INDIVIDUAL AND COLLECTIVE WAGE BARGAINING

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ABSTRACT

Wage negotiation is modeled as an "oceanic" game. The employer and the union or unions (if any) are represented as atoms while the unorganized workers form a non-atomic continuum. The workers are heterogeneous in their outside opportunities but, for simplicity, are assumed to appear homogeneously in the employer's production function. The surplus that each set of participants is capable of generating is a measure of that set's potential, and a cooperative game in characteristic-function form is thereby defined. Its Shapley-value solution—the marginal surplus attributable to each player averaged over all possible alignments—distributes the total surplus among the players so as to yield a plausible long-run wage settlement. Several different levels of unionization are examined and contrasted.

It is noteworthy that this solution is not tied to any particular set of bargaining rules or procedures but rests on the coalitional powers inherent in the economic model itself.

1. Introduction

The standard models of wage determination are generally concerned with the technological or organizational aspects of the labor market and tend to obscure the fact that, in the absence of monopoly on either side or governmental controls, both labor and management are in position to bring significant bargaining power to bear. Recent attempts to use game theory to study the bargaining aspects of wage determination have mostly considered only simple bilateral bargaining -- a game pitting one employer against one worker (or a sequence of such games) or one employer against one union. Since real labor markets are seldom that simple, a methodology for dealing with a richer class of institutional structures would appear to be a worthwhile adjunct to such investigations. Ideally, both actual and potential employees should be allowed into the game, perhaps represented at the bargaining table by a union or union(s), or perhaps representing themselves individually but gaining bargaining power from the possibility of *ad hoc* coalitional action. On the other side of the table there might be one or several employers, in various postures of association or competition. Our focus in the present paper, however, will be on the labor side.

In particular, our models allow for a certain amount of heterogeneity among the workers. Though equally productive on the job in question, they may have different capabilities and opportunities in other occupations. We represent this differentiation by a parameter called the "alternative wage"; it is sufficient for our purposes since it gives individual workers a variety of different incentives and fall-back positions in the bargaining game and similarly makes a union's power depend not only on its size but also on the composition of its membership.

An interesting problem of scaling arises when one tries to accommodate in the same model both "big" players (e.g. employers or unions) and "little" players (e.g. employees and potential employees). While the latter ought to have a significant effect on the outcome due to their large numbers, as individual actors they must be regarded as infinitesimal. The modeller must steer between two extremes: 1) insisting on equal negotiating rights for all players, to the extent that the influence of the major players is diluted to the vanishing point, and 2) allowing the minor players to be reduced to passive price-taking dummies, with no status at all as negotiators. This problem seems first to have been addressed by Milnor, Shapiro and Shapley in three Rand Corporation reports (1960-61), in which they borrowed from measure theory the idea of a mass distribution consisting of a finite set of "atoms" of positive mass together with a density function defined over an infinite continuum of other points, the latter called an "ocean" to suggest the lack of any *a priori* order or cohesion among its members. The original application was to weighted-majority voting games, as in a large publicly owned corporation[1].

An important feature of "oceanic game" theory is its focus on *capabilities* rather than *strategies*. It begins by constructing a "characteristic function" that describes what each subset of players can accomplish by joint action. But the characteristic function does not incorporate any assumption about the procedures for coalition forming or for the subsequent bargaining. Such a cooperative-game model (which is especially suitable when there are many players) contrasts sharply with the strategic-form or extensive-form models that are used in Nash equilibrium analyses, where the interactive moves (offers, responses, compromises, etc.) must all be spelled out explicitly. Such models are inevitably sensitive to the details of protocol, and this restricts their application. Of course, a price is also paid for omitting detail. Broader assumptions yield less specific answers. So our models do not deal with specific wage disputes and their resolution. But hopefully they can give some indication of the long- or middle-run resolution of the forces at work in the labor market, based on the underlying economic and institutional data and the inherent bargaining power wielded by the participants.[2]

Let us sketch some of the results that our models do provide. 1) As different institutional structures are compared we find that the members of a partial union will under certain conditions, but not always, do better by bargaining collectively than as individuals. But when *all* workers are organized and bargain as a unit, their total payoff is quite generally higher than what they would get bargaining as

[1] Shapiro and Shapley (1978) and Milnor and Shapley (1978) reproduce almost verbatim the still-available Rand reports. For related work see Hart (1973), Guesnerie (1977) and Fogelman and Quinzii (1980). Models with continuum of non-atomic agents are now commonplace in mathematical economics, following the lead of Aumann (1964, 1966); see also Aumann and Shapley (1974) for value theory and Hildenbrand (1974) for core theory. The term "oceanic game" however has come to denote the case that mixes atomic players with the non-atomic continuum.

[2] Our approach may be compared in its level of abstraction with the classical equilibrium model of exchange and production, which takes for its data only preferences, endowments and production possibilities, not the particulars of buying and selling.

individuals, though whether each worker is better off depends on how freely the union is able to distribute its total gain. 2) The ability to replace incumbent workers with potential workers definitely enhances the employer's bargaining position; how effective this will be will naturally depend on the number of potential employees and their alternative wages. 3) Finally, the individually negotiated wages of the workers correlate positively to their alternative wages and negatively to the size of the total labor force. While there are no big surprises in the above, it is nevertheless worth noting that our models do capture these effects and to a certain extent quantify them.

The paper is organized as follows: Section 2 describes the game-theoretic tools we employ. Section 3 presents the no-union case in which the only atom is the employer; Section 4 then introduces partially-organized and fully-organized labor force; Concluding remarks are given in Section 5.

2. Cooperative Games in Coalitional Form

The coalitional form of a game (N,v) is based on a *player set* N and a *characteristic function* v defined on the subsets of N, where $v(S)$ is meant to indicate in some way what the players in S can accomplish if they agree to act as a coalition. There are several different ways to formulate this, but the essential ingredient is a description of the *assurable S-payoffs*, i.e., the utility levels that S can obtain for its members against "worst-case" behavior by the members of $N\backslash S$. The cooperative solutions of the game are generally assumed to be Pareto optimal in the set of assurable N-payoffs, which is simply the set of all payoff vectors that are feasible under full cooperation -- the presumption being that however bitter the dispute over the distribution of surplus, the "grand coalition" will ultimately take hold and achieve Pareto optimality.

When there is a common unit of utility (e.g., $$$) as well as a way to transfer it among the players more or less freely, then the various assurable sets may be so "flat-topped" in the utility space that $v(S)$ can be defined simply as the maximum total utility that S can assured in the worst case.[3] This

[3] A detailed account of the players' strategies is often unnecessary in the cooperative theory when the game enjoys the so-called *fixed threat* property, which holds when the worst case occurs for each S when $N\backslash S$ just leaves it alone. This type of "no-externalities" condition is familiar from the classical Walras/Edgeworth model, but is also satisfied in bargaining situations whenever the "disagreement payoff" is unique.

is called a "TU" characteristic function (for "transferable utility"), and when its use can be justified it represents a great simplification of the models, both analytical and conceptual, and even brings games with a continuum of players within reach of elementary mathematical tools.

In our current application the monetary utilities, though interpersonally commensurable, are only imperfectly transferable, and so the "TU" assumption must be used with care. In particular, we may find that the solution of the game calls for payments to be made to workers in the labor pool who influence the wage settlement by their presence but who do not in the end get hired at the production level that yields the efficient outcome $v(N)$. These side payments are typically quite small, but it is not clear how or whether they might be implemented in practice. One possible interpretation might be to regard them not as actual payments but as a measure of unrealized bargaining power, expressed in terms of what the available but unhired workers *could* claim at the bargaining table *if* utility were freely transferable[4]. For the present, however, our position will be that the TU assumption in this paper is only a simplifying approximation, similar in degree and effect to various other idealizations that are commonplace in abstract microeconomic theory.[5]

Given the TU characteristic function, our solution concept will be the Shapley value as it applies to oceanic games. This solution has a well-known axiomatic basis in the finite case as well as the purely non-atomic case (see Shapley 1953; Aumann and Shapley 1974). But for oceanic games, the axiomatic approach is not so effective (see Hart 1973), and a better formal definition is obtained through approximations by finite games -- the so-called asymptotic approach (see Shapiro and Shapley 1961, 1978; Fogelman and Quinzii 1980). For actual calculations, however, a third approach -- called "random order" -- is by far the most effective; it derives from the observation (Shapley 1953) that in

[4] Imperfect transferability might also be treated by using the more general "NTU" value theory applied to a model in which certain forms of indirect or imperfect transfer are provided as strategic options for employer. Thus, the basic model could be elaborated by allowing the firm to allocate some of its surplus to programs that indirectly benefit the population of potential employees, e.g., scholarship awards, contribution to community amenities, travel expenses for job interviews, etc. But such elaborations would not be well-matched to the abstract, non-strategic character of the basic model.

[5] In this connection, we might point out that there is also an NTU value theory which, although more complex in its definition and application, is nevertheless a *continuous* extension of the TU value. This makes it permissible to regard the TU solution as an approximation to the NTU solution provided the deviations from perfect transferability are small.

finite games the value of a player is just his *expected marginal contribution* when the players enter the grand coalition in a random order.[6]

Unfortunately it is theoretically impossible to randomly order a measurable continuum of players in such a way that measurability is preserved.[7] For oceanic games, however, a procedure is available that has much the same effect: Imagine the continuum of minor players to be ordered initially in some definite way -- say spread out uniformly in the unit interval [0,1] representing a period of time during which the ocean is "flowing" at a constant rate into the grand coalition. Then insert the major players $A_1, A_2, \cdots, A_p$ at times $t_1, t_2, \cdots, t_p$ respectively, where the latter are random variables chosen independently according to the uniform distribution on [0,1]. By this device we suitably randomize the entry of the major players.

So far so good, but how do we randomize the entry of the minor players? If the ocean happens to be homogeneous there is no problem, since the value solution does not discriminate among identical players, anyway. But even an inhomogeneous ocean can be handled if each player can be fully described by a *profile* consisting of a finite set of parameters (e.g. reservation wage, productivity, seniority, ...). In that case, if the ocean is sufficiently well mixed[8] the minor players in any subinterval of [0,1] of positive length will, with probability approaching 1, be as close as we please to a "faithful sample" of the whole population.

Accordingly, only a certain class of orders will have to be considered. Without loss of generality we may assume that the random numbers t_j are all different from each other and from 0 and 1. Let $t_{j_1}, \cdots t_{j_p}$ be these numbers arranged in increasing order. Then the only orderings we shall need to consider have the following form: First a minor-player block of size t_{j_1} with a mix of profiles in exact proportion to the total mix of profiles -- i.e., a faithful sample of the ocean as a whole. Denote this block by "$samp(t_{j_1})$". Then add the first atomic player A_{j_1} and pay him an amount equal to his contri-

[6] The interplay between these three mutually supportive approaches to value theor is a recurring theme throughout Aumann and Shapley (1974).

[7] Op. cit., Chapter 2.

[8] Imagine [0,1] chopped up into a large finite number of intervals, which are then shuffled like a deck of cards and returned to [0,1] in the new order.

bution to the worth of the growing coalition, namely,

$$v\left[samp(t_{j_1})\cup A_{j_1}\right] - v\left[samp(t_{j_1})\right].$$

Next add another faithful sample of the ocean, of size $t_{j_2}-t_{j_1}$, followed by the second atom A_{j_2}, and pay him

$$v\left[samp(t_{j_1})\cup A_{j_1}\cup samp(t_{j_2}-t_{j_1})\cup A_{j_2}\right] - v\left[samp(t_{j_1})\cup A_{j_1}\cup samp(t_{j_2}-t_{j_1})\right]$$

and so on, until all the atoms are accounted for.

The value for any given atomic player is then defined to be his expected payment under this scheme. To calculate it, integrate that player's contributions over the p-dimensional unit cube of the t_j's, taking note of the fact that for any k, the number l such that $j_l=k$ is a random variable that depends on the relative positions of the t_j's in [0,1].

Finally, to obtain the *oceanic value density* for any given profile, introduce a small set of new players of size $\delta>0$, endow each of them with that profile, then merge them together into a single, $(p+1)$st atom for which we can determine the value as above. To get the desired density, merely divide this value by δ and let δ go to zero.

This whole procedure will be amply illustrated in the sequel.

3. First Model - Unorganized Labor

Consider a market consisting of an employer E and an ocean $[0, n]$ of workers. The assumption will be that the integer n is so large that treating $[0, n]$ as a real interval in our mathematical models is a reasonable approximation. Let f denote the labor demand function and g the labor supply function, both functions defined on $[0, n]$ or perhaps some larger interval in $\mathbb{R}_+$. We assume f to be continuous and strictly decreasing up to some point x_0 where $f(x_0)=0$, and identically 0 thereafter, and we assume g to be nonnegative and nondecreasing, but not necessarily continuous. Without loss of generality we may assume that $g(n)\le f(0)$, since workers x with $g(x)>f(0)$ always turn out to be dummies in the game.

The characteristic function v defines the worth $v(S)$ of any coalition S to be the total surplus it is capable of achieving by its own efforts. In particular, $v(S)=0$ for any S not containing E, and if S is the grand coalition $E \cup [0, n]$ then $v(S)$ is the shaded area in Figure 3.1.

FIGURE 3.1 ABOUT HERE

Consider now a faithful sample of $[0, n]$ of size tn, $0 \le t \le 1$. Since we only deal with such a sample as a block of players, the internal order is irrelevant and their membership can be described by a function g_t defined on $[0, tn]$ by $g_t(x)=g(x/t)$ as shown in Figure 3.2. Figure 3.2 also shows the curve $w(tn)$, the equilibrium wage associated with the intersection of f and g_t; we denote the number of workers employed at that point by $k(tn)$. Note that $w(x) > f(x)$ and $k(x) < x$ over the whole domain $[0, n]$ except for x=0.

FIGURE 3.2 ABOUT HERE

We can now express the worths of the types of coalitions needed in our calculations. Specifically, the surplus available to a coalition consisting of E and a faithful sample of size tn is given by

$$S(t) = \int_{x=0}^{k(tn)} [f(x)-g_t(x)]dx = \int_{x=0}^{k(tn)} [f(x)-g(x/t)]dx \qquad (3.1)$$

This is indicated by the shaded area in Figure 3.2. Of course, as already mentioned, any coalition that does not include E has worth 0.

Thus, to obtain the Shapley value Φ_E of the employer we merely select a random number $t_E \in [0,1]$ representing the "time" at which he enters the coalition and award him the expected value of $S(t_E)$ -- this being his incremental contribution. The expected value is then

$$\Phi_E = \int_0^1 S(t)dt = \int_0^1 \int_0^{k(tn)} [f(x)-g(x/t)]dxdt. \qquad (3.2)$$

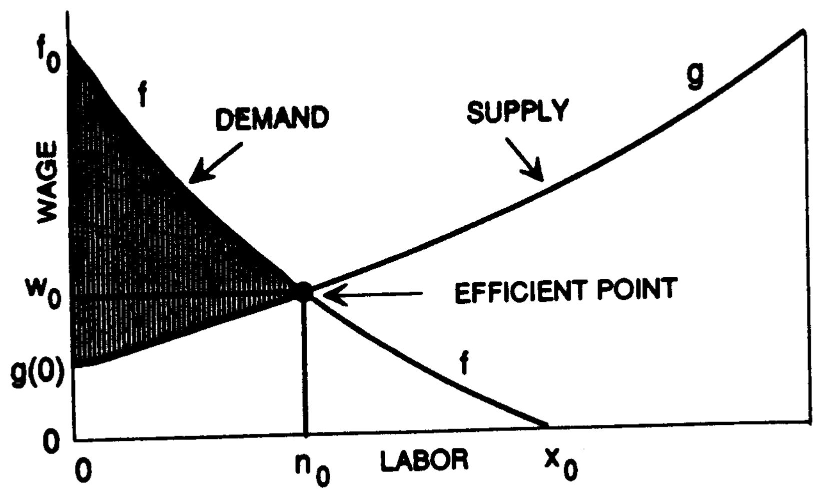

Figure 3.1. Illustrating the surplus attainable by the grand coalition $E \cup [0, n]$.

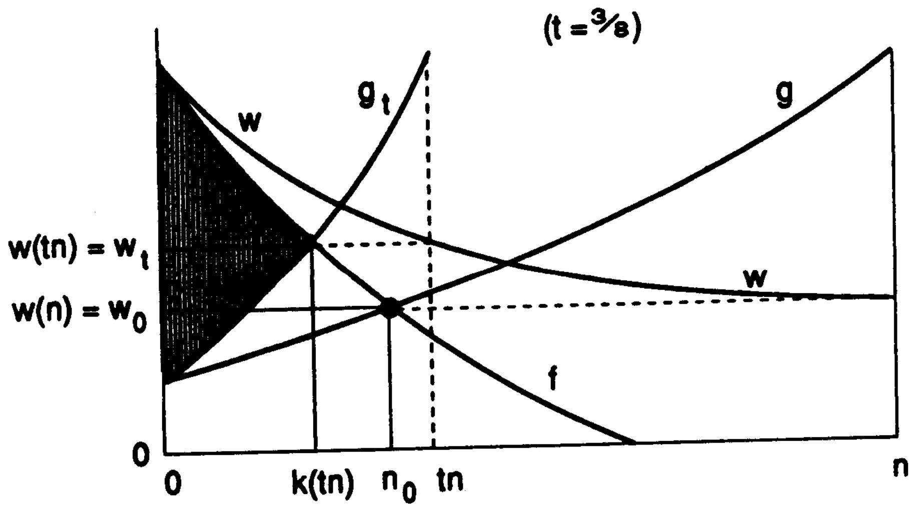

Figure 3.2. Surplus attainable by E with a faithful sample of size tn.

This double integral may be visualized in three dimensions as the volume of a solid whose base is the region in the plane bounded by f and g and the axis $x=0$ (Figure 3.3), and whose height above the plane varies from 1 (on the left boundary) to 0 (on the curve g), the level sets being given by the curves g_t. In this representation, we can think of the height of the solid above a given point is the probability that E will obtain the bit of revenue represented by that point.

Figure 3.3 ABOUT HERE

The Shapley values of the oceanic players will be represented by a cumulative distribution function on $[0, n]$. Thus, if $\Phi(x)$ is the total payoff to the oceanic interval $[0, x]$, then the derivative $\phi(x) = \frac{d\Phi(x)}{dx}$ is the "payoff density" to a worker whose alternative wage is $g(x)$.

Let t be fixed and let $\hat{y}$ be a fixed alternative wage. We must determine the increment to the surplus when a small atom $\Delta_{\hat{y}}$ consisting of additional workers having alternative wage $\hat{y}$ is added to the set of workers who are on hand at the time t, i.e., the workers represented by the domain $[0, tn]$ of the function g_t. Let $\delta>0$ denote the size of $\Delta_{\hat{y}}$. There are two possible configurations. The first is illustrated in Figure 3.4; it assumes $g(0) \leq \hat{y} < w(tn)$. The increment due to $\Delta_{\hat{y}}$, assuming E is present, is the shaded area on the left. This is equal by horizontal translation to the simpler shaded area on the right which is approximately $[w(tn) - \hat{y}]\delta + o(\delta)$. The second configuration (not illustrated) assumes $w(tn) \leq \hat{y} \leq g(n)$; in this case the increment due to $\Delta_{\hat{y}}$ is nil.

Figure 3.4 ABOUT HERE

In order to obtain the value-density ϕ, let $\hat{x}$ be the worker in $[0, n]$ for whom we wish to evaluate ϕ, so $\hat{y}=g(\hat{x})$, and let t_E denote the time of E's arrival. Then, letting $\delta \to 0$, we have[9]

[9] The variable t is the time at which $\Delta_{g(\hat{x})}$ enters the grand coalition. The lower limit t_E in the second integral ensures that E is present when this occurs.

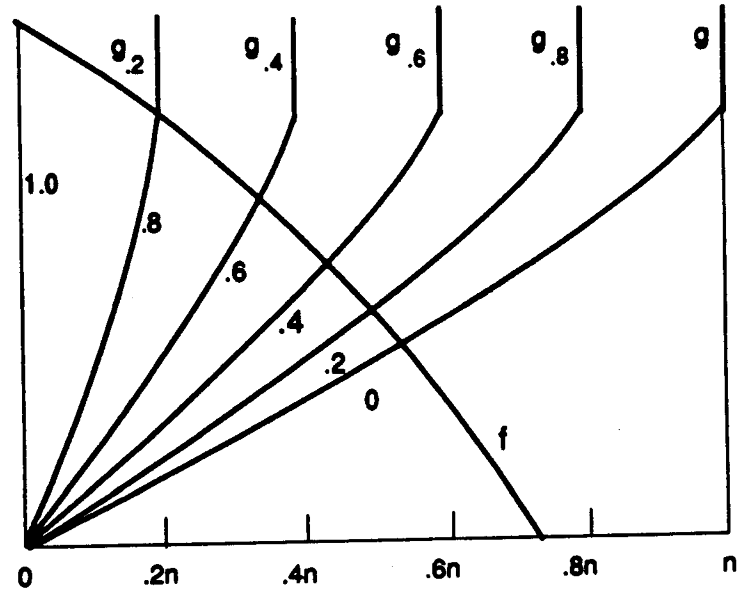

Figure 3.3. Illustrating the level sets in (3.2).

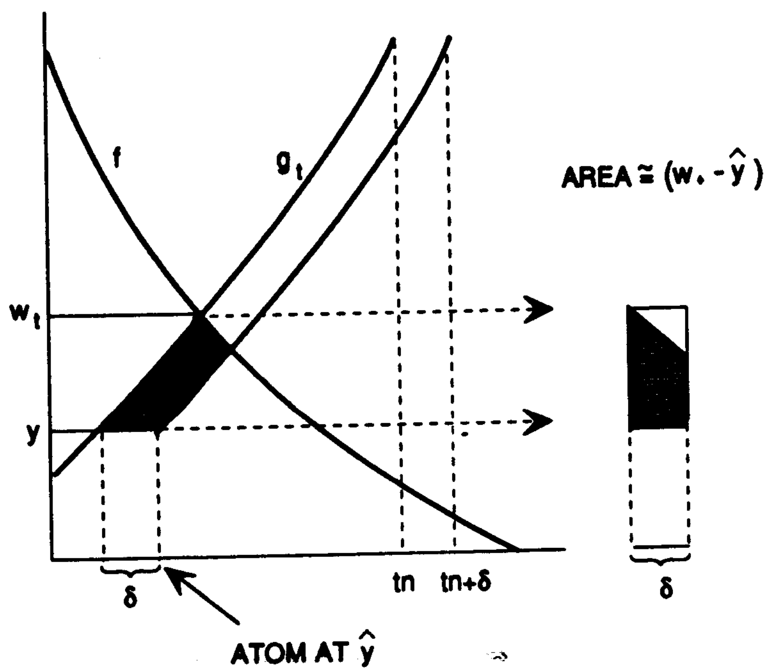

Figure 3.4. Illustrating the surplus attainable by the grand coalition $E \cup [0, n]$.

$$\phi(\ell) = \int_{t_g=0}^{1} \int_{t=t_g}^{\bar{t}} \frac{1}{n}[w(tn) - g(\ell)]dt\, dt_g \tag{3.3}$$

where the upper limit $\bar{t}$ breaks out into three cases as shown in Figure 3.5. Case 1: If $g(\ell) \leq w(n)$, then $\bar{t} = 1$. Case 2: If $w(n) \leq g(\ell) \leq w(t_g n)$, then $\bar{t} = w^{-1}(g(\ell))/n$.[10] Case 3: If $w(t_g n) \leq g(\ell)$, then $\bar{t} = t_g$ -- in other words, the integral collapses. Note that $\bar{t}$ is a function of both ℓ and t_g. From (3.3) we obtain

$$\phi(\ell) = \int_0^1 \int_{t_g}^{\bar{t}(\ell)} w(tn)dt\, dt_g - g(\ell)\int_0^1(\bar{t}-t_g)dt_g\,. \tag{3.4}$$

Figure 3.5 ABOUT HERE

In case 1 this simplifies considerably, since $\bar{t}=1$:

$$\begin{aligned}\phi_h(\ell) &= \int_0^1 \int_{t_g}^{1} w(tn)dtdt_g - g(\ell)\int_0^1(1-t_g)dt_g \\ &= \frac{1}{n}\int_0^1[W(n)-W(t_g n)]dt_g - \frac{1}{2}g(\ell),\end{aligned} \tag{3.5}$$

where $W(x) = \int_0^x w(x)dx$ and the subscript "h" signifies "hired". Note that the first term is independent of ℓ, and that the second term shows that the value-density of the workers who are hired decreases as their alternative wage increases, as one would expect.

Cases 2 and 3 are best handled together, since the case distinction between II and III depends on t_g. Again there is a considerable simplification; we omit the details and state the result:

$$\phi_{nh}(\ell) = \frac{1}{n}\int_0^{\bar{t}}[W(\bar{t}n)-W(t_g n)]dt_g - \frac{1}{2}g(\ell)\bar{t}^2, \tag{3.6}$$

where the subscript "nh" signifies "not hired". At the boundary with case 1, i.e., when $g(\ell)=w(n)$ and $\bar{t}=1$, we see that (3.5) and (3.6) agree; indeed, if g is continuous, then the value-density function ϕ will be continuous throughout its domain.

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10 Note that w is continuous and strictly monotonic, so w^{-1} is well defined.

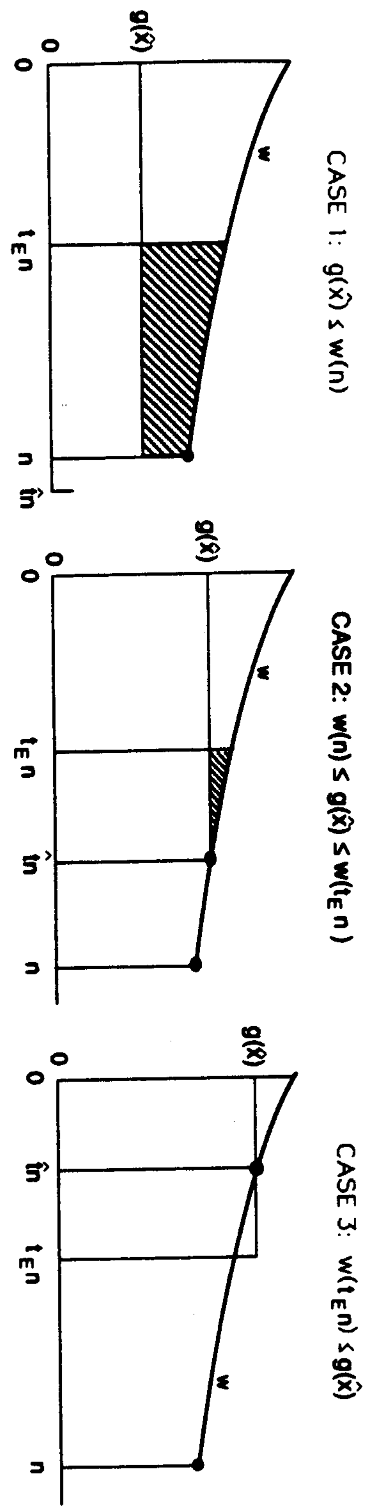

Figure 3.5. Case distinctions in (3.3).

3.1. First Model - Examples

In the first example the workers are fully heterogeneous in opportunities, with g as well as f linear. In the second example there are only two types of workers: the ones hired at the efficient production level having one alternative wage and the ones not hired have higher alternative wage. We omit the details of the algebraic calculations and state the results.

EXAMPLE 1: Let $f(x) = 1-x \quad (0 \le x \le 1)$, $g(x) = x/2 \; (0 \le x \le n)$, and $n = 2$. The value payoff for the employer is:

$$\Phi_g = \int_0^1 S(t)dt = \int_0^1 \frac{t}{1+2t}dt = \frac{1}{2} - \frac{1}{4}\ln 3 = 0.22535.$$

Now we compute the value density to the workers. In case 1 we have $g(\hat{x}) \le w(n)$, that is, $\hat{x} \le 2/3$. Using equation (3.5):

$$\phi_n(\hat{x}) = \frac{1}{2} - \frac{1}{4}\ln 3 - \frac{\hat{x}}{4}.$$

In the remaining case (combining the previous cases 2 and 3) we have $2/3 \le \hat{x} \le 2$. So, from equation (3.6)

$$\phi_{nh}(\hat{x}) = \frac{1}{2\hat{x}} - \frac{\hat{x}}{8} - \frac{1}{2}\ln(2/\hat{x}).$$

Figure 3.6 ABOUT HERE

In Figure 3.6, we have plotted both the supply curve $g(x)$ and the total wage $\phi(x)+g(x)$. The difference between the two is the oceanic value $\phi(x)$, which is a decreasing but positive function of x. In addition, the competitive wage is plotted, which in this case is 0.333 for $x \le 2/3$. Also plotted is the monopolistic solution. In this case, the union wage is 0.6 for $0 < x < 0.4$. Further discussion of these results is given at the end of this section.

EXAMPLE 2: In this example there are just two alternative wages. Let $f(x) = 1 - x \; (0 \le x \le 1)$, let

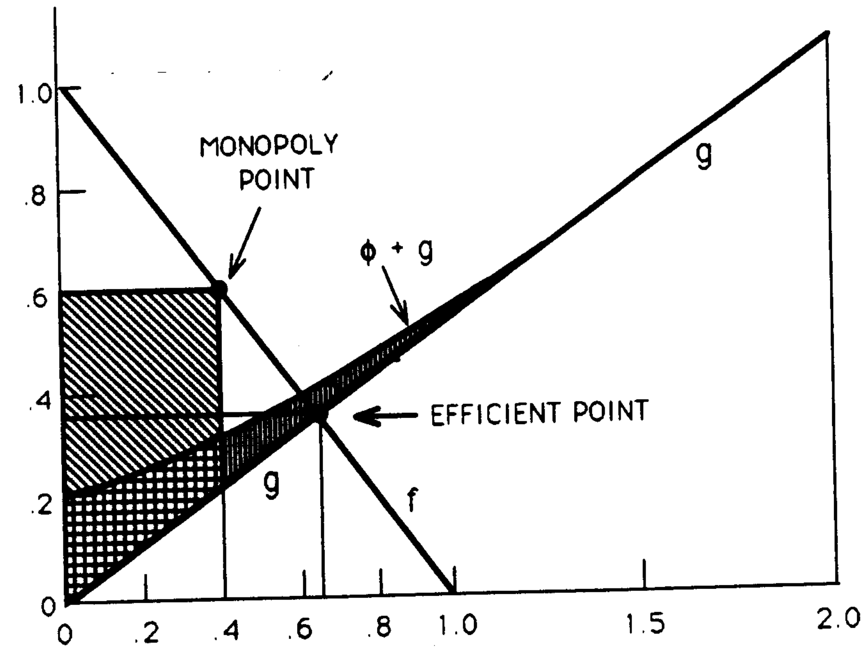

Figure 3.6. Solution of Example 1.

$c < 0.6$, let $n \geq 0.5$, and let

$$g(x) = \begin{cases} c & \text{if} \quad 0 < x < 1/2 \\ 0.6 & \text{if} \quad 1/2 < x < n \\ \infty & \text{if} \quad n < x \end{cases}$$

Figure 3.7 indicates the surplus for faithful samples of various sizes t. We see that if $t = 1$, the total surplus $S(1)$ is $3/8 - c/2$. Omitting the details we state the results:

$$\Phi_E = 0.23 - 0.25c - \frac{0.01}{n}$$
$$\phi_h(\ell) = 0.29 - \frac{c}{2} + \frac{0.01}{n^2}$$
$$\phi_{nh}(\ell) = \frac{0.01}{n^2} \tag{3.7}$$

As a check, we note that the sum of all the players' payoffs is

$$(0.23 - 0.25c - \frac{0.01}{n}) + \frac{1}{2}(0.29 - \frac{c}{2} + \frac{0.01}{n^2}) + (n - \frac{1}{2})(\frac{0.01}{n^2}) \ = \ 0.375 - \frac{1}{2}c,$$

which is equal to the total surplus $S(1)$.

Figure 3.7 ABOUT HERE

3.2. First Model: Discussion

These examples reveal that the individual worker has bargaining power, which despite being infinitesimal is not negligible. Though workers are unorganized, the results still indicate variations in wages which do not correspond to variations in the workers' productivity on their current job.

Specifically, Example 1 demonstrates the positive relationships between the value of the workers' outside opportunities and their wage (although the Shapley values of the workers decrease as their alternative wage increase, the wage, which includes the value of the opportunities, increases with opportunities). We analyzed a case where workers are relatively more heterogeneous in outside opportunities than on the present job. For such instances, the theory predicts that the wage variations will overstate variations in productivity (which in this case are zero). For the more general case where workers may

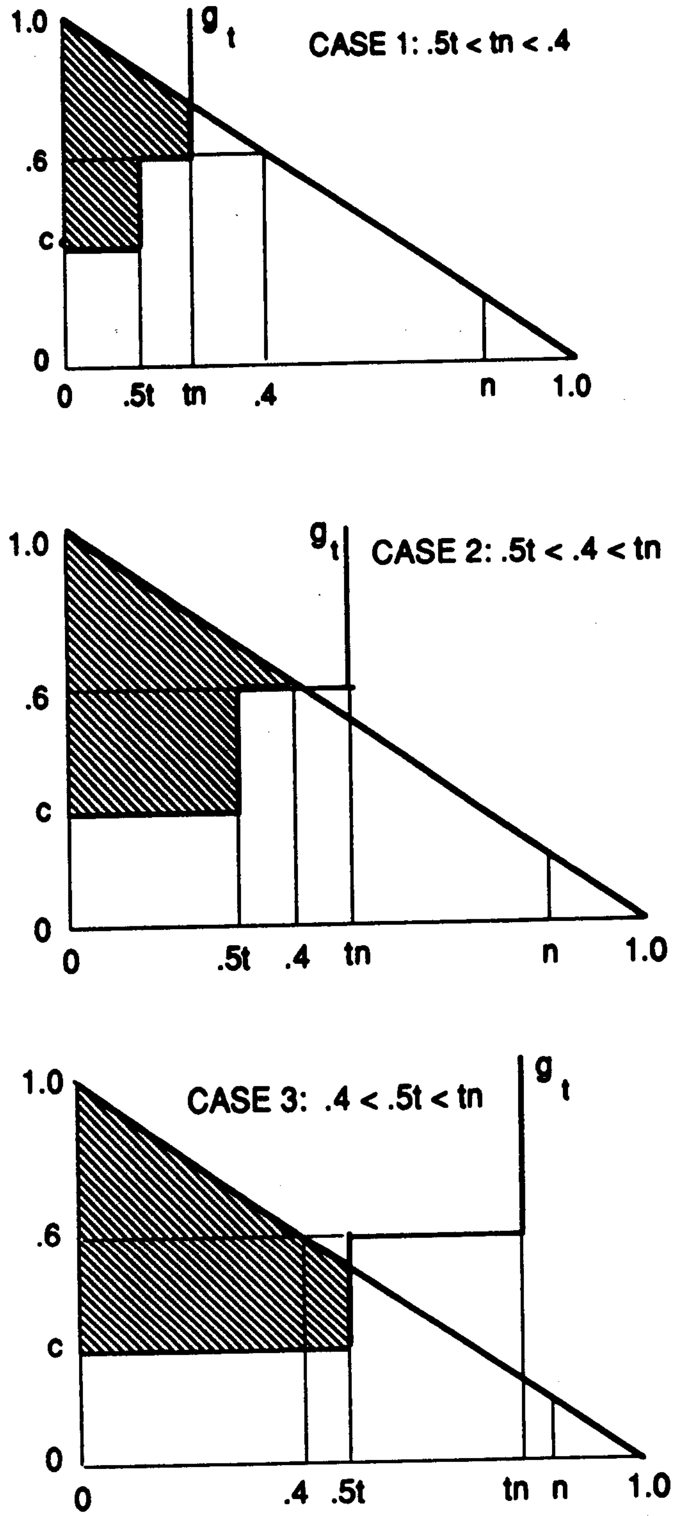

Figure 3.7. Case distinctions for Example 2.

have different productivity on the current job, and are homogeneous in outside opportunities, we conjecture that wage variations will understate productivities variations. The model provides a consistent way for analyzing wage outcome as function of outside opportunities. It also captures the observation that in certain occupations wages are determined on individual basis as an outcome of negotiations affected by the party's bargaining powers.

From Example 2 we learn that the relationship between the employer's payoff and the institutional structure ("oceanic game" vs. competitive), depends on the size of the labor force n and on the distribution of the workers' opportunities c. In particular, if n is small and c large, the worse is the employer's bargaining payoff relative to the competitive solution. Thus, the stronger is the employer's incentive to keep a competitive structure. This is clearly reversed as we examine the workers' incentives. The workers' wage is higher under the bargaining structure if n is small and c large. (Note that in the classical model, as long as n is greater then the equilibrium employment level, its size does not have any effect on the wage outcome. Also, the distribution of the workers' opportunities doesn't affect the competitive wage; only the opportunity of the marginal worker matters.) Actual wages were shown to be a direct function of outside opportunities (for a given employer and a given on-the-job productivity). Note that although our model recognizes the bargaining power of the workers (in contrast to the competitive model), the wages are not necessarily higher than in the competitive model; indeed they will be lower when the labor pool is large and the alternative wage is low.

In both examples the workers not hired are able to extract some benefit from the situation, which is not unreasonable because their presence keeps the actual wage below what it would otherwise be. This extracted benefit is usually very small. Indeed, in equation 3.7 we see that it goes to zero *in total amount* as the size of the unhired pool goes to infinity. In Equation (3.4) we see that it goes to zero as n (the total number of workers) goes to infinity. Finally, we emphasize that our model is not a competitive structure. The wage solution obtained is not stable in the competitive sense: some workers are getting more than the value of their marginal productivity. The employer is persuaded to agree to such a contract, by the threat of coalitional action by the various possible subsets of the worker pool.

4. Second Model: Labor Partially Organized

Our second model is an extension of the first. We assume that a certain subset U of the workers have already formed a union in order to bargain as a unit with the employer. If some but not all of the workers are so organized we still have an oceanic game, but now there are two atoms: E and U.

For simplicity, we assume that U consists of the workers who occupy a certain interval $[a, b]$ of the alternative-wage scale. This is admittedly a special assumption, but it is not entirely unreasonable since union formation is likely to be a highly selective process. We shall also assume (most of the time) that

$$a < b < w_0 \tag{4.1}$$

where $w_0 = g(n_0)$ is the equilibrium wage rate. Thus U consists entirely of people who would be employed at equilibrium if there were no union.

Figure 4.1 ABOUT HERE

Figure 4.1 displays some further notation. Thus, u is the size of the union, $\hat{g}$ is the supply function that characterizes the unorganized workers -- i.e.,

$$\hat{g}(x) = \begin{cases} g(x) & \text{if } 0 \le x \le d \\ g(x+u) & \text{if } d < x \le \hat{n} \end{cases} \tag{4.2}$$

and $\hat{w}_0$ is the corresponding equilibrium wage rate.

To determine the Shapley value of this two-atom oceanic game by the "random order" method, we shall require two independent uniformly-distributed random variables, say t_E and t_U, representing the time of entry of the atoms E and U into the ordered continuum of unorganized workers. Our probability space is therefore a square, as shown in Figure 4.2. The number of unorganized ("oceanic") workers who are present when E arrives on the scene is $t_E \hat{n}$ and their alternative wage distribution is given by the compressed curve $\hat{g}_{t_E}$, defined like the g_t of Section 3 (Figure 3.2). Note the discontinuity along the diagonal. If $t_U > t_E$, the union's entry is responsible for a substantial increase in the surplus,

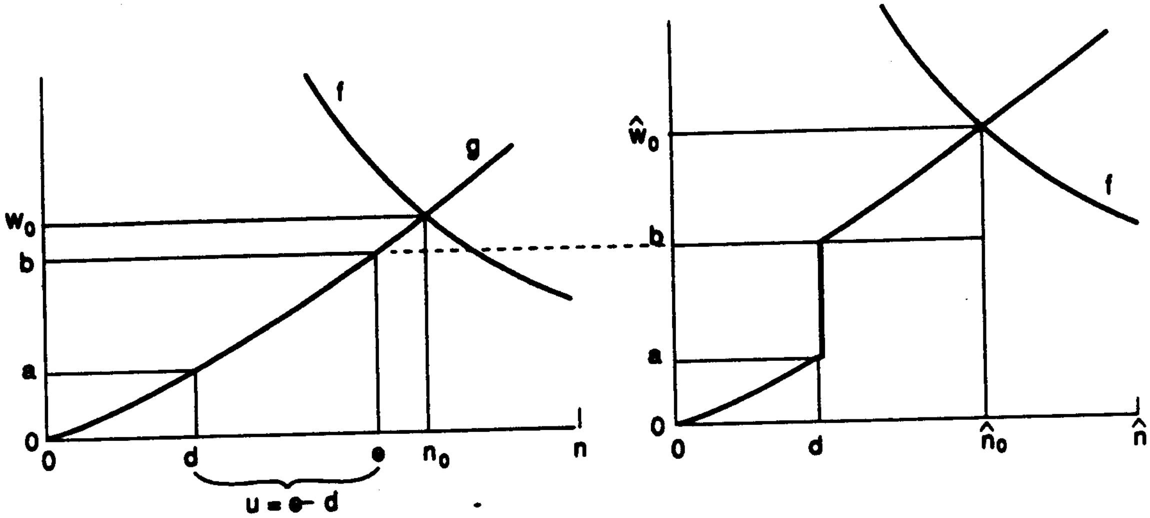

Figure 4.1. Notation for the second model.

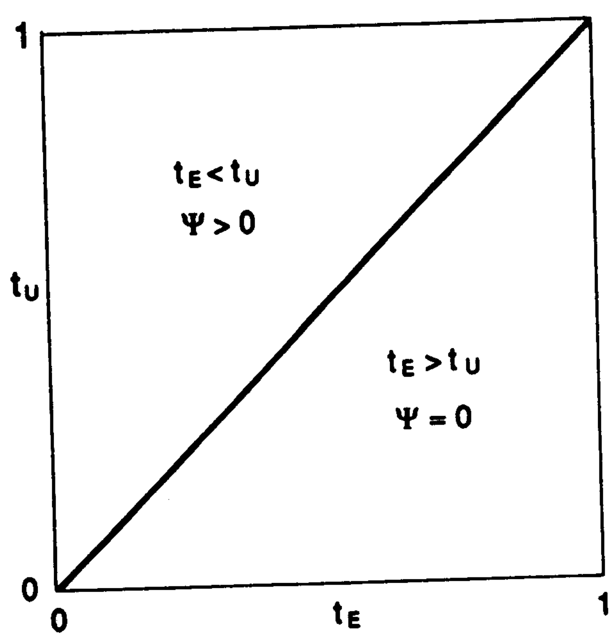

Figure 4.2. The probability space for two atoms.

but if $t_U < t_E$, U brings in nothing. The boundary case $t_U = t_E$ can be ignored, since it has probability 0.

Figure 4.2 ABOUT HERE

Let us now develop a formula for the union's value Φ_U, extending to the two-atom case the geometrical techniques we used in Model 1 (see the discussion in Section 2). To reduce notational clutter we shall write t for t_U until further notice.

Let $\Psi(t)$ denote U's contribution to the surplus if it enters at time t and E is already present, i.e., $t > t_E$. (Note that $\Psi(t)$ in this case is independent of t_E.) Then U's value is given by

$$\Phi_U = \int_{t=0}^{1} \int_{t_E=0}^{t} \Psi(t)\,dt_E\,dt = \int_{t=0}^{1} \Psi(t)\,t\,dt. \tag{4.3}$$

Figure 4.3 provides a geometric representation of the function $\Psi(t)$, namely, the area bounded by $\overline{ABCDFA}$. Here, $\overline{ABC}$ is a portion of the t-compressed version of g, defined by $g_t(x) = g(x/t)$, $\overline{CD}$ is a portion of the graph of f, and $\overline{AFD}$ is a portion of the graph of a function we shall call g'_t, defined by

$$g'_t = \begin{cases} g_t(x) & \text{if } 0 \le x \le td \\ g(x+(1-t)d) & \text{if } td \le x \le td+u \\ g_t(x-u) & \text{if } td+u \le x \le t\hat{n}+u \end{cases} \tag{4.4}$$

which represents the labor supply at time t with *all* the members of U included. (Thus, the segment $\overline{AF}$ is *not* compressed by a factor of t.) From the wage level b on up the graphs of g_t and g'_t are parallel with a horizontal separation of u.[11] In order to obtain an analytical expression, we have divided the area representing $\Psi(t)$ into three parts, as shown in the figure, whose separate areas are:

$$\begin{aligned} \Psi_I &= \int_{td}^{td+u} [b - g'_t(x)]\,dx = bu - \int_{d}^{d+u} g(x)\,dx, \\ \Psi_{II} &= u(\hat{w}'_t - b), \\ \Psi_{III} &= \int_{\hat{w}'_t}^{\hat{w}_t} [f^{-1}(y) - g_t^{-1}(y)]\,dy. \end{aligned} \tag{4.5}$$

[11] By assumption (4.1) we ensure that F lies below D in the figure, whatever the value of

Here $\hat{w}_t$ is the equilibrium wage for $\hat{g}_t$ and $\hat{w}'_t$ is the equilibrium wage for $\hat{g}'_t$. Combining (4.3) and (4.5), we obtain

$$\Phi_U = \int_0^1 \Psi(t_U) t_U \, dt_U = \int_0^1 (\Psi_I + \Psi_{II} + \Psi_{III}) t_U \, dt_U$$

$$= \int_0^1 \left\{ u\hat{w}'_{t_U} - \int_d^e g(x)dx + \int_{w'_{t_U}}^{\hat{w}_{t_U}} [f^{-1}(y) - \hat{g}_{t_U}^{-1}(y)]dy \right\} t_U \, dt_U ,$$

where we have now restored the subscript "U".

Figure 4.3 ABOUT HERE

By a similar calculation, which we omit, it can be shown that the corresponding expression for the employer is

$$\Phi_E = \int_{t_U=0}^{1} \left\{ \int_{t_E=0}^{t_U} \int_0^{\hat{h}_{t_E}} [f(x) - \hat{g}_{t_E}(x)] dx dt_E + \int_{t_E=t_U}^{1} \int_0^{\hat{h}'_{t_E}} [f(x) - \hat{g}'_{t_E}(x)] dx dt_E \right\} dt_U ,$$

where $\hat{h}_t = f^{-1}(\hat{w}_t)$ and $\hat{h}'_t = f^{-1}(\hat{w}'_t)$.

Figure 4.4 ABOUT HERE

Finally we calculate $\phi(x)$, the value-density function for the unorganized workers, as described at the end of section 2. The probability space is now three dimensional, but we can represent it adequately in two dimensions by treating t_x, the arrival time of a typical infinitesimal oceanic player x, as a variable marker on the t_E and t_U scales, as shown in Figure 4.4. The six possible order of entry of E, U and the infinitesimal player x are conveniently grouped into three cases by the double lines. If $t_x < t_E$ (at the right -- total probability $1-t_x$), the oceanic player contributes nothing. If $t_x > t_E$ but $t_x < t_U$ (upper left -- total probability $t_x(1-t_x)$) he contributes $\max\{0, \hat{w}_{t_x} - g(x)\} dt_x$. If $t_x > t_E$ and $t_x > t_U$ (lower left -- total probability t_x^2) he contributes $\max\{0, \hat{w}'_{t_x} - g(x)\} dt_x$. So we obtain, writing "t" for

t. Without this assumption, additional case distinctions would appear as t approaches 1.

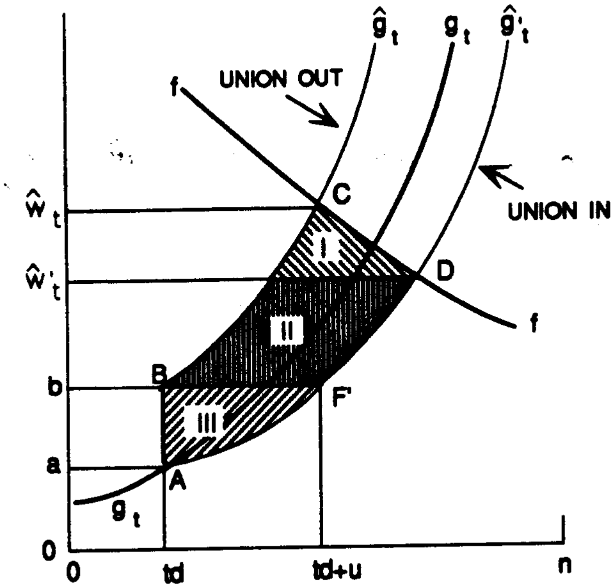

Figure 4.3. Guide to (4.5).

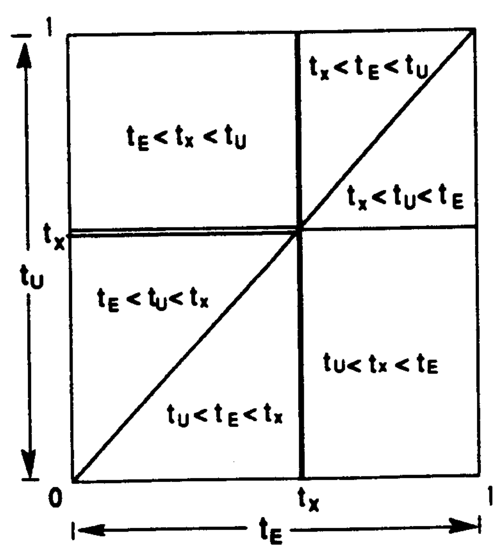

Figure 4.4. Adding an infinitesimal atom.

"t_x" and integrating,

$$\phi(x) = \int_0^1 [(t-t^2)\max\{0, \hat{w}_t - g(x)\} + t^2 \max\{0, \hat{w}'_t - g(x)\}]dt.$$

4.1. Collective vs. Individual Bargaining -- I

As an application of this analysis we shall show that it is better for the members of U to bargain as a union than as individuals – at least if the functions f and g are linear. Thus, we shall be comparing Φ_U (above) with

$$\int_{x \in U} \bar{\phi}(x)dx, \tag{4.6}$$

where $\bar{\phi}$ is the value-density function for the "ocean" of our first, un-unionized model.

Figure 4.5 ABOUT HERE

Figure 4.5 shows the comparison. As in Figure 4.3, the integrand $\Psi(t)$ where there is a union is given by $\overline{ABCDFA}$. The corresponding integrand for an infinitesimal set "dx" of unorganized workers in our first model is given by a narrow strip along the g_t curve (see Figure 3.4). Its vertical extent is from $g_t(\alpha)$ to w_t while its horizontal extent is everywhere dx, so the area (disregarding second-order infinitesimals) is given by $(w_t - g_t(t\alpha))dx$. Since $g_t(t\alpha)$ is just $g(x)$, the combined contributions of all the members of U, when not organized, is represented by the area of $\overline{ABC'D'FA}$, say $\bar{\Psi}(t)$, which by horizontal translation simplifies to

$$\bar{\Psi}(t) = \int_d^e (w_t - g(x))dx,$$

as shown in inset #1. We must therefore examine the difference[12] $\Psi(t) - \bar{\Psi}(t)$. But this is just the difference between the two triangles IV_t and V_t. Our claim is not that IV_t is always larger than V_t, but

12 In order to set up this comparison we have changed the order of integration, bringing the $\int dx$ integral inside the double integral $\iint dt_U dt_E$.

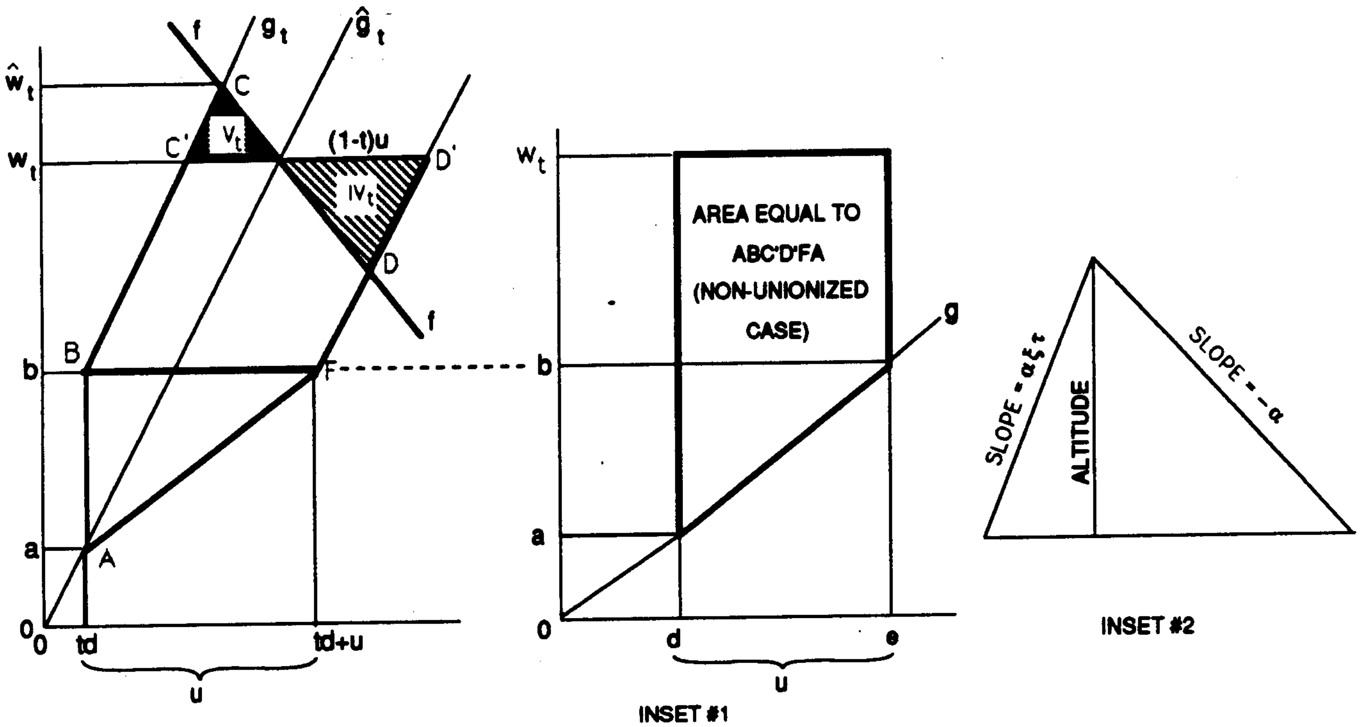

Figure 4.5. Comparison of partial union and no union.

that it is *larger on average* when all values of $t_g \le t$ are taken into account.

Note first that IV_t and V_t are similar triangles, with bases tu and $(1-t)u$ respectively. Let

$$g(x) = \alpha x \ , \qquad f(x) = \beta - \gamma x,$$

where α, β, γ are positive constants. Then, as shown in inset #2, we have

$$IV_t = \frac{\alpha\gamma t^2 u^2}{2(\alpha+\gamma t)} \ , \qquad V_t = \frac{\alpha\gamma(1-t)^2 u^2}{2(\alpha+\gamma t)}. \tag{4.7}$$

Our claim is that

$$\int_{t=0}^{1} \int_{t_g=0}^{t} [IV_t - V_t] dt_g \, dt > 0.$$

From (4.7) we have

$$IV_t - V_t = \frac{\alpha\gamma u^2 (2t-1)}{2(\alpha+\gamma t)}$$

In particular, for t between 0 and 1/2 we have,

$$\int_0^t [IV_t - V_t] dt_g = \frac{\alpha\gamma u^2 (2t-1)t}{2(\alpha+\gamma t)}, \quad \text{and} \quad \int_0^{1-t} [IV_{1-t} - V_{1-t}] dt_g = \frac{\alpha\gamma u^2 (1-2t)(1-t)}{2(\alpha+\gamma(1-t))}.$$

The sum of these two expressions is

$$\frac{\alpha\gamma u^2 (2t-1)}{2} \left[\frac{t}{\alpha+\gamma t} - \frac{1-t}{\alpha+\gamma(1-t)} \right] = \frac{\alpha^2 \gamma u^2 (2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))},$$

which we see is always nonnegative, and in fact is positive everywhere except at $t=1/2$. So we conclude:

$$\Phi_U - \int_U \phi(x) dx = \left[\int_{t=0}^{1/2} + \int_{1/2}^{1} \right] \int_{t_g=0}^{t} [IV_t - V_t] dt_g \, dt = \int_{t=0}^{1/2} \int_{t_g=0}^{t} [IV_t - V_t + IV_{1-t} - V_{1-t}] dt_g \, dt$$

$$= \int_0^{1/2} \frac{\alpha^2 \gamma u^2 (2t-1)^2}{2(\alpha+\gamma t)(\alpha+\gamma(1-t))} dt > 0.$$

This completes the proof that in the linear case the members of U are better off organized than unorganized.[13]

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[13] On the basis of several examples we have calculated, we conjecture that this remains true for all monotonic functions g and f and for any measurable set U consisting of hired workers. But we have found that it is not true in general if at the efficient production levels some of the workers in U are not hired, even if g and f are linear. Indeed, forming a union out of such a mixed set is inherently inefficient since it would result in the employer either hiring some who should not have been hired, or not hiring some who should have been hired. Such inefficiencies - reducing the total surplus - can diminish the value payoffs to the members

4.2. Collective vs. Individual Bargaining -- II

We shall now prove that when *all* workers (hired and unhired) are unionized and bargain as a unit, their total payoff is higher than what they get in Model 1, where none are unionized and the employer E is the only atom. This result does not require that g and f be linear functions.

Figure 4.6 ABOUT HERE

Consider Figure 4.6. The total surplus is

$$S = \int_0^{n_0} [f(x)-g(x)]dx, \tag{4.8}$$

and the value of the game for the employer (as derived in Section 3) is

$$\Phi_E = \int_{t=0}^{1} \int_{x=0}^{n_t} [f(x)-g_t(x)]dxdt = \int_0^{1/2} \left\{ \int_0^{n_t} [f(x)-g_t(x)]dx + \int_0^{n_{1-t}} [f(x)-g_{1-t}(x)]dx \right\} dt$$

Taking y in place of x as the independent variable, we can rewrite this as

$$\Phi_E = \int_0^{1/2} \left\{ \int_{y=0}^{f(0)} [A_1+A_2]dy \right\} dt$$

where

$$A_1 = \min\{g_t^{-1}(y), f^{-1}(y)\}$$
$$A_2 = \min\{g_{1-t}^{-1}(y), f^{-1}(y)\}.$$

There are then three cases:

$$\begin{aligned} &\textit{for } 0 \le y \le w_{1-t}: && A_1 = g_t^{-1}(y), \; A_2 = g_{1-t}^{-1}(y), \\ &\textit{for } w_{1-t} \le y \le w_t: && A_1 = g_t^{-1}(y), \; A_2 = f^{-1}(y), \\ &\textit{for } w_t \le y \le f(0): && A_1 = f^{-1}(y), \; A_2 = f^{-1}(y), \end{aligned}$$

and we observe that

$$\int_0^{w_{1-t}} g_t^{-1}(y)dy = \int_0^{w_{1-t}} (g^{-1}(y)-g_{1-t}^{-1}(y))dy.$$

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of U as well as the other players.

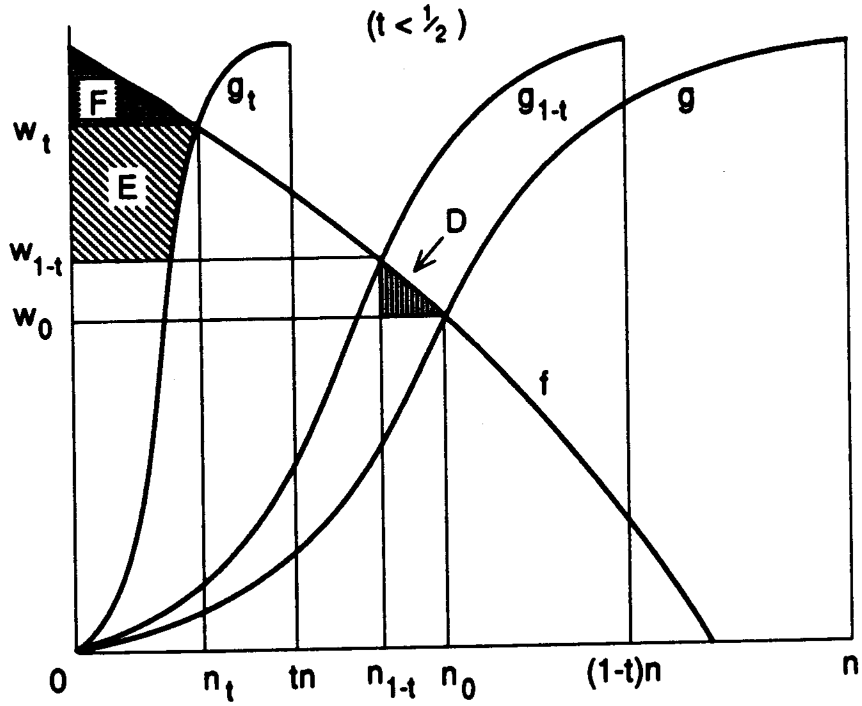

Figure 4.6. Advantage of one big union.

So the employer's value payoff may be calculated as follows:

$$\Phi_E = \int_{t=0}^{1/2}\left\{S + \int_{w_0}^{w_{1-t}} [g^{-1}(y) - f^{-1}(y)]dy + \int_{w_{1-t}}^{w_t} g_t^{-1}(y)dy + \int_{w_t}^{f(0)} f^{-1}(y)dy\right\}dt = \int_0^{1/2} Sdt + \int_0^{1/2} Kdt = \frac{S}{2} + \frac{K}{2},$$

where K denotes the sum of the areas of D, E and F in Figure 4.6. Since the total value to all players is the surplus S, the value to the workers must be

$$S - \Phi_E = S/2 - K/2.$$

On the other hand, if all the workers get together and bargain as a unit, then it is just a two-player simple bargaining game, and the value payoff to each side is just $S/2$. So if the entire labor force is unionized, their total wage is $K/2$ greater than if they had no union at all.

5. Concluding Remarks

Models of wage determination tend to ignore the fact that wages are directly or indirectly the outcome of negotiated contracts between the workers (individually or collectively) and the employers. As one recognizes the potential losses in terminating employment relationships, it becomes clear that bargaining power generated by the ability of each side to inflict costs on the other should be explicitly considered in the analysis of wage determination. Studies that do account for bargaining potentials are mainly in the framework of single employer and single worker. The complex problem of n-person bargaining has not yet explicitly modeled.

This is in spite the fact that the structures of labor markets are not uniform: they vary from the extreme situation where a single employer faces a single worker to the other extreme where many workers face a single large firm or many small firms. Since most real structures are at neither extreme but rather something in between, we have developed a framework for wage determination which can deal effectively with many intermediate structures on a consistent basis. In particular, the use of "oceanic" games allows us to consider labor contract bargaining even when the setting includes a continuum of unorganized but not helpless workers.

Our approach in this paper has stressed the role of the workers' alternative opportunities; as a result, our models are rather abstract. We do not attempt to represent the bargaining *process* as a game in strategic form, with proposals and counter proposals following according to some fixed protocol. Such process-specific models are suitable for formalized bargaining situations (e.g., auctions), but are too restrictive to do justice to the free-wheeling and essentially cooperative[14] nature of labor-management negotiations. Instead, we adopt the viewpoint that the wage determined will be the result of the underlying bargaining powers of the participants, irrespective of tactical considerations, and use a cooperative game in characteristic function form as the basic model with the Shapley value as the solution concept.

In the first model we dealt with unorganized labor. It was shown that for a given employers' structure and a given on-the-job productivity, the workers wage is inversely related to the size of the labor force and is positively affected by the workers' outside opportunities. The model is capable of predicting wage variations among workers that are independent of productivity variations, even when workers bargain multilaterally as individuals without a formal union. In the second model such a formal union was introduced, acting as a single agent to represent at least some of the workers. The Shapley value payoff to all the participants was derived, and the question of the wage to the union members under different institutional structures was addressed. The major predictions of the first model hold in the presence of a union as well. In addition it was shown, among other results, that when *all* available workers negotiate through a single union (bilateral monopoly), their total payoff is higher than what they would get as individual negotiators.

Another feature of our approach is that some payments are made to workers in the labor pool who do not in the end get hired. The reason is that their presence influences the wage settlement. These side payments are usually small (see for example equation (3.7)) It is not clear how these payments might be implemented in practice, but a possible interpretation might regard them as a measure of unrealized bargaining power, expressed in terms of what the recipients could get if utility were freely transferable. Using the NTU value theory would be another way to deal with the imperfect transferability, as discussed in Section 2.

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14 "Cooperative" in the game theory sense; i.e., when the antagonists come to terms, they can write any contract they please -- and then are bound by it.

At a more general level, there is an important distinction between the classical model and the present bargaining model in the factors they capture as affecting the wage outcome: In the classical models only *local* properties are important in affecting the outcome. For example, the size of the labor pool capable of working in a particular occupation does not affect the classical wage outcome provided that its reservation wages are above the equilibrium employment level. In contrast, the *size* and the *whole distribution* of *outside opportunities* of the labor force (not only around the equilibrium level) play a significant role in determining the outcome under the present bargaining model.

Finally, in subsequent research we wish to extend the present model in two directions: to allow for more than one type of labor, and to investigate in more detail the various institutional structures that might be adopted on the employer's side, there being a somewhat delicate balance to be struck between cooperation in wage negotiation and competition in the product market.

REFERENCES

Aumann R., "Markets with a Continuum of Traders," *Econometrica*, 32, pp. 39-50, 1964.

Aumann R., "Existence of Competitive Equilibria in Markets with a Continuum of Traders," *Econometrica*, 34, pp. 1-17, 1966.

Blair D. H. and Crawford D. L., "Labor Unions Objectives and Collective Bargaining," *Quarterly Journal of Economics*, pp. 547-565, August 1984.

Fogelman, F. and Quinzii, M., "Asymptotic Values of Mixed Games," *Mathematics of Operations Research*, 5 (1980), pp. 86-93.

Guesnerie R., "Monopoly, Syndicate and Shapley Value: About Some Conjectures," *J. Econom. Theory*, 15, pp. 235-251, 1977.

Hart S., "Values of Mixed Games," *International Journal of Game Theory*, 2, pp. 69-85, 1973.

Levy A., "Heterogeneity, Wage Bargaining and the Expected Union Size," mimeo, 1986.

Milnor J. W. and Shapley L. S., "Values of Large Games, II: Oceanic Games," RM-2649, The Rand Corporation, Santa Monica, California, 1961.

Milnor J. W. and Shapley L. S., "Values of Large Games, II: Oceanic Games," *Mathematics of Operations Research*, 3, pp. 290-307, 1978.

Shapiro N. and Shapley L. S., "Values of Large Games I: A Limit Theorem," RM-2648, The Rand Corporation, Santa Monica, California, 1960.

Shapiro N. and Shapley L. S., "Values of Large Games I: A Limit Theorem," *Mathematics of Operations Research*, 3, pp. 1-9, 1978.

Shapley L. S., "A value for n-person Games," *Annals of Mathematics Study* 28 (1953), pp. 307-317.

Shapley L. S., "Values of Large Games, III: A Corporation with Two Large Stockholders," RM-2650, The Rand Corporation, Santa Monica, California, 1961.

Shapley L. S., *The "Value of the Game" as a Tool in Theoretical Economics*, The Rand Corporation, P-3658, August 1967.

Shapley, L. S., and Shubik, M. "Ownership and the Production Function," *Quarterly Journal of Economics*, 81 (1967), pp. 88-111.