EQUILIBRIUM ASSET PRICING MODELS

AND

PREDICTABILITY OF EXCESS RETURNS

BY

M. HASHEM PESARAN
UNIVERSITY OF CAMBRIDGE AND UCLA

AND

SIMON M. POTTER
UCLA

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M. Hashem Pesaran

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Simon M. Potter

University of California. Los Angeles

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ABSTRACT

This paper considers the types of unobservable stochastic discount factors and economic structures that are compatible with a prediction of negative excess returns. Under fundamental pricing for stocks and assuming the existence of a safe asset, sufficient conditions for the conditional expectations of excess returns to be negative are derived. The paper also provides illustrations of the force of these conditions by investigating the properties of the expected excess return function in the case of a number of representative agent asset pricing models familiar in the literature. The effects of introducing stochastic bubbles or money into the analysis are also analyzed. Finally, using a specification derived from a particular representative agent asset pricing model with minimal restrictions on preferences, we present new evidence on the statistical significance of predictions of negative excess returns for the value weighted CRSP stock index.

1.   Introduction

There is now a wide body of empirical evidence suggesting that excess returns on stocks, defined as the difference between the return on a total stock market portfolio and a safe rate of return, are predictable.[1] There is also a large literature examining whether stock prices are excessively volatile compared to the volatility of fundamentals.[2] As argued by a number of authors, notably Cochrane (1991), it is always possible to explain findings of excess volatility and predictability of excess returns by a judicious choice of a stochastic discount factor in a general equilibrium framework. Since these general equilibrium stochastic discount factors are inherently unobservable it is difficult to statistically assess the credibility of such explanations.

The line of research initiated by Mehra and Prescott (1985) focusses on explaining <u>unconditional</u> moments of asset market returns and shows that simple representative agent general equilibrium asset pricing models do not provide satisfactory explanations of these moments. This can be seen most clearly in the general approach developed recently by Hansen and Jagannathan (1991). They calculate greatest lower bounds of the volatility of the stochastic discount factors and argue that under a representative agent asset pricing model the curvature of the agent's utility function must be unrealistically extreme to simultaneously match the first and second <u>unconditional</u> moments of excess returns. It is possible to adapt Hansen and Jagannathan's approach to take account of conditioning information and, therefore, relate it more directly to the evidence of predictability in excess return regressions (see Gallant, Hansen and Tauchen (1990)). In this paper, we follow an alternative approach and focus on the <u>conditional</u> moments of the excess return function.[3] This approach is in line with empirical research carried out on predicting excess returns in Breen, Glosten and Jagannathan (1990), Brock, Lakonishok and LeBaron

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[1]See, for example, Fama and French (1988), Breen, Glosten and Jagannathan (1990) and Brock, Lakonishok and LeBaron (1992).

[2]See, for example, the recent surveys by LeRoy (1989) and West (1988).

[3]In Gallant et al. (1990) conditioning information can be used to produce more efficient estimates of <u>unconditional</u> moments, which is different from the approach adopted in this paper which focusses directly on the conditional moments.

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(1992) and Pesaran and Timmermann (1992) (BGJ, BLL and PT respectively hereafter). Further, we are interested in examining the validity of the representative agent asset pricing model for conditional moments when the model is capable of accounting for unconditional moments (i.e., the preference parameters are chosen to fit the unconditional moments).

We provide characterization of the types of unobservable discount factors and economic structures that are compatible with an equilibrium prediction of <u>negative</u> excess returns, and are therefore, able to provide an interpretation of the empirical evidence on predictability of excess returns found in BGJ and BLL and in our own empirical work below. We construct equilibrium explanations of negative expected excess returns by considering a dynamic stochastic general equilibrium asset pricing model with minimal restrictions on preferences, technology and stochastic processes for the endowments.[4] We analyze economies without and with money.

We find that in economies where asset payoffs are denominated in terms of a numeraire consumption good and there exists a safe asset, the probability of observing negative expected excess returns is positive if the stock market portfolio can have the property of acting as a hedge at certain times. We argue, by developing a general expression for expected excess returns in terms of the "<u>ex post</u> rational price" used in the excess volatility literature, that such a hedging property appears unlikely if the only determinant of asset prices are "fundamentals."

In order to relate our results more directly to the previous literature, we proceed to consider some specific examples using representative agent asset pricing models. We consider two polar cases, when the utility function in the Lucas (1978) "tree" model is logarithmic, but the dividend process is unrestricted; and when the utility function is unrestricted but the dividend process is assumed to be conditionally serially independent. In both cases we show that the expected excess return function is always non-negative. We also consider an intermediate case and derive sufficient conditions for the non-negativity of the expected excess return function for constant relative risk aversion (CRRA hereafter)

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[4]Partial equilibrium asset pricing models are incapable of producing negative expected excess returns. One merely has to consider the form of the security market line in the Capital Asset Pricing Model (CAPM) to realize this.

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utility functions and the dividend processes where the growth of dividends are conditionally log-normally distributed.

Of course, such specific asset pricing models assume that all risk is marketed and there is no distinction between aggregate consumption, output or dividends. Therefore, it is possible to imagine cases where the amount of non-marketed risk is large enough, and its correlation with the stock market is such that a hedging property could occur. This is reminiscent of parts of the Roll (1977) critique of empirical testing of the Capital Asset Pricing Model. We examine an alternative mechanism of introducing a hedging element into the holding of the stock market portfolios, namely stochastic bubbles. The requirement that the bubbles allow predictions of negative excess returns restricts their structure severely. In particular, these bubbles have the interesting characteristic of implying large drops in stock market prices that are unrelated to "fundamentals" in order to give the stock market portfolio a potential hedging property.

We next turn to the case where asset payoffs are nominal (i.e., denominated in current money prices) and there is no asset offering a unit of numeraire good next period for sure. In such economies, the possibility of equilibrium predictions of negative nominal excess returns is increased, in the case that inflation can be negatively correlated with the performance of the real economy. Thus, the source of the hedging property is now the relative riskiness of the bond market as compared to the stock market. We also show that the use of excess returns measured in current dollars biases empirical analysis against finding evidence of negative expected excess returns compared to the use of excess returns measured in real terms.

We base our empirical analysis on a functional form for the predicted excess return derived from a representative agent asset pricing model where the logarithm of the (exogenous) dividend process follows a geometric random walk. This assumption on the dividend process is frequently made in the literature. However, we do not place any restrictions on the preferences of the representative agent within the CRRA class. Under this model, the predicted excess return would be a positive linear function of the dividend yield for all choices of the preferences of the representative agent, and hence no other variables

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should prove significant in predicting excess returns. We provide statistical tests of both of these implications. The latter implication is examined simply by testing the significance of additional regressors in the excess return regressions.

Our main attention is focussed on the statistical evidence against non-negative predicted excess returns. Previous empirical research by BGJ, BLL and PT have concentrated on examining the profits earned on switching portfolios that move in and out of the stock market or the bond market depending on the sign of the predicted excess return. We examine the statistical significance of the predictions, (based on the dividend yield and other regressors) themselves rather than the success of the predictions in correctly predicting the sign of the realized excess return. We view our empirical results as complementing this earlier literature. The strength of our approach lies in the fact that it is not dominated by particular outliers in the data. The negative skewness in stock market returns tends to over-reward switching portfolios for moving out of the stock market for a small negative prediction, followed by a large downward movement (for example, October 1987): this does not, however, apply to our work. Here we utilize recursive estimation techniques and present time series of t-ratios for recursive predictions of the annual, quarterly and monthly excess returns on the CRSP value weighted index. We consider the statistical significance of these predictions rather than merely their signs.

It is found that over the whole sample (1954-1990) in addition to the dividend yield variable, other regressors such as lagged nominal interest rates, and inflation rates are also significant. Next it is established that the evidence for predictions of negative excess returns is weak if one conditions only on the dividend yield, the variable used by Fama and French (1989). However, once one conditions on an interest rate, as in BGJ and PT, there is much stronger evidence in favor of predictions of negative excess returns at certain periods.

The plan of the paper is as follows: Section 2 derives sufficient conditions for a nonnegativity restriction to hold on the conditional expectations of the excess return in a general class of competitive general equilibrium asset pricing models. Section 3 provides some specific examples of restrictions implied by the non-negativity condition. Section 4 proceeds to consider the role of stochastic speculative

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bubbles in producing negative expected excess returns. Section 5 examines the excess return function in the case where asset prices are quoted in nominal values. Section 6 sets out the statistical testing framework for the nonnegativity restriction and presents the empirical evidence. Section 7 concludes the paper.

2. Negative Expected Excess Returns in Dynamic Competitive General Equilibrium Models

In investigating the possibility of predicting negative excess returns in a dynamic general equilibrium model we make the following assumptions:

(i) expectations are rational;

(ii) asset portfolios with identical payoffs have the same price;

(iii) all arbitrage possibilities are exhausted;

(iv) payoffs have bounded second moments.

As shown in Hansen and Richard (1987) and Hansen and Jagannathan (1991) these assumptions allow us to assume the existence of a (strictly positive) stochastic discount factor, γ_t, that in conjunction with the equilibrium expectations of the economy gives prices today to claims to future random payoffs.

It is also useful for the development of our later arguments to formally introduce a probability space and a law of motion on this probability space. Once again we follow the approach of Hansen and Richard. Let (Ω, F, Pr) be a probability space. Let S be measurable, and a measure-preserving transformation mapping of Ω into itself. Let ω be an element of Ω, then ω will be the state at time zero and $S^t(\omega)$ will be the state at time t. That is, S^t is the transformation S applied t times. We assume that ω is not directly or indirectly observable by agents in the economy. Any random variable in the economy is a function of the particular ω drawn from Ω, though we will only explicitly state the functional dependence on ω when it is necessary for avoiding possible confusions. The conditional expectations operator $E_t[\bullet]$ used below is conditioned on the information available to agents at time t when the draw from the sample space is ω.

We will work directly with rates of return and write the ex ante arbitrage condition in the capital

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market as

$$E_t[\gamma_{t+1}(R_{t+1} - r_t)] = 0,$$ (1)

where

$$1 + R_{t+1} = \frac{Q_{t+1} + D_{t+1}}{Q_t} > 0,$$

and

$$1 + r_t = \frac{1}{E_t[\gamma_{t+1}]} > 0,$$

with Q_t being the ex-dividend price of the stock market portfolio at time t, D_t being the aggregate dividend paid at time t and r_t being the one period safe rate of return, that is the return on an asset that pays a unit of the numeraire consumption good next period for sure.

Using the formula for conditional covariance we obtain the following expression for the predicted excess return

$$E_t[R_{t+1} - r_t] = \frac{-1}{E_t[\gamma_{t+1}]} \, Cov_t(\gamma_{t+1}, R_{t+1} - r_t) = \frac{-1}{E_t[\gamma_{t+1}]} \, Cov_t(\gamma_{t+1}, R_{t+1}).$$ (2)

The above relationship shows clearly that equilibrium predictions of negative excess returns require the stock market portfolio to have the potential of performing as a hedge against future bad times, that is, covary positively with the stochastic discount factor. Before examining whether such a hedging property is compatible with "fundamental" pricing, we note some properties of the conditional covariance relationship when the stochastic discount factor can be given the interpretation of a marginal rate of substitution for a consumer.

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If one specializes to the case of a representative agent, single good economy with time separable preferences we have $\gamma_{t+1} = \beta u'(c_{t+1})/u'(c_t)$, and the requirement for equilibrium predictions of negative excess returns is that marginal utility of the numeraire consumption good [i.e., $u'(c_{t+1})$] can covary positively with the realized return on the stock market:

$$E_t(R_{t+1} - r_t) = \frac{-\text{Cov}_t[u'(c_{t+1}), R_{t+1}]}{E_t[u'(c_{t+1})]} . \tag{3}$$

However, without further restrictions on the underlying stochastic processes of the economy non-negativity of predicted excess returns is not necessarily ensured (as it is often presumed in the literature) by requiring a positive conditional covariance between consumption and the return on the stock market.[5] In order for $\text{Cov}_t(R_{t+1}, c_{t+1}) > 0$ to imply that the conditional covariance in equation (3) is negative, one needs stronger requirements on the conditional joint distribution of consumption and stock returns. One possibility would be to assume that conditional on Ω_t, c_{t+1} and R_{t+1} have a bivariate normal distribution. In this case as shown, for example, by Rubinstein (1976, Appendix) we have:[6]

$$\text{Cov}_t[u'(c_{t+1}), R_{t+1}] = E_t[u''(c_{t+1})] \, \text{Cov}_t(c_{t+1}, R_{t+1}),$$

and under the assumption that the utility function is strictly concave establishes the sufficiency of the condition $\text{Cov}_t(c_{t+1}, R_{t+1}) \geq 0$ for the non-negativity of the predicted excess returns. The assumption of joint normality of consumption and returns is not, however, supported by the evidence. There is substantial empirical evidence that the marginal distribution of stock returns display important departures from the normal distribution.

An alternative assumption which delivers the same result is to restrict the conditional joint distribution of (R_{t+1}, c_{t+1}) to belong to the class of Positively Quadrant Dependent (PQD) distributions,

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[5]See, for example, Abel (1992, p. 12).

[6]Also see Stein (1973).

namely that

$$\text{Prob}_t(R_{t+1} \le a, \, c_{t+1} \le b) \ge \text{Prob}_t(R_{t+1} \le a) \, \text{Prob}_t(c_{t+1} \le b),$$

for all a and b, where $\text{prob}_t(\bullet)$ stand, for the probability operator conditional on information at time t. The concepts of positively quadrant dependent and the related concept of negatively quadrant dependence are originally due to Lehmann (1966). Unlike covariances, the property of positively (negatively) quadrant dependence is <u>invariant</u> under non-decreasing (non-increasing) transformations. Hence, the assumption that (R_{t+1}, c_{t+1}) is positively quadrant dependent simultaneously implies that $\text{Cov}_t(R_{t+1}, c_{t+1}) \ge 0$ and $\text{Cov}_t[R_{t+1}, u'(c_{t+1})] \le 0$, with equality holding if and only if R_{t+1} and c_{t+1} are conditionally independent.

Now, consider using the conditional covariance relationship (3) to develop a version of the capital asset pricing model. It is standard to do this by assuming that, conditional on information at time t, the return on the market portfolio is proportional to the marginal utility of consumption:

$$R_{t+1} = -B \, u'(c_{t+1}), \qquad\qquad B > 0.$$

Then if one is examining the excess return on the "market" portfolio it cannot be predicted to be negative. Thus, it is clear that one possible explanation for prediction of negative excess returns is that the observed market portfolio is not equivalent to the total portfolio that the consumer makes decisions over. For example, if human capital is not marketed and its return varies with the marginal utility of consumption negatively, then it would be possible for there to exist periods when the consumer holds the stock market portfolio as a hedge against bad times.

Although the conditional covariance equality is used a great deal in the literature discussing the properties of asset returns, it does not impose the condition of fundamental pricing: namely that stock prices represent claims to future dividend streams only. In order to impose this condition and develop

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its implications for predictions of negative excess returns we use a concept from the excess volatility literature, the "ex post rational price."

Assume that there exists an asset that pays out dividends equal to a weighted sum of dividends from the N basic assets. Call this asset the "market portfolio" and let its "fundamental" price be Q_t^F:

$$Q_t^F = E_t \left\{ \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \gamma_{t+j}(\omega) \right) D_{t+i}(\omega) \right\} , \tag{4}$$

where $D_s = \sum_{k=1}^{N} w_{sk} D_{ks}$, $\forall s$, and w_{sk} represents the weight of the kth asset in the market portfolio.[7] Define $Q_t^*(\omega)$ as the ex post rational price:

$$Q_t^*(\omega) = \sum_{i=1}^{\infty} \left(\prod_{j=1}^{i} \gamma_{t+j}(\omega) \right) D_{t+i}(\omega) . \tag{5}$$

The dependence of $Q_t^*(\cdot)$ on ω emphasizes the particular realization of the dividends and the discount factors used to calculate the ex post rational price, which is not revealed when the stock price is realized.[8] The "fundamental" excess return is defined by

$$R_{t+1}^F - r_t = \frac{E_{t+1}[Q_{t+1}^*(\omega)] + D_{t+1}}{Q_t^F} - \frac{1}{E_t[\gamma_{t+1}(\omega)]} . \tag{6}$$

However, $Q_t^*(\omega)$ in (5) satisfies the forward recursion

$$Q_t^*(\omega) = \gamma_{t+1}(\omega)\{Q_{t+1}^*(\omega) + D_{t+1}(\omega)\}, \quad t = 1,2,... \tag{7}$$

Thus, substituting out the dividend and stock price at time (t+1) and taking conditional expectations with

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[7] It is assumed that the conditional expectations in (4) are well defined. Given our concentration on excess returns this is not a crucial assumption. If necessary one could work with price dividend ratios without affecting the later arguments.

[8] Notice that $E_t[Q_t^*(\omega)] = Q_t^F$.

respect to the information available at time t, yields

$$E_t(R^F_{t+1} - r_t) = \frac{1}{Q^F_t} \left\{ E_t\left(\frac{Q^*_t(\omega)}{\gamma_{t+1}(\omega)}\right) - \frac{E_t[Q^*_t(\omega)]}{E_t[\gamma_{t+1}(\omega)]} \right\}. \tag{8}$$

Hence, in order to check for sufficient conditions under which the expected excess return is non-negative one can examine the conditions under which

$$E_t\left(\frac{Q^*_t(\omega)}{\gamma_{t+1}(\omega)}\right) \geq \frac{E_t[Q^*_t(\omega)]}{E_t[\gamma_{t+1}(\omega)]}, \qquad \text{for all } t. \tag{9}$$

This form is suggestive of a Jensen's Inequality argument that is used to prove the following Theorem:

Theorem 1: If assumptions (i) - (iv) above hold, and the price of the market portfolio is given by (4), a sufficient condition for expected excess return function (1) to be non-negative is given by

$$\frac{Q^*_t(\omega_1)}{\gamma_{t+1}(\omega_1)} \{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\} \geq \frac{Q^*_t(\omega_2)}{\gamma_{t+1}(\omega_2)} \{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\},$$

for all t and for all possible states of the world (ω_1, ω_2).

Proof: By Jensen's Inequality, the expected excess return in (8) will be nonnegative if $Q^*_t(\omega)/\gamma_{t+1}(\omega)$ is convex in $[Q^*_t(\omega), \gamma_{t+1}(\omega)]$. The ratio $Q^*_t(\omega)/\gamma_{t+1}(\omega)$ is convex if for some λ in the range $(0,1)$, and for all possible pair of states of the world at time t+1, (ω_1, ω_2), we have

$$\lambda\left(\frac{Q^*_t(\omega_1)}{\gamma_{t+1}(\omega_1)}\right) + (1-\lambda)\left(\frac{Q^*_t(\omega_2)}{\gamma_{t+1}(\omega_2)}\right) \geq \frac{\lambda Q^*_t(\omega_1) + (1-\lambda)Q^*_t(\omega_2)}{\lambda\gamma_{t+1}(\omega_1) + (1-\lambda)\gamma_{t+1}(\omega_2)}$$

Since $\gamma_{t+1}(\omega) > 0$, the above inequality can be written after some manipulations as

$$\frac{Q^*_t(\omega_1)}{\gamma_{t+1}(\omega_1)} \{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\} \geq \frac{Q^*_t(\omega_2)}{\gamma_{t+1}(\omega_2)} \{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\}$$

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The intuition behind condition (10) is as follows: in situations where ω_1 represent a "good" state of the world relative to ω_2, it is less costly in terms of period t consumption to transfer consumption from $t+1$ into other periods and hence $\gamma_{t+1}(\omega_1) < \gamma_{t+1}(\omega_2)$, and using (10) it follows that

$$\frac{Q_t^*(\omega_1)}{\gamma_{t+1}(\omega_1)} \geq \frac{Q_t^*(\omega_2)}{\gamma_{t+1}(\omega_2)}.$$ A similar result also holds if ω_1 represents a "bad" state of the world relative to ω_2. Therefore, if bad times in the future tend to increase γ_{t+1} but at the same time depress the stock market, then expected excess returns always have to be non-negative to compensate agents for bearing risk. However, if stock prices do not necessarily drop when the state of the economy is "bad" then it is possible for agents to hold the stock market portfolio as a hedge against future "bad" times in the equilibrium expectation that its return will be lower than the safe rate.

Under the restrictions that the equity premium (i.e., the <u>unconditional</u> expectation of the excess return) is positive and fundamental pricing holds, then one can rule out cases where the inequality in (9) is reversed for all (ω_1, ω_2). Therefore, predictions of negative excess returns must imply some conditional switching in the relationship between excess returns and the stochastic discount factor. The frequency of the switches will, however, be constrained by the positive sign and size of the unconditional mean of excess returns. On the other hand, the possibility of such switching would increase the volatility of predicted excess returns, thus making the second aspect of the equity premium puzzle, the volatility of the stochastic discount factor emphasized by Hansen and Jagannathan (1991), easier to obtain.

If attention is restricted to economies where there is a strictly monotonically decreasing relationship between aggregate dividends and the discount factor, a sharper result can be obtained.

Corollary: If there exists a strictly monotonically decreasing relationship between D_{t+1} and γ_{t+1} and if the <u>ex post</u> rational price $Q_{t+1}^*(\omega)$ is once continuously differentiable, then a sufficient condition for (9) to hold is given by

$$\frac{\partial R_{t+1}^*(\omega)}{\partial D_{t+1}(\omega)} > 0,$$

where $R^*_{t+1}(\omega) = \dfrac{Q^*_{t+1}(\omega) - Q^F_t + D_{t+1}(\omega)}{Q^F_t}$ is the ex post rational rate of return.

Proof: Here we give a proof for the case where $\gamma_{t+1}(\omega_2) > \gamma_{t+1}(\omega_1)$. A similar proof can be constructed if $\gamma_{t+1}(\omega_2) < \gamma_{t+1}(\omega_1)$.

Using (7) we have

$$\frac{Q^*_t(\omega_1)}{\gamma_{t+1}(\omega_1)} - \frac{Q^*_t(\omega_2)}{\gamma_{t+1}(\omega_2)} = [Q^*_{t+1}(\omega_1) - Q^*_{t+1}(\omega_2)] + [D_{t+1}(\omega_1) - D_{t+1}(\omega_2)]. \tag{11}$$

Also by the mean value theorem applied to the ex post rational price $Q^*_t(\omega)$ we have

$$Q^*_{t+1}(\omega_1) - Q^*_{t+1}(\omega_2) = \left[\frac{\partial Q^*_{t+1}(\bar{\omega})}{\partial D_{t+1}}\right] \{D_{t+1}(\omega_1) - D_{t+1}(\omega_2)\},$$

where $\bar{\omega}$ represents a state "between" ω_1 and ω_2 such that $D_{t+1}(\bar{\omega})$ lies in the interval $[D_{t+1}(\omega_2), D_{t+1}(\omega_1)]$. Using this result in (11) now yields

$$\frac{Q^*_t(\omega_1)}{\gamma_{t+1}(\omega_1)} - \frac{Q^*_t(\omega_2)}{\gamma_{t+1}(\omega_2)} = \left[1 + \frac{\partial Q^*_{t+1}(\bar{\omega})}{\partial D_{t+1}}\right] \{D_{t+1}(\omega_1) - D_{t+1}(\omega_2)\},$$

and hence, in the case where $\gamma_{t+1}(\omega_2) > \gamma_{t+1}(\omega_1)$, the sufficient condition for the expected excess return to be non-negative is given by:[9]

$$\left[1 + \frac{\partial Q^*_{t+1}(\bar{\omega})}{\partial D_{t+1}}\right] \{D_{t+1}(\omega_1) - D_{t+1}(\omega_2)\} \geq 0.$$

However, under the assumption of the corollary and in the case where $\gamma_{t+1}(\omega_2) > \gamma_{t+1}(\omega_1)$, we have

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[9]Note that in the case where $\gamma_{t+1}(\omega_2) > \gamma_{t+1}(\omega_1)$, condition (10) in Theorem 1 implies that

$Q^*_t(\omega_1)/\gamma_{t+1}(\omega_1) \geq Q^*_t(\omega_2)/\gamma_{t+1}(\omega_2).$

$D_{t+1}(\omega_1) \geq D_{t+1}(\omega_2)$, and the above inequality is satisfied so long as $\partial Q^*_{t+1}(\omega)/\partial D_{t+1} + 1 > 0$. Now, using the definition of <u>ex post</u> rational rate of return

$$R^*_{t+1}(\omega) = \frac{Q^*_{t+1}(\omega) - Q^F_t + D_{t+1}(\omega)}{Q^F_t},$$

we also have

$$\frac{\partial R^*_{t+1}(\omega)}{\partial D_{t+1}} = \frac{1}{Q^F_t} \left[\frac{\partial Q^*_{t+1}(\omega)}{\partial D_{t+1}} + 1 \right] > 0. \qquad \blacksquare$$

Simple equilibrium models of asset pricing in the literature tend to restrict consumption to be equal or proportional to aggregate dividends and use marginal rates of substitution of aggregate consumption between periods as the stochastic discount rate. Therefore, all of the models within this class will satisfy the condition on the stochastic discount factor and the aggregate dividend. Hence, if the <u>ex post</u> rational rate of return satisfies the condition in the corollary then such models are not capable of producing predictions of negative excess returns. Note that the sufficient condition of the corollary is a great deal weaker than the condition that the derivative of the <u>ex post</u> rational stock price being positive with respect to the current period dividend.

3. <u>Properties of Expected Excess Return Function in the Lucas "Tree" Model</u>

In this section we focus on applications of Lucas (1978) asset pricing model, popularly known as the "tree" model. We examine the class of utility functions and the dividend processes for which the expected excess return function is non-negative to build intuition for cases where the sufficient condition of Theorem 1 holds.

Consider first the simple case where the utility function is logarithmic, $u(c) = \log(c)$, and suppose that the stochastic process for the dividends (or consumption) is such that the asset pricing function (4) exists. Then standard techniques yield

$$Q_t^F = \beta(1-\beta)^{-1}D_t, \qquad \gamma_{t+1} = \beta D_t/D_{t+1},$$

and hence using (6) we have

$$E_t(R_{t+1}^F - r_t) = E_t(\gamma_{t+1}^{-1}) - \frac{1}{E_t(\gamma_{t+1})}.$$

which by virtue of Jensen's inequality is non-negative for all dividend processes that generate a well-defined price function. Therefore, for logarithmic preferences, irrespective of the stochastic process generating the dividends, the expected excess return is always non-negative under fundamental pricing.

Consider now the polar case where the utility function $(u'(c) > 0,\ u''(c) \leq 0)$ is unrestricted within the Von Neumann-Morgenstern expected utility function class, but the dividends D_{t+i}, conditional on the information at time t, Ω_t, are serially independent. Note that this assumption does not rule out time variations in the conditional means and variances of D_{t+i}. This case is a generalization of the example discussed by Abel (1988) where he establishes the non-negativity of the expected excess return function under constant relative risk-aversion utility functions and log-normality of the conditional distribution of the dividend process.[10]

Starting with the decomposition of the expected excess return function given by (8) we first note that using $\gamma_{t+1} = \beta u'(c_{t+1})/u'(c_t)$ in (5),[11] and recalling that in the Lucas tree model $c_t = D_t$, we have:

$$E_t\left(\frac{Q_t^*}{\gamma_{t+1}}\right) - \sum_{i=1}^{\infty} \beta^{i-1}E_t\left\{\frac{D_{t+i}u'(D_{t+i})}{u'(D_{t+1})}\right\}. \tag{12}$$

Similarly

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[10]Notice that in the derivation of the expected excess return function, Abel (1988) assumes that the conditional mean and the conditional variance of the log dividend process are distributed independently of the dividend process itself. This assumption, for example, rules out the specification of a geometric random walk for the dividend process.

[11]For notational simplicity we have suppressed explicit references to ω, the argument of $Q_t^*(\omega)$ and $\gamma_{t+1}(\omega)$.

$$\frac{E_t(Q_t^*)}{E_t(\gamma_{t+1})} - \sum_{i=1}^{\infty} \beta^{i-1} \left\{ \frac{E_t[D_{t+i}u'(D_{t+i})]}{E_t[u'(D_{t+1})]} \right\}. \tag{13}$$

Hence, using (12) and (13) in (8) yields

$$E_t(R_{t+1}^F - r_t) = \frac{1}{Q_t^F} \sum_{i=1}^{\infty} \beta^{i-1} \left\{ E_t \left[\frac{D_{t+i}u'(D_{t+i})}{u'(D_{t+1})} \right] - \frac{E_t[D_{t+i}u'(D_{t+i})]}{E_t[u'(D_{t+1})]} \right\}. \tag{14}$$

Under the conditional independence of D_{t+1} and D_{t+i} for $i = 2,3,....$, we have

$$E_t \left[\frac{D_{t+i}u'(D_{t+i})}{u'(D_{t+1})} \right] - \frac{E_t[D_{t+i}u'(D_{t+i})]}{E_t[u'(D_{t+1})]} = E_t[D_{t+i}u'(D_{t+i})] \left\{ E_t \left(\frac{1}{u'(D_{t+1})} \right) - \frac{1}{E_t[u'(D_{t+1})]} \right\},$$

which are non-negative by virtue of the Jensen's inequality, and noting that by assumption $D_{t+i}u'(D_{t+i}) > 0$. Consider now the first term in (14). For $i = 1$ we have

$$E_t(D_{t+1}) - \frac{E_t[D_{t+1}u'(D_{t+1})]}{E_t[u'(D_{t+1})]} = \frac{-Cov_t[D_{t+1},u'(D_{t+1})]}{E_t[u'(D_{t+1})]}.$$

However, since $u'(D_{t+1})$ is a monotonically declining function of D_{t+1}, we have $Cov_t(D_{t+1}, u'(D_{t+1})) < 0$, and the first term of (14) is also non-negative.[12] Thus it is established that for all strictly concave utility functions the expected excess return function will be non-negative so long as the dividend process is conditionally independent over time.

Despite their relative generality, neither of the two polar cases discussed above are particularly realistic. An alternative approach would be to confine the analysis to the class of constant relative risk

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[12]A simple proof of $Cov(x,u'(x)) < 0$ can be obtained using the property of Positively Quadrant Dependent (PQD) distributions discussed in Section 2. Let y have the same distribution as x and note that

$$Prob(x \leq a, y \leq b) = Max\{Prob(x \leq a), Prob(y \leq b)\} \geq Prob(x \leq a), Prob(y \leq b)$$

which establishes that (x,y) or (x,x) is PQD. Since $u'(x)$ is a non-increasing function then $(x, -u'(x))$ is also PQD. Therefore, using Lemma 3 in Lehmann (1966) it follows that $Cov(x, -u'(x)) > 0$, or $Cov(x,u'(x)) < 0$.

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aversion utility functions

$$u(c) = \frac{c^{1-\alpha} - 1}{1-\alpha}, \qquad \alpha > 0. \tag{15}$$

where α is the relative risk-aversion coefficient, but consider a wider class of dividend processes that allow for the serial dependence in the $\{D_t\}$ process that is observed in time series data. With this in mind, let $y_{t+1} = \Delta\log D_{t+1}$ and suppose that

$$y_{t+1} \mid \Omega_t \sim N(\mu_t, \sigma_t^2), \tag{16}$$

where μ_t and σ_t^2 could be time varying, but are assumed to be distributed independently of the level of the D_t process. This specification contains the geometric random walk model, which is popular in the finance literature, as a special case and also some recent models with Markov chains describing the movement of the conditional mean and variance (see Abel (1992) for a discussion of this literature). Under (15) and (16), the expected excess return function in (14) becomes

$$E_t(R_{t+1}^F - r_t) = \left(\frac{D_t}{Q_t^F}\right) \sum_{i=1}^{\infty} \beta^{i-1}[X_{t+1} - Y_{t+1}] \, Exp\{E_t(A_{t+i})\}$$

where

$$X_{t+i} = Exp\left\{\frac{1}{2} V_t[\alpha y_{t+1} + (1-\alpha)y_{t+1}^{t+i}]\right\},$$

$$Y_{t+i} = Exp\left\{\frac{1}{2} V_t[(1-\alpha)y_{t+1}^{t+i}] - \frac{1}{2} V_t[\alpha y_{t+1}]\right\},$$

$$A_{t+i} = \mu_{t+1} + (1-\alpha)(\mu_{t+2} + \mu_{t+3} + \dots + \mu_{t+i}),$$

$$y_{t+1}^{t+i} = y_{t+1} + y_{t+2} + \dots + y_{t+i}.$$

Hence, the non-negativity of the expected excess return function is ensured by the following sufficient conditions on the growth rates of the dividend process:

$$V_t(y_{t+1}) + (1-\alpha) \sum_{j=2}^{i} \text{Cov}_t(y_{t+1}, y_{t+j}) \geq 0, \qquad \text{for } i = 2,3,\dots$$

It is clear that under $\alpha = 1$ (the logarithmic utility case), as to be expected the above conditions are satisfied. Another polar case where these conditions are satisfied is when $\alpha \neq 1$, but $y_{t+1} = \Delta\log D_{t+1}$, are serially uncorrelated. Thus, we can see that we need a combination of high risk aversion for the representative consumer and high positive autocorrelation in dividend growth rates for the sufficient condition to fail. However, using the techniques of Tauchen and Hussey (1991) we found that the positive autocorrelation in dividend growth rates produced a prediction of excess returns that was negative for all states of the world (see also Tauchen and Hussey's own experiments with positively autocorrelated growth).

In the case where $\{D_t\}$ follows a geometric random walk with a drift, the above results can be used to derive an exact expression for the excess return function in terms of the dividend yield variable, D_t/Q_t^F. Under the geometric random-walk dividend model, we have[13]

$$E_t(R_{t+1}^F - r_t) = \phi(D_t/Q_t^F), \tag{17}$$

where

$$\phi = \frac{\lambda e^{\mu\alpha - \frac{1}{2}\sigma^2\alpha^2}(e^{\alpha\sigma^2} - 1)}{1 - \lambda\beta}, \quad \text{and} \quad \lambda = e^{\mu(1-\alpha) + \frac{1}{2}(1-\alpha)\sigma^2}.$$

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[13] Notice that in this case $\mu_t = \mu$, $\sigma_t = \sigma$, and y_t is serially independent.

Note that for the price function (4) to exist it is necessary that $\lambda\beta < 1$. Hence, in the case of this example the expected excess return function is a non-negative function of the dividend yield and a switching portfolio using predictions from excess return regressions based on the dividend yield will fail to outperform the market portfolio.

4. Bubbles and Negative Expected Excess Returns

In this section we consider the effect of the introduction of a "non-fundamental" or a "bubble" component into the equilibrium pricing function. Suppose that the market price is composed of two components:

(1)   The fundamental given by the expected discounted future stream of dividends from equation (4).

(2)   An additive bubble component, B_t, that is "rational" in the sense that the market price continues to satisfy the intertemporal asset pricing condition.

Let

$$Q_t = B_t + Q_t^F. \tag{18}$$

Under (2) above we have

$$Q_t = E_t[\gamma_{t+1}(Q_{t+1} + D_{t+1})],$$

or

$$B_t + Q_t^F = E_t[\gamma_{t+1}(Q_{t+1}^F + B_{t+1} + D_{t+1})],$$

$$= E_t[\gamma_{t+1}B_{t+1}] + E_t[\gamma_{t+1}(Q_{t+1}^F + D_{t+1})].$$

Now noting that Q_t^F is assumed to satisfy (4) we have

$$Q_t^F = E_t[\gamma_{t+1}(Q_{t+1}^F + D_{t+1})],$$

and hence the bubble component also needs to satisfy the condition $B_t = E_t[\gamma_{t+1}B_{t+1}]$. Thus, the observed market excess return becomes

$$R_{t+1} - r_t = \frac{Q_{t+1}^F + B_{t+1} + D_{t+1}}{Q_t^F + B_t} - \frac{1}{E_t[\gamma_{t+1}]}. \tag{19}$$

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After some manipulation one obtains the condition

$$Q_t E_t (R_{t+1} - r_t) = - \left\{ E_t \left[\frac{Q_t^*(\omega)}{\gamma_{t+1}} \right] - \frac{E_t[Q_t^*(\omega)]}{E_t[\gamma_{t+1}]} \right\} + \left\{ E_t[B_{t+1}] - \frac{E_t[B_{t+1}\gamma_{t+1}]}{E_t[\gamma_{t+1}]} \right\} \geq 0 . \qquad (20)$$

for the expected excess return to be non-negative.

The covariance properties of $\{B_{t+1}\}$ and $\{\gamma_{t+1}\}$ can be seen more clearly by rewriting (20) as:

$$E_t[R_{t+1} - r_t] = - \frac{1}{Q_t E_t[\gamma_{t+1}]} \left\{ Cov_t \left(\gamma_{t+1}, \frac{Q_t^*(\omega)}{\gamma_{t+1}} \right) + Cov_t(\gamma_{t+1}, B_{t+1}) \right\}. \qquad (21)$$

Comparing the above results with (8) of section 2, one can see directly the following properties of the predicted expected return function if one allows for a "bubble" or a non-fundamental component in asset pricing.

(1)   If $\{B_{t+1}\}$ is a deterministic sequence then neither the predicted, nor the realized excess returns are affected by the presence of a bubble.

(2)   If $\{B_{t+1}\}$ is conditionally independent of $\{\gamma_{t+1}\}$ then the presence of a bubble in the market price does not affect the predicted excess return.

(3)   If conditionally $\{B_{t+1}\}$ and $\{\gamma_{t+1}\}$ covary negatively and $\{Q_t^F, \gamma_{t+1}\}$ satisfy the sufficient condition of Theorem 1 then predicted excess returns must be non-negative.

(4)   If conditionally $\{B_{t+1}\}$ and $\{\gamma_{t+1}\}$ covary positively and $\{Q_t^F, \gamma_{t+1}\}$ satisfy the sufficient condition of Theorem 1, then it is possible for predicted excess returns to be negative.

Therefore, when the sufficient condition of Theorem 1 is satisfied, a prediction of a negative excess return is consistent with asset market equilibrium only if the bubble covaries positively with the discount factor. In these circumstances the introduction of a bubble allows the market portfolio to act as a hedge in economies whose "fundamental" structure would not support such behavior. Positive covariance between the bubble and the discount factor would tend to reduce volatility as it introduces negative covariance between "good" states of the fundamental economy and the bubble. In order to

illustrate the possibility that predictions of negative excess returns are compatible with economies satisfying the sufficient condition of Theorem 1, but containing non-fundamental components in the stock prices, consider the following construction:

$$B_{t+1} = \gamma_{t+1}^{-1} B_t + m_{t+1},$$ (22)

where $B_t > 0$ and $E_t[\gamma_{t+1} m_{t+1}] = 0$, $E_t[|m_{t+1}|] < \infty$.

Then

$$Cov_t(B_{t+1}, \gamma_{t+1}) = E_t[B_t + \gamma_{t+1} m_{t+1}] - B_t E_t[\gamma_{t+1}^{-1}]E_t[\gamma_{t+1}] - E_t[\gamma_{t+1}] E_t[m_{t+1}].$$

or

$$Cov(B_{t+1}, \gamma_{t+1}) = \{1 - E_t[\gamma_{t+1}]E_t[\gamma_{t+1}^{-1}]\}B_t - E_t[\gamma_{t+1}]E_t[m_{t+1}].$$

By Jensen's inequality $E_t[\gamma_{t+1}^{-1}] \geq \dfrac{1}{E_t[\gamma_{t+1}]}$. Hence to obtain a positive conditional covariance between B_{t+1} and γ_{t+1} one at least requires $E_t[m_{t+1}] < 0$. Note also that the larger B_t is, holding the properties of $\{\gamma_{t+1}\}$ constant, the more negative must be the conditional mean of m_{t+1} to produce a prediction of negative excess returns.

Thus, predictions of negative excess returns in economies whose fundamental structure satisfies the sufficient condition of Theorem 1 are possible when one allows for non-fundamentals to affect asset prices. Furthermore, one would expect to see large falls in stock prices in economies where prices contain positive "bubble" components that are <u>ex ante</u> positively correlated with the discount factor.

5. The Use of Nominal Excess Returns

The construction of the expression for the predicted excess return of Section 2 did not rely formally on all prices being denominated in a numeraire consumption good, however, the interpretation of the results did strongly rely on assuming the existence of an asset paying a unit of the numeraire consumption good next period for sure. In economies with money, excess returns will be denominated in money prices. In order to aid interpretation, notational amend equation (1) to

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$$E_t[\tilde{\gamma}_{t+1}(R_{t+1} - r_t)] = 0, \tag{23}$$

where $\tilde{\gamma}_{t+1} = \gamma_{t+1} \dfrac{P_t}{P_{t+1}},$ and P_t is the money price of the numeraire consumption good. One can proceed with exactly the same analysis as in Section 2 and rewrite condition (11) as

$$\frac{Q_t^*(\omega_1)}{\tilde{\gamma}_{t+1}(\omega_1)} \; (\tilde{\gamma}_{t+1}(\omega_2) - \tilde{\gamma}_{t+1}(\omega_1)) \geq \frac{Q_t^*(\omega_2)}{\tilde{\gamma}_{t+1}(\omega_2)} \; (\tilde{\gamma}_{t+1}(\omega_2) - \tilde{\gamma}_{t+1}(\omega_1)). \tag{24}$$

Under the assumption that $\tilde{\gamma}_{t+1}(\omega_2) > \tilde{\gamma}_{t+1}(\omega_1)$, a sufficient condition for the expected excess return function to be nonnegative is

$$\frac{Q_t^*(\omega_1)}{Q_t^*(\omega_2)} > \frac{\tilde{\gamma}_{t+1}(\omega_1)}{\tilde{\gamma}_{t+1}(\omega_2)},$$

which implies that

$$\frac{P_{t+1}(\omega_2)}{P_{t+1}(\omega_1)} < \left\{\frac{Q_t^*(\omega_1)}{Q_t^*(\omega_2)}\right\}\left\{\frac{\gamma_{t+1}(\omega_2)}{\gamma_{t+1}(\omega_1)}\right\}.$$

Hence, for example, $Q_t^*(\omega_1) > Q_t^*(\omega_2)$ under the assumption $\tilde{\gamma}_{t+1}(\omega_2) > \tilde{\gamma}_{t+1}(\omega_1)$ would ensure positive predictions for nominal excess returns.

The main difference from previous expressions is that we now have an extra source of variation, the price level, that can have different effects on the bond market compared to the stock market. In other words, the bond market does not offer a method of ensuring a real unit of consumption good in the next period. Inequality (24) is likely to fail if bad times at time $(t+1)$ are associated with a high price level at time $(t+1)$. Intuitively we can think of the differing possibilities for the price level as representing a potential tax that might be placed on holders of bonds next period. The stock market could become a hedge if the tax is only implemented in bad times making bonds a relatively poor form of avoiding risk. Note that if bad times are associated with a much higher price level then the assumption $\tilde{\gamma}_{t+1}(\omega_2) >$

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$\overline{\gamma}_{t+1}(\omega_1)$ will no longer be valid. In such circumstances predictions of excess returns could still be positive even if

$$\frac{Q_t^*(\omega_1)}{\overline{\gamma}_{t+1}(\omega_1)} > \frac{Q_t^*(\omega_2)}{\overline{\gamma}_{t+1}(\omega_2)},$$

since the higher price level in state ω_2 could produce

$$\frac{Q_t^*(\omega_1)}{\overline{\gamma}_{t+1}(\omega_1)} < \frac{Q_t^*(\omega_2)}{\overline{\gamma}_{t+1}(\omega_2)}.$$

In the case where real excess returns are considered, $(R_{t+1}^F - r_t)P_t/P_{t+1}$, the expression for predicted excess returns would become

$$E_t\left[(R_{t+1}^F - r_t)\frac{P_t}{P_{t+1}}\right] - \frac{1}{Q_t^F}\left\{ E_t\left[\frac{Q_t^*(\omega)\frac{P_t}{P_{t+1}}}{\overline{\gamma}_{t+1}}\right] - \frac{E_t[Q_t^*(\omega)]E_t\left[\frac{P_t}{P_{t+1}}\right]}{E_t[\overline{\gamma}_{t+1}]}\right\}.$$

A sufficient condition for this expression to be positive is (using a similar Jensen's Inequality Approach to Section 2)

$$\frac{Q_t^*(\omega_1)}{\overline{\gamma}_{t+1}(\omega_1)}\{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\} \geq \frac{Q_t^*(\omega_2)}{\overline{\gamma}_{t+1}(\omega_2)}\{\gamma_{t+1}(\omega_2) - \gamma_{t+1}(\omega_1)\}.$$

Thus, in this case the sufficient condition is less likely to hold than the condition (24) obtained for nominal returns.

Further, in economic environments where inflation can be negatively correlated with the performance of the real economy, the equity premium calculated by adjusting nominal returns for inflation will have a downward bias, that is it will underestimate the true premium on equities. Thus, if the periods of conditional negative correlation between inflation and the performance of the economy are important it will tend to exacerbate the "equity premium puzzle" of Mehra and Prescott (see Labadie

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(1989)).

6. Negative Expected Excess Returns: Some Empirical Evidence

There are two main approaches one could take to the empirical analysis of the expected excess return function. If one knew both the correct form of the asset pricing model (e.g., representative agent with constant relative risk aversion preferences) and the information used to predict dividends then one could solve the model either analytically or numerically for an expression relating the expected excess return to observables. An example of this approach is given in Section 3, and involves explicit modelling of the conditional mean and the conditional variance of the growth of the dividends in terms of the observable.[14] One possibility would be to characterize the joint distribution of the dividend growth and the other related variables by means of a vector autoregression. The properties of the expected excess return function associated with this structural model can then be compared to unrestricted predictions obtained, for example, by linear or nonlinear functions of the predetermined variables. An alternative approach would be to approximate the expected excess returns by linear or nonlinear functions of the predetermined variables, and then test the non-negativity property of the expected excess returns. In this section we consider simple examples of both of these two approaches. In later work we intend to examine the structural approach in more detail using the VAR techniques.

A simple example of the structural approach is given by the representative agent asset pricing model with dividends following a geometric random walk and preferences in the CRRA class discussed in Section 3. Recall that in the case of this model expected excess returns are linear functions of the dividend yield alone, and do not depend on any other predetermined variables. This leads to the following regression equation:

$$R_{t+1} - r_t = y_{t+1} = \theta' x_t + u_{t+1}, \qquad t = 0,1,2,.....n \qquad (25)$$

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[14]See equation 14 and the subsequent discussions.

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where θ is 2×1 vector, $x_t = (1, D_t/Q_t)'$ and $E_t[u_{t+1}] = 0$. In this simple case the empirical adequacy of the underlying asset pricing model can be evaluated by testing for the significance of additional predetermined regressors in (25).

The reduced form approach does not require an explicit specification of the asset pricing model, but assumes that adequate approximations of the excess return function can be achieved using linear regression techniques. There is now a vast literature documenting the extent to which this can be done. However, there is little agreement on the set of variables that should be used as predictors of excess returns, or the importance of the linearity assumption. Our aim is not to resolve the question of the best set of predictors or make an argument for the use of linear predictors, but rather just to document the behavior of some simple predictors of excess returns of the CRSP value weighted index over the post-Second World War period. What is different about our analysis is the emphasis placed on the negative predictions and their individual statistical significance.

The previous studies by BGJ, BLL and PT that have concentrated on the negative predictions use a switching portfolio to evaluate their economic importance. Our theoretical analysis has something to say about the performance of such a switching portfolio if one of our sufficient conditions for non-negativity holds. However, switching portfolios are not an efficient method of assessing the statistical significance of predictions of negative excess returns. For example, if excess returns have negative skewness (as they do) then a switching portfolio might show large profits from moving out of the stock market based on only a weak and statistically insignificant prediction. Thus, our approach is to recursively generate predictions of excess returns and assess their statistical significance by means of simple t-tests.

Suppose the excess return function can be approximated by a particular linear combination of predetermined variables:

$$R_{t+1} - r_t = y_{t+1} = \theta' x_t + u_{t+1}, \qquad t = 0,1,2,\ldots,n \qquad (26)$$

where θ and x_t are $k\times 1$ vectors and $E[u_{t+1}x_t] = 0$. Define w_t to be the linear predictor of y_{t+1} based on x_t. If we recursively estimate (26) we have for each $t > k$:

$$\hat{w}_t = \hat{\theta}'_t x_t, \tag{27}$$

where $\hat{\theta}_t = \left[\sum_{s=0}^{t-1} x_s x'_s\right]^{-1} \sum_{s=0}^{t-1} x_s y_{s+1}.$

The construction of the standard error of the forecast will depend on whether the error term in (26) is conditionally heteroscedastic. Since we do not find evidence of conditional heteroscedasticity in the full sample estimates we compute the standard errors of the forecasts for the homoscedastic case:

$$\hat{SE}_t(\hat{w}_t) = \{\hat{\sigma}^2_t x'_t H^{-1}_{t-1} x_t\}^{0.5} \tag{28}$$

where $\hat{\sigma}^2_t = (t-k)^{-1} \sum_{s=0}^{t-1} \hat{e}^2_s,$ $H_{t-1} = \sum_{s=0}^{t-1} x_s x'_s,$ $\hat{e}_s = y_{s+1} - \hat{\theta}'_s x_s.$ The recursive t-ratios are then calculated as $\hat{w}_t / \hat{SE}_t(\hat{w}_t)$.

The data for the estimation of the excess return regressions are taken from PT (1992) which should be consulted for further details. We present results for annual, quarterly and monthly observations from the start of 1954 to the end of 1990. The excess returns are calculated from the CRSP value weighted index and the safe one period nominal interest rate for the relevant holding period. We carry out our analysis for all the three data frequencies in order to ascertain the robustness of our results to the choice of the holding period. The previous analyses of BGJ and BLL have concentrated on the monthly and daily data,respectively. We are wary of results based on holding periods shorter than a month because of the presence of transactions costs which we have abstracted away from. PT find that high transactions costs prevent switching portfolios from being profitable compared to the market portfolio at the monthly frequency. The very high frequencies, however, do have the advantage of reducing inflation risk and thus allow us to assess the effect of inflation in negative predictions by comparing different frequencies

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We consider two basic sets of pre-determined regressors in addition to the dividend yield:[15]

Set (1): The dividend yield at same t, the interest rate variable used in the computation of the excess

return, the change in the shortest period interest rate available at time t, and the change in an

annual average of industrial production at time t-1.

Set (2): The regressors in (1) plus the change in a long run inflation rate at time t-1.

The choice of regressors is motivated by a number of factors. Our theoretical development

suggests that the dividend yield alone is unlikely to produce predictions of negative excess returns but

is a potential predictor of excess returns. There is a well-documented negative correlation between

interest rates and excess returns. Therefore, we wish to see whether the correlation is strong enough to

produce statistically significant negative predictions. There are a large number of possible interest rates

one could choose, so to keep things simple we chose the interest rate used in the excess return definition

and the change in the shortest period interest rate available at time t, when the forecast of the excess

return for time t+1 is made. The latter is designed to capture changes in the real economy and monetary

movements just before the start of the holding period. The change in industrial production is frequently

included in excess returns regressions to capture aspects of business cycle fluctuations not contained in

the dividend yield variable (Balvers et al. (1990)). Finally, we include a measure of "long run" inflation

to see if negative predictions are more likely during periods of stagflation when one might believe that

a worsening of the real economic situation could be associated with higher inflation. The second lags of

these latter two variables were included to allow for the fact that at the monthly and quarterly frequencies,

observations on these variables are announced with a lag, so that they are not in the public information

set at time t.

The full sample estimates of the regression coefficients are summarized in Tables 1, 2 and 3 for

the monthly, quarterly and annual frequencies, respectively. In all the regressions, the dividend yield has

a positive and statistically significant coefficient. The other variables that are statistically significant

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[15]Exact descriptions of the variables are given at the foot of Tables 1, 2 and 3 where the full sample
regression results are summarized.

across the equations are the level of the interest rates at the monthly and the quarterly frequencies. Apart from the dividend yield most of the other variables enter the excess return equation with a negative coefficient.[16] Overall, it appears that even without restrictions on preferences the geometric random walk model (for the dividend process) is not supported by the data.

We now turn to the assessment of whether the recursively generated forecasts are significantly negative. Figures 1, 2 and 3 present recursive t-ratios for the monthly, quarterly, and the annual predictions, respectively. We initialize the recursive estimates with the observations up to 1960 (with the exception of regression (ii) for the annual data which requires observations up to 1961). One can see by examining the first graph in each figure that the dividend yield variable alone, although statistically significant for the whole sample, is not capable of producing a statistically significant negative prediction. Adding the extra regressors of set (i) above does produce some evidence of statistically significant negative predictions. Further, in the majority of cases statistically significant evidence of negative as well as positive predictions are generally found during the volatile periods in the 1970s and 1980s.

The null hypothesis for the prediction of excess returns to be positive in a particular period, say t, would be, H_0: $w_t \geq 0$. Of course, if one wanted to test for non-negativity of the function used to approximate the conditional expectation of excess returns, then the null hypothesis of interest would become, H_0: $w_t \geq 0$ for all t, and the alternative H_1: $w_t < 0$, for some t. If the latter null hypothesis was true we would expect to find violations if we checked each prediction individually at the rate of the size of the test. One can bound from below the size of such a test that examines all of the predictions by using a Bonferroni type procedure. A more appropriate technique would be to compare the constrained and the unconstrained estimates of the expected excess return function. In this paper we present a series of t-ratios for the recursive predictions and quote a Bonferroni Bound for the test of H_0: $w_t \geq 0$ against H_1: $w_t < 0$, for some t.

A simple application of the Bonferroni Bound procedure works as follows: Let $P_1,...,P_M$ be the

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[16]Notice that the diagnostic tests for conditional heteroscedasticity support our use of the standard error of the forecast constructed under the assumption of an homoscedastic error.

p-values corresponding to the M test statistics for each w_t, $t = 1,2,...,m$, the recursive predictions centered at zero. Let $P_{(1)},...,P_{(M)}$ be the ordered p-values. One would reject H_0 at the α level if $P_{(1)} \leq \alpha/M$, therefore, $\alpha = M P_{(1)}$ (see Savin (1980) for more details). It is clear that the procedure can be very conservative if there are many small p-values and ignores information in the joint distribution of the predictions.

The (one-sided) Bonferroni Bounds at the 5 percent significance level are -3.64, -3.35 and -2.95 for the monthly, quarterly and annual frequencies respectively. Only the monthly recursive predictions based on regression equation (1) violate the Bonferroni bound. The periods that are associated with this finding are from the end of the 1970s to the beginning of the 1980s. These results are suggestive of the explanation advanced in Section 5. The switch in monetary regime in the United States during this period might have been associated with a belief that worsening states of the economy would be associated with higher levels of inflation.

7. Conclusions

In this paper we show that general equilibrium asset pricing models must have a very special structure to generate predictions of excess returns that are negative. By special we mean something different from the recent literature, which has focused on the possibility of matching the unconditional moments of excess returns implied by representative agent general equilibrium asset pricing models with the estimates of these moments obtained from historical observations, without needing to resort to "implausible" values for the preference parameters. Our theoretical results show that for predictions of negative excess returns to be consistent with the predictions of intertemporal general equilibrium asset pricing models it is necessary for agents to hold the stock market portfolio as a hedge, which runs counter to a long tradition of viewing the stock market as a leading indicator of economic activity. It is shown that this conclusion is reasonably robust to the choice of preference parameters and holds even for implausible values of the preference parameters. so long as asset prices reflect fundamentals only. Two possible explanations of the hedging property are advanced in the paper, namely non-fundamental pricing

and the presence of risk in the bond market which is hedged in the stock market. We intend to investigate the empirical importance of these possible explanations more fully in future research.

Under assumptions that are common in the literature of CRRA preferences and a geometric random walk for dividends, the expected excess return is a non-negative linear function of the dividend yield. Furthermore since we did not restrict preferences within the CRRA class such a specification of the expected excess return function is capable of explaining the equity premium puzzle of Mehra and Prescott. In the present paper we are able to generate negative predictions of excess returns using linear regression techniques when the regressors contain predetermined variables in addition to the lagged dividend yield. These predetermined variables are also statistically significant over the whole sample. If we had merely considered unconditional moments of the excess return, such as the equity premium, no evidence against the dividend yield specification of the excess return function would have been found. In using conditional moments of the excess returns not only was evidence found against the particular specification of the asset pricing model, but also against all asset pricing models that restrict the expected excess return to be non-negative.

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TABLE 2

QUARTERLY EXCESS RETURN REGRESSIONS[*,†]

(Sample Period 1954(1)-1990(4))

(i) $\widehat{EVW}_{t+1} = -0.077 + 9.800\ YVW_t$
         $(0.032)\quad (3.232)$

$$R^2 = 0.059,\quad \bar{R}^2 = 0.053,\quad \hat{\sigma} = 0.082,\quad DW = 1.71$$

$\chi^2_{SC}(4) = 4.23,$ $\chi^2_{FF}(1) = 0.01,$ $\chi^2_N(2) = 22.54,$ $\chi^2_H(1) = 0.01$
         $[0.376]$ $[0.922]$ $[0.000]$ $[0.936]$

(ii) $\widehat{EVW}_{t+1} = -0.091 + 18.3\ YVW_t - 0.011\ I3_t + 0.001\ \Delta I1_t - 0.179\ \Delta_4 IP_{t-1}$
         $(0.03)\quad (3.597)\qquad (0.002)\qquad (0.010)\qquad (0.125)$

$$R^2 = 0.189,\quad \bar{R}^2 = 0.166,\quad \hat{\sigma} = 0.075,\quad DW = 1.84$$

$\chi^2_{SC}(4) = 2.763,$ $\chi^2_{FF}(1) = 0.374,$ $\chi^2_N(2) = 26.44,$ $\chi^2_H(1) = 1.995$
         $[0.598]$ $[0.541]$ $[0.000]$ $[0.158]$

(iii) $\widehat{EVW}_{t+1} = -0.105 + 20.19\ YVW_t - 0.0085\ I3_t - 0.002\ \Delta I1_t - 0.262\ \Delta_4 IP_{t-1} - 0.389\ \Delta_4 P_{t-1}$
         $(0.032)\quad (3.758)\qquad (0.003)\qquad (0.009)\qquad (0.134)\qquad (0.240)$

$$R^2 = 0.204,\quad \bar{R}^2 = 0.176,\quad \hat{\sigma} = 0.075,\quad DW = 1.83$$

$\chi^2_{SC}(4) = 3.18,$ $\chi^2_{FF}(1) = 1.10,$ $\chi^2_N(2) = 23.03,$ $\chi^2_H(1) = 2.17$
         $[0.529]$ $[0.295]$ $[0.000]$ $[0.141]$

[*] EVW_{t+1} is the quarterly value CRSP portfolio excess return,
        YVW_t is the quarterly dividend yield variable,
        $I3_t$ is the three month T-bill rate measured at the end of the quarter,
        $\Delta I1_t$ is the change in the one month T-bill rate from the middle of the quarter to the end of the quarter,
        Δ_4 is the fourth difference operator,
        IP_t is the logarithm of the 12 month average of industrial production,
        P_t is the logarithm of the 12 month average of the producer price index.

[†]   See Table 1.

TABLE 3

ANNUAL EXCESS RETURN REGRESSIONS[*],[†]

(Sample Period 1954-1990)

(i) $\widehat{EVW}_{t+1} = -0.210 + 7.081 \ YVW_t$
$$(0.108) \quad (2.763)$$

$$R^2 = 0.158, \quad \bar{R}^2 = 0.134, \quad \hat{\sigma} = 0.135, \quad DW = 2.09$$

$$\chi^2_{SC}(1) = 0.173, \qquad \chi^2_{FF}(1) = 0.046, \qquad \chi^2_{N}(2) = 0.477, \qquad \chi^2_{H}(1) = 0.477$$
$$[0.678] \qquad\qquad [0.830] \qquad\qquad [0.788] \qquad\qquad [0.491]$$

(ii) $\widehat{EVW}_{t+1} = -0.227 + 10.21 \ YVW_t - 0.015 \ I12_t - 0.030 \ \Delta I1_t - 0.176 \ \Delta IP_{t-1}$
$$(0.107) \quad (2.781) \qquad (0.010) \qquad (0.037) \qquad (0.392)$$

$$R^2 = 0.348, \quad \bar{R}^2 = 0.266, \quad \hat{\sigma} = 0.125, \quad DW = 2.25$$

$$\chi^2_{SC}(1) = 2.714, \qquad \chi^2_{FF}(1) = 0.151, \qquad \chi^2_{N}(2) = 2.342, \qquad \chi^2_{H}(1) = 1.793$$
$$[0.099] \qquad\qquad [0.698] \qquad\qquad [0.310] \qquad\qquad [0.181]$$

(iii) $\widehat{EVW}_{t+1} = -0.266 + 10.94 \ YVW_t - 0.007 \ I12_t - 0.046 \ \Delta I1_t - 0.332 \ \Delta IP_{t-1} - 0.881 \ \Delta P_{t-1}$
$$(0.112) \quad (2.844) \qquad (0.012) \qquad (0.040) \qquad (0.414) \qquad (0.782)$$

$$R^2 = 0.373, \quad \bar{R}^2 = 0.272, \quad \hat{\sigma} = 0.124, \quad DW = 2.42$$

$$\chi^2_{SC}(1) = 6.711, \qquad \chi^2_{FF}(1) = 1.465, \qquad \chi^2_{N}(2) = 2.395, \qquad \chi^2_{H}(1) = 4.351$$
$$[0.010] \qquad\qquad [0.226] \qquad\qquad [0.302] \qquad\qquad [0.037]$$

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[*] EVW_{t+1} is the annual weighted CRPS portfolio excess return,
       YVW_t is the annual dividend yield variable,
       $I12_t$ is the twelve month T-bill rate at the end of January,
       $DI1_t$ is the change in the one month T-bill from December to January,
       Δ is the first difference operator.
       IP_t is the logarithm of industrial production,
       P_t is the logarithm of the producer price index.
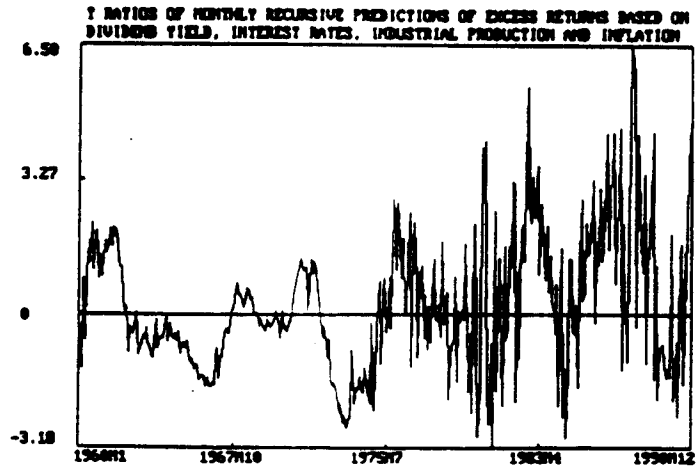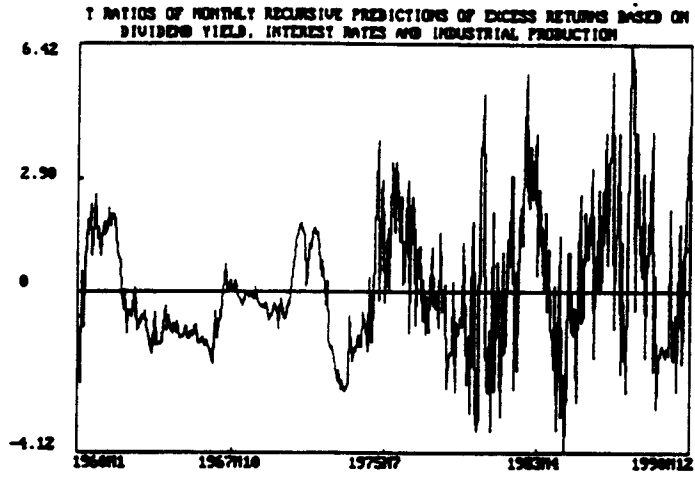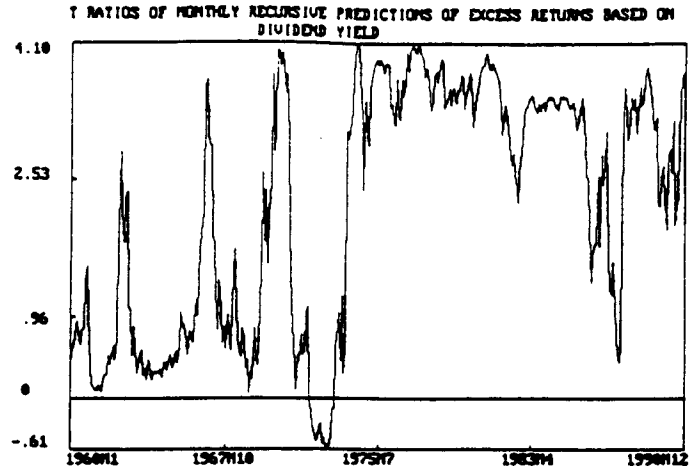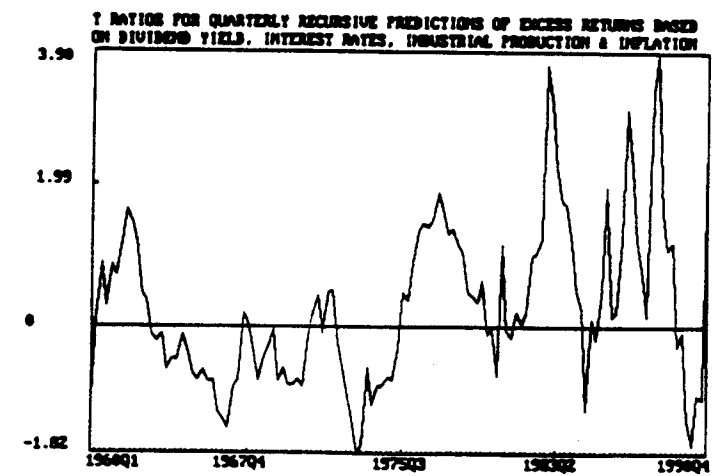
[†]   See Table 1.

FIGURE 1

T RATIOS OF MONTHLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON
DIVIDEND YIELD

4.10

2.53

.96

0

-.61

1960M1   1967M10   1975M7   1983M4   1990M12

T RATIOS OF MONTHLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON
DIVIDEND YIELD, INTEREST RATES AND INDUSTRIAL PRODUCTION

6.42

2.90

0

-4.12

1960M1   1967M10   1975M7   1983M4   1990M12

T RATIOS OF MONTHLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON
DIVIDEND YIELD, INTEREST RATES, INDUSTRIAL PRODUCTION AND INFLATION

6.50

3.27

0

-3.18

1960M1   1967M10   1975M7   1983M4   1990M12

FIGURE 2

T RATIOS FOR QUARTERLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD

T RATIOS FOR QUARTERLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD, INTEREST RATES AND INDUSTRIAL PRODUCTION

T RATIOS FOR QUARTERLY RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD, INTEREST RATES, INDUSTRIAL PRODUCTION & INFLATION
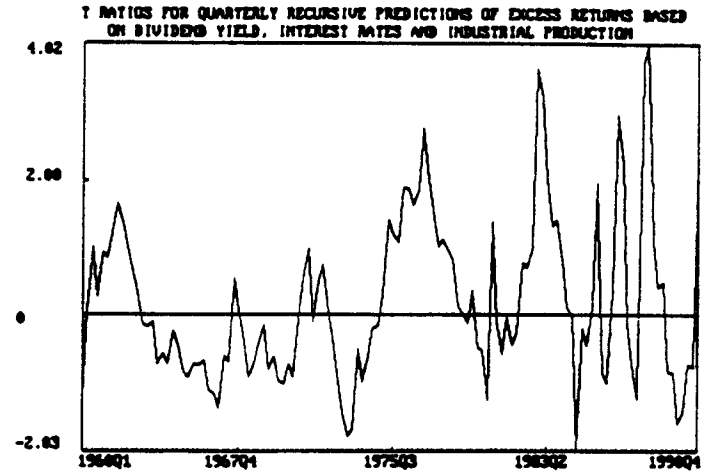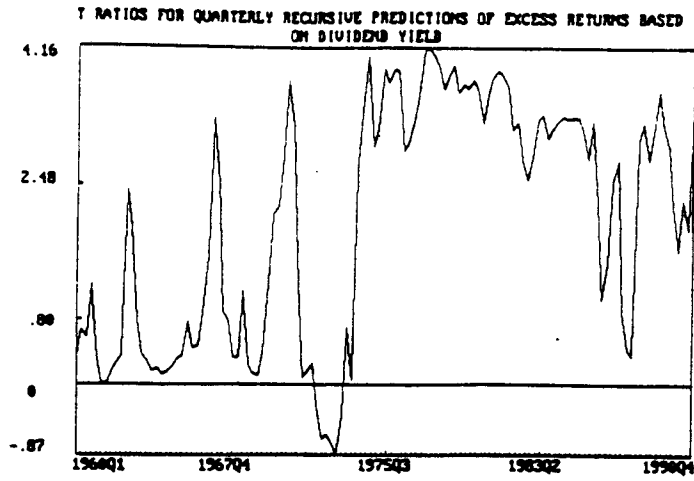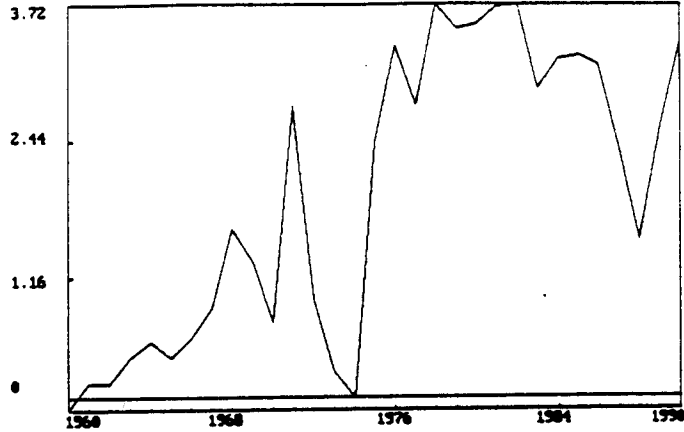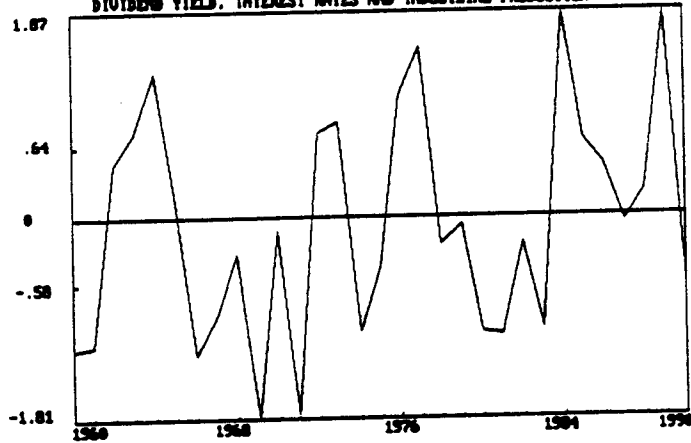
FIGURE 3

T RATIOS FOR ANNUAL RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD

T RATIOS OF ANNUAL RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD, INTEREST RATES AND INDUSTRIAL PRODUCTION

T RATIOS OF ANNUAL RECURSIVE PREDICTIONS OF EXCESS RETURNS BASED ON DIVIDEND YIELD, INTEREST RATES, INDUSTRIAL PRODUCTION AND INFLATION