THE ECONOMETRICS OF INDETERMINACY:

AN APPLIED STUDY

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Abstract

This paper presents evidence from the US economy on the propagation mechanism and on the impulses that have been responsible for business cycles in the United States over the period from 1929 through 1988. Our results support the view, advanced in earlier work, that a general equilibrium model with an indeterminate steady state does a good job of accounting for the propagation mechanism in US data. In addition to suggesting a novel explanation for the propagation mechanism of business fluctuations, the method by which we estimate our model is able to measure the relative importance of different sources of fluctuations by identifying different shocks with the residuals to our estimated equations. We divide the impulses to the business cycle into supply shocks (shocks that affect productivity) and demand shocks (unexplained fluctuations in private and government consumption). We find that demand shocks are roughly twice as important as supply shocks in the US data over the period from 1929 through 1988; in the post-war period, however, supply shocks and demand shocks have both been of roughly equal magnitude and the variances of both kinds of disturbances have been five times lower than in the pre-war period.

1) Introduction

This paper is about the impulse and propagation mechanism of US business cycles. We present estimates of the parameters of a model of the US economy that was studied in earlier work by Benhabib and Farmer [1994] and Farmer and Guo [1994]. The model is a modified version of the real business cycle economy pioneered by Long and Plosser [1983] and Kydland and Prescott [1982] that allows for the effects of increasing returns in production of the kind studied by Baxter and King [1991]. Our earlier work showed that economies with increasing returns to scale are able to perform at least as well as real business cycle economies at explaining the second moments of employment, consumption, gdp and capital when business cycles are driven by "animal spirits" rather than by fundamental shocks to technology. We also demonstrated that that a model driven by animal spirits looked to be a more promising way of accounting for the dynamic patterns among real variables in the post-war quarterly data. This study pushes our earlier work further by formerly estimating the model as opposed to calibrating the values of the parameters; our estimation technique applies classical simultaneous equation methods to a linerarized version of the model using US annual time series. Our linearization method follows the techniques laid out in King, Plosser, and Rebelo [1988].

The Benhabib-Farmer [1994] paper showed that a sufficient condition for animal spirits to be a potentially important source of business cycle fluctuations was the requirement that the labor demand curve should slope *up* with a slope that is greater than the slope of the labor supply curve. Although it has been known for some time that externalities can potentially cause the labor demand curve to slope up (Robert Hall [1991]), the possibility that this phenomenon might occur in practice has typically been considered to be unlikely. Our parameter estimates of labor demand imply that this phenomenon is not only theoretically possible but it also is consistent with US data.[1]

In theoretical work on the possibility of animal spirits as a driving force of economic fluctuations, Benhabib and Farmer found that the labor demand curve should be steeper than the labor supply

[1] Our work is in the same spirit as recent papers that apply econometric techniques to real business cycle models. Examples include work by Altug [1989], Braun [1994], Christiano and Eichenbaum [1992], Leeper and Sims [1994] and McGrattan [1994]. Unlike the work in this tradition, our theoretical model admits the possibility that the equilibrium of the economy may be indeterminate if one fails to explicitly model a process for beliefs that is independent of preferences, endowments and technologies.

curve however, in their theoretical work they maintained the assumption that labor supply as a function of the real wage *holding constant the level of consumption* should have a non-negative slope. In our econometric work, we find evidence that this condition is also violated; in other words, not only does the labor demand curve slope up, but we find that the *labor supply curve slopes down.*

We view the work presented in this paper as preliminary. Our main purpose in embarking on this exercise was to show that a model in which beliefs independently influence outcomes is capable of being formally compared with data. Our general impression, on discussing the idea with our colleagues, is that there is a good deal of skepticism to the idea that models with indeterminate equilibria have *any* use in helping to explain the data. It is often asserted that models of multiple equilibria can explain anything; that they are incapable of being refuted and that they are therefore bad science. We have tried to show in this paper, provided one is prepared to parameterize the process by which agents form beliefs, that models with multiple equilibria *can* be formulated and tested in much the same way as standard intertemporal general equilibrium models.

Although we believe that we have met with some success in demonstrating that formal econometric methods can be applied to models with indeterminate equilibria; our work has also raised a number of puzzles that need to be explained. There are two related issues that we discuss in section (11) of the paper. The first is that shocks to the Euler equation of our economy are autocorrelated – since Euler equation errors should be unforecastable, this finding is inconsistent with the maintained assumptions of the model. Second: we find that when we identify the labor supply curve, using exclusion restrictions, that labor supply as a function of the wage slopes down. This finding violates our maintained assumption of a representative agent with separable convex preferences. We discuss, in section (11), a number of possible resolutions of these puzzles of which the most promising, in our view, is to drop the assumption of separability.

2) The Data

We begin with a description of the data that we used for the study. All time series are taken from the US National Income and Product Accounts for the period 1929 through 1988. Since our definitions are not standard, we present time series plots of the data in Figures 1 through 3. Our main deviation from standard practice has been to aggregate private consumption and government consumption into a single consumption series. Similarly, to construct our capital stock data, we aggregated private investment and government investment into a single aggregate investment series and we integrated the total investment series using the capital accumulation identity assuming an annual depreciation rate of 6% and an initial 1929 value of aggregate capital of 1,869 billion 1987 dollars.[2] In other words, our capital series is constructed from the equations:

$$K_{t+1} \equiv (0.94)K_t + I_t,$$
$$K_{1929} = 1{,}869.$$

Our gdp series is also non-standard and is constructed from the identity:

$$Y \equiv I + C,$$

where I and C are the total (including government) investment and consumption series described above. This construction differs from the national income and product definition by excluding net exports although the difference is not great as net exports never account for more than 4% of gdp over the period that we study.

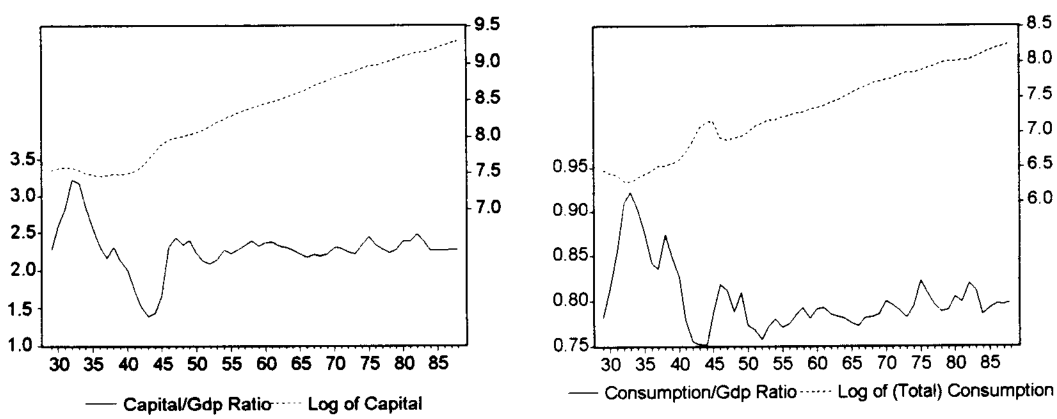

Figure 1: Capital and Consumption

[2] We chose the starting value of capital by integrating values of investment for data from 1890. Our earlier investment series is constructed from consumption and gdp data in Balke and Gordon [1986] assuming that government investment and consumption is divided in the ration 4 to 1 as in the post 1929 NIPA accounts. Government expenditure data for the pre 1929 period is from *Historical Statistics of the United States: Colonial Times to 1970, U.S. Government Printing Office, 1975.* We experimented with various capital stock series using alternative assumptions about depreciation and alternative methods of estimating the initial 1929 level of capital with little difference to our parameter estimates.

Figure 1 graphs our constructed capital series on the left hand panel and our total consumption series on the right-hand panel. These two figures also plot the ratio's of capital to gdp and consumption to gdp. Both of these ratios pass a Dickey-Fuller test for stationarity at the 5% level whereas capital, gdp and consumption are each individually non-stationary.

The employment series that we use in our work is a measure of full time equivalent employees that is available from the NIPA accounts on an annual basis from 1929. Figure 2 graphs this series as a ratio of total US population. The employment to population ratio has trended upwards over the period as a consequence of increases in female participation in the post-war period. We allow for this upward trend in our model specification by allowing a non-stationary taste shifter that permits the supply of labor to drift up over time.

Figure 2: Employment Per Person

Figure 3 presents data on growth rates of various series used in the formal model that we describe in section 3. The left panel of the figure plots the growth rates in gdp, population growth and employment growth against time. The right panel plots gdp growth, consumption growth and net investment (capital growth) against time. These five series are used in our econometric analysis to disentangle the slopes of labor demand and supply curves.

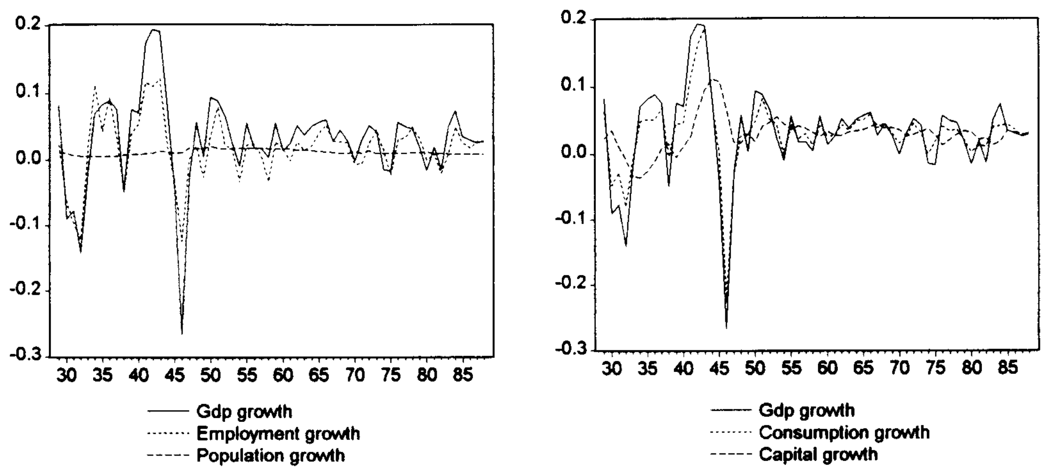

Figure 3: Growth Rates of Data Series Used in the Analysis

The main econometric innovation in our paper is to use classical exclusion restrictions to identify the slopes of the labor demand and supply curves. Capital enters the labor demand curve, but not the labor supply curve. Similarly, consumption enters the labor supply curve but not the labor demand curve. Unlike more standard examples of rational expectations models in which forward looking expectations prevent one from using these exclusion restrictions to identify parameters; we are able to exploit the theory of rational expectations in indeterminate systems. Roughly speaking, the property of indeterminacy invalidates the cross equation restrictions of classical rational expectations econometrics and allows us to use lagged values of endogenous variables as instruments to identify parameters of labor demand and supply equations. This procedure would not be valid in a standard real business cycle model because one would expect *not* to be able to find instruments for the endogenous variables that were simultaneously uncorrelated with the error terms to each equation. We explain below, in section (6), why this criticism does not apply to the case of an indeterminate rational expectations equilibrium.

3) The Equations of the Model

Our theoretical model builds on work by Benhabib and Farmer [1994] and Farmer and Guo [1994]. We assume that the data can be described "as if" it were chosen by a representative family that solves the problem:

$$\text{Max } U = \sum_{t=0}^{\infty} E_t \left[\frac{1}{1+\rho}\right]^t \left(\log\left(\frac{C_t}{N_t}\right) - V_t \left(\frac{Z_t}{N_t}\right)^{1+\gamma} \frac{1}{1+\gamma} \right), \qquad \rho > 0,$$

subject to the constraints:

(1) $$K_{t+1} = K_t(1-\delta) + Y_t - C_t,$$

(2) $$\frac{Y_t}{N_t} = A\left(\frac{K_t}{N_t}\right)^{\alpha}\left(\frac{Z_t}{N_t}\right)^{\beta} U_t.$$

The notation K_t refers to aggregate capital, Y_t is aggregate gdp, Z_t is aggregate employment, N_t is population, C_t is aggregate consumption, A, α, β and δ are parameters and U_t and V_t are aggregate shocks that we specify more completely below. Unlike the standard real business cycle model, we allow the parameters α and β to sum to more than one. In our earlier work we showed how either externalities at the level of the economy or individual increasing returns plus imperfect competition can reconcile this specification of the production function with a theory of distribution. The key to this reconciliation is to distinguish between the social technology and the private technology.

Equation (2) represents the *social* technology which explains how aggregate gdp will respond if all families in the economy expand their use of inputs simultaneously. This function is conceptually distinct from the *private technology* given by equation (2').

(2') $$Y_t^i = A\left(K_t^i\right)^a\left(Z_t^i\right)^b U_t X_t,$$

where the superscript "i" indexes the individual family. In an economy in which externalities are important the term X_t, which is taken as given by the representative family, is equal to:

(2'') $$X_t \equiv \left(\frac{K_t}{N_t}\right)^{\alpha - a}\left(\frac{Z_t}{N_t}\right)^{\beta - b},$$

where (K/N) and (Z/N) represent the average inputs of capital and labor by *other* families in the economy. In a symmetric equilibrium, when all families take the same actions, K^i and Z^i will be equal to (K/N) and (Z/N) and (2'') may be substituted into (2') to give the social technology, (2).[3]

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[3] The use of externalities to explain social increasing returns were first used in the endogenous growth literature by Paul Romer (1986). Our use of externalities to explain pro-cyclical productivity in models

Equations (1) and (2) comprise two of the key equations of our model. Equation (1) is the gdp accounting identity; this identity holds in our data set, by construction, with a value of $(1\text{-}\delta)$ equal to 0.94. Equation (2) is the social production function; these two equations are supplemented by the first order conditions of the representative family's optimizing problem:

$$V_t \frac{C_t}{N_t}\left(\frac{Z_t}{N_t}\right)^{\gamma} = b\,\frac{Y_t}{Z_t}, \tag{3}$$

$$\frac{N_t}{C_t} = E_t\left\{\frac{N_{t+1}}{C_{t+1}}\left(\frac{1}{1+\rho}\right)\left[0.94 + a\frac{Y_{t+1}}{K_{t+1}}\right]\right\}. \tag{4}$$

Equation (3) is an intra-temporal first order condition that instructs the representative family to equate the slope of its indifference curve (the left hand side of equation (3)) to the private marginal product of labor (the right hand side of equation (3)). If the economy's allocation problem were to be decentralized, both sides of this equation would be set equal to the real wage. In section (10) of the paper we introduce data on the real wage and present evidence on the magnitude of the parameters β and γ that is important to understanding our explanation of the propagation mechanism of business cycles.

Equation (4) is an inter-temporal first order condition that is often referred to as a "stochastic Euler equation". The left side of this expression represents the utility that would be given up by a representative family that decides to allocate an additional commodity unit to future consumption. The right side expresses the expected utility gain from this reallocation; the future expected marginal utility is discounted by the time preference factor $(1/(1+\rho))$ and multiplied by the additional return gained by waiting and using the resources in production in the future period.

A key to our results is the distinction between the parameters a, b, α, and β. Equation (3) equates the slope of the representative family's indifference curve:

$$V_t\left(\frac{C_t}{N_t}\right)\left(\frac{Z_t}{N_t}\right)^{\gamma},$$

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driven by demand shocks follows the work by Baxter and King (1991) although the Baxter King specification does not result in indeterminate dynamics. An alternative explanation of social increasing returns based on monopolistic competition is worked out in Benhabib and Farmer (1994) and similar specifications are discussed in Gali (1994) and Chatterjee and Cooper (1993).

to the *private* marginal product of capital,

$$b\frac{Y_t}{Z_t}.$$

In a decentralized allocation the parameter b would represent labor's share of gdp and in real business cycle economies this statistic is typically used to calibrate the value of labor's marginal product. In our economy, however, the private agent does not take into account the effect of its employment decision on the productivity of other firms, (the effect of Z on X), and as a consequence labor's share, "b" will typically be less than the social marginal product, "β".

Just as the parameter "b" measures the share of gdp going to labor, so "a" measures the share going to capital and in an economy with externalities, "α" will typically exceed "a". In the real business cycle version of the model, on the other hand, one has the parameter restrictions:

$$\alpha = a, \quad \beta = b, \quad \alpha + \beta = 1.$$

The key to the earlier work by Benhabib and Farmer and Farmer and Guo was to show that when the parameter β is bigger than $(1+\gamma)$, it becomes possible for the model to display very different behavior from that of a standard rational expectations real business cycle model. One of the contributions of our current work is to demonstrate that there is considerable evidence to suggest that this condition is satisfied in practice.

4) Modeling Fundamental Uncertainty

In earlier work we calibrated a version of the model discussed in section (3) and, in this earlier work, we followed the route that that is typical in the calibration literature by using data that had been passed through the Hodrick-Prescott filter[4]. Since our object in this paper is to estimate the parameters of the model using more standard econometric methods, we have chosen not to filter the data and we have instead chosen to pursue the implications of our model economy for low frequency movements as well as high frequency movements.

Preliminary tests indicate that the logarithms of consumption, investment and gdp are all I(1) time series – we also are unable to reject the hypothesis that the logarithm of employment per person

[4] Robert Hodrick and Edward Prescott [1980].

is I(1).[5] Theoretically, it is not possible for the logarithm of employment per person to be an integrated series since it is bounded above by the time endowment of the representative family; nevertheless, the series may be well approximated by an I(1) variable over finite samples. In our econometric work we make the working assumption that both U and V are I(1) series and we model them with the stochastic process:

$$\text{(5)} \qquad \begin{aligned} \log(U_{t+1}) &= \log(U_t) + u_{t+1} \\ \log(V_{t+1}) &= \log(V_t) + v_{t+1} \end{aligned}.$$

We impose the assumptions;

$$\text{(6)} \qquad E_t\begin{bmatrix} u_{t+1} \\ v_{t+1} \end{bmatrix} = \begin{bmatrix} \bar{u} \\ \bar{v} \end{bmatrix}, \quad Var(u_{t+1}, v_{t+1}) = \Omega, \qquad \text{for all } t.$$

The variable U represents a non-stationary productivity shock; the variable V is a non-stationary taste shifter that we hope will pick up the effect of increasing labor force participation over time. In words, our assumption about the fundamental uncertainty in the model is that the productivity shock and the taste shock are each logarithmic random walks with drift. We allow for the innovations in these variables to be correlated with covariance matrix Ω although it is our prior belief that the covariance between innovations in the taste shock V and innovations in the productivity shock U should be zero. To model population growth, we assume that N_t also is a logarithmic random walk with drift and we model the stochastic properties of population growth with the statistical model:

$$\text{(7)} \qquad \log(N_{t+1}) = \log(N_t) + dn_{t+1},$$

where dn is an observable stationary random variable with mean $\bar{v}$.

5) The Structural Form

For the purposes of estimating the parameters of the model, we first find a representation of the system in which the errors to each equation are stationary. It is relatively easy to show that, in the absence of uncertainty, the model has a unique balanced growth path in which the ratio of capital to gdp is a constant, gdp per capita, capital per capita and consumption per capita grow at a fixed

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[5] We conform to standard notation in referring to a time series as I(p) if the p'th difference of the series is covariance stationary.

rate, and employment per capita grows a different fixed rate. Appendix A shows how to find the equations that represent this balanced growth path and it derives a representation of the model in which the ratio of gdp to capital, and the growth rates of consumption, capital, gdp and employment are all stationary variables. This result is important for our econometric work since, under the maintained hypothesis that the model represents the true data generation process, it implies that in the representation of the system expressed by equations (8) – (11); equations (8) (9) and (10) each describe a relationship between stationary variables:

$$dy_t - \beta dz_t - \alpha dk_t - (1-\alpha-\beta) dn_t - u_t = 0, \tag{8}$$

$$dy_t - (1+\gamma) dz_t - dc_t + (1+\gamma) dn_t - v_t = 0. \tag{9}$$

Equation (8) expresses the production function and equation (9) the labor market equilibrium condition in terms of logarithmic differences of the state variables. The variables dy, dk, dc, dn and dz represent logarithmic differences of gdp, capital, consumption, population and labor supply respectively. Equations (10) and (11) express the representative family's Euler equation (4) and the accounting identity (1) in terms of logarithms and logarithmic differences of the state variables:[6]

$$dc_t = dn_t - \rho + \log\left(0.94 + a\exp(y_t - k_t)\right) + w_t, \tag{10}$$

$$k_t \equiv \log\left\{0.94\exp(k_{t-1}) + \exp(y_{t-1}) - \exp(c_{t-1})\right\}, \tag{11}$$

where y and k are the logarithms of gdp and capital and w_t is the Euler equation error defined by the identity:

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[6] Taking logarithms of equations (2) and (3) we can express the logarithm of gdp, as a function of the logarithms of consumption and capital and of the logarithms of the disturbance terms U and V:

$$y_t = \lambda_0 + \lambda_1 c_t + \lambda_2 k_t + \lambda_3 n_t + \lambda_4 \log(U_t) + \lambda_5 \log(V_t).$$

Linearizing equation (4) leads to:

$$n_t - c_t = E_t\left\{n_{t+1} - c_{t+1} - \rho + \log\left[0.94 + a\exp(y_{t+1} - k_{t+1})\right] + O(x^2)\right\},$$

which given the definition of w_t in equation (12) leads to the structural equation:

$$dc_{t+1} = dn_{t+1} - \rho + \log\left(0.94 + a\exp(y_{t+1} - k_{t+1})\right) + w_{t+1}.$$

(12)

$$w_t \equiv \left[c_t - E_{t-1}\{c_t\}\right] - \left[n_t - E_{t-1}\{n_t\}\right] + \left[Q_t - E_{t-1}\{Q_t\}\right] + O(x^2),$$

where

$$Q_t \equiv \log\left(0.94 + a\exp(y_t - k_t)\right),$$

and $O(x^2)$ represents terms of order x^2 or higher where x is the vector of deviations of c, y and k from their non-stochastic balanced growth paths.

6) Obtaining Instruments

To recover estimates of the parameters $\{\alpha,\beta,\gamma,\rho,a,\bar{u},\bar{v}\}$ we use an instrumental variables estimator on the system:

(13) $$dy_t = \alpha\, dk_t + \beta\, dz_t + (1-\alpha-\beta)dn_t + \bar{u} + (u_t - \bar{u}),$$

(14) $$dc_t = dy_t - (1+\gamma)\,dz_t + (1+\gamma)\,dn_t - \bar{v} - (v_t - \bar{v}),$$

(15) $$dc_t = dn_t - \rho + \log\left(0.94 + a\exp[y_t - k_t]\right) + w_t.$$

Since the capital stock series is constructed from the identity (1), it is predetermined at date t; we also treat the logarithm of population, n_t as exogenous. Equations (13), (14) and (15) thus constitute three simultaneous equations in differences of the three endogenous variables dc_t, dy_t, dz_t, the level of the endogenous variable y_t and the two exogenous variables k_t and dn_t. To estimate the system, we need instruments for the differences of the endogenous variables dy and dz and the level of the endogenous variable y, that are highly correlated with dy, dz and y but are uncorrelated with the error terms u,v and w.

To find instruments for the endogenous variables we show, in appendix B, that the reduced form of the system can be written as a pair of non-linear difference equations in the state variables c_t and k_t:

(16)
$$dc_{t+1} = f1\left(c_t, k_{t+1}, n_{t+1}, dn_{t+1}, w_{t+1}, u_{t+1}, v_{t+1}\right),$$
$$dk_{t+1} \equiv f2\left(k_t, c_t, n_t; U_t, V_t\right)$$

The work of Benhabib and Farmer [1994] showed that when there are no stochastic disturbances, that is when {w,u,v,dn} is identically zero, this system is locally stable around the balanced growth path if the parameters of the system satisfy the condition:[7]

(17) $$\beta > 1+\gamma.$$

As we illustrate in section (9), this condition is equivalent to the statement that the labor demand curve slopes up with a slope that is steeper than the slope of the labor supply curve. The importance of the condition is that, when it holds in practice, the economy no longer displays a unique rational expectations equilibrium. For any given initial stock of capital there will be a continuum of paths each converging back to the steady state. In stochastic versions of this economy, condition (17) is sufficient for the existence of multiple *stationary* rational expectations equilibria each of which is associated with an arbitrary time series process for the Euler equation error w_t.

In more standard examples of rational expectations models, it is typical to find that the reduced form of the system (16) displays "saddle path stability". In other words, the linearization of the dynamics around the balanced growth path leads to a dynamical system for which the matrix that governs stability has one root outside, and one root inside, the unit circle. In this more familiar case one must solve one root of the linearized system "forwards" to find the unique initial value of consumption that is consistent with paths that converge back to the stationary state. The non-linear reduced form of a standard real business cycle model has a simpler structure than (16) given by: [8]

(16') $$\begin{aligned} dc_{t+1} &= \tilde{f}1\left(k_{t+1}, n_{t+1}, dn_{t+1}; u_{t+1}, v_{t+1}\right), \\ dk_{t+1} &\equiv f2\left(k_t, c_t, n_t; U_t\ V_t\right). \end{aligned}$$

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[7] The Benhabib–Farmer condition for indeterminacy is necessary and sufficient only in the continuous time model. In discrete time it is a necessary condition, but the gap between β and $1+\gamma$ that is sufficient to allow for indeterminacy increases as the period of the model gets larger.

[8] Farmer [1993] discusses the theoretical issues involved with multiple rational expectations equilibria in more depth than we are able to cover in this paper. If the model was exactly linear, the function $\tilde{f}1(\cdot)$ would correspond to the eigenvector associated with the root of the difference equation that was within the unit circle in the representation for which the state variables are written as functions of their own *future* values. In non-stochastic systems, the solution process is equivalent to choosing c_{t+1} as a function of k_{t+1} and n_{t+1} to place the system on the convergent branch of a saddle path.

The first equation of the system (16') (the real business cycle reduced form) is simpler than (16) (the Benhabib-Farmer reduced form) since lagged consumption "c_t" and the Euler equation error, "w_{t+1}" do not independently enter the function that determines consumption in period t+1. In the real business cycle reduced form, consumption does not vary independently of the exogenous variables k and n and consequently lagged consumption, would not be expected to be a good instrument for current consumption. In indeterminate systems, on the other hand, there is information in the movement of lagged consumption that helps to predict current consumption and which is uncorrelated with the contemporaneous shocks to u, v and w. In our econometric work we exploit this idea to use lagged values of the endogenous variables as instruments for current values of these variables. Our instruments are valid under the maintained hypothesis that the system is indeterminate, that is, under the hypothesis that β is greater than $1+\gamma$.

7) Estimating the Parameters of the Propagation Mechanism

In the introduction to the paper we claimed that our study would shed light on both the propagation mechanism and the relative importance of different impulses to the business cycle in the United States. We begin by reporting estimates of the structural model that are potentially responsible for propagating an impulse through the system. We are particularly interested in the hypothesis that β is greater than $1+\gamma$. Sections (7) and (8) report our findings on this matter which we refer to as estimates of the parameters of the propagation mechanism. In addition to uncovering the propagation parameters, our study is also capable of discovering the relative magnitude of different impulses to the system by estimating the parameters of the variance covariance matrix of different shocks. We refer to this evidence as estimates of the impulses to the system and we report our evidence on the impulse to the US business cycle in section (9).

Our data consists of 60 annual observations for the period from 1929 through 1988 on the logarithms of consumption, gdp, capital, employment, and population which we refer to as c, y, k, z and n. We also constructed the logarithmic differences of each series which we refer to as dc, dy, dk, dz and dn. Our study estimates equations (13) through (15) by weighted two-stage least squares using lagged values of dc, dz, and dy, lagged values of c, y, z, and k, and contemporaneous values of dk and dn as instruments. Attempts to estimate the system by iterated three stage least squares, a method that is asymptotically equivalent to full information maximum likelihood, were unsuccessful as the variance-covariance matrix of the residuals is near singular. We report our estimates of the variance matrix of the driving shocks of the model in section (9). Our weighted two-stage least squares estimates suggest that one of our equations is mis-specified which is further reason to avoid using a systems technique that might contaminate parameter estimates of correctly specified equations. The method of weighted two-stage least squares is equivalent to two stage least squares on each equation with the exception that the objective

Table 1: Estimated Equations	*Unrestricted and Restricted Equations Estimated by Weighted Two-Stage Least Squares*	*Determinant of Residual Covariance*
Unrestricted	(i) $dy = C(1) + C(2)\,dz + C(3)\,dk + C(4)\,dn$ (ii) $dc = C(5) + C(6)dz + C(7)\,dy + C(8)\,dn$ (iii) $dc = C(9) + C(10)\,dn + \log[0.94 + C(11)\exp(y - k)]$	1.81E-10
Restricted	(i) $dy = \bar{u} + \beta dz + \alpha\,dk + (1-\alpha-\beta)dn$ (ii) $dc = -\bar{v} - (1+\gamma)dz + dy + (1+\gamma)dn$ (iii) $dc = -\rho + dn + \log[0.94 + a\exp(y - k)]$	1.91E-10

function is weighted by the inverse of the estimated variance of the residuals from each equation. The choice of weighted two stage least squares as an estimation technique allows us to conduct joint hypothesis tests on parameter restrictions across equations of the system whilst avoiding the cross-equation contamination from mis-specification referred to earlier.[9]

Table 1 lists the equations that we estimated in restricted and unrestricted versions and it reports the determinant of the residual covariance matrix that results from applying weighted two stage least squares to the unrestricted and restricted systems. Notice that this determinant is not very different in the two cases implying that the restrictions are likely to be accepted by formal hypothesis tests. The parameter estimates for the full sample of 60 observations are reported in Table 2 for the unrestricted case and for the case in which we impose the parameter restrictions

[9] We also tried estimating each equation by two-stage least squares on each equation separately; this method gives very similar parameter estimates to weighted two-stage least squares.

implied by the theory. The unrestricted system contains eleven parameters and the restricted system contains seven.

Table 2	*Coefficient*	*Std. Error*	*T-Statistic*
	Unrestricted Parameter Estimates		
C(1)	-0.013005	0.012303	-1.057099
C(2)	1.234075	0.106475	11.59032
C(3)	0.060988	0.153604	0.397048
C(4)	1.681520	1.053550	1.596051
C(5)	0.012323	0.005978	2.061332
C(6)	-0.255425	0.145206	-1.759044
C(7)	0.898716	0.110620	8.124381
C(8)	-0.385579	0.503193	-0.766263
C(9)	-0.043405	0.046257	-0.938352
C(10)	0.409386	1.494178	0.273987
C(11)	0.294383	0.108484	2.713608

	Coefficient	*Std. Error*	*T–statistic*
	Restricted Parameter Estimates		
α	0.158928	0.153418	1.035914
β	1.403088	0.119683	11.72340
γ	-0.557008	0.059129	-9.420291
ρ	0.049086	0.045193	1.086134
a	0.289822	0.107108	2.705872
$\bar{u}$	0.006822	0.004829	1.412639
$\bar{v}$	-0.003052	0.001938	-1.574523

Wald tests for the four implied parameter restrictions are reported in Table 3. The restrictions that generate hypothesis H1 and hypothesis H3 are equivalent to the assumption that only per-capita variables are important in the labor market. One would not necessarily expect these hypotheses to hold in an economy in which externalities matter since population growth may generate scale economies that cause the use of factors to become more efficient as the economy adds more people. There is some evidence for this possibility in the fact that H1 has a p–value of only 5% suggesting that the specification of externalities at the level of the individual family, as maintained in our study, may be incorrect. The per capita specification is more parsimonious than an alternative specification that would allow population to enter the production function with a separate coefficient and since it is not is not wildly at odds with the data we have chosen to maintain the hypothesis in the remaining part of our study.

Table 3: Wald Tests	*Null Hypothesis*	*Chi-Square*	*P– Value*
H1:	C(4)=1-C(2)-C(3)	3.82	0.05
H2:	C(7)=1	0.84	0.36
H3:	C(6)+C(8)=0	1.24	0.26
H4:	C(10)=1	0.15	0.69
Joint Test of H1 - H4	C(4)=1-C(2)-C(3) C(7)=1 C(6)+C(8)=0 C(10)=1	9.02	0.06

Hypothesis H2 tests the restriction that gdp enters the labor market clearing equation with a unit coefficient. This restriction is implied by the Cobb-Douglas specification of the production function and the fact that it has a relatively high p–value (36%) is supportive of this specification. Finally, the hypothesis H4 tests the assumption that it is per capita consumption that enters the Euler equation; this hypothesis has a p–value of 69%. A further indication of how well these restrictions fit the data is given in Figures 4 and 5 which graph the residuals from the restricted and unrestricted estimates of the equations of the system.

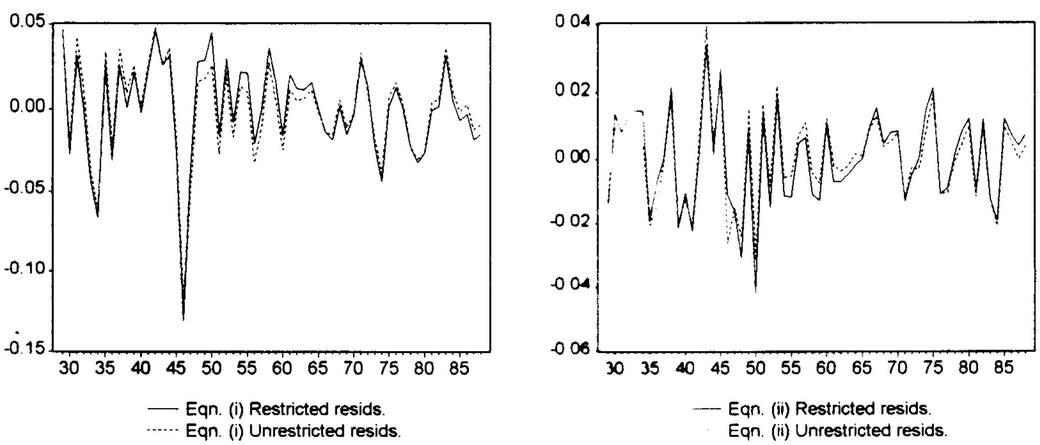

Figure 4: Restricted and Unrestricted Residuals from Equations (i) and (ii)

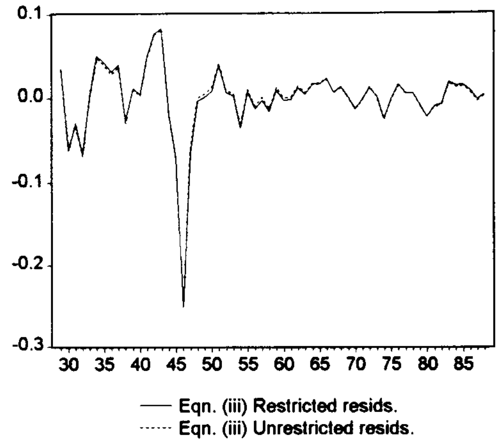

Figure 5: Restricted and Unrestricted Residuals from Equation (iii)

Although we have stressed the fact that the four restrictions reported in Table 3 fare relatively well by the standards of much of the literature that tests equilibrium models of the business cycle, it should be noted that our model is already relatively loosely parameterized by the standards of real business cycle models. In particular, we have relaxed the restrictions α=a, β=b and $\alpha+\beta=1$. We did not estimate the parameter b in this study although estimates in the calibration literature range from 0.5 to 0.7 based on labor's share of gdp. The Wald test of the hypothesis $\beta=0.7$ has a zero p–value based on our estimates. The hypothesis α=a fares much better with a p-value of 67% but this is mainly because there is very little information in the sample on the value of α. The constant returns to scale assumption, $\alpha+\beta=1$ also fares badly with a p-value of 0.3% and the combined hypothesis of constant returns, with α=a, fares even less well with a p–value of 0.1%. Table 4 collects together the information on these additional tests of the restricted model.

Table 4: Wald Tests of the RBC restrictions	*Null Hypothesis*	*Chi-Square*	*P– Value*
H6:	$\beta = 0.7$	29.2	0.00
H7:	$\alpha = a$	0.17	0.67
H8:	$\alpha + \beta = 1$	8.67	0.003
H9:	$\alpha + \beta = 1$, $\alpha = a$	17.69	0.001

We have reported the results of two sets of hypothesis test of our model – and we have argued that our specification fares relatively well using the criterion of tests of implied parameter restrictions. Further statistics of goodness of fit for the two sets of regressions are reported in Table 5. For the most part these statistics are supportive of the maintained hypothesis that the structural form is a good representation of the data; with one important exception. For the structural estimates to be valid, the errors to each equation should be serially uncorrelated. Diagnostic tests on the residuals from equations (i) and (ii) support the hypothesis that u and v are serially uncorrelated, but the residuals from equation (iii) show a very low value of the Durbin Watson statistic. This statistic should be close to 2 under the hypothesis of no serial correlation.

Table 5	*EQN. (i)*		*EQN. (ii)*		*EQN. (iii)*	
	Unrest.	Restr.	Unrest.	Restr.	Unrest.	Restr.
R-squared	0.83	0.81	0.93	0.92	0.30	0.29
Adjusted R-squared	0.82	0.80	0.93	0.92	0.28	0.28
S.E. of regression	0.029	0.031	0.014	0.015	0.044	0.044
Sum squared resid	0.049	0.053	0.011	0.013	0.113	0.114
Durbin-Watson stat	1.60	1.55	2.13	2.09	1.11	1.093

To check the hypothesis of uncorrelated errors, we report the Ljung-Box Q statistics for the first ten lags from the residuals to each equation in Table 6. Under the hypothesis of no-serial-correlation the Q statistic has a chi-squared distribution. Table 6 reports the p–value for this hypothesis at each of the first ten lags for the three sets of residuals from the restricted regressions. The properties of the residuals from equation (i), the production function, and equation (ii), the labor market clearing equation, allow one to accept the hypothesis of no serial correlation. The residuals from the Euler equation (equation (iii)), however, display significant auto-correlation implying that this equation may be mis-specified. We are aware of more general versions of general equilibrium models that nest the model studied in this paper by generalizing equation (iii). In particular, versions of the overlapping generations model studied by Blanchard [1985] and Weil [1989]. In future work, we plan to estimate models in this class in the hope of explaining the failure of the representative agent's Euler equation. In this study, however, we merely note that out estimates of the parameters ρ and a that appear in equation (iii) are likely to be biased.

Table 6:	*EQN. (i)*		*EQN. (ii)*		*EQN. (iii)*	
Lag	Q	P- Value	Q	P- Value	Q	P- Value
1	1.7	0.18	0.19	0.66	12.64	0.00
2	2.7	0.26	1.19	0.55	13.23	0.00
3	5.8	0.12	5.78	0.12	19.11	0.00
4	6.9	0.13	5.78	0.22	26.84	0.00
5	7.3	0.20	7.72	0.17	32.16	0.00
6	8.3	0.22	8.22	0.22	32.17	0.00
7	8.3	0.30	8.26	0.31	32.35	0.00
8	9.2	0.32	8.52	0.39	35.51	0.00
9	9.5	0.39	8.56	0.48	35.80	0.00
10	11.2	0.34	8.57	0.57	37.01	0.00

8) Parameter Constancy

As a further check on the consistency of our structural model, we estimated the structural parameters of our system for recursive samples beginning for the period 1929 through 1948 and adding one year of data at a time. Figures 6 and 7 report the results of these recursive estimates. The solid line in each figure plots the point estimate of each parameter for successively larger samples and the dashed lines plot two-standard error bounds. We find a high degree of parameter stability over the sample period and for the most part our estimates fall within a range that conforms with our prior. The main exception is the finding of a negative value of γ which does not conform to our initial expectations. However, as we report below, although the finding of negative γ is inconsistent with globally convex indifference curves, it *is* nevertheless possible to make economic sense of the value of γ that we find in practice.

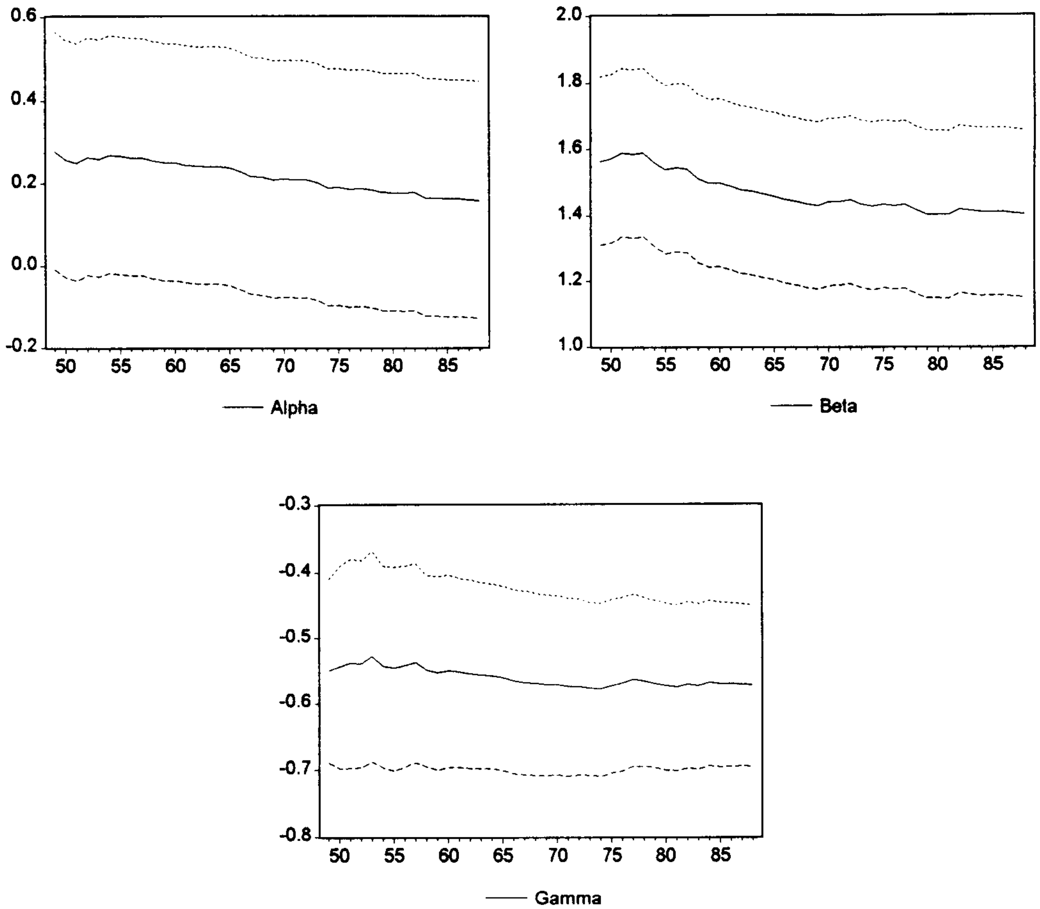

Figure 6: Recursive Estimates of α, β and γ

The parameter α represents the elasticity of real gdp with respect to capital in the *social* production function. Our point estimate of α is low at 0.16 although the parameter is very imprecisely estimated. The parameter β is important for our study since it is key to understanding the dynamics of an indeterminate system. Our earlier work showed that a value of β greater than $1+\gamma$ is a necessary condition for the system to display an indeterminate steady state; our point estimate of β is a little more than 1.4 and we are able to bound this away from 1 for the entire sample period by two standard errors. Since our estimate of γ is negative and less than one in absolute value, we find that in the US data the indeterminacy condition is satisfied. Our finding of a value of β greater than unity means that, not only does estimate of the social production function violate the assumption of constant returns to scale in capital and labor, it also satisfies the property of increasing returns to scale in labor alone. It has long been known that a regression

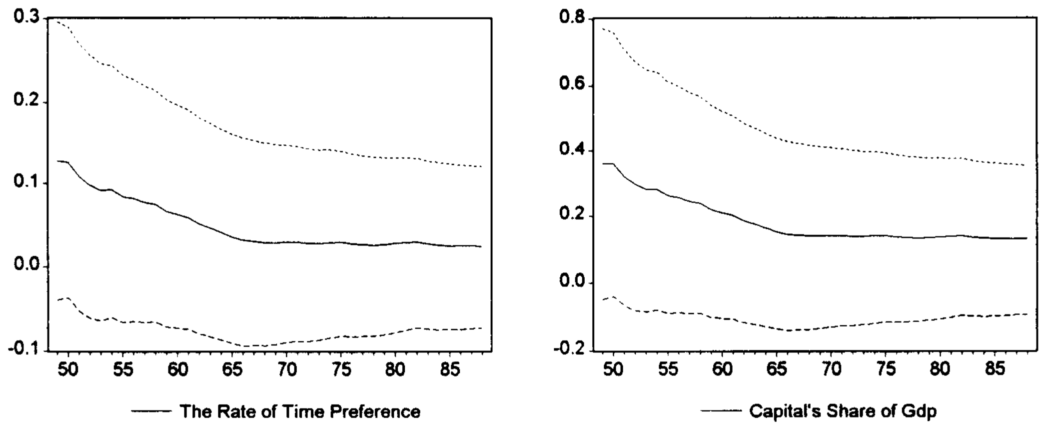

Figure 7: Recursive Estimates of ρ and a

of gdp on employment leads to a coefficient bigger than unity but the large magnitude of the least squares regression coefficient has usually been ascribed to simultaneous equations bias; employment is correlated with the error term in an OLS regression. Our analysis suggests that this interpretation of the relative magnitudes of employment and output fluctuations at business cycle frequencies is mistaken since we continue to estimate high values of β *even after accounting for simultaneous equations bias with instrumental variables*[10].

Figure 7 reports recursive estimates of "ρ", the rate of time preference and "a", the share of gdp going to capital. Although these estimates fall within bounds that we find a priori reasonable, 0.04 for the rate of time preference and 0.2 for capital's share of gdp, there are reasons to be skeptical of the specification of this equation since we find the Euler equation error to be autocorrelated, a fact that is inconsistent with the assumption that it is unforecastable.

[10] This result is consistent with the work reported by Caballero and Lyons [1992] for disaggregated data and the results of Baxter and King [1991] using quarterly US data.

9) Estimating the Impulse

Along with the parameters of the propagation mechanism, we also estimated the means and the variance-covariance matrix of the disturbances to our model. The two parameters that explain growth and the change in labor supply per person over time are represented by the "drift" parameters $\bar{u}$ and $\bar{v}$. We estimated $\bar{u}$ to be equal to 0.6% per year and $\bar{v}$ to equal -0.3% per year although both parameters have large standard errors. A negative drift for V implies that the disutility of labor effort has been falling over time which is how our specification accounts for the slow upward movement in labor supply over the sample period.

To estimate the variance-covariance matrix of the shocks u,v and w, we regressed the squared residuals and the cross products of residuals from each equation on constants for successively increasing samples beginning with 1929-1948 and ending with 1929-1988. This gave us a set of six regressions on constants that we estimated by ordinary least squares. The results of these six recursive regressions are reported in Figures 9 and 10. There is a strong tendency for the diagonal elements of the variance matrix (Figure 9) to fall over time reflecting the fact that the economy has been more stable in the post-war period at least using measured statistics. The covariance terms (graphed in Figure 10) are all small and the covariances of w with u and v are both zero. Our prior belief was that all of the covariance terms would be zero but we cannot reject the fact that u and v are negatively correlated.[11]

[11] We believe that this negative correlation indicates possible misspecification in our labor market equations. It occurs because productivity shocks are associated with upward shifts of the labor demand curve. Since the model is setting the slope of the labor demand curve upward sloping *more steeply than the slope of the labor supply curve*, a positive productivity shock tends to *reduce* employment. To the extent that productivity shocks are responsible for business fluctuations, the model tries to offset this effect by causing the labor supply curve to shift at the same time that the labor demand curve shifts.

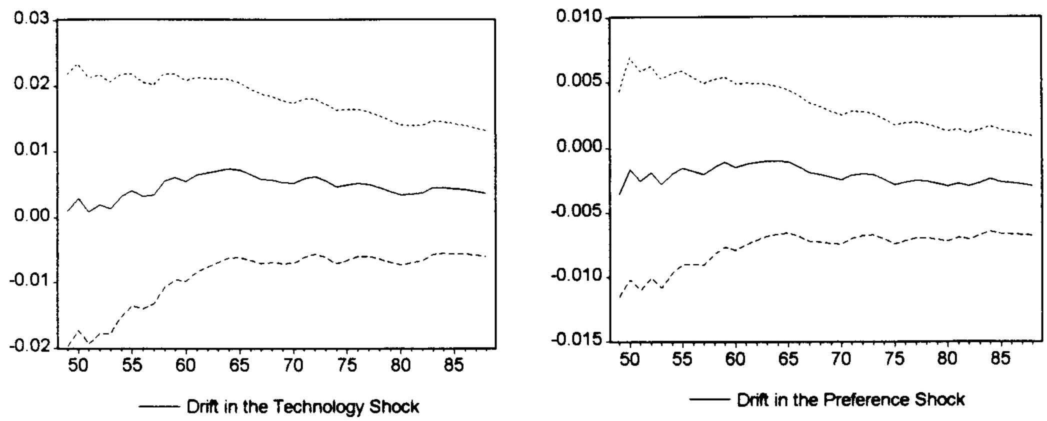

Figure 8: The Drift Parameters of the Random Walks u and v

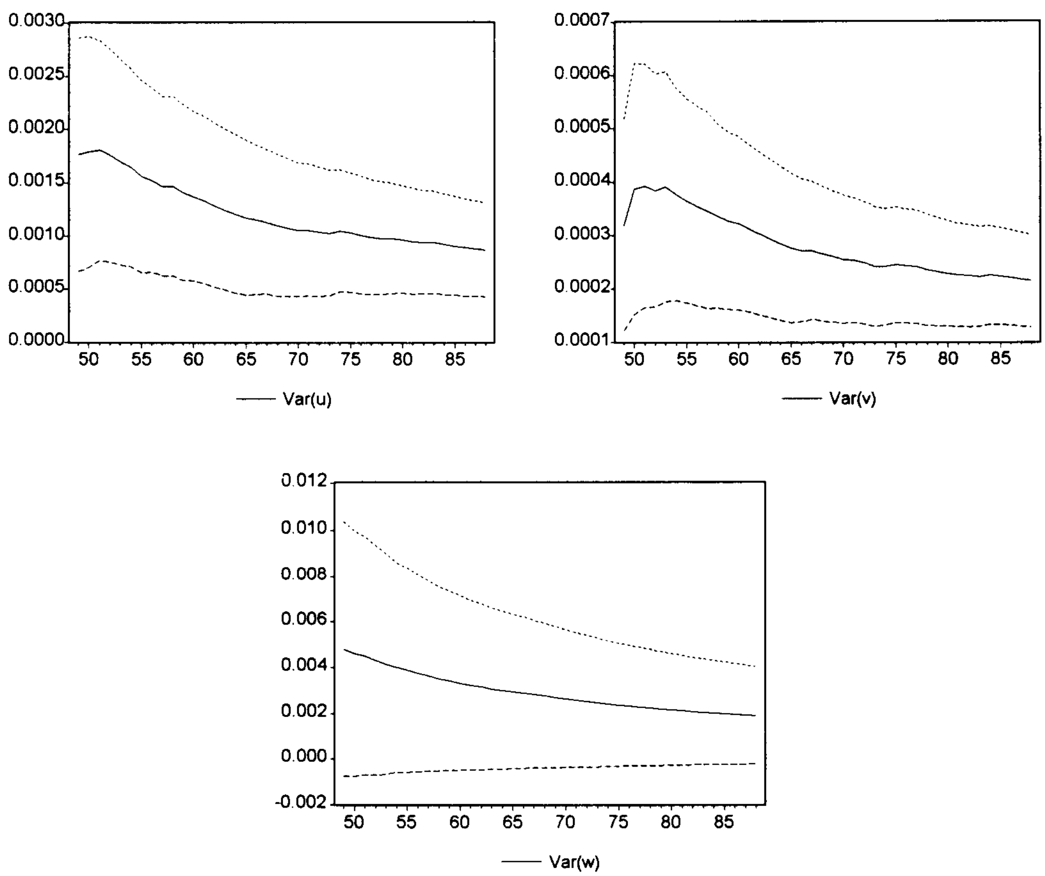

Figure 9: Recursive Estimates of the Diagonal Elements of the VCV Matrix of the Shocks

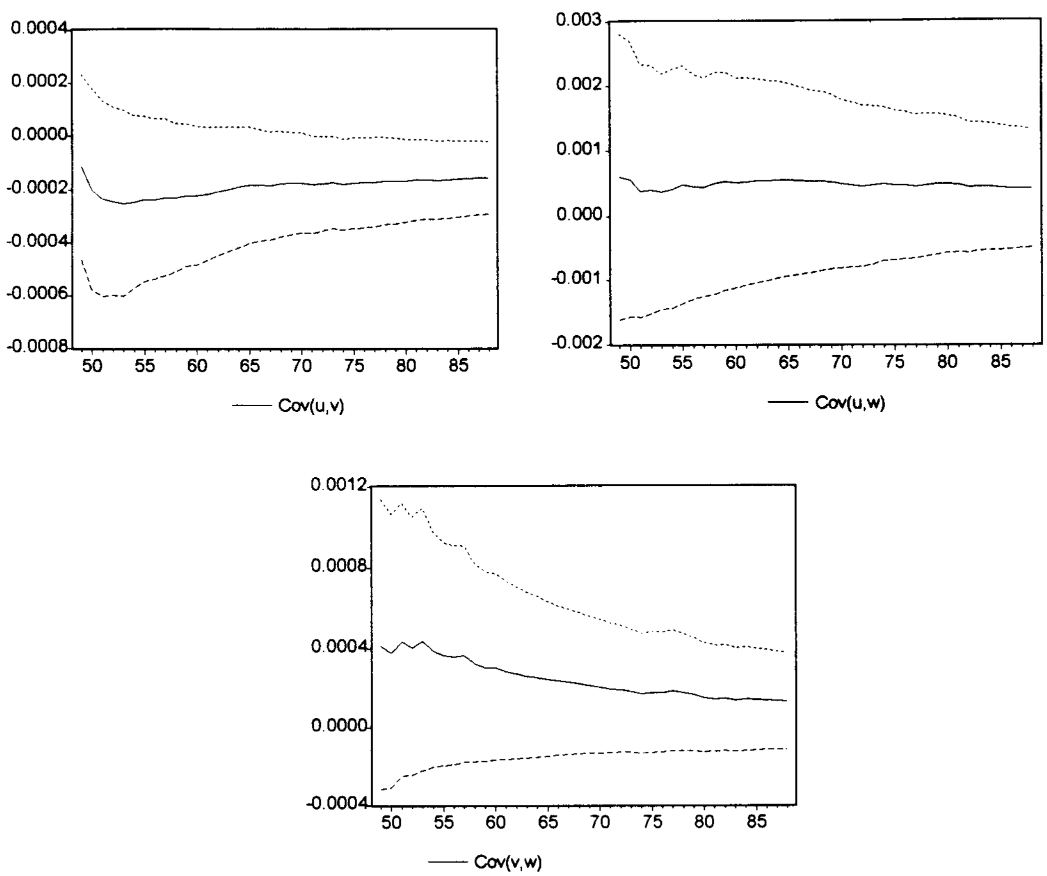

Figure 10: Recursive Estimates of the Off-Diagonal Elements of the VCV Matrix of the Shocks

Tables 7, 8 and 9 report our estimates of the elements of the variance covariance matrix of the residual together with t-statistics (in parentheses). Table 7 reports estimates for the entire sample and Tables 8 and 9 split the sample in 1950 to illustrate the point that the variance of the shocks to the economy has dropped dramatically in the post-war period. We suspect that the main reason for this drop in variance is the fact the second World War and the Great Depression, the two most significant macroeconomic events of the century, both occurred in the first part of the sample although it is also possible that changes in the methods of data collection are partly responsible for this effect as suggested by the work of Christina Romer [1986]. Each of the tables reports an "approximate" variance covariance matrix of the shocks that rounds up the point estimates, scales them by a factor of 10^4 and sets the insignificant elements to zero. For the entire sample, shocks to productivity, u, and Euler equation errors, w, dominate the preference shocks "v" to the labor market clearing equation. If we identify "u" with "supply shocks" and "w" with "demand shocks"

Exact VCV Matrix[12]

1929-1988

Table 7	u	v	w
u	0. 000871 (3.882359)	-0.000156 (2.529995)	0.000424 (1.044602)
v	-0.000156 (2.529995)	0.000214 (5.193638)	0.000131 (0.763579)
w	0.000424 (1.044602)	0.000131 (0.763579)	0.001908 (1.803317)

Approx. VCV Matrix

1929-1988

	u	v	w
u	10	-1	0
v	-1	2	0
w	0	0	20

Exact VCV Matrix

1929-1949

Table 8	u	v	w
u	0.001797 (3.227344)	-0.000174 (0.981370)	0.000381 (1.044977)
v	-0.000174 (0.981370)	0.000335 (3.312943)	0.000565 (0.524830)
w	0.000381 (1.044977)	0.000565 (0.524830)	0.004919 (1.749583)

Approx. VCV Matrix

1929-1949

	u	v	w
u	20	-2	0
v	-2	3	0
w	0	0	50

Exact VCV Matrix

1950-1988

Table 9	u	v	w
u	0.000234 (3.981910)	-0.000106 (3.181049)	5.45E-05 (1.543076)
v	-0.000106 (3.181049)	0.000161 (3.517864)	-1.35E-05 (0.558053)
w	5.45E-05 (1.543076)	-1.35E-05 (0.558053)	0.000162 (3.459460)

Approx. VCV Matrix

1950-1988

	u	v	w
u	2	-1	0
v	-1	2	0
w	0	0	1.5

[12] For the exact matrices t-statistics are reported in parentheses. Approximate elements of the VCV are obtained by rounding the exact figures, scaling by 10^4, and setting insignificant elements to zero.

our model finds that over the entire sample demand shocks are roughly twice as important as supply shocks. The breakdown into two sub-periods indicates, however, that since the war both sources of disturbances have been of comparable magnitude and both are an order of magnitude smaller than in the pre-war period.[13]

In earlier work (Farmer and Guo [1994]) we identified Euler equation errors with "animal spirits". We are more reluctant to make this connection in the present study since our consumption series adds together private and government consumption and, to the extent that these two variables are not perfect substitutes, our model is mis-specified. This mis-specification will show up as fluctuations in w that arise as a result of changes in government purchases. A good example of this phenomenon is the increase in government spending during the second World War, an increase that generates the largest residuals in our sample to both u and w. But although we are reluctant to identify Euler equation errors with animal spirits, it *is* possible, nevertheless, to identify fluctuations in w with "demand shocks" and fluctuations in u with "supply shocks" and using this identification our estimates suggest that demand shocks are roughly twice as important as supply shocks over the entire sample in the sense that the variance of w is twice that of u. In the post-war period, on the other hand, this situation is reversed and since 1950 supply shocks, identified as the variance of u, have been a little more important than demand shocks as causes of business cycle fluctuations.

10) The Labor Market

Although we have talked in terms of labor demand curves and labor supply curves, the estimates of the parameters β and γ reported in sections (6) and (7) did not make use of wage data. As a check on the consistency of our explanation we estimated the equations:

$$dw_t = \gamma\left(dz_t - dn_t\right) + \left(dc_t - dn_t\right) + \bar{v} + v_t, \tag{17}$$

$$dw_t = \alpha\left(dk_t - dn_t\right) + (\beta - 1)\left(dz_t - dn_t\right) + \bar{\varepsilon} + \varepsilon_t, \tag{18}$$

[13] Robert Hall [1986] has argued that consumption shocks have been responsible for considerable portions of post-war US business cycle fluctuations. Our evidence suggests that the role of consumption shocks may be even greater in the pre-war data.

where dw is the log difference of the real wage. Equation (17) equates the real wage to the slope of the representative family's indifference curve and equation (18) equates the real wage to the slope of the production function. Both equations are estimated in log differences.

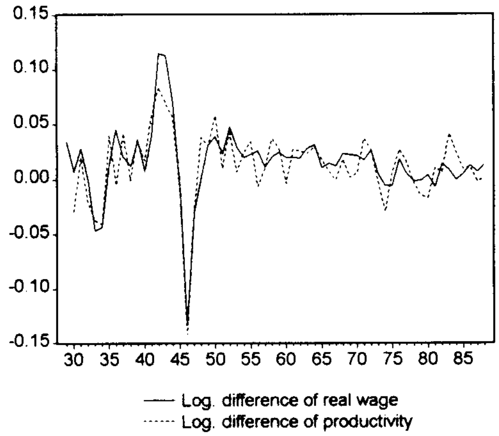

Figure 11: The Real wage and Labor Productivity

To construct the real wage series used to estimate these equations, we divided compensation to employees, from the national income and product accounts, by full time equivalent employees to give a measure of the nominal wage and we weighted this series by the gdp deflator to give our measure of the real wage, w. Figure 11 plots the first difference of our real wage series against labor productivity (y-n). These series were assumed to be identical in our earlier work since we ascribed all sources of disturbance in the labor market to the taste shift parameter, v. Although the hypothesis that the series are identical is clearly false, productivity and the real wage have moved closely together with a correlation coefficient of 0.85.

Table 10	*Coefficient*	*Std. Error*	*T-Statistic*
$\bar{\varepsilon}$	-0.011980	0.001849	-6.480692
γ	-0.602510	0.056224	-10.71630
$\bar{v}$	0.004390	0.004483	0.979103
$\beta-1$	0.465429	0.144056	3.230889
α	0.326190	0.110273	2.958030

Table 10 reports the results of estimating equations (17) and (18) by two-stage least squares. The key parameters β and γ are estimated at -0.6 and 1.46 using real wage data as opposed to point estimates of -0.56 and 1.4 in the work reported in sections (6) and (7). Since the estimates in these sections did not make use of data on the real wage, the finding that our estimates in Table 10 are remarkably close to our previous estimates is further confirmation of the fit of our theory with the evidence.

To provide visual evidence of the fit of our labor demand and supply curves, in Figure 12 we have broken down the two stage least squares estimates into stages and graphed the implied labor demand and supply curves. The points on these curves are constructed by running the first stage regressions to generate instruments for consumption and labor supply. The second stage equations are then estimated in steps by regressing forecast labor supply and the wage on the remaining variables in each equation and plotting the unexplained components of the wage and employment against each other. The figure illustrates the fact that this procedure leads to estimates of a labor *demand curve that slopes up* (β is bigger than one) and an estimate of *labor supply that slopes down* (γ is negative).

Figure 12: Estimates of Demand and Supply Curves for the US Labor Market

Since the assumption that $\gamma > 0$ is required for the period utility function

$$(19) \qquad U = \log(C) - \frac{Z^{1+\gamma}}{1+\gamma}$$

to be quasi-concave the finding that γ is negative in our estimates is potentially a problem since it implies that the households maximization problem may not admit an interior solution. In the following section we discuss two potential resolutions of this puzzle.

(11) Puzzles and Extensions

The finding that "labor supply slopes down" follows directly from our identification assumption. Capital enters the labor demand curve and not the labor supply curve. Consumption, on the other hand, enters labor supply but not labor demand. Our reaction to this finding was to try to defend it by plotting the indifference curves each period implied by our estimates. These curves are plotted in Figure 13 and our initial reaction was to be encouraged, since the one period problem implied by these preferences has a well defined interior solution. However, this is not true for the intertemporal problem since an individual with the preferences in Figure 13 would try to bunch his labor supply in one period.[14]

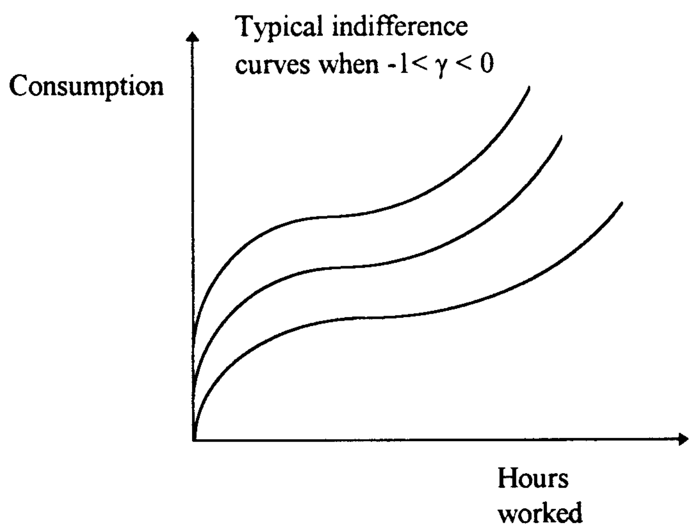

Figure 13: Typical Indifference Curves Implied by Our Parameter Estimates

A second possibility that we think deserves consideration is that the spot model of the labor market is simply wrong. Simple search models suggest that the probability of a match in the

[14] We thank Michael Woodford for pointing this out to us in private correspondence.

labor market depends not only the number of suppliers but also on the number of demanders. We think that simple search models, if grafted onto models of the kind presented in this paper, will likely invalidate our identification assumptions and might potentially resolve the labor supply issue. However, at the present time we are unable to formulate such a model in a way that nests existing approaches in a simple way.

As a final possibility we have begun to pursue the idea that supply slopes down because preferences are non-separable in consumption and labor supply. A simple example of a period utility function with the right properties is described by equation (20)[15]:

$$U = \frac{(C-A)^{1-\theta}}{1-\theta} Z^{\chi}, \qquad \theta > 1, \quad \chi > 0, \tag{20}$$

where the parameter "A" measures subsistence consumption, C is consumption and Z is labor supply. The period indifference curves for this utility function are graphed in Figure 14.

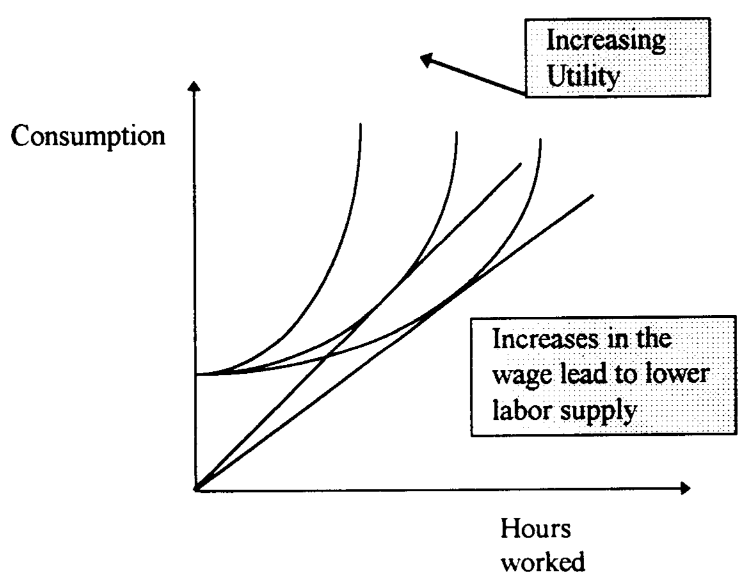

Figure 14: The Preference Map for the Non-Separable Preferences Described by Equation (20)

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[15] Note that U is an increasing concave function of C, for C greater than minimum consumption, A, and it is a decreasing convex function of labor supply, Z.

These preferences imply that labor supply will fall as the real wage increases as the income effect dominates the substitution effect everywhere. In the one period problem in which consumption equals the real value of labor supply, this is easily seen in Figure 14 – as the budget line rotates up so labor supply drops. It is also true that the labor supply curve generated by these preferences, holding constant consumption, is a decreasing function of the real wage.

Our current view is that non-separability of preferences is a promising avenue for future work although pursuing this idea further is well beyond the scope of this paper. We are unaware, as yet, of the theoretical properties of models in this class; for example, we do not know the theoretical conditions for indeterminacy in this case. Non-separable preferences of the kind described in equation (20) also introduce non-stationary elements to a growth model since the effects of subsistence consumption become smaller as society grows. Finally, preferences of this kind lead to an Euler equation in which not only consumption growth, but also employment growth, affects the intertemporal conditions. As a preliminary check on the likely implications of moving to a non-separable specification, we tried re-estimating the model including labor supply in the Euler equation. We found that employment was significant in this estimation, and more

Table 11 *Lag*	*Q-Stat*	*Prob*
1	0.1825	0.669
2	6.5462	0.038
3	6.6118	0.085
4	6.6145	0.158
5	6.7109	0.243
6	6.9864	0.322
7	6.9899	0.430
8	7.1560	0.520
9	7.4039	0.595
10	7.4580	0.682

importantly, we found that including labor supply growth in the Euler equation goes some way to removing the residual auto-correlation in this equation. Table 11 presents the Box-Ljung statistics for the first ten lags of the Euler equation amended to include employment growth.

Recall that these statistics provide evidence of the null hypothesis of no auto-correlation in the residuals at each lag. Compare the P-values in Table 11 with the p-values for the third equation in Table 6 earlier in the paper. Without including employment growth, the p-value of no autocorrelation was zero at every lag. With the inclusion of labor supply growth in the Euler equation these residuals considerably more white. This result gives us some hope that a move to the non-separable case may well solve two of the puzzles thrown up by our work in one step.

11) Conclusion

This paper has argued that annual US data is well described by an intertemporal general equilibrium model in which there is an indeterminate balanced growth path. It has been known for some time that general equilibrium models could display indeterminate equilibria but this result has typically been taken to be a theoretical curiosity. In earlier work with Michael Woodford (Farmer and Woodford [1984]), Farmer has argued that indeterminacy is important because in models with indeterminate equilibria it is relatively easy to construct examples of business fluctuations that are driven by "belief shocks"[16]. The Farmer-Woodford example, however, was a long way from resembling an economic model that might be convincingly matched with time series data. Benhabib and Farmer provided an additional reason to be interested in models with indeterminacy by showing that the familiar real business cycle model of the economy, when supplemented by empirically plausible externalities, could provide an explanation of the propagation mechanism of US business cycles. Farmer and Guo [1994] calibrated an economy in this class and showed that it is capable of providing an explanation of business fluctuations with the same degree of precision as the real business cycle model. Our current paper takes this previous work several steps further by providing a complete econometric model of the economy and estimating the parameters of this model using annual data. Our estimated model remains structurally stable over sixty years of US data and it provides an explanation of the impulses and the propagation mechanism of US business cycles that can be interpreted in terms of the rational choices of goal seeking individual agents in a well specified economic environment.

[16] Azariadis [1981] calls these belief shocks "self-fulfilling prophecies". Cass and Shell [1983] use the term "sunspots" and Howitt and McAfee [1992] prefer the term "animal spirits". In all cases these terms refer to allocations in general equilibrium models that are different across states of nature even though preferences, endowments and technologies remain unaltered.

The fact that a theoretical model displays indeterminacy has often been cited as a reason for avoiding such a theory; it might be thought that indeterminacy means that one is unable to use a model to make predictions about the future path of the variables that one is trying to explain. This argument is, however, incorrect since as long as agents use the same method in each period to forecast the future the prices and quantities that they forecast will be qualitatively no less predictable than in an environment with a unique rational expectations equilibrium. The effect of indeterminacy is to admit an additional possible source of fluctuations to the economy; in addition to shocks to preferences and technologies, business cycles may also be driven by the self-fulfilling beliefs of individual agents.

We believe that general equilibrium models that permit the existence of indeterminate equilibria may potentially explain a wide range of business cycle phenomena that are otherwise difficult to understand. In recent related work Beaudry and Devereux [1993] and Jong-Yann Lee [1994] have shown that models in the same class we study here, supplemented with a motive to hold money, can explain the phenomenon of "sticky prices" in the context of a general equilibrium market clearing model. In the international context, Guo and Sturzenegger [1994] have calibrated an international model in which there is an indeterminate equilibrium and shown that this model performs well in comparison with an international real business cycle model at explaining the covariances of consumption, employment and gdp across countries. We hope that the arguments presented here will persuade our colleagues that general equilibrium models with indeterminate equilibria will not only provide interesting qualitative explanations of a wide range of phenomena but that these explanations may also be fleshed out with empirical evidence to provide quantitative assessments of the way that the macroeconomy works.

Appendix A

This appendix derives a stationary representation of the model and shows how to find the balanced growth path. First, define the transformed variables:

(A1) $$\hat{Y} \equiv \frac{Y_t}{N_t\Phi_t}, \quad \hat{C} \equiv \frac{C_t}{N_t\Phi_t}, \quad \hat{K} \equiv \frac{K_t}{N_t\Phi_t} \text{ and } \hat{Z}_t \equiv \frac{\hat{Z}_t}{N_t\Psi_t},$$

where Φ and Ψ are defined implicitly by the equations:

(A2) $$\Phi^{1-\alpha}\Psi^{-\beta} \equiv U, \quad \Psi^{-(1+\gamma)} \equiv V.$$

Using these definitions one may rewrite equations (1) to (4) in terms of transformed variables:

(A3) $$G_{t+1}\hat{K}_{t+1} = \hat{K}_t(1-\delta) + \hat{Y}_t - \hat{C}_t,$$

(A4) $$\hat{Y}_t = A\,\hat{K}_t^{\alpha}\hat{Z}_t^{\beta},$$

(A5) $$\hat{C}_t\hat{Z}_t^{1+\gamma} = b\,\hat{Y}_t,$$

(A6) $$\frac{1}{\hat{C}_t} = E_t\left[\frac{1}{\hat{C}_{t+1}}\left(\frac{1}{1+\rho}\right)\frac{1}{F_{t+1}}\left(0.94 + a\frac{\hat{Y}_{t+1}}{\hat{K}_{t+1}}\right)\right],$$

where $\{F_t\}$ and $\{G_t\}$ are *stationary* stochastic processes defined by the equations:

(A7) $$G_{t+1} \equiv \frac{\Phi_{t+1}N_{t+1}}{\Phi_t\,N_t}, \qquad F_{t+1} \equiv \frac{\Phi_{t+1}}{\Phi_t}.$$

When there is no uncertainty in the model, the variables U and V and N grow at constant rates given by the expressions:

(A8) $$\frac{U_{t+1}}{U_t} \equiv \exp(\bar{u}), \quad \frac{V_{t+1}}{V_t} \equiv \exp(\bar{v}), \quad \frac{N_{t+1}}{N_t} \equiv \exp(\bar{v}),$$

and since Φ is a function of U and V implicitly defined by equation (A2) it follows that, in the non-stochastic steady state, F_t and G_t are also constant. We denote the stationary values of F_t and G_t as F and G. G represents the growth factor of gdp, capital and consumption on the balanced growth path and F represents the exogenous growth factor of population. To derive the balanced growth path of the model, set $F_{t+1} = F$, $G_{t+1} = G$, and solve equations (A3) – (A6) for the steady state values of $\{\hat{K}, \hat{Z}, \hat{C}, \hat{Y}\}$ as functions of the parameters of the model.

Appendix B

This Appendix derives the reduced form equations (16) discussed in section (6). First write the transformed equations (A4) and (A5) in terms of logarithms (using lowercase letter to denote logs.)

(B1) $\hat{y}_t = \alpha \hat{k}_t + \beta \hat{z}_t + \log(A)$,

(B2) $\hat{c}_t + (1+\gamma)\hat{z}_t = \log(b) + \hat{y}_t$,

and solve these equations to write $(\hat{y}_t - \hat{k}_t)$ as a linear function of $\hat{z}_t$ and $\hat{c}_t$:

(B3) $(\hat{y}_t - \hat{k}_t) = \lambda_0 + \lambda_1 \hat{c}_t + \lambda_2 \hat{k}_t$.

Similarly write the transformed Euler Equation (A6) replacing $(\hat{y}_{t+1} - \hat{k}_{t+1})$ from (B3):

(B4) $-\hat{c}_t = E_t\left[-\hat{c}_{t+1} - \rho + \log\left(0.94 + a\exp\left(\lambda_0 + \lambda_1 \hat{c}_{t+1} + \lambda_2 \hat{k}_{t+1}\right)\right) + O\left(x^2\right)\right]$,

which can be rewritten in the form:

(B5) $-\hat{c}_t = -\hat{c}_{t+1} - \rho + \log\left(0.94 + a\exp\left(\lambda_0 + \lambda_1 \hat{c}_{t+1} + \lambda_2 \hat{k}_{t+1}\right)\right) + e_{t+1}$,

where the error term "e" is an expectational difference defined by the expression

(B6) $e_{t+1} \equiv E_t\left[H\left(\hat{c}_{t+1}, \hat{k}_{t+1}\right)\right] - H\left(\hat{c}_{t+1}, \hat{k}_{t+1}\right)$,

and the function $H(\cdot)$ is the negative of the first three terms on the right hand side of (B5). Dropping terms of $O(x^2)$ it follows from the definitions of $\hat{c}$ and $\hat{k}$ that e_{t+1} depends only on expectational differences of consumption and on the innovations to population growth, to U and to V. It follows from the implicit function theorem that around the balanced growth path there exists a function f such that:

(B7) $dc_t = f1\left(c_{t-1}, k_t, n_t, dn_t; w_t, u_t, v_t\right)$, which is the function $f1(\cdot)$ in section (6). The function $f2(\cdot)$ is found directly by expressing the capital accumulation equation in terms of logarithms of the state variables consumption and capital and solving for gdp using (B1) and (B2).

Substituting this expression into (10) and combining it with equation (11) generates the nonlinear reduced form of the system in the two state variables c and k:

$$\text{(B2)} \qquad \begin{aligned} dc_t &= f1\left(c_{t-1,}\, k_t, n_t, dn_t; w_t, u_t, v_t\right), \\ dk_t &\equiv f2\left(k_{t-1}, c_{t-1}, n_{t-1}; U_{t-1}, V_{t-1}\right). \end{aligned}$$

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