INTERACTIONS OF REAL GNP BUSINESS CYCLES IN A THREE COUNTRY TIME SERIES MODEL

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Abstract

The paper describes a procedure for examining short-run dynamic interactions among macroeconomic models by constructing aggregate state space submodels for dynamic modes corresponding with short-run response patterns. Using the quarterly real GNP from the U.S., West Germany and Japan for 1974.III to 1991.I, we examine dynamic interactions in the frequency ranges roughly comparable with a range of business cycle frequencies. We find that there is no (world wide) shock common to the three countries, even though the West German and Japanese real GNP are hit by a common shock.

1. Introduction

Intercountry interactions of economic variables have been examined in the context of policy coordination in general, and of business cycles analysis in particular. For example, the notion of world wide versus country specific shocks have been examined in the context of policy coordination questions in Aoki (1973). Gerlach and Klock (1988) use an unobservable component model with one international and several domestic components to model international business cycles. Stockman (1988) decomposes the growth rates of several countries into components specific to countries, those specific to industries plus idiosyncratic components and concludes that most macroeconomic fluctuations may not be attributable to technical shocks alone. Solow residuals of several countries have been compared by Costello (1989) using similar decomposition, i.e., error component models, in an attempt to shed light on real business cycles.

These works all focus on co-movements of real GNP fluctuations, by assuming at the beginning the existence of significant or non-negligible amounts of world wide shocks. In this paper we identify several structural models of short-run dynamic interaction patterns of real GNP of the U.S., West Germany and Japan.[1] These models are then used to examine the question of relative contributions to real GNP movements (in the business cycle frequencies) of common and country-specific shocks. This approach is more general and informative than that which posits *a priori* a particular decomposition of shocks into world wide and idiosyncratic components which is shown to be a special case of our approach. We also introduce several tools of analysis which are useful in this type of investigation involving

[1] A sequential Chow test as well as a more sophisticated test indicate a structural shift in the Japanese real GNP in the period before 1974II. To avoid a possible structural break we use data from 1974III to 1991I.

trending time series.

By dynamic interaction in the short-run, or in business cycle frequency ranges, we mean dynamic phenomena which occur in a time span of roughly four to six years. Responses (called dynamic mode(s) in this paper) of the eigenvalues with magnitude close to one are taken to be the trend component by definition. Dynamic modes due to eigenvalues less than .9 in magnitude, says, are the ones responsible for the phenomena in the business cycle frequencies. To see this, observe that in dynamic modes with eigenvalues larger than .93, say, effects of initial disturbances persist, i.e., do not disappear in the time span of interest, while those with eigenvalues .9 or less do in quarterly models. Effects of initial disturbances are reduced to less than 8% in 25 quarters. Note that $.99^{25} = .75$, $.95^{25} = .28$, but $.90^{25} = .07$ and $.89^{25} = .05$.

These numbers suggest that main features of short-run dynamic interactions in macroeconomic models are produced by dynamic modes with eigenvalues not greater than .9 in magnitude, and are captured by short-run dynamic models with dynamic modes in these ranges. To focus on dynamic phenomena in business cycle frequencies, then, it is convenient to decompose or pre-filter time series data to retain only dynamic modes that correspond to these frequency ranges. Such a use of decomposition amounts to aggregating full models to build submodels which inherit only dynamic modes with eigenvalues of magnitude less than .9, say.[2]

Taking the first difference of the logarithms of the trending data series to eliminate "trend" components, such as taking the logarithms of the ratios of data values one year apart, has been a common practice for a long

[2] Aoki (1968) proposes a procedure of dynamic aggregation to retain only a prescribed subset of eigenvalues of the original model. Aoki (1988) describes an improved procedure to accomplish the same.

time. This practice has been criticized for loss of longer-run information and alternative detrending schemes have been proposed to deal with the so-called unit root or random trends. Unfortunately, test for unit roots are frequently inconclusive because of low power of tests.[3] These tests usually can't distinguish time series with eigenvalues of unit magnitude from those near one with magnitude .98 or greater, for example. For our purposes it is not necessary to single out eigenvalues of unit magnitude. Dynamic modes with eigenvalues near and at one, if such exists, all contribute to response patterns which die out slowly if at all, i.e., trend movements, which are not immediately relevant to shorter-run behavior of real GNPs. We use a procedure for separating out this broader class of low frequency components from higher frequencies ones in data, called a two-step method in Aoki (1990).

Section 2 briefly describes this two-step method, and two other ways of extracting trends from data in models of three real GNPs. Once is to extend the average-difference way of representing dynamic interactions of two country models described in Aoki (1981) to models of three countries when the three countries are not necessarily specified symmetrically, as in the original two-country model. The original method has been applied by others[4] and later extended.[5] The extension described in this paper is different. We show that the dynamic model for the average is a special case of models obtained by the dynamic aggregation in the sense of Aoki (1988), i.e., by the first way of defining trends as behavior patterns with the

[3]There is a large body of literature. Sims, Stock and Watson (1990) is a recent example.

[4]See Miller and Salmon (1984), and Buiter (1990), for example.

[5]See Fukuda and Hamada (1988) which describes n-country models in the average-difference scheme suitably generalized.

eigenvalues of largest magnitude. The third way is to modify the reduced rank regression models for VAR time series with unit roots, such as in Johansen (1988), to eliminate (near) unit roots shared by all the components of the data vector. The relation of this approach to the first is also indicated.

Section 3 summarized the estimated model from the second step. Dynamic relations among the three real GNPs are conveniently summarized by the impulse response patterns of the estimated model to structural shocks. This requires that we solve the identification problem after we estimate the dynamic model. This is taken up in Section 4. Two models are chosen as "best" ones from all the identified models. In both models West Germany and Japan share common shocks in the business cycles, but shocks to the U.S. are not shared by the other two countries. Thus, there is no world-wide co-movements in the real GNP business cycles, while a model which presupposes the decomposition into common and idiosyncratic shocks indicates a significant presence of common shocks. The paper concludes with Section 5.

2. Trend Determination[6]

We first summarize a state space common trend representation following Aoki (1990, Ch. 11). This representation is the one actually used to estimate the short-run model for the business cycles.

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[6]A scheme for extracting a trend, not discussed here, is due to Whittaker (1923) who defined the trend as that time series which minimizes a weighted sum of the squared deviation of the data series from the trend series and squares of a measure of smoothness of the trend series in terms of the k^{th} difference. Much later Akaike (1980) gave a Bayesian framework for choosing the weight and k. This scheme is also used by Hodrick and Prescott (1981).

The trend series thus constructed may be smoother than that obtained by the two-step procedure described in this paper since the former uses the data in the whole sample period, while the latter uses only the data up to the point where the trend is being extracted.

Common Trend Representation of Nearly Cointegrated Series

In the above subtitle, by nearly cointegrated we mean that data series share a common slow dynamic mode due to the eigenvalue with the largest magnitude.

Let ρ be the largest eigenvalue of a dynamic matrix A in a state space representation for "trending" data series $\{y_t\}$,

$$x_{t+1} = Ax_t + \text{noise} \tag{1}$$

$$y_t = Cx_t + \text{noise}.$$

Let p be the corresponding normalized column eigenvector

$$Ap = p\rho,\ p'p = 1.$$

All the other eigenvalues of Matrix A are strictly smaller in magnitude than ρ. Let

$$Q'A = Q\Lambda,\ Q'Q = I$$

be the corresponding row eigenvector expression, i.e., the row space of Q' spans the subspace corresponding to all other eigenvalues of A. Note that $p'Q = 0$.

Change the coordinate system so that the components of vector x_t change into τ_t and η_t

$$x_t = [p,Q] \begin{bmatrix} \tau_t \\ \eta_t \end{bmatrix}.$$

Since $\tau_t = p'x_t$ and $\eta_t = Q'x_t$ they are called aggregated subvectors of x_t, each of which inherits dynamic modes associated with ρ and Λ, respectively. To see this, note that the dynamics are now represented by

$$\begin{bmatrix} \tau_{t+1} \\ \eta_{t+1} \end{bmatrix} = \begin{bmatrix} p' \\ Q' \end{bmatrix} x_{t+1}$$

$$= \begin{bmatrix} p' \\ Q' \end{bmatrix} A[p,Q] \begin{bmatrix} \tau_t \\ \eta_t \end{bmatrix} + \text{noise}.$$

Note that

$$\begin{bmatrix} p' \\ Q' \end{bmatrix} A[PQ] = \begin{bmatrix} \rho & S \\ 0 & \Lambda \end{bmatrix}$$

where

$$S = p'AQ.$$

From (1), the data series are related to the newly introduced variables by

$$y_t = Cp\tau_t + CQ\eta_t + \text{noise}.$$

The variable τ_t represent a slow dynamic mode shared by all the components of y_t, i.e., the lower frequency elements in y_t, and the second and the third term higher frequency components.

This common trend representation of the data shows that any vector orthogonal to Cp can be used to eliminate the slow dynamic modes, i.e., the trend in the data series. Suppose β is such that $\beta' Cp = 0$. Then

$$\beta' y_t = \beta' CQ\eta_t + \text{noise}.$$

Since η_t is generated by a dynamic process with spectrum of Λ, $\beta' y_t$ exhibits only those dynamic modes of Λ and not that due to the largest one ρ. The space $(Cp)^{\perp}$ is the space of cointegrating vectors in this extended sense. Its dimension is two in the real GNP series of this paper.

In the first step of the two-step procedure, the trend component can be interpreted as the asymptotically most efficient instrument in estimating the disaggregation matrix $H = Cp$, where $y_t = H\tau_t$ + residuals (Aoki, 1990, Ch. 9). It is given by

$$\tau_t = S y^{-}_{t-1}$$

where

$$S = \Sigma V' R^{-1}, \quad R = \mathrm{cov}(y_{t-1}^{-}),$$

with the data stacked as $y_{t-1}, y_{t-2}, \ldots$ denoted by

$$y_{t-1}^{-} = \begin{bmatrix} y_{t-1} \\ y_{t-2}^{-} \end{bmatrix},$$

and where matrices Σ and V' appear in the singular value decomposition of $E(y_t y_{t-1}^{-\prime}) = U\Sigma V'$. Since the orthogonal projection of y_t onto the subspace spanned by y_{t-1}^{-} denoted by $\hat{E}$ is

$$\hat{E}(y_t; y_{t-1}^{-}) = U\Sigma V' R^{-1} y_{t-1}^{-}$$

$$= U\tau_t,$$

this definition amounts to selecting a common trending component as τ_t and H by U in decomposing the data into a slow-moving common trend y_{ct} and higher frequency residuals, $y_t = Hy_{ct}$ + residual.

Generalized Average-Difference Representation

We next describe how the average-difference framework originally proposed in Aoki (1981) for a two-country model can be extended to this trivariate real GNP series. In a symmetrically specified two-country structural or bivariate time series model, the average and the difference of the two components are defined by dividing the sum and the difference by 2. The original purpose of this transformation is to render the joint dynamics for these newly defined variables recursive or decoupled, i.e., to transform the dynamic matrix into (block) triangular or (block) diagonal forms in the special case of two countries having the same model parameters (elasticities). In the case of time series, the average and the difference becomes uncorrelated if the data is normalized to have the same variance.

To illustrate the nature of the extension simply, consider a simple model VAR(1),

$$y_t = \phi y_{t-1} + \epsilon_t$$

with the dynamic matrix

$$\phi = \begin{bmatrix} a & b & c \\ b & a & b \\ c & b & a \end{bmatrix}. \tag{2}$$

With this dynamic matrix ϕ (dropping time subscript), $y_a = (1,1,1)y$ possesses eigenvalue of ϕ, $\rho = a + b + c$ under some conditions on the parameter values. (As we later show this symmetric interaction pattern is not consistent with the time series data.) The generalized averaging vector for a non-symmetric ϕ is now defined as the row eigenvector for the largest eigenvalue of the dynamic matrix ϕ, exactly as described in the previous subsection on the common trend representation by calculating the eigenvector corresponding to the largest eigenvalue of ϕ.

A related procedure is the following: Let ρ be the largest eigenvalue of the dynamic matrix ϕ, not necessarily symmetrically specified. Let q' be its normalized row eigenvector

$$q'\phi = q'\rho, q'q = 1.$$

Then let $y_{at} = q_t$.

Its dynamics are

$$y_{at} = \rho y_{at-1} + q'\epsilon_t.$$

Let $\phi R = R\Lambda$, $R'R = I_2$, $\Lambda = \mathrm{diag}(\lambda_1, \lambda_2)$,

where $|\lambda_i| < \rho$, $i = 1,2$.

Note that $q'R = 0$. Define

$$d_t = R'y_t.$$

Then $\begin{bmatrix} y_{at} \\ d_t \end{bmatrix} = \begin{bmatrix} q' \\ R' \end{bmatrix} y_t,$

i.e., $$y_t = \begin{bmatrix} q' \\ R' \end{bmatrix}^{-1} \begin{bmatrix} y_{at} \\ d_t \end{bmatrix} = [q,R] \begin{bmatrix} y_{at} \\ d_t \end{bmatrix}$$

and hence $$d_t = (R'\phi q)y_{at-1} + \Lambda d_{t-1} + R'\epsilon_t.$$

The data vector is now representable as

$$y_t = qy_{at} + Rd_t \tag{3}$$

where y_{at} is the "average" and d_t is the difference for the three country model. When ϕ is given by (2), q' is proportional to (1,1,1). This procedure extends naturally to higher order VAR(q), $q \geq 2$ as well.

Reduced-Rank Regression Method

This last subsection relates a VAR representation of cointegrated series to the state space one described earlier. We do not use this representation in this paper because of the inherent limitations noted at the end of this section. It is included here since this type of model is widely known to the profession.

Let a VAR representation of y_t be given by[7]

$$\phi(L)y_t = n_t. \tag{4}$$

with a mean zero, weakly stationary n_t. Suppose that ρ is the largest eigenvalue of this model and, $\phi(1/\rho) \neq 0$ is of reduced rank, $\rho\phi(1/\rho) = \alpha\beta'$, say, where α and β are rank 2 matrices.[8] Define

$$\phi^*(L) = \frac{\phi(L)-\rho L\phi(1/\rho)}{1-\rho L}.$$

and rewrite (4)

7 Although VAR models are almost exclusively used in the literature on macroeconomic time series, state space representations easily allow for ARMA models, as shown in Aoki (1990), for example. The MA models for noises are sometimes important.

8 If the rank is one, then the eigenvalue ρ is repeated. This is highly unlikely to occur and is excluded.

$$\phi(L)y_t = \rho L\phi(1/\rho)y_t + \phi^*(L)\delta y_t = n_t,$$

where $\delta y_t = (1-\rho L)y_t$,

i.e., $\phi^*(L)\delta y_t = -\alpha\beta' y_{t-1} + n_t$. Since δy_t and n_t are weakly stationary $\beta' y_{t-1}$ must also be so, i.e., the components of y's are cointegrated.[9] As an example consider VAR(2)

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + n_t. \tag{5}$$

The model becomes

$$\delta y_t = -\alpha\beta' y_{t-1} + \phi_2^* \delta y_{t-2} + n_t$$

where $\phi_1 + \phi_2/\rho - \rho I = -\alpha\beta'$

and $\phi_2^* = -\phi_2/\rho$.

Note that the vector β must be orthogonal to Cp in the common trend state space representation. To see the relation between the latter two representations more concretely, consider the example above, with the state vector defined by

$$x_t = \begin{pmatrix} y_t \\ y_{t-1} \end{pmatrix}.$$

The dynamic representation (5) becomes

$$x_t = Ax_{t-1} + \begin{pmatrix} n_t \\ 0 \end{pmatrix}, \quad y_t = [I_3 \ 0]x_t$$

with
$$A = \begin{bmatrix} \phi_1 & \phi_2 \\ I & 0 \end{bmatrix}. \tag{6}$$

The eigenvalue-eigenvector relation

$$\begin{pmatrix} \phi_1 & \phi_2 \\ I & 0 \end{pmatrix} \begin{pmatrix} p_1 \\ p_2 \end{pmatrix} = \rho \begin{pmatrix} p_1 \\ p_2 \end{pmatrix}$$

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[9] This is true only when the contemporaneous components of y_t are cointegrated by assumption. There are examples in which some components of y_t and y_{t-1} are cointegrated but not y_t and y_{t-1}.

yields $y_t = (I_3\ 0)x_t = p_1\tau_t + Q_1 y_t + \eta_t$,

where $Q_1 = (I_3\ 0)Q$,

we see that $\beta' y_t$ is weakly stationary if and only if $\beta' p_1 = 0$. In words, the two dimensional subspace orthogonal to p_1 is the space of cointegrating vector β's. Put differently, $\tau_t = p' x_t$ can be used as a trend which is shared by the components of y_t as $C\tau_t$ and the residuals $y_t - C\tau_t$ have only dynamic modes of to Λ, with magnitude less than ρ.

Comparisons

The previous subsections are meant to convey to the reader our view that use of the row-eigenvector corresponding to the slowest dynamic mode of data vectors as the aggregation vector is a convenient organizing device conceptually. Although this is not the place to comparatively discuss numerical behavior of the alternative methods for trend selections, our experience indicates that the combination of the trend extraction by the first step of the two-step procedure with the use of left-eigenvector to change the coordinate system in which to represent the model improves, in some cases, statistics of the residuals from the second step of the two-step procedure. This device is found to be useful in some modeling context with exogenous signals also, even though we have not touched on the issue of exogenous signals in this paper.

3. Estimated Model

All series are divided by the first data value component-wise, so that the logarithms of the series start with zero values in the three components The first step of the common trend representation procedure extracts the first order dynamics with the eigenvalue .97. The residuals from this first step, denoted by w_t, are then modeled in state-space form

$$z_{t+1} = Fz_t + Ge_t;\ w_t - \bar{w} = Hz_t + e_t$$

where $\bar{w}$ is the sample mean of the residuals from the first step. A three-dimensional state vector z_t is chosen to be sufficient, i.e., a three-dimensional dynamics produce the innovation vector e_t which is sufficiently uncorrelated over time to approximate a white noise sequence, measured by the DW statistics which are 2.01, 2.11 and 1.93 respectively for the three components of e_t.

The dynamic matrix, F, of this second step has eigenvalues .89 and a pair of complex eigenvalues with the real part .84 and the imaginary part .22. The period of the pair is 24.4 quarters.

The impulse responses to innovation shocks are given by

$$HF^{k-1}G;\ k = 1,2,\ldots$$

In this paper we are primarily interested in the impulse response patterns of this short-run model with respect to structural shocks. In the next section we relate the structural shocks to the innovations so that we can use the above formula to calculate the effects of structural shocks.

4. Identification Problem

Having retained for further modeling purposes only faster dynamic modes corresponding to the eigenvalues of less than ρ in magnitude, and having estimated a dynamic submodel for them, we next relate the estimated model to a structural model in order to analyze effects of structural shocks. More specifically, we relate the innovations in the estimated time series model for short-run dynamics of the data series to structural shocks. We do not use a common but somewhat arbitrary procedure of variance decomposition for VAR models, and adapt the procedure used by Bernanke (1986) and Sims (1986) to state space models. This section describes this identification method and the results for the three real GNP series.

A structural relation for the data implies a particular relation between the contemporaneous components of the three real GNPs. When only contemporaneous variables are collected from a structural model, it defines a relation

$$y_t = \psi n_t + \text{lagged terms} \tag{7}$$

where y_t has three components y_{ut}, y_{gt} and y_{jt}, the real GNP of the USA, West Germany and Japan, in that order, and similarly for n_t. This relation may be thought of as follows.

Let $\phi(L)y_t = \theta(L)n_t$ describe the unknown structural model, where $\phi(\bullet)$ and $\theta(\bullet)$ are some finite order matrix polynomials of the lag operator L, which governs the three dimensional data vector, y_t. We assume that structural shocks are such that its means, conditional on the past data, are zero. Subtract the expectation of both sides of the structural equation, conditional on the past data, from it to obtain the contemporaneous relation between the innovation of the data vector with respect to its own past and the structural noise $\phi_0 e_t = \theta_0 n_t$ where the ϕ_0 and θ_0 are the constant matrices of the respective lag matrix polynomials. Assuming that ϕ_0 is non-singular, we derive a relation equivalent to (7) where $\psi = \phi_0^{-1}\theta_0$,

$$e_t = \psi n_t$$

which is basic to what follows. The matrix ψ, therefore, relates the structural shock to the innovation vector, and determines how the structural shock is distributed among the components of the data vector, as shown in (7).

The estimated covariance matrix of the innovation vector, Δ, is therefore related to the covariance matrix of the structural shock, Σ, by

$$\Delta = \psi\Sigma\psi' . \tag{8}$$

A three-dimensional model has been estimated for the detrended real GNP series. The estimated short-run dynamics has one real and a pair of complex eigenvalues with the period of about 24.4 quarters, and the innovation covariance matrix

$$10^3 \times \Delta = \begin{bmatrix} .1215 & .0368 & .0293 \\ .0368 & .1067 & .0337 \\ .0293 & .0337 & .0970 \end{bmatrix}$$

The estimated innovation vector series has the DW statistics, 2.01, 2.11 and 1.93 respectively, and the estimated model is deemed satisfactory.[10]

The unknown matrices ψ and Σ must be estimated subject to (8). Since (8) provides only six independent relations, the elements of the matrices ψ and Σ must be constrained in some ways to allow for only six independent parameters if the parameters are to be (uniquely) determined (just-identified).

Without loss of generality, the diagonal elements of the matrix ψ can be normalized to be one, and we assume that the three components of the structural noise are uncorrelated, i.e., the matrix Σ is diagonal. This uses up three of the six relations. In the just identified cases, we must introduce three zero restrictions to determine the matrix ψ. There are 20 such ψ matrices. Of these, 6 of them are eliminated outright since these imply that an off-diagonal element of Δ is zero, which is not empirically supported by the estimated Δ as shown above. For example, matrix ψ

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[10]Estimated dynamics and statistics of the innovations vary as the size of the Hankel matrix is changed. The size is chosen to produce a pair of complex eigenvalues with the period of about 6 years or less.

specified as

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & \bullet \\ \bullet & \bullet & 1 \end{bmatrix}$$

together with a diagonal Σ implies that $\Delta_{1,1}$ is zero.

The remaining 14 possible matrices and the associated diagonal elements of matrix Σ are solved.[11] A particular matrix ψ implies the relation among the contemporaneous real GNPs

$$\Omega y = n \tag{9}$$

where the matrix Ω is ψ^{-1} premultiplied by a diagonal matrix D in such a way that the diagonal elements of Ω are all ones. There are six models with the values of the trace of $D\Sigma D$ closer to each other.

The best in terms of the value of the trace of shocks in (7), denoted as m1, has the estimated ψ

$$\begin{pmatrix} 1 & .280 & .302 \\ 0 & 1 & .347 \\ 0 & 0 & 1 \end{pmatrix}$$

with the variances $s_1 = .105 \times 10^{-3}$, $s_2 = .095 \times 10^{-3}$, and $s_3 = .097 \times 10^{-3}$, where $s_i = \mathrm{var}(n_i)$, $i = 1,2,3$, i.e., the innovations in the real GNPs of this model are related to structural shocks by

$$e_u = n_1 + .28n_2 + .30n_3;\ e_g = n_2 + .35n_3;\ e_j = n_3,$$

where the subscript refers to a particular country, u, for the U.S., and so forth. This matrix implies that (9) becomes in this model

$$y_u = .28y_g + .20y_j + n_1$$

$$y_g = .34y_j + n_2$$

[11] Some of the resulting equations need be solved using a symbol manipulating software.

$$y_j = n_3$$

with the trace value of $.297 \times 10^{-3}$.[12] Although n_3, which is identical to the Japanese real GNP innovation, appears in the other two country's real GNPs the percentages in the innovation variances are small; in the U.S. case $.30^2 \times s_3/\Delta_{11}$ is about .072, and in the German innovation $.35^2 \times s_3/\Delta_{22}$ is about .111. The shock n_1 is specific to the U.S. real GNP.

Another class of matrices ψ introduces four unknown parameters with the constraint that the second and the third components of Σ are equal to achieve the just identification. The best model in this class, denoted as m2, has the form

$$\psi = \begin{bmatrix} 1 & .286 & .294 \\ 0 & 1 & .318 \\ 0 & .030 & 1 \end{bmatrix}.$$

with $s_1 = .105 \times 10^{-3}$, $s_2 = s_3 = .097 \times 10^{-3}$.

Note that it is very similar to the matrix ψ estimated for model m1. The relation (9) of this model is

$$y_u = .28y_g + .20y_j + n_1;\ y_g = .32y_j + .99n_2;\ y_j = .03y_g + .99n_3$$

with the trace $D\Sigma D$ being $.295 \times 10^{-3}$.[13] Comments similar to the one above apply to this class of models, i.e., no dominant common shock can be

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[12]The second best is

$$y_u = .30y_j + n_1;\ y_g = .24y_u + .04y_j + n_2;\ y_j = n_3$$

with the trace value of $.298 \times 10^{-3}$. The third is

$$y_u = .34y_g + n_1;\ y_g = n_2;\ y_j = .16y_u + .26y_g + n_3$$

with the trade at $.299 \times 10^{-3}$.

A close fourth is

$$y_u = n_1;\ y_g = 30y_u + n_2;\ y_j = .16y_u + .26y_g + n_3.$$

[13]A close second of this class has the form:

$$y_u = n_1;\ y_g = .24y_u + .25y_j + .99n_2;\ y_j = .23y_u + .025y_g + .99n_3$$

found.

A third class of models incorporates the assumption that structural shocks are composed of common (world) shock plus country-specific shocks and posit

$$n = hn_c + n_s$$

where n_c is a common shock and the vector h distributes it among the three real GNPs, and n_s is uncorrelated with n_c and has a diagonal covariance matrix. Together with (7), this assumption posits that

$$y_t = \psi(hn_c + n_s)_t + \text{lagged terms}$$

As it is, this model has too many parameters. If we set ψ to be the identity matrix and normalize the vector h to be $[1\ h_1\ h_2]'$, then there are exactly six parameters and six relations: $var(n_c) = .032 \times 10^{-3}$, $var(n_u) = .090 \times 10^{-3}$, $var(n_g) = .064 \times 10^{-3}$, $var(n_j) = .070 \times 10^{-3}$, where the subscript indicates common or country-specific shocks, and $h_1 = 1.150$, and $h_2 = .916$.

If this representation is used, then about 26% in the U.S. and Japan, and about 40% in Germany of the noise variances are due to common shocks. It must be admitted, however, that this representation is rather special since ψ is constrained to be the identity matrix.

5. Concluding Discussion

If the third class of model specifications is adopted *a priori*, this amounts to assuming that the coefficient ϕ_0 in the original ARMA model is not of the full rank. We can see this because the assumption implies that there are only two linear relations among the components of y's to eliminate the term depending on the common shocks,

$$\alpha' y = \alpha' n_s; \quad \text{and} \quad \beta' y = \beta' n_s$$

where the vectors α and β are independent, and are orthogonal to the vector h. There is no third linearly independent relation like this. In other words, there is no full rank matrix such that

$$\phi_0 y_t = \theta_0 h_t + \ldots$$

for some coefficient matrices with ϕ_0 nonsingular as we have assumed at the beginning of Section 4.

We have examined all possible contemporaneous relations among the data consistent with the estimated innovation covariance matrix and selected two best models to conclude that the U.S. real GNP series contains country-specific shocks not shared by the other two series.

We have already shown that the best two models do not show the existence of any significant common shocks. Let us examine the implication of another model which is the second best model in the first class. It states that the innovations are related to the structural shocks as follows:

$$e_u = n_1 + .30n_3; \; e_g = n_2 + .24n_1 + .35n_3; \; e_j = n_3$$

which is seen to have n_3 in common. However, $.3^2 \times s_3/\Delta_{11}$ is about .066, although $.35^2 \times s_3/\Delta_{22}$ is about .102 where $s_3 = .089 \times 10^{-3}$. Therefore the effects of n_3 on e_u is negligible.

We therefore conclude that the real business cycles in the U.S. contain behavior patterns not shared by the other two business cycles, at least for the period of data examined in this paper.

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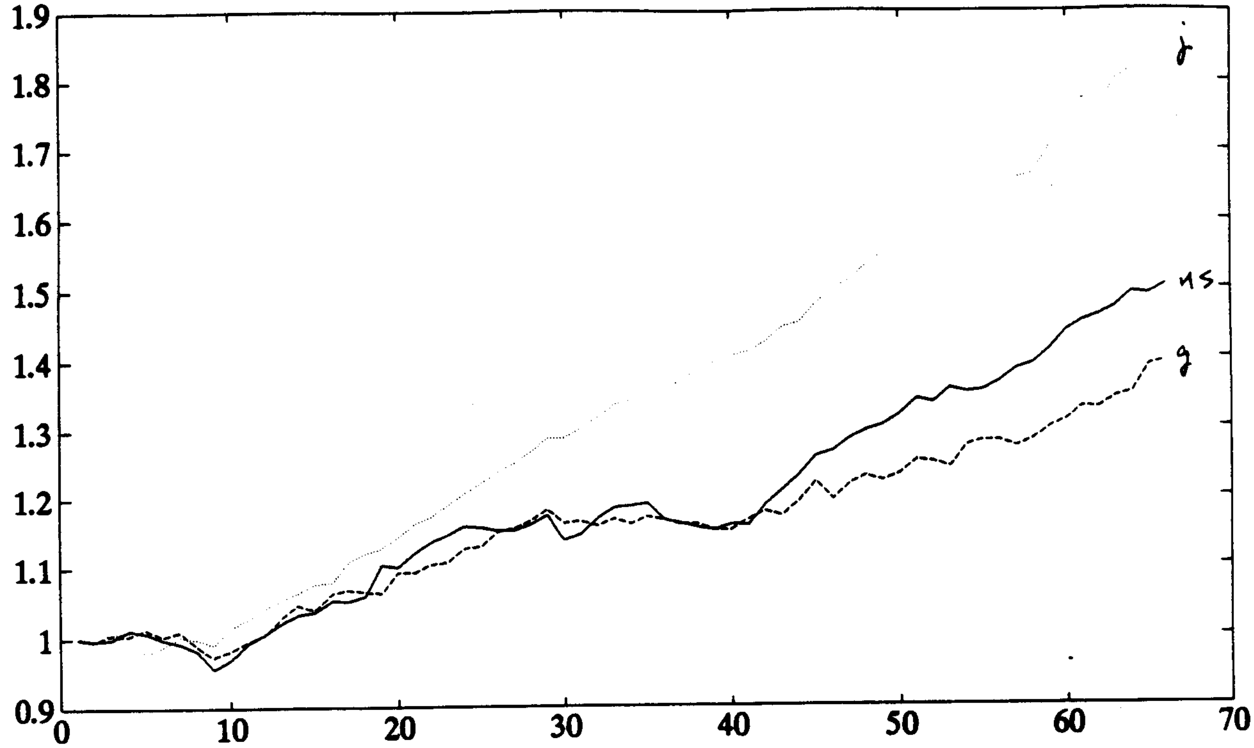
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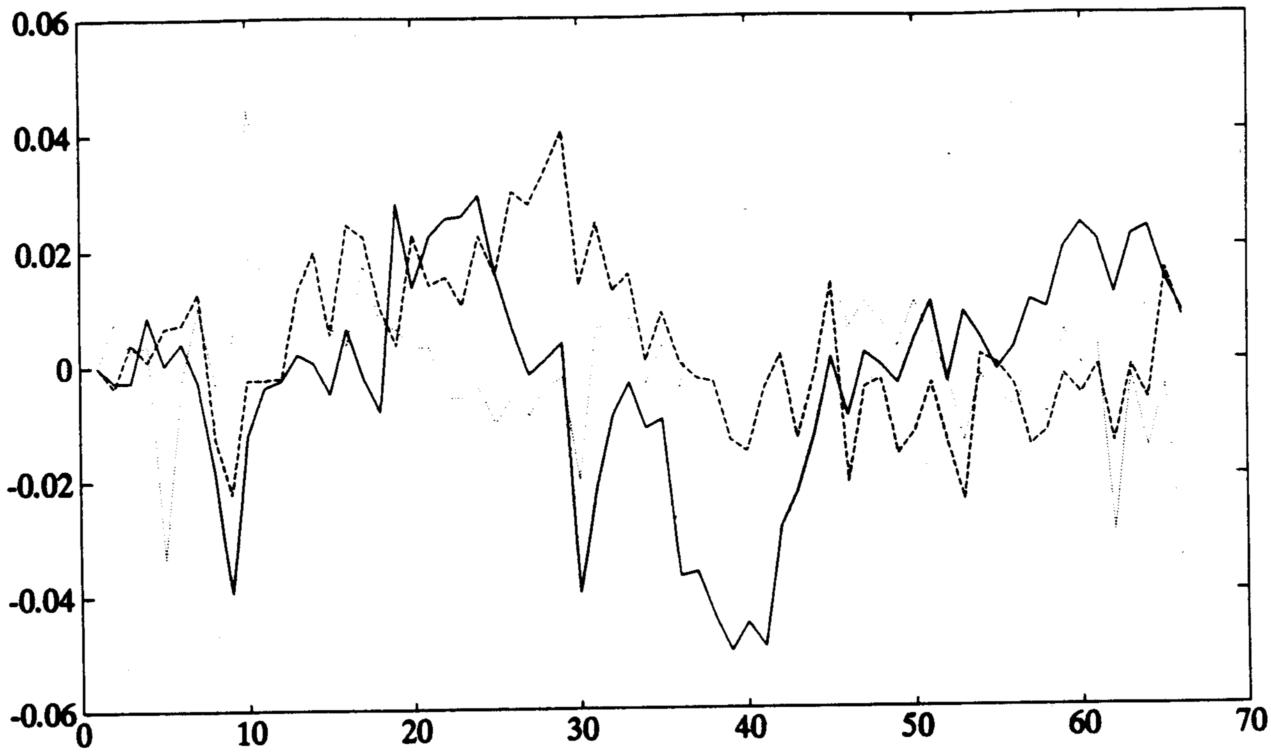

1-3

Result of deflation

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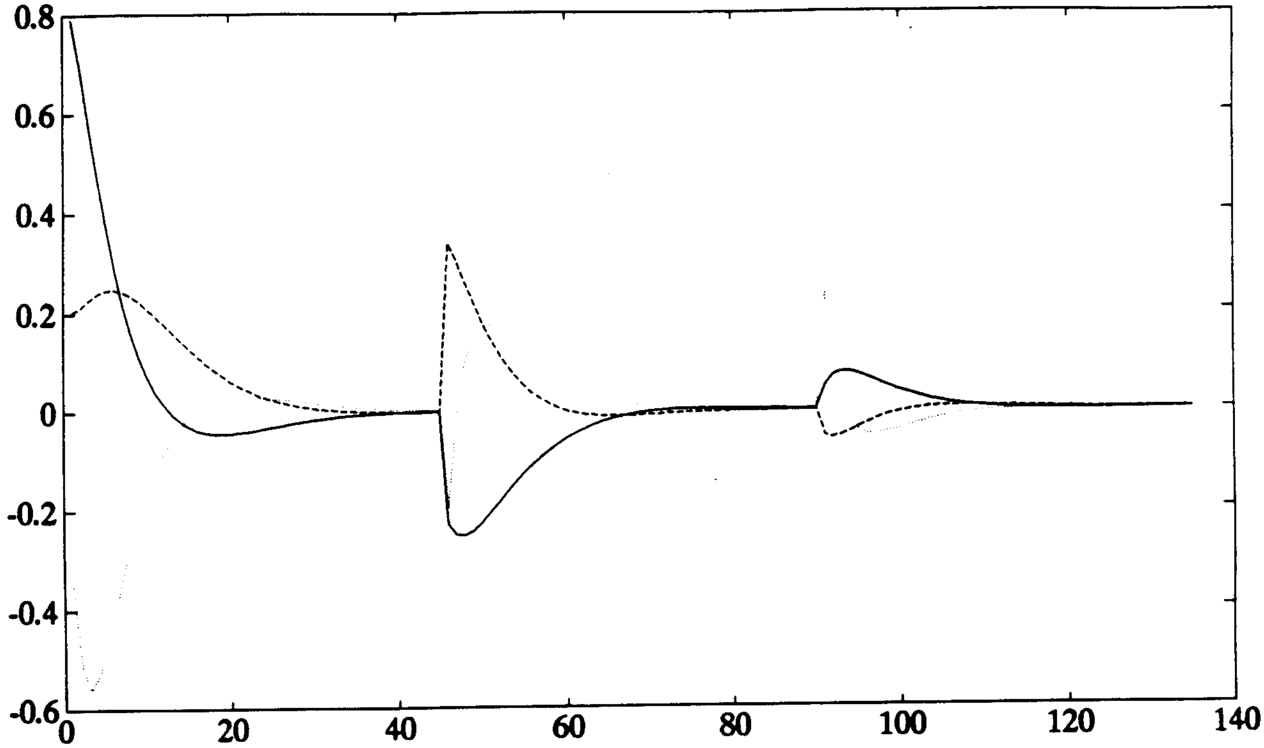
residual



US-shock G-shock Jap-shock expectation revision

innovation in real GNP of
US G J

expectation revision
~~innovation~~
multiplier
$C\hat{A}B$
3 multipliers
plotted
together
$B(:,i)$ $i=1,2,3$