EXPERIMENTAL TESTS OF THE PARADOX OF POWER

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Abstract

Rational decision-makers will be equalizing the marginal returns of cooperative effort in production versus fighting over the distribution of output. Given both cooperative and conflictual opportunities, our experimental subjects overwhelmingly arrived at the Nash rather than the cooperative solution -- though with some slippage toward cooperation in experimental treatments that permitted learning. As predicted by the analytic model, more resources were devoted to conflict when decisiveness was high. With one significant exception, the results supported the model's 'paradox of power' predictions, as to when an initially weaker party will improve its position relative to a stronger opponent.

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Individuals, groups, or nations -- if rational and self-interested -- will be equalizing the marginal returns of two main ways of generating income: (1) production and exchange, versus (2) 'appropriative' efforts designed to capture resources previously controlled by other parties (or to defend against the latter's attempts to do the same). This balancing between two ways of making a living has been examined in a number of theoretical studies, among them Haavelmo (1954), Skogh and Stuart (1982), Hirshleifer (1989, 1991), Skaperdas (1992), and Grossman and Kim (1994).

How decision-makers choose between productive and conflictual activities has not heretofore, so far as we could determine, been addressed experimentally. The model offered in Hirshleifer (1991) provides a convenient format for such an investigation. In that model decision-makers simultaneously interact in two ways: through joint production (the cooperative element) and distributive struggle (the conflictual element). As a real-world example, management and labor jointly generate the aggregate output of the firm, yet at the same time contend with one another over the distribution of the firm's net revenues.

The implication of the model that constitutes our central subject is "the paradox of power". It might have been expected that in appropriative struggles between stronger and weaker contenders, the strong would grow ever stronger and the weak always weaker still. The paradox is the observation that in actual contests, poorer or smaller combatants often actually end up improving their position relative to richer or larger opponents. A notable instance is the political struggle over income redistribution. Although citizens in the upper half of the income spectrum surely have more political strength than those in the lower half, modern governments have mainly been transferring income from the former (stronger) to the latter (weaker) group.[1]

The theoretical explanation is that initially weaker or poorer contenders are typically motivated <u>to fight harder</u>, that is, to devote relatively more effort to appropriative (conflictual) effort. Put another way, the marginal payoff of appropriative effort relative to productive effort is typically greater at low levels of income.[2] Thus, while the rich always have the capability of exploiting the poor, it often does not pay them to do so.

Nevertheless, in some social contexts initially richer and more powerful contestants do exploit weaker rivals. Affluent aristocracies often use their power to extort even more

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[1] See. e.g., Browning and Browning (1987), p. 241.

[2] When agricultural prices fell to extraordinarily low levels in the great Depression of 1929-33, Kansas farmers were urged by their leaders to "raise less corn and raise more hell".

resources from the lower classes.[3] So the question is, when does and when does not the paradox of power hold? In the model, the governing factor is a parameter m reflecting the decisiveness of conflictual effort. When decisiveness m is low, the rich are content to concentrate upon producing a larger social pie of income even though the poor will be gaining an improved share thereof. But when conflictual preponderance makes a sufficiently weighty difference for achieved income -- at the extreme, when the battle is "winner take all" -- the rich cannot afford to let the poor win the contest over distributive shares.

Certain game-theoretic and implementational concerns are also addressed in our experimental design. In the strict game-theoretic sense the noncooperative equilibrium is about strangers who meet once, interact strategically in their self-interest, and will never meet again. In such a random-meetings economy, the antagonists have no history or future. Yet in many contexts individuals interact in repeated games, where they must think about the future and possibly learn from the past. In the particularly simple version where the one-shot game is iterated with the same payoffs each round, we have a supergame. The study of such games has been motivated by the

[3] "In Rwanda ... the masses of the people were peasants who were forced to contribute goods and services to the support of a vast and complex political administration. ... although they are said to have gloried in their subjugation, which is a matter in doubt, they received little beyond the minimum reallocations in return for almost the entirety of their production..." Codere (1968), p. 242.

intuition or "folk theorem" that repetition makes cooperation possible (Mertens, 1984). But formal theorems to this effect for finite horizons have not been forthcoming, and interest has settled on experimental studies of both single-play games and supergames, and on variations in the protocol for matching players in repeat play.

McCabe, Rassenti and Smith (1995) study a class of extensive-form games[4] in which the parties move sequentially in a series of rounds. In any round the first-mover can forward signal the desire to cooperate, but the other player can defect. In one game the first player can punish such defections, in the other he/she has no such recourse. If pairs are matched at random for each play, in a repeated sequence of unknown length, subjects gradually learn to cooperate when the punishment option is available; when this option is not available they tend to play non-cooperatively. If instead the same pairs remain matched up for the entire length of the supergame, they tend to achieve cooperation whether or not the opportunity for direct punishment or defection is available. (With the punishment opportunity, however, learning is faster.)

Our experiments, in addition to testing the substantive predictions associated with the paradox of power, will be addressing some of these issues arising in the experimental and

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[4] This term denotes repeated games in which players see only the actually played moves, and thus are not in a position to know what the opponent would have done at points in the game tree that never were reached.

game-theoretic traditions. Specifically, we will be comparing the results of experiments in which the partners are randomly varied in each round with experiments in which the partners are fixed throughout the supergame. As suggested by the preceding discussion, we anticipate that the condition of fixed partners will favor somewhat more cooperative behavior.

Section I below outlines the analytic model. Section II describes the experimental procedures and outcomes. Section III discusses the results and summarizes.

I. The Model

Each of two contenders $i = 1,2$ must divide his/her exogenously given resource endowment R_i between productive effort E_i and appropriative ('fighting') effort F_i:

$$\begin{aligned} E_1 + F_1 &= R_1 \\ E_2 + F_2 &= R_2 \end{aligned} \qquad (1)$$

The E_i efforts are inputs to a joint production function. A convenient form for this function, characterized by constant returns to scale and constant elasticity of substitution, is:

$$I = A(E_1^{1/s} + E_2^{1/s})^s \qquad (2)$$

where A is an index of total productivity and s is an index of complementarity between E_1 and E_2.[5] However, for utmost

[5] This form of the production function is entirely symmetrical as between the parties. It would be simple to adjust for productive asymmetry, for example multiplying E_1 and E_2 by respective 'efficiency coefficients' e_1 and e_2. We do not explore this generalization here.

simplicity here we have assumed $A = s = 1$, so that (2) reduces to the simple additive equation:

$$I = E_1 + E_2 \tag{2a}$$

Thus, the parties can cooperate by combining their productive efforts so as to generate a common pool of income available to the two of them jointly. But the respective shares p_1 and p_2 (where $p_1 + p_2 = 1$) are determined in a conflictual process. In particular, the Contest Success Function (CSF) takes the fighting efforts F_i as inputs, yielding the distributive shares as outputs:

$$\begin{aligned} p_1 &= F_1^m/(F_1^m + F_2^m) \\ p_2 &= F_2^m/(F_1^m + F_2^m) \end{aligned} \tag{3}$$

Here m is a 'decisiveness parameter' controlling the mapping of the input ratio F_1/F_2 into the success ratio p_1/p_2. For $m \leq 1$ the CSF is characterized by diminishing marginal returns as F_1 increases with given F_2, or vice versa. However, for $m > 1$ there will be an initial range of increasing returns before diminishing marginal returns set in.[6] [7]

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[6] Like the production function, the Contest Success Function is (by assumption) entirely symmetrical as between the inputs on both sides. To reflect an asymmetrical situation where one player is intrinsically a better fighter than the other, the inputs F_1 and F_2 could be adjusted by multiplicative 'efficiency coefficients' f_1 and f_2.

[7] There are alternative possible forms for the Contest Success Function. As one important distinction, it might be expressed as a function either of the ratio F_1/F_2 of the fighting efforts (as assumed here) or alternatively as a function of the difference F_1-F_2 (see Hirshleifer (1989)). When the difference form is used, contender i generally will be able to attain a positive level of income even when setting

As a simplifying assumption, we postulate that conflict is non-destructive, i.e., there is no "battle damage". Choosing fighting activity over productive activity involves some opportunity loss of potential output, but the struggle does not itself damage the resource base or otherwise reduce the aggregate of income attainable.

Finally, the incomes accruing to the contestants are:

$$I_1 = p_1 I \qquad I_2 = p_2 I \tag{4}$$

For each level of fighting effort by contender 2, there is a corresponding optimal effort for contender 1 (and vice versa). Thus, 1's optimization problem is to choose $F_1 \geq 0$ so as to solve:

$$\text{Max } I_1 = p_1(F_1 | F_2) \times I(E_1 | E_2) \quad \text{subject to} \quad E_1 + F_1 = R_1$$

and similarly for side 2. Assuming neither party's resource constraint is binding, and using the simplified production function (2a), the Nash-Cournot reaction functions are:

$$\frac{F_1}{F_2^m} = \frac{m(E_1 + E_2)}{F_1^m + F_2^m} \qquad \frac{F_2}{F_1^m} = \frac{m(E_1 + E_2)}{F_1^m + F_2^m} \tag{5}$$

The right-hand sides being identical, $F_1 = F_2$ is always a solution of these equations. That is, the reaction curves

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$F_i = 0$. Since we wanted to make some degree of fighting effort indispensable for achieving positive income, for our purposes the ratio form was more appropriate.

intersect along the 45° line on F_1, F_2 axes. In fact, this is the sole intersection in the positive quadrant.

If however the boundary constraint is binding for the poorer side (which we always take to be contender 2), the second equation would be replaced by:

$$F_2 = R_2 \tag{5a}$$

In that case, at equilibrium $F_1 \neq F_2$ in general, but the intersection of the reaction functions still determines the Nash-Cournot equilibrium values of the fighting efforts.

As indicated above, the experiments were intended in part to challenge a number of specific predictions derived from the model. And in particular:

(i) <u>Fighting intensities</u>: As the decisiveness parameter m exogenously increases, it pays both sides to 'fight harder', i.e., the equilibrium fighting efforts F_i will rise. (Implying, of course, that the ultimate achieved incomes I_i must fall.)

(ii) <u>Conflict as equalizing process (paradox of power), strong form vs. weak form</u>: For sufficiently low values of the decisiveness parameter m, the "paradox of power" will tend to moderate income disparities as compared to resource endowments. Letting contender 1 be the initially better endowed side:

$$R_1/R_2 > I_1/I_2 \geq 1 \tag{6}$$

When the <u>equality</u> on the right holds (i.e., when the achieved incomes of the initially richer and initially poorer sides end up exactly equal) we have the 'strong form' of the paradox of power. As already noted, for any interior solution (that is, when the poorer side does not run into its resource constraint) we must have $F_1 = F_2$ -- so that the strong form of the paradox necessarily applies.[8] It can be shown that there will be interior solutions up to some critical value ρ of the resource ratio:

$$\rho = (2 + m)/m \tag{7}$$

Thus specifically, in our experiments employing the low value $m = 1$ for the decisiveness parameter, the prediction is that the strong form of the POP will hold for low resource ratios, specifically for $R_1/R_2 \leq 3$. For resource ratios larger than $\rho = 3$ only the weak form, i.e., the strict inequality on the right of equation (6), is predicted.

(iii) Conflict as inequality-aggravating process: The model also indicates that, for sufficiently high values of the decisiveness coefficient m and the resource ratio R_1/R_2, the paradox of power will not apply. The rich would get richer and the poor poorer. Specifically, for our experiments using the high decisiveness coefficient $m = 4$, the critical value τ of the resource ratio for this condition is approximately 2.18.[9] Also, from (7), when $m = 4$ the critical ρ separating the

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[8] This result does not hold for the more general CES production function (2), but only for our simplified special case (2a) where the productive complementarity coefficient is set at $s = 1$.

[9] The value of τ was obtained by finding the resource ratio where the condition $I_1/I_2 = R_1/R_2$ was met for $m = 4$.

weak from the strong forms of the paradox of power equals 1.5. Thus in our experiments using the low resource ratio 25/15 = 1.67 we expect the weak form of the paradox of power to hold, since 1.67 lies between ρ and τ. But for the experiments with $R_1/R_2 = 32/8 = 4$, since this ratio exceeds $\tau = 2.18$ the prediction is that the initially better endowed party will improve its relative position compared to the less well endowed side:

$$I_1/I_2 = (F_1/F_2)^m > R_1/R_2 \tag{8}$$

II. Experimental Procedures and Outcomes

A. Experimental Design

We conducted 24 experiments using a total of 574 subjects. No subject participated in more than one experiment. There were 6 bargaining pairs in each experiment, except for a few cases with only 4 or 5 pairs. Each experiment involved iterated play, the payoffs being constant in each round. Also, play was simultaneous in each round. Subjects were not informed how many rounds would take place; in fact, in each experiment there were 16 or 17 rounds before termination. Subjects were recruited for two-hour sessions but the experiments took much less time, making credible the condition of an unknown horizon.[10]

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[10] This technique was also used in McCabe, Rassenti and Smith (1995) and found to be effective in leading to cooperation, even on the "last" repetition.

As indicated in the attached INSTRUCTIONS, in every round each subject allocated his/her initial endowment of tokens between an "Investment Account" (IA) and a "Rationing Account" (RA). Tokens contributed to the IA corresponded to productive effort E_i in the theoretical model: the paired IA contributions generated an aggregate pool of income (in the form of 'experimental pesos') in accordance with equation (2a) above. Funds put into the RA corresponded to fighting effort F_i and determined the respective distributive shares p_1 and p_2 in accordance with equations (3). For simplicity, only integer choices were permitted. (More precisely, each subject could allocate, within his/her resource constraint, amounts in integral hundreds of tokens to invest in the IA, the remainder of course going into the RA.) The totals of pesos ultimately achieved were converted into actual dollars at the end of the experiment, so subjects had a substantial motivation to make self-interested choices. (The payoffs ranged from $.25 to $75.25, not including the $5 show-up fee. The average payoff was $17.66.)[11]

To challenge the implications of the model, we manipulated the resource endowments R_1 and R_2 and also the decisiveness coefficient m. Four experiments were run with each of the

[11] The INSTRUCTIONS appended to this paper apply to the experiments involving: fixed partners, identical resource endowments ($R_1 = R_2 = 20$), and the "low" value of the decisiveness coefficient ($m = 1$). For the other experimental treatments, the INSTRUCTIONS were modified accordingly.

three endowment vectors (R_1,R_2) = (20,20), (25,15), and (32,8) -- first using a low value $m = 1$ of the decisiveness parameter, and then using a high value $m = 4$. Thus there were 24 experiments in all.

Also, in view of earlier experimental results showing that cooperation is promoted by repeated play with the same partners, each group of four experiments was further subdivided into alternative matching protocols. In the first ('varying partners') protocol, partners were randomly changed each round. Under the second ('fixed partners') protocol, subjects were randomly paired at the beginning of the experiment but played repeatedly with the same partner throughout.

So overall there were 8 experiments under each of the three endowment conditions. Four of the 8 involved varying partners, and four fixed partners. There was an analogous but orthogonal subdivision between experiments conducted using $m = 1$ and using $m = 4$. The upshot is that there were exactly two experiments for each of the 12 sets of experimental conditions or "treatments". The treatment design is summarized in Table 1.

[Table 1 here]

B. Results -- Nash versus Cooperative Solutions

The theoretical model described in the previous Section derived the Nash-Cournot noncooperative equilibrium. However, the experimental literature has intensively investigated conditions under which subjects might arrive at a more cooperative outcome. This is the first issue we addressed:

H_o: the null hypothesis is that $(F_1,F_2) = (C_1,C_2)$

H_a: the alternative hypothesis is that $(F_1,F_2) = (N_1,N_2)$

Here the N_i signify the respective fighting efforts F_i under the Nash solution, while the C_i are the fighting efforts under the cooperative solution. As indicated above, we anticipated that the Nash solution would be more strongly supported under the 'varying partners' protocol than under the 'fixed partners' protocol.

The theoretical Nash solution is generated by the intersection of the reaction functions of equations (5) above for an interior outcome -- or in the case of a boundary outcome (where the poorer contender 2's resource constraint is binding), substituting (5a) for the second equation in (5). The cooperative solution was defined as $(C_1,C_2) = (1,1)$. That is, where each side devotes the minimal allowable positive amount to fighting effort.[12]

The outcomes from a subset of the experiments can be found in Figures 1 to 12. They plot, on axes representing the

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[12] As can be seen in equations (3), the relative shares p_1 and p_2 are indeterminate when $F_1 = F_2 = 0$. To remedy the indeterminacy, the Profit Table shown in the INSTRUCTIONS to subjects provided for zero payoff whenever player i put zero into his RA, i.e., whenever $F_i = 0$ was chosen. Thus, while it is possible for either member of the pair to choose $F_i = 0$, whenever both do so each receives zero payoff. So the available cooperative combination maximizing the aggregate payoff, under the integer constraint, is $(C_1,C_2) = (1,1)$.

respective fighting efforts F_1 and F_2, the sixteenth (the last or next-to-last) observation for each bargaining pair and the average of these observations in each of the experiments. One experiment from each of the twelve treatments is included. Each diagram shows the reaction functions (the jagged "staircases"), the computed Nash noncooperative equilibrium at the intersection of these functions, the postulated cooperative solution at (1,1), and the actual experimental outcomes. Table 2 is a summary that allows a comparison between the Nash equilibrium and the average of the 6 (or, in a couple of cases, 4 or 5) observations in each of the 24 experiments.

[Figures 1-12, and Table 2 here]

Instead of the traditional apparatus of 'tests of significance', we used the likelihood ratio[13] to test the alternative Nash hypothesis against the cooperative null hypothesis. The likelihood ratio is particularly convenient for Bayesian conversion of prior beliefs p' into posterior beliefs p'' in the light of the experimental evidence. The relevant version of Bayes' Theorem is:

$$\frac{p_o''}{p_a''} = \frac{\text{Likelihood of evidence under } H_o}{\text{Likelihood of evidence under } H_a} = \frac{p_o'}{p_a'} \qquad (9)$$

Assuming that the actual observations for the 'fighting

[13] Following the lead of Mosteller and Wallace (1984).

efforts' F_i in any round are distributed randomly about the C_i solutions if the null hypothesis H_0 is true, and similarly about the N_i in the opposite case, then for any given treatment the likelihood ratio is:

$$\lambda = \frac{\dfrac{1}{(2\pi)^{Tn/2} s_i^{Tn}} \exp\left[- \sum_i \dfrac{1}{2s_i^2} \sum_t (F_i(t) - C_i)^2 \right]}{\dfrac{1}{(2\pi)^{Tn/2} s_i^{Tn}} \exp\left[- \sum_i \dfrac{1}{2s_i^2} \sum_t (F_i(t) - N_i)^2 \right]}$$

$$= \exp\left\{ \sum_i \frac{1}{2s_i^2} \left[2\sum_t F_i(t)\,(C_i - N_i) + n(N_i^2 - C_i^2) \right] \right\} \tag{10}$$

Here the t subscript indexes the rounds from 1 to T, while the i subscript indexes the individual participants from 1 to n.

A $\lambda < 1$ would indicate that, for this particular treatment, the observed choices had a higher probability of occurring under the under the alternative (Nash) hypothesis than under the null (cooperative) hypothesis.[14] Whatever the prior beliefs, if $\lambda < 1$ then a person's posterior beliefs should now place higher credence than before upon the

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[14] As a technical qualification, a strict Bayesian would have to deal with the fact that the true normal variance σ is unknown. In principle one ought to specify prior beliefs about the variance and deal with it as a "nuisance parameter". However, we have taken the liberty of simply employing the observed sample variance s for σ. Doing so provides an enormous computational saving without substantially affecting the results.

alternative hypothesis, and the reverse for $\lambda > 1$.[15]

The twelve rows of Table 3 show the λ's for all the treatments, expressed for convenience in terms of logs (the log likelihood ratios). Positive entries in the table represent results favoring the null hypothesis while negative entries favor the alternative hypothesis.

[Table 3 here]

The columns toward the left of Table 3 identify the conditions for each of the 12 treatments. The remaining columns show the results for "All Rounds" and also for the "Sixteenth Round" (that is, the last or next-to-last round) separately. (For the "Sixteenth Round" columns, equation (10) is modified by simply dropping the indexing over t.) From the statistical point of view, the "All Rounds" reports provide a larger sample size and thus are less influenced by random fluctuations. On the other hand, the "Sixteenth Round" reports are more likely to isolate the mature behavior of the experimental subjects. Finally, F_1 refers to the subject having the larger, and F_2 the smaller resource endowment. (In the equal-endowment cases, the assignment of F_1 versus F_2 was random.)

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[15] Since the likelihood ratio is itself distributed as chi-squared, it would be possible to find the 'level of significance' associated with any given divergence of the observed λ from that predicted under the null hypothesis. While possible, we regard such a calculation as rather pointless and do not attempt it. However, it will be evident that our results would, in common parlance, be "highly significant" in rejecting the null hypothesis.

The results summarized in Tables 2 and 3 overwhelmingly support the Nash as opposed to the cooperative solution. The predominantly negative values of the log likelihood ratios correspond of course to likelihood ratios less than unity in equation (10) above. Thus, any observer, regardless of prior beliefs, should revise those beliefs so as to attach greater confidence than before to the Nash outcome.

As a quantitative illustration, take the -87 appearing in the first row under the 16th Round, F_1 column. This implies a likelihood ratio of e^{-87}, which is a decimal fraction beginning with some 37 zeros! Thus, even if a person's prior belief favored the null hypothesis by a hundred billion to one, in equation (9) this 10^{11} factor for p_o'/p_a' on the right-hand-side would be utterly swamped by the 10^{-37} likelihood ratio. So after observing these data, enormously high credence would have to be awarded to H_a even if one's hypothesized initial belief overwhelmingly favored H_o.

Apart from the generally negative signs for the log λ's, two features of Table 3 stand out. First, in all 24 possible comparisons the log likelihood ratios for the "16th Round" columns are somewhat less negative than in the corresponding "All Rounds" columns. This may in part be the consequence of smaller sample size, but that is evidently not the entire story -- since the only two instances of positive values both fall under the "16th Round" headings. So, there is a strong suggestion that participants had, to some degree, "learned to cooperate" (or at any rate, to fight less) by the 16th round of interaction. Second, again for all 24 comparisons, the results under the 'fixed partners' (F) condition are noticeably less negative than the corresponding 'varying partners' (V) results. Since the 'fixed partners' condition facilitates the development of mutual understanding, this feature of the data is also consistent with the "learning to cooperate" interpretation.

Inspection of the Figures also suggests that while the observed data tend to fall considerably closer to the Nash than to the cooperative outcome, on average the data points do seem to lie somewhere between the two. This pattern, of a persistent tendency to diverge from the Nash solution in the direction of cooperation, parallels that found in a number of other experimental studies, as noted earlier.

We can quantify the average percent deviations in the direction of cooperation by defining "slippage fractions" S_1 and S_2:

$$S_i \equiv \frac{N_i - F_i}{N_i - C_i} \qquad (11)$$

In Table 4, a positive number in the two right-hand columns indicates slippage in the direction of cooperation. A negative number indicates slippage in the direction of conflict beyond that called for by the Nash solution. As expected, the positive numbers far outweigh the negative numbers. Also, consistent with the "learning to cooperate" interpretation, the positive numbers predominate more under the 'fixed partners' (F) condition. Finally, there is a noticeable <u>positive correlation</u> between the S_1 and S_2 numbers on each row of the Table: when one subject behaves cooperatively, his/her partner is likely to do so as well. Once again, as expected this positive correlation holds particularly for the 'fixed partners' condition. And in addition, it holds noticeably more strongly for the cases with equal resource endowments $(R_1,R_2) = (20,20)$.

[Table 4 here]

C. Results -- The Paradox of Power

The parameters employed in the experiments led to several specific predictions derived from the analytic model.

<u>Prediction 1</u> Higher values of the decisiveness parameter m will lead to larger fighting efforts on both sides.

Here the prediction is that the fighting efforts F_1 and F_2 will both be greater at the higher decisiveness level $m = 4$ than at $m = 1$. The upper half of Table 2 shows the $m = 1$ data, and the lower half the results for $m = 4$. There are 48 comparisons, of which a remarkable 45 are in the direction predicted.

<u>Prediction 2a</u> At the specific low value of $m = 1$ of the decisiveness parameter, the initially poorer side will always end up improving its position.

For $m = 1$ the prediction is that the attained income ratio I_1/I_2 (which for $m = 1$ simply equals the ratio of fighting efforts F_1/F_2) will exceed the resource ratio R_1/R_2. The requirement of unequal initial endowments limits the relevant data to rows 5 through 12 of Table 2. Here all 8 of the 8 comparisons showed the predicted relative improvement -- that is, $I_1/I_2 < R_1/R_2$ -- almost always by quite a wide margin.

<u>Prediction 2b</u> For $m = 1$ the poorer side should attain approximate equality of income (strong form of the POP) for initial resource ratios $R_1/R_2 < 3$, but only some relative improvement -- $1 < I_1/I_2 < R_1/R_2$ -- for larger resource ratios (weak form of the POP).

Looking once again only at the unequal endowments cases, rows 5 through 12 of Table 2 are relevant. The average of the tabulated results is $I_1/I_2 = 1.125$, on the high side of the predicted $I_1/I_2 = 1$. By way of comparison, for rows 9 through 12 where only the weak form $I_1/I_2 > 1$ is predicted, the average outcome is $I_1/I_2 = 1.43$. So, at least <u>relatively</u>, the predicted comparison of the strong form versus weak form predictions is supported.

<u>Prediction 2c</u> At the specific high value $m = 4$ of the decisiveness coefficient, the paradox of power should continue to hold (in its weak form) for $\rho < R_1/R_2 < \tau$, where $\rho = 1.5$ and $\tau = 2.18$. But for higher resource ratios the richer side should end up actually improving on its relative position. That is, in this range $I_1/I_2 = (F_1/F_2)^4$

should exceed R_1/R_2.

Once again, the equal-endowment cases are not relevant for this prediction. For rows 17 though 20 of Table 2, the resource ratio is $R_1/R_2 = 25/15 = 1.67$, lying between ρ and τ, so we expect the paradox of power to hold in these cases. But for rows 21 through 24 the resource ratio is $R_1/R_2 = 32/8 = 4 > 2.18 = \tau$, so we expect the rich to become richer still.

Taking up the latter group first, 3 of the 4 cases support the prediction $I_1/I_2 = (F_1/F_2)^4 > 4$. In fact, the average of the observed results was $I_1/I_2 = 12.19$. Turning to the first group, however, we see that all 4 cases violate the prediction! Quantitatively, the predicted Nash outcome $(N_1,N_2) = (16,15)$ implies $I_1/I_2 = (16/15)^4 = 1.29 < 1.67$ while the average of the observed results was $I_1/I_2 = 2.32 > 1.67$.

III. Discussion and Summary

This experimental investigation deals with a mixed-incentive iterated-play bilateral interaction. In each of some 16 rounds paired individuals had to strike a balance between production and appropriation: more explicitly, between investing resources in joint production versus engaging in a distributive struggle over the respective shares.

We tested two main kinds of predictions:

(1) The first group dealt with issues common to much of the game-theoretic and experimental literature. Of these, the major question was the degree to which the experimental

outcomes approximated the non-cooperative Nash solution, as opposed to a more cooperative outcome generating a larger income for the group as a whole. We also compared protocols with randomly varying partners each round as opposed to fixed partners over the entire sequence of play.

(2) The second group of predictions dealt with inferences from the specific model of conflict in Hirshleifer (1991), and specifically those associated with the 'paradox of power'. The paradox is that, in many situations, an initially poorer side will end up gaining in relative position in comparison with an initially richer and thus stronger opponent.

With regard to the first group of predictions, the experimental observations overwhelmingly supported the Nash as opposed to the cooperative solution. However, while the Nash solution is much better supported in a dichotomous comparison between the two, the experimental results typically displayed some degree of slippage in the direction of cooperation. The divergence toward cooperation was stronger under the fixed-partners as opposed to the varying-partners protocol, and also was more evident in the mature (16th round) choices than the overall behavior. Together with an observed tendency toward positive correlation of the deviations from the Nash equilibrium, these results are all consistent with a "learning to cooperate" interpretation. Fixed partners over multiple rounds of interaction favor the development of mutual understanding relative to varying partners. Still, we must reemphasize, overall the results were dominated by non-cooperative (Nash) behavior.

With regard to the underlying conflict model, the central prediction (Prediction 1) was that relatively larger fighting efforts on both sides would be observed for higher values of the 'decisiveness coefficient' m -- a parameter that indicates the degree to which the fighting efforts as inputs determine the relative shares of incomes attained. Prediction 1 was confirmed in 45 of 48 comparisons: when fighting became a more decisive determinant of relative income shares, both sides invested more in the struggle.

The evidence was more mixed concerning certain of the predictions bearing more specifically upon the 'paradox of power' thesis -- the circumstances under which the poorer side would improve its relative position. For the experiments employing a low value $m = 1$ for the decisiveness coefficient, Prediction 2a was that at least the "weak form" of the paradox should always hold, that is, that $I_1/I_2 > R_1/R_2$. And in fact 8 of 8 possible comparisons confirmed Prediction 2a. Prediction 2b was more stringent, specifying that for the 4 cases where the initial resource ratio was sufficiently small ($R_1/R_2 = 25/15 = 1.67$) the "strong form" should hold: $I_2 = I_2$. While the observed average income ratio for these cases was $I_1/I_2 = 1.125 > 1$, the average was closer to the predicted value than the average 1.43 obtained in the 4 cases with a high resource ratio $R_1/R_2 = 32/8 = 4$ (where the "strong form" was not predicted). So there was at least some relative confirmation of Prediction 2b.

For the high value $m = 4$ of the decisiveness coefficient, Prediction 2c was that the paradox of power would hold in its weak form for the low resource ratio $R_1/R_2 = 1.67$, but should be violated for the high resource ratio $R_1/R_2 = 4$. The latter part of this prediction was substantially confirmed. For an already high resource ratio it was predicted that the rich would get richer, and in fact they did so. But they also did so for the relatively lower initial resource ratio, when according to the theory the richer side should have 'held back' and devoted almost all of its superiority to productive rather than fighting effort. So this part of Prediction 2c was definitely contradicted by the experimental results.

As a possible speculative explanation, the better endowed players in this (32,8) case might have become so attracted by the prospect of increasing their <u>relative</u> shares that they did so even at the expense of <u>absolute</u> incomes achieved. But this explanation seems in conflict with the observations in Table 3 showing that the <u>only</u> outcomes favoring the cooperative solution occurred under one of the (32,8) experimental conditions. So at this point the puzzle remains unresolved.

Other intriguing aspects of the data also call for future exploration and explanation, among them: (1) The extent of <u>slippage toward cooperation</u>, and the correlation between the slippages on each side suggest a possible 'learning' explanation that needs to be modelled more explicitly for testing purposes. (2) Whether the anomalous 2 of 48 comparisons in Table 3, that support the cooperative over the Nash hypothesis, are only random fluctuations (which seems hard to believe given the substantial positive log likelihood ratios)[16] or whether there may be a more systematic explanation.

Summing up, this investigation is a new departure in demonstrating the fruitfulness of experimental investigations into the conflictual processes that are so salient a feature of the observed world.

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[16] In Table 3 the two positive log likelihoods 0.8 and 8.5 translate into respective likelihood ratios of 2.23 and 4915 favoring the null (cooperative) hypothesis.

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*Table 1

TREATMENTS
# of experiments (# of subjects)*

Endowments (R_1, R_2)	Decisiveness m	Variable Pairing	Fixed Pairing	Totals
20,20	1	2(24)	2(24)	4(48)
25,15	1	2(24)	2(24)	4(48)
32,8	1	2(22)	2(24)	4(46)
20,20	4	2(22)	2(22)	4(44)
25,15	4	2(20)	2(24)	4(44)
32,8	4	2(24)	2(24)	4(48)
	TOTALS	12(288)	12(286)	24(574)

*In some experiments only 10 subjects (5 pairs) were used. Each experiment in a treatment was run for either 16 or 17 rounds.

TABLE 2
EXPERIMENTAL RESULTS

	Treatment Parameters				Nash solution		Av. Results (16th obs.)		
Exp.#	m	Pair-ing	R_1,R_2	R_1/R_2	N_1,N_2	N_1/N_2	F_1,F_2	F_1/F_2	I_1/I_2
1	1	V	20,20	1	10,10	1	7.83,6.83	1.15	1.15
2	1	V	20,20	1	10,10	1	8,9	.89	.89
3	1	F	20,20	1	10,10	1	8.67,6.67	1.30	1.30
4	1	F	20,20	1	10,10	1	4,5	.8	.8
5	1	V	25,15	1.67	10,10	1	10.83,8.5	1.27	1.27
6	1	V	25,15	1.67	10,10	1	9,9.17	.98	.98
7	1	F	25,15	1.67	10,10	1	10.17,9	1.13	1.13
8	1	F	25,15	1.67	10,10	1	7.67,6.83	1.12	1.12
9	1	V	32,8	4	10,8	1.25	11.83,7.67	1.54	1.54
10	1	V	32,8	4	10,8	1.25	10.33,7.5	1.38	1.38
11	1	F	32,8	4	10,8	1.25	5.17,3.17	1.63	1.63
12	1	F	32,8	4	10,8	1.25	5.4,4.6	1.17	1.17
13	4	V	20,20	1	16,16	1	10.33,12.83	.81	.42
14	4	V	20,20	1	16,16	1	14.67,15.33	.96	.84
15	4	F	20,20	1	16,16	1	11.67,13.83	.84	.51
16	4	F	20,20	1	16,16	1	10,9.2	1.09	1.40
17	4	V	25,15	1.67	16,15	1.07	15.5,11.83	1.31	2.95
18	4	V	25,15	1.67	16,15	1.07	16.5,12.5	1.32	3.04
19	4	F	25,15	1.67	16,15	1.07	16.17,13.83	1.17	1.87
20	4	F	25,15	1.67	16,15	1.07	13.5,12.33	1.10	1.44
21	4	V	32,8	4	12,8	1.5	11.67,7.33	1.59	6.42
22	4	V	32,8	4	12,8	1.5	11.67,7	1.67	7.72
23	4	F	32,8	4	12,8	1.5	10.5,7.5	1.4	3.84
24	4	F	32,8	4	12,8	1.5	11,4.67	2.36	30.76

Table 3

LOG LIKELIHOOD RATIOS
Nash vs. Cooperative Bargaining Solution

Treatment Parameters and Matching Protocol				All Rounds ($T \cdot n=198$)		16th Round ($n=12$)	
			Pair-	Types		Types	
R_1	R_2	m	ing	F_1	F_2	F_1	F_2
20	20	1	V	- 873	- 654	- 87	- 30
20	20	1	F	- 545	- 169	- 3.8	- 0.7
25	15	1	V	- 939	-1302	- 31	- 133
25	15	1	F	- 262	- 218	- 20	- 23
32	8	1	V	- 879	-2583	- 31	- 111
32	8	1	F	- 123	- 73	0.8*	8.5*
20	20	4	V	-1260	-1376	- 114	- 78
20	20	4	F	- 68	- 92	- 6.3	- 10
25	15	4	V	-1475	- 541	-1001	- 115
25	15	4	F	- 860	- 796	- 48	- 69
32	8	4	V	-2756	-1643	-1008	- 114
32	8	4	F	- 373	- 332	- 43	- 23

*2 results favoring Cooperative over Nash solutions (of 48)

TABLE 4
SLIPPAGE TOWARD COOPERATION

Treatment Parameters					Nash solution	Av. Results		Av. Slippage*	
Exp.#	m	Pair-ing	R_1,R_2	R_1/R_2	N_1,N_2	F_1	F_2	S_1	S_2
1	1	V	20,20	1	10,10	7.83	6.83	.24	.35
2	1	V	20,20	1	10,10	8	9	.22	.11
3	1	F	20,20	1	10,10	8.67	6.67	.15	.37
4	1	F	20,20	1	10,10	4	5	.67	.56
5	1	V	25,15	1.67	10,10	10.83	8.5	-.09	.17
6	1	V	25,15	1.67	10,10	9	9.17	.11	.09
7	1	F	25,15	1.67	10,10	10.17	9	-.02	.11
8	1	F	25,15	1.67	10,10	7.67	6.83	.26	.41
9	1	V	32,8	4	10,8	11.83	7.67	-.20	.05
10	1	V	32,8	4	10,8	10.33	7.5	-.04	.07
11	1	F	32,8	4	10,8	5.17	3.17	.54	.69
12	1	F	32,8	4	10,8	5.4	4.6	.51	.46
13	4	V	20,20	1	16,16	10.33	12.83	.38	.21
14	4	V	20,20	1	16,16	14.67	15.33	.09	.04
15	4	F	20,20	1	16,16	11.67	13.83	.29	.15
16	4	F	20,20	1	16,16	10	9.2	.40	.45
17	4	V	25,15	1.67	16,15	15.5	11.83	.33	.23
18	4	V	25,15	1.67	16,15	16.5	12.5	-.03	.18
19	4	F	25,15	1.67	16,15	16.17	13.83	-.01	.08
20	4	F	25,15	1.67	16,15	13.5	12.33	.17	.19
21	4	V	32,8	4	12,8	11.67	7.33	.03	.10
22	4	V	32,8	4	12,8	11.67	7	.03	.40
23	4	F	32,8	4	12,8	10.5	7.5	.14	.07
24	4	F	32,8	4	12,8	11	4.67	.09	.48

* $S_i = (N_i - F_i)/(N_i - C)$

Figure 1

20x20 B1

m = 1; Varying Partners

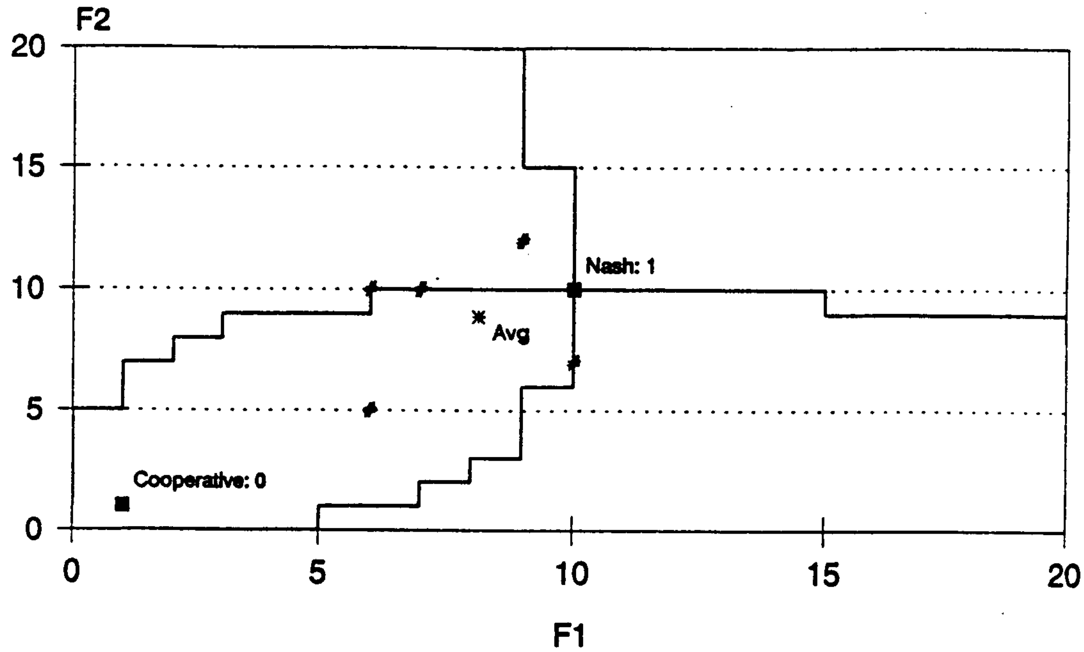


Figure 2

20x20 D1

m = 1; Fixed Partners

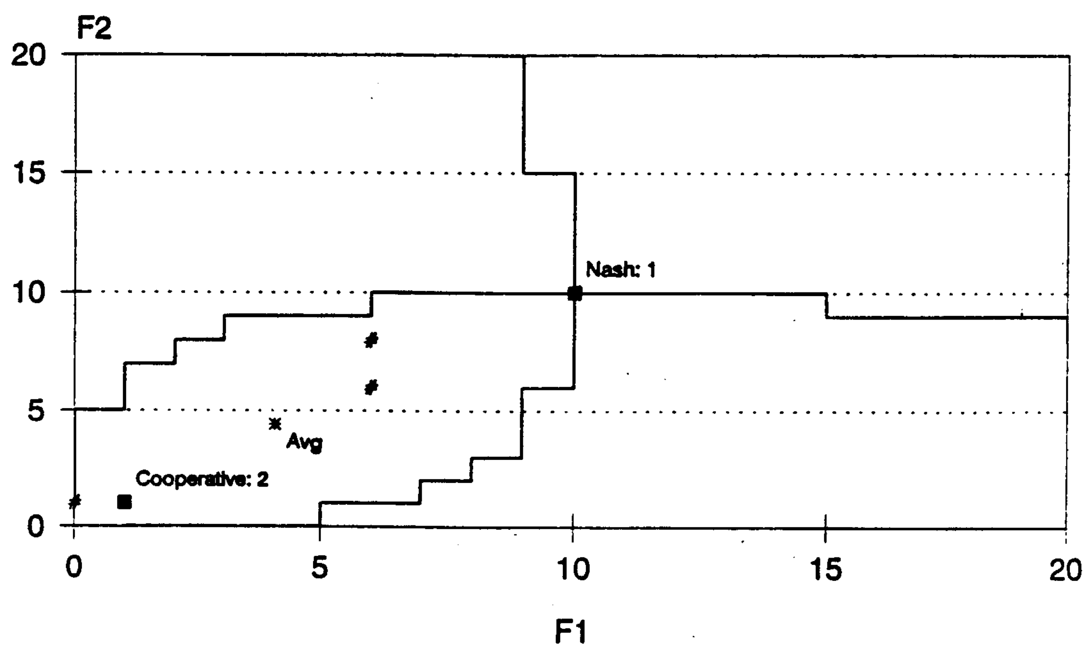


Figure 3

25x15 A1

m = 1; Varying Partners

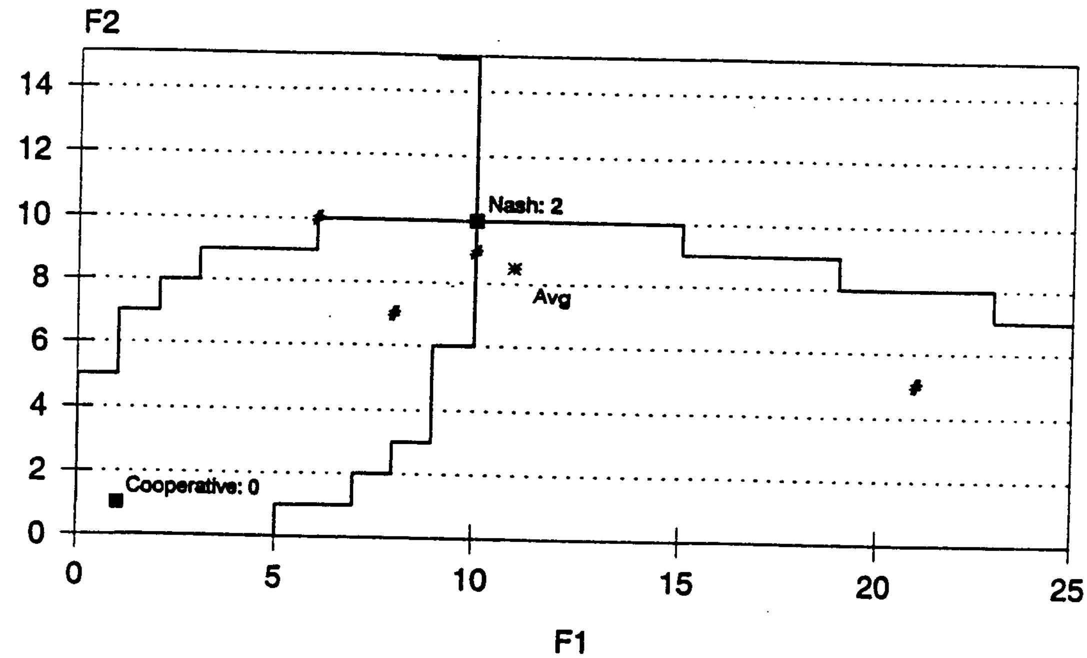
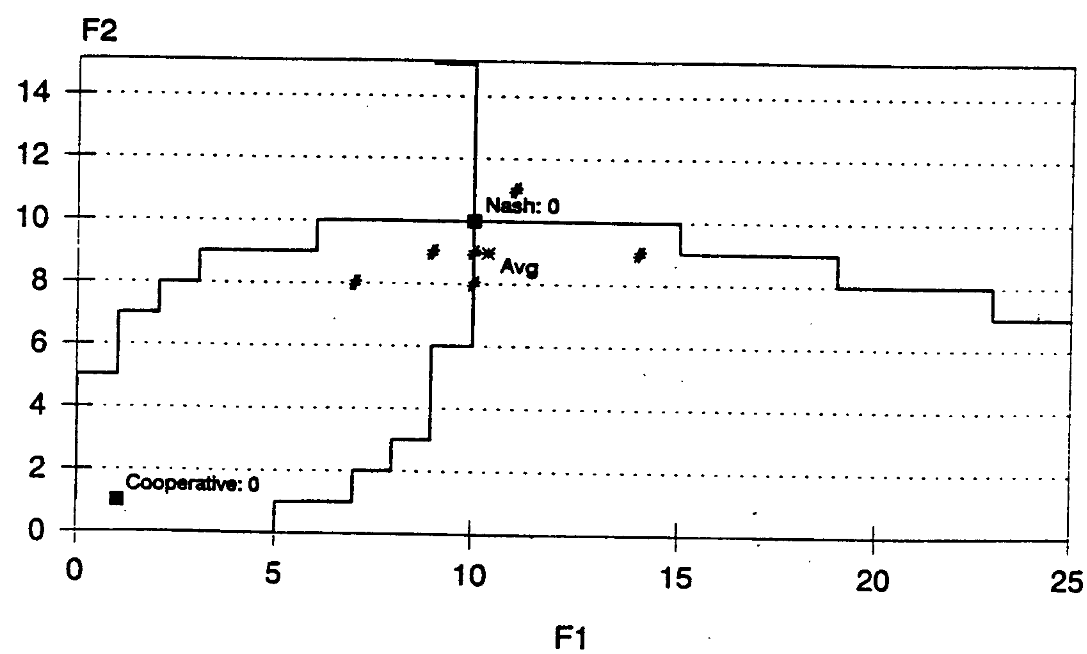


Figure 4

25x15 C1

m = 1; Fixed Partners



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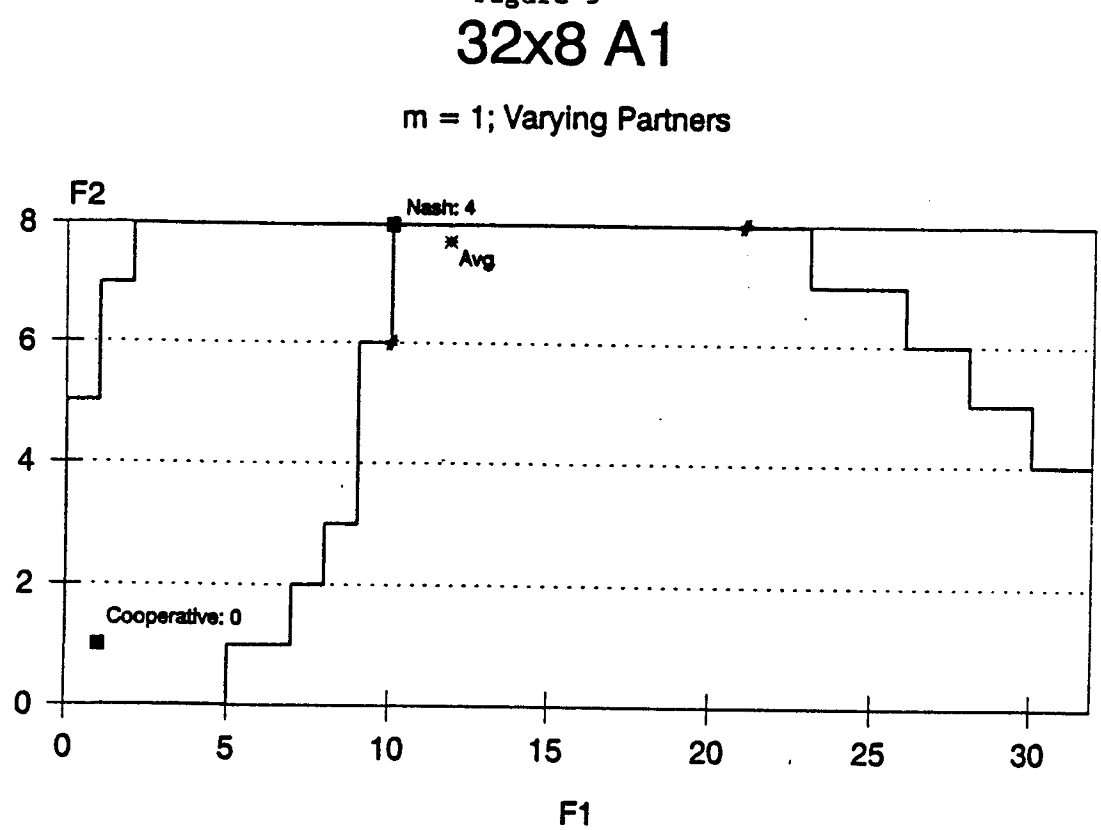


Figure 6

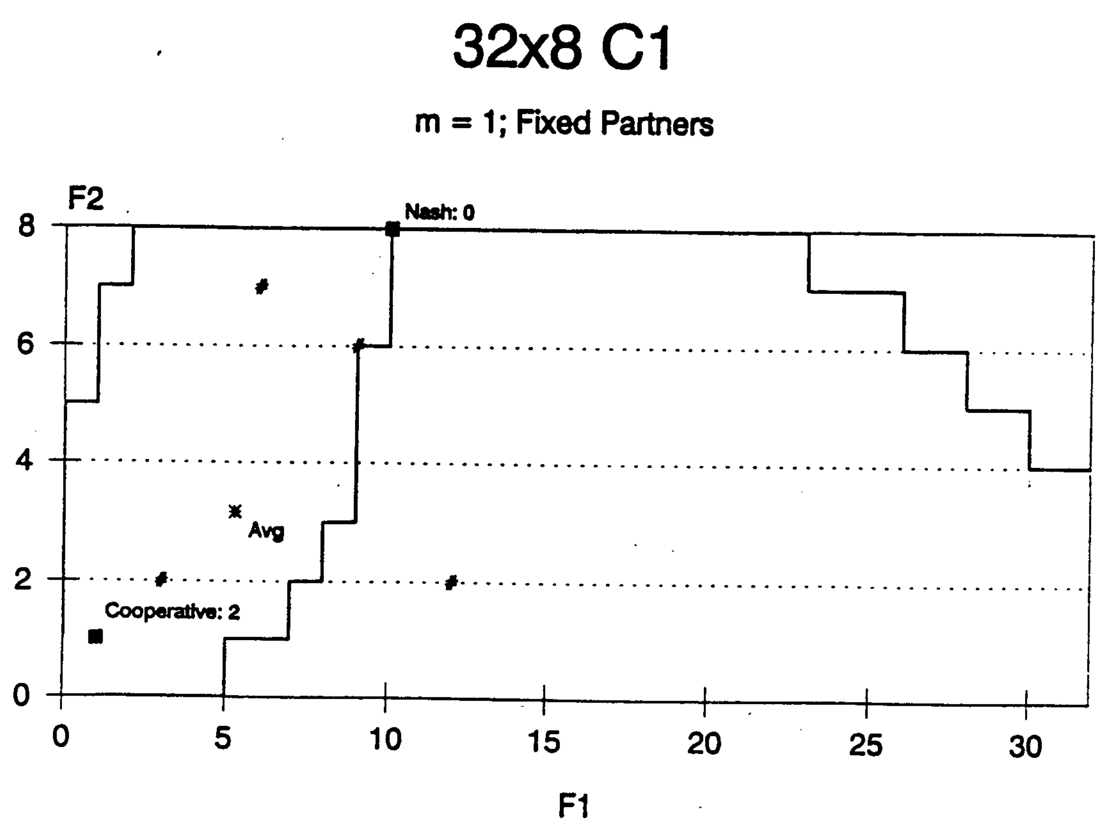


Figure 7

20x20 B4

m = 4; Varying Partners

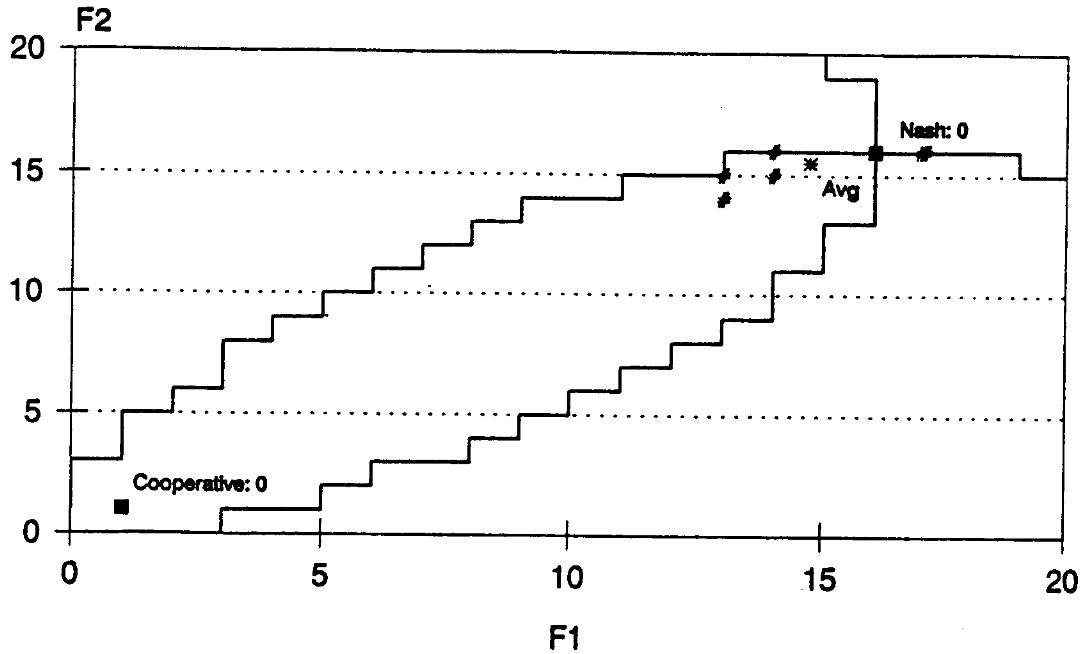


Figure 8

20x20 C4

m = 4; Fixed Partners

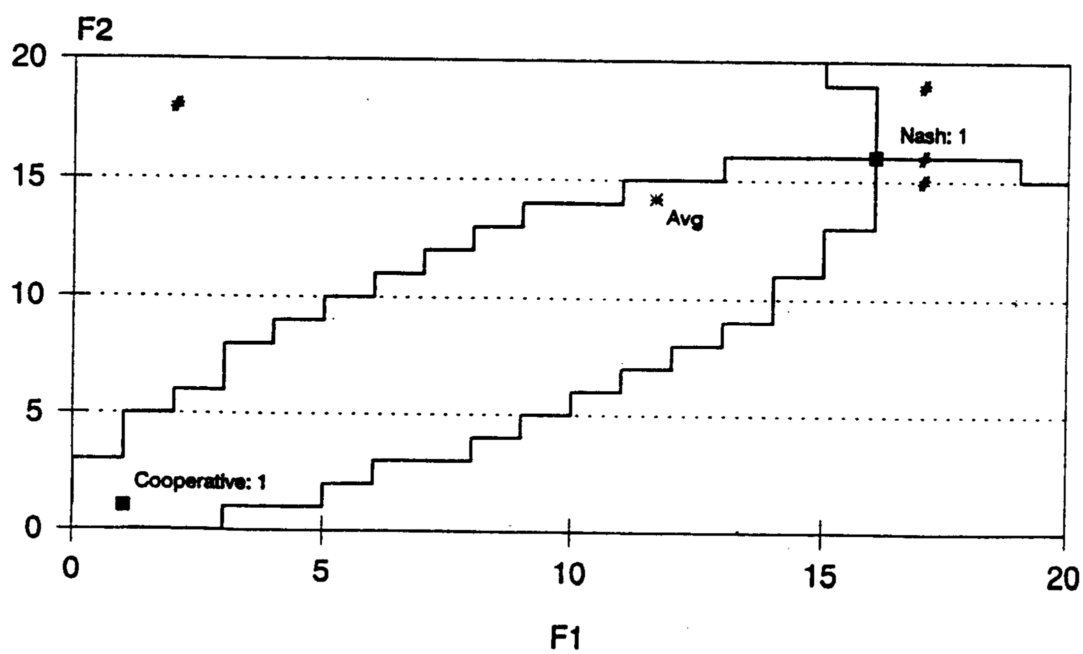


Figure 9

25x15 A4

m = 4; Varying Partners

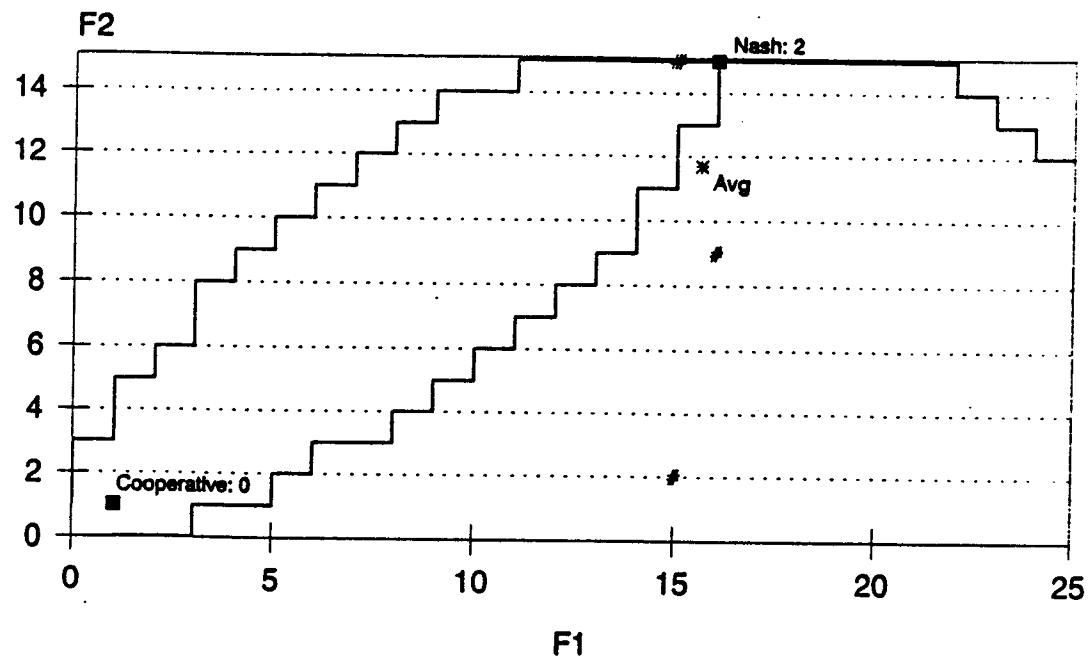


Figure 10

25x15 C4

m = 4; Fixed Partners

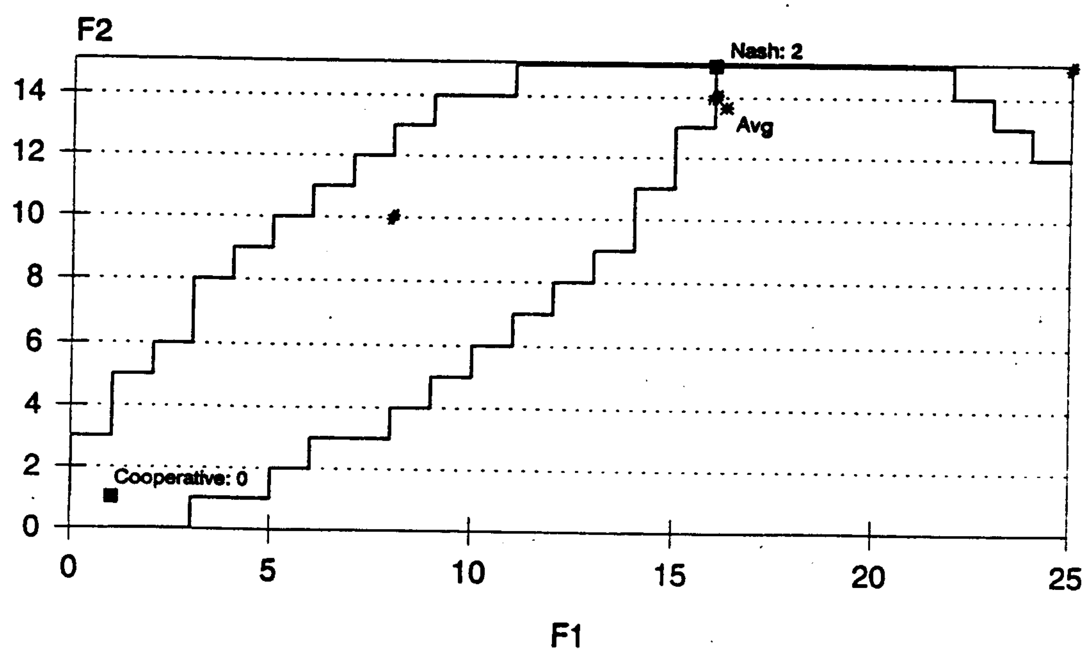


Figure 11

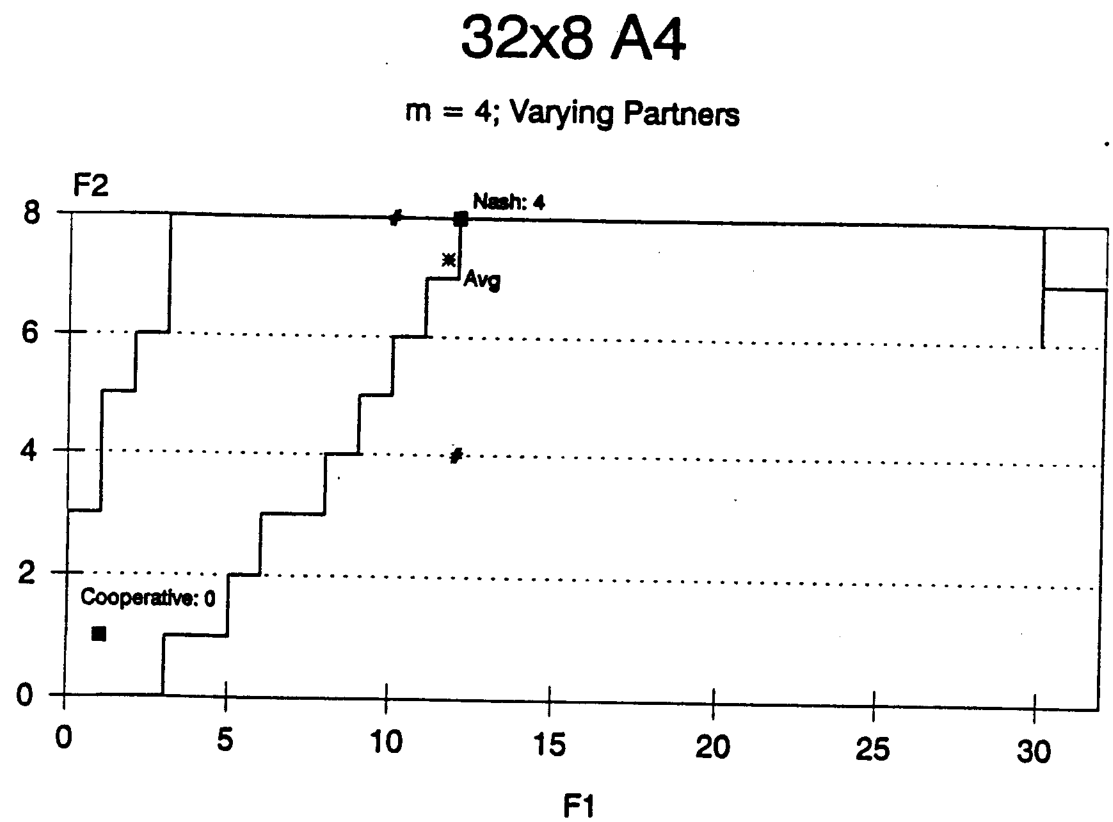
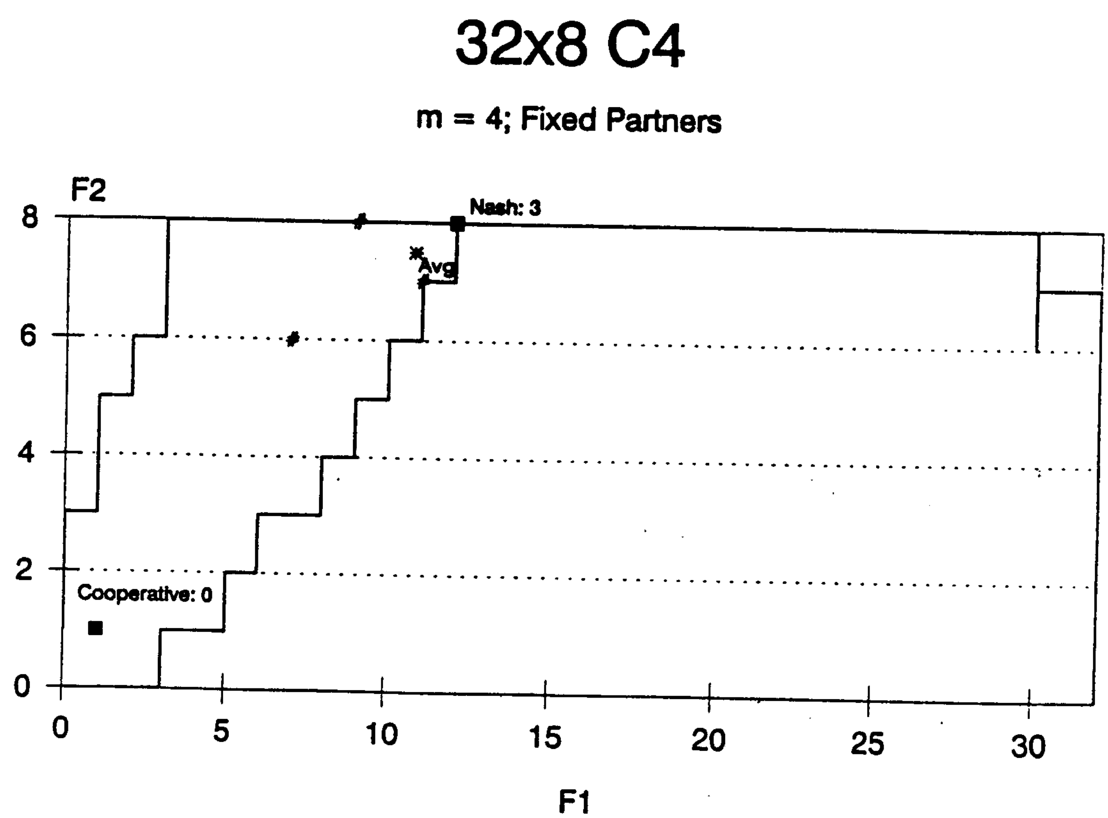


Figure 12



INSTRUCTIONS

This is an experiment in the economics of decision making. Funds for this experiment have been provided by various research foundations. If you read the instructions carefully and make wise decisions, you may earn a considerable amount of money which will be paid to you in cash at the end of the experiment. All references to money during the experiment will be made in terms of "experimental pesos." You will be paid $1 for every 1,200 experimental pesos you earn. The more experimental pesos you earn, the more U.S. dollars you will take home with you.

This experiment will consist of several decision-making periods. At the beginning of the experiment, you will be randomly paired with another person in the experiment. This person will be referred to as your "counterpart." At the beginning of every period, you and your counterpart will each be given a certain number of tokens. You and your counterpart will have two joint accounts, an **Investment Account** and a **Rationing Account.** Each of your tokens must be put into one of your two accounts. You must individually decide how many of your own tokens to put into each account. The tokens put into your Investment Account will earn experimental pesos. You and your counterpart will then divide the earnings from your Investment Account between you. Your share of the Investment Account Earnings will be determined by how many tokens you and your counterpart each put into the Rationing Account. Therefore, the tokens put in the Investment Account will determine the number of pesos earned, while the tokens put in the Rationing Account will determine how those earnings will be divided between you and your counterpart. The details of how each of your two accounts works are explained below.

Investment Account (IA) -- The tokens that you and your counterpart put into your Investment Account will earn experimental pesos. Each period, the number of pesos earned from your Investment Account will be determined as follows:

Total Investment Account Earnings = Your Tokens in IA + Counterpart's Tokens in IA.

In other words, each token put into your Investment Account, either by you or your counterpart, will earn 1 experimental peso. Therefore, if you put 300 tokens in your Investment Account one period and your counterpart puts 500 tokens in, your Investment

Account will earn 800 experimental pesos. These 800 pesos will be divided between you and your counterpart according to the number of tokens you each put in your Rationing Account. Your Period Profit will be equal to the number of pesos you are able to obtain from the Investment Account Earnings. Note that regardless of which one of you puts the tokens in your Investment Account, the earnings from the account will be available to both of you. The more tokens you put into your Investment Account, the more pesos it will earn, and therefore, the more pesos you and your counterpart will have available to divide between you.

Rationing Account (RA) -- The number of tokens each of you puts in your Rationing Account will determine how the Investment Account Earnings will be divided between you. A Rationing Fraction will be determined for you as follows:

$$\text{Rationing Fraction} = \frac{\text{(Your Tokens in RA)}}{\text{(Your Tokens in RA)} + \text{(Counterpart's Tokens in RA)}}.$$

Your share of the Investment Account Earnings will be determined by your Rationing Fraction. For example, assume that you put 400 tokens in your Rationing Account and your counterpart puts 800 tokens in your Rationing Account. Your Rationing Fraction would then be 400/(400 + 800) = 400/1200 = 1/3. Therefore, you would get 1/3 of the Investment Account Earnings, and your counterpart would get the other 2/3. Let's say that instead of putting 400 tokens in the Rationing Account, you put 800. Your Rationing Fraction would then be 800/(800 + 800) = 800/1600 = 1/2, so you would get 1/2 of the Investment Account Earnings, and your counterpart would get the other 1/2.

Given that your counterpart has placed a certain number of tokens in your Rationing Account, you will increase the size of your share of the Investment Account Earnings by placing more tokens in your Rationing Account.

There are two types of agents in this experiment, Type A agents and Type B agents. You will be a **Type B** agent today. At the beginning of the experiment, you will be randomly paired with a Type A agent who will be your counterpart. This Type A agent will be your counterpart for the entire experiment. The Type A agents and Type B agents are identical in today's experiment. All Type A agents and all Type B agents will be given 2000 tokens at the beginning of each period. Therefore, you will have 2000 tokens each period to put in either your Investment Account or your Rationing Account. Your counterpart will also have 2000 tokens each period. You MUST put each of your tokens in one of your two accounts. Your tokens must be placed in each account in groups of 100. Therefore, you could put 800 tokens in your Investment Account and 1200 tokens in your Rationing Account or 100 tokens in your Investment Account and 1900 in your Rationing Account, etc. However, you could not put 801 tokens in your Investment Account and 1199 tokens in your Rationing Account. Remember, the more tokens you put in your Rationing Account, the bigger your share of the Investment

Account Earnings. However, the more tokens you put in your Rationing Account, the fewer you have to put in your Investment Account, so the smaller the Investment Account Earnings will be.

Each period, you will be asked to decide the number of tokens you wish to place in your Investment Account. You will write this number on your decision slip (see attached packet of decision slips) for that period, and it will be collected by the experimenter. The experimenter will automatically place the remainder of your tokens in your Rationing Account. Your counterpart will make his/her decision at the same time as you make yours. At the end of each period, you will be given a results slip which will notify you of how many tokens your counterpart placed in the Investment Account. You will then be able to determine your Period Profit from the Profit Table given to you with these instructions.

Your Profit Table can help you make your decision each period. For any given choice of tokens placed in the Investment Account by you and your counterpart, the table tells you the profit each of you would earn. Along the left side of the chart the possible choices you could make are written. Along the top of the table, the possible choices your counterpart could make are written. To determine what your profit would be, look at the row of the table corresponding with your choice of tokens. Look at the particular entry in that row that corresponds with the choice you think your counterpart will make. There are two numbers at those coordinates of the Profit Table. The top number, which is written in black, is the number of pesos you would earn with those token choices. The bottom number, which is written in red, is the number of pesos your counterpart would earn with those token choices.

For example, suppose you choose to put 200 tokens into the Investment Account. Look down the left side of the Profit Table where it says "Your Tokens in IA" and find 200. The profit you would make by choosing to put 200 tokens in your Investment Account will be found in this row of the Profit Table. You can see that the profit you would make depends on your counterpart's choice. If your counterpart puts 300 tokens in your Investment Account, you would make 257 pesos and your counterpart would make 243 pesos. If your counterpart puts 600 tokens in the Investment Account, you would make 450 pesos and your counterpart would make 350 pesos.

Let's look in detail at how your profit is determined. If you put 200 tokens in your Investment Account and your counterpart put 600 tokens in, your Total Investment Account

Earnings would be 800 pesos. Given that you put 200 tokens in your Investment Account, you must have put 1800 tokens in the Rationing Account. Your Counterpart must have put 1400 pesos in the Rationing Account. Therefore, your Rationing Fraction must be 1800/(1400+1800) = 9/16. So you would get 9/16 of the 800 pesos or 450 pesos, and your counterpart would get the remaining 7/16 or 350 pesos. Please note that this was just an example, and the numbers used in the example do not indicate what you should do in the experiment.

You have been given a Record Sheet which you will use to keep track of your profit throughout the experiment. The period number is listed in column (1). The number of tokens you place in the Investment Account in that period goes in column (2). You will write the number of tokens your counterpart places in the Investment Account in that period in column (3). Your Period Profit, which you can determine from your Profit Table, will go in column (4). Column (5) is the Total Profit column, which is a running total of your profit from each period.

There will be one practice period in order for you to get used to the mechanics of the experiment and filling out your Record Sheet. You will NOT be paid for the practice period. However, you WILL be paid for every period following that. Remember that you will be paired with the same Type A agent <u>each</u> period. Note that all Type A agents and all Type B agents in this experiment were given a set of instructions and a Profit Table that are identical to yours.

The questions on the following page were designed to test your understanding of your Profit Table. Please answer them carefully. Raise your hand if you have any questions.

Suppose you put 1600 tokens in your Investment Account and your counterpart put 1300 tokens in your Investment Account.

1. How many tokens did each of you put in your Rationing Account?

2. What would the Total Investment Account Earnings be?

3. How much profit would you earn?

4. How much profit would your counterpart earn?

PROFIT TABLE : *** Your Profit *** Counterpart's Profit

Counterpart's Tokens in IA

Your Tokens in IA	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300	1400	1500	1600	1700	1800	1900	2000
0	0 0	51 49	105 95	162 138	222 178	286 214	353 247	424 276	500 300	581 319	667 333	759 341	857 343	963 337	1077 323	1200 300	1333 267	1478 222	1636 164	1810 90	2000 0
100	49 51	100 100	154 146	211 189	271 229	335 265	403 297	475 325	552 348	633 367	721 379	814 386	915 385	1023 377	1140 360	1267 333	1404 296	1555 245	1719 181	1900 100	2100 0
200	95 105	146 154	200 200	257 243	318 282	382 318	450 350	523 377	600 400	683 417	771 429	867 433	969 431	1080 420	1200 400	1330 370	1473 327	1629 271	1800 200	1989 111	2200 0
300	138 162	189 211	243 257	300 300	361 339	425 375	494 406	567 433	645 455	729 471	819 481	915 485	1020 480	1133 467	1257 443	1391 409	1538 362	1700 300	1879 221	2078 122	2300 0
400	178 222	229 271	282 318	339 361	400 400	465 435	533 467	607 493	686 514	770 530	862 538	960 540	1067 533	1183 517	1309 491	1448 452	1600 400	1768 332	1956 244	2165 135	2400 0
500	214 286	265 335	318 382	375 425	435 465	500 500	569 531	643 557	722 578	808 592	900 600	1000 600	1109 591	1227 573	1357 543	1500 500	1658 442	1833 367	2029 271	2250 150	2500 0
600	247 353	297 403	350 450	406 494	467 533	531 569	600 600	674 626	754 646	840 660	933 667	1035 665	1145 655	1267 633	1400 600	1547 553	1711 489	1894 406	2100 300	2333 167	2600 0
700	276 424	325 475	377 523	433 567	493 607	557 643	626 674	700 700	780 720	867 733	961 739	1064 736	1176 724	1300 700	1437 663	1589 611	1759 541	1950 450	2167 333	2414 186	2700 0
800	300 500	348 552	400 600	455 645	514 686	578 722	646 754	720 780	800 800	887 813	982 818	1086 814	1200 800	1326 774	1467 733	1624 676	1800 600	2000 500	2229 371	2492 208	2800 0
900	319 581	367 633	417 683	471 729	530 770	592 808	660 840	733 867	813 887	900 900	995 905	1100 900	1216 884	1344 856	1488 812	1650 750	1833 667	2043 557	2285 415	2567 233	2900 0
1000	333 667	379 721	429 771	481 819	538 862	600 900	667 933	739 961	818 982	905 995	1000 1000	1105 995	1222 978	1353 947	1500 900	1667 833	1857 743	2077 623	2333 467	2636 264	3000 0
1100	341 759	386 814	433 867	485 915	540 960	600 1000	665 1035	736 1064	814 1086	900 1100	995 1105	1100 1100	1218 1082	1350 1050	1500 1000	1671 929	1869 831	2100 700	2373 527	2700 300	3100 0
1200	343 857	385 915	431 969	480 1020	533 1067	591 1109	655 1145	724 1176	800 1200	884 1216	978 1222	1082 1218	1200 1200	1333 1167	1486 1114	1662 1038	1867 933	2109 791	2400 600	2756 344	3200 0
1300	337 963	377 1023	420 1080	467 1133	517 1183	573 1227	633 1267	700 1300	774 1326	856 1344	947 1353	1050 1350	1167 1333	1300 1300	1454 1246	1633 1167	1845 1055	2100 900	2411 689	2800 400	3300 0
1400	323 1077	360 1140	400 1200	443 1257	491 1309	543 1357	600 1400	663 1437	733 1467	812 1488	900 1500	1000 1500	1114 1486	1246 1454	1400 1400	1582 1318	1800 1200	2067 1033	2400 800	2829 471	3400 0
1500	300 1200	333 1267	370 1330	409 1391	452 1448	500 1500	553 1547	611 1589	676 1624	750 1650	833 1667	929 1671	1038 1662	1167 1633	1318 1582	1500 1500	1722 1378	2000 1200	2357 943	2833 567	3500 0
1600	267 1333	296 1404	327 1473	362 1538	400 1600	442 1658	489 1711	541 1759	600 1800	667 1833	743 1857	831 1869	933 1867	1055 1845	1200 1800	1378 1722	1600 1600	1886 1414	2267 1133	2800 700	3600 0
1700	222 1478	245 1555	271 1629	300 1700	332 1768	367 1833	406 1894	450 1950	500 2000	557 2043	623 2077	700 2100	791 2109	900 2100	1033 2067	1200 2000	1414 1886	1700 1700	2100 1400	2700 900	3700 0
1800	164 1636	181 1719	200 1800	221 1879	244 1956	271 2029	300 2100	333 2167	371 2229	415 2285	467 2333	527 2373	600 2400	689 2411	800 2400	943 2357	1133 2267	1400 2100	1800 1800	2467 1233	3800 0
1900	90 1810	100 1900	111 1989	122 2078	135 2165	150 2250	167 2333	186 2414	208 2492	233 2567	264 2636	300 2700	344 2756	400 2800	471 2829	567 2833	700 2800	900 2700	1233 2467	1900 1900	3900 0
2000	0 2000	0 2100	0 2200	0 2300	0 2400	0 2500	0 2600	0 2700	0 2800	0 2900	0 3000	0 3100	0 3200	0 3300	0 3400	0 3500	0 3600	0 3700	0 3800	0 3900	0 0