THE CONSEQUENCES OF PRICE INSTABILITY
ON SEARCH MARKETS
Towards Understanding the Costs of Inflation

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ABSTRACT

Real price variability depreciates the information about future prices contained in current ones. Repeat purchase customers have, then, less incentives to acquire price information. The fact that consumers are less well informed allows firms to increase their markups and permits inefficient producers to increase their sales. Production gets reallocated towards higher-cost firms. Given the well documented correlation between inflation and relative price variability, these results help us understand some of the costs of inflation.

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There is considerable disparity between popular opinion and formal estimates of the welfare consequences of inflation. This paper attempts to partially fill that gap, by studying the welfare effects of one of the pervasive characteristics of inflationary processes -- relative price variability.

There is extensive evidence that inflation is positively correlated with the variability of prices across markets (see Domberger 1987 and Marquez and Vinig 1984). More recently, inflation has been shown to be positively related to the variability of prices across sellers of the same good.[1] Furthermore, Tommasi (1992) shows that the duration of *real* prices greatly diminishes at high inflation, i.e., that inflation lowers the informativeness of current prices about future prices. The evidence surveyed in Weiss (1992) indicates that no real world inflation has been a smooth process in which all nominal prices change in lockstep without

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[1]Van Hoomissen (1988), Lach and Tsiddon (1992) and Tommasi (1992). The latter two papers also find that most price variability in high inflation is *intragood* rather than *intergood* variance.

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effects on relative prices. It also suggests that in a highly inflationary environment, it is hard to establish who the low-price sellers are, since the price observed today is not a good predictor of future prices. Insofar as there is a spurious (not related to changes in relative scarcities) component to that relative price variability, we should include the effects of this real instability in our assessment of the welfare effects of inflation.[2]

Inflation affects the economic system through many avenues, and different studies have concentrated on different aspects of the phenomenon. Cukierman (1984), the first chapters of Fischer (1986) and Driffill, Mizon and Ulph (1990) list and evaluate the effects of inflation considered in the literature. While this literature recognizes the positive relationship between inflation and relative price variability, it does not formalize its welfare implications.[3] It is a folk theorem that inflation-induced "excess" price variability generates inefficiencies in resource allocation. Here I formalize that result in the context of an imperfect information framework.

Previous models that attempt to evaluate the consequences of inflation on imperfectly competitive (search) markets include Benabou (1988) and (1992), Benabou and Gertner (1990) and Diamond (1990). Inflation is

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[2]Let me note at the onset that the model developed in this paper is better tailored to compare situations of high inflation (high instability of relative prices) with more stable situations, than to study the effects of short run (business cycle) variations in the inflation rate. This is the case because I model long-term relationships and I do not emphasize costs of changing nominal prices (which in high inflation economies are pretty low).

[3]Fisher (1986, p. 42-43) shows that relative price variability will increase consumer welfare in a world of costless information. His result is a restatement of the quasiconvexity of the indirect utility function in prices. I show later that such result need not hold in a world of imperfect information.

introduced in these models as a common shock (deterministic or stochastic) to the cost of producers. The deterministic case is used to evaluate the effects of the average (expected) rate of inflation, and the stochastic case to evaluate the effects of the variance of (unanticipated) inflation. In this paper, I follow a somewhat similar strategy and introduce inflation as a idiosyncratic shock to firms' costs, in order to study the effects of relative price variability.[4] The other main difference with previous work, is that I emphasize the intertemporal dimension of relationships as opposed to having short-lived consumers who purchase the good only once.

I analyze the market for a homogeneous good being sold by a continuum of firms. Buyers purchase the good every period. Each period they can visit as many stores as they like. Each visit entails a (search) cost, which is different for different consumers. This heterogeneity maps into downward sloping demands for individual sellers, who set prices to maximize expected profits. Sellers face downward sloping demand curves because buyers are not fully informed; if they were, all output would be produced by the lowest cost firms. In such a world, inflation exacerbates the informational problem by depreciating the information that current relative prices convey about future relative prices. Buyers react by holding smaller information stocks. As I show below, this translates into higher reservation (acceptance) prices; that is, consumers become less choosy. The total amount of resources spent on search may either increase or decrease.[5]

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[4]This is one significant modeling short-cut: the relation between inflation and instability in costs is assumed rather than derived. The relation between inflation and price instability is well documented, and its sources are not well understood; thus, it seems a reasonable modeling strategy to simply posit this relationship and focus on its consequences.

[5]As it is well known in capital theory, a higher depreciation rate

The increase in consumers' acceptance prices shifts up the demand curve faced by the individual seller. This has several effects. First, sellers charge higher prices (they increase their markups). Second, high-cost producers are able to charge prices high enough to cover their costs. That is, there are real resource costs on top of a redistribution against consumers. If individual buyers' demands are elastic, we get the standard deadweight loss. Additionally, production gets allocated toward high-cost producers, increasing overall production costs. Contrary to the "administered inflation" hypothesis that establishes causality from markups to higher inflation, this model predicts the causal relationship to run *from* inflation *to* market structure and performance. Price instability moves the economy away from perfect competition, generating the loss of many of its efficiency properties.

Section I presents the general setup. Section II analyzes consumers' optimization and Section III looks at sellers' behavior. Section IV presents the equilibrium and Section V studies the effects of price instability. The results are obtained under the assumption that firms price "myopically", i.e., ignoring intertemporal linkages. Section VI explains the technical difficulties involved in the general case, and argues that the intertemporal pricing behavior of firms would tend to reinforce the predictions of the model. Section VII concludes.

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implies smaller stocks, but the investment flow of information may move in either direction. This clarifies the point about the "search cost" of price instability. Fischer (1981, p. 391) mentions that "excessive search is believed to be the mechanism through which monetary disturbances produce misallocations of resources." This paper shows that the total amount of resources devoted to search does not necessarily increase with inflation. Instead, there is a decrease in the stock of information available when making a decision. This loss of information induces buyers to enter into less adequate transactions, and that is what causes a welfare loss.

I. THE MODEL

The analysis is phrased in terms of a consumer good market, but it is applicable to firms minimizing the cost of acquiring inputs or to any other market interaction between buyers that search and sellers that set prices. I study the endogenous determination of equilibrium price distributions, which depends on the characteristics of buyers and sellers. There is heterogeneity on both sides of the market. Consumers belong to a continuum [0,1] and are characterized by their search cost c, distributed uniformly on [0,C]. *Potential* sellers are also a continuum [0,1] and have unit production costs θ_i, which follow a stochastic process to be specified below. Consumers go from store to store and observe the price tags -- the process of information transmission is such that quotations are received instantaneously. Sequential search is optimal in such case. The model can be seen as an extension of models with heterogeneity and sequential search (as MacMinn 1980 and Carlson and McAfee 1983) to intertemporal consumption (repeated purchase).

The model is set up in real terms. I assume that aggregate inflation is (either deterministic or) known to every agent. I do so to concentrate on the uncertainty about relative prices and to isolate its effects from other consequences of inflation which have been already analyzed.[6] As stated in

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[6]Uncertainty or incomplete information about the price index could be part of the reason why relative prices are less stable at higher inflation rates (Cukierman 1984). I do not attempt here to explain the inflation-variability correlation. For surveys on that topic see Fisher (1981) and Driffill *et al.* (1990).

the Introduction, I follow previous inflation-search literature in modeling inflation as affecting producers' costs.[7] To emphasize the role of relative price instability in markets with repeated relationships, I assume a very stylized first order Markov process for the unit cost of firms,

$$\theta_{it} = \begin{cases} \theta_{it-1} & \text{with probability } \rho \\ \text{a drawing from } \Psi(\theta) & \text{with probability } (1-\rho) \end{cases} \tag{1}$$

where $\Psi(\theta)$ is the cumulative density function of unit costs with domain $[\underline{\theta}, \bar{\theta}]$. In order to simplify the proofs, it is assumed that the p.d.f. $\psi(\theta)$ has no mass points. It is shown below that, in equilibrium, the cost process (1) maps into a similar process for output prices. The correlation of real costs over time, ρ, is assumed to be a decreasing function of the inflation rate, π; that is, $\rho = \rho(\pi)$, where $\rho' < 0$. This is based on the aforementioned empirical findings that inflation is positively correlated with relative price variability across sellers of the same good[8] and that inflation lowers the intertemporal correlation of real prices. Implicit in this formulation is the assumption that productivity shocks are stationary and thus all instability is induced by inflation. Insofar as inflation introduces spurious elements in pricing, the qualitative properties of the results will be the same if we allow the inflation shocks to interact with real shocks. The simple specification in (1), in the spirit of the "uncertain recall" models of Landsberger and Peled (1977) and Karni and Schwartz (1977), allows us to derive very clear comparative statics results.

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[7] At the cost of some notational and mathematical clustering I could obtain the same results by introducing inflation from the demand side; for instance by adding "noise traders" with demands tied to monetary-induced redistributions.

[8] Notice that $\text{Var}_i(\theta_{it+1}/\theta_{it})$ is decreasing in ρ (increasing in π) in this formulation.

II. THE CONSUMER PROBLEM

Consumers are infinitely lived. Each consumer has a real cost per search of c. He purchases one unit of the good per period, and his objective is to minimize the discounted present value of expected total expenditure per period (price plus search cost), or

$$\min E \sum_{t=0}^{\infty} \beta^t (p_t + N_t c),$$

where $\beta \in (0,1)$ is a discount factor and p_t and N_t are the price paid and the number of stores visited at t.

The known and time-invariant distribution of *real* prices has cumulative density function F(p). The location of each individual seller on that distribution follows the stochastic process

$$p_{t+1} = \begin{cases} p_t & \text{with probability } \rho \\ \text{a drawing from } F(p) & \text{with probability } (1-\rho) \end{cases} \quad (1')$$

The equilibrium distribution F and the stochastic process for individual prices will be derived later. I will show that in the simplified setting of this paper, the probability of the real price being unchanged equals the parameter ρ of the cost process.

What distinguishes this model from the standard consumer search problem, as described in Sargent (1987 Ch. 2), is that consumption takes place every period. Consumers can visit several stores per period. Real prices follow the process (1') over time. The probability of finding any given store charging the same real price next period is equal to ρ. In terms of the "recall problem" in the search literature, recall is "uncertain" over time.

Given the recursive structure of the problem, it can be solved using

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dynamic programming techniques. I will use the equivalence between the sequence problem expressed in the objective function above and Bellman's equation. Let $V(x)$ be the value function reflecting the present value of expected total expenditure when the consumer is at a store offering price x. At that point, there are two possible choices: acceptance or rejection. Acceptance implies paying x today and behaving optimally starting tomorrow. Free recall from last period's store is assumed for simplicity. Hence, coming back tomorrow is optimal, and the value of accepting x today equals $x+\beta[\rho V(x)+(1-\rho)EV]$, where $EV = \int_0^\infty V(\mu)dF(\mu)$. If the consumer rejects the quotation of x, he pays a search cost and gets a new drawing, with expected value c+EV. So that:

$$V(x) = \min\left\{x+\beta[\rho V(x)+(1-\rho)EV],c+EV\right\}. \qquad (2)$$

Proposition 1 shows that the optimal search strategy each period, consists of setting a reservation price p and accepting the first offer below p that comes along.

Proposition 1. The unique solution to the functional equation (2) is

$$V(x) = \frac{\min\{x,p\}}{(1-\rho\beta)} + A, \qquad (3)$$

where

$$A = \frac{\beta(1-\rho)}{(1-\beta)(1-\rho\beta)}[p-(1-\rho\beta)c], \qquad (4)$$

and p solves

$$c = \frac{1}{1-\rho\beta}\int_0^p (p-x)dF(x). \qquad (5)$$

Proof. **See the Appendix.**

Equation (5), which defines the reservation (indifference) price, is a familiar condition. The marginal cost of search is equated to the marginal benefit, which in this case equals the discounted present value of the expected price reduction due to an additional search. Notice that ρ

affects reservation prices in the same way as consumers' discount factor, capturing the idea that inflation shortens agents' horizons.

Differentiating (5) with respect to ρ gives

$$\frac{\partial p}{\partial \rho} = - \frac{c\,\beta}{F(p)} < 0. \tag{6}$$

This is a crucial result: reservation prices are decreasing in the correlation coefficient ρ. We can interpret $(1-\rho)$ as a measure of (inflation-induced) price variability. The more unstable the environment is, the higher real reservation prices are (the less choosy consumers become). If we think -metaphorically- of $(1-\rho)$ as a depreciation coefficient on the stock of information (Van Hoomissen 1988), we predict consumers holding smaller stocks (measured by the inverse of p). In this vein, the expected number of stores visited, n, can be interpreted as the "flow demand" for information, i.e., investment. The effect of ρ on the demand for information is uncertain. To show that, I first compute n as

$$n = \rho + (1-\rho)\,F(p) \sum_{i=1} i\,[1-F(p)]^{i-1} = \rho + (1-\rho)\,\frac{1}{F(p)}, \tag{7}$$

which reflects the possibility of stopping after a successful recall plus the possibility of having to rebuild the stock of information in case recall is unsuccessful. Notice that the expected number of quotations asked is inversely related to the reservation price --a buyer with a high acceptance price (high search cost) will be likely to find an acceptable price early. Deriving with respect to ρ,[9]

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[9]The analysis in the text refers to the interior case of $\rho\in(0,1)$. From (7), we see that $\rho=0 \rightarrow n=1/F$, and $\rho=1 \rightarrow n=1$. If $\rho=0$, there is no intertemporal link, and each period's search behavior is as described in the standard (static) search model. When $\rho=1$, all search is undertaken in the initial period; so, in all subsequent periods, the consumer makes only one trip, to purchase from the same store (confirming the intuition in Stigler 1961).

$$\frac{\partial n}{\partial \rho} = 1 - 1/F + (1-\rho) \, c\beta f/F^3. \qquad (8)$$

There are two forces at work in (8). On the one hand, a higher ρ increases the likelihood of a successful recall and hence of an early stopping. On the other, if recall happens to be unsuccessful, the lower reservation price (from (6)) makes the consumer more likely to search longer. Little more can be said without further specification of F. If the distribution of prices is uniform, search will be a concave function of inflation: increasing at low inflation (the Fischer intuition) and decreasing at very high inflation (as observed in the Argentine episodes of hyperinflation). This establishes my earlier claim that the total amount of resources spent on search (nc) may either increase or decrease as a response to (inflation-induced) price instability. Notice that the above is a partial equilibrium derivation; it is easy to show that the result carries over in "general" equilibrium (when we allow the distribution F to change), since $F(p(c))$ is decreasing in ρ.

III. DEMAND AND FIRM OPTIMIZATION

The purpose of this and the next two sections is to show (in a simplified setting where equilibrium can be explicitly solved for) how changes in the stock of information held by consumers affect the equilibrium price distribution once sellers' behavior is taken into account. It is assumed that firms maximize expected profits each period, ignoring intertemporal links. This strong assumption is made to gain mathematical tractability. After presenting the results under the "myopia" assumption, Section VI explains the difficulties involved in the general

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case. There, I argue that the results in the paper are unlikely to be overturned in the "exact" solution.

It is easy to show (MacMinn 1980) that the expected quantity, q(p), sold by a firm charging price p equals

$$q(p) = \frac{1}{\delta C} \int_{c(p)}^{C} 1/F(p(c)) \, dc. \tag{9}$$

where c(p) is the search cost of buyers with reservation price p, and δ represents the measure of sellers that are in operation -- high-cost sellers may fail to make any sale, generating a truncation at the upper tail of the cost distribution, in which case $\delta<1$, otherwise $\delta=1$.

From (5), $\dfrac{dc}{dp} = \dfrac{F(p)}{1-\rho\beta}$. Hence (by the implicit function theorem),

$$\frac{1}{F(p)} = \frac{1}{1-\rho\beta} \frac{dp}{dc},$$

which implies , using change of variable, that

$$q(p) = \frac{1}{\delta C} \int_{p}^{\bar{p}} \frac{1}{1-\rho\beta} \, dp = \frac{1}{\delta C} \frac{1}{1-\rho\beta} (\bar{p} - p), \tag{10}$$

where $\bar{p}$ is the reservation price of the buyer with the highest cost of search. We can now write the profit function as

$$(p-\theta)q(p) = \frac{(p-\theta)(\bar{p}-p)}{\delta(1-\rho\beta)C}.$$

Optimizing this quadratic profit function gives the pricing rule

$$p(\theta) = \frac{\bar{p} + \theta}{2}. \tag{11}$$

An equilibrium is defined as a price distribution F such that

(i) Consumers follow the reservation price rule (5) and

(ii) Firms price according to (11).

IV. EQUILIBRIUM

Let [m,M] be the range of equilibrium prices. From demand curve (10), it is clear that M, the highest price in the market, should satisfy $\bar{p} \geq M$. The upper bound of the price distribution is the reservation price of the consumers with the highest marginal cost of search. If a firm charges more, it won't make any sales (will be out of the market). There are therefore, two possible situations which depend on the relation between $\bar{p}$ and $\bar{\theta}$. When $\bar{\theta} \leq \bar{p}$, all potential sellers are indeed in the market every period ($\delta = \Psi(\bar{p}) = 1$). Otherwise, there will be truncation and only firms with $\theta \leq \bar{p}$ will be in the market ($\delta = \Psi(\bar{p}) < 1$). Applying the definition of a reservation price (5) to consumers with the highest cost, C, gives

$$\bar{p} = (1-\rho\beta)C + Ep,$$

where Ep stands for the expectation of the price. From (11), we get

$$Ep = \frac{\bar{p} + E(\theta|\theta \leq \bar{p})}{2}.$$

It follows that

$$\bar{p} = 2(1-\rho\beta)C + E(\theta|\theta \leq \bar{p}). \tag{5'}$$

Using (5'), the pricing function (11) can be rewritten as

$$p(\theta) = (1-\rho\beta)C + \frac{E(\theta|\theta \leq \bar{p})+\theta}{2}. \tag{11'}$$

The pricing function (11') is an equilibrium - notice the fixed point in $\bar{p}$ - mapping from unit cost θ to output price p. Conditions for existence and uniqueness are discussed in the Appendix. Each firm prices as a function of a weighted average of its unit cost and the average unit cost in the market. Together with the cost process (1), equation (11') determines the process for output price, which is stated in (1').

The change of variable formula can be used to express the distribution of prices $f(p)$, as a transformation of the probability density function of unit costs, $\psi(\theta)$. We get

$$f(p) = \psi(\theta)\ \frac{\partial\theta}{\partial p} = 2\ \psi(\theta(p)),$$

where $\theta(p) = 2p-2(1-\rho\beta)C-E(\theta|\theta\leq\bar{p})$. In the truncation case, $f(p)$ will need to be normalized by the proportion of firms in operation, $\Psi(\bar{p})$. All of the above is summarized in the following proposition.

Proposition 2. The equilibrium distribution of prices, $f(p)$, is equal to

$$f(p) = \begin{cases} \dfrac{2\psi(2p-2(1-\rho\beta)C-E(\theta|\theta\leq\bar{p}))}{\Psi(2(1-\rho\beta)C+E(\theta|\theta\leq\bar{p}))} & \text{for } p\in[m,M] \\[2ex] 0 & \text{elsewhere,} \end{cases} \qquad (12)$$

where $m = (1-\rho\beta)C + \dfrac{E(\theta|\theta\leq\bar{p})+\underline{\theta}}{2}$ and $M = \min\left\{(1-\rho\beta)C + \dfrac{E\theta+\bar{\theta}}{2}, \bar{p}\right\}$. The distribution $f(p)$ always exists. It will be unique if ψ is continuous and the hazard rate $\psi/_\Psi$ is decreasing.

Proof. See the Appendix.

V. THE EFFECTS OF PRICE INSTABILITY

We know from the analysis of the consumer problem that acceptance prices are decreasing in ρ (increasing in price instability). Such comparative statics was performed in the partial equilibrium sense of maintaining constant the distribution F. Here, I study the effects on the distribution as a whole. Now we have the reinforcing (equilibrium) effect of the whole distribution shifting to the right in response to lower ρs.

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Mathematically, the exercise consists in looking at the comparative statics in the fixed point to the expression

$$g(x) \equiv 2(1-\rho\beta)C + E(\theta|\theta \leq x),$$

which determines $\bar{p}$. See the proof of Proposition 2 in the Appendix for the details for uniqueness (and hence stability) of that solution.

We look first at the effects of changing ρ on the intercept $\bar{p}$. It is clear that the sign of $\frac{\partial \bar{p}}{\partial \rho}$ is the same as the sign of $\frac{\partial g}{\partial \rho} = -2\beta C < 0$. This implies that in situations of price instability (low ρ), the intercept of the demand curve will be higher. If we are in a truncated equilibrium, this will allow the operation of "marginal" firms. This agrees with the conventional wisdom about chronic inflation countries -- that inefficient producers can survive in such environment.

From the pricing function (11'), the effect on markups is

$$\frac{\partial p(\theta)}{\partial \rho} = -\beta C + \frac{1}{2} \frac{\partial E(\theta|\theta < \bar{p})}{\partial \rho} < 0,$$

all prices increase as ρ falls. This implies that the *mean* of the price distribution increases in an unstable environment($\frac{\partial Ep}{\partial \rho} < 0$). This is a differentiating prediction of this model with respect to Benabou (1988). All real prices in the market increase, and consumer welfare decreases at higher inflation rates (at lower ρ). In particular, even the lowest price in the market (m in this model, s in menu-cost models) does increase, which is consistent with the empirical finding of Sheshinski *et al.* (1981) for Israel.[10] Villanueva and Echeverry 1991 (and references there) found markups increasing in the inflation rate for the case of Argentina.

To shorten the exposition, from now on I will assume that movements are

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[10]That finding represented a puzzle to the authors, in terms of menu-cost models.

within the non truncation range, so that the unconditional $E\theta$ can be used instead of $E(\theta|\theta<\bar{p})$. Using (10) and (11'),

$$q(\theta) = 1 + \frac{(E\theta-\theta)}{2C(1-\rho\beta)}. \tag{13}$$

Equation (13) shows that producers with cost equal to the mean $E\theta$, will produce one unit (remember that the size of buyers and sellers is unity). High-cost producers will produce less than one, and low-cost producers will produce more than one. In highly unstable environments, production will get redistributed towards high-cost producers,[11] since

$$\frac{\partial q(\theta)}{\partial \rho} = \beta \frac{(E\theta-\theta)}{2C(1-\rho\beta)^2}.$$

The change in quantity is positive for firms with $\theta<E\theta$ and negative for firms with unit cost above average. This means that a low ρ (high π), will redistribute output in the wrong direction.

As a consequence of such redistribution of output, total production costs in the industry will be higher -- they are decreasing in ρ. Total cost equals

$$TC = \int_{\underline{\theta}}^{\bar{\theta}} \theta \, q(\theta) \, d\Psi(\theta) = \int_{\underline{\theta}}^{\bar{\theta}} \left[\theta + \theta \frac{(E\theta-\theta)}{2C(1-\rho\beta)}\right] d\Psi(\theta)$$

$$= E\theta + \frac{(E\theta)^2-E(\theta^2)}{2C(1-\rho\beta)} = E\theta - \frac{Var(\theta)}{2C(1-\rho\beta)} \, ;$$

so that

$$\frac{\partial TC}{\partial \rho} = - \beta \frac{Var(\theta)}{2C(1-\rho\beta)^2} < 0$$

as claimed. Notice that given that output is fixed in the model, prices determine the partition of gains between firms and consumers, and costs of

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[11]"High-cost producers" is somewhat misleading since in the particular Markov process (1), there is no innate component to firms' costs. As explained before, (1) is just a stylized version of a more general process. The general point is: as long as inflation adds noise to the inference problem of consumers and induces them to be less well informed, they will be more likely to engage in less efficient transactions.

search and production represent aggregate performance. It can be shown that the increase in the industry total cost is the dominant effect, constituting a dead weight loss from instability.

The effect of price variability (and hence, of the degree of consumer information) on dispersion, is more subtle. Contrary to common belief, the main equilibrium effect of information is on average prices and not on its cross-sectional dispersion: smaller information stocks (or reduced search intensity in a static model), do not necessarily imply higher price dispersions. In this model, it does so only in case of truncation, via its effect on $M=\bar{p}$. In the non-truncation case, there is a problem with the commonly assumed link between information and price dispersion. It seems that price dispersion is not necessarily a measure of ignorance in the market. In this model, the price distribution is a transformation of the production cost distribution. The transformation takes place through the aggregation of consumer search strategies, as reflected by demand curve (10). Demand here is linear because the costs of search are uniformly distributed. In such a case, the shift in the demand curve induced by a smaller ρ will have no effect on dispersion (for a given distribution of production costs). More generally, the effect on dispersion will depend on the shape of the demand curve, which is equal to the cumulative density function of c (with the axis changed). Notice that if one defines dispersion in terms of a coefficient of variation, the increase in the mean price *reduces* dispersion in the non truncation case.

The equilibrium effects of a lower ρ (higher inflation or price instability) are summarized in Figure 1. There is a clockwise rotation in the demand curve for the individual firm. This movement contains several effects. First, the upward movement in the whole locus induces an increase

in the real price charged by any firm (markups increase, real wages decrease). Second, the steepening of demand reflects the fact that high-cost (high-price) firms expand their expected sales at the expense of the more "efficient" firms. Third, the intercept being higher, firms that otherwise would be out of the market can charge profitable prices and make positive sales.

VI. DYNAMIC BEHAVIOR OF FIRMS

The previous results were obtained for the case in which firms ignore intertemporal links when making their pricing decisions. Such a modelling choice was made for tractability. In this section I explain the difficulties involved when firms take these dynamic links into account, and I argue that a consideration of the general case would tend to reinforce the main predictions.

Let p_{t-1} be the price a particular firm charged last period . By charging p_{t-1}, the firm drove away all potential customers with reservation price below p_{t-1}. The stock of customers who will be returning at t will not be a random sample of the whole consumer population, but a truncated sample of customers with search cost greater than the one that induces p_{t-1} as reservation price. The firm's demand at t will be composed of demand from old customers plus demand from newcomers. Such demand will have a kink at p_{t-1}, since by increasing its price the firm will be driving away not only newcomers, but also part of its customer base. This will be a source of price rigidity: small changes in the firm's marginal cost will not be reflected in its price. By the same reasoning, the demand curve

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will have a kink at every price that (going backwards in time) was a maximum (the highest price charged in the last 2 periods, 3 periods, 4 periods, etc.). It is clear that solving the firms' optimization problem will be much harder here than under the assumption of myopia. Also, the probability of keeping the same price will in general be different from the parameter ρ of the cost process. Furthermore, when the firm does change its price, the new price will no longer be, from the perspective of its clients, a random price from $F(p)$. Customers will have to remember the history of the firm's prices. This complicates the analysis of the consumer problem and equilibrium. These difficulties suggest that we should adopt a strategy of simplifying some aspects of the model in order to tackle the case of non-myopic firms.[12] This exercise is the natural extension of this work.[13] Below, I provide an heuristic explanation of why the inclusion of "full optimization" by firms will tend to reinforce the predictions of the model.

We established that a lower ρ implies: higher markups, production shifted toward high cost firms (and hence higher total industry cost), and possibility of survival of "inefficient" firms (from the higher intercept of demand curve). These results were caused by the increase of the reservation price for any search cost. The missing link in our analysis was intertemporal pricing. We now analyze a simplified scenario,[14] and ask

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[12]Having overlapping generations of consumers who live for two periods will certainly help, by limiting the time links to only adjacent periods.

[13]See McMillan and Morgan (1988), Benabou (1990) and Fishman and Rob (1991) for models where firms price intertemporally in the context of information-induced long-term trading relationships.

[14]The crucial "assumptions" of the scenario are: 1) existence and some form of uniqueness that enables us to perform comparative statics; 2) intertemporal dependence of only first order; and 3) differentiability of

whether the "intertemporal pricing effect" will work in the same direction as the "reservation price effect". Imagine that we can write the firm's demand curve as $q(p_t, p_{t-1})$. We already know that q depends negatively on p_t. It is the case that q also depends negatively on p_{t-1}: a lower price today attracts more repeat buyers for tomorrow (intertemporal complementarity). This is a force for prices to be lower in the full fleshed model than in the myopic case. The marginal cost of today is equated not just to today's marginal revenue, but to a "full marginal revenue" (Benabou 1990) inclusive of the effect of p_t on q_{t+1}. It remains to be shown that the strength of this elasticity (q_{t+1} with respect to p_t) is an increasing function of ρ. Consider the extreme case of $\rho=0$. In that case, intertemporal links are absent,[15] we just play a one-purchase game every period, and the myopic and full solution coincide. For $\rho>0$, intertemporal considerations become important for the firm, and prices are lower than under $\rho=0$ (or myopia). More generally, intertemporal considerations will be increasingly important as we increase ρ, which plays (once more) a role similar to the discount factor β. In terms of the inflation interpretation of the paper, also on the firm side, inflation shortens agents' horizons. From there we conclude that at lower ρ, firms will charge higher markups, not only indirectly due to consumers' increased reservation prices, but also directly because of the diminished incentive to attract repeat buyers.

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demand functions. The first two conditions could be obtained under suitable assumptions. The third one is just an artifact to simplify the exposition. See Benabou (1990) for the details of optimization with kinked demand curves.

[15]Assuming that the first search of each period has the same (zero) cost whether you go back to your old store or just go to a new one; and assuming away any gaming by firms (such as trying to establish a reputation for low prices).

VII. CONCLUSION

The next step to take, along the path suggested by this paper is to relax the assumption of "myopic" firms, in order to fully characterize the equilibrium in the repeated purchase game. We should also proceed to incorporate this type of "market structure" considerations into general equilibrium models, to be able to provide more adequate quantitative assessments of the effects of inflation and/or macroeconomic instability,[16] as well as explicitly linking inflation in the macroeconomy to diminished information at the micro level. Ball and Romer (1992) and Fishman and Tommasi (1993) are preliminary attempts in that direction.

APPENDIX

Proof of Proposition 1

Equation (2) can be rewritten as

$$V(x) = \min \left\{ \frac{x}{1-\rho\beta} + \frac{\beta(1-\rho)}{1-\rho\beta} EV, \ c + EV \right\} \tag{A1}$$

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[16]Ball and Romer (1992) provide some preliminary numbers in the context of a model similar in spirit to ours. They suggest potentially significant effects of inflation in the context of long term relationships -- which in their model originate from set up costs, rather than from costs of acquiring information.

First, I will demonstrate that (3)-(4)-(5) is indeed a solution to (A1). Then, I will prove that (A1) has a unique solution.

(i) Solution of (A1)

Claim: The value function defined by (3)-(4)-(5) solves functional equation (A1).

Proof

From (3),

$$EV = \frac{1}{1-\rho\beta} \left[\int_0^p x \, dF(x) + \int_p^\infty p \, dF(x) \right] + A.$$

From (5),

$$\frac{p}{1-\rho\beta} = c + \int_0^p \frac{x}{1-\rho\beta} \, dF(x) + \int_p^\infty \frac{p}{1-\rho\beta} \, dF(x) \qquad (A2)$$

Using (A2), $EV = \frac{p}{1-\rho\beta} - c + A$. Substituting in (A1) we get, after some manipulation,

$$V(x) = \min \left\{ \frac{x}{1-\rho\beta} + A, \ \frac{p}{1-\rho\beta} + A \right\}$$

as claimed. □

(ii) Uniqueness of the solution to (A1)

Claim: The functional equation (A1) admits a unique solution.

Proof

For very high search cost c, the unique optimal strategy consists of accepting any quotation. Below, I prove uniqueness in the more interesting case in which search is possible.

Clearly, any solution to (A1) should belong to the set:

$$\Gamma = \left\{ z:\mathbb{R}_0^+ \to \mathbb{R} \mid z(x) = \min \ [bx+hv, c+v] \right\}$$

$$b = \frac{1}{1-\rho\beta} \quad , \quad h = \frac{\beta(1-\rho)}{1-\rho\beta} \quad , \quad v \in \mathbb{R}$$

Define the distance between two elements of Γ as $d(z,w) = \sup_{x \in [0,\bar{p}]} |z(x)-w(x)|$,

where $\bar{p} = 2(1-\rho\beta)C+E\theta$.

Define the operator $T:\Gamma \to \Gamma$

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$$Tz = \min [bx + hEz, c + Ez]$$

From $0 \leq h < 1$, it is clear that $d(Tz,Tw) = |Ez-Ew|$. Hence, for any two different elements of Γ, $d(Tz,Tw) < \sup |z(x)-w(x)| = d(z,w)$.

Now, suppose that $V(x)$ and $V'(x)$ are two solutions to (A1), i.e., $V(x)=TV$ and $V'(x)=TV'$. Then, $d(V,V') = d(TV,TV')$. But we have already established that if $V \neq V'$, $d(TV,TV') < d(V,V')$. Hence we arrive at a contradiction. $\square$

Proof of Proposition 2

The equilibrium is a fixed point in $\bar{p}$. I will discuss the non-truncation and truncation cases separately.

I. Non-truncation case: $2(1-\rho\beta)C + E\theta \geq \bar{\theta}$. In this case $E(\theta|\theta \leq \bar{p})=E\theta$ and $\bar{p} = 2(1-\rho\beta)C+E\theta$. Both are functions of parameters and they exist uniquely.

II. Truncation case: $2(1-\rho\beta)C + E\theta < \bar{\theta}$.
Define $g(x) \equiv 2(1-\rho\beta)C + E(\theta|\theta \leq x)$. We have to prove that there exists a unique fixed point to g, which will be called $\bar{p}$.

II.1. Existence:
Notice that, if ψ has no mass points, then g is continuous. Notice also that:[17]

$$g(\underline{\theta}) = 2(1-\rho\beta)C + \underline{\theta} > \underline{\theta} \tag{A3}$$

$$g(\bar{\theta}) = 2(1-\rho\beta)C + E\theta < \bar{\theta} \tag{A4}$$

From continuity of g and the above conditions, it follows that $g(x)$ has at least one fixed point.

II.2. Uniqueness:[18]

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[17] I am indebted to R. Benabou for pointing out that, to be completely rigorous we should replace (A3) by a limit argument, since it involves the ratio of two expressions that converge to 0 as $x \to \underline{\theta}$. See Benabou (1990).

[18] I am indebted to a referee for pointing out this existing result, which is more intuitive than my previous formulation. Notice that the condition is satisfied by most usual distributions.

Theorem II.2 of in Benabou (1990, p.14) establishes that a sufficient condition for uniqueness (and hence stability) is that the distribution ψ have a continuous density ψ and a decreasing hazard rate ψ/Ψ. It is straightforward to show that this implies $g'(x) \in (0,1)$. □

REFERENCES

Ball, Laurence and D. Romer (1992). "Inflation and the Informativeness of Prices." NBER Working Paper 4267.

Benabou, Roland (1988). "Search, Price Setting and Inflation." *Review of Economic Studies* 55, 353-376.

Benabou, Roland (1990). "Search Market Equilibrium, Bilateral Heterogeneity and Repeat Purchases." Mimeo. MIT. October.

Benabou, Roland and R. Gertner (1990). "The Informativeness of Prices: Search with Learning and Cost Uncertainty." Working Paper. University of Chicago GSB.

Benabou, Roland (1992). "Inflation and Efficiency in Search Markets." *Review of Economic Studies* 59, 299-329.

Carlson, John and R. McAfee (1983). "Discrete Equilibrium Price Dispersion." *Journal of Political Economy* 91, 480-493.

Cukierman, Alex (1984). *Inflation, Stagflation, Relative Prices, and Imperfect Information.* Cambridge University Press.

Diamond, Peter (1990). "Search, Sticky Prices and Inflation." MIT Working Paper. Forthcoming, *Review of Economic Studies.*

Domberger, Simon (1987). "Relative Price Variability and Inflation: A Disaggregated Analysis." *Journal of Political Economy* 95, 547-566.

Driffill, John, G. Mizon and A. Ulph (1990). "Costs of Inflation." Chapter 19 in *Handbook of Monetary Economics.* B. Friedman and F. Hahn (eds.). Amsterdam: North-Holland.

Fischer, Stanley (1981). "Relative Shocks, Relative Price Variability, and Inflation." *Brookings Papers Econ. Activity* 2, 381-431.

Fischer, Stanley (1986). *Indexing, Inflation, and Economic Policy.* MIT Press.

Fishman, Arthur and Rafael Rob (1991). "The Durability of Information, Market Efficiency and the Size of Firms; Search with Repeated Transactions."

Mimeo, University of Pennsylvania.

Fishman, Arthur and M. Tommasi (1993). "Inflation and the Informational Content of Real Prices." Mimeo, Tel Aviv University, May.

Karni, Edi and Aba Schwartz (1977). "Search Theory: The Case of Search with Uncertain Recall." *Journal of Economic Theory* 16, 38-52.

Lach, Saul and Daniel Tsiddon (1992). "The Behavior of Prices and Inflation: An Empirical Analysis of Disaggregated Price Data." *Journal of Political Economy* 100, 349-389.

Landsberger, Michael and Dan Peled (1977). "Duration of Offers, Price Structure and The Gain from Search." *Journal of Economic Theory* 16, 17-37.

MacMinn, Richard (1980). "Search and Market Equilibrium." *Journal of Political Economy* 88, 308-327.

Marquez, Jaime and Daniel Vinig (1984) "Inflation and Relative Price Behavior: A Survey of the Literature." in Ballabou (ed.) *Economic Perspectives: An Annual Survey of Economics* vol 3. Harwood.

McMillan , John and Peter Morgan (1988). "Price Dispersion, Price Flexibility and Repeated Purchasing. " *Canadian Journal of Economics* 21, 883-902.

Sargent, Thomas (1987). *Dynamic Macroeconomic Theory*. Harvard University Press.

Sheshinski, Eytan, A. Tishler and Y. Weiss (1981). "Inflation and Costs of Adjustment, and the Amplitude of Real Price Changes." In Flanders and Razin (eds.) *Development in an Inflationary World*. Academic Press.

Stigler, George (1961) "The Economics of Information." *Journal of Political Economy* 69, 213-225.

Tommasi, Mariano (1992). "Inflation and Relative Prices: Evidence from Argentina." In Sheshinski and Weiss (eds) *Optimal Pricing, Inflation and Cost of Price Adjustment*. MIT Press. Forthcoming.

Van Hoomissen, Theresa (1988). "Price Dispersion and Inflation: Evidence from Israel." *Journal of Political Economy* 96, 1303-1314.

Villanueva, Juan, and J. Echeverry (1991). "Fijacion de Precios Bajo Condiciones de Hiperinflacion: el Caso Argentino." *Desarrollo Economico* 121, 73-89. Buenos Aires.

Weiss, Yoram (1992). "Inflation and Price Adjustment: A Survey of Findings from Micro-Data." Working Paper No 3-92, Tel-Aviv University.

FIGURE 1

Demand Curve
for an Individual Firm
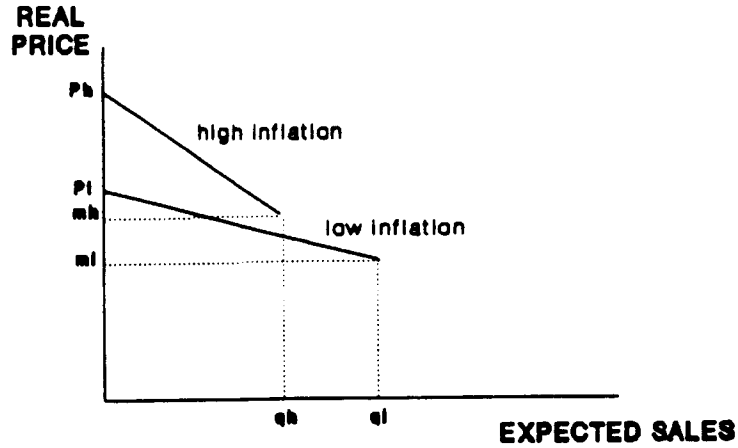
NOTE
Ph: maximum acceptance price (p(C)) when inflation is high.
Pl: maximum acceptance price (p(C)) when inflation is low.
mh: lowest real price in the market at high inflation.
ml: lowest real price in the market at low inflation.
qh: expected sales of firms with the lowest cost at high inflation.
ql: expected sales of firms with the lowest cost at low inflation.