods decreases. This and a positive income effect caused by an output boom in the traded goods sector imply that p/q falls.

Recalling the results derived in section 2, it is clear that an increase (decrease) in e (Ep^*/p) increases the wedge between the relative prices of traded versus nontraded goods faced by the consumers and the producers. Thus the quantity of the nontraded goods demanded at any level of q/p goes down. It straightforwardly follows that when e is positive, then p/q and Ep^*/p move in the opposite directions.

4. Implications for the Stochastic Properties of the Data

As it stands, the model cannot explain the observed movements over time of output, consumption, real exchange rate, trade balance and the stock of foreign assets. This deficiency can be remedied by explicitly allowing for an uncertainty. Randomness can be introduced by, for example, assuming that each period the production and/or trading technologies are subject to random shocks.[5] It is clear that shocks to arbitrage costs may play an important role in generating the observed variability of the real exchange rates (see e.g., Cumby and Huizinga (1990), Engel (1991)). These cost shocks may be identified with changes not only in the marketing and communication technologies but also in the government's regulations affecting the international exchange of goods (e.g. the regulations on safety/product liability, pollution standards, health hazards labelling, etc.). Barring Arrow-Debreu securities, it is clear that in the absence of capital, the international propagation of disturbances would be due solely to agents' desire for consumption smoothing and the derivative impact of changes in the real exchange rate on the aggregate supply and demand.

When augmented by an explicit recognition of uncertainty the model would, in particular, deliver the following seven predictions on the statistical properties of the data. First, the real exchange rate should follow a stationary stochastic process. This is because the long

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[5] The resulting setup would share many common features with the currently undertaken research program on the international business cycles (see e.g., Backus, Kehoe, and Kydland (1991a) and Stockman and Tesar (1991)).

run real exchange rate is determined by the initial asset position which is (trivially) stationary. If the nominal exchange rate, the domestic and foreign prices are integrated processes, then they should be cointegrated. Second, the real exchange rate should be positively correlated with the trade surplus (expressed in terms of the home-market goods) but it may or may not be positively correlated with the trade deficit. Third, there is no general presumption about the correlation of the current account balance and the real exchange rate as there is no presumption about the contemporaneous correlation of the trade balance and the interest payments (receipts). Fourth, consumption should be positively (negatively) correlated with the real exchange rate in periods when the (normalized) real rate is above (below) unity. That implies that one may expect a weak overall correlation of consumption and the real exchange rate. Fifth, output (and employment) should be positively correlated with the real exchange rate (and thus the trade balance) in periods when the (normalized) real rate is above unity (i.e., when there is a trade surplus) but this correlation in the other periods is ambiguous. Given three above, .this implies that a weak correlation of output and the current account. Sixth, consumptions should be positively correlated across countries. Seventh, outputs may or may not be positively correlated internationally.

How well do the predictions one through seven match the available empirical evidence? Several studies using a long span of data have found that the real exchange rates are stationary (see e.g., Frankel (1986), Hakkio and Jones (1990) and Diebold, Husted, and Rush (1991), and Edison and Melick (1992)). Stockman and Tesar (1990) report that their earlier investigation failed to detect a consistent pattern of correlations between the trade balance and the terms of trade. Backus, Kehoe and Kydland (1991b) find this correlation to be negative in 9 out of 11 developed countries. However, except for the U.S. (which displays a positive correlation), the correlations are relatively small. Strictly, these empirical findings are not inconsistent with the second prediction. Backus, Kehoe and Kydland (1991b) explain the negative comovements of the terms of trade and net exports by stressing the productivity-shocks-induced fluctuations in investment. This channel is closed in the current model. Instead, as argued above (see (18H')), such negative correlation may be caused by a combination of low elasticities of labor supply and consumption demand.

In accordance with prediction three, direct and indirect statistical tests have often failed to establish any regularity between the current account and the real exchange rate (see e.g., Campbell and Clarida (1987)).

I am not aware of any study that reports the empirical correlations of consumption and the real exchange across countries (prediction four). Backus and Kehoe (1991), and Stockman and Tesar (1991) present the evidence that the net exports are most often countercyclical. This finding supports the Keynesian view that income plays the major role in the determination of imports, and seems to contradict the models in which trade reflects consumption smoothing in response to the productivity shocks. Strictly, it does not completely contradict my prediction five. Recall that in the model, if the consumption demand and the foreign supply are, respectively, inelastic and elastic, then a consumption/import boom would coincide with a fall in the domestic output. Prediction six agrees with a strong international evidence on the positive correlations of consumption (see Backus and Kehoe (1991), and Stockman and Tesar (1991)). As explained above, in the current model, these correlations are expected to be positive for a reason which is different than the one stressed by the international real business cycle models (risk-sharing through complete markets).

Backus and Kehoe (1991) and Stockman and Tesar (1991) have found a very strong positive correlation of outputs across countries. Prediction seven is thus validated by the evidence if one is willing to assume that the consumption demands (supply) are elastic (inelastic). This implies that the evidence cannot be taken as supporting both predictions five and seven.

5. Conclusions

I have studied a two-country general equilibrium model in which selling and buying on the foreign market involve paying convex costs in terms of domestic labor. The model predicts that the (long run) real exchange rate depends, in an easily-testable way, on the initial stock of net foreign assets. I have discussed the implications of the convexity of the trading costs for the relationships between the real exchange rate and output, employment, consumption and the trade balance. The predictions on the stochastic properties of the data derived from the model are reasonably successful in matching the available evidence.

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